

The Relationship between Horizontal and Vertical Velocity Wavenumber Spectra in Global Storm-Resolving Simulations

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(Manuscript received 29 April 2022, in final form 8 January 2023)

ABSTRACT: Several studies have reported vertical kinetic energy spectra almost white in horizontal wavenumber space with evidence of two maxima at synoptic scales and mesoscales, leaving the explanation of these maxima open. Processes known to influence the shape of the horizontal kinetic energy spectra include the superposition of quasi-linear inertia–gravity waves (IGWs), quasigeostrophic turbulence, and moist convection. In contrast, vertical kinetic energy has been discussed much less, as measuring vertical velocity remains challenging. This study compares the horizontal and vertical kinetic energy spectra and their relationships in global storm-resolving simulations from the DYAMOND experiment. The consistency of these relationships with linear IGW theory is tested by diagnosing horizontal wind fluctuations associated with IGW modes. Furthermore, it is shown that hydrostatic IGW polarization relations provide a quantitative prediction of the spectral slopes of vertical kinetic energy at large scales and mesoscales, where the intrinsic frequencies are inferred from the linearized vorticity equation. Our results suggest that IGW modes dominate the vertical kinetic energy spectra at most horizontal scales, whereas an incompressible, isotropic scaling of the continuity equation captures the relationship between horizontal and vertical kinetic energy spectra at small scales.

KEYWORDS: Atmosphere; Inertia-gravity waves; Vertical motion; Cloud resolving models; Dynamics; Kinetic energy

1. Introduction

Atmospheric motions span a wide range of horizontal scales, from large-scale geostrophically balanced flows and long atmospheric waves to three-dimensional turbulent dissipation scales. The atmospheric kinetic energy is not distributed randomly across horizontal scales. Instead, the energy spectrum of horizontal motions as a function of horizontal wavenumber κ obeys canonical power laws (Gage 1979; VanZandt 1982; Nastrom and Gage 1985). This spectrum consists of a shallow-sloped region at global scales (~40000-10000 km), a steeper-sloped κ^{-3} regime at intermediate wavenumbers, and the mesoscale regime with a transition to a shallower $\kappa^{-5/3}$ slope at scales of ~300–600 km (Nastrom et al. 1984; Nastrom and Gage 1985; Lindborg 1999). The prevailing explanation for what shapes the κ^{-3} portion rests on applications of quasigeostrophic (QG) turbulence theory, which shows that this region of the spectrum is consistent with a downscale enstrophy cascade (Charney 1971).

The cause of the $\kappa^{-5/3}$ behavior of the mesoscale spectrum (scales ≤ 600 km) is still subject to debate. Several competing theories have been proposed during the last decades to

explain the dynamic origin of the mesoscale spectrum. Dewan (1979) first suggested that a superposition of weakly nonlinear inertia-gravity waves dominates the mesoscale energy. Later, Dewan (1997, hereafter D97) presented the hypothesis that wave saturation and cascade can explain the spectral slopes of horizontal and vertical velocity fluctuations, as well as those of temperature and density. Conversely, other studies interpreted the mesoscale range to be predominantly turbulent. Some of the explanations for the mesoscale spectrum consist of different types of QG turbulence theories (Tung and Orlando 2003; Tulloch and Smith 2006), and strongly stratified turbulence (Lindborg 2006). The shape of atmospheric energy spectra is not just of theoretical interest but has practical implications for atmospheric predictability. Lorenz (1969) proposed that a turbulent flow with $\kappa^{-5/3}$ has a finite predictability limit, which means that more accurate knowledge of the initial state cannot improve forecasts significantly. However, if the mechanism underlying the energy spectrum is linear gravity waves, predictability may not be limited to power-law characteristics alone (Malardel and Wedi 2016) since linear gravity waves do not propagate errors in the same way as turbulent flows.

Understanding the dynamic coupling between horizontal and vertical atmospheric motions is essential to unraveling the mechanisms that shape mesoscale kinetic energy spectra. However, a critical piece of the puzzle is missing: What

DOI: 10.1175/JAS-D-22-0105.1

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mechanisms control vertical kinetic energy at mesoscales? There is strong observational and numerical evidence of vertical kinetic energy spectra (E_w) relatively flat at mesoscales with a local maximum at small scales, leaving the explanation of this maximum open. Global storm-resolving simulations have shown that E_w peaks at synoptic scales (Terasaki et al. 2009; Skamarock et al. 2014, hereafter S14), which can be associated with long atmospheric waves; however, validating this feature with observations is unattainable. High-resolution simulations with state-of-the-art general circulation models (GCMs) provide an opportunity to test proposed theories, as they compare well with observations (Hamilton et al. 2008; Terasaki et al. 2009; S14; Selz et al. 2019). The newest generation of these models is now running at kilometer scales, explicitly resolves deep convection, and can therefore be expected to represent mesoscale dynamics realistically. The availability of three-dimensional data has proven valuable for the interpretation of one-dimensional aircraft observations across a wide range of scales (Bierdel et al. 2016). However, observational validation of simulated vertical velocities remains a challenge, as observations of vertical velocities across different horizontal scales, particularly on mesoscales, are scarce (Bacmeister et al. 1996; Bony and Stevens 2019; Stephan and Mariaccia 2021). For this reason, vertical velocity spectra have been studied much less compared to horizontal energy spectra (Bacmeister et al. 1996; Callies et al. 2016; Schumann 2019, hereafter **S19**).

This study examines whether different kilometer-scale global GCMs agree on the relationship between vertical and horizontal kinetic energy spectra and how existing theoretical models explain the relationship. For this purpose, we employ storm-resolving global simulations from the Dynamics of the Atmospheric general circulation Modeled On Nonhydrostatic Domains (DYAMOND) experiment (Stevens et al. 2019), which explicitly model deep convection. There are several aspects related to model design and configuration that are known to affect the kinetic energy spectrum (S14). These include the convective parameterizations, microphysics, vertical resolution, numerical filters, the representation of subgrid processes that account for unresolved turbulent motions, and subgrid-scale orography. Horizontal motion spectra and their dependence on model formulation are discussed in detail in Stephan et al. (2022), including the simulations analyzed here. The representation of explicit versus parameterized convection and their effects on convectively generated inertia-gravity waves (IGWs) and the vertical velocity spectrum for several configurations of the Integrated Forecasting System (IFS) model are discussed in Polichtchouk et al. (2022). Foremost, we focus on the relationship between the models' horizontal and vertical kinetic energy spectra rather than comparing the components in isolation. This relationship between spectra can shed light on the underlying physical processes, as revealed in the analysis. In particular, we are interested in whether or not the properties of resolved IGWs matter for how the vertical velocity spectrum relates to the horizontal motion spectrum.

This paper is structured as follows: Section 2 introduces the numerical models and describes the analysis methods. Section 3 compares the horizontal and vertical kinetic energy spectra

between models and examines their vertical dependence. Furthermore, we investigate the contribution of balanced and unbalanced circulations to the total horizontal kinetic energy using two approaches. One is based on a Helmholtz decomposition, which yields the horizontal wind's purely divergent and rotational components. The other is based on a normal mode function decomposition, which yields the contribution of IGWs to the horizontal kinetic energy spectra. To estimate the contribution of IGWs to the vertical velocity spectra, we numerically solve the mass continuity equation in physical space. Furthermore, we discuss if the shape of the vertical kinetic energy spectrum can be estimated from knowledge of the horizontal kinetic energy spectrum at the same level but without invoking information about other levels. Finally, section 4 contains a summary of the results and conclusions.

2. Data and methods

a. DYAMOND models

To explore the relationships between the horizontal (E_h) and vertical (E_w) kinetic energy spectra, we analyze numerical outputs from high-resolution global simulations of four different model members of the DYAMOND experiment. The DYAMOND experiment consists of two phases of simulations, referred to as "summer" and "winter," respectively, each spanning 40 days and 40 nights. The horizontal grid spacing of the models is <5 km. We analyze winter simulations initialized at 0000 UTC 20 January 2020. Most DYAMOND models solve the Navier-Stokes system of compressible equations, except for the IFS, which uses primitive hydrostatic equations. The numerical methods employed by the different models to solve their governing equations depend on the choice of the grid and the time integration methods and therefore vary considerably. The advantage of using DYAMONDtype models is that these models are global while resolving deep convection explicitly. We use numerical outputs from the Icosahedral Nonhydrostatic (ICON) model, Goddard Earth Observing System (GEOS), Nonhydrostatic Icosahedral Atmospheric Model (NICAM), and IFS with horizontal resolutions of 2.5, 3.3, 3.5, and 4.0 km, respectively. Detailed information about the model configurations can be found in Stevens et al. (2019). Our analysis period spans 12 days, starting 1 February 2020. We use 6-hourly outputs of 10 days after initialization to exclude the model spinup period. This well exceeds previous estimates of spinup time based on numerical models and theory (Skamarock 2004; Hamilton et al. 2008) and ensures that the energy spectra are in equilibrium. The preparation of the numerical outputs of the models for analysis consists of averaging the models' three-dimensional wind fields within a target grid cell. The target grid is a regular Gaussian grid with 8192×4096 grid cells in the zonal and meridional direction, respectively, corresponding to a horizontal grid spacing of approximately 4.88 km at the equator. In the analysis of section 3a, no vertical interpolation of model outputs is performed prior to the computation of the kinetic energy spectra. Instead, we select the model levels closest to the level of interest, a reasonable approximation in the stratosphere, where model levels correspond to constant height surfaces in ICON and NICAM and constant pressure surfaces in IFS and GEOS. As noted by S14, kinetic energy spectra computed on surfaces of constant height and constant pressure have the same qualitative character.

b. Spherical harmonics and Helmholtz decomposition

All spectral transformations performed here rely on spherical harmonics analysis. To calculate the power spectrum, we use Parseval's theorem in spherical geometry, which for each wind component $\mathbf{v} = (u, v, w)$ relates the sum of its squares in physical space to the sum of the squared Fourier coefficients. The 2D spectrum of horizontal kinetic energy per unit mass is

$$E_{l,n} = \frac{1}{2} (|\hat{u}_{l,n}|^2 + |\hat{v}_{l,n}|^2), \qquad (1)$$

where *l* is the spherical wavenumber, and *n* is the zonal wavenumber, $\hat{u}_{l,n}$ and $\hat{v}_{l,n}$ are the spherical harmonics coefficients of the zonal and meridional wind components. These coefficients are obtained by expanding the horizontal velocity in a triangularly truncated series of spherical harmonics basis functions Y_n^l (Baer 1972). The basis functions are $Y_n^l = P_{l,n} e^{in\theta}$, where $P_{l,n}$ are the Legendre polynomials and θ is longitude.

To shed light on the dynamics that underlie the horizontal kinetic energy spectra, we calculate the contributions of divergent and rotational energies by performing a Helmholtz decomposition (Bierdel et al. 2016; Li and Lindborg 2018). An alternative expression for the horizontal kinetic energy is as follows:

$$E_{l,n} = \frac{1}{2} \frac{a^2}{l(l+1)} (|\hat{\zeta}_{l,n}|^2 + |\hat{\delta}_{l,n}|^2), \qquad (2)$$

where *a* is Earth's radius (Lambert 1984), and $\hat{\zeta}_{l,n}$ and $\hat{\delta}_{l,n}$ are the spherical harmonic coefficients of vorticity and horizontal divergence. Equations (1) and (2) yield almost identical results for l > 10. From (2) one can define the horizontal wavenumber as $\kappa = \sqrt{l(l+1)/a} \approx l/a$.

The horizontal spectrum of kinetic energy per unit mass $E_h(\kappa)$ is defined as the sum of (2) over the zonal wavenumber. Similarly, the vertical kinetic energy per unit mass $E_w(\kappa)$ is expressed in terms of the spherical harmonics coefficients of vertical velocity as $E_w(\kappa) = |\hat{w}(\kappa)|^2/2$. From (2), it follows that $E_h = E_r + E_d$, where $E_d = |\hat{\delta}(\kappa)|^2 / (2\kappa^2)$ is the horizontal spectrum of divergent kinetic energy, and $E_r = |\hat{\zeta}(\kappa)|^2 / (2\kappa^2)$ is the horizontal spectrum of rotational kinetic energy. The spectral coefficients of the wind field, vorticity, and horizontal divergence are calculated using Climate Data Operator (CDO) (Schulzweida 2022). Given the number of latitudinal samples, the transform is exact if the function is bandlimited to spherical wavenumber $l_{\text{max}} = N - 1 = 4095$. Since we are not interested in dissipative scales related to the model filters, we analyze spectra with a triangular truncation at the spherical wavenumber $l = 2048 (\lambda_h \sim 10 \text{ km}).$

c. Normal mode function decomposition

We perform a normal mode function (NMF) decomposition using the MODES software, described in detail in

Žagar et al. (2015) to distinguish between balanced and unbalanced horizontal motion. MODES performs a multivariate linear projection of the horizontal winds on balanced and unbalanced eigensolutions of the primitive equations, linearized around a resting background state (Kasahara and Puri 1981). The orthogonal basis functions of the projection satisfy the dispersion relationships for Rossby waves (including the mixed Rossby-gravity wave mode) and inertia-gravity waves, including the Kelvin mode (Kasahara 2020). In the following, we will refer to the "balanced" component of the flow as that which projects onto the low-frequency linear Rossby modes, as opposed to the standard definition of a flow in which the three-dimensional velocity field is functionally related to the mass field (McIntyre 2015). The "unbalanced" component of the flow is defined as that which projects onto the linear IGWs. Given the linearity of the decomposition, the IGW modes may contain some ageostrophic imbalance, not only freely propagating IGWs.

First, we interpolate the required input fields (three-dimensional horizontal winds, temperature, specific humidity, topography, and surface pressure) to a regular N256 Gaussian grid. The horizontal resolution at the equator is \sim 39 km. As the set of NMFs implemented in MODES is defined on sigma levels (Kasahara and Puri 1981), we next interpolate the threedimensional fields vertically to 68 hybrid sigma-pressure levels extending from the surface to ~ 10 hPa (about 32 km). The NMF decomposition is carried out at individual time steps and provides the spectrum of the horizontal kinetic plus available potential energy as a function of the zonal wavenumber and the meridional and vertical wave indices, which define the Hough harmonics. Since MODES is computationally expensive, we use a zonal wavenumber truncation of l = 320, which resolves horizontal wavelengths ($\lambda_h \sim 125$ km). By projecting back to physical space, we isolate the wind field associated with the balanced and unbalanced circulation, respectively, as demonstrated, for instance, in Žagar et al. (2017).

Figure 1 illustrates the modal decomposition performed on ICON outputs in the lower stratosphere (24 km), corresponding to 0600 UTC 3 February 2020. The inverse projection of horizontal wind associated with the Rossby and IGW modes is shown in Figs. 1a and 1b. Large-scale features dominate the balanced circulation at this level, i.e., the stratospheric polar vortex, while the IGW circulation contains contributions from large scales at high latitudes and smaller scales in the tropics. The large-scale IGW energy seems to be associated with spontaneously generated waves around the polar vortex, at least in the stratosphere. In addition, the gradient wind balance may contribute to the IGW energy at planetary scales in the winter stratosphere (Žagar et al. 2015). In energetic terms, the large-scale portion of the horizontal kinetic energy spectrum (E_h) is mainly explained by the kinetic energy spectrum of the Rossby modes $(E_{RO_{\mu}})$, which is purely rotational $(E_{\rm RO} \sim E_r)$, at scales $L \gtrsim 600$ km (see Fig. 1c). At mesoscales $(L \leq 600 \text{ km})$, the horizontal kinetic energy spectrum E_{IG} , of the IGW component and the purely vortical energy E_r have comparable magnitudes. Section 3b examines the contributions of balanced and unbalanced components to the energy spectra in more detail. The following section describes



FIG. 1. Modal decomposition of the atmospheric circulation performed on ICON at 24 km corresponding to 0600 UTC 3 Feb 2020. (a),(b),(d),(e) Maps in an orthographic projection centered at the North Pole and 57°W. The maps in (a) and (b) show the horizontal winds of Rossby and IGW modes from the inverse NMF decomposition. The vertical velocities of (d) Rossby and (e) IGW modes are calculated by solving mass continuity (3), discussed in detail in section 2d. (c) The horizontal kinetic energy spectra associated with Rossby modes E_{RO_h} (dashed red) and IGWs E_{IG_h} (dashed black); the rotational E_r and divergent E_d kinetic energy spectra from the Helmholtz decomposition are shown in solid red and green, respectively. (f) The vertical kinetic energy spectra E_w (solid black) and estimated IGW vertical kinetic energy spectra E_{IG_w} (dashed black). The spectra shown in (c) and (f) are averaged over 24 h on 3 Feb 2020, and the shaded area around each line indicates the standard deviation. Reference slopes for κ^{-3} , $\kappa^{-5/3}$, and $\kappa^{2/3}$ are shown in dashed lines. The gray shaded area indicates horizontal wavelengths < 20 km.

the method for estimating the vertical velocity from horizontal IGW and Rossby modes shown in Fig. 1.

d. Estimating vertical velocity from the horizontal wind

To diagnose the vertical velocity field from horizontal wind, we start with the mass continuity equation in hybrid-sigma vertical coordinates (Simmons and Burridge 1981). The diagnostic equation for vertical pressure velocity ω is expressed as follows:

$$\omega(\eta) = -\int_0^{\eta} \nabla \cdot \left(\mathbf{u} \frac{\partial p}{\partial \eta} \right) d\eta + \mathbf{u} \cdot \nabla p, \qquad (3)$$

where $\mathbf{u} = (u, v)$ is the horizontal wind vector, and p is pressure. The vertical coordinate $\eta(p, p_s)$ is a monotonic function of pressure, and depends on the surface pressure p_s such that $\eta(p_s, p_s) = 1$ and $\eta(0, p_s) = 0$. A detailed description of the

vertical coordinate system is given in Untch and Hortal (2003). We solve (3) numerically using the IFS vertical discretization (ECMWF 2021) since the vertical grid used for the modal decomposition is a subsample of the IFS vertical grid L137. Finally, assuming hydrostatic balance, the vertical velocity w is estimated using $w = -\omega/(\rho g)$, where ρ is the air density and g is the acceleration of gravity.

3. Results

We begin with comparisons of the horizontal kinetic energy spectra (E_h) and the vertical kinetic energy spectra (E_w) between the different simulations before investigating how they relate to each other in section 3c. The spectra differ substantially between the troposphere and the stratosphere, so we mainly show 6 km as representative of the free troposphere and 24 km as representative of the stratosphere.



FIG. 2. Horizontal kinetic energy spectra as a function of spherical wavenumber for ICON, NICAM, IFS, and GEOS (left) in the free troposphere at 6 km and (right) in the stratosphere at 24 km. Reference slopes of κ^{-3} and $\kappa^{-5/3}$ are shown in dashed lines. The gray shaded area indicates horizontal wavelengths < 20 km.

a. Kinetic energy spectra

Figure 2 shows E_h as a function of the spherical wavenumber for ICON, IFS, GEOS, and NICAM. The models reproduce the observed shape of the Nastrom-Gage spectrum to first order. The models agree well in spectral power across all scales at 6 km and scales of 1000-2000 km at 24 km. Overall, the greatest differences exist in the mesoscale region in the stratosphere. ICON shows similar mesoscale energy per unit mass in the troposphere and the stratosphere; these results are in agreement with the results of S14, based on global MPAS simulations with a horizontal resolution of 3 km. The GEOS and IFS models have slightly less energy in the stratosphere, whereas NICAM has greater mesoscale energy than in the troposphere. The scale at which dissipative effects become visible varies considerably between the models (\sim 20–50 km wavelength). As noted by Skamarock (2004), the effective resolution can be affected by numerical damping and various filters. Spectral power decays already at scales <100 km in the IFS. The related absence of the observed spectral slope -5/3 at mesoscales in the IFS model has been pointed out in previous studies and linked to the effects of parameterized energy transfer of subgrid-scale processes (Shutts 2005; Malardel and Wedi 2016).

Table 1 lists the spectral slopes obtained by performing a piecewise linear regression of each spectrum in logarithmic space for the intervals $20 \text{ km} \le \lambda_h < L$ and $L \le \lambda_h \le 2000 \text{ km}$, where λ_h is the horizontal wavelength. The synoptic-to-mesoscale transition scale (*L*) is the intermediate point in 20–2000 km that minimizes the sum of the squared errors of both intervals. Tropospheric spectral slopes vary slightly from model to model and are consistently shallower than -3 in the wavelength range 200–2000 km, ranging from -2.49 to -2.58. Stratospheric slopes are steeper than -3 for ICON (-4.12), slightly steeper for GEOS (-3.52), and NICAM (-3.59), while the IFS slopes

remain close to -3. The mesoscale slopes are consistently steeper than -5/3 in the troposphere ranging from -2.16 to -1.9. In the stratosphere, the slopes of IFS and GEOS remain close to -1.7 and shallower in ICON (-1.24) and NICAM (-1.31).

Figure 3 illustrates how E_h varies with height. The transition scale varies between 112 and 194 km in the troposphere and between 663 and 948 km in the stratosphere, agreeing with S14 results. The increase with altitude of the transition scale is not gradual but occurs abruptly at the tropopause somewhere between 12 and 16 km. The vertical variation of E_h in the IFS compares favorably to the results of Burgess et al. (2013) based on T799 ECMWF operational analysis.

TABLE 1. Regression of spectral slopes for ICON, IFS, GEOS, and NICAM at 6 and 24 km. The vertical grid spacing Δz is given for each model. The transition scale from synoptic to mesoscale is denoted as *L*. Estimated slopes and standard errors (std. err.) are shown for wavelength intervals of 20 km $\leq \lambda_h < L$ and $L \leq \lambda_h \leq 2000$ km.

| | | | $2000 \text{ km} \ge \lambda_h \ge L$ | | $L > \lambda_h \ge 20 \text{ km}$ | |
|------------------------|----------------|--------|---------------------------------------|-----------|-----------------------------------|-----------|
| | Δz (m) | L (km) | Slope | Std. err. | Slope | Std. err. |
| $z \sim 6 \text{ km}$ | | | | | | |
| ICON | 400.0 | 122 | -2.58 | 0.012 | -1.92 | 0.003 |
| IFS | 291.0 | 194 | -2.53 | 0.004 | -1.91 | 0.008 |
| GEOS | 251.5 | 132 | -2.57 | 0.005 | -2.16 | 0.004 |
| NICAM | 400.0 | 112 | -2.49 | 0.003 | -1.94 | 0.003 |
| $z \sim 24 \text{ km}$ | l | | | | | |
| ICON | 1008.0 | 663 | -4.12 | 0.011 | -1.24 | 0.036 |
| IFS | 521.5 | 948 | -3.09 | 0.021 | -1.74 | 0.097 |
| GEOS | 360.5 | 740 | -3.52 | 0.012 | -1.79 | 0.042 |
| NICAM | 986.0 | 794 | -3.59 | 0.012 | -1.31 | 0.046 |



FIG. 3. Horizontal kinetic energy spectra as a function of spherical wavenumber for ICON, IFS, NICAM, and GEOS at heights of 6, 8, 12, 16, 20, and 24 km. Reference slopes of κ^{-3} and $\kappa^{-5/3}$ are shown in gray dashed lines. The gray shaded area indicates horizontal wavelengths < 20 km.

The vertical velocity spectra show evidence of two different power-law behaviors and have approximately five orders of magnitude less energy integrated across resolved scales than their horizontal counterpart (note that Fig. 4 contains fewer orders of magnitude on the ordinate than Fig. 2). The results shown in Fig. 4 are in good agreement with previous findings regarding the spectral slopes of E_w at mesoscales (≤ 100 km) from observations (Bacmeister et al. 1996; Gao and Meriwether 1998) and from high-resolution numerical simulations (Terasaki et al. 2009; S14; Craig and Selz 2018; Müller et al. 2018). All models predict a similar spectral power for the maximum found at large scales in the troposphere. As in the case of E_h , most of the differences between the models occur in the mesoscale range.

Figure 5 shows E_w at various altitudes. The tropospheric and stratospheric spectra differ on several points. First, we observe a transition from slopes near -1 at large scales (10 < l < 40) toward slopes of about 1/3 at the mesoscale in the troposphere. In contrast, the large-scale slopes are steeper than -1 for all models in the stratosphere. Regarding the mesoscale region in the stratosphere, ICON presents slopes steeper than 1/3 of around 2/3, while GEOS's slopes flatten after spherical wavenumber $l \sim 200$ and NICAM closely follows a 1/3 scaling at all vertical levels. Finally, the E_w slopes in IFS show signs of energy dissipation similar to those of E_h , namely, flattening of the slopes and a rapid energy decay with wavenumber in the stratosphere at scales l > 200.

The evident diversity between models regarding the E_h and E_w scaling raises the question of whether the models' disagreement comes from differences in the underlying dynamics or model formulation. While beyond the scope of this paper, we recognize that aspects of a model's formulation that can influence the shape of the mesoscale kinetic energy spectrum include vertical resolution and vertical turbulent diffusion (Waite 2016; Skamarock et al. 2019), which vary substantially in our simulations. Additionally, convective parameterizations also affect the kinetic energy spectrum at small scales since convection is a crucial IGW source (Polichtchouk et al. 2022; Stephan et al. 2022).



FIG. 4. Vertical kinetic energy spectra as a function of spherical wavenumber for ICON, NICAM, IFS, and GEOS (left) in the free troposphere at 6 km and (right) in the stratosphere at 24 km. Reference slope of κ^{-1} , $\kappa^{1/3}$, and $\kappa^{2/3}$ are shown in dashed lines. The gray shaded area indicates horizontal wavelengths < 20 km.

The vertical grid spacings Δz for each model at levels 6 and 24 km are listed in Table 1. In the lower stratosphere, Δz is coarser for ICON (~1 km) and NICAM (~980 m) compared to IFS (~520 m) and GEOS (~360 m). Insufficient vertical resolution might explain some differences between models, even at well-resolved horizontal scales. For example, ICON and NICAM exhibit shallower mesoscale spectral slopes compared to IFS and GEOS in the stratosphere (see Fig. 1), which might indicate that the spectra are not fully converged at this level, consistent with the results of Skamarock et al. (2019), where convergence is approached for $\Delta z \leq 200$ m in MPAS simulations. Waite (2016) indicated that the sensitivity of model spectra to vertical resolution depends on the vertical mixing scheme; with no vertical mixing or weak, stabilitydependent mixing, the mesoscale spectra are artificially amplified by low resolution. Our simulations may show signs of amplification at the coarser vertical resolutions since ICON and NICAM, which have similar prognostic turbulent kinetic energy (TKE) schemes, show higher mesoscale energy magnitudes in the stratosphere than IFS and GEOS with a diagnostic eddy diffusivity scheme.

The shape of the mesoscale energy spectrum is often interpreted in terms of the different dynamics of balanced circulations and IGWs. Therefore, we next explore balanced and unbalanced dynamics contributions to E_h and E_w .

b. Contributions of IGWs to E_h and E_w

This section examines the contributions to E_h from rotational (E_r) and divergent (E_d) energy spectra obtained by Helmholtz decomposition, as well as the spectra of IGW wind fluctuations (E_{IG_h}) . In addition, E_{IG_h} is further decomposed into its divergent (E_{IG_d}) and rotational (E_{IG_f}) components. Finally, we present the energy spectra of vertical velocity (E_{IG_w}) estimated from IGW horizontal winds.

In the following, we analyze the modal decomposition of DYAMOND simulations using MODES presented in Stephan et al. (2022). Since IGW fields are unavailable for NICAM, we only show energy spectra of IGW modes corresponding to the ICON, GEOS, and IFS models. Figure 6 shows all horizontal energy components for ICON, IFS, and GEOS at 6 and 24 km. Model results are consistent with the established understanding that E_r dominates the planetary and synoptic ranges of E_h . E_d dominates the mesoscale energy in the stratosphere, while E_d and E_r approach the same order of magnitude toward smaller scales in the troposphere, in agreement with Skamarock and Klemp (2008). The models do not show large deviations from a $\kappa^{-5/3}$ scaling of E_d for spherical wavenumbers l > 10 with slopes -1.6 ± 0.02 , except for ICON in the stratosphere (1.28 \pm 0.01). E_r follows κ^{-3} over a wide range but flattens toward the smaller scales. The flattening of E_r slopes is present in all models in the troposphere at scales ≤ 100 km, confirming the results of Waite and Snyder (2013) based on idealized baroclinic wave simulations with 12.5 km resolution. Meanwhile, in the stratosphere, the flattening of E_r occurs at scales of about 400–500 km in ICON, agreeing with Hamilton et al. (2008). However, it is not evident in IFS and GEOS.

Oftentimes E_{IG_h} is approximated by E_d . However, IGWs can have nonzero rotational energy. As shown in Fig. 6, $E_{IG_d} \leq E_{IG_h}$, where the equality holds at mesoscales. The IFS's stratospheric E_{IG_h} at large scales shows different behavior compared to ICON and GEOS in that a greater fraction of E_r projects into IGW modes (see Fig. 6). Žagar et al. (2017) showed for the ERA-Interim and ECMWF operational analyses that the excess rotational energy in the IGW modes stems from the gradient wind balance within the stratospheric polar vortex (Žagar et al. 2015). Figure 6 suggests that ICON's shallow mesoscale slope found in the lower stratosphere, where $E_h \propto \kappa^{-1.24}$, is not explained by linear



FIG. 5. Vertical kinetic energy spectra as a function of spherical wavenumber for ICON, IFS, NICAM, and GEOS at heights of 6, 8, 12, 16, 20, and 24 km. Reference slopes of κ^{-1} and $\kappa^{1/3}$ are shown in gray dashed lines. The gray shaded area indicates horizontal wavelengths < 20 km.

IGW modes since E_{IG_h} has a significantly smaller magnitude than E_h , and follows slopes close to $\kappa^{-5/3}$. The modal decomposition filters some divergent energy at small scales due to the insufficient vertical truncation, i.e., the number of vertical modes is smaller than the number of model levels (Žagar et al. 2017). Note that the stratospheric mesoscale magnitudes and slopes of E_{IG_h} and E_r are of the same order in ICON, whereas E_{IG_h} dominates the mesoscale energy in the other models.

Fⁱgure 7 shows E_w and E_{IG_w} in the troposphere (6 km) and the stratosphere (24 km). E_w is almost fully explained by the horizontal IGW circulation, as expected, because the spectral shapes of E_{IG_h} and E_d are similar for most scales (Fig. 6). Deviations exist where the spectra of E_{IG_h} and E_d differ, as is the case, for example, at planetary scales in ICON and GEOS and at the mesoscales in ICON. At planetary scales, $E_{IG_h} > E_d$ in all models due to contributions from E_{IG_r} to E_{IG_h} , which is required to explain the large-scale peak of E_w at spherical wavenumbers 4–10, as will be discussed in section 3c. Our results agree with previous high-resolution numerical simulations that explicitly diagnose IGWs (Kitamura and Matsuda 2010; Terasaki et al. 2011; Žagar et al. 2015) or use divergent energy to approximate IGWs in the mesoscale (Callies et al. 2014). These results suggest that IGWs dominate the mesoscale range on average in the stratosphere, while the mesoscale IGW and balanced components have comparable magnitudes in the troposphere. However, in the stratosphere, ICON shows fractions of E_d and E_r to E_h of around 2/3 and 1/3 at mesoscales, in contrast to IFS and GEOS where E_d dominates.

Differences in the divergent to rotational and horizontal kinetic energy fractions may hint at differences in the underlying dynamics between the models. However, we do not exclude the possibility that the underlying dynamics are not represented correctly due to inadequate vertical resolution or insufficient/excessive vertical mixing, which may lead to spurious gravity waves or noise at small horizontal scales



FIG. 6. Kinetic energy spectra of the total horizontal wind field (solid black); rotational (red) and divergent (green) kinetic energies for ICON, IFS, and GEOS at (top) 6 and (bottom) 24 km. The total IGW energy spectra (E_{IG_h}) are shown in dashed black lines, along with divergent (dashed green) and rotational (dashed red) IGW kinetic energy components. Vertical dotted lines denote the crossing scale (L_c) where E_r and E_d intersect. The gray shaded area indicates horizontal wavelengths < 20 km.

(Waite 2016). In addition, the overlap between NMF and Helmholtz decomposition and missing information on nonlinear energy transfer makes it difficult to interpret the results in terms of physical processes directly. The following section turns to concepts that allow us to infer the relationship between E_h and E_w without requiring knowledge of the three-dimensional circulations.

c. Simplified models linking E_w and E_h

This section begins with exploring the relationship between E_w and E_{IG_h} at large scales based on the hydrostatic IGW polarization relation. Next, we discuss the prospect of extending the IGW interpretation of E_w to the mesoscale. Finally, we examine the kinematic link between E_w and E_d through mass continuity at mesoscales, providing a 1D description of the E_w spectrum from divergent horizontal winds at the same vertical level.

1) LARGE SCALES

As shown in Fig. 7, E_{IG_w} matches E_w reasonably well at most horizontal scales. D97 introduced the saturated-cascade theory (SCT), which provides predictions for the observed

 $\kappa^{-5/3}$ form of the mesoscale kinetic energy spectra. Additionally, the saturated-cascade theory predicts a scaling for E_{IG_w} directly from the wave polarization relation. For linear inertia–gravity waves, the hydrostatic polarization relation yields

$$E_{\mathrm{IG}_{w}}(\kappa,\,\hat{\omega}) = \frac{\hat{\omega}^{2}}{N^{2} - \hat{\omega}^{2}} \left(\frac{\hat{\omega}^{2} - f^{2}}{\hat{\omega}^{2} + f^{2}} \right) E_{\mathrm{IG}_{h}}(\kappa,\,\hat{\omega}),\tag{4}$$

where $\hat{\omega}$ is the intrinsic frequency, and f and N are the inertial and the Brunt–Väisälä frequencies, respectively. D97 further assumes $f^2 \ll \hat{\omega}^2 \ll N^2$ and $\lambda_z < H$, where λ_z is the vertical wavelength, and $H \sim 8$ km is the density scale height. The polarization relation under the medium-frequency approximation then takes the simple form

$$E_{\mathrm{IG}_{w}}(\kappa,\,\hat{\omega}) = \frac{\hat{\omega}^{2}}{N^{2}} E_{\mathrm{IG}_{h}}(\kappa,\,\hat{\omega}). \tag{5}$$

The saturated-cascade condition given by (55) in D97 relates the intrinsic frequency with the horizontal wavenumber as $\hat{\omega}^2 = c \varepsilon^{2/3} \kappa^{4/3}$, where *c* is a constant and ε is the wave dissipation rate, which implies that only waves with specific frequencies



FIG. 7. Vertical kinetic energy spectra E_w (solid black), and estimated IGW vertical kinetic energy spectra E_{IG_w} (red) for ICON, IFS, and GEOS at (top) 6 and (bottom) 24 km. Reference crossing scales (L_c) are shown as vertical dotted lines. The gray shaded area indicates horizontal wavelengths < 20 km.

contribute to the spectrum. The spectral relationships in SC theory are strictly one-dimensional so that $E_{IG_{\mu}}(\cdot) = (\hat{\omega}^2/N^2)E_{IG_{\hbar}}(\cdot)$, where (·) could be κ , $\hat{\omega}$, or the vertical wavenumber *m*. Eliminating $\hat{\omega}$ in (5) gives

$$E_{\mathrm{IG}_{w}}(\kappa) = c \frac{\varepsilon^{2/3}}{N^{2}} \kappa^{4/3} E_{\mathrm{IG}_{h}}(\kappa) \propto \kappa^{-1/3}.$$
 (6)

The prediction of E_w slopes based on (6) is inconsistent with the simulated slopes in all models. In ICON, which exhibits a significantly shallower mesoscale slope $E_{\text{IG}_h} \propto -1.24$, (6) predicts a flat E_w instead of the observed $E_w \propto \kappa^{2/3}$. This disagreement, however, does not invalidate the interpretation of gravity waves controlling E_w . Instead, the saturation and cascade conditions may not cooccur, and the relationship between the wave intrinsic frequency and the horizontal wavenumber may differ from $\hat{\omega} \propto \kappa^{2/3}$. Dewan and Good (1986) introduced the linear instability theory (LIT), which assumes that the saturation amplitude of each wave packet is *N/m* regardless of the frequency or horizontal wavenumber, which leads to the prediction of $E_h(m) \sim m^{-3}$. Several observational studies have corroborated this prediction (Smith et al. 1987; Allen and Vincent 1995; Zhang et al. 2017), but not necessarily confirm either the LIT or the SCT. This assumption implies that the shape of the vertical wavenumber spectrum does not depend on wave frequency; therefore, the joint $(m, \hat{\omega})$ spectrum of horizontal and vertical winds are separable. We follow this assumption of separability using a one-dimensional frequency spectrum of the form $B(\hat{\omega}) \propto \hat{\omega}^{-p}$, where $p \sim 5/3$ (Gardner 1996). Using the standard Jacobian transformation, one can obtain the one-dimensional spectrum $E_{\mathrm{IG}_{h}}(\kappa) = E_{\mathrm{IG}_{h}}(m)|dm/d\kappa|$, and similarly for $E_{\mathrm{IG}_{w}}(\kappa)$. These assumptions are rather crude, and in fact, some studies have indicated the nonseparability of the joint $(m, \hat{\omega})$ spectrum (Gardner 1996; Gardner et al. 1998). However, they allow us to relate $E_{\mathrm{IG}_{w}}(\kappa, z)$ and $E_{\mathrm{IG}_{h}}(\kappa, z)$ at fixed heights using (4), and compare them to the model's spectra. Next, we suggest an alternative derivation of $\hat{\omega}(\kappa)$.

From the linear vorticity equation, we have for inertiagravity waves (Li and Lindborg 2018)

$$R = \frac{E_{\mathrm{IG}_d}}{E_{\mathrm{IG}_e}} = \frac{\hat{\omega}^2}{f^2},\tag{7}$$

which is true for each Fourier mode of a wave field regardless of its vertical structure. Equation (7) implies that the relationship $\hat{\omega}(\kappa)$ is determined by $R(\kappa)$, provided that $E_{IG_{\ell}} \ge E_{IG_{\ell}}$



FIG. 8. Ratio *R* of divergent $E_{IG_{4}}$ to rotational $E_{IG_{2}}$ kinetic energies as a function of spherical wavenumber for ICON, IFS, and GEOS at 6, 8, 12, 16, 20, and 24 km. Vertical wavelengths are shown in dashed lines for the troposphere (gray) and the stratosphere (black). Reference slopes for $\kappa^{4/3}$ and κ^{2} are shown in gray dashed lines. The horizontal dashed line corresponds to R = 1. The gray shaded area indicates horizontal wavelengths < 20 km.

or $R \ge 1$, so that $\hat{\omega} \ge f$. The equality R = 1 holds at large scales for pure inertial waves. The scale at which E_{IG_d} becomes larger than E_{IG} is defined as L_{IG} .

Figure 8 shows $R(\kappa)$ at different altitudes in the troposphere and stratosphere. We focus on the $R \ge 1$ region in what follows. The IFS shows a scaling of $R(\kappa)$ that follows $\kappa^{4/3}$ closely at scales $l \ge 10$ in the stratosphere, which implies $\hat{\omega} \propto \kappa^{2/3}$, consistent with the saturated-cascade hypothesis. In the troposphere, the slope is only slightly flatter than $\kappa^{4/3}$. Meanwhile, ICON and GEOS deviate sconer from $\kappa^{4/3}$, following a scaling closer to κ^2 at scales ≥ 800 km. Models show more similar slopes in the troposphere than the stratosphere, with *R* being approximately an order of magnitude smaller in the troposphere compared to the stratosphere. Furthermore, it is possible to verify that the vertical wavelengths are within the applicability limits of (5), namely, $\lambda_z < H$, by using estimates of the intrinsic frequency from $R(\kappa)$ in the gravity wave hydrostatic dispersion relation:

$$m^{2} = \frac{\kappa^{2}(N^{2} - \hat{\omega}^{2})}{\hat{\omega}^{2} - f^{2}},$$
(8)

where $m = 2\pi/\lambda_z$ is the vertical wavenumber. In the troposphere, the models present $\lambda_z \sim 4$ km at mesoscales, while λ_z ranges from 4 to around 6 km in the stratosphere.

As a consequence of (7), it follows that the intrinsic frequency can be approximated using $\hat{\omega}/f = \sqrt{R}$. Figure 9 shows $\hat{\omega}/f$ estimated from the zonally averaged ratio of divergent and rotational kinetic energies in physical space at 6 and 24 km for ICON, IFS, GEOS, and the ERA5 reanalysis. These results show near-inertial frequencies in the lower stratosphere (2.0*f*-2.5*f*) and higher (2.0*f*-3.5*f*) in the troposphere. These estimates of $\hat{\omega}$ are consistent with the mediumfrequency approximation of the polarization relation. The models show considerable differences regarding the meridional distribution of $\hat{\omega}/f$; however, they consistently exhibit higher intrinsic frequencies in the troposphere compared to the stratosphere at midlatitudes in the Northern Hemisphere and the opposite behavior in the Southern Hemisphere. Further, ICON and GEOS show values of $\hat{\omega}/f$ approximately constant at midlatitudes in the troposphere. In contrast, in the lower stratosphere, $\hat{\omega}/f$ systematically decreases with latitude in the Southern Hemisphere and from the equator to around 60°N. In the IFS and ERA5, $\hat{\omega}/f$ are almost identical and consistent with linear IGW theory ($\hat{\omega}/f > 1$) at the latitude band 40°S-40°N. To verify these estimates, we compare the meridional distribution of $\hat{\omega}/f$ shown in Fig. 9 to the results of Geller and Gong (2010, their Fig. 1a), which were calculated using kinetic to potential energy ratios based on radiosonde data (1998–2006). This comparison indicates that ICON and GEOS provide a better match to radiosonde observations, at least in the Northern Hemisphere.

Geller and Gong (2010) showed that the intrinsic frequency computed from averaged energy ratios using polarization relations is consistently smaller than the average intrinsic frequencies calculated with the hodograph method for each radiosonde sounding by approximately a constant factor. We assume here that $\hat{\omega}$ in (4) is proportional to that obtained from (7) resulting in $\hat{\omega}^2 = \alpha R f^2$, where $\alpha > 0$. For convenience we define $R' = \alpha R$. The proportionality factor α accounts for the effect of wave superposition modulating the wave frequencies, and amplitudes since (4) is only exact for monochromatic waves (Fritts 1984).

Eliminating the intrinsic frequency in (5) using $\hat{\omega}^2 = R' f^2$, we obtain the following approximation for the IGW vertical kinetic energy:

$$E_{\mathrm{LS}_{w}}(\kappa, z) = \frac{f^{2}}{N^{2}} R'(\kappa, z) E_{\mathrm{IG}_{h}}(\kappa, z).$$
(9)

Note that (9) is highly sensitive to the values of Prandtl's ratio f/N. We use the value of the Coriolis frequency f at midlatitudes (i.e., f at 45°), and N(z) is approximated by a stepwise function of altitude, which takes values N = 0.012 rad s⁻¹ in the troposphere and N = 0.026 rad s⁻¹ in the stratosphere.



FIG. 9. Meridional distribution of zonally averaged ratio of intrinsic to inertial frequency $\hat{\omega}/|f| = \sqrt{R}$ at 6 (solid) and 24 km (dashed) for models ICON, GEOS, and IFS. ERA5 is shown for reference in black. Dashed gray lines delimit $\hat{\omega} = |f|$. The standard deviation is shown as a shaded area for each line.

Figure 10 shows the prediction of (9) and E_w . In a statistical sense, the analytical model derived in this section explains to first order the vertical velocity spectra for a wide range of horizontal scales and predicts the average vertical kinetic energy at large scales (500–2000 km), save for the proportionality factor α . We estimate α using a nonlinear least squares regression of (9) to the models' spectra. The parameter α consistently decreases with height; however, it varies significantly between models. In ICON, α ranges from approximately 0.26 in the stratosphere to 0.65 in the troposphere, in GEOS from 0.2 (stratosphere) to 1.0 (troposphere), and from 0.36 (stratosphere) to 2.0 (troposphere) in IFS.

The tropospheric slopes of $E_{\rm LS_w}$ range from -1 to -1/3 at scales 400 km $\leq \lambda_h < L_{\rm IG_c}$, which matches the slopes of E_w in all models. In the stratosphere, the predicted slopes are consistent with the E_w slopes in GEOS, while for ICON and IFS, the prediction fails to capture the large-scale slopes. In addition, (9) captures the observed slope transition of E_w in the stratosphere, mainly through changes in the slope of $R(\kappa)$ since E_h does not deviate significantly from -5/3 for spherical wavenumbers l > 10. In the stratosphere, ICON and GEOS exhibit a slope transition to the mesoscale with slopes close to 2/3 and 1/3, respectively. In contrast, IFS shows a scaling of $\kappa^{-1/3}$ consistent with the wave saturation hypothesis.

We note that (9) largely underestimates the magnitude of E_w at mesoscales. In Polichtchouk et al. (2022), it is demonstrated that most of the mesoscale vertical velocity variance is owing to the tropical region. At the same time, the large-scale peak in the global E_w is associated with extratropical IGWs. Consistent with the estimates of $\hat{\omega}$ shown in Fig. 9, the models agree on the occurrence of higher averaged intrinsic frequencies

in the tropics and near-inertial frequencies toward the poles. It is therefore not surprising that (9), which includes waves $f^2 \ll \hat{\omega}^2 \ll N^2$ and is less sensitive to high-frequency IGWs than (4), is not representative of the mesoscale E_w .

S14 suggested that the synoptic-scale peak in the vertical kinetic energy spectra is related to vertical motions associated with large-scale waves. Most of the large-scale vertical kinetic energy in the stratosphere seems to be associated with spontaneously generated IGWs from imbalances around the polar vortex (see Fig. 1e), which are persistent throughout the analysis period. In the free troposphere, orographically generated waves might be significant in explaining some of the large-scale vertical kinetic energy. However, the fact that gravity wave polarization relations well describe the large-scale E_w through (9) does not imply that freely propagating IGWs dominate the synoptic scales.

An alternative explanation is that the synoptic-scale peak in the free troposphere comes from balanced vertical velocity associated with midlatitude baroclinic waves, which project onto the linear IGW modes. From a scaling analysis of the linearized QG equations, considering only the leading-order terms, we have for the balanced vertical kinetic energy (Dritschel and McKiver 2015):

$$E_w \sim \operatorname{Ro}^2 \frac{f^2}{N^2} \left(\frac{fL}{NH}\right)^2 E_h,$$
 (10)

where L and H are the horizontal and vertical characteristic length scales. The Rossby number Ro can be approximated as the ratio of ageostrophic velocity u_a to geostrophic velocity u_g , i.e., Ro $\sim |u_a|/|u_g|$. At large scales, the horizontal kinetic



FIG. 10. Vertical kinetic energy spectra E_w as a function of spherical wavenumber for ICON, IFS, and GEOS in the troposphere (6 km) and stratosphere (24 km). The vertical velocity spectra E_{LS_w} calculated with (9) are shown in red. The shaded area around each line indicates the 95% confidence bands from the uncertainties in the model parameters. Vertical dotted lines denote the crossing scale L_{IG_c} where E_{IG_v} and E_{IG_v} intersect. The gray shaded area indicates horizontal wavelengths < 20 km.

energy E_h is dominated by geostrophic flow $(E_h \sim u_g^2)$, while E_{IG_h} is mostly ageostrophic $(E_{IG_h} \sim u_a^2)$. Therefore, it follows that $E_{IG_h} \sim \text{Ro}^2 E_h$. This relationship allows us to express (9) in terms of E_h as $E_w \sim \text{Ro}^2 (f/N)^2 R' E_h$. This expression is consistent with (10) when $R' \sim (fL/NH)^2$. The validity of the QG approximation requires $(fL)/(NH) \sim 1$, implying that $\alpha R \sim O(1)$. As shown in Fig. 8, R ranges from 0.5 to 4 at scales $L \sim 2000$ –3000 km, which is consistent with the values of α^{-1} independently estimated for each model at different levels. This scaling analysis suggests that the observed large-scale peak in E_w may result from QG balanced vertical motions that still satisfy (9).

Wang and Bühler (2020, hereafter WB20) developed a method to incorporate weakly nonlinear ageostrophic corrections into the linear wave–vortex decomposition from onedimensional aircraft measurements using a statistical QG omega equation. This approach was motivated by the fact that nonlinearities can cause a nonzero vertical velocity field associated with the balanced flow that projects onto linear IGW modes. Their results suggest that IGW modes are robust to nonlinear effects in the lower stratosphere, even at large scales. However, it still needs to be determined whether linear IGW modes are also robust in the upper troposphere. Because we cannot directly quantify the nonlinear projection of vortical energy onto the IGW modes, our analysis does not allow for a definitive conclusion on the cause of the large-scale peak in the vertical kinetic energy spectrum. Applying WB20's approach to analyze 3D global DYAMOND-like simulations might be valuable to shed light on whether the large-scale E_w in the upper stratosphere is due to linear IGWs rather than vertical motions associated with the balanced ageostrophic flow.

The following section discusses a general interpretation of the relationship E_w/E_d based on mass continuity in the incompressible limit. Additionally, we show that the E_w positive slopes in the mesoscale end of the spectrum also emerge from the hydrostatic IGW polarization relation if one allows for higher-frequency IGWs.

2) MESOSCALES

Vertical velocity w is related to the horizontal wind components u and v by mass continuity. A scale analysis of the

continuity equation shows that for large-scale motions the mass flux is nondivergent, $\nabla \cdot (\rho \mathbf{v}) = 0$, also known as the anelastic approximation, where $\mathbf{v} = (u, v, w)$ and ρ is the air density. Neglecting horizontal variations in density at surfaces of constant height, [i.e., $\rho = \rho_0(z)$] gives $\nabla \cdot \mathbf{v} - w/H_\rho = 0$, where $H_\rho = -\rho_0(\partial\rho_0/\partial z)^{-1}$ is the density vertical length scale (~8 km). If we make the additional assumption that the vertical length scale of the circulation is much smaller than H_ρ , then $\nabla \cdot \mathbf{v} = 0$ (i.e., incompressible flow). This kinematic link between horizontal and vertical motions provides a framework for deriving a quantitative model of vertical velocity spectra for a wide range of spatial scales from the surface layer to the lower stratosphere. Such models have been discussed in previous studies (e.g., Peltier et al. 1996; Tong and Nguyen 2015; S19).

Following S19, integrating the continuity equation from the ground (z = 0) to a height z = h with boundary conditions w(0) = 0 yields

$$w(h) = -\int_0^h \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz = -h\left(\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y}\right),\tag{11}$$

where \overline{u} and \overline{v} denote the vertically averaged wind components.

The Fourier modes of the wind components $(\hat{u}, \hat{v}, \hat{w})$, also satisfy (11), from which follows that

$$\hat{w}\hat{w}^* = h^2 [\kappa_x^2 \hat{\overline{u}} \, \hat{\overline{u}}^* + \kappa_x \kappa_y (\hat{\overline{u}} \, \hat{\overline{v}}^* + \hat{\overline{v}} \, \hat{\overline{u}}^*) + \kappa_y^2 \hat{\overline{v}} \, \hat{\overline{v}}^*].$$
(12)

The second term on the rhs of (12) accounts for the mean correlations between u and v, which are small in the mesoscales. We can eliminate the cross-correlation term using the vertical component of vorticity (ζ) in Fourier space. The Fourier coefficients of ζ relate to the horizontal wind through $\hat{\zeta} = i\kappa_x \hat{v} - i\kappa_y \hat{u}$. After vertically integrating $\hat{\zeta}$ using the same limits as in (11) and multiplying by its complex conjugate, one obtains the horizontal wavenumber spectrum of the vertical vorticity as follows:

$$\hat{\zeta}\,\hat{\zeta}^* = \kappa_x^2\hat{\upsilon}\,\hat{\upsilon}^* - \kappa_x\kappa_y(\hat{u}\,\hat{\upsilon}^* + \hat{\upsilon}\,\hat{u}^*) + \kappa_y^2\hat{u}\,\hat{u}^*, \qquad (13)$$

where $\hat{\zeta}$ relates to the rotational kinetic energy as $E_r = \hat{\zeta}\hat{\zeta}^*/(2k^2)$ and the divergent kinetic energy is simply $E_d = E_h - E_r$. Inserting (13) into (12) gives

$$E_w(\kappa, h) = (h\kappa)^2 \overline{E}_d(\kappa), \qquad (14)$$

where $E_w = \hat{w}\hat{w}^*/2$ is the horizontal wavenumber spectrum of vertical velocity at height h, $\overline{E}_d = (\hat{u}_d \hat{u}_d^* + \hat{v}_d \hat{v}_d^*)/2$ denotes the kinetic energy spectra computed from vertically averaged spectral coefficients of the divergent winds. Note that (14) is only exact in a horizontally isotropic atmosphere with constant density at height h.

To allow comparisons of (14) with modeled $E_w(k, h)$ and $E_d(k, h)$ at a given h, S19 proposed that $\overline{E}_d(k, h)$ and the horizontal spectra of divergent kinetic energy $E_d(k, h)$ are

proportional, at sufficiently large scales ($h\kappa \ll 1$). Considering $\overline{E}_d(k, h) = \beta^2 E_d(k, h)$, and inserting in (14) gives

$$E_{\mathrm{MC}_{w}}(\kappa, h) = (h_{e}\kappa)^{2}E_{d}(\kappa, h), \text{ for } h\kappa \ll 1,$$
(15)

where $h_e = \beta h$ denotes the "effective height" controlled by the parameter β and measures the depth of layers with effectively uniform divergent flow (S19). The physical interpretation of β depends on the application. In S19's interpretation, β encodes the vertical coherence of the profiles of divergent horizontal velocities. For example, in a barotropic flow in a layer of depth h, $\beta \rightarrow 1$ and $\overline{E}_d(k, h) \approx E_d(k, h)$. In Peltier et al. (1996), a similar parameter was associated with surface layer stability. These two interpretations are equivalent in the free convective regime where the mean vertical wind shear decreases (Businger 1973), and $\beta \rightarrow 1$. In the following, we investigate to what extent E_{MC_w} is a good approximation of mesoscale E_w for the different models.

Figure 11 shows the ratio E_w/E_d scaled by $(h\kappa)^2$ at different model levels for ICON, IFS, and GEOS. This ratio shows a scaling close to κ^2 at mesoscales as predicted by (15). However, this scaling breaks at scales ~ 100 km in the troposphere and larger scales in the lower stratosphere. These breaks presumably occur at scales where the spatial variability of density is not negligible, and therefore, the assumption of incompressibility does not hold. From a nonlinear least squares regression of (15) to model spectra, we estimate β at each vertical level. The value of β varies from approximately 0.49–0.66 in the troposphere to around 0.11-0.13 in the stratosphere. The parameter β decreases with height due to small vertical correlations of horizontal motions between the stratosphere and the troposphere. S19 reported values of $\beta = 0.5$ at h = 9.5and 0.05 at 17 km, resulting in $h_e = 5$ and 1 km, respectively, based on MPAS 3 km simulations. In the DYAMOND simulations, we observe less pronounced variations of h_e , which slowly decrease with height ranging between 2.6 and 4 km in all models.

According to (15) and assuming that E_d scales as $\kappa^{-5/3}$ at mesoscales, the prediction for the scaling of E_w is $\kappa^{1/3}$. In the troposphere, we observe positive slopes closer to 1/3, except for ICON, with a steeper slope at scales < 100 km (see Fig. 7). In the stratosphere, IFS and GEOS show a slope close to 2/3 for scales (~200–1000 km) and significantly shallower slopes at scales < 200 km, while ICON shows the 2/3 slope throughout the stratosphere's mesoscale. ICON's E_w steeper slopes are explained by the shallow E_d slopes of about -4/3 (see Table 1).

In the following, we explore the relationship between E_w and E_{IG_h} in the mesoscale region. Analytical models of the form (15) must also apply to the ratio E_{IG_w}/E_{IG_h} at mesoscales since linear IGW modes satisfy the incompressible continuity equation by definition. A simple approximation for E_{IG_w} can be derived from (4) and the dispersion relation (8):

$$E_{\rm MS_{w}}(\kappa) = \tilde{\kappa}^2 h_e^2(\kappa) E_{\rm IG_{k}}(\kappa), \qquad (16)$$

where $\tilde{\kappa} = \kappa/2\pi$ is the scaled wavenumber in units (m⁻¹), and the "effective height" parameter is redefined in terms of gravity wave vertical wavelengths and intrinsic frequencies as



FIG. 11. The ratio of vertical to horizontal divergent kinetic energy scaled by $(h\kappa)^2$, computed for ICON, IFS, and GEOS at levels 6, 8, 12, and 24 km. Horizontal dashed lines correspond to the predictions of E_{MC_w} from the S19 analytical model, and the corresponding β coefficient is depicted to the right of each line. Effective height is shown in the inlet, along with the corresponding altitude.

$$h_e = \lambda_z \left(\frac{\hat{\omega}^2}{\hat{\omega}^2 + f^2}\right)^{1/2} = \lambda_z \left(\frac{R'}{R' + 1}\right)^{1/2}.$$
 (17)

For near-inertial waves $\hat{\omega}/|f| \sim 1$, (17) predicts $h_e \sim 0.7\lambda_z$, while in the high-frequency range $\hat{\omega} \sim N$, it gives $h_e \sim \lambda_z$. At midlatitudes in the stratosphere, where $\hat{\omega}/|f| \sim 2$ (see Fig. 9), we have $h_e \sim 0.9\lambda_z$. The estimates of $h_e \sim 0.8\lambda_z$ are consistent with those shown in Fig. 11, where h_e is calculated from fitting (15) to model spectra, and λ_z is calculated from the hydrostatic dispersion relation (see Fig. 8).

Note that (16) is similar to (15), except that E_w is related to E_{IG_h} and the parameter h_e is a function of horizontal wavenumber as it depends on E_{IG_d} and E_{IG_r} . Considering $\alpha = 1$, (16) simplifies to $E_{\mathrm{IG}_w} = (\lambda_z \tilde{\kappa})^2 E_{\mathrm{IG}_d}$, which is consistent with the incompressible mass-continuity scaling of IGW wind components. In the high-frequency limit $\hat{\omega} \to N$, (16) is less sensitive to α , since $E_{\mathrm{IG}_h} \to E_d$ and $h_e \to \lambda_z$. For practical applications of (16), we use an averaged effective height in the mesoscale region (20–500 km) and values for α of 0.5 and 1.2 in the stratosphere and troposphere, respectively.

Figure 12 shows $E_{\rm MS_w}$ and E_w at 6 and 24 km. Notably, $E_{\rm MS_w}$ approximates E_w with high accuracy regarding mesoscale spectral slopes in all models. In particular, the stratospheric large-scale slopes of E_w are captured by $E_{\rm MS_w}$ in IFS. These results suggest that $E_{\rm IG_h}$ is a better predictor of E_w compared to E_d in the large-scale portion of the mesoscale (200–1000 km). Equation (15) accurately predicts the slopes of E_w provided that E_d remains close to $E_{\rm IG_h}$ (see Fig. 6). The vertical kinetic energy $E_{\rm MC_w}$ calculated with (15) predicts steeper slopes than E_w , and therefore a faster energy increase toward small scales. In the troposphere, $E_{\rm MC_w}$ converges toward E_w at scales $\lambda_h < 100$ km. In the stratosphere, especially for IFS and GEOS, one could obtain a better match between $E_{\rm MC_w}$ and E_w at scales ~200–1000 km by increasing h_e to approximately $h_e \sim \lambda_z$; however, this results in an overestimation of E_w at shorter scales ($\lambda_h < 100$ km).

Simplified analytical models based on linear IGW polarization relations of the form (9) and (16) together provide a quantitative description of E_w for a wide range of horizontal scales in the troposphere and the stratosphere. These results are consistent with those obtained by integrating the continuity Eq. (3) from horizontal IGW modes. These results suggest that IGW properties, namely, the dominant vertical wavelength and intrinsic frequency, control the effective height and, therefore, the magnitude of E_w . The main benefit of the IGW interpretation of h_e is that it links vertical and horizontal kinetic energy spectra, invoking only local wind field information, which can be validated with observations. In principle, we can constrain the h_e parameter at horizontal scales ~200 km using vertical wavelengths estimated from vertical profiles of horizontal winds and vertical velocities estimated from dropsonde data as demonstrated, e.g., by Bony and Stevens (2019).

4. Summary and conclusions

This study compared the relationship between horizontal and vertical kinetic energy spectra calculated from global storm-resolving simulations of four numerical models of the DYAMOND experiment. The data analyzed consist of numerical outputs from the ICON, IFS, GEOS, and NICAM models with horizontal grid spacings < 5 km, covering 12 days of the winter experiment. We focus primarily on the relationships between E_h and E_w across all resolved horizontal scales $(\lambda_h > 20 \text{ km})$. We investigate the role of balanced and unbalanced circulations obtained utilizing normal mode function decomposition, which yields the contribution of IGWs to the horizontal kinetic energy spectra. To estimate the contribution of IGWs to the vertical velocity spectra, we numerically solve the mass continuity equation in physical space from horizontal IGW modes. Additionally, we analyze $E_{IG_{1}}$ and $E_{IG_{4}}$ associated with the unbalanced IGW component. Furthermore, we consider the linearized vorticity equation and hydrostatic IGW polarization relations to link E_w and E_h at large scales and discuss the prospect of extending the IGW interpretation



FIG. 12. Vertical kinetic energy spectra E_w as a function of spherical wavenumber for ICON, IFS, and GEOS at (top) 6 and (bottom) 24 km. The vertical kinetic energy spectra E_{MC_w} calculated with (15) and E_{MS_w} are shown in green and red, respectively. The shaded area around each line indicates the 95% confidence bands from the uncertainties in the model parameters. Vertical dotted lines denote the crossing scale (L_{IG_v}) where E_{IG_v} and E_{IG_v} intersect. The gray shaded area indicates horizontal wavelengths < 20 km.

to the mesoscale region. In addition, we explore the kinematic link between E_w and E_h at mesoscales and shorter scales using an incompressible, isotropic scaling of the continuity equation.

All models exhibit a high degree of agreement on spectral power in the large-scale regime for wavelengths greater than 600-800 km in the free troposphere. The stratospheric spectral slope, however, is slightly steeper than κ^{-3} —with a similar transition in spectral slopes from large scales to a shallower mesoscale regime in the stratosphere. The mesoscale transition region varies slightly from model to model and occurs consistently at longer wavelengths in the stratosphere compared to the troposphere. In the mesoscale region, the models differ in their magnitudes of kinetic energy per unit mass in the stratosphere, while these differences are less significant in the troposphere. Model results are consistent with the observation that the rotational flow dominates the synoptic range. In contrast, the rotational and divergent components are of the same order in the mesoscale range in the troposphere, and the divergent IGW energy dominates E_h in the stratosphere.

The vertical kinetic energy spectra are relatively flat across all resolved horizontal scales, with evidence of two peaks, one at synoptic scales (~2000 km) and one at the smallest resolved scale (~20 km). All models predict a similar spectral power related to the maxima found at large scales, while most differences occur in the mesoscale. For example, E_w mesoscale slopes are close to 1/3 in the troposphere for all models and slightly steeper (2/3) in the lower stratosphere in ICON, while in IFS and GEOS, the slopes flatten for $\lambda_h < 100$ km. We show that vertical kinetic energy spectra are explained, to a good approximation, exclusively by horizontal winds over a wide range of horizontal scales.

At the mesoscale, the vertical and horizontal kinetic energy spectra are linked kinematically, as shown by S19. This kinematic link between the horizontal and vertical motions provides a framework for deriving a quantitative analytical model of E_w from knowledge of E_d at a given vertical level. The relationship of E_w to E_d on the mesoscale is best explained by mass continuity in the incompressible limit at scales < 100 km, and the ratio E_w/E_d scales to a good approximation as $(h_e \kappa)^2$. The "effective height" is approximately within 2–4 km in all models, but depends weakly on height for each model independently. This variation of h_e is approximately 1 km between the troposphere and stratosphere, consistent with variations of the vertical wavelengths shown in Fig. 8 estimated from the dispersion relation. Our results suggest that the properties of IGWs, namely, the dominant vertical wavelength and the intrinsic frequency, control the h_e parameter and hence the magnitude of E_w . The main benefit of this interpretation of h_e is that it links E_w and E_h , invoking only the wind field information at the same level. IGW characteristics can, in principle, be estimated directly from observations.

At large scales, the proportionality $E_w/E_d \propto \kappa^2$ breaks since the transition in the E_w slopes from negative to positive between global and synoptic scales passing through an energy minimum (at $l \sim 20$ in the stratosphere), has no counterpart in E_d . The large-scale maxima found in E_w can be explained to a good approximation by the hydrostatic IGW polarization relation in the midfrequency limit, where the intrinsic frequencies are inferred from the energy ratio $E_{IG_{d}}/E_{IG_{r}}$. A simple analytical model relating E_w and E_{IG_v} save for a proportionality factor α is presented. The value α decreases with altitude from approximately 1.2 in the troposphere to around 0.5 in the stratosphere. The estimates of $\hat{\omega}/f$ from the ratio of divergent to rotational IGW energies are consistent with the results presented in Geller and Gong (2010) based on radiosonde observations. These results show $\hat{\omega}/f$ of around 1.5-2.5 in the stratosphere and a higher ratio of 2-3 in the troposphere, which would be consistent with the hypothesis that IGWs control E_w at large scales. However, the largescale E_w peak also seems consistent with QG scaling, and additional analysis is required to determine its cause. Nevertheless, the simplified analytical models derived here describe vertical kinetic energy for a wide range of spatial scales.

The results obtained from the partitioning into IGW and balanced modes in the lower stratosphere suggest that IGWs dominate mesoscale spatial variability in IFS and GEOS, while in ICON, these components are of the same order. In the troposphere, the contributions from IGWs and vortical modes to E_h are similar in all models. The IGW modes explain differences in E_h , and to some degree, differences in E_w because E_{IG} , governs most of E_w kinematically and through the hydrostatic polarization relation at most resolved scales. Alternatively, E_w could explain the magnitudes of E_d and E_r since energy converts from available potential energy to the kinetic energy of the divergent flow through vertical motions and then to rotational kinetic energy (Lorenz 1960; Chen and Wiin-Nielsen 1976). However, a quantitative analysis of these energy conversion processes and the interactions involving rotational and divergent modes in global storm-resolving simulations is missing. Regardless of the model discrepancies in the underlying dynamics of horizontal winds, the vertical velocity seems to be consistent with quasi-linear dynamics. In light of these results, we believe that a detailed analysis of the spectra of the physical tendencies in highresolution simulations and their impacts on the representation

of IGW sources are desirable to elucidate energy transfer between horizontal and vertical motions.

Acknowledgments. This work was supported by the International Max Planck Research School on Earth System Modelling (IMPRS-ESM). DYAMOND data management was provided by the German Climate Computing Center (DKRZ) and supported through the projects ESiWACE and ESiWACE2. The projects ESiWACE and ESiWACE2 have received funding from the European Union's Horizon 2020 research and innovation programme under Grant Agreements 675191 and 823988. This work used resources of the Deutsches Klimarechenzentrum (DKRZ) granted by its Scientific Steering Committee (WLA) under Project IDs bk1040 and bb1153. We thank Bjorn Stevens for valuable discussions in the early stages of this manuscript. Further, we acknowledge Daniel Klocke's valuable comments during the internal review. We also thank Chris Snyder and two anonymous reviewers for their insightful comments and suggestions.

Data availability statement. The model outputs from the DYAMOND initiative can be accessed at the project website https://www.esiwace.eu/services/dyamond-initiative. Access to the MODES software can be requested at https://modes.cen.uni-hamburg.de/software.

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