



On marginal deformations of $(0, 2)$ non-linear sigma models

Ilarion V. Melnikov^{a,*}, Eric Sharpe^b

^a Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Am Mühlenberg 1, D-14476 Golm, Germany

^b Physics Department, Virginia Tech, Blacksburg, VA 24061, USA

ARTICLE INFO

Article history:

Received 16 October 2011

Accepted 21 October 2011

Available online 25 October 2011

Editor: M. Cvetič

ABSTRACT

An $N = 1$, $d = 4$ supersymmetric compactification of the perturbative heterotic string is described by a $d = 2$ $(0, 2)$ superconformal field theory. The first order marginal deformations of the internal $(0, 2)$ SCFT are in 1 to 1 correspondence with massless gauge-neutral scalars in the spacetime theory. Working at tree-level in the α' expansion, we describe these first order deformations for SCFTs with a $(0, 2)$ non-linear sigma model description. Our results clarify the structure of deformations of heterotic Calabi–Yau compactifications and more general heterotic flux vacua.

© 2011 Elsevier B.V. All rights reserved.

1. Introduction

A perturbative heterotic string compactification that preserves $N = 1$ super-Poincaré invariance in four dimensions has a worldsheet description as a unitary $(0, 2)$ superconformal field theory (SCFT) with integral R-charges [1] orbifolded by a heterotic GSO projection. The resulting massless spectrum consists of the minimal supergravity multiplet, the axio-dilaton chiral multiplet, vector multiplets for the spacetime gauge group G , “matter” chiral multiplets charged under G , as well as a number of G -neutral chiral multiplets. The latter parametrize \mathcal{V} – the space of first order deformations of the $(0, 2)$ SCFT, which consists of right-chiral primary SCFT states with conformal weights $(h, \bar{h}) = (1, \frac{1}{2})$.

A well-understood example is offered by a theory with $(2, 2)$ worldsheet supersymmetry [2,3], in which case \mathcal{V} has a decomposition into three types of states with respect to the $(2, 0)$ superconformal algebra: $\mathcal{V}_{(a,c)}$, $\mathcal{V}_{(c,c)}$, and \mathcal{V}' . The first two are $N = 2$ descendants of elements of the (a, c) and (c, c) rings of the $(2, 2)$ SCFT, while \mathcal{V}' denotes any additional $(0, 2)$ chiral primaries. When the $(2, 2)$ theory is well-approximated by a non-linear sigma model (NLSM) with a Calabi–Yau target-space M , the decomposition has a geometric interpretation in terms of certain cohomology groups, leading to the familiar terminology of “Kähler, complex structure, and bundle moduli”. While useful on the $(2, 2)$ locus, the decomposition relies on the accidental $(2, 0)$ supersymmetry, and in generic $(0, 2)$ theories the familiar terminology becomes a less than useful misnomer. This can be clearly seen in F-theory constructions, where the “bundle” and “complex structure” de-

mations enter on a more symmetric footing [4]. In the heterotic context, first order deformations have been recently explored from the supergravity point of view in heterotic flux vacua [5], as well as in compactifications involving a choice of a stable holomorphic bundle over a Calabi–Yau manifold [6].

The aim of this Letter is to examine the space of first order deformations \mathcal{V} from the worldsheet point of view in the context of a $(0, 2)$ NLSM. Working at tree-level in α' and using some simple $(0, 2)$ superspace techniques, we will find a hands-on description of \mathcal{V} . While involving ingredients familiar from the usual “Kähler, complex structure, bundle” decomposition, and reducing to known results on the $(2, 2)$ locus, we will see that in general \mathcal{V} differs markedly from its $(2, 2)$ form.

Our results agree with and generalize the supergravity analysis of heterotic Calabi–Yau compactifications. Formally they also apply to heterotic flux vacua without a large radius Calabi–Yau limit. To the extent that the NLSM and geometry are good guides to such vacua,¹ our results provide a starting point for describing the moduli space of heterotic flux compactifications.

The rest of the Letter is organized as follows: in Section 2 we set up the tree-level $(0, 2)$ NLSM; in Section 3 we describe the first order deformations; Section 4 is devoted to checking the analysis by comparing to known cases, and we end with some concluding remarks in Section 5.

2. The $(0, 2)$ NLSM

In this section we will review some basic properties of $(0, 2)$ NLSMs relevant for heterotic compactification. Throughout, the

* Corresponding author.

E-mail addresses: ilarion.melnikov@aei.mpg.de (I.V. Melnikov), ersharpe@vt.edu (E. Sharpe).

¹ One might expect this to hold in vacua with extended spacetime supersymmetry.

geometric setup will be a stable holomorphic bundle $E \rightarrow M$ satisfying the usual anomaly cancellation conditions $\text{ch}_2(E) = \text{ch}_2(T_M)$, where M is a Hermitian 3-fold with trivial canonical bundle. To be concrete, we will restrict attention to models with $G = G' \times E_8$ and $c_1(E) = 0$. These theories possess an additional structure on the worldsheet: a non-anomalous left-moving $U(1)$ symmetry $U(1)_L$, and as in Gepner's original construction [7], the GSO projection ensures that the $SO(k)$ gauge symmetry associated to k free left-moving fermions combines with $U(1)_L$ to form G' .² Since we will be interested in the gauge-neutral sector, we will from now on focus on the internal theory. Apart from a few small details of conventions, we are following the standard treatment, as reviewed in, e.g. [8].

2.1. (0, 2) Superspace and the NLSM Lagrangian

We work in Euclidean signature with (0, 2) superspace coordinates $(z, \bar{z}, \theta, \bar{\theta})$, with covariant derivatives $\mathcal{D}, \bar{\mathcal{D}}$ and supercharges $\mathcal{Q}, \bar{\mathcal{Q}}$ given by³

$$\begin{aligned} \mathcal{D} &= \frac{\partial}{\partial \theta} + \bar{\theta} \bar{\partial}, & \bar{\mathcal{D}} &= \frac{\partial}{\partial \bar{\theta}} + \theta \partial, \\ \mathcal{Q} &= -\frac{\partial}{\partial \theta} + \bar{\theta} \bar{\partial}, & \bar{\mathcal{Q}} &= -\frac{\partial}{\partial \bar{\theta}} + \theta \partial, \end{aligned}$$

where $\bar{\partial} \equiv \partial/\partial \bar{z}$. The non-trivial anti-commutators are

$$\{\mathcal{D}, \bar{\mathcal{D}}\} = +2\bar{\partial} \quad \text{and} \quad \{\mathcal{Q}, \bar{\mathcal{Q}}\} = -2\bar{\partial}.$$

Note that \mathcal{D} and \mathcal{Q} have $U(1)_R$ charge $\bar{q} = -1$, while $\bar{\mathcal{D}}$ and $\bar{\mathcal{Q}}$ have $\bar{q} = +1$. Our basic fields are the chiral matter and chiral Fermi fields

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta\bar{\theta}\bar{\partial}\phi, \quad \Gamma = \gamma + \sqrt{2}\theta G + \theta\bar{\theta}\bar{\partial}\gamma,$$

as well as the anti-chiral conjugate fields

$$\bar{\Phi} = \bar{\phi} - \sqrt{2}\bar{\theta}\bar{\psi} - \theta\bar{\theta}\bar{\partial}\bar{\phi}, \quad \bar{\Gamma} = \bar{\gamma} + \sqrt{2}\bar{\theta}\bar{G} - \theta\bar{\theta}\bar{\partial}\bar{\gamma}.$$

By construction, $\bar{\mathcal{D}}$ (\mathcal{D}) annihilates the chiral (anti-chiral) fields.

To build the NLSM Lagrangian, we take 3 chiral multiplets Φ^i , r Fermi multiplets Γ^β and their conjugates. Assuming the NLSM will describe a superconformal theory, each Φ (Γ) multiplet contributes (2, 3) ((1, 0)) to the central charge (c, \bar{c}) ; furthermore the $U(1)_L$ symmetry can be taken to act just on Γ and $\bar{\Gamma}$, assigning charges 1 and -1 , respectively, while $U(1)_R$ symmetry leaves both Γ and Φ invariant. With these assumptions, the most general (0, 2) supersymmetric Lagrangian is

$$\begin{aligned} 4\pi\alpha'\mathcal{L} &= \mathcal{D}\bar{\mathcal{D}} \left[\frac{1}{2} (\mathcal{K}_i(\Phi, \bar{\Phi})\partial\Phi^i - \bar{\mathcal{K}}_{\bar{i}}(\Phi, \bar{\Phi})\partial\bar{\Phi}^{\bar{i}}) \right. \\ &\quad \left. - \mathcal{H}_{\beta\bar{\alpha}}(\Phi, \bar{\Phi})\bar{\Gamma}^{\bar{\alpha}}\Gamma^\beta \right]. \end{aligned} \quad (2.1)$$

Here $\mathcal{H}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})$ is a Hermitian metric on the fibers of the bundle $E \rightarrow X$, while \mathcal{K}_i and $\bar{\mathcal{K}}_{\bar{i}}$ satisfy a reality condition $(\bar{\mathcal{K}}_{\bar{i}})^* = \mathcal{K}_i$. The \mathcal{K}_i should be thought of as a locally defined (1, 0) form $\mathcal{K} = \mathcal{K}_i d\phi^i$, and the action is invariant under shifts $\delta\mathcal{K} = \omega$ for any holomorphic (1, 0) form ω , as well as under $\delta\mathcal{K} = i\partial f$ for some real function $f(\phi, \bar{\phi})$. In addition, setting $\mathcal{H}' = U\mathcal{H}U^\dagger$ for any

² The k free fermions and the "hidden" E_8 current lead to a modular-invariant critical string.

³ Our conventions have the advantage of not being cluttered by factors of i ; however, the price to pay is a non-standard charge conjugation action on the fermions: $\mathcal{C}(\gamma) = \bar{\gamma}$, and $\mathcal{C}(\bar{\gamma}) = -\gamma$.

unitary transformation U leads to an equivalent theory. The free action with canonically normalized fields corresponds to $\mathcal{K}_i = \bar{\phi}^i$ and $\mathcal{H}_{\beta\bar{\alpha}} = \delta_{\beta\bar{\alpha}}$.

2.2. Equations of motion and component expansion

The equations of motion following from (2.1) can be derived by two well-known results: first, if X is a general (0, 2) superfield, then

$$\mathcal{D}\bar{\mathcal{D}}(AX)|_{\theta, \bar{\theta}=0} = 0 \quad \forall X \quad \Rightarrow \quad A = 0;$$

second, any chiral (anti-chiral) superfield, say $\delta\Phi^i$ ($\delta\bar{\Phi}^{\bar{i}}$), can be expressed as $\bar{\mathcal{D}}X$ ($\mathcal{D}X$) for some general superfield X . Varying the action in (2.1), we obtain, up to total derivatives,

$$\begin{aligned} 8\pi\alpha'\delta\mathcal{L} &= \mathcal{D}\bar{\mathcal{D}} \left[\mathcal{K}_{i,j}\partial\Phi^i - \partial\mathcal{K}_j - \bar{\mathcal{K}}_{\bar{i},j}\partial\bar{\Phi}^{\bar{i}} \right. \\ &\quad \left. - 2\mathcal{H}_{\beta\bar{\alpha},j}\bar{\Gamma}^{\bar{\alpha}}\Gamma^\beta \right] \delta\Phi^j - 2\bar{\Gamma}^{\bar{\alpha}}\mathcal{H}_{\beta\bar{\alpha}}\delta\Gamma^\beta + \text{h.c.}, \end{aligned}$$

which leads to the equations of motion

$$\begin{aligned} 0 &= E_j^\Phi = \bar{\mathcal{D}} \left[(\mathcal{K}_{j,i} + \bar{\mathcal{K}}_{\bar{i},j})\partial\bar{\Phi}^{\bar{i}} \right] + (\mathcal{K}_{j,ik} - \mathcal{K}_{i,jk})\partial\Phi^i \bar{\mathcal{D}}\bar{\Phi}^{\bar{k}} \\ &\quad + 2\bar{\mathcal{D}}(\mathcal{H}_{\beta\bar{\alpha},j}\bar{\Gamma}^{\bar{\alpha}})\Gamma^\beta, \\ 0 &= E_\beta^\Gamma = \bar{\mathcal{D}}[\mathcal{H}_{\beta\bar{\alpha}}\bar{\Gamma}^{\bar{\alpha}}]. \end{aligned} \quad (2.2)$$

The lowest component of E_β^Γ and its conjugate yield the equations of motion for the auxiliary fields G and \bar{G} :

$$\bar{G}^{\bar{\alpha}} = -\bar{\mathcal{A}}_{\bar{\beta},j}^{\bar{\alpha}}\bar{\gamma}^{\bar{\beta}}\bar{\psi}^{\bar{j}}, \quad G^\alpha = \mathcal{A}_{\beta,j}^\alpha\gamma^\beta\psi^j,$$

where \mathcal{A} and $\bar{\mathcal{A}}$ denote components of the Hermitian connection on E constructed from the metric \mathcal{H} and its inverse:

$$\mathcal{A}_{\beta j}^\alpha = \mathcal{H}^{\bar{\beta}\alpha}\mathcal{H}_{\beta\bar{j}}, \quad \bar{\mathcal{A}}_{\bar{\beta},j}^{\bar{\alpha}} = \mathcal{H}^{\alpha\beta}\mathcal{H}_{\beta\bar{j}}.$$

With a little work we can also obtain the component expansion of the Lagrangian. Up to boundary terms, we find

$$\begin{aligned} 2\pi\alpha'\mathcal{L} &= \frac{1}{2}g_{i\bar{j}}(\partial\phi^i\bar{\partial}\bar{\phi}^{\bar{j}} + \partial\bar{\phi}^{\bar{j}}\partial\phi^i) + \frac{1}{2}B_{i\bar{j}}(\partial\phi^i\bar{\partial}\bar{\phi}^{\bar{j}} - \partial\bar{\phi}^{\bar{j}}\partial\phi^i) \\ &\quad + g_{i\bar{j}}\bar{\psi}^{\bar{j}}\partial\psi^i + \bar{\psi}^{\bar{i}}[\partial\phi^k\Omega_{ikj}^- + \partial\bar{\phi}^{\bar{k}}\Omega_{\bar{i}\bar{k}\bar{j}}^-]\psi^j \\ &\quad + \bar{\gamma}_\mu(\bar{\partial}\gamma^\mu + \bar{\partial}\phi^j\mathcal{A}_{\beta j}^\mu\gamma^\beta) + \bar{\gamma}_\mu\mathcal{F}_{\beta\bar{j}k}^\mu\gamma^\beta\psi^k\bar{\psi}^{\bar{j}}, \end{aligned} \quad (2.3)$$

where $\bar{\gamma}_\mu \equiv \mathcal{H}_{\mu\bar{\beta}}\bar{\gamma}^{\bar{\beta}}$, and $\mathcal{F}_{\beta\bar{j}k}^\mu = \mathcal{A}_{\beta k, \bar{j}}^\mu$ is the (1, 1) component of the curvature for the connection \mathcal{A} ; the metric g and B-field are given by

$$g_{i\bar{j}} = \frac{1}{2}(\mathcal{K}_{i,\bar{j}} + \bar{\mathcal{K}}_{\bar{j},i}), \quad B_{i\bar{j}} = \frac{1}{2}(\mathcal{K}_{i,\bar{j}} - \bar{\mathcal{K}}_{\bar{j},i}),$$

and Ω^- denotes the H -twisted connection

$$\Omega_{ikj}^- = \Gamma_{ikj} - \frac{1}{2}H_{ikj}, \quad \Omega_{\bar{i}\bar{k}\bar{j}}^- = \bar{\Gamma}_{\bar{i}\bar{k}\bar{j}} - \frac{1}{2}H_{\bar{i}\bar{k}\bar{j}}, \quad (2.4)$$

where $H = dB$ is the tree-level torsion and Γ is the Hermitian Christoffel connection for g . As expected from the spacetime analysis [9], the torsion is determined by the Hermitian form: $H_{ijk} = g_{k\bar{j},i} - g_{\bar{k},i,j}$. For what follows it will be useful to recast the super-space equations of motion in terms of g and Ω^- :

$$\begin{aligned} g_{i\bar{j}}\bar{\mathcal{D}}\partial\bar{\phi}^{\bar{j}} &= -\Omega_{i\bar{j}\bar{k}}^- \partial\bar{\phi}^{\bar{j}}\bar{\mathcal{D}}\bar{\phi}^{\bar{k}} - \Omega_{i\bar{j}k}^- \partial\phi^j\bar{\mathcal{D}}\bar{\phi}^{\bar{k}} - \mathcal{F}_{\beta\bar{k}i}^\alpha \bar{\mathcal{D}}\bar{\phi}^{\bar{k}}\bar{\Gamma}_\alpha^\beta\Gamma^\beta, \\ \bar{\mathcal{D}}\bar{\Gamma}_\alpha &= 0, \quad \bar{\Gamma}_\alpha \equiv \mathcal{H}_{\alpha\bar{\alpha}}\bar{\Gamma}^{\bar{\alpha}}. \end{aligned} \quad (2.5)$$

2.3. Symmetries of the classical action

By construction the action is (0, 2)-supersymmetric. The action of the supercharges \mathbf{Q} and $\bar{\mathbf{Q}}$ on any superfield X is defined by

$$\sqrt{2}(\xi \mathbf{Q} + \bar{\xi} \bar{\mathbf{Q}}) \cdot X \equiv -\xi \mathbf{Q}X - \bar{\xi} \bar{\mathbf{Q}}X,$$

where ξ and $\bar{\xi}$ denote constant Grassmann parameters. After eliminating the auxiliary fields, the non-trivial transformations are as follows:

$$\begin{aligned} \bar{\mathbf{Q}} \cdot \bar{\phi}^i &= -\bar{\psi}^i, & \bar{\mathbf{Q}} \cdot \psi^i &= \bar{\partial} \phi^i; \\ \mathbf{Q} \cdot \phi^i &= \psi^i, & \mathbf{Q} \cdot \bar{\psi}^i &= -\bar{\partial} \bar{\phi}^i, \\ \mathbf{Q} \cdot \gamma^\beta &= -\mathcal{A}_{\nu j}^\beta \psi^j \gamma^\nu, & \mathbf{Q} \cdot \bar{\gamma}_\alpha &= \mathcal{A}_{\alpha j}^\nu \psi^j \bar{\gamma}_\nu. \end{aligned}$$

It is not hard to see that $\mathbf{Q}^2 = \bar{\mathbf{Q}}^2 = 0$ and $\{\mathbf{Q}, \bar{\mathbf{Q}}\} = \bar{\partial}$; the latter relation requires the use of the γ equations of motion, while the former hold off-shell.

It is also easy to see that corresponding to the $U(1)_L \times U(1)_R$ symmetries we have the conserved currents $J_L = \gamma^\alpha \bar{\gamma}_\alpha$, $J_R = g_{i\bar{j}} \psi^i \bar{\psi}^{\bar{j}}$, satisfying $\bar{\partial} J_L = 0$ and $\partial J_R = 0$ up to equations of motion. Similarly, we have the classical left-moving energy-momentum tensor

$$\begin{aligned} T &= -\frac{1}{\alpha'} \left\{ g_{i\bar{j}} \partial \phi^i \bar{\partial} \bar{\phi}^{\bar{j}} + \frac{1}{2} (\bar{\gamma}_\beta \partial \gamma^\beta + \gamma^\beta \partial \bar{\gamma}_\beta) \right. \\ &\quad \left. + \mathcal{A}_{\beta j}^\mu \partial \phi^j \bar{\gamma}_\mu \gamma^\beta \right\}. \end{aligned} \quad (2.6)$$

T , like J_L , is annihilated by both \mathbf{Q} and $\bar{\mathbf{Q}}$ and hence conserved: $\bar{\partial} T = 0$.

3. Massless G -neutral states via the (0, 2) NLSM

If we assume that the NLSM describes a (0, 2) SCFT, then we have all of the tools necessary for constructing the massless spectrum of the corresponding heterotic vacuum. A typical approach is to determine the massless fermions and infer the rest of the spectrum via supersymmetry. That is, we work in the (NS, R) and (R, R) sectors of the theory and identify right-moving ground states with L_0 eigenvalue of $+1$ for (NS, R) states and $L_0 = 0$ for (R, R) states. When working at tree-level in the NLSM, it is possible to construct the states in the Born–Oppenheimer approximation, where the mode expansion of the fields is truncated to right-moving zero modes and first excited modes on the left [10]. Working in this truncated Fock space, we can then classify the states annihilated by \mathbf{Q} and $\bar{\mathbf{Q}}$ and having $L_0 = +1$. Imposing the GSO projection, we will obtain the tree-level spectrum of massless fermions.

The procedure sounds straightforward, and it would be surprising if it had not already been applied to the (0, 2) NLSM some time ago. Indeed, the computation is presented in [11], where the massless spectrum is determined with one caveat: “*To be consistent, we should include the first excited modes of $[\phi]$, but as we are primarily interested in the gauge degrees of freedom, we will omit them.*” That the excited modes of ϕ should contribute to the analysis is reasonably clear, for instance from the last term in the classical energy–momentum tensor in (2.6). While this mixing is indeed unimportant in the charged matter sector,⁴ it does affect the spectrum of neutral massless scalars arising from the (NS, R) sector.

⁴ The (NS, R) charged matter states involve a free left-moving fermion tensored with γ or $\bar{\gamma}$ and a wavefunction of the bosonic zero modes; there are no additional ϕ excitations.

Table 1

The classical charges and weights of NLSM fields.

	X	Γ	$\bar{\Gamma}$	$\mathcal{D}\phi$	$\bar{\mathcal{D}}\bar{\phi}$	$\partial\phi$	$\bar{\partial}\bar{\phi}$
\bar{q}	1	0	0	-1	+1	0	0
q	0	1	-1	0	0	0	0
\bar{h}	1/2	0	0	1/2	1/2	0	0
h	1	1/2	1/2	0	0	1	1

Our goal is to determine the neutral massless spectrum, keeping track of all the necessary left-moving excitations. However, instead of pursuing the Born–Oppenheimer approach, we will attack the problem in a slightly different fashion by studying equivalence classes of chiral operators in the NLSM.

3.1. First order deformations of a (0, 2) SCFT

The G -neutral massless scalars of the four-dimensional effective theory have a simple interpretation in the internal (0, 2) SCFT as marginal $U(1)_L$ -preserving first order deformations: in the language of conformal perturbation theory, the action is deformed by the integrated zero-momentum vertex operator for the emission of the scalar. The form of marginal supersymmetric deformations of a unitary SCFT is tightly constrained. For instance, in [12] it is shown that in an $N = 1$, $d = 4$ superconformal theory the deformation must be an F-term

$$\Delta S = \int d^4x d^2\theta \mathcal{O} + \text{h.c.},$$

where \mathcal{O} is a chiral primary operator with R-charge 2; there are no non-trivial marginal D-term deformations. A similar result holds in unitary (0, 2) SCFTs in two dimensions⁵: a marginal supersymmetric deformation must take the form

$$\Delta S = \int d^2z \mathcal{D}X + \text{h.c.},$$

where X is a (0, 2) chiral primary operator with $h = 1$ and right-moving R-charge $\bar{q} = +1$; as in the four-dimensional case, a marginal deformation that is expressed as an integral over all of superspace is necessarily trivial.

3.2. Marginal superpotential deformations of the NLSM

We will now assume that the (0, 2) SCFT in question is well approximated by a weakly coupled (0, 2) NLSM. Let X be an operator in the SCFT of the type we just described. Then in a classical (i.e. large radius) limit, X must reduce to X_c – a chiral superfield constructed from the NLSM fields with their classical dimensions and charges listed in Table 1. In other words, X_c must be of the form

$$\begin{aligned} X_c &= [\bar{\Gamma}_\alpha \Gamma^\beta \Lambda_{\beta\bar{i}}^\alpha(\Phi, \bar{\Phi}) + \partial\phi^i Y_{i\bar{i}}(\Phi, \bar{\Phi}) \\ &\quad + \partial\bar{\phi}^{\bar{j}} g_{i\bar{j}} Z_{\bar{i}}^i(\Phi, \bar{\Phi})] \bar{\mathcal{D}}\bar{\phi}^{\bar{i}}. \end{aligned}$$

The NLSM fields are of course only defined in local coordinate patches, with transition functions relating the fields in different patches. X will be well-defined across the patches if Λ , Y , and Z take values in sections of certain bundles:

$$\begin{aligned} \Lambda &\in \Gamma(\text{End } E \otimes \Omega_M^{0,1}), & Y &\in \Gamma(\Omega_M^{1,1}), \\ Z &\in \Gamma(T_M \otimes \Omega_M^{0,1}), \end{aligned}$$

where $\Omega_M^{p,q}$ denotes the (p, q) forms on the target-space M .

⁵ This is a consequence of the (0, 2) SCFT unitarity bounds [13].

We have yet to impose that X_c is chiral, i.e. $\bar{D}X_c = 0$. We need not require X_c to be chiral off-shell – indeed, such a requirement would be too strong; instead, as in [14,15], we only require $\bar{D}X_c = 0$ up to the equations of motion of the unperturbed NLSM. Computing $\bar{D}X_c$ and using (2.5) to eliminate $\bar{D}\bar{F}$ and $\partial\bar{D}\bar{\Phi}$ terms, we obtain

$$\begin{aligned} \bar{D}X_c = & g_{i\bar{j}}\partial\bar{\Phi}^{\bar{j}}\bar{D}\bar{\Phi}^{\bar{k}}\bar{D}\bar{\Phi}^{\bar{l}}Z_{i,\bar{k}}^{\bar{l}} + \partial\Phi^i\bar{D}\bar{\Phi}^{\bar{k}}\bar{D}\bar{\Phi}^{\bar{l}}(Y_{i\bar{l},\bar{k}} - H_{j\bar{i}\bar{k}}Z_{\bar{l}}^j) \\ & + \bar{\Gamma}_\alpha\Gamma^\beta\bar{D}\bar{\Phi}^{\bar{k}}\bar{D}\bar{\Phi}^{\bar{l}}(\Lambda_{\beta\bar{l},\bar{k}}^\alpha - \mathcal{F}_{\beta\bar{k}i}^\alpha Z_{\bar{l}}^i). \end{aligned}$$

As there cannot be cancellations between the three terms, $\bar{D}X_c = 0$ requires

$$\begin{aligned} Z_{i,\bar{k}}^{\bar{l}} - Z_{\bar{k},i}^{\bar{l}} &= 0, \\ Y_{i\bar{l},\bar{k}} - Y_{i\bar{k},\bar{l}} &= Z_{\bar{k}}^j H_{j\bar{i}\bar{l}} - Z_{\bar{l}}^j H_{j\bar{i}\bar{k}}, \\ \Lambda_{\beta\bar{l},\bar{k}}^\alpha - \Lambda_{\beta\bar{k},\bar{l}}^\alpha &= \mathcal{F}_{\beta\bar{l}i}^\alpha Z_{\bar{k}}^i - \mathcal{F}_{\beta\bar{k}i}^\alpha Z_{\bar{l}}^i. \end{aligned} \quad (3.1)$$

Of course not all solutions to (3.1) correspond to distinct first order deformation of the SCFT – a good thing, since the solution space is infinite dimensional; instead, only certain equivalence classes of solutions correspond to deformations.

To identify the equivalence relations, we first consider another SCFT operator X' with classical limit $X'_c = X_c + \bar{D}W_c$ for some well-defined superfield W_c . If X' and X are distinct deformations of the theory, then their difference is a non-trivial deformation; however, the latter would be a marginal deformation given as an integral over the full $(0, 2)$ superspace. Since such deformations do not exist in the SCFT, we conclude that X and X' define isomorphic deformations of the theory. Conversely, if a classical chiral superfield X_c corresponds to a chiral primary operator X in the SCFT, then $X_c + \bar{D}W_c$ must correspond to the same first order deformation.

Thus, to count the first order deformations in the classical limit, we must consider chiral superfields X_c modulo the equivalence relation $X_c \sim X_c + \bar{D}W_c$. In fact, there is another manner in which we can shift X_c without affecting the deformation: $X_c \rightarrow X_c + \partial W'_c$ for some chiral superfield $\partial W'_c$ leaves ΔS invariant. As we will see, this additional equivalence will be trivial in most cases of interest. So, to summarize, in the classical limit we expect the first order deformations to correspond to X_c that solve (3.1), modulo the equivalence relation

$$X_c \sim X_c + \bar{D}W_c + \partial W'_c, \quad \bar{D}\partial W' = 0.$$

It is not difficult to make the equivalence more explicit – we simply need to expand W_c and W'_c in terms of the component fields. Since we will now just work with the classical NLSM Lagrangian, we will drop the c subscripts on the fields. Dimensional analysis and the $U(1)_L \times U(1)_R$ symmetry constrain W and W' to be

$$W = \bar{\Gamma}_\alpha\Gamma^\beta\lambda_\beta^\alpha + \partial\Phi^i\mu_i + \partial\bar{\Phi}^{\bar{i}}g_{i\bar{i}}\zeta^{\bar{i}}, \quad W' = \bar{D}\bar{\Phi}^{\bar{i}}\xi_{\bar{i}},$$

where

$$\lambda \in \Gamma(\text{End } E), \quad \mu \in \Omega_M^{1,0}, \quad \zeta \in \Gamma(TM), \quad \xi \in \Omega_M^{0,1}.$$

$\partial W'$ will be chiral up to the NLSM equations of motion provided that $\xi_{\bar{i}}$ satisfies

$$\begin{aligned} \nabla_{\bar{k}}^-\xi_{[\bar{l},\bar{j}]} &= 0, \quad \nabla_{\bar{k}}^-\xi_{[\bar{l},\bar{j}]} = 0, \\ g^{\bar{j}\bar{i}}(\mathcal{F}_{\beta\bar{m}\bar{i}}^\alpha\xi_{[\bar{l},\bar{j}]} - \mathcal{F}_{\beta\bar{i}\bar{i}}^\alpha\xi_{[\bar{l},\bar{j}]}) &= 0, \end{aligned}$$

where the ∇^- connection is defined with the twisted connection Ω^- given in (2.4). These conditions are solved by any $\bar{\partial}$ -closed ξ ;

this is the most general solution for an $SU(3)$ structure target-space (see, e.g. [16]). Otherwise $\bar{\partial}\xi$ would be a non-trivial ∇^- -constant form, in addition to the Hermitian form and the $(3, 0)$ form that define the $SU(3)$ structure; this would lead to a further reduction of structure. Thus, we must have $\xi \in H_{\bar{\partial}}^{0,1}(M)$. It is easy to see that when ξ is cohomologically trivial, it can be eliminated by redefining μ and ζ .

Expanding out $\bar{D}W + \partial W'$, we find the equivalence relation on (Λ, Y, Z) :

$$\begin{aligned} Z_{\bar{l}}^i &\sim Z_{\bar{l}}^i + (\zeta^i + g^{i\bar{j}}\xi_{\bar{j}})_{,\bar{l}} + g^{i\bar{k}}(\xi_{\bar{l},\bar{k}} - \xi_{\bar{k},\bar{l}}), \\ Y_{\bar{i}\bar{l}} &\sim Y_{\bar{i}\bar{l}} + \mu_{i,\bar{l}} + \xi_{\bar{l},i} + H_{\bar{i}\bar{l}j}(Z^j + g^{j\bar{k}}\xi_{\bar{k}}), \\ \Lambda_{\beta\bar{l}}^\alpha &\sim \Lambda_{\beta\bar{l}}^\alpha + \lambda_{\beta,\bar{l}}^\alpha - \mathcal{F}_{\beta\bar{l}i}^\alpha(\zeta^i + g^{i\bar{j}}\xi_{\bar{j}}). \end{aligned} \quad (3.2)$$

Eqs. (3.1) and (3.2) constitute our main result: in a large radius limit the G -neutral first order deformations of a supersymmetric heterotic vacuum correspond to solutions of (3.1) modulo the equivalence relations in (3.2). The Z , Y and Λ are familiar from the textbook treatment of (2, 2) compactifications and their deformations. For instance, setting the right-hand sides of (3.1) to zero, we see that (Z, Y, Λ) define cohomology classes

$$Z \in H^1(M, T_M), \quad Y \in H^1(M, T_M^*), \quad \Lambda \in H^1(M, \text{End } T_M).$$

However, the non-trivial right-hand sides indicate that for generic $(0, 2)$ theories the notion of splitting the deformations into “complex structure, Kähler, and bundle” is misleading.

4. Examples

Having obtained the general conditions, we can now check that they lead to the expected structure in familiar limits of (2, 2) theories and more general Calabi–Yau compactifications. Having verified this, we will be in a better position to discuss the implications for the general heterotic flux compactification.

4.1. The (2, 2) locus

On the $(2, 2)$ locus, $E = T_M$, $H = 0$ and g is Kähler. Moreover, since $H_{\bar{\partial}}^{0,1}(M)$ is trivial (we assume M has the full $SU(3)$ holonomy), we can set $\xi = 0$ without loss of generality. Thus, the equations reduce to

$$\begin{aligned} \bar{\partial}Z &= 0, \quad Z_{\bar{l}}^i \sim Z_{\bar{l}}^i + \zeta_{,\bar{l}}^i; \\ \bar{\partial}Y &= 0, \quad Y_{\bar{i}\bar{l}} \sim Y_{\bar{i}\bar{l}} + \mu_{i,\bar{l}}; \\ \Lambda_{\bar{n}\bar{l},\bar{k}}^m - \Lambda_{\bar{n}\bar{k},\bar{l}}^m &= R_{\bar{n}\bar{k}i}^m Z_{\bar{l}}^i - R_{\bar{n}\bar{l}i}^m Z_{\bar{k}}^i, \\ \Lambda_{\bar{n}\bar{l}}^m &\sim \Lambda_{\bar{n}\bar{l}}^m + \lambda_{\bar{n},\bar{l}}^m - R_{\bar{n}\bar{l}i}^m \zeta^i, \end{aligned} \quad (4.1)$$

where $R_{\bar{m}\bar{n}\bar{k}i}$ is the Riemann tensor for the Kähler metric g .

As expected, deformations correspond to $Z \in H^1(M, T_M)$, $Y \in H^1(M, T_M^*)$; however, the conditions on Λ still appear a little bit puzzling. The puzzle is easily resolved. Let

$$\tilde{\Lambda}_{\bar{n}\bar{l}}^m \equiv \Lambda_{\bar{n}\bar{l}}^m - \nabla_{\bar{n}}Z_{\bar{l}}^m, \quad \tilde{\lambda}_{\bar{n}}^m \equiv \lambda_{\bar{n}}^m - \nabla_{\bar{n}}\zeta^m.$$

Recasting the last line of (4.1) in terms of $\tilde{\Lambda}$ and $\tilde{\lambda}$, we obtain

$$\tilde{\Lambda}_{\bar{n}\bar{l},\bar{k}}^m - \tilde{\Lambda}_{\bar{n}\bar{k},\bar{l}}^m = R_{\bar{n}\bar{k}i}^m Z_{\bar{l}}^i - R_{\bar{n}\bar{l}i}^m Z_{\bar{k}}^i - (\nabla_{\bar{n}}Z_{\bar{l}}^m)_{,\bar{k}} + (\nabla_{\bar{n}}Z_{\bar{k}}^m)_{,\bar{l}}, \quad (4.2)$$

$$\tilde{\Lambda}_{\bar{n}\bar{l}}^m \sim \tilde{\Lambda}_{\bar{n}\bar{l}}^m + \tilde{\lambda}_{\bar{n},\bar{l}}^m + g^{m\bar{m}}[\nabla_{\bar{l}}\nabla_{\bar{n}}\zeta_{\bar{m}} - \nabla_{\bar{n}}\nabla_{\bar{l}}\zeta_{\bar{m}} - R_{\bar{m}\bar{n}\bar{l}i}\zeta^i]. \quad (4.3)$$

The square bracket in (4.3) is

$$[\nabla_{\bar{l}}, \nabla_{\bar{n}}]\zeta_{\bar{m}} - R_{\bar{m}\bar{n}\bar{l}i}\zeta^i = (R_{\bar{l}\bar{n}\bar{m}i} - R_{\bar{m}\bar{n}\bar{l}i})\zeta^i = 0,$$

where the last equality follows from the symmetry $R_{i\bar{m}\bar{n}i} = R_{\bar{m}\bar{n}i}$ enjoyed by the Riemann tensor for a Kähler metric. The vanishing of the right-hand side of (4.2) follows from similar manipulations and $\bar{\partial}Z = 0$. Thus, in terms of the $\tilde{\Lambda}$ and $\tilde{\lambda}$ variables, we recover the expected result:

$$\bar{\partial}\tilde{\Lambda} = 0, \quad \tilde{\Lambda} \sim \tilde{\Lambda} + \bar{\partial}\tilde{\lambda}.$$

The first order deformations for a (2, 2) compactification do have the canonical split

$$(Z, Y, \tilde{\Lambda}) \in H^1(M, T_M) \oplus H^1(M, T_M^*) \oplus H^1(M, \text{End } T_M).$$

4.2. Calabi–Yau compactifications

A more generic (0, 2) vacuum is obtained by taking $E \rightarrow M$ to be a stable holomorphic bundle over a (conformally) Calabi–Yau manifold. In this case, the deformation space still has a familiar description. Working at tree-level we still have $H = 0$, and as in the (2, 2) case ξ must be $\bar{\partial}$ -exact and hence can be absorbed into ζ and μ . Thus, $Z \in H^1(M, T_M)$, $Y \in H^1(M, T_M^*)$, and the remaining non-trivial condition is

$$\Lambda_{\beta\bar{i},\bar{k}}^\alpha - \Lambda_{\beta\bar{k},\bar{i}}^\alpha = \mathcal{F}_{\beta\bar{k}i}^\alpha Z_{\bar{i}}^i - \mathcal{F}_{\beta\bar{i}i}^\alpha Z_{\bar{k}}^i.$$

Since $Z \in H^1(M, T_M)$ and \mathcal{F} is the (1, 1) curvature for the holomorphic connection, the right-hand side defines a class in $H^2(M, \text{End } E)$. If this class is trivial, then the equation can be solved for Λ ; otherwise, the deformation is obstructed. As discussed at length in [6], this is encoded in a long exact sequence in cohomology [17], associated to the short exact sequence

$$0 \longrightarrow E \otimes E^* \longrightarrow Q \xrightarrow{\pi} T_M \longrightarrow 0, \\ \cdots \longrightarrow H^1(M, Q) \xrightarrow{d\pi} H^1(M, T_M) \xrightarrow{\alpha} H^2(M, E \otimes E^*) \longrightarrow \cdots,$$

where the map α is given by contracting $Z \in H^1(M, T_M)$ with \mathcal{F} .

4.3. Application to heterotic flux vacua

More generally, we hope to apply our results to heterotic compactifications on non-Kähler manifolds. These backgrounds are characterized by a tree-level H background, the most studied examples being T^2 bundles over K3 [18–20]. The NLSM α' expansion is rather formal for these backgrounds, as they generically contain string-scale cycles. However, to the extent to which an α' expansion can be used, our tree-level analysis describes the infinitesimal moduli of heterotic flux vacua. The qualitative structure is quite sensible: for instance, the deformations of the complexified Hermitian form (the $Y_{i\bar{i}}$) now have a non-trivial mixing with the complex structure deformations, and the “breathing mode,” corresponding to taking Y proportional to the Hermitian form appears to be obstructed.

It would be useful to clarify the geometry behind (3.1) and (3.2). For instance, is it possible to prove that the space of these first order deformations is finite dimensional for a smooth and compact flux background? Do $SU(3)$ structure examples admit non-trivial ξ equivalences?⁶ How is this presentation of deformations related to the infinitesimal perturbations of solutions to the one-loop supergravity equations examined in [5]?

⁶ Examples with extended spacetime supersymmetry and hence reduced structure certainly possess non-trivial ξ , see e.g. [21].

5. Concluding remarks

We have carried out the tree-level analysis of gauge-neutral massless scalars in a perturbative heterotic vacuum based on a (0, 2) NLSM. Of course this is a far cry from providing a complete analysis of even first order deformations, let alone a picture of the (0, 2) moduli space, and it is worthwhile to review the limitations of our results.

First, our analysis has been carried out for compactifications based on $SU(n)$ bundles over M – this is the source of the $U(1)_L$ symmetry of the internal theory. While this covers many vacua, it is certainly not the most general situation, and there are certainly interesting compactifications based on $U(n)$ bundles, as well as more general constructions, e.g. [22,23]. Second, while it is natural (even technically so) to restrict to gauge-neutral scalars, at least as far as the string perturbative limit is concerned, the Higgs deformations where G is broken to some sub-group should be considered on par with the neutral scalars we described. Fortunately, at least the massless charged spectrum has already been described in [11].

Modifications are also expected in going beyond tree-level in the α' expansion. In heterotic Calabi–Yau compactifications the possible lifting of states is constrained by the axionic symmetries associated to shifts of the NLSM B -field [2,3]; in more general heterotic flux compactifications analogous constraints are not well understood. At any rate, we certainly expect additional G -neutral massless scalars associated to stringy enhanced symmetries, as well as lifting of states by world-sheet non-perturbative effects.⁷

Although the general structure of deformations is complicated, since our analysis is just a simple application of (0, 2) supersymmetry, it should be a good starting point for a systematic expansion in α' away from the large radius limit. For instance, it is reasonable to expect that at one loop in α' the conditions will be modified by replacing H with its gauge-invariant form. It would be interesting to see whether this expectation is borne out and to attempt to extend it to an all orders result.

Other fruitful directions include applying these results to heterotic vacua with extended spacetime supersymmetry (their NLSM description has been recently explored in [26]), as well relating them to gauged linear sigma model constructions. The latter would be especially interesting for the linear sigma models appropriate for flux backgrounds [27–29].

Acknowledgements

IVM would like to thank Ido Adam and Ronen Plesser for a fruitful (but as yet unpublished) collaboration on aspects of (0, 2) SCFT that have been very useful for the present work. Some of this work was completed while IVM was visiting the Virginia Tech Center for Neutrino Physics, and he would like to thank the group for their hospitality. The work of ES is partially supported by NSF grants PHY-0755614 and PHY-1068725.

References

- [1] T. Banks, L.J. Dixon, D. Friedan, E.J. Martinec, Nucl. Phys. B 299 (1988) 613.
- [2] M. Green, J. Schwarz, E. Witten, Superstring Theory, Vol. 2, Cambridge University Press, 1987.
- [3] J. Polchinski, String Theory, Vol. 2, Cambridge University Press, Cambridge, UK, 1998.
- [4] R. Donagi, M. Wijnholt, Higgs bundles and UV completion in F-theory, arXiv:0904.1218 [hep-th].

⁷ Examples of these phenomena have recently been investigated in Landau–Ginzburg vacua [24,25].

- [5] M. Becker, L.-S. Tseng, S.-T. Yau, *Nucl. Phys. B* 786 (2007) 119, arXiv:hep-th/0612290.
- [6] L.B. Anderson, J. Gray, A. Lukas, B. Ovrut, The Atiyah class and complex structure stabilization in heterotic Calabi–Yau compactifications, arXiv:1107.5076 [hep-th].
- [7] D. Gepner, *Phys. Lett. B* 199 (1987) 380.
- [8] J. Distler, Notes on $(0, 2)$ superconformal field theories, arXiv:hep-th/9502012.
- [9] A. Strominger, *Nucl. Phys. B* 274 (1986) 253.
- [10] R. Rohm, E. Witten, *Annals Phys.* 170 (1986) 454.
- [11] J. Distler, B.R. Greene, *Nucl. Phys. B* 304 (1988) 1.
- [12] D. Green, Z. Komargodski, N. Seiberg, Y. Tachikawa, B. Wecht, *JHEP* 1006 (2010) 106, arXiv:1005.3546 [hep-th].
- [13] I. Adam, I.V. Melnikov, M.R. Plesser, On marginal deformations of $(0, 2)$ SCFTs, unpublished.
- [14] C. Beasley, E. Witten, *JHEP* 0501 (2005) 056, arXiv:hep-th/0409149.
- [15] C. Beasley, E. Witten, *JHEP* 0602 (2006) 060, arXiv:hep-th/0512039.
- [16] M. Graña, *Phys. Rept.* 423 (2006) 91, arXiv:hep-th/0509003.
- [17] M.F. Atiyah, *Trans. AMS* 85 (1) (1957) 181.
- [18] K. Dasgupta, G. Rajesh, S. Sethi, *JHEP* 9908 (1999) 023, arXiv:hep-th/9908088.
- [19] K. Becker, M. Becker, J.-X. Fu, L.-S. Tseng, S.-T. Yau, *Nucl. Phys. B* 751 (2006) 108, arXiv:hep-th/0604137.
- [20] K. Becker, S. Sethi, *Nucl. Phys. B* 820 (2009) 1, arXiv:0903.3769 [hep-th].
- [21] E. Goldstein, S. Prokushkin, *Commun. Math. Phys.* 251 (2004) 65, arXiv:hep-th/0212307.
- [22] R. Blumenhagen, G. Honecker, T. Weigand, *JHEP* 0506 (2005) 020, arXiv:hep-th/0504232.
- [23] J. Distler, E. Sharpe, *Adv. Theor. Math. Phys.* 14 (2010) 335, arXiv:hep-th/0701244.
- [24] P.S. Aspinwall, I.V. Melnikov, M.R. Plesser, $(0, 2)$ elephants, arXiv:1008.2156 [hep-th].
- [25] P.S. Aspinwall, M.R. Plesser, Elusive worldsheet instantons in heterotic string compactifications, arXiv:1106.2998 [hep-th].
- [26] I.V. Melnikov, R. Minasian, *JHEP* 1109 (2011) 065, arXiv:1010.5365 [hep-th].
- [27] A. Adams, M. Ernbjerg, J.M. Lapan, Linear models for flux vacua, arXiv:hep-th/0611084.
- [28] M. Blaszczyk, S. Groot Nibbelink, F. Ruehle, Green–Schwarz mechanism in heterotic $(2, 0)$ linear sigma models, arXiv:1107.0320.
- [29] C. Quigley, S. Sethi, Linear sigma models with torsion, arXiv:1107.0714.