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Planck Scale Boundary Conditions

And

The Standard Model

This diploma thesis has been carried out by

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at the

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under the supervision of

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Diplomarbeit

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vorgelegt von

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Randbedingungen der Planck-Skala

und das Standardmodell

Die Diplomarbeit wurde von

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Randbedingungen der Planck-Skala und das Standardmodell

Das Standardmodell (SM) der Elementarteilchenphysik könnte eine effektive Quantenfeldtheorie (QFT) bis zur Planck-Skala sein. In dieser Arbeit wird diese Situation angenommen. Wir untersuchen, ob die Physik der Planck-Skala Spuren in Form von Randbedingungen des SMs hinterlassen haben könnte. Zuerst argumentieren wir, dass das SM-Higgs-Boson kein Hierarchieproblem haben könnte, wenn die Physik der Planck-Skala aus einem neuen, nicht feld-theoretischen Konzept bestehen würde. Der Higgs-Sektor wird bezüglich der theoretischer und experimenteller Einschränkungen analysiert. Die notwendigen mathematischen Methoden aus der QFT, wie z.B. die Renormierungsgruppe, werden eingeführt um damit das Laufen der Higgs-Kopplung von der Planck-Skala zur elektroschwachen Skala zu untersuchen. Einige physikalisch motivierte Randbedingungen der Higgs-Kopplung werden implementiert, um eine Vorhersage der Higgs-Masse zu erhalten. Die notwendigen Anpassungsbedingungen zwischen der Higgs-Kopplung und der Higgs-Masse werden angewandt, und wir analysieren die experimentallen und theoretischen Fehler für die Vorhersagen der Higgs-Masse. Zum Schluss diskutieren wir Möglichkeiten um verschiedenen Randbedingungen am LHC zu unterscheiden sowie die Gültigkeit des SMs bis zur Planck-Skala.

Planck scale boundary conditions and the Standard Model

The Standard Model (SM) of particle physics could be an effective quantum field theory (QFT) valid up to the Planck scale. In this thesis we assume that this scenario is true, and investigate the remnant of boundary conditions on the Higgs quartic coupling left by Planck scale physics. We will first argue that the SM Higgs might not suffer from a hierarchy problem if the Planck scale physics consists of a new physical concept which is non-field theoretic. The Higgs sector will then be analyzed, both from experimental searches and theoretical constraints. Necessary QFT mathematical tools such as renormalization group equation (RGE) will be introduced in order to investigate the running of the Higgs quartic coupling from Planck scale to the electroweak scale. We will then impose various physically motivated boundary conditions on the Higgs quartic coupling, and investigate their Higgs mass prediction. Proper matching of the Higgs quartic coupling to the Higgs mass will be applied. The experimental error and theoretical uncertainties will be analyzed for our Higgs mass prediction due to Planck scale boundary conditions. Prospects of the LHC experiment to differentiate different boundary conditions will be discussed and subsequently the validity of the SM up to the Planck scale will be analyzed.

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Contents

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Heidelberg, winter 2011 — Kher Sham Lim

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Chapter 1

Introduction

1.1 Motivation

Since ancient times, man has always asked the fundamental question: What does the universe consist of? This question has long been tackled by philosophers and scientists. From the discovery of classical mechanics to quantum theory and general relativity, physicists have pieced up the solution to this puzzle bit by bit. After years of progress, both from theoretical perspective and the breakthrough of experiments, we have come to a phase where we can now ask, how far are we in understanding our universe completely? The Standard Model (SM) of particle physics for instance, which consists of the Glashow-Weinberg-Salam theory of electroweak interaction [1, 2] and Quantum Chromodynamics (QCD) [3, 4, 5], has served as a successful benchmark for describing the subatomic physics for the last forty years. The SM has produced remarkable results in precision beyond the tree level computation, i.e. it allows the quantum effects to be probed. However, we know from a different perspective, whether it is from observation, experimental results or aesthetic point of view, that the SM is necessary to be extended. From cosmological observations, extra ingredients beyond the SM (BSM) are needed to explain the nature of dark matter, expansion of the universe and the early universe physics. A simple argument from the particle physicist's point of view on the need to extend the SM, is the observation of neutrino mass and oscillation.

From the collider physics perspective, to date the SM is confirmed to a high degree of precision. The Large Hadron Collider (LHC) has started to collect data since the year 2009. With the center-of-mass energy $\sqrt{s} = 7\,\mathrm{TeV}$ and integrated luminosities of more than $1\,\mathrm{fb}^{-1}$, there is no clear hint on the signatures of BSM physics to date. The SM consists of the gauge bosons and fermions, transforming in accordance to the adjoint and the fundamental representation of SM gauge group $SU_{\mathrm{C}}(3)\times SU_{\mathrm{L}}(2)\times U_{\mathrm{Y}}(1)$

respectively. So far all the ingredients of the SM have been found, except for the part that is responsible for the Electroweak Symmetry Breaking (EWSB), namely the Higgs boson. In the next couple of years, the ATLAS and CMS detector at the LHC should be able to rule out or discover the Higgs boson. With no sign of BSM physics so far in the LHC and a large portion of the allowed SM Higgs mass region being ruled out [6, 7, 8], one is prompted to think about the status of the SM (with the extension of neutrino mass) with no new extra physics. That is, *could the SM be an effective theory up to the Planck scale?*

From the traditional aesthetic point of view, the SM cannot be a complete theory. This is because the Higgs boson, being the only fundamental scalar postulated in nature, does not have any symmetry to protect its mass value from being destabilized by a heavy particle, possibly dictated by a new physics scale. The SM is a renormalizable quantum field theory (QFT) on itself, however when it is extended by a new physics in the framework of QFT, it is not possible to understand how the electroweak scale could be many orders of magnitude lighter than the scale of an embedding QFT. Conventional solutions of the hierarchy problem stay within QFT. One solution is to postulate a new symmetry such as supersymmetry, which cancels the problematic quadratic divergences. Alternatively the scalar sector may be considered as an effective condensate such that form factors remove large quadratic divergences. Another idea is that the Higgs particle could be a pseudo-Goldstone-boson such that it is naturally somewhat lighter than a scale where richer physics exists, or we can saturate the Planck scale near the TeV scale with extra dimensions. However, none of these ideas has shown up in experiments as yet [9].

Could the SM be an effective theory up to an arbitrary energy scale? This view is untenable, as we know that the SM is only a theory without the interaction of gravity being taken into account. So far, the general theory of relativity is not a viable field theory to be quantized, as general relativity itself is not renormalizable. One could speculate that the new physics, combining the SM and gravity interaction, could be a new physical concept that is non-field theoretic. From the renormalizable theory point of view, this might look like a drawback, however we have to keep the disadvantage of embedding a QFT in a chain of larger theory in mind. First, QFT does not predict any absolute mass and coupling for a given theory. Embedding the SM into another larger group will only shift the problem to another scale, but not solve it. The problems of gravity may be a sign of physics based on new concepts which may ultimately allow to determine absolute masses, mixing and couplings. There is no need for the SM to be directly embedded into gravity as various layers of conventional gauge theories, such as grand unified theory (GUT) or left-right symmetry could be in-between. However with such a chain of embedment, the Higgs boson will usually suffer from fine-tuning problem. The second reason why an embedding into a new concept beyond renormalizable QFTs might be good is that it might offer new solutions to the hierarchy problem. The point is that the unknown new physics may allow for mechanisms which stabilize a low-lying effective QFT from the perspective of the Planck scale. From the perspective of the low-lying effective QFT this may then appear to be a hierarchy problem if one tries to embed the SM into a renormalizable QFT instead of the theory which is based on the new, extended concepts.

The above considerations prompt us to speculate that the SM might be valid up to the Planck scale, where it is embedded directly into new concepts without any intermediate energy scale. The new concepts behind the Planck scale physics might then offer a solution to the hierarchy problem which is no longer visible when one looks at the SM only. The only way how the SM would "know" about such an embedding could be special boundary conditions similar to compositeness conditions or auxiliary field conditions in theories where redundant degrees of freedom are eliminated in embeddings. In fact, this embedment of the SM QFT into a completely new physical concept is not a new idea after all. In the theory of superconductivity for instance, the macroscopic superconducting phenomenon can be described by the Ginzburg-Landau theory [10], where the energy of a superconductor is given by:

$$E \approx \alpha|\Phi|^2 + \beta|\Phi|^4 + \ldots \tag{1.1}$$

The phenomenological parameter α and β in the framework of Ginzburg-Landau theory itself has to be determined by experiment. However, the underlying microscopic physics of superconductivity, namely the BCS theory [11], can determine these phenomenological parameters from the microscopic theory. In this sense, the phenomenological parameters in Ginzburg-landau theory "know" the boundary condition set by the microscopic theory[1].

If we assume that the SM is a valid effective QFT up to the Planck scale, in analogy to the superconducting physics mentioned above, it would require the full new underlying microscopic quantum gravity to determine the Higgs mass and all the physical couplings. The full quantum gravity theory could in principle dictate all the values of SM coupling at Planck scale. These couplings then run to the electroweak scale and yield the spectrum of low energy effective field theory. This scenario is roughly depicted in Fig. (1.1). To

[1]See the derivation of Ginzburg-Landau theory from BCS theory by Gorkov [12].

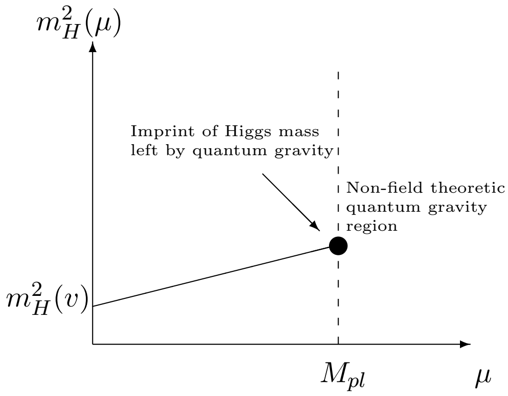


Figure 1.1: The SM Higgs mass is determined and fixed by unknown quantum gravity. The running of the Higgs mass from the Planck scale down to the electroweak scale is fully dictated by the SM coupling.

construct such a complete theory is not an easy task, and to date there is no satisfying model[2] that can yield accurate prediction for the SM parameters. All the couplings and masses from the SM are experimentally known by now, except the Higgs mass, which is related to the Higgs quartic coupling. So far, there is no sign of new physics appearing in the LHC, except for a 2σ deviation at the low Higgs mass region that might indicate the existence of the SM Higgs boson [8]. Therefore, the spirit of this thesis is to assume that the SM is an effective QFT up to the Planck scale, without any intermediate energy scale. To specify the physically motivated boundary conditions on the Higgs quartic coupling at the Planck scale, left as an imprint from the complete theory of quantum gravity, is the main task of this work. Some phenomenological implications on Higgs searches due to certain boundary conditions will subsequently be discussed.

1.2 Outline

This thesis is organized as follows: First we will review the concept of regularization and renormalization in Sec. (2), introducing the necessary tools such as the beta function and the concept of running coupling, which is needed for Higgs mass prediction due to some boundary conditions imposed for the Higgs quartic coupling. The SM Lagrangian and the hierarchy problem will then be reviewed in Sec. (3). Some of the phenomena that require the extension of the SM will be discussed in the subsequent section. We will argue that the existence of only two scales, namely the Fermi and the Planck scale,

[2]The anthropic principle in string theory and M-theory is not considered as a solution here.

is more advantageous than introducing an intermediate energy scale in QFT framework from the fine-tuning perspective. Once we have all the theoretical tools needed for our analysis, we will then review some facts concerning the SM Higgs boson, both from experimental searches and theoretical consideration, in Sec. (4). Two of the theoretical constraints, namely the triviality and vacuum stability bound, will be plotted by solving the renormalization group equation (RGE) numerically. We will then utilize the running couplings and RGE to investigate the Veltman condition, which is then extrapolated to the Planck scale to illustrate its overlap with the vacuum stability band of the Higgs potential. This intriguing scenario will let us to contemplate on the possible solution of hierarchy problem at Planck scale. In Sec. (5), we will turn this argument around and demand other boundary conditions imposed on the Higgs quartic coupling. The Higgs mass prediction due to different boundary conditions will be examined. The matching of the physical masses to the renormalized coupling in the minimal subtraction scheme with its uncertainty will be thoroughly discussed in between. At the end, we will discuss the fate of the SM up to the Planck scale, if LHC manages to find a Higgs mass with a specific low value.

1.3 Unit convention

For the rest of this work, we will adopt natural units, where the natural constants $\hbar = c = 1$. All the necessary values of physical quantities in natural units are given in Appx. (C). We adopt the Einstein summation convention in this work, i.e. repeating indices are summed over, unless stated otherwise.

Chapter 2

Renormalization Group Equation, Beta Function and Running Coupling

In this section we will review the concept of renormalization and renormalization group methods that we need in order to proceed with our analysis later. We will first introduce the notion of generating functional and the idea of an effective potential, which we will use later in one of the analysis. In order to convey our idea in a clear and simple way we will just discuss the concept of regularization and renormalization with the help of a real scalar field, i.e. ϕ^4-theory. This toy model is particularly useful as it resembles the Higgs sector in the SM. For generalizations to fermionic and gauge fields we refer the reader to textbook like Bailin & Love [13] or Cheng & Li [14]. We will then study the meaning of renormalization scale invariance and demonstrate the invariance of physical quantities with varying renormalization scale. We will show that under certain renormalization schemes, the coupling constant will not correspond to the physical quantity that we can measure in experiments. These quantities are easier to be computed in determining the running of couplings. However, in order to make physical predictions, matching of these quantities with physical couplings are necessary. We postpone the investigation of the matching conditions to Sec. (4.3) when higher precision in Higgs mass prediction is needed. Improved perturbation theory will be discussed after the introduction of the beta functions and the running of couplings.

2.1 Generating functional and Green's functions

We first introduce the concept of generating functionals and Green's functions. Consider a QFT with a generic field ϕ, its interaction being encoded in the Lagrangian $\mathcal{L}(\phi, \partial_\mu \phi)$. For simplicity we suppress the spacetime and spinor indices for vector and spinor fields.

From the path-integral formalism in QFT, the generating functional for the Green's function is given by:

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \exp\left(i \int \mathrm{d}^4x \mathcal{L}(\phi, \partial_\mu \phi) + J\phi\right), \tag{2.1}$$

where the term J is the source of the field ϕ. With the generating functional we can compute the Green's function:

$$G^n(x_1, \dots, x_n) = \langle 0|T\phi(x_1)\dots\phi(x_n)|0\rangle = \frac{(-i)^n}{\mathcal{Z}[J=0]} \frac{\delta^n \mathcal{Z}[J]}{\delta J(x_1)\dots\delta J(x_n)}\bigg|_{J=0}, \tag{2.2}$$

where the operator T stands for the time-ordering operator. The above generating functional generates a whole class of interactions without discriminating the topologically connected or disconnected one. Often we are only interested in the connecting part of the Green's function. Therefore it is useful to consider the connected-part generating functional:

$$\mathcal{W}[J] = -i \log \mathcal{Z}[J], \tag{2.3}$$

and the connected n-point function is generated via:

$$G^n_c(x_1, \dots, x_n) = (-i)^n \frac{\delta^n \mathcal{W}[J]}{\delta J(x_1)\dots\delta J(x_n)}\bigg|_{J=0}. \tag{2.4}$$

With the connected-part generating functional defined, we can now define an effective action, i.e. an action that incorporates quantum corrections, via Legendre transformation. The effective action is defined as:

$$\Gamma[\phi_c] = \mathcal{W}[J] - \int \mathrm{d}^4x J(x)\phi_c(x), \qquad \phi_c(x) := \frac{\delta \mathcal{W}[J]}{\delta J(x)}. \tag{2.5}$$

Taking the functional derivative w.r.t. the effective action, we obtain the *one particle irreducible* (1PI) as followed:

$$\Gamma^n(x_1, \dots, x_n) = (-i)^n \frac{\delta^n \Gamma[\phi_c]}{\delta\phi_c(x_1)\dots\delta\phi_c(x_n)}. \tag{2.6}$$

A 1PI diagram is a set of diagrams that cannot be subdivided into two disconnected diagrams by a cut on any internal propagator. What experiments measure is actually the connected Green's function generated together with the 1PI vertex, that one can usually associate with parameters of the theory. Since the 1PI diagrams are generators of the effective action, we can express $\Gamma[\phi_c]$ in terms of the 1PI vertex. By Fourier transformation we obtain:

$$\Gamma[\phi_c] = \sum_{n=0}^{\infty} \frac{1}{n!} \int \mathrm{d}p_1 \ldots \mathrm{d}p_n \delta^4(p_1 + \ldots + p_n) \Gamma^n(p_1, \ldots, p_n) \tilde{\phi}_c(p_1) \ldots \tilde{\phi}_c(p_n). \tag{2.7}$$

Suppose that we perform a derivative expansion of the effective action:

$$\Gamma[\phi_c] = \int \mathrm{d}^4x \left[-V_{\text{eff}}(\phi_c) + \frac{1}{2} \partial_\mu \phi_c \partial_\mu \phi_c Z(\phi_c) + \ldots \right] \tag{2.8}$$

and set the classical field $\phi_c(x)$ as a constant field, only the *effective potential* V_{eff} remains, as the derivative on constant field vanishes. Comparing this term with Eq. (2.7) in the limit of all vanishing external momenta, we obtain:

$$V_{\text{eff}}(\phi_c) = -\sum_{n=0}^{\infty} \frac{1}{n!} \phi_c^n \Gamma^n(p_i = 0). \tag{2.9}$$

The infinite sum of the 1PI vertices with no external momenta gives an effective potential, which we will use later in Sec. (5.1.1) to improve the Higgs mass bound due to vacuum stability. But for now, let us review the concept of renormalization and the running of coupling.

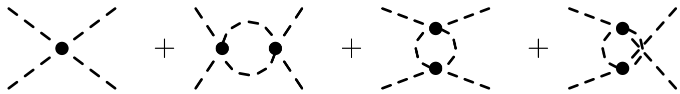
2.2 Concepts of renormalization

With a given Lagrangian one can compute the interaction of a theory, i.e. the 1PI vertex and compare it with the experiment dictated by the S-matrix. For simplicity we consider a real scalar field with a ϕ^4-interaction, i.e.

$$\mathcal{L}_B = \frac{1}{2} \partial_\mu \phi_B \partial^\mu \phi_B - \frac{1}{2} m_B^2 \phi_B^2 - \frac{1}{4!} \lambda_B \phi_B^4. \tag{2.10}$$

The subscript B, labelled for our field ϕ, mass term m and quartic coupling λ, stands for bare quantities, which we will explain later. For now we can think of the bare quantities as parameters that do not include quantum corrections. We will see later that these quantities are infinite and hence not observable, prompting us to apply renormalization to associate the couplings with observables from experiments. With the given Lagrangian we can calculate the quantum correction for the mass term m_B and quartic coupling λ_B via loop diagrams. The radiative correction for m_B will be discussed in Sec. (3.2). For now, we will just focus on computing the one-loop order of 1PI for a scattering process with initial four-momenta p_1, p_2 and final four-momenta p'_1, p'_2:

$$i\Gamma^4(p_1, p_2, p'_1, p'_2) =$$



$$= -i\lambda_B + V(s) + V(t) + V(u). \tag{2.11}$$

The term $V(p^2)$ defined as:

$$V(p^2) := \frac{(-i\lambda_B)^2}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{i}{k^2 - m_B^2} \frac{i}{(p+k)^2 - m_B^2} \tag{2.12}$$

and s, t and u are the Mandelstam variables:

$$s = (p_1 + p_2)^2,\ t = (p_1 - p'_1)^2,\ u = (p_1 - p'_2)^2. \tag{2.13}$$

The integral above is logarithmically divergent. One simple way to see this is to Wick-rotate the intergal to Euclidean space:

$$k^0 := ik_E^0, \quad \boldsymbol{k} := \boldsymbol{k}_E \tag{2.14}$$

and perform the integration:

$$\begin{aligned}
V(p^2) &= -\frac{(-i\lambda_B)^2}{2} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m_B^2} \frac{1}{(p+k)^2 - m_B^2} \\
&= -\frac{(-i\lambda_B)^2}{2} \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{1}{(l^2 + x(1-x)p^2 - m_B^2)^2} \qquad (l^2 = k + xp) \\
&= \frac{i\lambda_B^2}{2} \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^4 l_E}{(2\pi)^4} \frac{1}{(l_E^2 - x(1-x)p^2 + m_B^2)^2} \qquad (l^0 = il_E^0) \qquad (2.15a) \\
&= \frac{i\lambda_B^2}{32\pi^2} \int_0^1 \mathrm{d}x \left[\frac{m^2 - x(1-x)p^2}{m^2 - x(1-x)p^2 + l_E^2} + \log(l_E^2 + m^2 - x(1-x)p^2) \right] \Bigg|_0^\infty . \qquad (2.15b)
\end{aligned}$$

Clearly $V(p^2)$ diverges logarithmically as we take the limit of l_E to infinity, yielding a non-sensible result for our four-point scattering amplitude when we take quantum effects into account. This divergence occurs because we are using unrenormalized quantities in our loop calculation, i.e. bare quantities which do not include radiative corrections, in computing our scattering amplitude. In real life we want to compute cross sections and other observable quantities as a function of renormalized quantities and compare them with the experimental results, which are finite and measurable. Therefore it is necessary to obtain these renormalized quantities and such procedure goes under the name of regularization and renormalization. First we will discuss the regularization methods in the following.

2.3 Regularization

It is imperative to regularize our integral in Eq. (2.15a) before we proceed to renormalization. In this thesis we will discuss two regularization methods which we will need later in analyzing the divergence for scalar mass parameter and the dimensionless couplings. The first regularization method is by imposing an *ultraviolet cut-off* on the upper integral limit in Eq. (2.15b). After some algebraic simplifications we obtain:

$$\begin{aligned} i\Gamma^4(s,t,u) = -i\lambda_B + \frac{i\lambda_B^2}{32\pi^2}\int_0^1 \mathrm{d}x \Bigg[\log\left(\frac{\Lambda^2}{m_B^2 - sx(1-x)}\right) \\ + \log\left(\frac{\Lambda^2}{m_B^2 - tx(1-x)}\right) + \log\left(\frac{\Lambda^2}{m_B^2 - ux(1-x)}\right) - 3\Bigg], \end{aligned} \tag{2.16}$$

with the ultraviolet cut-off Λ, a scale that quantifies the breakdown of our effective theory. Clearly we have dropped terms that decay like $1/\Lambda^2$ as Λ is approaching infinity. Our integral is trivially regularized but the disadvantage of utilizing an ultraviolet cut-off is that Lorentz symmetry and local gauge invariance are not *manifestly* preserved. In our simple ϕ^4 example, the theory does not have gauge symmetries, however the SM contains gauge fields and we would have trouble maintaining the gauge symmetries should we use the cut-off regularization. A hard cut-off on the radial direction of a four-dimensional sphere breaks Lorentz invariance. In order to prevent the loss of gauge and spacetime symmetry, we can introduce another regularization method, the so-called *dimensional regularization* [15].

Dimension regularization has the advantage of preserving all symmetries of the theory and it is very convenient to apply. The idea is simple, we extend the integral measure to a d-dimensional measure, generalizing spacetime to d dimensions. However we need to keep the Dirac gamma matrices four dimensional in order to obtain a consistent regularization. After performing certain integral tricks, the divergence of our integral will appear as pole in four dimension of the Laurent series, and because of this, certain divergences will not show up in dimensional regularization. Later in Sec. (3.2), we will calculate the radiative correction for our scalar particle mass and we will see that the so-called quadratical divergence will not appear in dimensional regularization. The quadratic divergence appears as a pole only if we reduce the integral dimension to two.

We will come to this later in Sec. (3.2.1).

In our example, we first generalize the spacetime dimensions of Eq. (2.15a) to d dimensions:

$$V(p^2) = \frac{i(\mu^\epsilon \lambda_B)^2}{2} \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 - x(1-x)p^2 + m_B^2)^2}, \tag{2.17}$$

where we have added an appropriate power of arbitrary energy scale μ to λ_B in order to keep λ_B dimensionless in d-dimensional spacetime,

$$\lambda_B \to \mu^\epsilon \lambda_B. \tag{2.18}$$

The term ϵ is defined as a small deviation of the spacetime dimension from four, i.e.

$$\epsilon := 4 - d. \tag{2.19}$$

By regularizing the integral in Eq. (2.17) with the trick:

$$\int \frac{\mathrm{d}^d l_E}{(2\pi)^d} \frac{1}{(l_E^2 + \Delta)^2} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma(2 - d/2)}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-d/2}, \tag{2.20}$$

we obtain the following result:

$$\begin{aligned} V(p^2) &= \frac{i(\mu^\epsilon \lambda_B)^2}{2} \int_0^1 \mathrm{d}x \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \frac{1}{(m_B^2 - x(1-x)p^2)^{2-d/2}} \\ &= \frac{i(\mu^2)^{2-d/2}\lambda_B^2}{2} \int_0^1 \mathrm{d}x \frac{\Gamma(2-\frac{d}{2})}{(4\pi)^{d/2}} \left(\frac{\mu^2}{m_B^2 - x(1-x)p^2}\right)^{2-d/2} \\ &\xrightarrow[d\to 4]{} \frac{i\lambda_B^2}{32\pi^2} \int_0^1 \mathrm{d}x \left[\frac{2}{\epsilon} - \gamma + \log(4\pi\mu^2) - \log(m_B^2 - x(1-x)p^2)\right] \\ &= \frac{i\lambda_B^2}{32\pi^2} \int_0^1 \mathrm{d}x \left[\frac{2}{\epsilon} - \gamma + \log\left(\frac{4\pi\mu^2}{m_B^2 - x(1-x)p^2}\right)\right], \end{aligned} \tag{2.21}$$

where $\gamma = 0.577\ldots$ is the Euler-Mascheroni constant. The appearance of an arbitrary mass scale μ might seem worrisome at first, as we started off with ϕ^4-theory with only one dimensionful parameter m and through dimensional regularization we obtained another parameter μ with mass dimension one. It turns out that μ is just a mathematical parameter which we have to carry along when doing our regularization and renormalization. It has no direct physical meaning other than fixing our renormalization scale, i.e. the energy scale where we perform our perturbation calculation for our theory. It is crucial to understand that we can choose our renormalization scale at will in order to improve our perturbation series. We will come to that later in Sec. (2.5.1).

2.4 Renormalization

As mentioned above all the parameters and field content of our original Lagrangian (2.10) are bare quantities which diverge. The basic idea of renormalization is to remove these divergences by applying a certain renormalization scheme, i.e. a prescription on how the divergent and also finite parts of the bare quantities are systematically subtracted. In general we first define a relation between bare and renormalized quantities λ, m and ϕ as follows:

$$\phi_B = Z^{1/2}\phi, \quad \delta Z := Z - 1, \tag{2.22a}$$

$$m_B^2 = Z^{-1}(m^2 + \delta m^2), \tag{2.22b}$$

$$\lambda_B = \mu^\epsilon Z^{-2} Z_\lambda \lambda = Z'_\lambda \lambda, \quad \delta\lambda := Z_\lambda - 1, \tag{2.22c}$$

with the wave function renormalization constant Z. Quantum effects for our parameters and field are encoded in δm^2, $\delta\lambda$ and δZ respectively. By plugging the bare quantities in terms of the renormalized ones in our original Lagrangian, our theory in four dimension can be split in two parts:

$$\mathcal{L}_B = \mathcal{L} + \delta\mathcal{L}, \tag{2.23a}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4, \tag{2.23b}$$

$$\delta\mathcal{L} = \frac{1}{2}\delta Z\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\delta m^2\phi^2 - \frac{\lambda}{4!}\delta\lambda\phi^4, \tag{2.23c}$$

where $\mathcal{L}$ is the Lagrangian of ϕ^4-theory written in terms of renormalized parameters and field while $\delta\mathcal{L}$ consists of *counterterms* that absorb the infinities from our loop calculations. Counterterms can be considered as a set of interaction terms that we introduce in order to get rid of the infinities. With these "new" interaction terms added to our Feynman rules, perturbative calculation can be performed by using only renormalized and counterterm parameters. The quantities δm^2, $\delta\lambda$ and δZ may be expanded as power series in λ or Laurent series in ϵ:

$$\delta m^2 = \sum_{i=1}^{\infty} \delta m_i^2 = m^2 \left(a_0(\lambda, \mu/m) + \sum_{i=1}^{\infty} \frac{a_i(\lambda, \mu/m)}{\epsilon^i} \right), \tag{2.24a}$$

$$\delta Z = \sum_{i=1}^{\infty} \delta Z_i = b_0(\lambda, \mu/m) + \sum_{i=1}^{\infty} \frac{b_i(\lambda, \mu/m)}{\epsilon^i}, \tag{2.24b}$$

$$\delta\lambda = \sum_{i=1}^{\infty} \delta\lambda_i = c_0(\lambda, \mu/m) + \sum_{i=1}^{\infty} \frac{c_i(\lambda, \mu/m)}{\epsilon^i}, \tag{2.24c}$$

where a_0, b_0 and c_0 are regular when $\epsilon \to 0$. The coefficients a_i, b_i and c_i can only depend on λ and μ/m as they are dimensionless quantities.

Continuing our calculation of $\Gamma^4(s,t,u)$ from the previous section we see that by introducing the counterterm $\delta\lambda$ in our evaluation, we obtain the following result:

$$\begin{aligned} i\Gamma^4(s,t,u) &= -i\lambda + V(s) + V(t) + V(u) - i\lambda\delta\lambda_1 \\ &= -i\lambda + \frac{i\lambda^2}{32\pi^2}\left\{-3\gamma + \frac{6}{\epsilon} + \int_0^1 \mathrm{d}x \left[\log\left(\frac{4\pi\mu^2}{m^2 - x(1-x)s}\right)\right.\right. \\ &\left.\left. + \log\left(\frac{4\pi\mu^2}{m^2 - x(1-x)t}\right) + \log\left(\frac{4\pi\mu^2}{m^2 - x(1-x)u}\right)\right]\right\} - i\lambda\delta\lambda_1. \end{aligned} \tag{2.25}$$

As our calculation is performed up to one-loop order, only one-loop counterterm $\delta\lambda_1$ from $\delta\lambda$ is needed. Remember that we have to drop the subscript B in Eq. (2.25) as we are evaluating the loop diagram in terms of renormalized parameters. In order for $\Gamma^4(s,t,u)$ to be finite, the counterterm $\delta\lambda_1$ must cancel the divergence coming from the ϵ-pole:

$$\frac{\lambda}{32\pi^2}\frac{6}{\epsilon} - \delta\lambda_1 \xrightarrow[\epsilon\to 0]{} \text{constant}. \tag{2.26}$$

This procedure however does not determine the finite part of $\delta\lambda_1$ in a unique way, hence a specific rule is needed to fix it. Such a rule is called a *renormalization scheme*. Physical results should not depend on the scheme that one has chosen, i.e. the S-matrix of a system is renormalization scheme independent. Once a scheme is chosen and fixed, the renormalized parameters can be inferred from physical quantities and one is not allowed to change the scheme anymore in order to obtain predictions of subsequent experimental results, unless proper matching conditions between the different schemes are applied. We will only focus on one renormalization scheme, namely the $\overline{\text{MS}}$ or *modified minimal subtraction* scheme, by demanding that $\delta\lambda_1$ subtracts the ϵ-pole and the constant term:

$$\delta\lambda_1 = \frac{\lambda}{32\pi^2}\left(\frac{6}{\epsilon} - 3\gamma + 3\log(4\pi)\right), \tag{2.27a}$$

$$c_0(\lambda, \mu/m) = \frac{\lambda}{32\pi^2}\left(-3\gamma + \log(4\pi)\right), \tag{2.27b}$$

$$c_1(\lambda, \mu/m) = \frac{6\lambda}{32\pi^2}. \tag{2.27c}$$

The reason why we apply the $\overline{\text{MS}}$ scheme in our analysis is due to the fact that most of the SM couplings collected by Particle Data Group (PDG) [16] are calculated in this scheme. Furthermore the coefficients a_i, b_i and c_i are mass-independent, i.e. they do not depend on the ratio μ/m. This mass-independence will simplify our task in calculating the running of couplings in next section. The mass term m in the $\overline{\text{MS}}$-scheme is not the physical mass that one obtains in experiments. Similarly the renormalized coupling λ in this scheme is not a physical quantity that can be directly measured from experiments. This is because the $\overline{\text{MS}}$ coupling is not directly matched to the physical observable. In order to associate the renormalized parameter to its corresponding physical quantity, the *on-shell* subtraction scheme has to be imposed. Alternatively one can convert the $\overline{\text{MS}}$ coupling to the on-shell physical coupling via matching. We will thoroughly discuss the matching procedure of $\overline{\text{MS}}$ coupling to the corresponding physical quantity later in Sec. (4.3).

2.5 Renormalization group equation

Most of the time even though the infinite part our 1PI is subtracted by counterterms, the perturbative result is not valid up to arbitrary high energy scale. By looking at Eq. (2.25) we notice that should the Mandelstam variables be way larger than renormalization scale μ, our perturbative approach breaks down as the large external momentum can generate a large logarithmic contribution for our one-loop quantum correction. Therefore it would be wiser if we choose another renormalization scale which has the same magnitude of the given external momentum. The change of energy scale and its implication for the renormalized quantities can be summarized in the *renormalization group equation* (RGE).

First we examine the relation between general bare and renormalized 1PI. From the scaling property of 1PI we know that:

$$\Gamma^n_B(\{p_i\}, \lambda_B, m_B) = Z^{-n/2}\Gamma^n(\{p_i\}, \lambda(\mu), m(\mu), \mu), \tag{2.28}$$

where p_i stands for a collection of n external momenta. Notice that bare 1PI does not depend on the renormalization scale. Hence differentiating both equations w.r.t. μ we have:

$$\begin{aligned} 0 &= \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\Gamma^n_B(\{p_i\}, \lambda_B, m_B) \\ &= \mu\frac{\mathrm{d}}{\mathrm{d}\mu}\left(Z^{-n/2}\Gamma^n(\{p_i\}, \lambda(\mu), m(\mu), \mu)\right), \end{aligned} \tag{2.29}$$

and by applying the chain rule the term becomes:

$$\left(\mu\frac{\partial}{\partial\mu}+\beta\frac{\partial}{\partial\lambda}-\frac{n}{2}\gamma+\gamma_m\frac{\partial}{\partial m^2}\right)\Gamma^n(\{p_i\},\lambda(\mu),m(\mu),\mu)=0, \tag{2.30}$$

with the *beta function* β and *anomalous dimension* of mass (γ_m) and field (γ) defined as:

$$\beta(\lambda,m/\mu,\epsilon):=\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu}\xrightarrow[\epsilon\to 0]{}\beta(\lambda,m/\mu), \tag{2.31a}$$

$$\gamma(\lambda,m/\mu,\epsilon):=\frac{1}{Z}\mu\frac{\mathrm{d}Z}{\mathrm{d}\mu}\xrightarrow[\epsilon\to 0]{}\gamma(\lambda,m/\mu), \tag{2.31b}$$

$$\gamma_m(\lambda,m/\mu,\epsilon):=\mu\frac{\mathrm{d}m^2}{\mathrm{d}\mu}\xrightarrow[\epsilon\to 0]{}\gamma_m(\lambda,m/\mu). \tag{2.31c}$$

Note that all the bare couplings do not depend on the renormalization scale. The beta function and anomalous dimensions are dimensionless and in general depend on λ and m/μ. If we adopt a mass-independent renormalization scheme, e.g. the $\overline{\text{MS}}$-scheme, the dependence on m/μ drops out and thus simplifies the solution of our RGE [17, 18].

2.5.1 Beta function and running coupling

Having chosen a renormalization scheme[1] and the subtraction of infinities by counterterms being performed, we are ready to evaluate the four-point scattering amplitude:

$$\begin{aligned} i\Gamma^4(s,t,u)=-i\lambda(\mu_0)+\frac{i\lambda^2(\mu_0)}{32\pi^2}&\left[\int_0^1\mathrm{d}x\log\left(\frac{\mu_0^2}{m^2-x(1-x)s}\right)\right.\\ &\left.+\log\left(\frac{\mu_0^2}{m^2-x(1-x)t}\right)+\log\left(\frac{\mu_0^2}{m^2-x(1-x)u}\right)\right]. \end{aligned} \tag{2.32}$$

However perturbation theory breaks down if we have chosen a renormalization scale μ_0 which deviates in large order of magnitude compared to the external momentum. The large value of μ_0 will induce a large logarithmic factor to our loop correction, which will invalidate the assumption that we can perform perturbation theory. The solution to this problem is to adjust our effective coupling to a scale which has the same order as the external momentum. This running of coupling is justified as our renormalized coupling is a function of the renormalization scale and its change w.r.t. μ is dictated by beta function. Most of the time we have to calculate the beta function perturbatively and in general we only have to know the one-loop beta function in order to study the

[1] $\overline{\text{MS}}$ scheme in our case.

asymptotic behavior of the running couplings. In our example consider again Eq. (2.22c) and expand Z'_λ as:

$$\lambda_B = \mu^\epsilon Z'_\lambda \lambda = \mu^\epsilon \left(1 + \sum_i^\infty \frac{d_i(\lambda)}{\epsilon^i}\right) \lambda, \tag{2.33}$$

where again the coefficient d_i depends only on λ as we are only considering the mass-independent renormalization scheme. Since the bare coupling does not depend on μ, we find:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \lambda_B = \epsilon\lambda + \beta(\lambda, \epsilon) + \lambda Z_\lambda^{'-1} \mu \frac{\mathrm{d}}{\mathrm{d}\mu} Z'_\lambda = 0, \tag{2.34}$$

and by applying chain rule we obtain:

$$\beta(\lambda, \epsilon) \left(1 + \lambda \frac{\mathrm{d} \log Z'_\lambda}{\mathrm{d}\lambda}\right) + \epsilon\lambda = 0. \tag{2.35}$$

Usually one needs to evaluate the beta function perturbatively and in general it can be expressed as a perturbation series:

$$\beta(\lambda) = \frac{\beta^{(1)}(\lambda)}{16\pi^2} + \frac{\beta^{(2)}(\lambda)}{(16\pi^2)^2} + \dots \tag{2.36}$$

Our goal is to evaluate the one-loop beta function for our theory. For that we need to evaluate $\log Z'_\lambda$ with the help of Eq. (2.22c), where we need to take the one-loop quantum correction of Z and Z_λ into account. It is generally known that for ϕ^4-theory, δZ only contains corrections of $\mathcal{O}(\lambda^2)$ onwards and hence we need to consider only the one-loop correction of λ:

$$\begin{aligned} Z'_\lambda &= Z^{-2} Z_\lambda \\ &= 1 + \frac{\lambda}{32\pi^2} \left(\frac{6}{\epsilon} - 3\gamma + 3\log(4\pi)\right) + \mathcal{O}(\lambda^2). \end{aligned} \tag{2.37}$$

Plugging Eq. (2.37) into Eq. (2.35) we get:

$$\beta(\lambda, \epsilon) \left(1 + \frac{3\lambda}{16\pi^2\epsilon}\right) + \epsilon\lambda = 0 \tag{2.38}$$

and hence the one-loop beta function is given by:

$$\begin{aligned} \beta(\lambda, \epsilon) &= -\epsilon\lambda \left(1 + \frac{3\lambda}{16\pi^2\epsilon}\right)^{-1} \\ &= -\epsilon\lambda + \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3) \end{aligned}$$

$$\xrightarrow[\epsilon\to 0]{} \frac{3\lambda^2}{16\pi^2} + \mathcal{O}(\lambda^3). \tag{2.39}$$

With the beta function obtained we can solve Eq. (2.31a) and acquire:

$$\lambda(\mu) = \frac{\lambda(\mu_0)}{1 - \dfrac{3\lambda(\mu_0)}{16\pi^2}\log\left(\dfrac{\mu}{\mu_0}\right)}. \tag{2.40}$$

The asymptotic behavior of this coupling in the SM under the scaling of μ will be discussed in the Sec. (4.2.2).

2.5.2 Improved perturbation theory

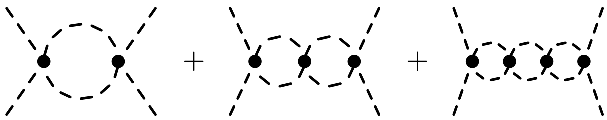
We would like to use the running coupling to improve our perturbation calculation in the four-point scattering amplitude. Consider again Eq. (2.32) but this time let us assume that we would like to know the behavior of our scattering amplitude in the limit of large external momenta:

$$s = t = u \to k\mu_0^2 = \mu^2, \qquad k \gg 1. \tag{2.41}$$

As mentioned above our perturbation theory breaks down due to the large logarithmic contribution. But with the running coupling one can improve the perturbation by running up the coupling constant w.r.t. the scale factor. Substituting Eq. (2.40) into Eq. (2.32) we acquire the improved version of our scattering amplitude:

$$\begin{aligned} i\Gamma^4(s=t=u=\mu^2) &= -i\lambda(\mu_0) + \frac{i\lambda^2(\mu_0)}{32\pi^2}\left[\int_0^1 \mathrm{d}x \log\left(\frac{\mu_0^2}{m^2 - x(1-x)s}\right)\right. \\ &\quad \left. + \log\left(\frac{\mu_0^2}{m^2 - x(1-x)t}\right) + \log\left(\frac{\mu_0^2}{m^2 - x(1-x)u}\right)\right] + \mathcal{O}(\lambda^3) \\ &= -i\lambda(\mu)\left[1 - \frac{3\lambda(\mu)}{16\pi^2}\log\left(\frac{\mu}{\mu_0}\right)\right] + \frac{3i\lambda^2(\mu)}{32\pi^2}\times \\ &\quad \int_0^1 \mathrm{d}x \log\left(\frac{\mu_0^2}{-x(1-x)\mu^2}\right) + \mathcal{O}(\lambda^3) \\ &= -i\lambda(\mu) + \frac{3i\lambda^2(\mu)}{32\pi^2}\int_0^1 \mathrm{d}x \log\left(\frac{3}{-4x(1-x)}\right) + \mathcal{O}(\lambda^3). \end{aligned} \tag{2.42}$$

Observe that μ and μ_0 drop out from the integral and hence we have recovered the validity of perturbation theory. Actually, the advantage of utilizing the running coupling comes from the summation of all the leading-logarithmic terms. In terms of Feynman diagram, the leading-logarithms are represented by:



$$+\ldots \sim \sum_n \lambda^n(\mu_0)\log^n\left(\frac{\mu}{\mu_0}\right). \tag{2.43}$$

By summing all the leading-logarithmic terms, part of the higher order loop corrections which are generated by the nested one-loop radiative corrections are resummed in an effective coupling defined on another energy scale. Similarly the subleading-logarithmic terms will also be resummed, if the two-loop beta function is taken into account. Now we can understand how the perturbation theory is improved when a running coupling is used. The running coupling gives us some partial informations on higher order loop correction with only the one-loop computations. We will use the improved perturbation techniques to investigate its effect on Higgs mass quadratic quantum correction later in Sec. (4.4).

Chapter 3

The Standard Model and Beyond

In this section we will review the successful theory of elementary particle physics based on QFT, namely the Standard Model of particle physics. We will first define our convention for the SM Lagrangian and investigate the quantum corrections for the Higgs mass. We argue that if the SM is not a fundamental theory but is instead embedded into a high energy QFT, then the Higgs mass necessarily obtains large radiative corrections, yielding a large hierarchy between quantum corrections and the tree level value. Possible solutions to the hierarchy problem will be discussed and we will argue that it is more advantageous to have physics on two fundamental energy scales, namely the Fermi and Planck scale. Phenomena that require BSM physics will be subsequently discussed. We will then provide some arguments that the SM with simple extension of three right-handed neutrinos might be an effective theory valid up to the Planck scale and still able to accommodate physics beyond the Standard Model without any need of intermediate energy scale.

3.1 The Standard Model Lagrangian

In the SM, particles are classified according to their fundamental interactions. It consists of the gauge group $SU_{\mathrm{C}}(3) \times SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$ which dictates the fundamental forces (excluding gravity) in subatomic realms. The particle content consists of the vector gauge bosons in the adjoint representation and the fermionic sector in fundamental representations of gauge group. In addition we need to introduce a scalar Higgs sector to break $SU_{\mathrm{L}}(2)\times U_{\mathrm{Y}}(1)$ to $U_{\mathrm{EM}}(1)$ in order to generate masses for fermionic particles and weak bosons. Our SM Lagrangian and notation throughout this work follow basically the convention by Borodulin et al. [19]. We will use Dirac spinor as the fermionic Lorentz group representation, see Appx. (A.3).

The SM Lagrangian can be divided into three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kin} + \mathcal{L}_{\rm H} + \mathcal{L}_{\rm y}, \tag{3.1}$$

where the component of the sub-Lagrangian $\mathcal{L}_{\rm kin}$ contains the kinetic terms of the SM gauge and fermionic fields:

$$\begin{aligned} \mathcal{L}_{\rm kin} &= -\frac{1}{4}G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} - \frac{1}{4}W^{a}_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{L}_m \not{D} L_m \\ &\quad + i\bar{E}_m \not{D} E_m + i\bar{Q}_m \not{D} Q_m + i\bar{U}_m \not{D} U_m + i\bar{D}_m \not{D} D_m, \end{aligned} \tag{3.2}$$

in which $G^{\alpha}_{\mu\nu}$, $W^{a}_{\mu\nu}$ and $B_{\mu\nu}$ are the gauge field strengths of $SU_{\rm C}(3) \times SU_{\rm L}(2) \times U_{\rm Y}(1)$ given in terms of their respective gauge fields:

$$G^{\alpha}_{\mu\nu} = \partial_\mu G^{\alpha}_{\nu} - \partial_\nu G^{\alpha}_{\mu} + g_3 f^{\alpha\beta\gamma} G^{\beta}_{\mu} G^{\gamma}_{\nu}, \tag{3.3a}$$

$$W^{a}_{\mu\nu} = \partial_\mu W^{a}_{\nu} - \partial_\nu W^{a}_{\mu} + g_2 \epsilon^{abc} W^{b}_{\mu} W^{c}_{\nu}, \tag{3.3b}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{3.3c}$$

Note that $\alpha = 1, ..., 8$ and $a = 1, 2, 3$. G^{α}_{μ} describes the gluon gauge field while the linear combination of W^{a}_{μ} and B_μ describe the weak bosons W^{+}_{μ} ,W^{-}_{μ} ,Z_μ and photon field A_μ. We will see below how the weak bosons obtain their masses once we introduce the Higgs sector. Fermions constitute only doublets (L_m and Q_m) and singlets (E_m, U_m and D_m) in $SU_{\rm L}(2) \times U_{\rm Y}(1)$ representation, where the index m stands for the type of flavor. L_m represents the left-handed lepton doublet, E_m the right-handed electron type lepton, Q_m for left-handed quark doublet, U_m and D_m as right-handed up-type and down-type quark respectively:

$$L_m = \begin{pmatrix} (\nu_m)_L \\ (e_m)_L \end{pmatrix}, \qquad Q_m = \begin{pmatrix} (u_m)_L \\ (d_m)_L \end{pmatrix}, \tag{3.4a}$$

$$U_m = (u_m)_R, \qquad D_m = (d_m)_R, \qquad E_m = (e_m)_R. \tag{3.4b}$$

The particle contents sorted in generations are given as below:

$$\nu_1 = \nu_e, \qquad \nu_2 = \nu_\mu, \qquad \nu_3 = \nu_\tau, \tag{3.5a}$$

$$e_1 = e, \qquad e_2 = \mu, \qquad e_3 = \tau, \tag{3.5b}$$

$$u_1 = u, \qquad u_2 = c, \qquad u_3 = t, \tag{3.5c}$$

$$d_1 = d, \qquad d_2 = s, \qquad d_3 = b. \tag{3.5d}$$

The notation $\not{D}$ in the SM Lagrangian represents the covariant derivative of the fermionic fields w.r.t. to the SM gauge group. Depending on which field the covariant derivative acts upon, proper gauge fields must be included in the covariant derivative in accordance to the transformation of the field. The gauge transformation properties for the gauge fields and for one generation of fermionic fields w.r.t. to the SM gauge group are given in Appx. (A.4).

Both the gauge fields and fermions described in Eq. (3.2) have zero mass, contradicting the observation that all charged fermions and weak bosons are massive. To obtain the mass term one can use the Higgs mechanism [20, 21] to break $SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$ down to $U_{\mathrm{EM}}(1)$ spontaneously. In the SM we have a Higgs $SU_{\mathrm{L}}(2)$ doublet Φ with hypercharge $Y = 1$ and its Lagrangian term is given by:

$$\mathcal{L}_{\mathrm{H}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \tag{3.6}$$

with the terms:

$$D_\mu \Phi = \left(\partial_\mu - i\frac{g_1}{2} B_\mu - i g_2 \frac{\sigma^a}{2} W_\mu^a \right) \Phi, \tag{3.7}$$

$$V(\Phi) = m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \tag{3.8}$$

The 2×2 hermitian matrices σ^a are the usual Pauli matrices. In a well defined QFT, the potential term must be bounded from below, i.e. $\lambda > 0$. When m^2 is positive, the potential has only a minimum at $\Phi = 0$. In order for $V(\Phi)$ to obtain a minimum at a non-zero value of Φ, we have to demand that $m^2 < 0$. We say that the doublet acquires a non-zero vacuum expectation value (vev) and we can parameterize its value to be at the neutral component of the doublet:

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \tag{3.9}$$

where v is determined by minimizing the potential in Eq. (3.8):

$$v^2 = \frac{|m^2|}{\lambda}. \tag{3.10}$$

After acquiring a non-zero vev, the Higgs doublet can be written as:

$$\Phi = \begin{pmatrix} i\omega^+ \\ \frac{1}{\sqrt{2}}(v + H - iz) \end{pmatrix}, \tag{3.11}$$

where H and z are real scalar fields while ω^+ describes a complex scalar field. Notice that by shifting the degrees of freedom, Φ is not manifestly invariant under $SU_L(2) \times U_Y(1)$ transformations. We say that the symmetry is broken spontaneously, as the vacuum state in Eq. (3.9) is not $SU_L(2) \times U_Y(1)$ invariant. We should keep in mind that the SM Lagrangian is still invariant under such transformation despite the nomenclature-abuse of broken symmetry. By going to unitary gauge:

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H) \end{pmatrix}, \tag{3.12}$$

three of the scalar fields ($\omega^\pm$, z) are eaten by the weak bosons to form their longitudinal components. This can be demonstrated by substituting Eq. (3.9) into Eq. (3.6):

$$\begin{aligned}(D_\mu \Phi)^\dagger (D^\mu \Phi) \supset \frac{1}{8} v^2 g_2^2 (W_\mu^1 - i W_\mu^2)(W^{1\mu} + i W^{2\mu}) \\ + \frac{1}{8} v^2 (-g_2 W^{3\mu} + g_1 B^\mu)(-g_2 W_{3\mu} + g_1 B_\mu).\end{aligned} \tag{3.13}$$

Diagonalizing the mass matrix we obtain the mass terms for the weak bosons:

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2), \qquad m_W = \frac{1}{2} g_2 v\,, \tag{3.14a}$$

$$\begin{aligned}Z_\mu &\equiv \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2 W_\mu^3 - g_1 B_\mu) \\ &= W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W, \qquad m_Z = \frac{1}{2}\sqrt{g_1^2 + g_2^2}\, v\,,\end{aligned} \tag{3.14b}$$

$$\begin{aligned}A_\mu &\equiv \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1 W_\mu^3 + g_1 B_\mu) \\ &= W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W, \qquad m_A = 0,\end{aligned} \tag{3.14c}$$

while the photon A_μ remains massless. Note that we have defined the weak mixing angle θ_W as:

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}, \quad \sin\theta_W = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \tag{3.15}$$

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}. \tag{3.16}$$

The second line of the equation above is only valid in tree level calculations, as loop calculations will in general generate extra corrections to it. However we can define on-shell $\sin^2\theta_W$ and $\cos^2\theta_W$ terms which are valid to all order of perturbation [22]:

$$\sin^2\theta_W^{\rm OS} \equiv s_w^2 = 1 - \frac{M_W^2}{M_Z^2}, \tag{3.17a}$$

$$\cos^2\theta_W^{\rm OS} \equiv c_w^2 = \frac{M_W^2}{M_Z^2}. \tag{3.17b}$$

The explicit use of capital letter M_i for the mass term of particle species i is needed to distinguish the physical mass from the Lagrangian parameter. We will discuss their difference and the matching procedure between these quantities in Sec. (4.3).

As for the Higgs boson, the only remaining physical component of the Higgs doublet after spontaneous symmetry breaking is H, which has the following potential terms:

$$\begin{aligned} V &= -|m^2|\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2 \\ &= \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4 - \frac{\lambda}{4}v^4. \end{aligned} \tag{3.18}$$

The mass term of Higgs boson defined as:

$$m_H^2 = 2\lambda v^2. \tag{3.19}$$

Now we come to the last piece of the SM Lagrangian, namely the Yukawa sector $\mathcal{L}_{\rm y}$:

$$\mathcal{L}_{\rm y} = -Y_{mn}^E \overline{L}_m \Phi E_n - Y_{mn}^D \overline{Q}_m \Phi D_n - Y_{mn}^U \overline{Q}_m (i\sigma^2\Phi^*) U_n + h.c \tag{3.20}$$

where the Yukawa matrices $\boldsymbol{Y}^E$, $\boldsymbol{Y}^D$ and $\boldsymbol{Y}^U$ are arbitrary complex 3×3 matrices. By applying the bi-unitary transformation:

$$\boldsymbol{\lambda}^E = \boldsymbol{V}_E^\dagger \boldsymbol{Y}^E \boldsymbol{W}_E = \mathrm{diag}(\lambda_e,\, \lambda_\mu,\, \lambda_\tau), \tag{3.21a}$$

$$\boldsymbol{\lambda}^U = \boldsymbol{V}_U^\dagger \boldsymbol{Y}^U \boldsymbol{W}_U = \mathrm{diag}(\lambda_u,\, \lambda_c,\, \lambda_t), \tag{3.21b}$$

$$\boldsymbol{\lambda}^D = \boldsymbol{V}_D^\dagger \boldsymbol{Y}^D \boldsymbol{W}_D = \mathrm{diag}(\lambda_d,\, \lambda_s,\, \lambda_b), \tag{3.21c}$$

and substituting Eq. (3.9) into Eq. (3.20), we obtain the masses of fermions at tree level:

$$\boldsymbol{\mathcal{M}}_E = \frac{v\boldsymbol{\lambda}_E}{\sqrt{2}},\ \boldsymbol{\mathcal{M}}_U = \frac{v\boldsymbol{\lambda}_U}{\sqrt{2}},\ \boldsymbol{\mathcal{M}}_D = \frac{v\boldsymbol{\lambda}_D}{\sqrt{2}}. \tag{3.22}$$

Note that there is no summation over the labels E, U, D involved in Eq.(3.21a-3.21c). The combination of $\boldsymbol{V}_U^\dagger \boldsymbol{V}_D$ yields the so-called *Cabibbo-Kobayashi-Maskawa* (CKM) matrix [23], which will influence the flavor-changing charged currents in the electroweak theory. Since in this thesis we do not concern ourselves with the electroweak currents, but rather with the Higgs sector, we will not discuss the CKM matrix further.

Later in our analysis we will need to know how the Higgs boson interacts with other elementary particles. As we can see from Eq. (3.12), the Higgs field H is similar to the vev except that it is a dynamical field. Therefore all the mass terms in the SM Lagrangian obtained via spontaneous symmetry breaking will automatically induce a coupling of particles with Higgs field via simple substitution of the vev v with the Higgs field H in the gauge and Yukawa sector. One will also obtain cross terms like particles coupled to vH if one performs the expansion carefully. We summarize the interaction of the Higgs with the gauge fields and fermions as follows:

$$\mathcal{L}_{\mathrm{SM}} \supset \left(\frac{H}{v} + \frac{H^2}{2v^2}\right)\left(2m_W^2 W_\mu^+ W^{-\mu} + m_Z^2 Z_\mu Z^\mu\right) - \sum_f \frac{m_f}{v}\bar{f} f H, \tag{3.23}$$

where f denotes the fermions.

Throughout this work we assume that neutrinos are massless, although it is known today that neutrinos are indeed massive as confirmed by neutrino oscillation experiments. If the neutrino masses are generated by the Higgs mechanism via introducing right-handed component of neutrinos ν_R:

$$\mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{M_\nu} \supset -\frac{1}{2}\bar{\nu}_R^c \boldsymbol{\mathcal{M}}_M \nu_R - \bar{\nu}_L \boldsymbol{\mathcal{M}}_D \nu_R + \text{h.c}, \tag{3.24}$$

the Majorana mass matrix $\boldsymbol{\mathcal{M}}_M$ can be also included besides the usual Dirac mass term $\boldsymbol{\mathcal{M}}_D$, as ν_R is a singlet under the transformation of SM gauge group. Diagonalizing the masses of neutrinos we would obtain the famous see-saw formula:

$$\boldsymbol{\mathcal{M}}_\nu = -\boldsymbol{\mathcal{M}}_D^T \boldsymbol{\mathcal{M}}_M^{-1} \boldsymbol{\mathcal{M}}_D. \tag{3.25}$$

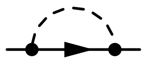
The formula above is only valid in the limit of $m_M \gg m_D$, where $m_{M/D}$ are the scales of $\boldsymbol{\mathcal{M}}_{M/D}$ respectively. For the case of one neutrino generation we can estimate the order of the neutrino Yukawa coupling. Since $m_\nu \lesssim \mathcal{O}(1\,\mathrm{eV})$, the Yukawa coupling of neutrino would be around $\mathcal{O}(1)$ for Majorana mass scale of order $10^{15}\,\mathrm{GeV}$. This large coupling between Higgs boson and neutrino could alter the prediction on Higgs mass later via the running coupling, particularly in the triviality and vacuum stability condition of Higgs sector. We will discuss more on the effects of neutrino on the SM Higgs mass prediction when we encounter the running coupling later in Sec. (4.2.2).

3.2 Hierarchy problem and fine-tuning

The SM's predictive power has withstood all the precision measurements up to the level of radiative correction for the last forty years. With all the ingredients of the SM discovered except for the Higgs particle, this prompts us to rethink about its status in the SM. At present we do not know about its properties, except that it is one of the candidates to generate mass terms for quarks, leptons, and the weak gauge bosons via EWSB. This mechanism is simple in the sense that it only requires one scale parameter, namely the vev $v = 264\,\mathrm{GeV}$, to generate masses for all the massive elementary particles in the SM. However this simple and elegant method of generating masses for elementary particles does come with a price, namely that the Higgs mass is not stable against radiative corrections. Every quantity that we can calculate in QFT receives radiative corrections, should the theory come with interaction terms. However, the mass term of a scalar field receives a large radiative correction that can exceed more than its initial value. Before we discuss this problem, let us first look at the fermionic and vector theory, and see that this class of theory has a certain symmetry to protect its mass value. Consider part of our SM fermionic Lagrangian after spontaneous symmetry breaking:

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} &\supset i\bar{t}\not{D}t - \frac{\lambda_t}{\sqrt{2}} H\,\bar{t}_L t_R + h.c \\ &= i\bar{t}\not{D}t - \frac{m_t}{v} H\,\bar{t}_L t_R + h.c.\end{aligned} \tag{3.26}$$

We consider only the top quark coupled to the SM Higgs for simplicity. Interesting things happen when we calculate the one-loop effect for the top mass correction:

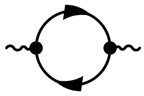
$$-i\delta m_t|_{\mathrm{yukawa}} =$$ 

$$\begin{aligned}&= \left(-\frac{m_t}{v}\right)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(p-k)^2 - m_H^2} \frac{\not{k} + m_t}{k^2 - m_t^2} \\ &\propto \frac{i m_t^3}{16\pi^2 v^2} \int_0^1 \mathrm{d}x \, \log\left(\frac{\Lambda^2}{x m_H^2 + m_t^2(1-x) - x(1-x)p^2}\right),\end{aligned} \tag{3.27}$$

where we have used the cut-off regularization. We observe that the tree level mass term m_t controls the breaking of chiral symmetry, i.e. the radiative correction above vanishes if $m_t \to 0$. The fermionic theory exhibits the so-called *chiral symmetry*, should the mass term m_t vanish. Without the mass term, the left- and right- handed fermions of the SM transform differently under a global $SU_{\mathrm{L}}(3) \times SU_{\mathrm{R}}(3)$ symmetry. By adding a mass term, we break the global symmetry into its vector subgroup $SU_{\mathrm{V}}(3)$. Therefore, the crucial parameter that controls such symmetry breaking must be proportional to the mass term. Notice that if we treat the cut-off Λ as a scale where the SM breaks down, we would only receive a small radiative correction for the top mass due to the logarithm nature. The important point is that the correction to the fermion mass is of the order of its mass. Note that there exists also flavor hierarchy problem in the flavor sector, but we will not discuss it further in this thesis. Another important point that we should keep in mind, is the interpretation of Λ. We will discuss this quantity more detailed in the next section.

What about the gauge boson sector? Let us consider the case after the spontaneous symmetry breaking of $SU_{\rm L}(2) \times U_{\rm Y}(1) \to U_{\rm EM}(1)$. Calculating the one-loop correction of the photon propagator, we find the self-energy contribution to be:

$$-i\Pi^{\mu\nu}(q) =$$



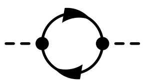
$$\begin{aligned} &= (-1)\left(\frac{2}{3}e\right)^2 \int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{tr}\left[\gamma^\mu \frac{1}{\not{k}-m}\gamma^\nu \frac{1}{\not{k}+\not{q}-m}\right] \\ &\propto -\frac{\alpha}{\pi}\left(\frac{2}{\epsilon} - \log\frac{m^2}{4\pi\mu^2}\right)\left[q^2 g^{\mu\nu} - q^\mu q^\nu\right]. \end{aligned} \tag{3.28}$$

The photon remains transversal and hence massless after adding the radiative correction. Notice that we have performed dimensional regularization instead of an ultraviolet cut-off, as a hard cut-off on the integral will violate the *Ward identity*, which corresponds to the violation of gauge symmetry. A naive cut-off regularization will yield a mass term for the photon, which is proportional to Λ[1]. Therefore, the vanishing mass of the photon is protected by the gauge symmetry.

3.2.1 The problem with scalar fields

Now consider the case for a scalar field. In the SM, only the Higgs particle is a scalar boson. Evaluating the one-loop top correction to the Higgs mass with the ultraviolet cut-off regularization we obtain:

$$-i\delta m_H^2\big|_{\rm yukawa} =$$



$$= (-1)N_c\frac{m_t^2}{v^2}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\mathrm{Tr}\left[\frac{1}{\not{k}-m_t}\frac{1}{\not{k}-m_t}\right]$$

[1]See for instance the derivation in Peskin & Schroeder [24].

$$= \frac{iN_c m_t^2}{4\pi^2 v^2}\left(\underbrace{\Lambda^2}_{\text{(A)}} \underbrace{- 3m_t^2 \log\left(1 + \frac{\Lambda^2}{m_t^2}\right)}_{\text{(B)}}\right), \tag{3.29}$$

where N_c is the number of colors, which corresponds to 3 in the SM. Suppose that the SM is only an effective theory up to a very high energy scale, 10 TeV or above, we would have a problem stabilizing the Higgs mass, which is supposed to be around the Fermi scale according to the electroweak precision fit. This unnatural separation between the high energy scale and the electroweak scale is known as the *hierarchy problem.* Confusion usually occurs on how to interpret such a problem, or how to treat the large splitting between these two scales. Note that the SM is a renormalizable theory, and in the framework of SM, such divergence must be renormalized. The hierarchy problem only appears when we try to embed the SM into another UV-complete theory. In order to present the problem more clearly, we split the r.h.s. of Eq. (3.29) into two parts. We first analyze part A, commonly known as the quadratic divergence of the scalar field. In general there are two different views on the hierarchy problem caused by the quadratic divergence:

1. **Believer approach**: The Λ^2 term represents the scale where the SM is UV-completed by another theory. This scale signals the breakdown of the SM, and it should be replaced by its UV-complete counterpart. In fact, one can find an analogy of such a cut-off in solid state physics, where the cut-off represents the physical distance of two lattice points. One cannot observe the fluctuation with an effective distance smaller than the lattice scale. The UV cut-off method adheres the spirit of Wilsonian view on renormalization. Any new BSM physics must provide a solution to cancel or to cancel quadratic divergence.

2. **Atheist approach**: The quadratic divergence appears only in cut-off regularization, therefore it is an artifact or a signal that we are using a not well-controlled regularization scheme. If on the contrary a symmetry-preserving regularization method is applied, such as dimensional regularization, the quadratic divergence[2] does not appear. The cut-off method also poses a problem for the loop calculations of effective field theory, as it could devoid the power suppression of the new physics[3]. Although the cut-off procedure adheres the Wilsonian picture of QFT,

[2] Of course the cut-off does not appear in dimensional regularization. What we mean here is the structure of the divergence.

[3] A beautiful example is illustrated in lecture notes of Pich [25].

one has to be careful in applying it directly to particle physics. The reason is that Lorentz and gauge invariance are the fundamental symmetries of the SM, which are absent in condensed matter physics. The cut-off method violates Lorentz and gauge invariance manifestly, as it truncates the upper integration limit on one of the spacetime axis[4].

Confusion usually arises for the "believers" on the role of Λ. As we have mentioned before, if the SM is treated as an isolated theory on its own, the cut-off Λ only plays a role of integral-regulator, which has to be sent to infinity at the end of the perturbative calculation. The term Λ can only be interpreted naively as a scale where a new physics appears when the Wilsonian view of QFT is considered. However one should be careful with the subtleties involved. If the SM is UV-completed by a new physics of QFT, the new theory will introduce a new generic mass scale, dictated by the mass of the new intermediate particle that couples to the SM Higgs boson. The radiative correction of the Higgs boson will appear to be:

$$-i\delta m_H^2|_{M_{\text{new}}} \sim \Lambda^2 - cM_{\text{new}}^2 \log\left(1 + \frac{\Lambda^2}{M_{\text{new}}^2}\right) \tag{3.30}$$

Roughly speaking we have substituted the top quark in part B of Eq. (3.29) with another particle which is of orders of magnitude heavier than the Higgs boson. Notice that with the SM embedded in a larger QFT, the cut-off Λ still exist. This is clear as we have again regularize the loop-integral with a hard cut-off; the scale of a new physics is actually encoded in part B, represented by the mass of the heavy intermediate particle. Therefore the usual statement that the cut-off represents a scale of new physics in many of the literatures has to be taken with extra care, as it is actually the mass of the new physics that give rise to the hierarchy problem for the SM Higgs boson. In general the radiative correction for the Higgs boson mass due to the heavy particle will be of the same order as the mass of this heavy particle. That is, *if the SM Higgs boson is coupled to a particle with a mass scale higher than a few orders of magnitude compared to the SM mass parameter m, then the hierarchy between the SM Higgs mass and the heavy particle mass exists.* The general statement of the naturalness condition is that the symmetry of a theory must increase when one of the couplings of a theory is set to zero, which is not the case when a scalar mass term vanishes [27, 28]. A generic scalar field mass will be pulled to the same order of mass by the heaviest particle that it couples to via radiative corrections. The Higgs mass has to be finely tuned to cancel out the large radiative correction induced by the heavy intermediate particle, unless

[4] This is illustrated in the book by Weinberg [26].

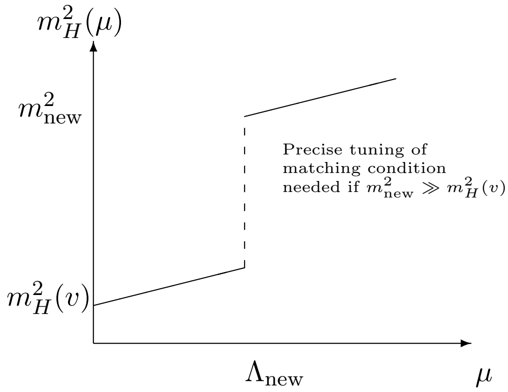


Figure 3.1: The running of the Higgs mass with respect to the energy. In the $\overline{\text{MS}}$ renormalization scheme for instant, the running of the Higgs mass has to be matched to a heavy intermediate particle mass m^2_{new}, if the SM Higgs is coupled to a new physics at a higher energy scale. This precise matching condition is highly fine-tuned if the intermediate mass scale of the new physics is $m_{\text{new}} \gg$ TeV.

this cancellation is provided by certain mechanisms or symmetries of the UV-QFT. This fine-tuning due to part B exist regardless of any regularization method used. Strictly speaking, every parameter in QFT would have to be renormalized, what is meant here is that the subtraction of the divergence for a scalar boson in certain scheme has to take the large finite part from the heavy intermediate particle into account, which is very obscure from the naturalness point of view. In a more refined analysis, the hierarchy problem is equivalent to the dependence of very precisely tuned initial conditions on the renormalized Higgs mass, which from the perspective of RGE matching is highly unnatural[5]. The running of Higgs mass at high energy scale is dictated by the heavy intermediate particle mass, and its matching to the low energy running is highly fine-tuned when the mass of the intermediate scale is of orders of magnitude larger than the electroweak scale, see Fig. (3.1). This argument is true within the framework of QFT.

If however, the next new physics that the Higgs boson can couple to is at the Planck scale, then the previous argument might not apply, as Planck scale physics might be a new concept different from QFT. The field theoretical cut-off Λ might not have a physical interpretation in this case [30]. Therefore it is possible that one is not allowed to use the Wilsonian approach to integrate out the heavy modes of quantum fields, as quantum gravity might not be a QFT[6]. The SM Higgs mass in this sense, could be a remnant of this new quantum gravity concept, and its value at electroweak scale is dictated by

[5] See the lectures of Barbieri [29].
[6] This view is also shared by Meissner and Nicolai [31].

the running of all SM coupling from the Planck scale, compare Fig. (1.1) and Fig. (3.1) for a clearer depiction. From Fig. (1.1) we can see that if the SM is an imprint left by the quantum gravity, then it is possible that the SM Higgs mass does not suffer from fine-tuning problem, as there might not be any large mass scale coming from the heavy intermediate particle that one has to match it with the running of the Higgs mass. This is possible as there might be no heavy particle that the SM Higgs boson can couple to at the Planck scale. We do not know whether such a destabilization of Higgs mass by the Planck scale exists, as a full theory of quantum gravity is not understood yet. However, forbidding any kind of new intermediate quantum field scale between weak and Planck scale might be a viable way to stabilize the Higgs mass, if we assume that Planck scale physics requires a new concept other than QFT[7].

3.2.2 Possible solutions for the hierarchy problem

Before we study the possibility of the SM left as traces of quantum gravity, we first review some of the conventional idea in QFT to solve the hierarchy problem. To see how we can find possible solutions to the hierarchy problem, we consider a generic action of extended SM with generic gauge group, more fields and extra dimensions:

$$S \sim \int \mathrm{d}^d x \, i\bar{q}_{L/R}(\partial\!\!\!/ - ig_i T^a W\!\!\!\!/^a) q_{L/R} - \frac{\lambda_q}{\sqrt{2}} H \, \overline{q}_L q_R + \frac{1}{2}\partial_\mu H \partial^\mu H - \frac{\lambda}{4} H^4 + \dots \tag{3.31}$$

We have dropped some terms, generalized the gauge group and its interaction and added some new fields just to illustrate a few possible solutions. In general the contending solutions can be classified as:

Supersymmetry: One possible solution is to extend the field content of the SM by *supersymmetry* partners [32, 33][8], where every bosonic particle has a corresponding fermionic partner and vice versa. Roughly speaking, we equate λ_q to λ such that the quadratic divergence disappears as each bosonic loop correction is cancelled by the respective fermionic loop due to the minus sign contributed by every fermionic loop. However such an elegant theory does come with its trade-off, as the breaking of supersymmetry is not entirely understood. Supersymmetry has to be broken, otherwise superpartners would have been observed already as they are required to have the same mass as the SM particles in the unbroken phase. The parameterization of our ignorance in supersymmetry breaking with soft mass terms gives the theory of the *Minimal*

[7] Note that from an opposite point of view, it could also be possible that there is no theory of quantum gravity, but rather gravity itself is an emergent effect, thus rendering the question of hierarchy pointless.

[8] Refer to the review from Martin [34] for more references.

Supersymmetric Standard Model (MSSM) a lot of free parameters, which one needs to constrain in order to have some interesting phenomenology at the LHC scale. This constrained version of the MSSM is called the cMSSM. Recent results of ATLAS and CMS from the collider physics perspective have put severe constraints on the cMSSM. However, with more than 100 parameters available for weak scale supersymmetry, portions of MSSM parameter space can circumvent the experimental tension. Perhaps an interesting question that we should ask is when should supersymmetry be given up as a solution for the hierarchy problem from naturalness point of view. The one-loop correction for one of the Higgs doublet parameters m_{H_U} in the MSSM receives a contribution from the stop. Without fine-tuning, one would expect the lightest stop mass to be around $\tilde{m}_t < 150\,\mathrm{GeV}$, which would give the lightest Higgs boson m_h in a range that has been excluded by LEP. Therefore, one needs to increase the stop mass value to about 500 GeV in order to have a reasonable light Higgs mass. This large stop mass however will contribute to large m_{H_U} and subsequently introduce a fine-tuning to the relation between Z boson mass and the mass of H_U.

Extra dimension: From Eq. (3.31) we have generalized the integral measure $\int \mathrm{d}^4x \to \int \mathrm{d}^dx$, i.e. we have *extra dimensions* such that the Planck scale that we obtained in four dimensions is an effective version of the actual Planck constant from higher extra dimensions. The idea that the Planck constant can be so large from the point of view of four dimensions comes from the fact that the fundamental Planck scale M_* is multiplied by a volumetric suppression of the compactified extra dimension R^n:

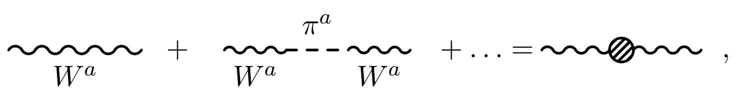
$$R^n \approx \frac{M_{pl}^2}{M_*^{n+2}}. \tag{3.32}$$

That is, the observed smallness of Newton's constant is ascribed to the propagation of the gravitational force in extra dimensions, while the SM fields remain at our three spatial dimensions, not propagating to the extra dimensions [35]. However, the task to explain the hierarchy problem has been turned into a difficult hunt for geometry and the number of extra dimensions. More complicated scenarios with hierarchical distribution of extra dimension's sizes are possible. Suppose we assume that there is only one fundamental scale at 1 TeV, i.e $M_* \approx 1\,\mathrm{TeV}$, we would have to accept that the number of extra dimension must be greater than two in order to concord with the experimental limits. One can go even further with *warped extra dimensions* [36] to explain the hierarchy between the Planck and Fermi scale by setting up effective theories in brane and bulk in such a way that the actual vev (at the order of Planck scale) is multiplied with a warp factor that runs exponentially with the characteristic length of the extra dimension. In this sense what we perceive as the weak scale vev is nothing but an effective description of Planck energy physics scaled by a warp geometry. The SM fields can also be localized on brane or bulk, yielding different phenomenologies. However, recent results from ATLAS and CMS have excluded some of the extra dimensional models up to TeV region [9].

Technicolor: *Technicolor* has been a proposed candidate by Susskind [37] and Weinberg [38] to generate the Fermi scale dynamically via dimension transmutation by the running of a new coupling from the technicolor gauge sector, mimicking Λ_{QCD} from the strong $SU(3)$ sector. That is to say, from the first term in Eq. (3.31) the $\bar{q}q$ forms a condensate, breaking the chiral symmetry. From Goldstone's theorem we know that massless pseudoscalars (technipions) accompany this chiral symmetry breaking and thus will provide a coupling to the gauge bosons. Roughly speaking, we have:

$$\begin{aligned} \bar{q}_L(g_i T^a \not{W}^a)q_L + \bar{q}_R(g_i T^a \not{W}^a)q_R + \ldots &\to W_\mu \bar{q}\gamma^\mu\gamma^5 T^a q + \ldots \\ &\sim g W^a_\mu \partial^\mu \pi^a + \ldots \end{aligned} \tag{3.33}$$

and summing up the interaction of the term above for the gauge boson's propagator:



(3.34)

the weak bosons acquire masses. In this sense a Higgs boson is not required, as the technifermions form a condensate to give the weak bosons their masses. Pure technicolor by itself suffers from certain experimental prediction problems, such as a large amount of flavor-changing neutral currents and a large S-parameter in electroweak precision tests. Generic technicolor models also cannot explain the masses of fermions, prompting us to embed it in a larger symmetry group, the *extended technicolor* group, in order to have a chance to tackle the problem of flavor and fermionic masses at same level. In order to describe the quark and lepton mass spectrum without running into phenomenological problems, such a model from extended technicolor is not easily constructed, and to date there is no accepted model in the literature. A proposal to extend and modify technicolor is by imposing the walking of technicolor coupling or conformality in order to avoid all the phenomenological difficulties. Such a theory has rich phenomenological consequences and can be studied via *Anti de Sitter/Conformal field theory* (AdS/CFT) correspondence parallel with extra dimensional models. In the AdS/CFT framework, the composite pseudo Nambu-Goldstone Higgs can be identified with the fifth component of a gauge field in 5-dimensional spacetime [39].

Anthropic: There is another "solution" to the hierarchy problem, namely by invoking the anthropic principle to explain the smallness of the observed vev, which is directly related to the physical mass spectrum of all elementary particles in the SM. The anthropic principle states that physical parameters that we observe have to have certain values such that intelligent life can exist to observe such values. It turns out that either neutron or proton cannot be formed if we adjust the vev in a certain way. Interested readers are referred to Ref. [40].

Despite all the well-motivated extensions of the SM to solve the hierarchy problem, none of the predicted extra signatures by each solution has shown up in the LHC. Therefore we may ask: can the SM be a valid theory up to the Planck scale?

3.3 Beyond the Standard Model?

Despite the success of the SM, it is known that the SM alone cannot be a complete theory. From the view of internal consistency alone, the SM $U_{\mathrm{Y}}(1)$ gauge group and the Higgs sector suffer from the Landau pole problem. It is also known that the SM is not complete to describe the observed natural phenomena. One needs to extend the SM to explain certain phenomena that have been verified by experiments. Several proposals have been put forward to solve different problem, either through bottom-up approach or top-down ansatz. Most of the solutions, whether they are motivated by symmetry, unification setting or why-not scenario, usually introduce an intermediate scale between weak scale and Planck scale. Such an intermediate scale will usually reintroduce the hierarchy or fine-tuning problem in another form, therefore we would like to forbid any high energy scale which the SM can couple to. However the SM cannot possibly be valid up to any arbitrary high energy scale, as we know that gravity exists and its significant effect on elementary particles starts to emerge when the SM is run to Planck scale, $M_{pl} \approx 10^{19}\,\mathrm{GeV}$. The effect of quantum gravity will influence all the fundamental couplings of elementary particles when physical interactions are considered at the Planck scale, and this might not lead to any fine-tuning should quantum gravity consist of new concept other than conventional QFT. From all the arguments given it is speculated that the SM might be valid up to this high energy scale without any need to be embedded into another theory at any intermediate energy scale. But before we motivate for the necessity of just having two energy scale as a possible physical scenario, we shall review some of the phenomena that require an extension of the SM.

Neutrino mass: For a more established evidence of BSM we can consider neutrino physics. In the SM neutrinos are treated as massless fields. However, neutrinos are proven to be massive by various neutrino oscillation experiments. Although we now know that neutrinos are massive, their mass hierarchy is still not known, nor is the absolute mass scale. There are different mechanisms to generate mass for the neutrinos. A simple extension of the SM with right-handed neutrino singlets for instance can accommodate the neutrino mass origin and oscillation. It is not known also whether neutrinos are Majorana or Dirac particles, therefore experiments like neutrinoless double beta decay are needed to verify it. For more review see Refs. [41, 42].

Dark matter: We can also consider physics beyond the SM on the cosmological scale. The SM has no appropriate candidate for a dark matter particle, yet the existence of dark matter has been supported by a number of observational evidences, such as the angular rotation curves of galaxies, gravitational lensing and the cosmic microwave background power spectrum. For a long time weakly interacting massive particles (WIMP) have been thought of as the leading candidate to explain the relic abundance of dark matter. However, much of the direct detection and indirect searches for WIMPs have run into disagreement between different experiments. Until today we are still not certain of the dark matter's nature. Several proposals such as modified Newtonian gravity or primordial black holes, which do not require a particle description of dark matter, have been examined, and they are still in tension with certain phenomenological observations[9]. For more information on dark matter we refer the reader to the reviews [44, 45].

Baryogenesis: On the aesthetic point of view in cosmology, the abundance of matter compared to anti-matter also demands a more dynamical solution of baryogenesis. To achieve baryon asymmetry it is necessary to satisfy the Sakharov condition [46]. However, CP-violation in the SM is not sufficient enough to achieve the baryon asymmetry ratio that we observe in cosmology today, therefore extra ingredients in BSM physics are required. Several proposals to explain baryogenesis have been put forward, such as GUT baryogenesis, Affleck-Dine mechanism [47] and leptogenesis [48][10]. Within the leptogenesis framework, one can invoke the vacuum structure of $SU_{\mathrm{L}}(2) \times U_{\mathrm{Y}}(1)$ to generate baryon number violation via sphalerons.

Strong CP-problem: From the perspective of vacuum topology, besides playing a role in leptogenesis, the complicated vacuum structure of non-abelian gauge theory also causes a fine-tuning problem in another gauge sector, namely in QCD. Such an extreme fine-tuning of vacuum θ-angle is commonly denoted as the strong CP-problem [50]. One can relate the dark matter with the strong CP-problem and try to find a solution addressing both the problem in one framework. Peccei and Quinn [51] for instance proposed a new degree of freedom which can solve both the problems mentioned. By

[9] Actually, modified Newtonian gravity can satisfy the baryonic Tully-Fisher relation better than ΛCDM, see the recent comparison fit of gas rich galaxies result by McGaugh [43].

[10] See Ref. [49] for more review.

postulating a new hypothetical pseudoscalar particle called axion, they managed to bring the θ-angle dynamically to zero, while at the same time providing a dark matter candidate. The strong-CP problem can also be solved in conjunction with the non-trivial topology in extra dimensions [52, 53].

Dark energy: From the cosmic microwave background we know that the universe is expanding, requiring a negative pressure or the *dark energy* for explanation. So far we do not know the nature of dark energy, as its contending candidate consists of a constant, some new fields, quintessence or topological effects. It could also be that we are using an oversimplified Friedmann-Robertson-Walker metric to describe our universe. The void model for instance can explain the expansion of the universe by giving up the assumption of homogeneity. For more reviews see Refs. [54, 55].

From all the arguments given above, the SM necessarily has to be extended. However, from all the precision tests conducted to test the model of Beyond Standard Model (BSM), experimental data has put some very tight constraints on most of the BSM models so far. As mentioned in Sec. (3.2), the Higgs boson mass will usually suffer the large fine-tuning of mass scale if the SM is embedded into a theory with higher energy scale, unless that the UV-theory also provides a mechanism to solve the hierarchy problem, or it consists of a new concept which is non-field theoretic. Therefore it is more advantageous from the naturalness perspective that the BSM physics can be accommodated without the need of introducing an intermediate scale between Fermi scale and Planck scale. One can be ambitious in constructing models which try to accommodate most of BSM phenomenon in one framework without the necessity for a new intermediate scale, for instance a minimal extension of three generations of neutrino like νMSM [30]. Such a minimal extension is surprisingly an economical way to describe all the phenomenology of fundamental interactions of elementary particles at the weak scale, as it provides solutions for problems in the SM such as neutrino oscillation and masses, dark matter, baryon asymmetry in one stroke. The strong CP-problem can be delayed to Planck scale physics while dark energy can be explained by one of the alternative models, such as the void model or back-reaction. The important point to be taken here is that one does not necessary need a new intermediate energy scale between the Planck and Fermi scale to accommodate the observational evidences that require an extension of the SM. The spirit of this thesis lies on the assumption that there is no other new physical scale other than the electroweak and Planck scale. In this work we will investigate the implications of this assumptions for the Higgs mass and the hierarchy problem.

Chapter 4

Higgs Sector in the Standard Model

In this section we will review some known experimental bounds on what we know about the SM Higgs particle. We will also examine some theoretical constraints on the Higgs mass and the quartic Higgs coupling λ based on theoretical consideration and renormalization group analysis. Although we will evaluate those constraints numerically, it is worth to investigate them analytically to predict their asymptotic behavior. We will then focus on the theoretical constraint from the fine-tuning perspective and examine the Veltman condition, which could be a solution for the hierarchy problem that we have not fully understood. But first let us investigate what we know about the Higgs particle from the experimental perspective.

4.1 Experimental bound on the Higgs mass

From the SM point of view, every field that acquires a mass due to spontaneous symmetry breaking will necessary couple to the Higgs scalar field. This is easy to see by substituting the vev with the Higgs field in the SM Lagrangian. For a tree level decay of Higgs particle, the main decay product would be a pair of particle and anti-particle. The decay rate of the Higgs is proportional to the mass of the decay products, as the coupling constants of such a tree level decay is dictated by the respective Yukawa coupling, which is proportional to the decay product's mass. Hence, Higgs particle prefers to decay into the heaviest particle subjected to kinematic constrains. The branching ratio for the Higgs particle is plotted in Fig. (4.1). For a Higgs mass lower than 130 GeV it tends to decay into a $b\bar{b}$ quark pair. Taking the off-shell effect into account, a SM Higgs particle will decay into a W^+W^- or ZZ pair predominantly if its mass is higher than 130 GeV. To extract the Higgs mass in a collider we need to reconstruct the four-momenta of its decay products.

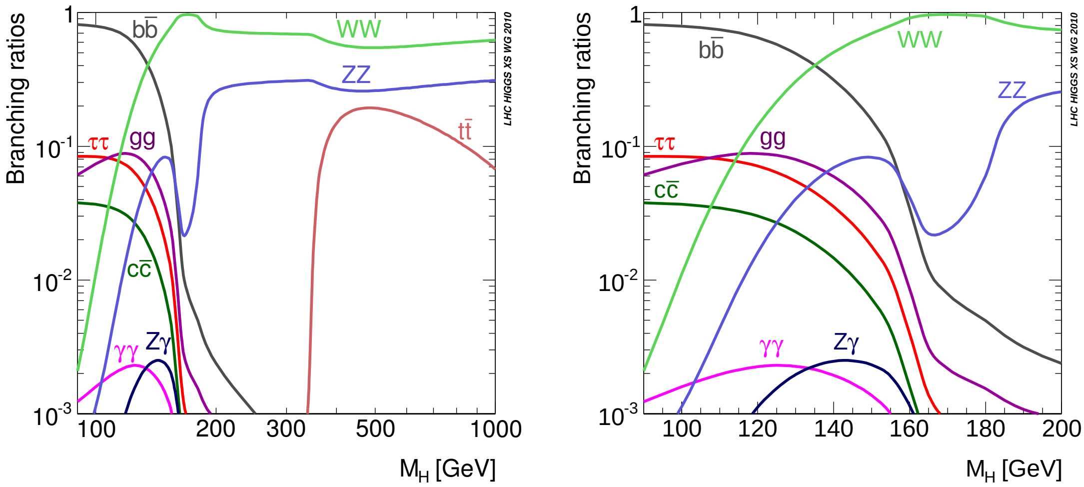


Figure 4.1: Branching ratio of the SM Higgs as a function of its mass. Off-shell effects are also included in the plot. The plot is taken from the LHC Higgs cross section working group [56].

4.1.1 Searches at LEP

The Large Electron-Positron (LEP) collider was built to study the physics of the electroweak scale, including the search for the Higgs particle. The main Higgs production mechanism in LEP is given by the Higgs strahlung process and WW fusion process, depicted in Fig. (4.2). Naively one might include the tree level $e^-e^+ \to H$ process, however as this coupling is exactly the Yukawa coupling, it is proportional to the mass of the electron, which is small compared to the coupling of the electron to the weak bosons. Searches at the first LEP experiment (LEP1) focused mainly on the decay of the Z resonance into a Higgs and two light fermions. At the same experiment one can also investigate the decay of a Z to a Higgs and a photon via triangular loop processes. The absence of any Higgs signal by collaborations at LEP1 allowed us to set the 95% confidence level (CL) limit on $M_H \geq 65.2\,\text{GeV}$. As for searches at LEP2, with the increased center of mass energy to 209 GeV, the dominant production process is Higgs-strahlung, i.e. the off-shell Z boson decays into a Higgs particle and a real Z boson. With no convincing evidence of a Higgs signature, LEP2 has put a lower bound on Higgs mass, $M_H > 115\,\text{GeV}$ [57]. It should be mentioned however that there is a 1.7σ excess of events for a Higgs boson in the vicinity of 116 GeV, but this excess is not enough for evidence (3σ) or discovery (5σ). For a future e^+e^- collider with a center of mass energy be-

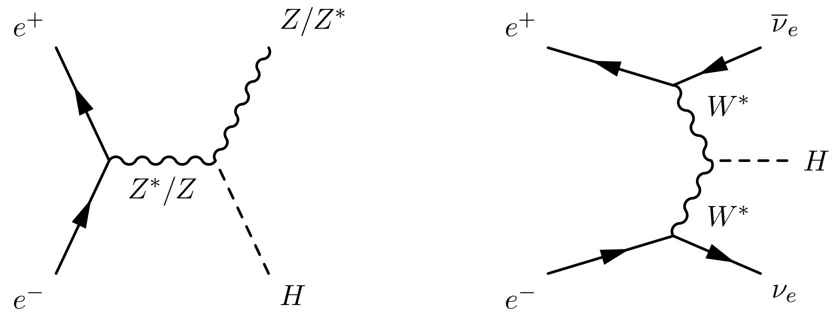


Figure 4.2: Higgs-strahlung process (with real Z boson at the end vertex) on the left. If we replace the on-shell Z with its off-shell counterpart, we would have a Z resonance decay. WW fusion process is depicted on the right.

yond LEP2, some other production channels will contribute to the Higgs searches. The sub-dominating production is contributed by the WW fusion method, see Fig. (4.2). There are also other production mechanism such as the ZZ fusion and double Higgs production, which are however of higher order in terms of the couplings and hence their production cross sections are smaller than Higgs-strahlung processes and WW fusion. Other proposals such as muon collider, are also very attractive phenomenologically, as the Higgs boson can be produced at tree level as a s-channel resonance. Of course any future linear collider should be built for precision measurements if the LHC manages to find the Higgs boson.

4.1.2 Searches at hadron colliders

Next we turn to hadron colliders and focus mainly on the decay channel for low Higgs mass searches, which will be our case of interest later. The presentation below follows the review from Djouadi [58] and lecture notes from Plehn [59]. First we discuss the production of Higgs bosons with associated production of electroweak gauge bosons. Producing a Higgs particle with this method requires quark and anti-quark, which is more preferable in Tevatron, compared to LHC which would rely on the anti-sea quarks in the proton. It has been shown that with W/Z decaying into leptons, this production channel is the most promising detection mode at Tevatron Run II for a light Higgs boson which decays predominantly into a $b\bar{b}$ pair. Some of the decay channels that are prominent in associated Higgs production with W/Z are given below:

- $H \to b\bar{b}$: This channel is the dominant decay mode for $M_H \leq 135\,\text{GeV}$ and its distinctive signature is dilepton/isolated lepton/missing energy plus two jets. This channel is utilized in Tevatron as the backgrounds are not too large and they have been calculated to next-to-leading (NLO) in QCD.

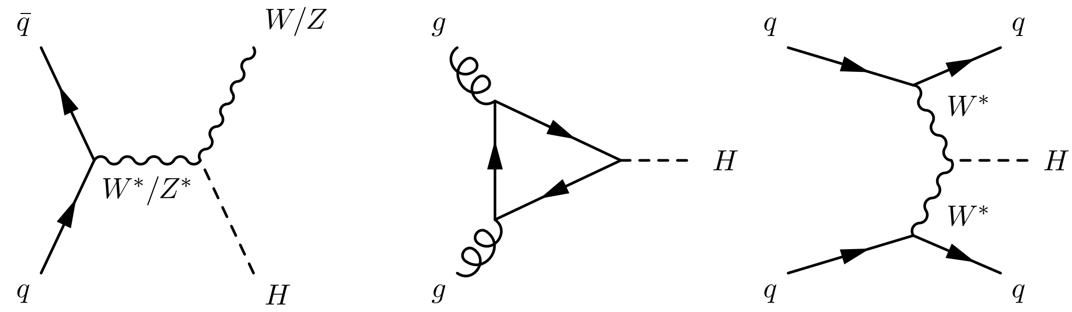


Figure 4.3: Main Higgs production channels in a hadron collider: Higgs production via associated production with W/Z (left), gluon fusion (center) and vector boson fusion (right).

- $H \rightarrow WW^{(*)}$: The channel is the dominant decay channel for $M_H \geq 135\,\text{GeV}$ and is more promising at LHC for mass range $M_H \approx 160 - 180\,\text{GeV}$, where the Higgs boson decays almost 100% into this state. Distinctive signatures include for instance trilepton events and like-sign dileptons with two jets.

- $H \rightarrow ZZ^{(*)}$: This channel has a too small branching ratio for $M_H \leq 180\,\text{GeV}$ when one of the Z is virtual. Above the ZZ threshold, the associated Higgs production with weak bosons yields a small cross section at the Tevatron. At the LHC this cross section is sizable only after we take the leptonic branching ratio of Z and W into account.

- $H \rightarrow \gamma\gamma$: This channel is not that useful in Tevatron as it is too rare but it is one of the main detection channel at the LHC. In fact the CMS detector is built and optimized to detect the two photon final state. The backgrounds are similar to the one where the Higgs boson is produced in gluon fusion, which we will discuss shortly.

In the LHC, protons are bombarded together to produce Higgs particles via gluon fusion, as gluons are relatively abundant inside the proton. However, the massless gluons do not couple directly to the Higgs, so the process occurs only at loop level. Nevertheless the large coupling of the $SU_{\text{C}}(3)$ gauge group can provide a compensation against the loop suppression factor. The gluon fusion vertex is depicted in Fig. (4.3). Since the Higgs particle prefers to couple to the heaviest particle in mass spectrum, we would expect that the dominant contribution comes from the top quark in the triangle loop. To discuss the production of Higgs we would need to understand the gluon emission inside proton, and good understanding of QCD background is also necessary for accurate Higgs production and decay rate estimation. It became apparent that this production channel will hinder us from looking at the hadronic decay mode due to the large QCD background. We will list the decay modes below:

- $H \to b\bar{b}$: In gluon fusion, the huge background signal of $gg \to b\bar{b}$ will render the search of Higgs boson in the decay channel $H \to b\bar{b}$ useless. This channel will not be triggered at the LHC.

- $H \to \gamma\gamma$: This channel is the so-called "silver channel" for Higgs boson detection with mass below 150 GeV. The main problem for this channel is due to detector background from pions mistaken for photons. Moreover one needs to discriminate the background produced directly from $q\bar{q} \to \gamma\gamma + X$ and also the loop induced $gg \to \gamma\gamma + X$. Since this type of decay is rare, a large amount of luminosity is needed and at low luminosity, a combination of all $H \to \gamma\gamma$ modes is required.

- $H \to ZZ^{(*)}$: The decay of $ZZ^{(*)}$ to four leptons is called the "golden channel" of LHC searches. The main background comes from a continuum of ZZ production which is known precisely. This channel can be utilized to search for a Higgs boson with mass down to 120 GeV, however it is limited by having a too small branching ratio when the Higgs mass is around twice the mass of the electroweak bosons.

- $H \to WW^{(*)}$: The decay of $WW^{(*)}$ to the final state $ll\nu\nu$ serves as one of the most promising detection channels for a light Higgs boson, ranging from $M_H \approx 120$ GeV to twice the Z mass. However it is a hard task to reconstruct the Higgs mass due to the missing energy from the neutrinos, one needs to observe a clear excess of events from background, which has to be known precisely. One can take advantage of the angular correlation to distinguish between the background produced by a W boson and the desired signal. This channel is important for the Tevatron, as combining the result from associated Higgs production they managed to put an exclusion limit for the Higgs mass between $156 - 177$ GeV [60], see Fig. (4.4) for more explanations.

Note that we did not discuss other decay channels like $H \to \tau^+\tau^-$ or $H \to \mu^+\mu^-$. The latter requires a huge amount of luminosity and is more appropriate for future hadron colliders like SLHC and VLHC, while the former is problematic for Higgs mass reconstruction. This channel is however more promising in the vector boson fusion Higgs production.

The vector boson fusion mechanism is depicted in Fig. (4.3). This channel is more viable in the LHC. Each quark-quark pair emits a W boson to create a Higgs boson and subsequently they hadronize into two forward jets, which can be used as a tagger. The

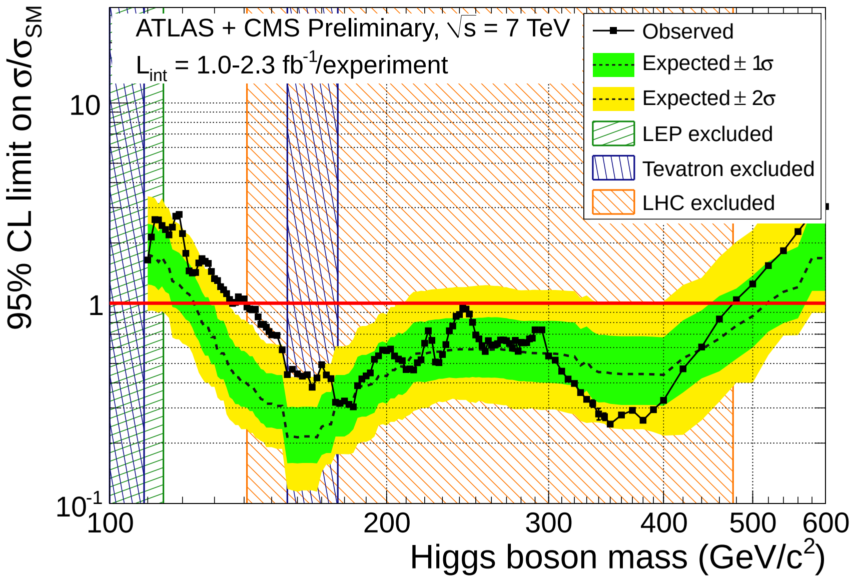


Figure 4.4: The Higgs mass exclusion regions by the LHC, Tevatron and LEP are shown [8]. The dashed line indicates the median expected 95% CL on the signal strength modifier $\sigma/\sigma_{\rm SM}$ value for the background-only hypothesis, while the green and yellow bands indicate the range expected to contain 68% and 95% of all observed limit excursions from the median. The observed data is represented by the black solid curve and once it dips down below 1, its Higgs mass region is excluded at 95% CL as we have measured less events than predicted by the SM Higgs. The region of $141 - 476$ GeV has been excluded at 95% CL by the LHC recently.

Higgs particle and its decay products are expected in the central detector. Therefore what experiments are looking for are events with two forward jets and decay products of the Higgs boson in the central detector. The signature of Higgs decay modes are slightly altered:

- $H \to \tau^+\tau^-$: This channel plays an important role as a discovery channel for a Higgs boson with $M_H \leq 130\,\text{GeV}$ if the luminosity is larger than $30\,\text{fb}^{-1}$. This channel is useful for BSM scenarios and it has been shown that the discovery of one Higgs boson over the entire supersymmetry parameter space is guaranteed.
- $H \to \gamma\gamma$: This decay mode is rare and has to be combined with other production processes that produce diphoton discussed above in order to allow for significance larger than 5σ.
- $H \to WW^{(*)}$: This channel can produce a significance larger than 3σ in the

end state of $e\mu + X$ and $ee/\mu\mu + X$ for $M_H \geq 130\,\text{GeV}$ with luminosity greater than $10\,\text{fb}^{-1}$. A significance of 5σ can be obtained when the channels above are combined with the $l\nu jj$ modes, hence $H \to WW^{(*)}$ could be a potential discovery channel.

- $H \to ZZ^{(*)}$: This channel receives a smaller rate and cannot be used if the Higgs mass is less than twice the Z mass.

Again the channel of $H \to b\bar{b}$ is problematic due to the large QCD 4-jet background. For a long time it has been thought that the reconstruction of a light Higgs mass via the $H \to b\bar{b}$ channel is not prominent in gluon fusion or weak boson fusion due to the sizable QCD background coming from off-shell gluon emitted by a quark decaying into $b\bar{b}$. The associated Higgs production with electroweak bosons was thought to be not that useful also in LHC, as we mentioned above, the associated Higgs production requires quark and anti-quark, which would not be the main source in a proton-proton collider. It turns out that we can turn these two difficulties into an advantage by looking for boosted Higgs bosons with the Higgs decaying into $b\bar{b}$ in a back-to-back associated Higgs production with electroweak bosons [61]. The transversely boosted Higgs' decay products have large transverse momentum to be tagged. Furthermore the $Z \to \nu\bar{\nu}$ channel produced by Higgs with associated Z bosons will be "visible" due to large missing transverse energy.

Now combining all the different production and decay channels mentioned above, CMS and ATLAS have managed to exclude the Higgs mass for a larger mass region. With an integrated luminosity of about $1\,\text{fb}^{-1}$, ATLAS has excluded the Higgs boson mass range of $146 - 232\,\text{GeV}$, $256 - 282\,\text{GeV}$ and $296 - 496\,\text{GeV}$ at 95% CL [6], while CMS has put a 95% CL exclusion range of $145 - 216\,\text{GeV}$, $226 - 288\,\text{GeV}$ and $310 - 400\,\text{GeV}$ [7]. By combining the ATLAS and CMS analysis, the LHC has excluded the SM Higgs mass range from $141 - 476\,\text{GeV}$ at 95% CL. In Fig. (4.4) the Higgs mass exclusion range by this combined results is plotted.

4.1.3 Electroweak precision test constraint

All quantum particles receive radiative corrections via loop diagrams and if the Higgs particle exists, it also contributes to electroweak observables that we have measured to high accuracy. From such high-precision experiments we can constrain the Higgs mass value and among all the highly-precise measured observables, the W boson mass and the effective leptonic weak mixing angle $\sin^2\theta_{\text{eff}}$ have imposed the most stringent constraint on the Higgs mass value. The W boson mass was measured by searching

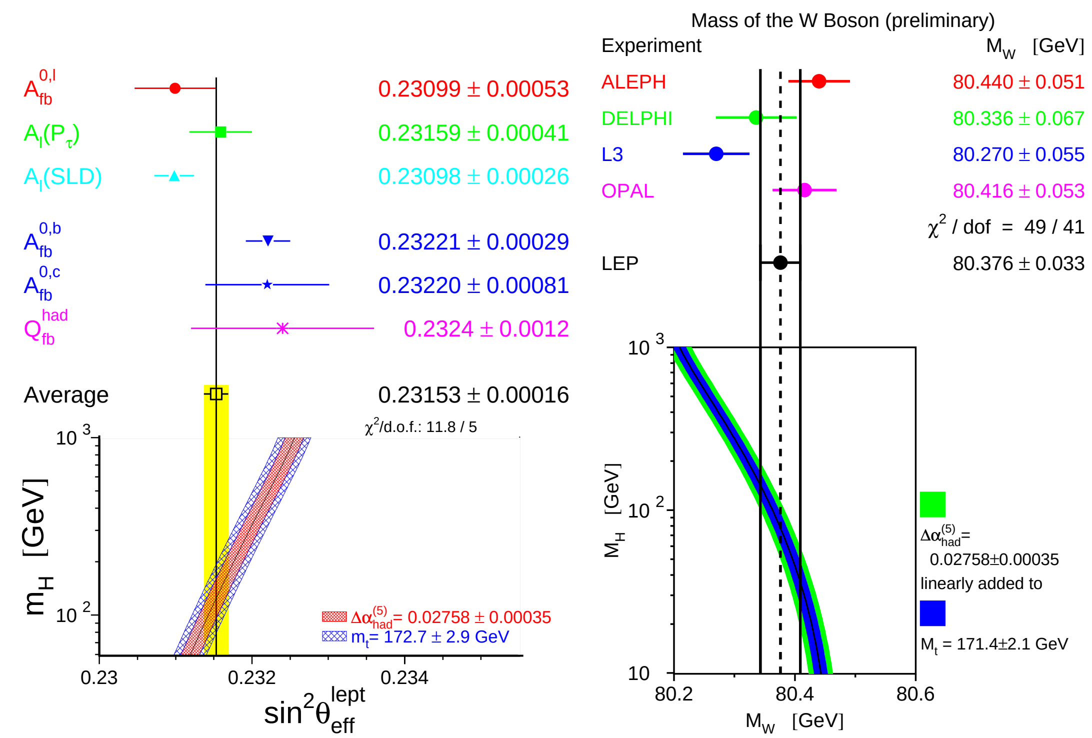


Figure 4.5: Constraints on the Higgs mass by measurements of electroweak precision observables. The hatched bands correspond to the theoretical prediction for forward-backward asymmetry and the W boson mass as a function of the Higgs boson mass. The experimentally measured value is projected down to the respective axis [57]. The measured values of M_W on the right plot have to be updated with the latest result from Tevatron searches, see Ref. [62].

for decay resonance of W boson while the effective weak mixing angle was measured in forward-backward asymmetries and also polarization asymmetries.

By including all the electroweak precision observables such as the Z boson mass, fine structure constant $\alpha(M_Z)$, strong coupling constant $\alpha_s(M_Z)$, and the top quark mass, one can perform a global fit to all the measurements to determine the most preferred Higgs mass value. In the complete fit performed by the GFitter group [63], the results of direct Higgs boson searches from LEP and Tevatron are also taken into account. With the uncertainties from $\alpha(M_Z)$, $\alpha_s(M_Z)$, hadronic contribution $\Delta^{\text{had}}\alpha(M_Z)$ and top mass M_t included, one obtains the predicted SM Higgs boson mass from electroweak precision constraint to be $M_H = 120^{+8[+21]}_{-10[-11]}$ GeV. The Fig. (4.6) shows $\Delta\chi^2$ as a function of M_H. The 95% CL upper limit can be approximated by $\Delta\chi^2 = 4$, which corresponds to 95%

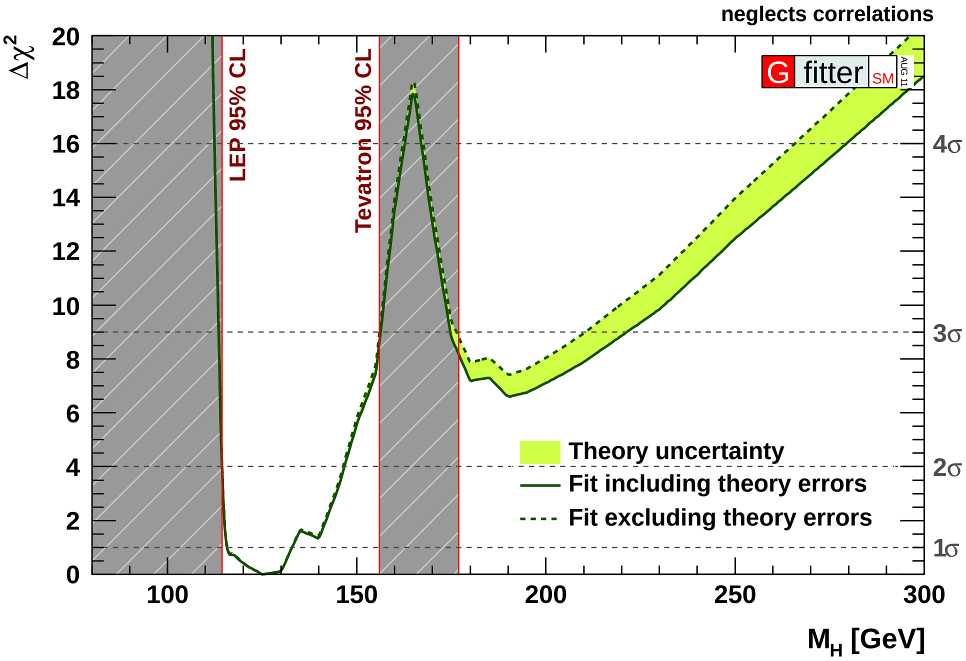


Figure 4.6: The $\Delta\chi^2$ of a fit to electroweak precision observables as a function of M_H by the GFitter group [63]. The solid line is produced with all the data from direct and indirect searches included. The yellow band is the estimated theoretical error from unknown higher order corrections. The gray region represents the excluded region by direct searches from LEP and Tevatron.

CL upper bound of Higgs mass lower than 145 GeV at present.

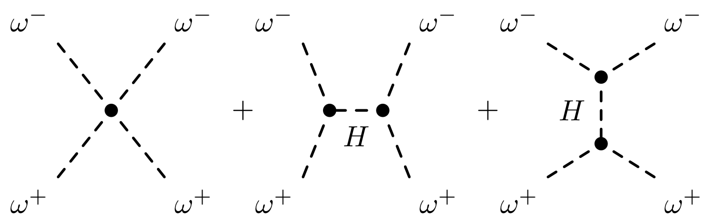
In Fig. (4.5) one can see that there are significant discrepancies between the measurements of leptonic and hadronic asymmetries. The former favor a light Higgs boson, which is also the case for W boson mass measurements while the latter yield a more favorable value for a heavier Higgs mass. Because of the large discrepancy between the hadronic asymmetries and the rest of the electroweak precision observables, the fit to the SM is rather poor. One can speculate a new physics effect which only affects the hadronic asymmetry result, namely the measurement on $Z \to b\bar{b}$, however this is in general difficult to implement without spoiling the agreement of the branching ratio of $Br(Z \to b\bar{b})/Br(Z \to \mu^+\mu^-)$ with the b-quark forward-backward asymmetry. If one removes the deviation of hadronic forward-backward asymmetry from the global fit due to its underestimated systematical errors, one would obtain a best value for the Higgs mass which is below 89 GeV. The conclusion is that from the fit of all the electroweak precision observables, we obtain a strong indication that the SM Higgs must be light.

4.2 Theoretical constraints on the Standard Model Higgs boson

By requiring the SM Higgs sector to be unitarity safe and perturbatively valid, one can derive some constraints on the Higgs mass. We start off by reviewing the issue of unitarity and then investigate the triviality and vacuum stability bound. The theoretical constraint on the SM Higgs mass from the fine-tuning perspective will be discussed afterwards.

4.2.1 Unitarity constraints

One of the main problems of the old Fermi theory of weak interactions was that unitarity is violated at energies close to the Fermi scale. One way to overcome this problem is by introducing intermediate massive vector boson $W^{\pm}$ and Z, however there is still a potential problem in the high energy region, where the interaction of longitudinal components of massive gauge bosons grow with their momenta and subsequently violate unitarity. Since the interactions of the longitudinal components of the massive gauge fields at high energies can be approximated by the interactions of Goldstone bosons $\omega^{\pm}$ and z due to the *equivalence theorem* [64], we will just focus on the amplitude of $W^+W^- \to W^+W^-$ in terms of Goldstone boson scattering $\omega^+\omega^- \to \omega^+\omega^-$. In order to prevent the growth of $\omega^+\omega^- \to \omega^+\omega^-$ with higher energy, it is necessary to introduce a Higgs boson to unitarize the Goldstone boson scattering. For tree level processes, there are three diagrams that contribute to the $\omega^+\omega^- \to \omega^+\omega^-$ amplitude:



(4.1)

These amplitudes can be calculated to be:

$$T = -\frac{im_H^2}{v^2}\left(2 + \frac{m_H^2}{s - m_H^2} + \frac{m_H^2}{t - m_H^2}\right), \tag{4.2}$$

and could still potentially violate the unitarity bound if the Higgs mass is large enough. To see this explicitly we shall examine the violation of unitarity in terms of the S-matrix and thus deduce an upper bound on the SM Higgs mass.

Every process in QFT involving external particles and interactions can be described by the S-matrix, which must be unitary in order to conserve probability. The S-matrix can be parameterized by two parts, consisting of interaction between in and out particles with amplitude T, and no interactions at all:

$$S = \mathbb{1} + iT. \tag{4.3}$$

In order for probability to be conserved, $S^\dagger S$ must be equal to unity. Hence we obtain:

$$\mathbb{1} \stackrel{!}{=} S^\dagger S = \mathbb{1} + i(T - T^\dagger) + T^\dagger T, \tag{4.4}$$

which can be simplified to:

$$T^\dagger T = -i(T - T^\dagger). \tag{4.5}$$

The operator given above only makes sense if we evaluate the correlation function with it. Assuming that we are looking for a forward scattering, the right hand side of Eq. (4.5) can be further simplified as follows:

$$-i\langle T - T^\dagger \rangle = -i\langle T - T^* \rangle = 2\mathrm{Im}T(\theta = 0), \tag{4.6}$$

where the angle $\theta = 0$ denotes the forward scattering process. We obtain the cross section of such a process as:

$$\sigma = \frac{1}{2s} T^\dagger T = \frac{1}{s} \mathrm{Im}T(\theta = 0). \tag{4.7}$$

This formula is known as the *optical theorem*. Next the transition amplitude is decomposed into its partial waves:

$$T = 16\pi \sum_{l=0}^{\infty} P_l(\cos\theta) a_l, \tag{4.8}$$

where P_l is the Legendre polynomial and a_l is the decomposition coefficient. For a $2 \to 2$ process, the cross section is given by $\mathrm{d}\sigma/\mathrm{d}\Omega = |T|^2/(64\pi^2 s)$, and by integrating the whole solid angle we get:

$$\sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1)|a_l|^2. \tag{4.9}$$

By applying the optical theorem above, we can equate the imaginary part of a_l with its absolute value and obtain the unitary bound:

$$(\mathrm{Re}\, a_l)^2 + \left(\mathrm{Im}\, a_l - \frac{1}{2}\right)^2 = \frac{1}{4} \quad \Rightarrow |\mathrm{Re}\, a_l| < \frac{1}{2}. \tag{4.10}$$

In order to calculate the perturbative unitary constraint to Goldstone scattering we just need to evaluate the a_0 term in partial wave expansion:

$$\begin{aligned} a_0 = \frac{1}{16\pi s}\int_{-s}^{0} |T| &= -\frac{m_H^2}{16\pi v^2}\left[2 + \frac{m_H^2}{s - m_H^2} - \frac{m_H^2}{s}\log\left(1 + \frac{s}{m_H^2}\right)\right] \\ &\xrightarrow[s \gg m_H^2]{} -\frac{m_H^2}{8\pi v^2}. \end{aligned} \tag{4.11}$$

Combining this result with the unitary bound above we can see that the Higgs boson mass should not be heavier than 870 GeV. This is true only if we take the limit of s to infinity, the bound changes had we chosen a finite value. In fact, the scattering of Goldstone bosons can couple to another channel such as $\omega^+\omega^- \to zz$, $\omega^+\omega^- \to HH$ and $\omega^+\omega^- \to zH$. Doing the analysis properly we obtain the Higgs mass bound $m_H \leq$ 710 GeV.

4.2.2 Triviality and vacuum stability

As mentioned in Sec. (2.5.1), every coupling constant in QFT runs with the energy. In the SM the running of the quartic Higgs coupling λ is dictated by the heaviest elementary particles, namely the Higgs boson, the top quark and the weak gauge bosons. By computing the one-loop beta function of λ one obtains:

$$\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = \frac{1}{8\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{8}g_2^4 + \frac{3}{16}(g_2^2 + g_1^2)^2\right]. \tag{4.12}$$

First we study the regime where λ becomes strong, rendering the Higgs sector non-perturbative. From Eq. (4.12) we know that if λ is dominating, then we could just keep the term of order λ^2 while discarding the rest. Hence we have:

$$\mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = \frac{24\lambda^2}{16\pi^2} + \mathcal{O}(\lambda). \tag{4.13}$$

Solving it would give us:

$$\lambda(\mu) = \frac{\lambda_0}{1 - \frac{3}{2\pi^2}\lambda_0 \log\left(\frac{\mu}{v}\right)}, \tag{4.14}$$

where $\lambda_0 = \lambda(\mu = v)$ and v is the vacuum expectation value. Notice that λ will hit a pole when the denominator in Eq. (4.14) becomes zero, i.e. the Higgs sector is non-perturbative at a scale

$$\begin{aligned} \Lambda_{\text{pole}} &= v \exp\left(\frac{2\pi^2}{3\lambda_0}\right) \\ &= v \exp\left(\frac{4\pi^2 v^2}{3m_H^2}\right). \end{aligned} \tag{4.15}$$

Such a pole is called *Landau pole* and it gives us the maximum scale where the SM Higgs sector is valid. The only way for a typical ϕ^4 theory to remain perturbative for all scales is to set $\lambda = 0$ identically, rendering the theory trivial, hence the name of this bound, *triviality bound* [65]. Note that this breakdown indication from a perturbation perspective does not mean that our quartic coupling constant does indeed diverge, as perturbations are no longer reliable for studying the evolution of λ. However numerical studies such as lattice computations do indicate that this coupling diverges approximately at the scale implied from the perturbative calculation above.

Another constraint, namely the *vacuum stability* [66, 67, 68] can be investigated by going back to our RGE in Eq. (4.12). We want to know how we can guarantee that $\lambda > 0$ such that the Higgs potential is bounded from below. Since $\lambda \approx 0$ we will neglect it in Eq. (4.12) and we obtain:

$$\begin{aligned} \mu\frac{\mathrm{d}\lambda}{\mathrm{d}\mu} &\approx \frac{1}{8\pi^2}\left[-3\lambda_t^4 + \frac{3}{8}g_2^4 + \frac{3}{16}(g_2^2 + g_1^2)^2\right] \\ \Rightarrow \lambda(\mu) &\approx \lambda_0 + \frac{1}{8\pi^2}\left[-3\lambda_t^4 + \frac{3}{8}g_2^4 + \frac{3}{16}(g_2^2 + g_1^2)^2\right]\log\left(\frac{\mu}{v}\right). \end{aligned} \tag{4.16}$$

Therefore for each $\lambda(\mu = \Lambda_{\text{stable}}) = 0$, we can obtain the corresponding Higgs mass via matching of λ_0. A Higgs mass below the vacuum stability curve can be excluded with this method. The triviality and vacuum stability bound can be neatly plotted in a diagram showing the allowed Higgs mass region, see Fig. (4.7). In our plot we did not use the approximation given in Eq. (4.16) and Eq. (4.13), instead we solved the beta function numerically including all the contributions. The theoretical uncertainty is represented by the width of the curves, which we define as the difference of the Higgs mass obtained via one-loop and two-loop beta function used for both for triviality and vacuum stability. This definition of theoretical uncertainty parameterizes the error of omitting three-loop beta function in our computation. We will describe the method for solving the running of λ and the possible uncertainties appearing later in Sec. (4.3-4.4).

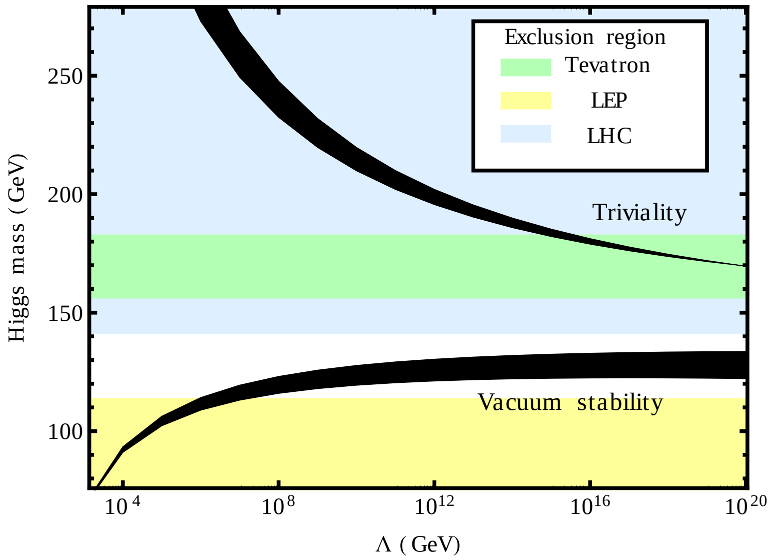


Figure 4.7: Triviality and vacuum stability bound for the SM Higgs boson. The yellow (green) band plotted describes the excluded Higgs mass region at 95% CL based on direct searches at LEP (Tevatron), while the light blue region represents the Higgs mass exclusion at 95% CL by the LHC.

The vacuum stability plotted in Fig. (4.7) does not take the effective potential of Higgs field into account, as only the condition $\lambda(\Lambda_{\text{stable}}) = 0$ is implemented. In order to obtain a more precise prediction for the Higgs mass from vacuum stability, the effective potential method has to be applied [67, 68]. Later in Sec. (5.1.1), when precision to $\mathcal{O}(1\,\text{GeV})$ accuracy plays a role in Higgs mass determination, we will use the effective potential method in order to be as precise as possible in extracting the Higgs mass from the vacuum stability bound.

Note that the question of tunneling rate w.r.t. the age of universe is not included in our consideration, refer to the paper from Ellis et al. [69] or Sher [66] for a more thorough review. The excluded regions based on the LEP, Tevatron and LHC searches [70, 71, 8] are also plotted in Fig. (4.7). If the Higgs boson mass is found to be between 130 GeV and 150 GeV, the validity of the SM can be extended up until the Planck scale $M_{pl} \approx 10^{19}$ GeV, opening up the possibility of a desert, i.e. no intermediate scale exists between the Planck scale and the electroweak scale. It is worth mentioning that the triviality and vacuum stability bounds overlap at energy scale of magnitude lower than the Planck scale, if the SM Higgs boson is coupled to generic fourth generation of quarks and leptons [72, 73]. Similar analysis can be carried out for a number of quark and lepton generation greater than four, and by the same reasoning one would also expect that the triviality and vacuum stability bounds overlap at low energy scale. Therefore, one can assume that a new physics must appear at energy lower than the Planck scale if the SM contains more than three generation of fermions. It is interesting that the SM could only be valid up to the Planck scale only if nature is endowed with three generation of chiral leptons and quarks.

As mentioned before in Sec. (3.1), if the Yukawa coupling of neutrino is of order $\mathcal{O}(1)$, it would alter the running of the triviality and vacuum stability bound. However this effect only influences the beta function at scales larger than the Majorana mass, which could be of the order of $\mathcal{O}(10^{15}\,\mathrm{GeV})$. We refer the reader to the paper from Casas et al. [74] for more information. As the inclusion of such heavy Majorana particles would increase the unknown parameters later in our Higgs mass prediction, we will just neglect it in order to obtain a generic SM prediction for the Higgs mass when we impose certain conditions on λ later in Sec. (5.1), but one should keep in mind that the above mentioned scenario could happen. It is worth mentioning that the predicted Higgs mass region for this scenario with extra inclusion of $\mathcal{O}(1)$ neutrino Yukawa coupling has been excluded by the recent ATLAS and CMS searches at 95% CL [7, 6].

4.3 Matching $\overline{\mathrm{MS}}$ coupling and physical mass

As we have encountered the triviality and vacuum stability bounds calculated via RGEs, it is worth to be more precise in this section about the numerics and prediction of the Higgs boson mass, since later in this work we would like to have a precise prediction of the Higgs mass due to boundary conditions set for λ at the Planck scale. Recall that the $\overline{\mathrm{MS}}$ couplings are not the physical couplings that can be directly observed from experiment, therefore one needs to match the $\overline{\mathrm{MS}}$ couplings to physical renormalized couplings and masses. In the SM most of the couplings are given in $\overline{\mathrm{MS}}$ scheme and it is convenient to calculate the running of couplings in this scheme. However in this work we would like to know the Higgs mass from certain conditions of the couplings. Therefore precise matching between the physical masses and the $\overline{\mathrm{MS}}$ couplings must be performed.

We start off by first discussing the exact matching of g_1 and g_2 electroweak couplings. It is customary to extract the $\overline{\mathrm{MS}}$ gauge coupling of g_1 and g_2 using the $\overline{\mathrm{MS}}$ definition of fine structure constant $\hat{\alpha}$ and electroweak mixing angle $\sin^2\theta_W^{\overline{\mathrm{MS}}}$, where:

$$\hat{\alpha}^{-1}(\mu) \equiv 4\pi \frac{g_1^2(\mu)+g_2^2(\mu)}{g_1^2(\mu)g_2^2(\mu)}, \tag{4.17a}$$

$$\sin^2\theta_W^{\overline{\text{MS}}}(\mu) \equiv \frac{g_1^2(\mu)}{g_1^2(\mu)+g_2^2(\mu)}. \tag{4.17b}$$

We follow the convention from PDG and define the electroweak mixing angle at scale $\mu = M_Z$ as $\sin^2\theta_W^{\overline{\text{MS}}}(M_Z) \equiv \hat{s}_Z$. The best known value for $\hat{\alpha}(M_Z)$ and $\hat{s}_Z^2$ is listed in Appx. (C). It is interesting to match the gauge coupling to electroweak boson's mass. As mentioned in a paper by Hambye & Riesselmann [75], the matching correction δ_W and δ_Z used in relating the $\overline{\text{MS}}$ couplings g_1 and g_2 with the physical masses of the weak bosons:

$$g_2^2(M_Z) = \frac{4M_W^2}{v^2}(1+\delta_W), \tag{4.18}$$

$$g_2^2(M_Z)+g_1^2(M_Z) = \frac{4M_Z^2}{v^2}(1+\delta_Z), \tag{4.19}$$

only contributes a correction of $\delta_W \approx -0.4\%$ and $\delta_Z \approx 0.7\%$ respectively.

The main uncertainty of matching conditions required for predicting Higgs mass comes from the matching error of the top Yukawa and the Higgs quartic coupling with their respective pole masses:

$$\lambda(\mu) = \frac{M_H^2}{2v^2}(1+\delta_H(\mu)), \tag{4.20}$$

$$\lambda_t(\mu) = \frac{\sqrt{2}M_t}{v}(1+\delta_t(\mu)). \tag{4.21}$$

In general should we want to compute the running coupling with the two-loop beta function, the matching conditions have to be imposed at one-loop level for consistency. Since our goal is to minimize the error of matching the physical and $\overline{\text{MS}}$-coupling, we will use the relevant matching conditions mentioned below for top and Higgs mass, keeping in mind that some of these higher order matching conditions already exceed the precision needed for our computations with the two-loop beta function.

We will now list the matching conditions for $\lambda(\mu)$ as calculated by Sirlin & Zucchini in Ref. [76]:

$$\delta_H(\mu) = \frac{M_Z^2}{32\pi^2 v^2}\left[\xi f_1(\xi,\mu) + f_0(\xi,\mu) + \xi^{-1} f_{-1}(\xi,\mu)\right], \tag{4.22}$$

where $\xi \equiv M_H^2/M_Z^2$ and each of the function f_i defined as:

$$f_1(\xi,\mu) = 6\log\frac{\mu^2}{M_H^2} + \frac{3}{2}\log\xi - \frac{1}{2}Z\left(\frac{1}{\xi}\right) - Z\left(\frac{c_w^2}{\xi}\right) - \log c_w^2 + \frac{9}{2}\left(\frac{25}{9} - \frac{\pi}{\sqrt{3}}\right), \tag{4.23a}$$

$$f_0(\xi,\mu) = -6\log\frac{\mu^2}{M_Z^2}\left[1+2c_w^2-2\frac{M_t^2}{M_Z^2}\right]+\frac{3c_w^2\xi}{\xi-c_w^2}\log\frac{\xi}{c_w^2}+2Z\left(\frac{1}{\xi}\right)$$
$$+\left(\frac{3c_w^2}{s_w^2}+12c_w^2\right)\log c_w^2-\frac{15}{2}(1+2c_w^2)-3\frac{M_t^2}{M_Z^2}\left[2Z\left(\frac{M_t^2}{M_Z^2\xi}\right)\right.$$
$$\left.+4\log\frac{M_t^2}{M_Z^2}-5\right]+4c_w^2 Z\left(\frac{c_w^2}{\xi}\right), \tag{4.23b}$$
$$f_{-1}(\xi,\mu) = 6\log\frac{\mu^2}{M_Z^2}\left[1+2c_w^4-4\frac{M_t^4}{M_Z^4}\right]-6Z\left(\frac{1}{\xi}\right)-12c_w^4 Z\left(\frac{c_w^2}{\xi}\right)-12c_w^4\log c_w^2$$
$$+24\frac{M_t^4}{M_Z^4}\left[\log\frac{M_t^2}{M_Z^2}-2+Z\left(\frac{M_t^2}{M_Z^2\xi}\right)\right]+8(1+2c_w^4). \tag{4.23c}$$

The terms s_w and c_w represent the on-shell sine and cosine of the Weinberg angle θ_W, as defined in Eq. (3.17a) and Eq. (3.17b), while the function $Z(z)$ is given by:

$$Z(z) = \begin{cases} 2A\arctan(1/A) & \text{if } z > 1/4 \\ A\log\left[(1+A)/(1-A)\right] & \text{if } z < 1/4 \end{cases} \qquad A = \sqrt{|1-4z|}\,. \tag{4.24}$$

With δ_H listed we can choose a suitable matching scale such that the correction δ_H is not larger than order one. An analysis for choosing a suitable μ has been carried out in Ref. [75] and we will just quote the result; the scale of $\mu \approx \max\{M_t, M_H\}$ is found to be appropriate for Higgs mass-coupling matching. We still need to know the matching correction for the top quark in order to find the best matching scale. It turns out that the most convenient matching scale for this work is $\mu = M_t$, as some of the matching conditions are only calculated specifically at this scale.

The matching correction δ_t has been calculated in Ref. [77] and its contribution can be decomposed into different parts:

$$\delta_t(\mu) = \delta_t^{\text{QCD}}(\mu)+\delta_t^{\text{QED}}(\mu)+\delta_t^{\text{W}}(\mu)+\delta_t^{\alpha\alpha_s}(\mu)+\dots \tag{4.25}$$

The terms δ_t^{QCD}, δ_t^{QED}, δ_t^{W} and $\delta_t^{\alpha\alpha_s}$ stand for the QCD contribution, QED contribution, electroweak part and electroweak-QCD mixing, respectively. The one-loop QCD contribution is given by:

$$\delta_{t(1)}^{\text{QCD}}(\mu) = \frac{g_3^2(\mu)}{12\pi^2}\left(3\log\frac{M_t^2}{\mu^2}-4\right). \tag{4.26}$$

We can also determine the changes of the predicted Higgs mass with the implementation of two- and three-loop QCD corrections, denoted by $\delta_{t(2)}^{\text{QCD}}$ and $\delta_{t(3)}^{\text{QCD}}$, in the top mass matching. Using the result given by Melnikov & Ritbergen [78], we obtain the two- and three-loop QCD corrections for $\mu = M_t$ as follows:

$$\delta^{\text{QCD}}_{t(2)}(M_t) = \left(\frac{g_3^2(M_t)}{4\pi^2}\right)^2 (1.0414 N_L - 14.3323)\,, \tag{4.27a}$$

$$\delta^{\text{QCD}}_{t(3)}(M_t) = \left(\frac{g_3^2(M_t)}{4\pi^2}\right)^3 \left(-0.65269 N_L^2 + 26.9239 N_L - 198.7068\right), \tag{4.27b}$$

with N_L as the number of light quarks (compared to the top quark), which we will take to be five. We can generalize the above two- and three-loop QCD matching conditions to arbitrary energy scale with the help of running coupling, but we will not need it for this work. With $\mu = M_t$ set, the two- and three-loop QCD corrections yield $\delta^{\text{QCD}}_{t(2)} \approx -0.01$ and $\delta^{\text{QCD}}_{t(3)} \approx -0.004$ with the top mass chosen to be $M_t = 173.2\,\text{GeV}$.

The matching corrections from QED and weak interactions with the approximated subleading corrections are summarized as[1]:

$$\begin{aligned}\delta^{\text{QED}}_t(M_t) + \delta^{\text{W}}_t(M_t) = {} & -\frac{4\hat{\alpha}(\mu)}{9\pi} + \frac{M_t^2}{16\pi^2 v^2}\left[\frac{11}{2} - \frac{M_H^2}{4M_t^2} - 8\frac{M_H^4}{16M_t^4}\text{sig}(M_H^2, M_t^2)\right. \\ & \left. +2\frac{M_H^2}{4M_t^2}\left(2\frac{M_H^2}{4M_t^2} - 3\right)\log\frac{M_H^2}{M_t^2} - \frac{9}{2}\log\frac{M_t^2}{\mu^2}\right] - 6.9\times 10^{-3} \\ & + 1.73\times 10^{-3}\log\frac{M_H}{300\,\text{GeV}} - 5.82\times 10^{-3}\log\frac{M_t}{175\,\text{GeV}},\end{aligned} \tag{4.28}$$

where the function $\text{sig}(M_H^2, M_t^2)$ is defined as:

$$\text{sig}(M_H^2, M_t^2) := \begin{cases} \left(1 - \frac{4M_t^2}{M_H^2}\right)^{3/2} \text{arccosh}\left(\frac{M_H}{2M_t}\right) & \text{if } M_H^2 \geq 4M_t^2 \\ \left(\frac{4M_t^2}{M_H^2} - 1\right)^{3/2} \text{arccos}\left(\frac{M_H}{2M_t}\right) & \text{if } M_H^2 < 4M_t^2 \end{cases}. \tag{4.29}$$

Observe that we have chosen $\mu = M_t$ such that we are allowed to use the approximation for the subleading term on the last line. Furthermore the electroweak matching condition simplifies, as terms with logarithmic dependence $\log(M_t/\mu)$ drop out. Such a choice $\mu = M_t$ is verified to be appropriate for $M_H/M_t \approx 0.8 - 1.7$ in Ref. [75], which will be the case of interest later.

We can analyze the magnitude of the correction for each individual term in Eq. (4.25); for $M_H/M_t \approx 0.8 - 2$ the maximal correction from the weak interaction part (second term in Eq. (4.28)) only gives $\delta^W_t \approx 0.01$. The QED part (first term in Eq. (4.28)) produces a correction of order $\mathcal{O}(10^{-3})$, which at first sight we could safely ignore compared

[1]The term -6.9×10^{-3} in the original paper from Hempfling and Kniehl [77] has a sign error, which has been corrected.

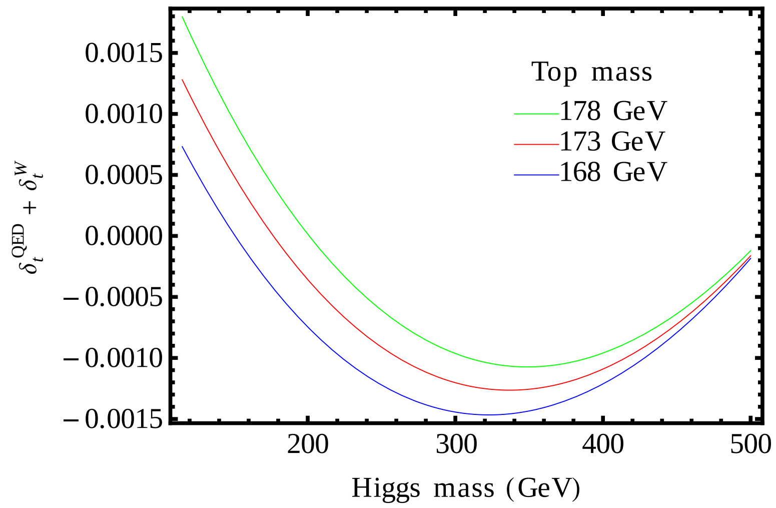


Figure 4.8: Variation of the electroweak and QED matching condition w.r.t. the Higgs mass. Different top masses are imposed for $\delta_t^W + \delta_t^{\text{QED}}$ to illustrate its variation by the top mass.

to the QCD correction $\delta_{t(1)}^{\text{QCD}} = -0.05$. However, the subleading terms in the last line of Eq. (4.28) compensate part of the leading order term of the weak interaction contribution, leaving the total contribution of the weak interaction part to the order of $\mathcal{O}(10^{-3})$. Hence we need to include the QED correction in order to obtain accurate prediction on Higgs mass. Notice that the three-loop QCD effect has the same order of magnitude as the one-loop weak correction, therefore we need to include the electroweak and QED contributions for consistency.

Throughout this thesis we will predict the Higgs mass with certain conditions imposed for λ at different energy scales, henceforth M_H is an output of the computation. However, the Higgs pole mass also appears in the top matching conditions, where M_t is our input parameter. In general it is difficult to implement $\delta_t^W + \delta_t^{\text{QED}}$ in a numerical solution on Higgs mass extraction, as an unknown Higgs pole mass value must be used in top pole mass matching beforehand. Furthermore $\delta_t^W + \delta_t^{\text{QED}}$ varies quite significantly w.r.t. the Higgs mass and the top mass, as illustrated in Fig. (4.8). A way out for the numerical difficulty mentioned above is by first imposing a fiducial value of the Higgs pole mass M_H^f, and iteratively the output Higgs mass value by solving all the RGEs will be used as an input parameter to deduce the subsequent Higgs mass value with all the other input parameters fixed. This way, the generated Higgs mass should converge to the actual physical Higgs mass. The algorithm stops when the difference between the obtained Higgs mass and the subsequent prediction does not exceed our uncertainty

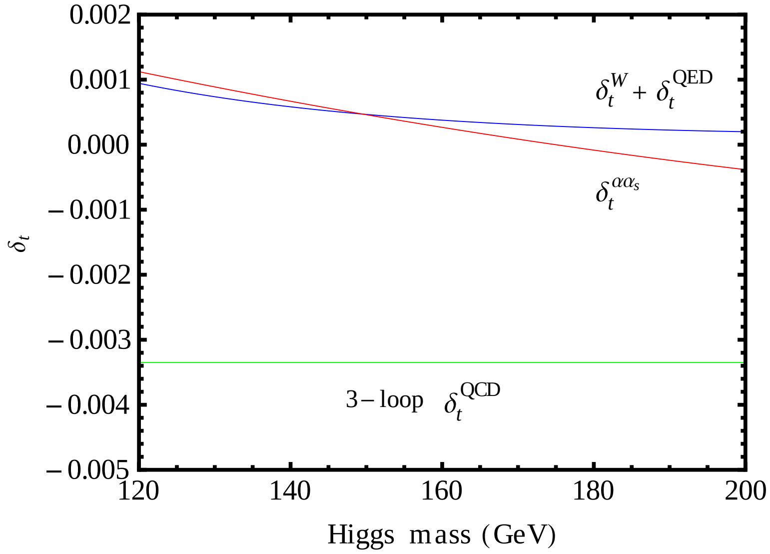


Figure 4.9: Comparison of three-loop QCD matching condition's magnitude with the contribution from $\alpha\alpha_s$ and the $\delta_t^W + \delta_t^{\text{QED}}$ term. The top mass is fixed to be 173.2 GeV. For the low Higgs mass region, the magnitude of electroweak and $\alpha\alpha_s$ contribution is lower than the three-loop QCD matching [79]. Notice that we have only included the first two terms in Eq. (4.30), corresponding to the omission of the last term from Eq. (4.30) which is not stated explicitly in Ref. [79].

tolerance level. We could have ignored the electroweak matching conditions, as this correction is just 2% of the one-loop QCD matching correction. One should be careful with this statement as we have chosen $\mu = M_t$. With higher Higgs mass, the choice $\mu = M_H$ would be more appropriate and this analysis would have to be redone. As we are interested in the precision of the low Higgs mass range later in Sec. (5.1), we can neglect this matching condition. From Fig. (4.9) one can deduce that for the low Higgs mass region, $\delta_t^W + \delta_t^{\text{QED}}$ contributes less than the three-loop QCD matching. For completeness however we include also the complete electroweak matching contribution towards δ_t. As we are interested in the precision of the low Higgs mass range later, i.e. $M_H \in \{115, 170\}$ GeV, a fiducial value of $M_H^f = 130$ GeV is sufficient enough for us to use only one-step iteration in obtaining the actual Higgs mass.

We now come to the last part of our matching condition, namely the matching order of $\mathcal{O}(\alpha\alpha_s)$ to the term $\delta_t^{\alpha\alpha_s}$. The formula is however only partially calculated in the paper by Jegerlehner & Kalmykov [79, 80]. The term $\delta_t^{\alpha\alpha_s}$ is calculated to be:

$$\delta_t^{\alpha\alpha_s} = \delta_{t(A)}^{\alpha\alpha_s} + \frac{1}{2}\delta_{t(1)}^{\text{QCD}} \left[Z_e^\alpha - Z_W^\alpha - Z_\theta^\alpha - \Delta r^\alpha\right]$$

$$+\frac{1}{2}\left[Z_e^{\alpha\alpha_s} - Z_W^{\alpha\alpha_s} - Z_\theta^{\alpha\alpha_s} - \Delta r^{\alpha\alpha_s}\right], \tag{4.30}$$

where the $Z_i^{\alpha(\alpha_s)}$ stems from the decomposition of renormalization constants from the following quantities:

$$\frac{\hat{\alpha}(\mu)}{\alpha(M_Z)} - 1 = Z_e^\alpha + Z_e^{\alpha\alpha_s} + \ldots, \qquad \frac{\sin^2\theta_W^{\overline{\text{MS}}}(\mu)}{s_w^2} - 1 = Z_\theta^\alpha + Z_\theta^{\alpha\alpha_s} + \ldots$$
$$\frac{m_W^2(\mu)}{M_W^2} - 1 = Z_W^\alpha + Z_W^{\alpha\alpha_s} + \ldots, \qquad 1 - \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F s_w^2} = \Delta r^\alpha + \Delta r^{\alpha\alpha_s} + \ldots \tag{4.31}$$

The best fit values of $\alpha(M_Z)$ and the Fermi decay constant G_F are given in Appx. (C). The term $\delta_{t(A)}^{\alpha\alpha_s}$ can be found in Ref. [80][2] while the second term in the first line of Eq. (4.30) can be manually determined via subtraction from terms given in Ref. [77]. The last term in Eq. (4.30) can be theoretically extracted from Refs. [81, 82, 83, 84, 85]. However, the total combination of all the terms has not been published yet, i.e. the last term of Eq. (4.30) is not known. From Fig. (4.9) we can see that by combining both the first two terms in Eq. (4.30), the magnitude of $\alpha\alpha_s$ contribution is only one-third of three-loop QCD contribution. We could assume that the last term in Eq. (4.30) will only contribute almost the same order of correction as in the case of the first two terms. For consistency one has to include the $\mathcal{O}(\alpha\alpha_s)$ term as it contributes the same order as the $\delta_t^W + \delta_t^{\text{QED}}$ terms, however as the complete formula is not yet known, we would necessary omit this contribution. Nevertheless we can estimate the discrepancy of the Higgs mass prediction due to this omission. Later we are only interested in the case for low Higgs mass region, and the omission of the three-loop QCD will yield approximately 1 GeV uncertainty to the Higgs mass prediction. Therefore we can assume that the $\mathcal{O}(\alpha\alpha_s)$ will yield an error less than $\mathcal{O}(1\,\text{GeV})$ to the Higgs mass obtained, and can be safely ignored.

As a conclusion to the matching analysis, we will use only the QCD matching up to three-loop order and the one-loop electroweak contribution for the top mass. The latter can actually be ignored, but we will keep it just for numerical sake. As for the Higgs mass matching condition, Eq. (4.22) will be applied. A complete analysis due to the inclusion and omission of certain matching conditions by different authors and the discrepancies induced for the Higgs mass predictions will be discussed throughly in Sec. (5.2), where the uncertainties become more important in the precision of Higgs mass prediction.

[2]Only the real part of the r.h.s. of Eq. (5.57) in Ref. [80] has to be taken into account. I would like to thank Fred Jegerlehner for pointing out the typo.

4.4 The fine-tuning constraint and the Veltman condition

We now come to the theoretical bound on the Higgs mass from the naturalness perspective, i.e. assuming that the Higgs mass is not finely-tuned, which value of the Higgs mass would this condition predict? With all the SM Lagrangian given we can compute all the loop diagrams in the SM that contribute to quadratic radiative corrections to the SM Higgs mass. A generic bare mass parameter m_{H_B} containing the quadratic divergence can be expressed as [86]:

$$m_{H_B}^2 = m_H^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i) \log^n \left(\frac{\Lambda}{\mu}\right) + \dots \tag{4.32}$$

where we have dropped the non-quadratic divergent contributions. The function c_n contains renormalized couplings that contribute to the self-energy computation. We have implicitly assumed that all the loop integrals are valid up to a common cut-off Λ, which represents the scale where a new physics appears. This is not necessary the case in general as new physics could appear at different energy scale for different type of particles. All the dimensionless coupling constants that appear in c_n are renormalized at a scale μ with the $\overline{\text{MS}}$-scheme. We need to keep the quadratic divergence of Higgs self-energy as we would like to investigate the overall divergence of the Higgs bare mass parameter. The SM cut-off scale Λ is naively interpreted as the scale where physics beyond the SM begins to appear, ranging from the TeV scale to the Planck scale. If the new physics is a QFT, then the term with a heavy intermediate particle mass should replace Λ. On the other hand if the next new physics that the SM couples to consists of a new concept, then it is more difficult to assign a meaning to Λ, however in general it can be interpreted as a field theoretical cut-off, see Sec. (3.2.1) for the discussion. Since m_{H_B} is invariant under renormalization scale change, taking the derivative of Eq. (4.32) w.r.t. μ yields:

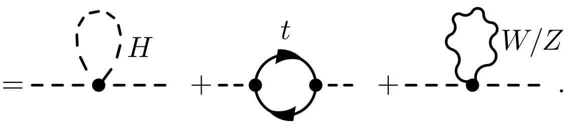
$$0 = \mu \frac{\mathrm{d}m_H^2}{\mathrm{d}\mu} + \Lambda^2 \sum_{n=0}^{\infty} \left((n+1)c_{n+1} - \mu \frac{\mathrm{d}c_n}{\mathrm{d}\mu} \right) \log^n \left(\frac{\Lambda}{\mu}\right) + \dots \tag{4.33}$$

The first term of the equation above yields the anomalous dimension of the mass parameter and is finite. In order for the sum of the second term to vanish, the recurrence relation between c_n and c_{n+1} must be satisfied:

$$(n+1)c_{n+1} = \mu \frac{\mathrm{d}c_n}{\mathrm{d}\mu} = \beta_i \frac{\mathrm{d}c_n}{\mathrm{d}\lambda_i}, \tag{4.34}$$

where β_i is the corresponding complete beta function for the i-th coupling λ_i. We would like to remind the reader that this subtraction is scheme-dependent. The absence of a quadratic divergence requires that $c_n = 0$ and thus we would need an infinite amount of constraints in order to ensure the absence of quadratic divergences to all orders. However, there is a way to circumvent this difficulty, that is we can resum the logarithm appearing in Eq. (4.32) via the running of the coupling constant. We shall come to that after we analyze the one-loop correction of Higgs self-energy.

For the SM, the dominant one-loop contributions for Higgs self-energy come from the Higgs, top quark and the weak boson masses. In terms of Feynman diagrams they are given as:

$\delta m_H^2 =$ 

. (4.35)

At one-loop level the quadratically-divergent contribution c_0 is calculated to be [87, 88],

$$c_0 = \frac{3}{64\pi^2}\left(3g_2^2 + g_1^2 + 8\lambda - 8\lambda_t^2\right). \tag{4.36}$$

Note that we have omitted all the Yukawa couplings except for the top quark in this calculation as they are small compared to the couplings mentioned above. In terms of masses we can write Eq. (4.36) as

$$c_0 = \frac{3}{16\pi^2 v^2}\left(2m_W^2 + m_Z^2 + m_h^2 - 4m_t^2\right) \equiv \mathrm{Str}\mathcal{M}^2, \tag{4.37}$$

where all the mass-coupling relations are given in Sec. (3.1). The term $\mathrm{Str}\mathcal{M}^2$ is defined here to match the convention in Ref. [89], where the *supertrace* Str assigns proper signs to bosons and fermions. In order to avoid fine-tuning, a reasonable requirement for an untuned Higgs mass would be for instance that the radiative correction to m_H in Eq. (4.37) should be of the same order as the Higgs mass itself. We can demand that the ratio between the radiative correction and the Higgs mass parameter in the Lagrangian is tuned to a given $100/\mathcal{F}$ percent:

$$\left|\frac{\delta m_H^2}{m_H^2}\right| \leq \mathcal{F}. \tag{4.38}$$

With the tuning parameter given, we can estimate the scale where a new physics should appear with a suitable choice of Higgs mass parameter. In Fig. (4.10) the relation of

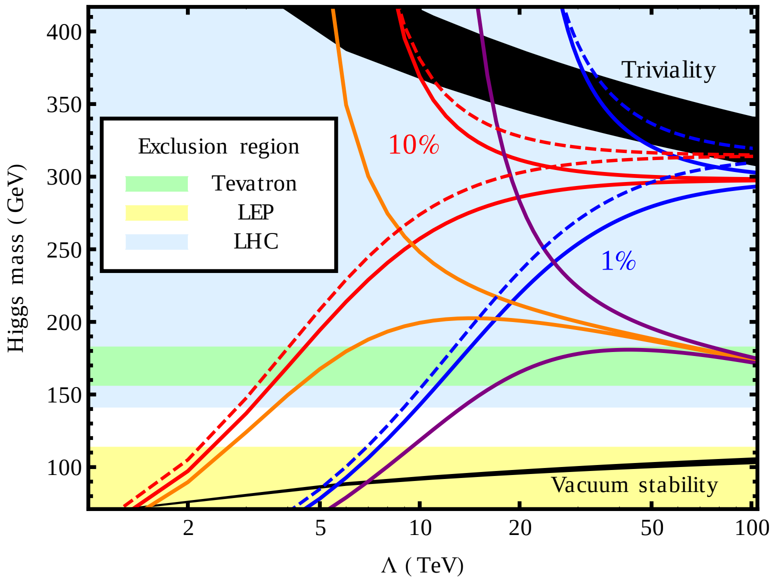


Figure 4.10: Higgs masses versus the cut-offs are plotted for the leading order of 10% (red) and 1% (blue) tuning conditions. Note that the Higgs masses plotted on the dashed curves are the $\overline{\text{MS}}$ masses. The physical Higgs masses in accordance to the cut-off are represented by the solid curves. Predicted physical Higgs masses for a given cut-off with two-loop leading-logarithm are plotted for 10% (orange) and 1% (purple) fine-tuning. One can interpret the cut-off given by tuning condition as the scale where a new physics should appear when a given Higgs mass is susceptible to certain percent of tuning. The Veltman condition (throat) is obtained for large cut-offs when all the tuning conditions converge to a line.

Higgs mass and cut-off is plotted pertaining to 10% and 1% tuning conditions. However one should be cautioned that the analysis represented by dashed curve only gives the $\overline{\text{MS}}$ renormalized mass parameter, as its matching with the physical pole mass has not been performed. The solid curves on the other hand give the real physical mass in relation to the cut-off, where the matching scale $\mu = M_t$ has been used, as mentioned in Sec. (4.3). Both versions of the plot with and without the matching are presented so that a comparison to the analysis made by Kolda and Murayama [89] and Casas et al. [90] can be made. Without the pole mass, the predicted Higgs mass deviates from the actual Higgs pole mass for $10 - 20\,\text{GeV}$. As we have analyzed the uncertainty caused by the omission of matching conditions of pole masses to their respective $\overline{\text{MS}}$ couplings in Sec. (4.3), we will use the proper matching condition prescribed in Sec. (4.3) and only the physical mass of the Higgs boson will be plotted in the rest of this thesis.

If the SM Higgs mass is found to be around 115-200 GeV and the tuning of self-energy correction to Higgs mass is about 10%, new physics would be expected to show up at $2-4\,\mathrm{TeV}$, which could be detected in the LHC. However, from electroweak precision data analysis, various fits from higher dimensional operators favor the cut-off Λ to be larger than $10\,\mathrm{TeV}$, this disagreement of new physics scale is called the *little hierarchy problem* [91]. One possible way to overcome this tension is to demand that the term inside the bracket in Eq. (4.37) vanishes.

$$2m_W^2 + m_Z^2 + m_h^2 - 4m_t^2 = 0 \tag{4.39}$$

such that Λ can be pushed to high energy scale without the Higgs mass suffering from severe fine-tuning. This condition is called the *Veltman condition.* Nevertheless for the naive Veltman condition (or Veltman "throat") plotted in Fig. (4.10), the triviality bound will void our Veltman condition at their crossing, producing a cut-off at $\Lambda > 100\,\mathrm{TeV}$ and the corresponding Higgs mass that we would obtain will be around $M_H \approx 300\,\mathrm{GeV}$. This predicted Higgs mass has been ruled out by the LHC up to 95% confidence level. This conclusion however only applies to our one-loop Higgs' self-energy correction.

We would like to investigate the case of two-loop quadratic divergence and its Higgs mass prediction from tuning argument. Recall from Eq. (4.34) that we can calculate the next-to-leading order c_1 via the recurrence relation. By using the one-loop beta function and applying them in Eq. (4.34), the coefficient c_1 is found to be:

$$\begin{aligned} c_1 = \frac{1}{(16\pi^2)^2} \Bigg[& \lambda(144\lambda - 54g_2^2 - 18g_1^2 + 72\lambda_t^2) - \frac{15}{2}g_2^2 + \frac{25}{2}g_1^2 \\ & + \frac{9}{2}g_1^2 g_2^2 + \lambda_t^2(27g_2^2 + 17g_1^2 + 96g_3^2 - 90\lambda_t^2) \Bigg] . \end{aligned} \tag{4.40}$$

Similar analysis for the Higgs mass prediction can be done for the two-loop tuning condition. From Fig. (4.10) we can observe that the Veltman throat is shifted below compared to the one obtained via one-loop computation. As a result a lower Higgs mass ranging from $115-200\,\mathrm{GeV}$, which is preferred by electroweak precision analysis and still not excluded by the experiments, can be obtained without severe fine-tuning while the cut-off is pushed to higher energy scale, hence ameliorating the little hierarchy problem. Since the Veltman condition predicts lower Higgs mass when the cut-off scale is increased, one might be tempted to push the cut-off to higher energy value, possibly to Planck scale in order to obtain a very low Higgs mass. However this two-loop analysis is only valid for $\Lambda^2 \log^2 \Lambda \leq (16\pi^2)^3 v^3$, which corresponds to $\Lambda < 50\,\mathrm{TeV}$. The reason is that large divergences would void the perturbative series, even if the two-loop Veltman condition is satisfied.

A proper way to prevent the large logarithm from spoiling our perturbation theory is to improve the perturbative calculation by running the coupling constants from renormalization scale $\mu = M_t$ to the cut-off scale Λ, that way the higher order loop appearing as the logarithmic term in the self-energy computation would drop out and our perturbative calculation is simplified:

$$\begin{aligned} m_{H_B}^2 &= m_H^2 + \Lambda^2 \sum_{n=0}^{\infty} c_n(\lambda_i(\mu)) \log^n\left(\frac{\Lambda}{\mu}\right) + \dots \\ &\xrightarrow[\mu\to\Lambda]{} m_H^2 + \Lambda^2 c_0(\lambda_i(\Lambda)). \end{aligned} \tag{4.41}$$

Of course all the logarithms are only resummed if the full beta function is known. As only two-loop beta functions for the SM couplings are known so far, only the leading and next-to-leading logarithm are resummed in our case. The Veltman condition is now a function of the cut-off, as our coupling constants are functions of Λ.

With the tools and techniques introduced in Sec. (2.5.1) we can use the beta functions for the SM coupling constants to obtain the effective running coupling constants. We will use the one-loop and two-loop beta functions in order to estimate the theoretical uncertainty of the Higgs mass prediction, i.e. we will define the difference of Higgs mass obtained via solving the running coupling with one-loop and two-loop beta function as our theoretical error. The two-loop beta functions for λ, λ_t and the gauge couplings are given in Appx. (B). Solving the coupling constants with the given one and two-loop beta functions, we can obtain the running Veltman condition as a function of the cut-off, see Fig. (4.11). Numerically what we did was to impose the Veltman condition or the tuning equation Eq. (4.38) as the boundary condition of $\lambda(\Lambda)$ in solving the coupled first-order partial differential equation of coupling constants. To solve a set of first order differential equation we would need a set of boundary conditions. The boundary condition for gauge couplings g_1 and g_2 at $\mu = M_Z$ is given by Eq. (3.17a) and Eq. (3.17b), while $g_3 = \sqrt{4\pi\alpha_s(M_Z)}$. The gauge couplings run to the scale $\mu = M_t$ without the top loop contribution in order to include the threshold effect. Their values at $\mu = M_t$ are then used as the boundary conditions for the complete one- and two-loop RGEs. With $\lambda(\Lambda)$ imposed we can solve the coupled running coupling equation and obtain the $\overline{\text{MS}}$ coupling of λ at a certain matching scale, which we took to be the top pole mass M_t. Once this is done we proceed to match the coupling $\lambda(M_t)$ with the physical Higgs mass. Although the Higgs mass can be found with this matching procedure, it is subjected to theoretical and experimental errors.

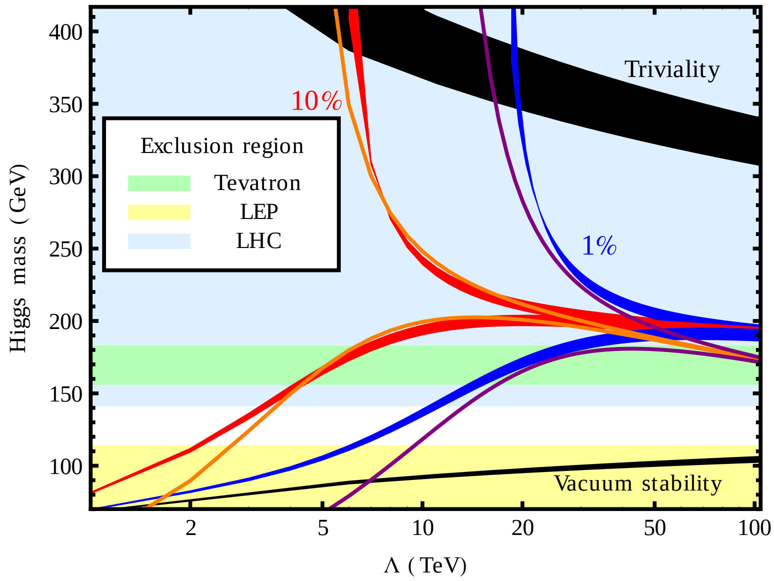


Figure 4.11: Improved tuning conditions for 10% (red) and 1% (blue) fine-tuning are plotted. Note that only the theoretical error, defined as the Higgs mass difference obtained from one-loop and two-loop beta function, has been taken into account in plotting the band, which corresponds to the bandwidth. Compared with the two-loop expansion of the tuning condition for 10% (orange) and 1% (purple), the Veltman condition obtained via the running couplings is shifted around 20 GeV above.

In general a solution obtained via higher order perturbative calculation might represent the true solution obtained by full computation. Therefore it is important to estimate the theoretical uncertainty due to the omission of higher order loop computation in the Veltman condition. We will consider the difference of the Higgs mass obtained via utilizing one- and two-loop beta function in RGEs as theoretical uncertainty to estimate the discrepancy of the Higgs masses caused by omission of the three-loop beta function. Optimistically we can approximate that the three-loop solution curve lies inside the difference between one and two-loop solution. Such theoretical uncertainty is represented by the width of the plotted bands in Fig. (4.11), where the middle of the band represent the solution on the Higgs mass obtained via two-loop running of the beta functions. From the bandwidth we can give a rough estimate of the theoretical uncertainty $\Delta M_H \approx \pm 5\,\text{GeV}$.

In Fig. (4.11) we notice that the solution obtained via running coupling deviates from the one that we have computed with the two-loop leading-logarithmic quadratic divergence contribution. At $\Lambda \lesssim 3\,\text{TeV}$ ($\Lambda \lesssim 20\,\text{TeV}$), the Veltman condition obtained using

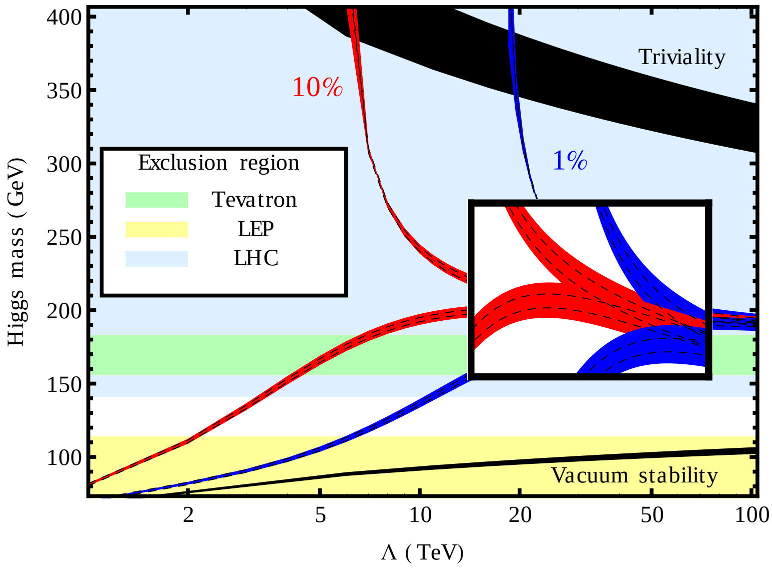


Figure 4.12: Veltman conditions are plotted with experimental uncertainties of M_t, governed by the black dashed lines. The Veltman throat is zoomed so that theoretical error and effect of top mass uncertainties can be compared.

the two-loop leading-logarithm with 10% (1%) tuning deviates quite significant compared to the one obtained via improved perturbation, as the higher order of logarithm is not considered. This is a typical discrepancy due to the omission of higher order terms. For instance if we expand the function $1/(1 - \log x) \approx 1 + \log x$ around $x = 1$, which resembles the expansion of leading-logarithm, both functions behave very differently when x is away from the expansion point. The Higgs mass predicted by the running Veltman condition never reaches the vacuum stability limit in our considered region; whereas the Veltman condition expanded with two-loop order crosses the vacuum stability bound. As we mentioned before that three-loop divergences are necessary relevant when $\Lambda > 50\,\text{TeV}$, so the analysis given by Kolda and Murayama's plot in Ref. [89] has to be taken carefully for the region of large cut-off. In general it is imperative to perform the resummation of the leading logarithm for a more accurate Higgs mass prediction from the Veltman condition, as the Veltman condition is only satisfied at one energy scale.

Besides the theoretical error, our prediction of the Higgs mass will be sensitive also to the experimental uncertainty. With g_1 and g_2 accurately determined by the measurement of weak mixing angle and fine structure constant, we can expect that their error propagation will not cause the uncertainty of the Higgs mass prediction more than

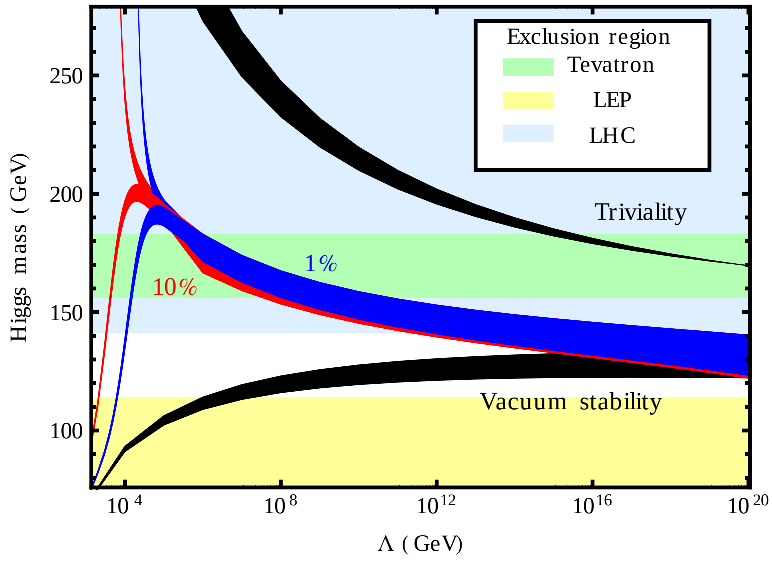


Figure 4.13: The Veltman condition is extrapolated to the Planck scale, where it overlaps with the vacuum stability bound.

0.1%. With that we expect that the only error source on the one-loop Veltman condition comes from the top mass. The current world best value of M_t determined with combined data from CDF and D0 is set to be $173.2 \pm 0.9\,\text{GeV}$. By considering only the experimental error and neglecting the theoretical one, we can plot the similar curve for the Veltman condition. Separating the two uncertainties and analyzing them independently, we observe from Fig. (4.12) that the uncertainty of the Higgs mass propagated by the experimental error of the top mass is roughly half of the magnitude of the theoretical uncertainty that we have defined above, that is $\Delta_{M_t} M_H \approx \pm 2\,\text{GeV}$. Therefore it is necessary to have a precision top measurement and analysis in order to reduce the Higgs mass uncertainty. We will discuss more detailed the significant influence of the top mass and its crucial role on the validity of SM to Planck scale later in Sec. (5.1).

The Veltman condition is one of the possible "solutions" to the little hierarchy problem. In a strict sense, it is not a solution because to date it has no UV-completion, i.e. we do not have a theory that triggers such a condition to solve the little hierarchy problem. However for curiosity sake we can try to extrapolate the Veltman condition to a high energy scale, to see if the Veltman throat crosses either the triviality line or the vacuum stability bound within a finite high energy scale before Planck scale. In Fig. (4.13) we plotted the Veltman condition up to the Planck scale, and it turns out that both the

Veltman condition and the vacuum stability overlap in the region of the Planck scale[3]! This is an intriguing scenario, as it suggests that the UV-complete theory for Veltman condition might come from the Planck scale physics itself, and the vacuum of the Higgs sector is just on the edge of stability. Interesting case could be that there exists a common origin for both the conditions. We know that from vacuum stability, $\lambda(M_{pl})$ has to be larger than zero in order to prevent the Higgs potential from becoming unbounded from below. The condition $\lambda(M_{pl}) = 0$ suggests that the Higgs quartic coupling or Higgs self-interaction is radiatively generated and is not required to be put in by hand. In this sense, Planck scale physics tries to suppress λ to prevent it from hitting the Landau pole. Moreover combining it with the Veltman condition from Eq. (4.39), we obtain:

$$3g_2^2(M_{pl}) + g_1^2(M_{pl}) - 8\lambda_t^2(M_{pl}) = 0. \tag{4.42}$$

i.e. it stabilizes the Higgs mass from obtaining a large radiative correction via a simple relation between gauge coupling and top Yukawa coupling. This result is interesting, as it suggests that there might be a common origin for the gauge and the Yukawa sector at the Planck scale. At the moment we cannot be sure of such a possibility, and one has to be careful and has to investigate whether this overlapping is physically motivated or just a random accident by chance. In the next section, we investigate this problem by turning the question another way round; given a certain boundary condition for λ at the Planck scale, which Higgs mass would it predict?

[3]Similar analysis has been carried out by Chaichian et al. [92], however the Higgs mass obtained by the authors deviates $\approx 30\,\mathrm{GeV}$ from our result. This is because they have not used the matching of the top pole mass to the $\overline{\mathrm{MS}}$ Yukawa coupling, and the top pole mass is directly assumed to be the $\overline{\mathrm{MS}}$ mass. This has caused some confusions initially as their wrongly obtained Higgs mass is long excluded by the Tevatron.

Chapter 5

Generic Boundary Condition for λ and its Phenomenology

In the previous section we found that the Veltman condition extrapolated to the Planck scale overlaps with the vacuum stability plot within the defined theoretical and experimental uncertainties. We may ask whether this overlap is just a coincidence, or if there are physical motivations behind it. Inspired by this, we can turn the question around and demand for more generic values of λ at the Planck scale, possibly due to some quantum gravity effect that we do not understand yet. With such generic boundary conditions imposed, the running coupling equation can be solved to determine the Higgs mass prediction for each case. We will examine the errors of such Higgs mass predictions to see whether it is possible for the LHC to discriminate them. We will see later that most of the conditions imposed will give a low Higgs mass, satisfying the electroweak precision test and even the recent LHC Higgs mass exclusion limit. We will argue that such a low mass prediction is naively improbable if we generate a range of random values for λ as boundary conditions at the Planck scale. Such a delicate interplay between λ at the Planck scale and the low energy Higgs mass will prompt us to rethink the status of the SM and possibly need to accept the non-existence of any intermediate scale, if the LHC do not find any new physics at the TeV scale. We may need to accept that the SM is a remnant of quantum gravity, opening up a new direction of model building of particle physics from a quantum gravity perspective.

5.1 Some other boundary conditions for λ

In the previous section we discovered that there is an overlapping region from the Veltman condition and the vacuum stability near the Planck scale. It is possible that this

Planck scale physics that prevents the hierarchy problem and the Landau pole problem might be just a mere coincidence. Therefore it is important for us to turn the question around and ask: Given a value of $\lambda(M_{pl})$, possibly due to some constraints or physical motivations, which Higgs mass would it predict at the weak scale. Guided by this principle, we can find some other boundary conditions imposed for λ at a high energy scale and try to see whether its Higgs mass prediction is allowed by the experimental bounds.

5.1.1 Vacuum stability revisited

Before we start to explore some other possible boundary conditions for λ, it is wise to revisit the vacuum stability and reintroduce the notion of effective potential, which we have derived from the effective action in Sec. (2.1). We want to be as precise as possible in the analysis of vacuum stability, as this condition is the most crucial one to determine whether the SM is a valid effective theory up to the Planck scale or not.

The general effective potential can be calculated as:

$$V_{\text{eff}}(\mu, \lambda_i(\mu); \phi(\mu)) = V_0 + V_1 + \dots \tag{5.1}$$

with λ_i representing all the generic field couplings contributing to the effective potential. We denote the classical Higgs field as ϕ in accordance with the convention given in [68, 67], where:

$$\phi(\mu) = \phi_c \xi(\mu) := \phi_c \exp\left(-\int_{M_Z}^{\mu} \frac{\gamma(\mu')}{\mu'} \mathrm{d}\mu'\right). \tag{5.2}$$

The function ξ dictates the scaling of the classical Higgs field ϕ_c defined at the renormalization point $\mu = M_Z$, see Appx. (B) for the full expression of the anomalous dimension γ used in our analysis. The term V_i corresponds to the expansion of the 1PI as explained in Sec. (2.1). In the SM the tree level potential and the one-loop correction are given as [68, 67, 93]:

$$V_0 = -\frac{1}{2} m^2(\mu)\phi^2(\mu) + \frac{1}{4}\lambda(\mu)\phi^4(\mu), \tag{5.3}$$

$$V_1 = \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\phi) \left[\log \frac{M_i^2(\phi)}{\mu^2} - d_i\right], \tag{5.4}$$

where the term $M_i^2(\phi) = \kappa_i \phi^2(\mu) - \kappa_i'$ represents the tree-level expression for masses of particles that enter in one-loop radiative correction. For our purpose only the coupling of gauge bosons, top Yukawa and Higgs quartic coupling term are considered. We list down the relevant M_i, κ, κ' and d_i in the Appx. (A.5).

It is known that the effective potential and its n-th derivative is scale-invariant under the change of renormalization scale:

$$\mu \frac{\mathrm{d}V_{\text{eff}}^n}{\mathrm{d}\mu} = 0, \tag{5.5}$$

where V_{eff}^n is given as:

$$V_{\text{eff}}^n = \xi^n(\mu) \frac{\partial^n}{\partial\phi(\mu)^n} V_{\text{eff}}(\mu, \lambda_i(\mu); \phi(\mu)). \tag{5.6}$$

Although the whole effective potential is scale-invariant, its one-loop approximation is not. Hence we need a criterion to choose an appropriate renormalization scale. Following our matching conditions discussed in Sec. (4.3), we will set $\mu = M_t$ for consistency.

By minimizing the effective potential, the value of ϕ at extrema can be obtained as:

$$\phi_{\text{ext}}^2 = \frac{m^2}{\tilde{\lambda}}, \tag{5.7}$$

where the effective quartic coupling $\tilde{\lambda}$ is defined as:

$$\tilde{\lambda} = \lambda + \sum_i \frac{n_i \kappa_i^2}{16\pi^2} \left[\log \frac{M_i^2(\phi)}{\mu^2} - d_i + \frac{1}{2} \right]. \tag{5.8}$$

The Higgs and Goldstone boson's contributions have been omitted as they are numerically irrelevant [68]. The structure of extrema for our effective potential can be studied at a scale μ where the effective potential is scale-invariant. As discussed in Refs. [68, 67], a wise choice would be $\mu \approx \phi$. If V_{eff} develops an extremum for a large field value of ϕ, then $\tilde{\lambda}$ must be small, but positive. Suppose that ϕ_{max} represents the field where the maximum of the potential is located; for any ϕ larger than ϕ_{max} the potential would be negative due to the fact that $\tilde{\lambda} < 0$ and the potential is dominated by the $\tilde{\lambda}\phi^4$ term. Therefore we need to demand that the maximum occurs at $\phi_{\text{max}} > \Lambda$, where Λ is the scale at which the SM is not a valid theory anymore. In our case we take $\Lambda = M_{pl}$. Hence the actual boundary condition needed for λ is:

$$\begin{aligned} &\tilde{\lambda}(M_{pl}) = 0 \\ \Rightarrow\ &\lambda(M_{pl}) = \frac{1}{16\pi^2} \left(\frac{3}{8} (g_1^2(M_{pl}) + g_2^2(M_{pl}))^2 \left[\frac{1}{3} - \log \frac{g_1^2(M_{pl}) + g_2^2(M_{pl})}{4} \right] \right. \\ &\qquad \left. + 6\lambda_t^4(M_{pl}) \left[\log \frac{\lambda_t^2(M_{pl})}{2} - 1 \right] + \frac{3}{4} g_2^4(M_{pl}) \left[\frac{1}{3} - \log \frac{g_2^2(M_{pl})}{4} \right] \right). \end{aligned} \tag{5.9}$$

To obtain the vacuum stability bound we consider two cases in solving the coupled differential equations [67, 68]:

1. We first impose the boundary conditions at tree-level, i.e. $\lambda(M_{pl}) = 0$ and apply the one-loop beta functions and anomalous dimension equations in our numerical RGE-solver. In this case the leading logarithms are resummed to all loop order in our effective potential.

2. Two-loop beta functions and anomalous dimension for m are considered in our RGEs and the effective potential is considered in one-loop approximation, as the boundary condition in Eq. (5.9) is imposed. The leading and next-to-leading logarithms are resummed to all loop order in the effective potential in this case.

The result is plotted as a blue band in Fig. (5.1) and Fig. (5.2), where the middle line of the band is obtained via case 2 mentioned above. The upper edge of the band is obtained via case 1 and the Higgs mass difference obtained from case 1 and 2 is considered as theoretical uncertainty, which is shaded as light blue region in Fig. (5.1). We also consider an additional source for the Higgs mass uncertainties, namely the error of strong coupling constant. We have separated the theoretical error from the discrepancies caused by $\Delta\alpha_s$ so that our analysis can be compared to Ref. [69]. We will comment more on the vacuum stability bound and the survival of the SM up to the Planck scale in Sec. (5.4). But before we do that, let us investigate some other boundary conditions which can be imposed for λ.

5.1.2 Higgs quartic coupling as a quasi fixed-point

Since we are interested in solving the running coupling via the beta function, it might be worth asking whether the beta function of λ at Planck scale is nearly conformal:

$$\beta_\lambda(M_{pl}) \approx 0. \tag{5.10}$$

In fact this condition is well motivated by other proposals, for instance in the paper from Shaposnikov and Wetterich [94], which can be summarized as follows: It is known that the SM coupled to general relativity is asymptotically safe, despite that general relativity is non-renormalizable. The SM with gravity could be valid up to arbitrarily high energy. Therefore it is possible for the coupling constant λ_j to run in arbitrary energy region in accordance with its beta function, which has to be supplemented with an additional contribution from gravity when the renormalization constant exceeds the

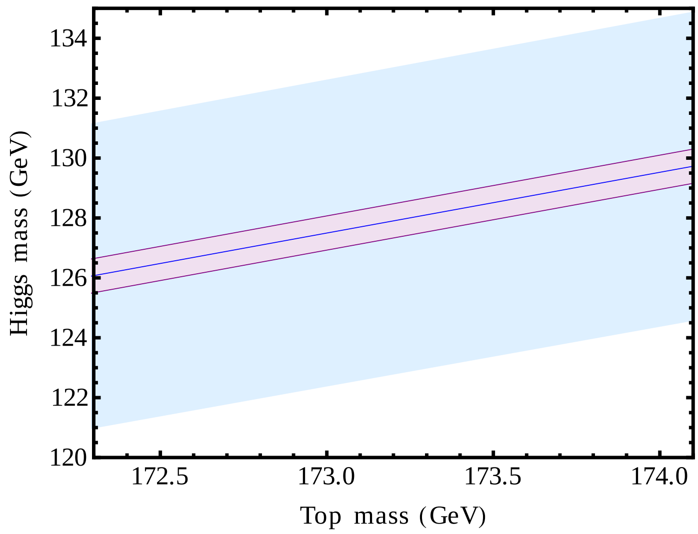


Figure 5.1: The vacuum stability bound is zoomed to region allowed by the top mass error. The blue line in the middle represents the vacuum stability bound obtained via the two-loop beta functions, which has been thoroughly discussed in main text. The purple band represents the uncertainties of the Higgs mass obtained via two-loop RGEs due to α_s uncertainties. The light blue region represents the theoretical error, defined as the Higgs mass difference obtained via one- and two-loop RGEs. With the best world average top pole mass 173.2 GeV given, the predicted Higgs mass from vacuum stability is around 128 GeV.

Planck scale:

$$\mu \frac{d\lambda_j}{d\mu} = \beta_{\lambda_j} + \beta_{\lambda_j}^{grav} \theta(\mu - \mu_{tr}), \tag{5.11}$$

with μ_{tr} defined as the transition scale

$$\mu_{tr} \approx \frac{M_{pl}}{\sqrt{2\xi_0}} \approx 10^{19}\,\text{GeV}, \tag{5.12}$$

with $\xi_0 \approx 0.024$. The gravitational contribution to the beta-function $\beta_{\lambda_j}^{grav}$ is typically given by:

$$\beta_{\lambda_j}^{grav} = \frac{a_j}{8\pi} \frac{\mu^2}{M_{pl}^2(\mu)} \lambda_j, \tag{5.13}$$

where a_j depends on the precise model which describes the high energy regime. Note that the Planck constant runs with energy due to effect of graviton loops. The authors of Ref. [94] analyzed some cases for a_j in the gauge sector, top quark and Higgs quartic coupling. They found that an interesting case in predicting the Higgs mass arises if λ_t^2 has an ultraviolet fixed point ($\lambda_t^2(\mu \to \infty) = 0$) while λ has an infrared fixed point

$(\lambda(\mu_{tr}) \approx 0)$. Near this region, the beta function of λ is nearly zero, yielding a quasi fixed point for λ. In Fig. (5.3) we can see the running behavior for $\lambda(\mu_{tr} \approx M_{pl}) = 0$; with this condition set we can see that λ does not change much until the top Yukawa coupling starts to grow and thus increases λ below $\mu = 10^{11}$ GeV. Therefore $\beta_\lambda(M_{pl}) = 0$ is a condition for such a theory. Since the authors of Ref. [94] require that $\lambda(M_{pl}) = 0$, the additional condition for the beta function will only yield the same Higgs mass prediction given solely from the vacuum stability condition. Their predicted Higgs mass is around 127 GeV for a given top mass of 171.3 GeV. This prediction however does not come with any error analysis in Ref. [94]. We will comment on the difficulty of distinguishing different Higgs mass prediction later in Sec. (5.2).

Besides Ref. [94], Froggatt and Nielsen [95] have imposed a similar condition, but with a different physical motivation. They demanded that the SM effective potential should have two degenerate vacua, one with generic value of $\langle\phi_1\rangle = 246$ GeV and the other placed at the Planck scale, $\langle\phi_2\rangle = M_{pl}$. The two minima are degenerate,

$$V_{\text{eff}}(\langle\phi_1\rangle) = V_{\text{eff}}(\langle\phi_2\rangle), \tag{5.14}$$

as motivated by the multiple point criticality principle, which states that nature prefers mixture of states. See Ref. [95] for a more elaborated argument. Suppose that the field energy acquires values of the order of the Planck scale, the effective potential can then be approximated as:

$$V_{\text{eff}}(\phi) \approx \tilde{\lambda}(\phi)\phi^4, \tag{5.15}$$

so the derivative of V_{eff} with respect to the field ϕ at the second minimum is given as:

$$\begin{aligned} \left.\frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}\phi}\right|_{\phi=\langle\phi_2\rangle} &= 4\tilde{\lambda}(\phi)\phi^3 + \frac{\mathrm{d}\tilde{\lambda}(\phi)}{\mathrm{d}\phi}\phi^4 \\ &= 4\tilde{\lambda}(\phi)\phi^3 + \beta_\lambda\phi^4 \overset{!}{=} 0. \end{aligned} \tag{5.16}$$

In order to obtain a vev at the weak scale for first vacuum, the coefficient of the ϕ^2 term must be of the order of the electroweak scale, therefore in order obtain degeneracy between these two vacua, the effective coefficient $\tilde{\lambda}(\langle\phi_2\rangle)$ must be zero so that the ϕ^4 term is suppressed. In that case, the $\tilde{\lambda}$ term is zero in Eq. (5.16), supplemented by the vanishing beta function. But since the second vacuum lies at the Planck scale, what the authors in Ref. [95] demand actually is:

$$\beta_\lambda(M_{pl}) = 0, \quad \tilde{\lambda}(M_{pl}) = 0. \tag{5.17}$$

From the analysis of Ref. [95] and Ref. [94] given above, one might be tempted to think that the condition $\beta_\lambda(M_{pl}) = 0$ is not necessary, as this condition overlaps with the vacuum stability. We would like to caution the reader that there exists another branch of solution where $\beta_\lambda(M_{pl}) = 0$ and vacuum stability do not overlap, which can be seen in Fig. (5.2).

5.1.3 Quasi-fixed point for bare mass parameter

Besides vanishing of β_λ, we can also try to impose quasi-fixed point solution on the bare mass parameter m in the Higgs sector, that is, we demand that $\gamma_m(M_{pl}) = 0$. The anomalous dimension of the mass parameter of the Higgs field is a quantity with dimension of mass, i.e. overall it must be proportional to some mass couplings, which can only be m itself as the Higgs doublet mass term is the only dimensionful parameter in the SM. The one-loop and two-loop anomalous dimension for m are given in Appx. (C). One has to be careful when implementing such conditions, as there are two ways to implement $\gamma_m(M_{pl}) = 0$. We observe that if $m^2(M_{pl}) = 0$, the anomalous dimension would also be zero. This alone does not provide sufficient information for our Higgs mass prediction, as $m^2(M_{pl}) = 0$ dictates $m^2(\mu) = 0$ for any arbitrary μ and no information about λ can be extracted. This result is easy to see if we write the solution of the beta function of m explicitly:

$$\mu \frac{\mathrm{d}m^2(\mu)}{\mathrm{d}\mu} = m^2(\mu) f(\lambda(\mu), g_i(\mu), \lambda_t(\mu)), \tag{5.18}$$

where the function $f(\lambda, g_i, \lambda_t)$ contains all the terms inside the bracket of the anomalous dimension of the mass parameter given in Eq. (B.8) and Eq. (B.14). Integrating the equation above we obtain:

$$\begin{aligned} &\Rightarrow \int_{m_0^2}^{m^2} \frac{\mathrm{d}m^{2\prime}}{m^{2\prime}} = \int_{M_{pl}}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} f(\lambda(\mu'), g_i(\mu'), \lambda_t(\mu')) \\ &\quad \Rightarrow m^2(\mu) = m_0^2(M_{pl}) \exp\left(\int_{M_{pl}}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} f(\lambda(\mu'), g_i(\mu'), \lambda_t(\mu')) \right) \xrightarrow[m_0^2(M_{pl})=0]{} 0. \end{aligned} \tag{5.19}$$

This alone does not imply any condition for λ at the Planck scale. Therefore in order to impose the condition of $\gamma_m(M_{pl}) = 0$ which will then provide us with information on λ at electroweak scale, we would necessarily have to demand that:

$$f(\lambda(\mu), g_i(\mu), \lambda_t(\mu)) = 0. \tag{5.20}$$

5.2 Analysis of boundary conditions imposed on λ

With the physically well motivated boundary conditions listed above, we can now solve the coupled beta function differential equation for each separate case. We choose the top quark pole mass as our input parameter in our RGE solving algorithm, as its deviation contributes most significantly to the change of the Higgs mass. The top quark pole mass will be matched to its corresponding $\overline{\mathrm{MS}}$ Yukawa coupling with the prescription given in Sec. (4.3). The known gauge couplings $g_i(M_Z)$ on the other hand run to the scale $\mu = M_t$ without including the top loop contribution, and then the value of $g_i(\mu = M_t)$ will be used in our complete one- and two-loop RGEs to predict the Higgs mass. With all of our gauge, Yukawa and in $\overline{\mathrm{MS}}$ value known, we can solve the set of RGEs numerically with $\lambda(M_{pl})$ imposed, as previously described in Sec. (4.4). The Higgs quartic coupling λ evolves from the Planck scale to the scale $\mu = M_t$, where M_t is our input top pole mass. We then match the Higgs quartic coupling at scale $\mu = M_t$ to its corresponding pole mass. The matching scale $\mu = M_t$ is chosen for a given input top pole mass value such that the matching error is minimized, as discussed in Sec. (4.3).

With all the points generated from solving the set of differential equation numerically, we can plot the Higgs mass dependence on the top mass, refer to Fig. (5.2). The gray hatched region of Fig. (5.2) represents the exclusion region for the Higgs boson mass by LEP and Tevatron [70, 71]. ATLAS and CMS have excluded the low mass range of Higgs boson from $146 - 232\,\mathrm{GeV}$ and $145 - 216\,\mathrm{GeV}$ at 95% CL respectively [7, 6], both of the exclusion regions are combined in the recent analysis to give a combined exclusion region of $141 - 476\,\mathrm{GeV}$ at 95% CL [8]. In order to obtain an experimentally allowed parameter space with correlation of Higgs boson, top quark and electroweak precision measurement taken into account, we have taken the plot points from the GFitter fit [63] and illustrate the 68%, 95% and 99% confidence interval in our plot. If such correlation is not taken into account, then the horizontal allowed region ranges from $114.4 - 141\,\mathrm{GeV}$ while the vertical domain is limited to $172.3 - 174.1\,\mathrm{GeV}$, corresponding to the best world average value of top quark mass $173.2 \pm 0.9\,\mathrm{GeV}$ [96]. We observe that most of the Higgs masses given by different conditions tend to overlap in the vicinity of the best determined value of the top mass, except the triviality bound. We use a more restrictive condition $\lambda(M_{pl}) = \pi$ to represent the triviality bound, and this condition yields a range of Higgs masses which is already excluded at 95% CL by the Tevatron and LHC. The Higgs masses generated by the rest of the conditions however are still allowed and not excluded yet by the experiments. The Veltman condition is truncated at the point where its Higgs mass calculated with two-loop beta functions starts to cross the

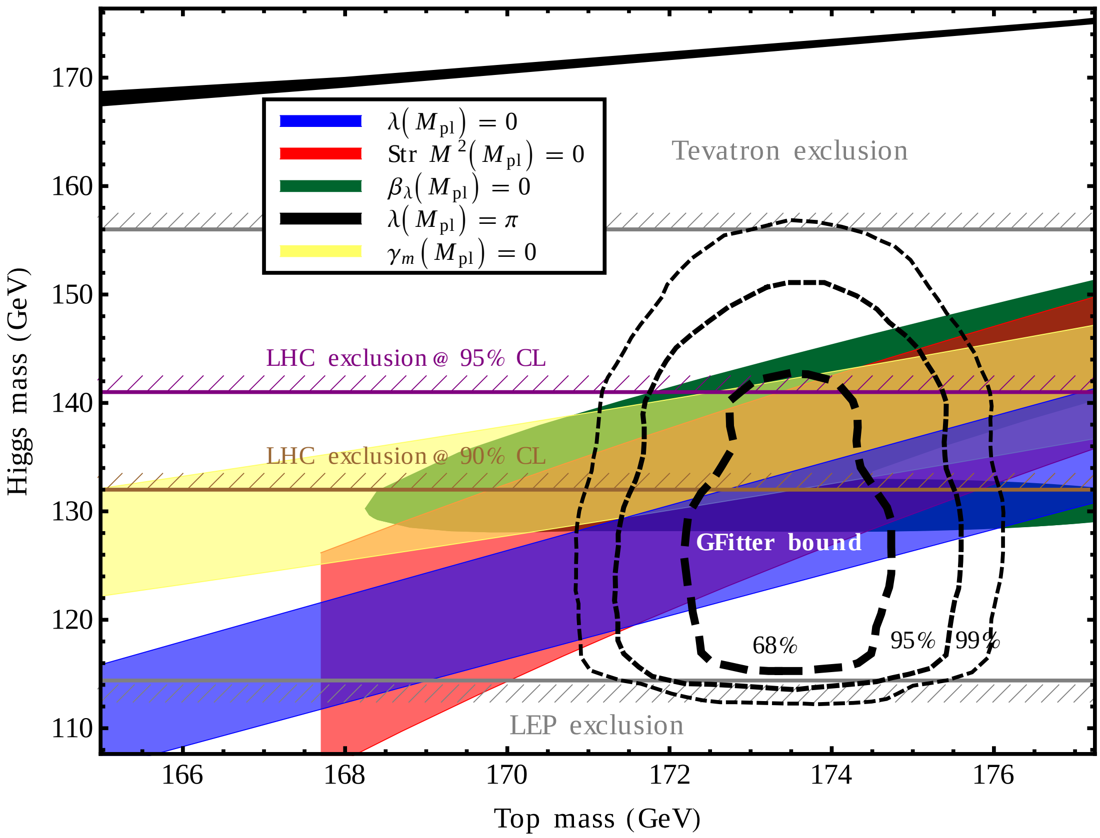


Figure 5.2: Higgs and top pole masses plotted for different boundary conditions at a high energy scale. The bandwidth plotted for each curve consists of the theoretical uncertainty due to the omission of higher order beta functions. The middle line of each band represents the Higgs masses obtained via solving the two-loop beta functions, while the upper edges consist of Higgs mass predictions from one-loop RGEs. The gray-hatched region is excluded by Higgs mass and top mass direct searchers from LEP and Tevatron. Similarly the region above the purple (brown) line is excluded by LHC Higgs searches at 95% (90%) CL. The electroweak precision fit from GFitter [63] is also included for 68%, 95% and 99% confidence intervals including the direct searches. The Veltman condition is truncated at the point where its Higgs mass calculated with two-loop beta functions starts to cross the vacuum stability bound obtained by two-loop RGEs.

vacuum stability bound obtained by two-loop RGEs. This is done in order to show the exact crossing point of these two conditions from two-loop RGEs. We purposely extend the bands to lower top mass region so that one can compare the predicted Higgs masses from certain boundary conditions between different values of the top mass used. We will see later in Sec. (5.4) that there are different means to extract the top mass, which will yield a lower top pole mass value compared to the best world average. It is therefore crucial to understand what type of the top mass is measured in the experiments.

We consider in our plot only theoretical errors in obtaining the Higgs pole mass pertaining to the difference of the Higgs masses extracted via one-loop and two-loop beta functions, in this sense we are estimating the error of running couplings with the omission of three-loop beta function. This theoretical uncertainty is represented by the bandwidth of each curve, with its middle line representing the Higgs mass obtained from two-loop RGE running. The upper edges of the bandwidths consist of the Higgs masses obtained from one-loop RGEs. We also consider the uncertainty on the curves due to the uncertainty of strong coupling constant $\alpha_s = 0.1184(7)$ [16] and we obtain ± 1 GeV uncertainty to the Higgs mass, which is negligible when quadratically added to the bandwidth on Fig. (5.2). Due to the relatively large theoretical uncertainty, the error propagation from the strong coupling constant can be safely ignored. Notice however that the α_s obtained from lattice simulation is not taken into account. The theoretical error on the Higgs mass due to the matching uncertainty [75, 77] between top Yukawa $\overline{\text{MS}}$ coupling and top pole mass is also considered. Comparing our vacuum stability band obtained with Casas et al. [67, 68], a discrepancy of around ± 7 GeV for the Higgs mass value obtained via two-loop RGEs is observed. This mismatch can be explained by the omission of two-loop QCD matching condition by the authors of Refs. [67, 68], as they have only considered one-loop QCD, electroweak and QED contribution in the top mass matching condition. Since we would like to consider only the uncertainties due to the number of loops of beta function used but not the errors caused by omission of better matching precision, we include the QCD matching between top Yukawa $\overline{\text{MS}}$ coupling and top pole mass up to three-loop, as we have discussed in Sec. (4.3). The resulting Higgs mass predicted by the vacuum stability with two-loop RGEs agrees with Ellis et al. [69] within 1 GeV. The $\alpha\alpha_s$ correction is neglected in our analysis due to its small contribution, see Sec. (4.3) for this justification. The Higgs pole mass is matched with λ at the top pole mass scale, i.e. the renormalization scale of λ is set to be at $\mu = M_t$. Since the higher order matching conditions for λ have not been calculated in the literature, only Eq. (4.22) will be used for our matching. If the matching of λ to the Higgs pole mass is not performed, the resulting error of Higgs pole mass is found to be less than 1 GeV. Therefore, we can safely assume that higher order matching conditions for λ will not yield a larger error.

The error estimation for the condition $\beta_\lambda(M_{pl}) = 0$ is not the same as for the rest of the boundary conditions due the reasons we will explain below, therefore a delicate treatment of extracting the Higgs mass for this case has to be implemented. The one-loop beta function of λ is a quadratic equation in the function λ, and for a given top mass we would obtain two positive values of λ at Planck scale, both are equally valid in the Higgs mass prediction. Negative values of $\lambda(M_{pl})$ are discarded. We have seen in Fig. (5.2) that the condition of $\beta_\lambda(M_{pl}) = 0$ generates a hook-like trajectory on the $M_H - M_t$ plane, overlapping with the vacuum stability bound. The hook ends where λ starts to take negative values. Due to the mismatch of the end of the trajectory when either one-loop or two-loop beta function is applied, we have to take a larger error into account. We generate error bars which cover the distance of the mismatch and plot a band to cover all the error bars. Besides the mismatch mentioned above, there exists also another source of error, namely number of loops of beta functions implemented in the $\beta_\lambda(M_{pl}) = 0$ condition. In principle one should apply the full beta function as the boundary condition, but in practise this is impossible and therefore we have to check the possible uncertainties which arise due to the number of loops in β_λ used as the boundary condition. The errors however lie within the band. As similar uncertainty due to number of loops used as boundary condition also appears in the $\gamma_m(M_{pl}) = 0$ condition, and the uncertainty is larger in comparison with the $\beta_\lambda(M_{pl}) = 0$ condition.

5.3 Distribution of Higgs mass with random Higgs coupling at Planck scale

In Fig. (5.2) we observe that all our imposed boundary conditions yield a range of Higgs mass which is still allowed by direct searches of LHC and Tevatron. This is intriguing, as almost all of our imposed boundary conditions favor a low Higgs mass value. Given that without a priori reason to select out a specific low value of λ at Planck scale, can we understand why the generic values on λ listed above tend to give a low Higgs mass prediction? The answer to that question is yes, and we shall demonstrate our reasoning here. The condition that $\lambda(M_{pl}) = 0$ will trivially generate the lightest Higgs mass provided that the SM is valid up to the Planck scale. As for the other conditions such as $\beta_\lambda(M_{pl}) = 0$ and $\mathrm{Str}\mathcal{M}^2(M_{pl}) = 0$, these are the conditions based on quantum loop correction for a Higgs bare mass parameter m and quartic coupling λ. In such cases where we impose the vanishing $\beta_\lambda(M_{pl})$ and $\mathrm{Str}\mathcal{M}^2(M_{pl})$, we can write λ as a function of gauge couplings and the top Yukawa coupling, i.e. $\lambda = f(g_i, \lambda_t)$. Since this function is obtained via loop calculation, the negative sign of the fermionic loop will compensate the contribution of the gauge fields, pushing the value of λ to be small, hence producing a low Higgs mass prediction at the Fermi scale. Perhaps we can ask a question at this stage: Why should the top quark be much heavier than other quarks in such a way

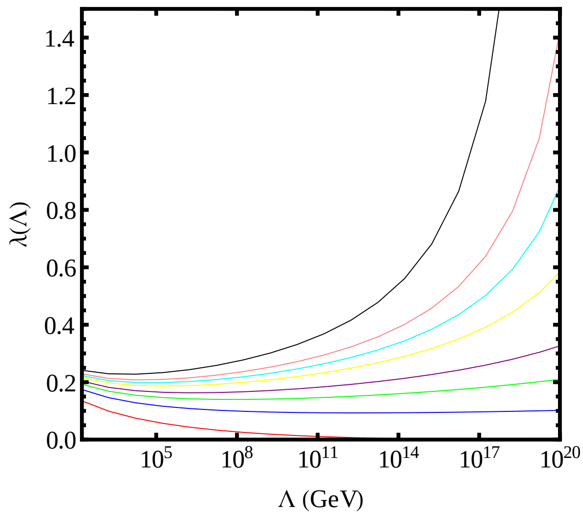


Figure 5.3: Running of λ from the Planck to the Fermi scale. In this example we set the top mass to be 173 GeV. A large parameter space of $\lambda(M_{pl})$ tends to produce $\lambda(v) > 0.2$, which is equivalent to $M_H > 150$ GeV.

to compensate all the other loop contributions that tend to drive Higgs boson heavier. This is intriguing, suppose that nature is only endowed with two physical energy scales, namely the Planck and Fermi scale, it is obvious that for a given theory of quantum gravity, it must provide a mechanism to suppress the value of λ at the Planck scale in order to concord with experimental evidence, possibly explaining also the sizable mass of the top quark. Of course this argument breaks down if the LHC or any future collider manages to detect any sign of new physics that couples directly to the Higgs boson. Even an indirect coupling, i.e. radiative correction to λ at loop level is severe enough to alter the running of λ drastically. But to date, there is no convincing evidence for additional physics at the TeV scale, whereas for the SM Higgs sector the search at the LHC is encouraging, as there are some excesses of Higgs-like event in the range of 120 GeV to 140 GeV, albeit at only about 2σ [7, 6, 8].

What is the motivation for choosing certain boundary conditions given above? We saw that most of our conditions tend to predict a Higgs mass lower than 150 GeV within the given error region. Furthermore a low Higgs mass is preferred by electroweak precision measurements, as mentioned in the previous section. Could this mean that all our generic values of $\lambda(\Lambda)$ considered do really describe the physics concorded to all experiment data, or are they just some random values on high energy scale that one can put in by hand and yet still be able to obtain the predictions allowed by the experiments. Suppose that

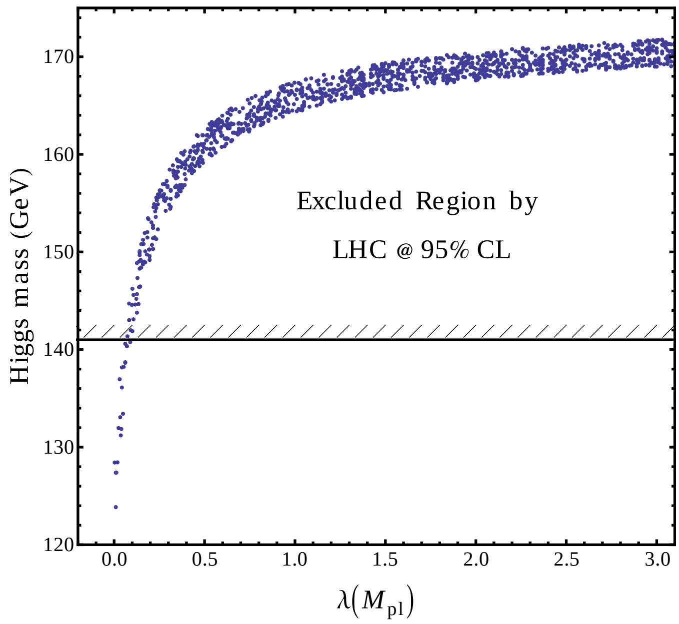


Figure 5.4: Scatter plot of Higgs mass at the Fermi scale determined by random λ at the Planck scale with random top mass constrained to the interval $[170, 175]$ GeV. Observe that only a relatively small percentage of generated $\lambda(M_{pl})$ yields a range of Higgs masses which is still not excluded by the LHC.

we have no a priori knowledge on the value of λ at the high energy scale, e.g. $\Lambda = M_{pl}$, we may then assume that $\lambda(M_{pl})$ is uniformly distributed at the Planck scale. Note that by distribution we do not mean a set of solutions provided by some high energy string landscape or some unknown quantum gravity, but rather we are just parameterizing our ignorance and lack of knowledge about the true solution of λ that would be produced by true quantum gravity. We may now ask, with such a uniformly distributed λ at the Planck scale, how would the Higgs mass at the Fermi scale be distributed? And consequently what can we say about the Higgs mass distribution at electroweak scale?

To answer this question, we can first randomly generate a set of $\lambda(M_{pl})$ with range of $\lambda \in [0, \pi]$. Theoretically one should push the upper limit to infinity, since there is no a priori reason for us to truncate λ at certain upper bound. But since we want to avoid the triviality bound, and technically it is more stable for the numerical program, we will just set the upper bound to be π. For our example, we randomly generate 600 values for $\lambda(M_{pl})$ and top pole masses ranging from $170 - 175$ GeV, then we subsequently determine their running in terms of different energy scales. The complete running of λ from the Planck to the weak scale can be found in Fig. (5.3). As we observe, a large portion of parameter space of λ at the Planck scale tends to yield λ at the Fermi scale greater than 0.2, which is equivalent to $M_H > 150\,\text{GeV}$. We generate a scatter plot for all the Higgs masses generated by the random seeding of $\lambda(M_{pl})$ and the top mass mentioned above. In Fig. (5.4) we can see that without imposing any preference on λ at the Planck scale, we obtain the Higgs mass from 160 GeV to 175 GeV with relative frequency of $\approx 90\%$. Clearly this region has been excluded by Tevatron and also recently by CMS and ATLAS experiments. Only less than 5% of the total predicted Higgs masses which are still allowed by experiment are obtained, yet all of the boundary conditions given in this work yield the Higgs mass in this region. Hence we can assume that it is unlikely that a full theory of quantum gravity generates any random value of λ as low energy imprint. Only physically well-motivated boundary conditions, e.g. those given in this work, can produce such a low Higgs mass value. These conditions could possibly be some remnant of symmetry in the full quantum theory of gravity. In other words, a full theory of quantum gravity needs to generate a small value of λ at the Planck scale, if the SM is valid up to the Planck scale. The Yukawa and λ couplings could have common origin from the Planck scale physics, as the top quark cancels the contribution of the Higgs boson in such a way that the SM could be extrapolated to the Planck scale. Indeed, this is an intriguing possibility, yet a more puzzling question would be: why do all the boundary conditions given in this work, which at first sight are independent from each other, tend to yield almost the same Higgs mass region? Could it be that some of the boundary conditions have to be satisfied simultaneously at the Planck scale? This possibility is intriguing, and possibly with such boundary conditions given for λ, one is urged to look for a quantum gravity solution that can satisfy parts of conditions simultaneously. For instance if we demand that $\lambda(M_{pl}) = 0$ and $\text{Str}\mathcal{M}^2(M_{pl}) = 0$ are satisfied then it is possible that Higgs quartic coupling is only generated radiatively and the quadratic divergence vanishes due to some unknown physics at Planck scale. This is obscure when looking from a low energy perspective, however from the Planck scale physics perspective this could be natural, as these two conditions could be a common trace of some unknown symmetry between the gauge, Yukawa and Higgs quartic coupling incorporated in quantum gravity physics. At the moment we cannot be sure of such

Given that it is possible to generate Higgs mass which is allowed by recent CMS and ATLAS exclusion limit, we may ask, should LHC only find the Higgs boson and no new physics, can we distinguish the boundary conditions listed above with present LHC sensitivity? ATLAS and CMS [97, 98, 99] are capable to detect Higgs mass to precision of 0.1% to 1% with integrated luminosity of $30\,\text{fb}^{-1}$. However with such an accuracy the determination of the high energy boundary conditions is still plagued by the relatively large theoretical uncertainty, unless higher order loop of beta function is calculated and is taken into account in reducing the error of the band in Fig. (5.2). A complete calculation of three-loop beta functions of the SM would also be very helpful in constraining different boundary conditions with smaller theoretical uncertainties.

5.4 Will the Standard Model live on?

In the next two years the LHC should be able to tell whether the SM Higgs boson exists, or exclude it. In general if the SM Higgs is found to be lower than 128 GeV, one can expect that the SM cannot be a valid theory up to the Planck scale as this value is below the Higgs mass allowed by the vacuum stability at the Planck scale. However this view must be taken with caution, as there are several factors that must be taken to account before jumping into this conclusion:

1. The Higgs mass of 128 GeV is obtained via two-loop beta function running from the vacuum stability condition at the Planck scale to the weak scale regime. As we can see from Fig. (5.1), the uncertainties due to the omission of higher order loop beta function span around ± 5 GeV. One can expect that the three-loop calculation lowers the predicted Higgs mass value and subsequently challenges the argument that the SM cannot be a valid QFT up to the Planck scale, should the LHC manage to find a Higgs boson with mass very near 128 GeV. Hence within the theoretical error, the SM still survives.

2. Precision top mass analysis is required to determine the exact value of the Higgs mass predicted via vacuum stability. The reason why we want to stress on this specific result is that to date, there is no general consensus on what type of top mass is actually measured via kinematic reconstruction [100]. In the Tevatron, the main method used for the top mass extraction actually “measures” the Pythia mass, which is a Monte-Carlo simulated template mass. Strictly speaking the top pole mass is not a well defined quantity, as the top quark does not exist as free parton. The top mass that the Tevatron has measured is based on the final state of the decay products. On the other hand the running $\overline{\text{MS}}$ top mass can be extracted directly from the total cross section in the top pair production. In this sense, one can obtain a complementary information of the top mass from the production phase. By converting the $\overline{\text{MS}}$ mass to the pole mass via matching conditions, the top pole mass value $168.9^{+3.5}_{-3.4}$ GeV extracted with this method by Langenfeld et al. [101] is found to be lower than the world best average value. The running top

mass is theoretically more well-defined for electroweak precision fits and RGE and therefore would be more preferred for Higgs mass prediction. However, this way of extracting the top mass suffers from larger numerical uncertainties. As we can infer from Fig. (5.2), a change of the top mass of 2 GeV can alter the Higgs mass prediction up to 6 GeV uncertainty. Suppose that the true top mass is actually significantly lower than the world best average, then the Higgs mass can be reduced to be as low as the LEP bound, well defined within the error, and still does not contradict the fact that the SM is a valid QFT up to the Planck scale.

3. The SM Higgs vacuum might not be a stable one, but rather metastable. Most of the parameter region of Higgs mass for the zero temperature metastability of SM vacuum has been almost ruled out by LEP, although not entirely excluded. The finite temperature metastability region however, with the local SM assumed to be stable against the thermal fluctuations up to the Planck scale temperature, fills out the the entire region from the LEP bound to the vacuum stability. Note that the theoretical error is always taken into account in this argument. Hence if the LHC discovers the SM Higgs boson with its mass lower than the one predicted by two-loop RGE vacuum stability bound, there is a possibility that the SM electroweak vacuum is not the stable one. Refer to Refs. [69, 102] for more detailed analysis.

The precise top mass and Higgs mass measurement will be important if we want to rule out the possibility that the SM surviving up to the Planck scale. A detailed analysis of the channel $H \to \gamma\gamma$ and $H \to b\bar{b}$ from associated Higgs production with the electroweak bosons will be crucial to detect the SM Higgs boson, should it lie in the low mass region. As we have shown in Fig. (5.2), the different Higgs masses predicted by a variety of Planck scale boundary conditions is difficult to be differentiated from the rest due to the large theoretical uncertainty. It would be very challenging experimentally to determine the precise boundary conditions that quantum gravity has set at the Planck scale, should the SM indeed be a valid QFT up to this scale. From the theoretical perspective however, this scenario opens up a new direction in model building on how to obtain the SM couplings from a new non-field theoretical concept. Possibly we would need a new idea to fully understand the quantum theory of gravity. The next challenge would be to understand how the SM exists as a remnant of quantum gravity, and why is there a big desert between the weak and Planck scale.

Chapter 6

Conclusion and Outlook

As we have argued in this work, the SM could be a valid effective QFT up to the Planck scale. This scenario should be considered as one of the possible outcomes that might describe the nature, should the LHC only find the SM Higgs boson and no signs of new physics. As we have mentioned in the previous sections, phenomena that cannot be adequately described by the SM, can find a possible solution in Planck scale physics or simple extension of the SM with sterile neutrinos (νMSM). So far we have adhered to the spirit of this thesis, namely that only two fundamental physics scales: Fermi and Planck scale, exist. We have argued from the fine-tuning perspective that it is more advantageous to forbid any intermediate scale in order to avoid a large hierarchy between its heavy particle mass and the SM Higgs mass.

With only Fermi scale and Planck scale allowed, the SM Higgs sector, even though at first sight might seem to suffering from a fine-tuning problem, can be viewed as natural from the Planck scale physics perspective, if quantum gravity consists of some new concepts which are non-field theoretic. To date, little is known about such a theory. However as we have argued in the introduction, the high energy quantum gravity theory could leave its low energy trace as certain boundary conditions for SM couplings. Motivated by this, we have examined a range of Higgs masses predicted by different boundary conditions of λ at the Planck scale by using the beta function in solving RGEs numerically. The matching error between $\overline{\text{MS}}$ coupling and respective pole mass has been thoroughly investigated and suitable matching conditions are used in order to minimize the matching error. We have seen that depending on the loop order of the beta function used to obtain the Higgs mass from a Planck scale boundary condition, we necessary obtain different values for the Higgs mass. We have defined the difference of the Higgs mass obtained via one- and two-loop beta functions used as our theoretical error. Experimental uncertainties however do not propagate significantly in the Higgs mass determination, except from the top mass.

We have seen in Fig. (5.2) that most of the physically well motivated boundary conditions tend to yield a range of low Higgs mass, which is to date still not excluded by ATLAS and CMS searches. In previous sections we have shown that only a small fraction of λ at Planck scale will yield such a low mass range, prompting us to ponder upon the role of the top quark and its interplay with the Higgs mass. It is argued that most of our conditions imposed stem from quantum loop corrections, which demonstrate a good cancellation between Higgs mass and top mass. This precise cancellation prompts us to think: why should the top mass be so heavier than other SM fermions? Perhaps an interplay between Higgs physics and Planck scale quantum gravity will shed light on this matter.

We have seen that with such a dense overlap of the boundary conditions imposed, it would be a challenge, experimentally and theoretically, to differentiate the correct boundary condition dictated by Planck scale physics. With around $5\,\mathrm{fb}^{-1}$ of integrated luminosity collected by ATLAS and CMS, it is possible to exclude the Higgs mass region from $114 - 600\,\mathrm{GeV}$ by the end of next year, if the SM Higgs does not exist in nature. A discovery of the SM Higgs boson on the other hand, will be significant for particle physics. If the LHC only finds the SM Higgs boson and nothing else new, then perhaps the spirit of this thesis is one of the correct descriptions for nature. The fate of the SM, will be decided by the discovery or exclusion of the SM Higgs boson.

After this work is carried out, it is announced that ATLAS and CMS will present the updates from the SM Higgs searches on 13.12.2011. Indeed exciting times are ahead.

Appendices

Appendix A

Notations and Conventions

A.1 Metric

Throughout the thesis we have used the "mostly minus" convention for our metric:

$$\eta_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.1}$$

A.2 Dirac Algebra

The Dirac matrices γ^μ satisfy the Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}, \tag{A.2}$$

with the components in chiral representation given as below:

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}, \quad \gamma^k = \begin{pmatrix} 0 & \sigma^k \\ -\sigma^k & 0 \end{pmatrix}, \quad \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}. \tag{A.3}$$

The $\mathbb{1}$ in this case is a two-by-two unit matrix. Defining γ^μ in terms of σ^μ and $\bar{\sigma}^\mu$, we have:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \tag{A.4}$$

where $\sigma^\mu = (\sigma^0, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\bar{\sigma}^0, \vec{\bar{\sigma}})$ are defined as:

$$\sigma^0 = \bar{\sigma}^0 = \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma^1 = -\bar{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\sigma^2 = -\bar{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = -\bar{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.5}$$

A.3 Spinors

Let ψ be a 4-component Dirac spinor. We define the Dirac conjugate of ψ as:

$$\bar{\psi} \equiv \psi^\dagger \gamma^0. \tag{A.6}$$

We can then proceed to decompose ψ into two two-component Weyl spinors, defined as:

$$\psi_L = \frac{1}{2}(1 - \gamma_5)\psi \quad \equiv \quad P_L \psi, \tag{A.7}$$
$$\psi_R = \frac{1}{2}(1 + \gamma_5)\psi \quad \equiv \quad P_R \psi, \tag{A.8}$$

which will be useful for some calculations.

A.4 Gauge transformation of the Standard Model fields

The infinitesimal gauge transformations of the gauge fields are given as:

$$\delta B_\mu = \partial_\mu \theta_1 \,, \tag{A.9}$$
$$\delta W^a_\mu = \partial_\mu \theta^a_2 - g_2 \epsilon^{abc} \theta^b_2 W^c_\mu \,, \tag{A.10}$$
$$\delta G^\alpha_\mu = \partial_\mu \theta^\alpha_3 - g_3 f^{\alpha\beta\gamma} \theta^\beta_3 G^\gamma_\mu, \tag{A.11}$$

while the fermionic fields transform under the SM gauge group as:

$$\delta L_m = \left(-\frac{i}{2}\theta_1 + \frac{\sigma^a}{2}\theta^a_2 \right) L_m \,, \tag{A.12}$$
$$\delta E_m = (-i\theta_1)\, E_m \,, \tag{A.13}$$
$$\delta Q_m = \left(\frac{i}{6}\theta_1 + \frac{\sigma^a}{2}\theta^a_2 + \frac{i}{2}\theta^\alpha_3 t^\alpha \right) Q_m \,, \tag{A.14}$$
$$\delta U_m = \left(i\frac{2}{3}\theta_1 + \frac{i}{2}\theta^\alpha_3 t^\alpha \right) U_m \,, \tag{A.15}$$

$$\delta D_m = \left(-\frac{i}{3}\theta_1 + \frac{i}{2}\theta_3^\alpha t^\alpha\right) D_m\,, \tag{A.16}$$

with t^α and σ^a representing the Gell-Mann and Pauli matrices respectively. The covariant derivatives of the SM fermionic fields are defined as:

$$D_\mu L_m = \left(\partial_\mu + i\frac{g_1}{2}B_\mu - ig_2\frac{\sigma^a}{2}W_\mu^a\right) L_m\,, \tag{A.17}$$

$$D_\mu E_m = (\partial_\mu + ig_1 B_\mu)\, E_m\,, \tag{A.18}$$

$$D_\mu Q_m = \left(\partial_\mu - i\frac{g_1}{6}B_\mu - ig_2\frac{\sigma^a}{2}W_\mu^a - ig_3\frac{t^\alpha}{2}G_\mu^\alpha\right) Q_m\,, \tag{A.19}$$

$$D_\mu U_m = \left(\partial_\mu - i\frac{2}{3}g_1 B_\mu - ig_3\frac{t^\alpha}{2}G_\mu^\alpha\right) U_m\,, \tag{A.20}$$

$$D_\mu D_m = \left(\partial_\mu + i\frac{g_1}{3}B_\mu - ig_3\frac{t^\alpha}{2}G_\mu^\alpha\right) D_m\,, \tag{A.21}$$

such that the SM Lagrangian is invariant under gauge transformations.

A.5 Effective potential of the Standard Model

In the SM the tree level potential and the one-loop correction are given as:

$$V_0 = -\frac{1}{2}m^2(\mu)\phi^2(\mu) + \frac{1}{4}\lambda(\mu)\phi^4(\mu), \tag{A.22}$$

$$V_1 = \sum_{i=1}^{5} \frac{n_i}{64\pi^2} M_i^4(\phi)\left[\log\frac{M_i^2(\phi)}{\mu^2} - d_i\right], \tag{A.23}$$

with the tree-level masses represented by $M_i^2(\phi) = \kappa_i\phi^2(\mu) - \kappa_i'$. The relevant parameters for the effective potential of the SM are given as below:

i	M_i	n_i	κ_i	κ_i'	d_i
1	M_W	6	$g_2^2/4$	0	5/6
2	M_Z	3	$(g_2^2+g_1^2)/4$	0	5/6
3	M_t	-12	$\lambda_t^2/2$	0	3/2
4	M_H	1	$3\lambda/2$	m^2	3/2
5	$M_{\text{Goldstone}}$	3	$\lambda/2$	m^2	3/2

Appendix B

Beta Functions and Anomalous Dimension

We give the beta function and anomalous dimension of the Higgs mass used in our calculation. The beta function for a generic coupling X is given as:

$$\mu \frac{\mathrm{d}X}{\mathrm{d}\mu} = \beta_X = \sum_i \frac{\beta_X^{(i)}}{(16\pi^2)^i}, \tag{B.1}$$

and the anomalous dimension for Higgs mass is given as:

$$\mu \frac{\mathrm{d}m^2}{\mathrm{d}\mu} = \gamma_m = \sum_i \frac{\gamma_m^{(i)}}{(16\pi^2)^i}. \tag{B.2}$$

The list of one-loop anomalous dimension of Higgs mass and beta functions of the relevant couplings are given below [103, 104, 105, 106, 93]:

$$\beta_\lambda^{(1)} = \lambda(-9g_2^2 - 3g_1^2 + 12\lambda_t^2) + 24\lambda^2 + \frac{3}{4}g_2^4 + \frac{3}{8}(g_1^2 + g_2^2)^2 - 6\lambda_t^4, \tag{B.3}$$

$$\beta_{\lambda_t}^{(1)} = \frac{9}{2}\lambda_t^3 + \lambda_t\left(-\frac{17}{12}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right), \tag{B.4}$$

$$\beta_{g_1}^{(1)} = \frac{41}{6}g_1^3, \tag{B.5}$$

$$\beta_{g_2}^{(1)} = -\frac{19}{6}g_2^3, \tag{B.6}$$

$$\beta_{g_3}^{(1)} = -7g_3^3, \tag{B.7}$$

$$\gamma_m^{(1)} = m^2\left(12\lambda + 6\lambda_t^2 - \frac{9}{2}g_2^2 - \frac{3}{2}g_1^2\right). \tag{B.8}$$

The two-loop γ_m and β_X are given as:

$$\begin{aligned}
\beta_\lambda^{(2)} &= -312\lambda^3 - 144\lambda^2\lambda_t^2 + 36\lambda^2(3g_2^2 + g_1^2) - 3\lambda\lambda_t^4 + \lambda\lambda_t^2\left(80g_3^2 + \frac{45}{2}g_2^2 + \frac{85}{6}g_1^2\right) \\
&\quad - \frac{73}{8}\lambda g_2^4 + \frac{39}{4}\lambda g_2^2 g_1^2 + \frac{629}{24}\lambda g_1^4 + 30\lambda_t^6 - 32\lambda_t^4 g_3^2 - \frac{8}{3}\lambda_t^4 g_1^2 - \frac{9}{4}\lambda_t^2 g_2^4 \\
&\quad + \frac{21}{2}\lambda_t^2 g_2^2 g_1^2 - \frac{19}{4}\lambda_t^2 g_1^4 + \frac{305}{16}g_2^6 - \frac{289}{48}g_2^4 g_1^2 - \frac{559}{48}g_2^2 g_1^4 - \frac{379}{48}g_1^6 \qquad \text{(B.9)} \\
\beta_{\lambda_t}^{(2)} &= \lambda_t\left(-12\lambda_t^4 + \lambda_t^2\left(\frac{131}{16}g_1^2 + \frac{225}{16}g_2^2 + 36g_3^2 - 12\lambda\right) + \frac{1187}{216}g_1^4\right. \\
&\quad \left. -\frac{3}{4}g_2^2 g_1^2 + \frac{19}{9}g_1^2 g_3^2 - \frac{23}{4}g_2^4 + 9g_2^2 g_3^2 - 108g_3^4 + 6\lambda^2\right), \qquad \text{(B.10)} \\
\beta_{g_1}^{(2)} &= g_1^3\left(\frac{199}{18}g_1^2 + \frac{9}{2}g_2^2 + \frac{44}{3}g_3^2 - \frac{17}{6}\lambda_t^2\right), \qquad \text{(B.11)} \\
\beta_{g_2}^{(2)} &= g_2^3\left(\frac{3}{2}g_1^2 + \frac{35}{6}g_2^2 + 12g_3^2 - \frac{3}{2}\lambda_t^2\right), \qquad \text{(B.12)} \\
\beta_{g_3}^{(2)} &= g_3^3\left(\frac{11}{6}g_1^2 + \frac{9}{2}g_2^2 - 26g_3^2 - 2\lambda_t^2\right), \qquad \text{(B.13)} \\
\gamma_m^{(2)} &= 2m^2\left(-30\lambda^2 - 36\lambda\lambda_t^2 + 12\lambda(3g_2^2 + g_1^2) - \frac{27}{4}\lambda_t^4 + 20g_3^2\lambda_t^2\right. \\
&\quad \left. +\frac{45}{8}g_2^2\lambda_t^2 + \frac{85}{24}g_1^2\lambda_t^2 - \frac{145}{32}g_2^4 + \frac{15}{16}g_2^2 g_1^2 + \frac{157}{96}g_1^4\right). \qquad \text{(B.14)}
\end{aligned}$$

The anomalous dimension of Higgs field is defined as:

$$\frac{1}{Z}\mu\frac{\mathrm{d}Z}{\mathrm{d}\mu} = \gamma = \sum_i \frac{\gamma^{(i)}}{(16\pi^2)^i}, \qquad \text{(B.15)}$$

with one-loop and two-loop $\gamma^{(i)}$ given by:

$$\begin{aligned}
\gamma^{(1)} &= 3\lambda_t - \frac{9}{4}g_2^2 - \frac{3}{4}g_1^2, \qquad \text{(B.16)} \\
\gamma^{(2)} &= 6\lambda^2 - \frac{27}{4}\lambda_t^4 + 20g_3^2\lambda_t^2 + \frac{45}{8}g_2^2\lambda_t^2 + \frac{85}{24}g_1^2\lambda_t^2 - \frac{271}{32}g_2^4 + \frac{9}{16}g_1^2 g_2^2 + \frac{431}{96}g_1^4. \qquad \text{(B.17)}
\end{aligned}$$

Appendix C

Experimental Values for the Parameters

The experimental values for the parameters used in the analysis are given below [16]:

Name	Symbol	Value
Z boson mass	M_Z	$91.1876 \pm 0.0021\,\text{GeV}$
W boson mass	M_W	$80.399 \pm 0.023\,\text{GeV}$
Strong coupling constant ($\overline{\text{MS}}$)	$\alpha_s(M_Z)$	0.1184 ± 0.0007
Top quark mass	M_t	$173.2 \pm 0.9\,\text{GeV}$ [96]
Fermi constant	G_F	$1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$
Gravitational constant	G_N	$6.70881(67) \times 10^{-39}\,\text{GeV}^{-2}$
Fine structure constant	$\alpha(M_Z)$	128.91 ± 0.02
Fine structure constant ($\overline{\text{MS}}$)	$\hat{\alpha}(M_Z)$	127.916 ± 0.015
Weak mixing angle ($\overline{\text{MS}}$)	$\sin^2\theta_W^{\overline{\text{MS}}}(M_Z) \equiv \hat{s}_Z^2$	0.23116 ± 0.00013

The vev of the SM Higgs is determined via:

$$v = \frac{1}{2^{1/4}\sqrt{G_F}} \approx 246.22\,\text{GeV}, \tag{C.1}$$

while the Planck scale is obtained from the following:

$$M_{pl} = \frac{1}{\sqrt{G_N}} \approx 1.22 \times 10^{19}\,\text{GeV}. \tag{C.2}$$

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Spring, Summer, Autumn, Winter...

and Spring.

Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 05.12.2011 ....

(Kher Sham Lim)