

## Iterative Entanglement Distillation: Approaching the Elimination of Decoherence

Boris Hage,<sup>1</sup> Aiko Sambrowski,<sup>1</sup> James DiGuglielmo,<sup>1</sup> Jaromír Fiurášek,<sup>2</sup> and Roman Schnabel<sup>1</sup>

<sup>1</sup>*Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik and Leibniz Universität Hannover, Callinstrasse 38, 30167 Hannover, Germany*

<sup>2</sup>*Department of Optics, Palacký University, 17. listopadu 12, 77146 Olomouc, Czech Republic*

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We experimentally demonstrate the two-step distillation of entanglement. The output of a first distillation stage underwent a second distillation step and was made available for subsequent steps. Our experiment displays the realization of the building blocks required for an entanglement distillation scheme that can fully overcome decoherence.

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If two or more subsystems are in an entangled state, the physical realities of these subsystems cannot be seen as independent from each other [1], although they do not interact and may be separated by an arbitrarily large distance. For maximally entangled states, measurements performed on individual subsystems exhibit peculiar correlations stronger than any classical correlations. This remarkable phenomenon is exploited in quantum communication [2] and quantum computing [3]. Examples are secure communication via quantum cryptography [4,5] or quantum teleportation [6–8].

Entanglement can be efficiently distributed by using entangled pairs of photons or entangled pairs of light modes. The transmission channels, be they optical fibers or free space as envisioned in satellite-based quantum cryptography [9], are however inevitably lossy, and the coupling to the environment leads to decoherence and entanglement degradation. The decoherence thus imposes a fundamental limit on communication distance, information processing time, and complexity beyond which the entanglement is completely destroyed and the advantages of using quantum—rather than classical—systems are lost.

It was therefore one of the great discoveries in the early years of quantum information science that the deleterious effect of noisy channels on entanglement distribution can be counteracted *a posteriori* by operations performed locally on each part of the shared entangled state and classical communication [10–13]. The conceptually simplest approach is the single-copy entanglement distillation whereby a local quantum filter is applied to one part of the shared entangled state [12,14–16]. Successful filtration results in a probabilistic increase of entanglement of the shared state. However, this method is intrinsically limited in its ability to overcome decoherence, enhance the entanglement, and purify the state [17]. A more sophisticated approach is a distillation protocol that uses the local interference of two copies of the entangled states [13,18]. Such entanglement distillation schemes were demonstrated recently [19–21], but a single application of these protocols does not allow us to completely eliminate decoherence.

A solution to the problem of decoherence is provided by the *iterative* distillation schemes [10,13,22]. Such schemes (i) involve the distillation of input states that have already been distilled in a previous step and (ii) enable the application of subsequent steps. The entanglement of the distilled state increases with each successful iteration, and under certain conditions the protocol asymptotically converges to a maximally entangled pure state.

Here we report on the first experimental demonstration of iterative entanglement distillation. We implemented a two-step distillation protocol that used three decohered copies of an entangled state shared between two parties *A* and *B*, as shown in Fig. 1. The first distillation step used two decohered copies of the shared entangled state. At both locations, the local parts of the two states were superimposed on a balanced beam splitter with one output port at each site detected by a balanced homodyne detector. A comparison of the measurement results via classical communication yielded a probabilistic signal for the success of distillation. The protocol provided one output copy with increased shared entanglement and partially eliminated decoherence [21]. The second two-copy distillation step then used the already distilled state and a third decohered copy. Again local measurements and classical communication were used to prepare an iteratively distilled state with even less decoherence and even higher entanglement. After successful two-step distillation the state was available for subsequent distillation steps or, alternatively, e.g., for a quantum teleportation protocol. The output state was characterized by means of a full two-mode quantum state tomography [23,24].

Our experiment used three copies of continuous-variable entangled pairs of continuous-wave laser beams. Each entangled pair was generated by splitting a squeezed laser mode on a balanced beam splitter. The squeezed states of light were generated in optical parametric amplifiers, which were constructed from second-order nonlinear crystals ( $\text{MgO}:\text{LiNbO}_3$ ) with type-I phase matching inside a degenerate doubly resonant cavity [25]. The optical parametric amplifier process was pumped with a

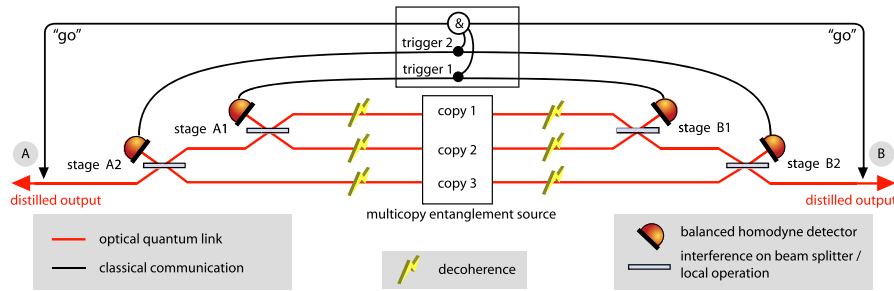


FIG. 1 (color online). Schematic of the experiment. The iterative three-copy distillation was demonstrated in the following way. When the entangled states of light left the source, they decohered due to phase fluctuations from coupling to the environment. Before they completely lost their entanglement, a first two-copy distillation protocol was realized by selecting states with low decoherence and strong entanglement by means of local measurements and classical communication. From the already distilled states, another subsequent two-copy distillation step further counteracted decoherence. For an infinite number of two-copy distillation steps, decoherence can be fully overcome.

frequency-doubled laser beam at 532 nm originating from a monolithic solid state laser (Nd:YAG) operating at 1064 nm. All three optical parametric amplifier outputs showed about 5 dB of squeezing and 9 dB of antisqueezing at modulation frequencies ranging from 5 to 15 MHz.

All three entangled pairs generated were distributed to two locations A and B and were intentionally exposed to independent random phase fluctuations which led to decoherence, *de-Gaussification*, and a degradation of the entanglement and the state purity [26]. In our experiment the phases of each of the six light fields involved were individually diffused by piezoactuated phase shifters driven by Gaussian quasirandom voltages that were generated by a personal computer sound card. Random phase fluctuations are a rather natural decoherence source, which produces non-Gaussian statistics of the states. This non-Gaussian property of phase noise allowed us to use Gaussian operations within our iterative distillation steps [13,18]. Note that for a purely Gaussian framework a no-go theorem for distillation applies [27,28]. The local operations and quantum measurements in our iterative distillation experiment involved the interference of 16 laser beams on eight beam splitters. Four beam splitters were required to pairwise interfere the distributed parts of the three entangled copies for the two distillation steps. Those in the first stage were balanced, while those in the second stage provided a 2:1 power *transmittance:reflectance* ratio. Another four beam splitters were integral parts of the four balanced homodyne detectors (BHDs) which are shown in Fig. 1. In the BHDs the beam splitters were used to generate an interference signal with a local oscillator beam, which then provided the local quantum measurements. Another two beam splitters were used in two more BHDs that were placed in the output ports of our setup (not shown in Fig. 1) in order to independently verify and characterize our distillation protocol by means of a full quantum tomography on the distilled outputs at A and B. The fringe contrasts achieved at the BHDs and at the distillation stages were between 97% and 99%. An important aspect of our experiment was the simultaneous phase control of

the two-times ten laser beam inputs to the beam splitters mentioned. In order to generate error signals for the in-phase interference at the distillation stages, small fractions (3%) of the beams were tapped in front of the balanced homodyne detectors in the distillation stages.

The "go" signal for successful iterative distillation in our experiment required (positive) trigger signals from both of the distillation stages. In each stage a trigger signal was generated from two BHDs' amplitude quadrature measurements  $X_{A1}$ ,  $X_{B1}$  and  $X_{A2}$ ,  $X_{B2}$ , respectively (Fig. 1). This provided information about how likely it was to have better than average entanglement on the second unmeasured output of the preceding distillation beam splitter, as shown in Refs. [18,21]. Successful iterative distillation was indicated if simultaneously  $|X_{A1} - X_{B1}| < Q$  and  $|X_{A2} - X_{B2}| < Q$ , where  $Q$  is a fixed but variable threshold. The lower the threshold was, the stronger the distillation and purification effect was and the lower the total distillation yield became.

In order to completely characterize the distillation protocol, we performed a full tomographic reconstruction of the iteratively distilled states at the output ports of the experiment for several different values of the trigger thresholds. The elements of two-mode density matrix in the Fock state basis  $\rho_{nk|lm} = \langle nk|\hat{\rho}|lm\rangle$  were obtained by averaging the appropriate pattern functions  $S_{nk}^{lm}$  over the recorded homodyne data [23,24]. Since the magnitude of the matrix elements decays rapidly for higher photon numbers, the reconstruction was truncated for  $\{n, k, l, m\} \geq 5$ . Figure 2 presents the result of our work derived from our tomography data and illustrates that our iterative distillation protocol increases the entanglement and purity and outperforms the corresponding single-step distillation protocol. We plotted the logarithmic negativity (a), the purity (b), and the total variance (c), respectively, versus the trigger threshold value. The latter was kept equal in both stages since our explicit data analysis showed that the best performance is generally achieved if the threshold values of both stages are the same. Our measurement data are given as a solid line including their error bars dominated by sample statistics.

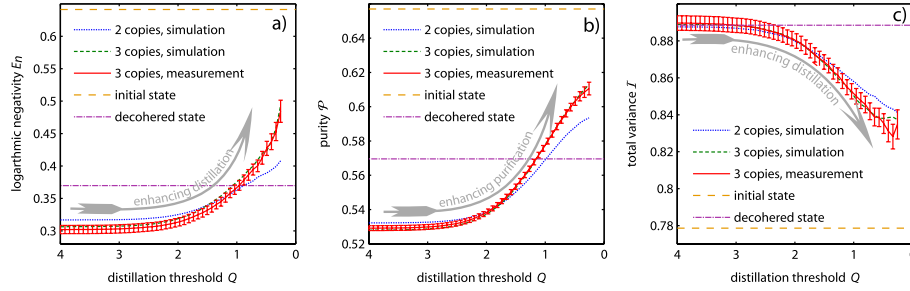


FIG. 2 (color online). Experimental data are given as solid lines including error bars. (a)–(c) show the logarithmic negativity, purity, and the total variance of the distilled states versus trigger threshold applied to both distillation stages. The short-dashed lines show the result of Monte Carlo simulations with the exact parameters of the experiment. Note that the error bars of the simulation (not shown) have the same magnitude as those of the measurement data, because both are dominated by the statistics of the sample number of  $3 \times 10^5$  for each of  $10^2$  tomography slices. The dotted lines represent the numerical simulation for the corresponding single-step two-copy protocol assuming exactly the same experimental parameters. The long-dashed lines represent the values before the decoherence, i.e., without any phase noise. The dot-dashed lines characterize the decohered input states before the distillation stages. All three quantities are improved by the distillation beyond their respective values for the input states. The iterative distillation outperforms the corresponding single-step protocol.

A precise Monte Carlo simulation of our experimental setting is given as a short-dashed line. Our model reproduces the measurement very well. Since the same sample size was used, the size of the statistical fluctuations also corresponded to the measurement error bars. The comparison with the corresponding single-stage distillation protocol is again given as a precise numerical simulation (dotted line). Note that a comparison with experimental data from a single-stage distillation protocol would be less accurate since a complete rearrangement of the experiment was necessary. The long-dashed lines characterize the initially prepared states without phase noise. The dot-dashed lines display the impact of the intentional phase diffusion. Note that the mixing of the input copies in the distillation stages further lowered the logarithmic negativity and the purity (see the left end of the individual figures).

The logarithmic negativity  $E_n$  as given in Fig. 2(a) is a computable entanglement measure [29] and the presence of entanglement is certified if  $E_n > 0$ . Figure 2(a) shows that the entanglement of the distilled states increased with lower (more strict) threshold values and exceeded the value of the input states (dot-dashed line). This break-even point corresponded to a probability of success of more than 70%/50% for the two/three copies protocols. Indeed, the iterative protocols yielded a faster increase than in the corresponding single-stage protocol. When changing the sample number  $N$  in our Monte Carlo simulations, we found that the logarithmic negativity slightly decreased with an increasing number of samples [30]. This effect was also observed in Ref. [16]. Since the results shown in Fig. 2(a) were all obtained for the same number of samples (measured and simulated), a fair comparison is guaranteed. Figure 2(b) demonstrates that also the purity of the distilled state, as given by  $\mathcal{P} = \text{Tr}(\rho^2)$ , did increase beyond the input states, again taking advantage from the iterative two-stage protocol for lower threshold values. In Fig. 2(c), we used another measure of entanglement which is

relevant for entanglement with a Gaussian statistics and for downstream applications within a purely Gaussian setting, such as the teleportation of Gaussian states [7,8]. Figure 2(c) shows the corresponding states' total variance  $I$  whose definition is based upon the variances of the difference and sum of the amplitude quadrature measurement results and the phase quadrature measurement results at parties  $A$  and  $B$  ( $X_A, X_B, P_A, P_B$ ), respectively:  $I = \text{Var}(X_A - X_B) + \text{Var}(P_A + P_B)$ . In this work we normalized the quadrature variance of an individual mode in its ground state to  $1/4$ . Then, according to Ref. [31], the state is entangled if  $I < 1$ . Note that for this entanglement measure a *smaller* value of  $I$  corresponds to stronger (Gaussian) entanglement. We also found the distilled states to be *Gaussified* in terms of the entropy-based Gaussianity measure introduced in Ref. [32].

An interesting question is how the improved performance due to the second distillation stage affects the overall rate of the distillation yield. Remarkably, Fig. 3 shows that in the relevant regime of a significant distillation effect the two-step scheme is superior even for a fixed requested total distillation yield. As an example, consider 3000 initially distributed decohered entangled states and a total distillation yield of 10%. With the single-stage protocol the distillate is the result from 1500 two-copy distillations and contains 300 states with a total variance of  $I = 0.843$ . With our two-stage protocol the distillate is the result of 1000 three-copy distillations and also contains 300 states but with stronger entanglement corresponding to a total variance of just  $I = 0.838$ . Note that in this plot the numerical simulations (dashed and dotted lines) used 300 times higher sample numbers ( $3 \times 10^6$ ), which resulted in statistical error bars considerably smaller than those of the measurement data (solid line). The simulated curves clearly suggest that our two-stage iterative protocol improved the entanglement into a regime not accessible for a single-stage protocol.

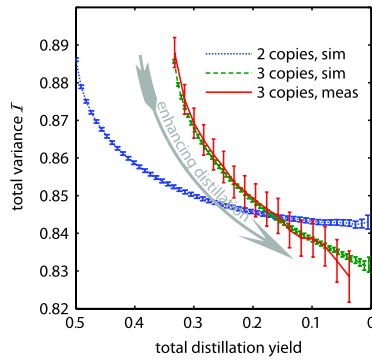


FIG. 3 (color online). Our three-copy iterative distillation is superior even under the more strict condition that not the trigger threshold(s) but the total distillation yield is kept constant. Here, the simulations used a 300 times higher sample number providing smaller error bars.

In our experiment, we quantitatively and qualitatively analyzed the entanglement shared between two separated locations  $A$  and  $B$  when a two-step distillation protocol was applied in order to counteract decoherence. We have successfully shown that an already distilled state could be further distilled when another decohered copy of shared entanglement was integrated. Complete evidence was provided by the first realization of a full two-mode continuous-variable tomographic reconstruction of the entangled states. A remarkable result of our experimental and theoretical analysis is that our entanglement distillation protocol, though iterative, provides a surprisingly high efficiency. The protocol provides a significant distillation and purification effect combined with a high total distillation yield of the order of 10%. We also emphasize that our distillation protocol does not depend on the characterization at the output ports and that the distilled states can be used in any downstream quantum protocol. In combination with a de-Gaussification protocol as recently used in Ref. [16], our iterative distillation scheme can be used in order to counteract also optical loss and not only phase diffusion as considered here. We therefore experimentally realized the necessary building blocks for achieving, in principle, full elimination of decoherence.

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