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The Unruh effect in general boundary quantum field theory

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We present an analysis of the Unruh effect from the perspective of the general boundary formulation of the quantum field theory of a massive scalar field in Minkowski and Rindler spacetimes. We underline the difficulty in identifying the Minkowski vacuum state with a superposition of multiparticle states defined in the double Rindler wedge. However, in contrast to this "global" version of the Unruh effect we show that a "local" version of it arises when comparing expectation values of observables quantized with the Feynman quantization prescription that have compact support restricted to one of the Rindler wedges. We find that even this notion of local Unruh effect does not exist when the same observables are quantized according to the Berezin-Toeplitz quantization scheme.

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I. INTRODUCTION

In his seminal paper [1] Oeckl proposed an axiomatic framework for the quantum theory that allows to formulate quantum field theory on general spacetime regions with general boundaries. This new formulation, named the General Boundary Formulation (GBF), assumes as an important ingredient (a particular version of) the mathematical framework of Topological Quantum Field Theory [1–4]. In particular the set of axioms that defines the GBF implements an assignment of algebraic structures to geometrical structures and guaranties the consistency of such an assignment. The physical interpretation relies on a generalization of the Born's rule to extract probabilities from the algebraic structures (in particular amplitudes and observable amplitudes discussed below).

The main motivation at the basis of the development of the GBF is the desire to render the formulation of quantum theory compatible with the symmetries of general relativity, in view of a possible future formulation of a quantum theory of gravity. From such perspective the GBF appears to be particularly advantageous with respect to the standard formulation of quantum theory since it does not require a spacetime metric for its formulation. Indeed the axioms of the GBF necessitate only a topological structure, and not a metric one. Moreover, it is worth noting that no (space)time notion enters in the definition of the generalized Born's rule. See [5] and [6] for more details on the relevance of the GBF for the problem of quantum gravity.

Although, as said, the spacetime background metric does not play any fundamental role in the GBF, evidently a general boundary quantum theory can be implemented for studying the dynamics of fields defined on a spacetime with a definite metric background. In that case the versatility of the GBF makes it possible to consider not only initial and final data on Cauchy surfaces as in the standard approach to quantum field theory but also the dynamics that take place in more general regions; the main interest will be represented by compact spacetime regions, namely regions whose boundaries have spacelike as well as timelike parts. A certain numbers of results, [7–14], have been obtained by applying the GBF for fields in Minkowski space and curved spaces, among which we cite a new perspective on properties of the standard S-matrix (in particular the crossing symmetry, that becomes a derived property within the GBF) and the proposals of new quantization schemes that allow a generalization of the standard S-matrix. In particular in Anti-de Sitter space, where the lack of temporal asymptotic regions obstructs the application of the traditional S-matrix techniques, involving temporal asymptotic *in* and *out* states, *spatial asymptotic states* have been

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rigorously defined within the GBF of a scalar quantum field theory and the corresponding amplitude that has been computed for these states can then be interpreted as a generalization of the standard S-matrix [15].

In this paper our main motivation is to consider from the GBF perspective the so called Unruh effect for a quantum scalar field. Usually the Unruh effect is understood as a particular relation between the notions of particle state in Minkowski and in Rindler spacetime. Rindler space is the spacetime naturally associated with uniformly accelerated observers and is isomorphic to a submanifold of Minkowski spacetime called the Rindler wedge. Then, the Unruh effect can be stated as follows: linearly uniformly accelerated observers perceive the Minkowski vacuum state (i.e. the no-particle state of inertial observers) as a mixed particle state described by a density matrix at temperature[1] $T = a/(2\pi k_B)$, a being the constant acceleration of the observer and k_B the Boltzmann constant. This effect was proposed by Unruh in 1976 [16] and has received a considerably amount of attention in literature because of its relation to other effects, like the particle creation from black holes (the so called Hawking effect) and cosmological horizons, and also because of recent proposals aimed at an experimental detection of the Unruh effect, see [17] for a review.

It is important to remark that the Unruh effect is a quantum field theory result that is derived by comparing the quantizations of a field in Minkowski and Rindler spacetime. We will refer to it as the *global Unruh effect* in order to distinguish it from a local notion of the effect defined later. The global Unruh effect is claimed to coincide with but must in fact be distinguished from the result that an uniformly accelerated Unruh-DeWitt detector responds as if submersed in a thermal bath when interacting with a quantum field in the Minkowski vacuum state [16]. In particular, the global Unruh effect is claimed to shed light on the relation between the quantum properties of the Minkowski vacuum state and the notion of particle in Rindler space: Indeed, the vacuum state in Minkowski is claimed to correspond to an entangled state between the modes of the field defined in the left and right Rindler wedges. Then tracing out the degrees of freedom in one of the wedges leads to a density operator describing a mixed thermal state at the Unruh temperature [16–22]. In the first part of this article, by comparing the general boundary quantum field theories in Minkowski and Rindler spacetimes, we point out the existence of an obstacle that prevents the acceptance of the previous sentence, at least in its full generality. As Belisnkii et al. showed in [23–26] the global properties of the field in the two spacetimes prevent the identification of the Minkowski vacuum state with states in Rindler spacetime. We recover the same obstruction to the global Unruh effect evidenced in these works by comparing the algebraic structures that define the general boundary quantum field theories of a free massive scalar field in Minkowski and Rindler spacetimes. In the second part of this paper the GBF quantization prescriptions for observables presented in [27], and further studied in [28], are applied to compute expectation values of local observables with compact support in a spacetime region contained in the right Rindler wedge. The result obtained suggests a notion of the Unruh effect different from the global one, that we call *local Unruh effect*. In particular the expression local Unruh effect stems from the coincidence of expectation values of local observables (i.e. observables with compact spacetime support) obtained in two different settings: in the first setting expectation values are computed on the vacuum state in Minkowski spacetime and in the second setting they are computed on a mixed state in Rindler spacetime[2].

The paper is structured as follows: In Sec. II we present a compact review of the GBF by specifying the two different representations so far implemented within the GBF, namely the Schrödinger representation in which the quantum states of the field are wave functionals of field configurations and the holomorphic representation where the states are holomorphic functions on germs of solutions to the field equations. In Sec. III A and Sec. III B we formulate the general boundary quantum field theory on Minkowski and Rindler spacetimes respectively for a massive Klein-Gordon field both in the Schrödinger and holomorphic representations. In Sec. IV we show that for the massive Klein-Gordon field the quantum field theory on Minkowski space and the one on Rindler space are inherently different and cannot be compared directly, thus invalidating the global Unruh effect. Nevertheless, in Sec. V we recover a version of the Unruh effect, that we call *local Unruh effect*, as the fact that for observables quantized according to the Feynman quantization prescription that are just defined on the interior of the right Rindler wedge expectation values in the Minkowski vacuum coincide with those calculated for a certain mixed state of the quantum theory in Rindler space. Finally, we summarize our conclusions and outlook in Sec. VI.

II. THE GENERAL BOUNDARY FORMULATION OF QUANTUM FIELD THEORY

This section presents a short review of the two representations in which the general axioms of [1] where implemented following the much more elaborate introduction given in [30]. These are the Schrödinger-Feynman representation [1] and the holomorphic representation [3]. We introduce the main structures that will be used in the rest of the paper,

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[1] We set $c = \hbar = 1$.

[2] This result is similar in some technical aspects to the one of Unruh and Weiss [29].

such as state spaces and amplitude maps for both representations.

As usual we start from an action $S[\phi] = \int_M d^N x\, \mathcal{L}(\phi, \partial\phi, x)$ which is considered to describe a linear real scalar field theory in a spacetime region M of an N-dimensional Lorentzian manifold $(\mathcal{M}, g)$. Denoting the boundary[3] of the region M with Σ, we associate with this hypersurface the space L_Σ of solutions of the Euler-Lagrange equations (derived from the action $S[\phi]$) defined in a neighborhood of Σ.[4] The symplectic potential on Σ results to be

$$(\theta_\Sigma)_\phi(X) := \int_\Sigma \mathrm{d}^{N-1}\sigma\, X(x(\sigma)) \left(n^\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi} \right)(x(\sigma)), \tag{1}$$

where n^μ is the unit normal vector to Σ. For every two elements of the space L_Σ there is the bilinear map $[\cdot,\cdot]_\Sigma : L_\Sigma \times L_\Sigma \to \mathbb{R}$ defined such that $[\xi, \eta]_\Sigma := (\theta_\Sigma)_\xi(\eta)$ and the symplectic structure, that is the anti-symmetric bilinear map $\omega_\Sigma : L_\Sigma \times L_\Sigma \to \mathbb{R}$ given by $\omega_\Sigma(\xi, \eta) := \frac{1}{2}[\xi, \eta]_\Sigma - \frac{1}{2}[\eta, \xi]_\Sigma$. The last ingredient for the quantum theory we need to specify is a complex structure J_Σ represented by the linear map $J_\Sigma : L_\Sigma \to L_\Sigma$ such that $J_\Sigma^2 = -\mathrm{id}$ and $\omega_\Sigma(J_\Sigma \cdot, J_\Sigma \cdot) = \omega_\Sigma(\cdot,\cdot)$. Remark, that all ingredients but the complex structure J_Σ are classical data uniquely defined by specifying the action.

These basic ingredients can now be used in different ways to specify the Hilbert spaces associated with the boundary hypersurface Σ. In the following subsection we introduce the two representations developed so far within the GBF, namely the Schrödinger representation, usually associated with the Feynman path integral quantization prescription, and the holomorphic representation.

A. The Schödinger-Feynman representation

In this representation quantum states are represented by wave functionals of field configurations. For its implementation it is convenient to introduce subspaces of the space L_Σ of solutions in a neighborhood of the hypersurface Σ. We start be defining what plays the role of the "space of momentum", denoted by $M_\Sigma \subset L_\Sigma$,

$$M_\Sigma := \{\eta \in L_\Sigma : [\xi, \eta] = 0\, \forall \xi \in L_\Sigma\}. \tag{2}$$

It can be shown that M_Σ is a Lagrangian subspace of L_Σ.[5] Next, we consider the quotient space $Q_\Sigma := L_\Sigma / M_\Sigma$ which corresponds the space of all field configurations on Σ. We denote the quotient map $L_\Sigma \to Q_\Sigma$ by q_Σ. The last ingredient needed for the Schrödinger representation is the bilinear map

$$\begin{aligned} \Omega_\Sigma :& Q_\Sigma \times Q_\Sigma \to \mathbb{C}, \\ & (\varphi, \varphi') \mapsto 2\omega_\Sigma(j_\Sigma(\varphi), J_\Sigma j_\Sigma(\varphi')) - i[j_\Sigma(\varphi), \varphi']_\Sigma, \end{aligned} \tag{3}$$

where j_Σ is the unique linear map $Q_\Sigma \to L_\Sigma$ such that $q_\Sigma \circ j_\Sigma = \mathrm{id}_{Q_\Sigma}$. Notice that the symplectic potential $[\cdot,\cdot]_\Sigma$ is equivalently seen as a map from $L_\Sigma \times Q_\Sigma$ to the complex numbers. The Hilbert space $\mathcal{H}_\Sigma^S$ (the superscript S refers to the Schrödinger representation) is now defined as the closure of the set of all coherent states

$$K_\xi^S(\varphi) = \exp\left(\Omega_\Sigma(q_\Sigma(\xi), \varphi) + i[\xi, \varphi]_\Sigma - \frac{1}{2}\Omega_\Sigma(q_\Sigma(\xi), q_\Sigma(\xi)) - \frac{i}{2}[\xi, \xi]_\Sigma - \frac{1}{2}\Omega_\Sigma(\varphi, \varphi) \right), \tag{4}$$

with respect to the inner product

$$\langle K_\xi^S, K_{\xi'}^S \rangle := \int_{Q_\Sigma} \mathcal{D}\varphi\, \overline{K_\xi^S(\varphi)}\, K_{\xi'}^S(\varphi), \tag{5}$$

where the bar denotes complex conjugation. The vacuum state K_0^S is then defined as the coherent state with $\xi = 0$.

So far we have defined the kinematical aspects and we now pass to the dynamical ones. Within the GBF the dynamics are encoded in an amplitude map $\rho_M : \mathcal{H}_\Sigma^S \to \mathbb{C}$ associated with the spacetime region M. In the Schrödinger

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[3] Notice that whether the boundary hypersurface Σ is a Cauchy surface (or a disjoint union of Cauchy surfaces) has no bearing on the following treatment.

[4] More precisely it is the space of germs of solutions at Σ which is the set of all equivalence classes of solutions where two solutions are equivalent if there exists a neighborhood of Σ such that the two solutions coincide in this whole neighborhood.

[5] It is this subspace M_Σ that defines the Schrödinger polarization of the prequantum Hilbert space constructed from L_Σ, see [30] for details.

representation for a state $\psi^S \in \mathcal{H}^S_\Sigma$, the amplitude ρ_M is defined in terms of the Feynman path integral prescription formally given by [28] (recall that Σ is the boundary of M)

$$\rho_M(\psi^S) := N_M \int_{L_M} \mathcal{D}\phi\, \psi^S(q_\Sigma(\phi)) e^{iS[\phi]}, \tag{6}$$

where L_M is the set of all field configurations in M that solve the Euler-Lagrange equations and with "$\mathcal{D}\phi$" we have denoted an hypothetical translation-invariant measure on L_M. As well known, in general no such measure exists in mathematical rigor. N_M is the normalization constant defined as

$$N_M := \int_{L^0_M} \mathcal{D}\phi\, e^{iS[\phi]}, \tag{7}$$

where L^0_M is the set of all field configurations in M that are zero on Σ. It is then possible, applying the generalized Born's rule [1, 31], to extract probabilities for the amplitude map ρ_M.

B. The holomorphic representation

From the complex structure J_Σ we define the symmetric bilinear form $g_\Sigma : L_\Sigma \times L_\Sigma \to \mathbb{R}$ as

$$g_\Sigma(\xi, \eta) := 2\omega_\Sigma(\xi, J_\Sigma \eta) \qquad \forall \xi, \eta \in L_\Sigma, \tag{8}$$

and assume that this form is positive definite. Next, we introduce the sesquilinear form

$$\{\xi, \eta\}_\Sigma := g_\Sigma(\xi, \eta) + 2i\omega_\Sigma(\xi, \eta) \qquad \forall \xi, \eta \in L_\Sigma. \tag{9}$$

The completion of L_Σ with the inner product $\{\cdot, \cdot\}_\Sigma$ turns it into a complex Hilbert space. The Hilbert space $\mathcal{H}^h_\Sigma = H^2(L_\Sigma, \mathrm{d}\nu_\Sigma)$,[6] namely the set of square integrable holomorphic functions on L_Σ, is the closure of the set of all coherent states [3]

$$K^h_\xi(\phi) := e^{\frac{1}{2}\{\xi, \phi\}}, \tag{10}$$

where $\xi \in L_\Sigma$ and the closure is taken with respect to the inner product

$$\langle K^h_\xi, K^h_{\xi'} \rangle := \int_{L_\Sigma} \mathrm{d}\nu_\Sigma(\phi)\, \overline{K^h_\xi(\phi)} K^h_{\xi'}(\phi), \tag{11}$$

where $\mathrm{d}\nu_\Sigma$ can be represented formally as $\mathrm{d}\nu_\Sigma(\phi) = \mathrm{d}\mu_\Sigma(\phi) e^{\frac{1}{4} g_\Sigma(\phi, \phi)}$ with a certain translation invariant measure $\mathrm{d}\mu_\Sigma$. The amplitude map for a state ψ^h is defined as

$$\rho_M(\psi^h) := \int_{L_{\tilde{M}}} \mathrm{d}\nu_{\tilde{M}}(\phi)\, \psi^h(\phi)\,, \tag{12}$$

where $L_{\tilde{M}} \subseteq L_\Sigma$ is the set of all global solutions on M mapped to L_Σ by just considering the solutions in a neighborhood of Σ.[7] The measure $\mathrm{d}\nu_{\tilde{M}}$ is a Gaussian probability measure constructed from the metric g_Σ [3].[8]

Independent of the representation the amplitude for coherent states turns out to be[9]

$$\rho_M(K_\xi) = \exp\left(\frac{1}{2} g_\Sigma(\xi^R, \xi^R) - \frac{1}{2} g_\Sigma(\xi^I, \xi^I) - \frac{i}{2} g_\Sigma(\xi^R, \xi^I) \right), \tag{13}$$

where $\xi^R, \xi^I \in L_{\tilde{M}}$ and $\xi = \xi^R + J_\Sigma \xi^I$.

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[6] To make this mathematically precise one actually has to construct $\mathcal{H}^h_\Sigma = H^2(\hat{L}_\Sigma, \mathrm{d}\nu_\Sigma)$ where $\hat{L}_\Sigma$ is a certain extension of L_Σ. For more details about the construction of $\hat{L}_\Sigma$ and $d\nu_\Sigma$ we refer the reader to [3].

[7] More precisely, global solutions are mapped to the corresponding germs at Σ.

[8] Again, we refer the reader to [3] where the constructions are given that make all the objects used here well defined.

[9] See equation (31) of [28] for normalized coherent states and equation (43) in [3]

III. GBF IN MINKOWSKI AND RINDLER SPACETIMES

We start with the action for the real massive Klein-Gordon field on $1+1$-dimensional Minkowski spacetime $\mathcal{M} = (\mathbb{R}^2, \eta = \mathrm{diag}(1,-1))$ which is given by

$$S[\phi] = \frac{1}{2}\int \mathrm{d}^2x \left(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right). \tag{14}$$

The resulting symplectic potential for a spacetime region M with boundary hypersurface Σ is

$$(\theta_\Sigma)_\xi(\phi) = \frac{\epsilon}{2}\int_\Sigma \mathrm{d}\sigma\, \xi(x(\sigma))\,(n^\mu\partial_\mu\phi)\,(x(\sigma)),$$

with n^μ the normalized hypersurface normal vector field pointing inside the region M and $\epsilon = +1/-1$ if Σ is everywhere spacelike/timelike respectively[10]. The derivative $\zeta := \frac{d}{d\sigma}\Sigma(\sigma)$ of the embedding function $\Sigma(\sigma)$ is normalized as $\eta^{\mu\nu}\zeta_\mu\zeta_\nu = 1$.

A. Minkowski spacetime

We want to investigate the GBF in a region $M \subset \mathcal{M}$ bounded by the disjoint union of two spacelike hypersurfaces represented by two equal time hyperplanes (this corresponds to the standard setting), which we denote as $\Sigma_{1,2} : \{t = t_{1,2}\}$, i.e. $M = \mathbb{R}\times[t_1,t_2]$. Then the boundary of the region M corresponds to the disjoint union $\Sigma := \Sigma_1 \cup \overline{\Sigma_2}$ (the bar denotes the inverted orientation). The set of solutions in the neighborhood of Σ decomposes in a direct sum as $L_\Sigma = L_{\Sigma_1} \oplus L_{\overline{\Sigma_2}}$ where L_{Σ_1} and $L_{\overline{\Sigma_2}}$ are the sets of solutions in the neighborhood of Σ_1 and $\overline{\Sigma}_2$ respectively each equipped with the corresponding symplectic form ω_{Σ_1} respectively $\omega_{\overline{\Sigma}_2}$ and a complex structure J_{Σ_1} respectively $J_{\overline{\Sigma}_2}$. The inversion of the orientation is implemented by the identification $[\phi,\phi']_{\overline{\Sigma}_2} = -[\phi,\phi']_{\Sigma_2}$ and $J_{\overline{\Sigma}_2} = -J_{\Sigma_2}$. The corresponding Hilbert space associated with Σ is given by the tensor product $\mathcal{H}_\Sigma = \mathcal{H}_{\Sigma_1}\otimes\mathcal{H}_{\overline{\Sigma}_2}$, where $\mathcal{H}_{\Sigma_1}$ and $\mathcal{H}_{\overline{\Sigma}_2}$ are the Hilbert spaces associated with the hypersurface Σ_1 and $\overline{\Sigma}_2$ respectively and the inversion of the orientation translates to the level of the Hilbert spaces by the map $\iota : \mathcal{H}_{\Sigma_2} \to \mathcal{H}_{\overline{\Sigma}_2}$, $\psi \mapsto \overline{\psi}$ as can be seen from the definition of the coherent states in section II.

In order to provide an explicit expression to the structures introduced in the previous section we expand the scalar field in a complete basis of solutions of the equation of motion,

$$\phi(x,t) = \int dp\ \left(\phi(p)\psi_p(x,t) + c.c.\right), \tag{15}$$

where $\psi_p(x,t)$ are chosen to be the eigenfunctions of the boost generator, namely the boost modes[11] [12]

$$\psi_p(x,t) = \frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty} \mathrm{d}q\, \exp\left(\mathrm{i}m(x\,\sinh q - t\,\cosh q) - \mathrm{i}pq\right) = e^{-\mathrm{i}\omega t}\frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty}\mathrm{d}q\,\exp\left(\mathrm{i}mx\,\sinh q - \mathrm{i}pq\right), \tag{16}$$

where we have introduced the operator $\omega = \sqrt{-\partial_x^2 + m^2}$. These modes are normalized as

$$\omega_{\Sigma_i}(\overline{\psi_p},\psi_{p'}) = \delta(p-p'), \qquad \omega_{\Sigma_i}(\psi_p,\psi_{p'}) = \omega_{\Sigma_i}(\overline{\psi_p},\overline{\psi_{p'}}) = 0. \tag{17}$$

The Hilbert space $\mathcal{H}_i$ of the quantum theory, associated to a hyperplane Σ_i $(i = 1,2)$ is defined by the vacuum state written in the Schrödinger representation as

$$K^S_{0,\Sigma_i}(\varphi_i) = N\exp\left(-\frac{1}{2}\int \mathrm{d}x\,\varphi_i(x)(\omega\varphi_i)(x)\right) \qquad (i=1,2), \tag{18}$$

[10] We stick here to the conventions used in [3] and earlier publicatons.
[11] It is assumed that an infinitely small imaginary part is added to t. Moreover, the integral over p in (15) must be extended from $-\infty$ to $+\infty$.
[12] Usually the expansion is given in the basis of plane wave solutions. However, it turns out to be more convenient for our purposes to use the boost modes.

N being a normalization constant and $\varphi_i \in Q_{\Sigma_i}$ are the boundary field configurations on the hypersurface Σ_i, namely $\varphi_i(x) = \phi(x,t)\big|_{t=t_i}$. This vacuum state corresponds to the standard Minkowski vacuum state[13], whose GBF expression has been given in [10], and it is uniquely defined by the complex structure [32]

$$J_{\Sigma_i} = \frac{\partial_t}{\sqrt{-\partial_t^2}}, \tag{19}$$

which defines a unitary complex structure on L_Σ in the sense that it is compatible with the dynamics of the field. The boost modes (16) are eigenfunctions of this complex structure, i.e. $J_{\Sigma_i}\psi_p = -\mathrm{i}\psi_p$. The structures introduced in the previous section, namely the symplectic form $\omega_{\Sigma_i}(\cdot,\cdot)$, the metric $g_{\Sigma_i}(\cdot,\cdot)$ and the inner product $\{\cdot,\cdot\}_{\Sigma_i}$, evaluated for two solutions $\phi, \phi' \in L_{\Sigma_i}$ $(i = 1, 2)$ take the form

$$\omega_{\Sigma_i}(\phi,\phi') = \frac{i}{2}\int_{-\infty}^{\infty} \mathrm{d}p \, \left(\overline{\phi(p)}\phi'(p) - \phi(p)\overline{\phi'(p)}\right), \tag{20}$$

$$g_{\Sigma_i}(\phi,\phi') = \int_{-\infty}^{\infty} \mathrm{d}p \, \left(\overline{\phi(p)}\phi'(p) + \phi(p)\overline{\phi'(p)}\right), \tag{21}$$

$$\{\phi,\phi'\}_{\Sigma_i} = g_{\Sigma_i}(\phi,\phi') + 2i\omega_{\Sigma_i}(\phi,\phi') = 2\int_{-\infty}^{\infty} \mathrm{d}p \, \phi(p)\overline{\phi'(p)}. \tag{22}$$

The dense subset of the Hilbert space associate to Σ_i, defined by the coherent states, as well as the amplitude map associated to the region M are implementable in terms of the above quantities.

B. Rindler spacetime

For the quantization of the scalar field in Rindler spacetime we consider again the action in equation (14) but restricted to the right wedge of Minkowski space, namely $\mathcal{R} := \{x \in \mathcal{M} : x^2 \leq 0, \, x > 0\}$, which is covered by the Rindler coordinates (ρ, η) such that $\rho \in \mathbb{R}^+$ and $\eta \in \mathbb{R}$. The relation between the Cartesian coordinates (x,t) and the Rindler ones is $t = \rho \sinh\eta$ and $x = \rho\sinh\eta$, and the metric of Rindler space results to be $ds^2 = \rho^2 d\eta^2 - d\rho^2$. We consider the region $R \subset \mathcal{R}$ bounded by the disjoint union of two equal-Rindler-time hyperplanes $\Sigma_{1,2}^R : \{\eta = \eta_{1,2}\}$, i.e. $R = \mathbb{R}^+ \times [\eta_1, \eta_2]$. In order to repeat the construct of the quantum theory implemented in Minkowski spacetime, we start by expanding the field in a complete set of solutions of the equation of motion,

$$\phi^R(\rho,\eta) = \int_0^{\infty} \mathrm{d}p \, \left(\phi^R(p)\phi_p^R(\rho,\eta) + c.c.\right), \tag{23}$$

where the Fulling modes [33] ϕ_p^R read

$$\phi_p^R(\rho,\eta) = \frac{(\sinh(p\pi))^{1/2}}{\pi} K_{\mathrm{i}p}(m\rho) e^{-\mathrm{i}p\eta}, \qquad p > 0, \tag{24}$$

$K_{\mathrm{i}p}$ is the modified Bessel function of the second kind, also known as Macdonald function [34]. The modes (24) are normalized as

$$\omega_{\Sigma_i^R}(\overline{\phi_p^R}, \phi_{p'}^R) = \delta(p - p'), \qquad \omega_{\Sigma_i^R}(\phi_p^R, \phi_{p'}^R) = \omega_{\Sigma_i}(\overline{\phi_p^R}, \overline{\phi_{p'}^R}) = 0. \tag{25}$$

The Hilbert space associated with the hypersurface Σ_i^R $(i = 1, 2)$ is characterized by the following vacuum state in the Schrödinger representation, expressed in terms of the boundary field configuration φ_i,

$$K^S_{0,\Sigma_i^R}(\varphi_i) = N \exp\left(-\frac{1}{2}\int \frac{\mathrm{d}\rho}{\rho}\, \varphi_i(\rho)(\omega\varphi_i)(\rho)\right) \qquad (i = 1, 2), \tag{26}$$

where ω now denotes the operator $\omega = \sqrt{(\rho\partial_\rho)^2 - m^2}$ and N is a normalization factor. This vacuum state is in correspondence with the following complex structure, defined by the derivative with respect to the Rindler time

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[13] In fact, the standard plane wave basis and the basis of the boost modes are related by a unitary transformation.

coordinate η,

$$J_{\Sigma_i^R} = \frac{\partial_\eta}{\sqrt{-\partial_\eta^2}}, \tag{27}$$

and the Fulling modes (24) are eigenfunctions of this complex structure: $J_{\Sigma_i^R}\phi_p^R = -\mathrm{i}\phi_p^R$. The algebraic structures defined on the hypersurface Σ_i^R, considered for two solutions $\phi^R, \psi^R \in L_{\Sigma_i^R}$ result to be

$$\omega_{\Sigma_i^R}(\phi^R, \psi^R) = \frac{i}{2}\int_0^\infty \mathrm{d}p \, \left(\overline{\phi^R(p)}\psi^R(p) - \phi^R(p)\overline{\psi^R(p)}\right), \tag{28}$$

$$g_{\Sigma_i^R}(\phi^R, \psi^R) = \int_0^\infty \mathrm{d}p \, \left(\overline{\phi^R(p)}\psi^R(p) + \phi^R(p)\overline{\psi^R(p)}\right), \tag{29}$$

$$\left\{\phi^R, \psi^R\right\}_{\Sigma_i^R} = g_{\Sigma_i^R}(\phi^R, \psi^R) + 2i\omega_{\Sigma_i^R}(\phi^R, \psi^R) = 2\int_0^\infty \mathrm{d}p \, \phi^R(p)\overline{\psi^R(p)}. \tag{30}$$

It is important to notice that in order for the quantum theory to be well defined the following condition must be imposed on the field in Rindler space: $\phi^R(\rho = 0, \eta) = 0$. Indeed the complex structure (27) is well defined except in the origin of Minkowski spacetime as can be seen by expressing (27) in terms of the Cartesian coordinates (x, t),

$$J_{\Sigma_i^R} = \frac{x\partial_t + t\partial_x}{\sqrt{-(x\partial_t + t\partial_x)^2}}. \tag{31}$$

The relevance of such a condition in the derivation of the Unruh effect has been emphasized, and discussed both in the canonical and algebraic approach to quantum field theory, by Belinskii et al. in [23–26, 35]. This condition plays indeed a fundamental role in the attempt to compare the quantum theories in Minkowski and Rindler spacetimes, as will be discussed in the next section.

IV. COMPARISON OF MINKOWSKI AND RINDLER QUANTIZATION - THERE IS NO GLOBAL UNRUH EFFECT

The Unruh effect can be expressed as the statement that an uniformly accelerated observer perceives the Minkowski vacuum state as a mixed thermal state at a temperature proportional to its acceleration. Such a claim relies on a comparison between the quantization of the field considered in Minkowski and in Rindler spacetimes; Rindler space being the spacetime naturally associated with an accelerated observer. The quantization scheme proposed by Unruh to implement such a comparison rests on the properties of particular linear combinations of the boost modes, known as the Unruh modes:

$$R_p(x,t) = \frac{1}{\sqrt{2\sinh(p\pi)}}\left(e^{p\pi/2}\,\psi_p(x,t) - e^{-p\pi/2}\,\overline{\psi_{-p}(x,t)}\right), \tag{32}$$

$$L_p(x,t) = \frac{1}{\sqrt{2\sinh(p\pi)}}\left(e^{p\pi/2}\,\overline{\psi_{-p}(x,t)} - e^{-p\pi/2}\,\psi_p(x,t)\right), \tag{33}$$

with $p > 0$, whose normalization is determined by the one of the boost modes (17). The key property of the Unruh modes is their behavior when evaluated in the right and left wedge of Minkowski spacetime[14]: the modes $R_p(x,t)$ (respectively $L_p(x,t)$) vanish for $(x,t) \in \mathcal{L}$ (respectively $(x,t) \in \mathcal{R}$) and moreover $R_p(x,t)$ coincide with the Fulling modes (24) in $\mathcal{R}$. Equations (32) and (33) are then interpreted as Bogolubov transformations connecting the expansion of the field in the basis of the boost modes and the one in the Unruh modes. The existence of such a Bogolubov transformation allows to relate the corresponding annihilation and creation operators defined within the canonical approach for the two quantization schemes and consequently the quantum states defined between the quantum theories. By inverting relations (32) and (33) and substituting in the expansion of the field in the basis of the boost modes provides an expression of the field in terms of the Unruh modes. Then, the restriction of the field to the right Rindler wedge allows a direct comparison, via the Bogolubov transformation, to the expansion of the field in Rindler spacetime in the basis of the Fulling modes.

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[14] The left wedge of Minkowski spacetime is the reflection of the right wedge with respect to the origin, namely $\mathcal{L} : \{x \in \mathcal{M} : x^2 \leq 0, x < 0\}$.

In particular, the expectation value of the canonical operator corresponding to the Rindler particle number operator, evaluated on the Minkowski vacuum state, is shown to be that of a thermal bath of Rindler particles. In their review [17] Crispino *et al.* provide the following definition (end of Section II.B): «The Unruh effect is defined in this review as the fact that the usual vacuum state for QFT in Minkowski spacetime restricted to the right Rindler wedge is a thermal state [...]». In the following we will refer to this result as *global Unruh effect*, because it relies on the properties of particle states and number particle operator which have a global nature[15].

However the derivation of the global Unruh effect has been criticized [35] in virtue of the existence of the boundary condition mentioned at the end of the preceding section. We will not repeat here the arguments presented in the cited papers but limit ourself to a few considerations. First, we notice that the correct expansion of the field in Minkowski spacetime in terms of the Unruh modes reads

$$\phi(x,t) = \lim_{\epsilon\to 0}\int_{\epsilon}^{\infty} \mathrm{d}p\,(r(p)R_p(x,t) + l(p)L_p(x,t) + c.c.) + \lim_{\epsilon\to 0}\int_{-\epsilon}^{\epsilon} \mathrm{d}p\,(\phi(p)\psi_p(x,t) + c.c.)\,, \tag{34}$$

which does not coincide with the expansion of the field in the basis of the Fulling modes for the world points located inside the right Rindler wedge, i.e. for $(x,t) \in \mathcal{R}$. The difference is due to presence of the last term in the r.h.s. of (34): for $(x,t) \neq (0,0)$ the integral vanishes in the limit $\epsilon \to 0$ because the boost modes take finite values[16], but for $(x,t) = (0,0)$ the contribution of this term cannot be neglected since the boost modes reduce to a delta function, as can be seen from expression (16), i.e. $\psi_p(0,0) = \delta(p)/\sqrt{2}$. The importance of this term is evident when considering the algebraic structures needed for the implementation of the GBF. For simplicity and without loss of generality, we consider the hyperplane Σ_0 at $t = 0$ and the structures (20), (21) and (22) defined on it. When restricting Σ_0 to the right wedge $\mathcal{R}$, by using (34) we obtain (we denote such restriction with the superscript $(\mathcal{R})$)

$$\omega^{(\mathcal{R})}_{\Sigma_0}(\phi,\phi') = \omega_{\Sigma_0^R}(\phi^R,\phi^{R'}) + \lim_{\epsilon\to 0} \mathrm{i}\int_0^{\epsilon} \mathrm{d}p\,\frac{\cosh(p\pi)}{\sinh(p\pi)}\left[\phi(p)\,\overline{\phi(p)'} - \overline{\phi(p)}\,\phi'(p)\right], \tag{35}$$

where Σ_0^R is the semi-hyperplane $\eta = 0$ (η being the Rindler time) corresponding to the intersection $\Sigma_0 \cap \mathcal{R}$. Analogue expressions are obtained for the restriction of (21) and (22). From (35) we notice that the restriction to $\mathcal{R}$ of the symplectic structure defined in Minkowski spacetime coincides with the symplectic structure in Rindler spacetime only if the second term in (35) vanishes, and for that we must impose $\phi(p) = 0$ for $p = 0$. However, requiring such a condition implies imposing the vanishing of the field at the left edge of the right wedge, namely at the origin of Minkowski spacetime. While this is a built-in boundary condition the field in Rindler has to satisfy[17], there is no reason to require the same condition for the field in Minkowski. Indeed imposing such condition translates in the exclusion of the zero boost mode from the set of modes on which the field is expanded, namely in the suppression of the second term in (34). But the remaining expression will then represent the expansion of a different field in Minkowski spacetime, namely a field that satisfies a zero boundary condition at $(x,t) = (0,0)$. The appearance of this condition is a consequence of the fact that the basis of the boost modes is not anymore complete without the zero boost mode. The consequence of this fact at the quantum level manifests in the loss of the translational invariance of the Minkowski vacuum state, see [35] for a detailed discussion on this point.

The claim that the Minkowski vacuum state can be written as an entangled state composed by multiparticle states defined in the left and right wedges is consequently not acceptable. For example Unruh and Wald [18] provide the following equality

$$|0_M\rangle = \prod_j N_j \sum_{n_j=0}^{\infty} e^{-\pi n_j \omega_j/a}|n_j,\mathcal{L}\rangle \otimes |n_j,\mathcal{R}\rangle, \tag{36}$$

where $|0_M\rangle$ is the vacuum state in Minkowski space, $N_j = (1 - \exp(-2\pi\omega_j/a))^{1/2}$, and $|n_j,\mathcal{L}\rangle, |n_j,\mathcal{R}\rangle$ represent the state with n_j particles in the mode j in the left, right wedge $\mathcal{L}$ and $\mathcal{R}$ respectively. From (36) it is then possible to

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[15] In the canonical approach particle states are determined by the action of the creation and annihilation operators whose definition involves the values that the field takes over a (non compact) Cauchy surface. In the GBF treatment, particle states are elements of the Hilbert space associated with the hypersurface under consideration and the construction of such a Hilbert space depends on the field configurations on this hypersurface, i.e. it depends on the global properties of the elements of the space L_Σ.

[16] We are here assuming that $|\phi(p)| < \infty$ in the limit $p \to 0$.

[17] This is usual decay condition at infinity for the field in Rindler spacetime since the origin of Minkowski spacetime corresponds to spatial infinity from the point of view of a uniformly accelerated observer.

obtain a reduced density matrix by tracing over the degrees of freedom in the left wedge,

$$\varrho^{(\mathcal{R})} = \prod_j N_j^2 \sum_{n_j=0}^{\infty} e^{-2\pi n_j \omega_j / a} |n_j, \mathcal{R}\rangle \otimes \langle n_j, \mathcal{R}|. \tag{37}$$

This reduced density matrix is interpreted as representing the restriction of the Minkowski vacuum to the region $\mathcal{R}$. As already mentioned, the left and right hand side of formulas (36) and (37) refer to states that belong to unitarily inequivalent quantum theories and are consequently not mathematically well-defined.

However, inspired by some results derived within the algebraic approach to quantum field theory[18], in the next section we present a result, obtained within the GBF, that suggests the existence of a relation between the Minkowski vacuum state and a particular mixed state of the quantum theory in Rindler spacetime (a relation that can be considered as the main insight of the Unruh effect). To be more precise, we compute the expectation value of a Weyl observable defined on a compact spacetime region in the interior of the right Rindler wedge in two different contexts: first on the vacuum state in Minkowski spacetime and then on a mixed state in Rindler space, whose form represents the analogue of the r.h.s of (37) in the GBF language. It turns out that these two expectation values are equal when the observables are quantized according to the Feynman quantization prescription and we refer to that equality as the *local Unruh effect*.

V. THE RELATION BETWEEN OPERATOR AMPLITUDES ON MINKOWSKI AND RINDLER SPACE - LOCAL UNRUH EFFECT

In this section, we make explicit what we mean with the expression *local Unruh effect*. This notion indicates the coincidence of the expectation values of local observables computed in Minkowski and Rindler spacetimes. The observables we consider have been called Weyl observables in [28] and are given by exponential of linear functional of the field,

$$F(\phi) = \exp\left(\mathrm{i} \int \mathrm{d}^2 x \, \mu(x) \phi(x)\right), \tag{38}$$

in our case $\mu(x)$ is assumed to have compact support in the interior of the right wedge $\mathcal{R}$ and (38) is consequently a well defined observable in both Minkowski and Rindler spacetime. The interest for looking at the Weyl observables is twofold: first, consistent quantization schemes have been established within the GBF, and second, general results concerning expectation values of these observables have been obtained in [28]. Here we consider the Feynman and Berezin-Toeplitz quantizations of (38) and compute for the corresponding quantum observables two different expectation values: one on the vacuum state in Minkowski spacetime and the other on a particular mixed state in Rindler spacetime.

In the GBF, as in algebraic quantum field theory, quantum observables O_M are associated with a spacetime region M. They are defined by a linear map, called observable map or observable amplitude, from (a dense subspace of) the Hilbert space associated with the boundary of the region to the complex numbers, $O_M : \mathcal{H}_\Sigma \to \mathbb{C}$, Σ being the boundary of the region M. A set of axioms establishes the properties of this map, in particular the spacetime composition of observables. The specific form of the observable map depends on the quantization scheme adopted. In the following two sections the Feynman and Berezin-Toeplitz quantization schemes combined with the Schrödinger and holomorphic representations are used for the Weyl observable in the settings specified above.

In the Feynman quantization prescription the observable map associated with an observable O_M evaluated on a state $\psi^S \in \mathcal{H}^S_\Sigma$ in the Schrödinger representation takes the form

$$\rho_M^{O_M}(\psi^S) = \int_{L_M} \mathcal{D}\phi \, \psi^S(q_\Sigma(\phi)) O_M(\phi) e^{\mathrm{i}S(\phi)}. \tag{39}$$

For the Berezin-Toeplitz quantization scheme we have, for a state $\psi^h \in \mathcal{H}^h_\Sigma$ in the holomorphic representation,

$$\rho_M^{\blacktriangleleft O_M \blacktriangleright}(\psi^h) = \int_{L_{\tilde{M}}} \psi^h(\xi) O_M(\xi) \mathrm{d}\nu_{\tilde{M}}(\xi), \tag{40}$$

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[18] We refer in particular to Fell's theorem [36] and the work of Verch [37].

where $\xi \in L_\Sigma$ and $L_{\tilde{M}}$ is the space of solutions of the equation of motion, defined in a neighborhood of the boundary hypersurface Σ that admit a well defined extension in the interior of the region M. $\mathrm{d}\nu_{\tilde{M}}(\xi)$ is a suitable measure on $L_{\tilde{M}}$ and we refer to [3, 27, 28] for details concerning the definition of such structures. In [28] Oeckl was able to quantify the difference between the observable maps computed in the Feynman quantization scheme and the one computed in the Berezin-Toeplitz quantization scheme. The result is presented in two propositions, in particular Proposition 4.3 and Proposition 4.7 of the cited paper, where the amplitude of a Weyl observable is derived for the two quantization prescriptions. We reproduce in the following formulas the statements of these propositions: For a coherent state K_τ we have

$$\rho_M^F(K_\tau) = \rho_M(K_\tau) F(\hat{\tau}) \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\, \mu(x)\eta_D(x) - \frac{1}{2} g_\Sigma(\eta_D,\eta_D)\right), \tag{41}$$

from the Feynman quantization (where $\hat{\tau}$ is a complex solution of the equation of motion determined by the coherent state K_τ and $\eta_D \in J_\Sigma L_{\tilde{M}}$, Σ being the boundary of the region M) and

$$\rho_M^{\blacktriangleleft F \blacktriangleright}(K_\tau) = \rho_M(K_\tau) F(\hat{\tau}) \exp\left(-g_\Sigma(\eta_D,\eta_D)\right), \tag{42}$$

from the Berezin-Toeplitz quantization.

A. Expectation values in the Schrödinger representation

1. *Observable maps from Feynman quantization*

Consider the spacetime region M defined in Sec. III A in Minkowski spacetime. We start by computing the observable amplitude $\rho_M^F : \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\overline{\Sigma}_2} \to \mathbb{C}$ for the Weyl observable (38) on the quantum state, in the Schrödinger representation, given by the tensor product of two copies of the vacuum state (18), namely $K^S_{0,\Sigma_1} \otimes \overline{K^S_{0,\Sigma_2}}$. Using the expression in equation (41) we arrive at

$$\rho_M^F(K^S_{0,\Sigma_1} \otimes \overline{K^S_{0,\Sigma_2}}) = \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\,\mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{M}}(x,x')\mu(x')\right). \tag{43}$$

where $G_F^{\mathcal{M}}$ is the Feynman propagator in Minkowski spacetime, which is evaluated only in the interior of the right Rindler wedge since the field $\mu(x)$ has support there. The explicit form of the Feynman propagator can be obtained in terms of the expression of the boost modes (16) in the right Rindler wedge, namely [38]

$$\psi_k(x,t)\big|_{(x,t)\in\mathring{\mathcal{R}}} = \frac{1}{\pi\sqrt{2}} \exp\left(\frac{\pi k}{2} - \mathrm{i}\frac{k}{2}\ln\left(\frac{x+t}{x-t}\right)\right) K_{\mathrm{i}k}(m\sqrt{x^2-t^2}), \tag{44}$$

where $\mathring{\mathcal{R}}$ denotes the interior of the right Rindler wedge. Then the Feynman propagator reads

$$\begin{aligned} G_F^{\mathcal{M}}(x,x')\big|_{x,x'\in\mathring{\mathcal{R}}} = \mathrm{i}\int_0^\infty \frac{\mathrm{d}k}{\pi^2} \Bigg\{ &\cosh(\pi k)\cos\left(\frac{k}{2}\left(\ln\left(\frac{x+t}{x-t}\right) - \ln\left(\frac{x'+t'}{x'-t'}\right)\right)\right) \\ &- \mathrm{i}\theta(t'-t)\sinh(\pi k)\sin\left(\frac{k}{2}\left(\ln\left(\frac{x'+t'}{x'-t'}\right) - \ln\left(\frac{x+t}{x-t}\right)\right)\right) \\ &- \mathrm{i}\theta(t'-t)\sinh(\pi k)\sin\left(\frac{k}{2}\left(\ln\left(\frac{x+t}{x-t}\right) - \ln\left(\frac{x'+t'}{x'-t'}\right)\right)\right)\Bigg\} K_{\mathrm{i}k}(m\sqrt{x^2-t^2}) K_{\mathrm{i}k}(m\sqrt{x'^2-t'^2}). \end{aligned} \tag{45}$$

Now, consider the region R defined in Sec. III B in Rindler spacetime. The evaluation of the observable map is now performed on the mixed state $D \in \mathcal{H}_{\Sigma_1^R} \otimes \mathcal{H}_{\overline{\Sigma}_2^R}$ given by expression

$$D = \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i/a}}{(n_i)!(2k_i)^{n_i}} \psi_{n_i} \otimes \overline{\psi_{n_i}}, \tag{46}$$

where ψ_{n_i} is the state with n_i particles defined in $\mathcal{H}_{\Sigma_i^R}$, $(i=1,2)$,[19] and $N_i = (1-\exp(-2\pi k_i/a))^{1/2}$. This state corresponds to the reduced density matrix (37) in the GBF framework. From now on we set $a=1$. Since, for the observable map evaluated on coherent states we can use the general result in equation (41)[20] it is convenient to express the mixed state (46) in terms of coherent states[21] ; the observable map in the region R for the state D then reads

$$\rho_R^F(D) = \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!(2k_i)^{n_i}} N^{-2} \int \mathrm{d}\xi_1\, \mathrm{d}\overline{\xi_1}\, \mathrm{d}\xi_2\, \mathrm{d}\overline{\xi_2}\, \rho_R^F(K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S})$$
$$\exp\left(-\frac{1}{2}\int \frac{\mathrm{d}k}{2k}|\xi_1(k)|^2\right)(\xi_1(k_i))^{n_i}\, \exp\left(-\frac{1}{2}\int \frac{\mathrm{d}k}{2k}|\xi_2(k)|^2\right)(\overline{\xi_2(k_i)})^{n_i}, \quad (47)$$

where the terms in the second line come from the scalar product of the n_i-particle states appearing in (46) and the coherent states $K_{\xi_1}^S$ and $\overline{K_{\xi_2}^S}$ respectively, see Sec. II.B of [10]. The observable map $\rho_R^F(K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S})$ has been shown to satisfy a factorization property, see Proposition 4.3 of [28], which corresponds to the amplitude map of the theory with a source field interaction [39],

$$\rho_R^F(K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S}) = \rho_R(K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S}) \exp\left(\int \mathrm{d}^2x\, \hat{\xi}(x)\mu(x)\right) \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right), \quad (48)$$

where we are now using x as global notation for the Rindler coordinates (η,ρ). The first term in the r.h.s. of (48) is the free amplitude map (6) for the state $K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S}$,

$$\rho_R(K_{\xi_1}^S \otimes \overline{K_{\xi_2}^S}) = \exp\left(\int_0^\infty \frac{\mathrm{d}k}{2k}\left(\xi_1(2)\overline{\xi_2(k)} - \frac{1}{2}|\xi_1(k)|^2 - \frac{1}{2}|\xi_2(k)|^2\right)\right). \quad (49)$$

$\hat{\xi}(x)$ is a complex solution of the equation of motion determined by the two coherent states $K_{\xi_1}^S$ and $\overline{K_{\xi_2}^S}$,

$$\hat{\xi}(x) = \mathrm{i}\int_0^\infty \mathrm{d}k\left(\phi_k^R(x)\,\xi_1(k) + \overline{\phi_k^R(x)}\,\overline{\xi_2(k)}\right), \quad (50)$$

where $\phi_k^R(\rho,\eta)$ are the Fulling modes (24). Finally, $G_F^{\mathcal{R}}(x,x')$ appearing in the last term of (48) is the Feynman propagator in Rindler spacetime and in the region R it reads[22]

$$G_F^{\mathcal{R}}(\rho,\eta,\rho',\eta') = \mathrm{i}\int_0^\infty \frac{\mathrm{d}k}{\pi^2}\left(\theta(\eta'-\eta)e^{-\mathrm{i}k(\eta'-\eta)} + \theta(\eta-\eta')e^{-\mathrm{i}k(\eta-\eta')}\right) K_{\mathrm{i}k}(m\rho)K_{\mathrm{i}k}(m\rho')\,\sinh(\pi k). \quad (51)$$

We now have at our disposal all the ingredients to compute the integrals in (47). It is convenient to proceed by expressing the powers of the modes $\xi_{1,2}(k_i)$ in (47) in terms of functional derivatives,

$$(\xi_1(k_i))^{n_i}\,(\overline{\xi_2(k_i)})^{n_i} = (2k_i)^{2n_i} \frac{\delta^{n_i}}{\delta\alpha(k_i)^{n_i}} \frac{\delta^{n_i}}{\delta\beta(k_i)^{n_i}} \exp\left(\int \frac{\mathrm{d}k}{2k}\left(\beta(k)\xi_1(k) + \alpha(k)\overline{\xi_2(k)}\right)\right)\Bigg|_{\alpha=\beta=0}. \quad (52)$$

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[19] Notice that the factor $(2k_i)^{n_i}$ appearing in the denominator of (46) comes from the normalization of the n_i-particle state,

$$\int \mathcal{D}\varphi\, \psi_{k_1,\dots,k_n}(\varphi)\overline{\psi_{k'_1,\dots,k'_n}(\varphi)} = \frac{1}{n!}\sum_{\sigma\in S_n}\prod_{i=1}^{n} k_i\,\delta(k_i - k'_{\sigma(i)}),$$

where the sum runs over all permutations σ of n elements.

[20] See also [39] for the expression of amplitude maps in terms of modes expansion.

[21] An important property satisfied by coherent states is the completeness relation expressed by the resolution of the identity operator id which, in a bra ket notation, takes the form

$$N^{-1}\int \mathrm{d}\xi\,\mathrm{d}\overline{\xi}\,|K_\xi^S\rangle\langle K_\xi^S| = \mathrm{id}, \qquad \text{with} \qquad N = \int \mathrm{d}\xi\,\mathrm{d}\overline{\xi}\,\exp\left(-\int \frac{\mathrm{d}k}{2k}\,|\xi(k)|^2\right).$$

[22] The general expression of the Feynman propagator for fields in (a wide class of) curved spacetimes has been obtained in [39], to which we refer also for details concerning the calculation presented here.

Substituting this in (47), the integrals are evaluated by the following shift of integration variables:

$$\xi_1 \to \xi_1 + \beta + \mu_1, \qquad \overline{\xi_1} \to \overline{\xi_1} + \alpha, \tag{53}$$

$$\xi_2 \to \xi_2 + \alpha, \qquad \overline{\xi_2} \to \overline{\xi_2} + \overline{\xi_1} + \mu_2, \tag{54}$$

where $\mu_1(k) = \mathrm{i} \int \mathrm{d}x^2\, 2k\, \phi_k^R(x)\mu(x)$ and $\mu_2(k) = \mathrm{i} \int \mathrm{d}x^2\, 2k\overline{\phi_k^R(x)}\mu(x)$. We arrive at

$$\begin{aligned}
\rho_R^F(D) &= \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!} (2k_i)^{n_i} \frac{\delta^{n_i}}{\delta\alpha(k_i)^{n_i}} \frac{\delta^{n_i}}{\delta\beta(k_i)^{n_i}} \exp\left(\int \frac{\mathrm{d}k}{2k} \left(\beta(k)\mu_1(k) + \alpha(k)\mu_2(k) + \alpha(k)\beta(k)\right)\right)\Big|_{\alpha=\beta=0} \\
&\quad \times \exp\left(\frac{\mathrm{i}}{2} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right), \\
&= \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!} \frac{\delta^{n_i}}{\delta\alpha(k_i)^{n_i}} (\alpha(k_i) + \mu_1(k_i))^{n_i} \exp\left(\int \frac{\mathrm{d}k}{2k}\, \alpha(k)\mu_2(k)\right)\Big|_{\alpha=0} \\
&\quad \times \exp\left(\frac{\mathrm{i}}{2} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right),
\end{aligned} \tag{55}$$

To compute the derivative with respect to α we use Rodrighues' formula, see 8.970.1 of [34], and obtain

$$\rho_R^F(D) = \prod_i N_i^2 \sum_{n_i=0}^{\infty} e^{-2\pi n_i k_i} L_{n_i}\left(-\frac{\mu_1(k_i)\mu_2(k_i)}{2k_i}\right) \exp\left(\frac{\mathrm{i}}{2} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right), \tag{56}$$

where L_{n_i} is the Laguerre polynomial of order n_i. According to formula 8.975.1 of [34], the sum over n_i gives

$$\rho_R^F(D) = \prod_i \exp\left(\frac{\mu_1(k_i)\mu_2(k_i)}{2k_i} \frac{e^{-\pi k_i}}{2\sinh(\pi k_i)}\right) \exp\left(\frac{\mathrm{i}}{2} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right). \tag{57}$$

Finally, the substitution of the expression of the quantities $\mu_1(k_i)$ and $\mu_2(k_i)$ leads to

$$\rho_R^F(D) = \exp\left(\frac{\mathrm{i}}{2} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) \left[\mathrm{i} \int \mathrm{d}k\, \phi_k^R(x)\overline{\phi_k^R(x')}\, \frac{e^{-\pi k}}{\sinh(\pi k)} + G_F^{\mathcal{R}}(x,x')\right] \mu(x')\right). \tag{58}$$

Noticing that only the symmetric component of the first term in the square bracket contributes to the integral and using (24) and (51), a straightforward calculation shows that the sum in the square bracket coincides with the Feynman propagator in Minkowski spacetime evaluated in the right Rindler wedge (45); and so do the observable maps (43) and (58). This coincidence of the observable maps computed in Minkowski spacetime on the vacuum state and in Rindler spacetime on the mixed state (46) for the *same local observable* supports the notion of the *local Unruh effect*. In the next section we present the same calculation performed according to the Berezin-Toeplitz quantization (40) of the Weyl observable.

2. *Observable maps from Berezin-Toeplitz quantization*

By examining the expressions (41) and (42) one can see that the difference between the observable maps of a Weyl observable in the two quantization schemes amounts to the last terms. Moreover it can be shown that the last exponential in (41) corresponds to the last one in (48), and consequently we have that

$$- g_\Sigma(\eta_D, \eta_D) = -\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x)\, \Im(G_F(x,x'))\, \mu(x'). \tag{59}$$

Hence, in Minkowski spacetime the observable map (42) on the vacuum state evaluated in the Berezin-Toeplitz quantization scheme in the spacetime region M, is given by

$$\rho_M^{\blacktriangleleft F \blacktriangleright}(\psi_0 \otimes \overline{\psi_0}) = \exp\left(-\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x)\, \Im(G_F^M(x,x'))\, \mu(x')\right). \tag{60}$$

In Rindler spacetime the observable map (42), in the same quantization scheme in the spacetime region R, takes the form

$$\rho_R^{\blacktriangleleft F \blacktriangleright}(D) = \exp\left(- \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) \left[\frac{1}{2} \int \mathrm{d}k\, \phi_k^R(x) \overline{\phi_k^R(x')}\, \frac{e^{-\pi k}}{\sinh(\pi k)} + \Im\left(G_F^R(x,x') \right) \right] \mu(x') \right). \tag{61}$$

As in (58) only the symmetric part of the terms in the square bracket contribute to the integral and the situation is similar to the one in the preceding section apart from the factor 1/2 appearing in (61). It is precisely this factor that prevents the coincidence of (60) and (61). We conclude that the Berezin-Toeplitz prescription for the quantization of observables gives no ground for the local Unruh effect.

B. Expectation values in the holomorphic representation

In this section we present the computation of the observable maps (41) and (42) for quantum states in the holomorphic representation[23]. First we notice that in Minkowski spacetime (41) for the vacuum state reduces to the same result obtained in Sec. V A 1, namely expression (43); this is a consequence of the equivalence between the Schrödinger and holomorphic quantizations shown in [30]. We now consider the same observable map in Rindler spacetime on the mixed thermal state in the holomorphic representation corresponding to the state (46) in the Schrödinger representation. For later convenience we write this state in terms of derivatives of coherent states,

$$D^h = \prod_k N_k^2 \sum_{n=0}^{\infty} e^{-2\pi n k} \frac{2^n}{n!} \frac{\delta^n}{\delta \xi_1(k)^n} \frac{\delta^n}{\delta \xi_2(k)^n} K_{\xi_1}^h \otimes \overline{K_{\xi_2}^h} \Bigg|_{\xi_1=\xi_2=0} \tag{62}$$

where $K_{\xi_1}^h \in \mathcal{H}_{\Sigma_1}^h$ and $K_{\xi_2}^h \in \mathcal{H}_{\Sigma_2}^h$ are the coherent states in the holomorphic representation defined by $\xi_i \in L_{\Sigma_i^R}$ $(i = 1, 2)$. Consequently D^h is a mixed state in $\mathcal{H}_{\Sigma_1}^h \otimes \mathcal{H}_{\overline{\Sigma}_2}^h$. The corresponding observable map for the Weyl observable (38) reads

$$\rho_R^F(D^h) = \prod_k N_k^2 \sum_{n=0}^{\infty} e^{-2\pi n k} \frac{2^n}{n!} \frac{\delta^n}{\delta \xi_1(k)^n} \frac{\delta^n}{\delta \xi_2(k)^n} \rho_R^F(K_{\xi_1}^h \otimes \overline{K_{\xi_2}^h}) \Bigg|_{\xi_1=\xi_2=0}. \tag{63}$$

We now specify the three terms appearing in the expression (41) for the observable map $\rho_R^F(K_{\xi_1}^h \otimes \overline{K_{\xi_2}^h})$:

- the free amplitude $\rho_R(K_{\xi_1}^h \otimes \overline{K_{\xi_2}^h})$ can be computed using (13), where in the present context $\xi^R = \xi_1 + \xi_2$ and $\xi^I = \xi_1 - \xi_2$, leading to

$$\rho_R(K_{\xi_1}^h \otimes \overline{K_{\xi_2}^h}) = \exp\left(\frac{1}{2} \int_0^{\infty} \mathrm{d}k\, \xi_1(k) \xi_2(k) \right), \tag{64}$$

- the Weyl observable evaluated on the complex solution $\hat{\xi}$ given in this case by[24]

$$\hat{\xi}(x) = \xi^R(x) - \mathrm{i}\xi^I(x) = \frac{1}{\sqrt{2}} \int_0^{\infty} \mathrm{d}k \left(\phi_k^R(x) \xi_1(k) + \overline{\phi_k^R(x)} \xi_2(k) \right), \tag{65}$$

- the last term in the r.h.s of (41) coincides with the last term in the r.h.s. of (48).

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[23] In [30] a one-to-one relation was established between the Schrödinger and the holomorphic representation in terms of an isomorphism between the corresponding Hilbert spaces. Thus, by using this result it will be possible to obtain the amplitude and observable maps in the holomorphic representation starting from those obtained in the Schrödinger representation. We shall elaborate on this elsewhere and follow here a different strategy: We start with the mixed state (62) and compute the observable map of the Weyl observable with the prescription suited for the holomorphic representation.

[24] As in the previous section x is used as global notation for the Rindler coordinates (ρ, η).

The observable map (63) can then be written as

$$\rho_R^F(D^h) = \prod_k N_k^2 \sum_{n=0}^{\infty} e^{-2\pi nk} \frac{2^n}{n!} \frac{\delta^n}{\delta\xi_1(k)^n} \frac{\delta^n}{\delta\xi_2(k)^n}$$
$$\times \exp\left(\frac{1}{2}\xi_1(k)\xi_2(k) + \frac{\mathrm{i}}{\sqrt{2}}\int \mathrm{d}^2x\, \mu(x)\left(\phi_k^R(x)\xi_1(k) + \overline{\phi_k^R(x)}\xi_2(k)\right)\right)\Bigg|_{\xi_1=\xi_2=0}$$
$$\times \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x) G_F^{\mathcal{R}}(x,x')\mu(x')\right). \tag{66}$$

We proceed by evaluating the first line in the r.h.s. of (66) by applying the general Leibniz rule

$$\frac{\mathrm{d}^n}{\mathrm{d}\gamma^n} f(\gamma)g(\gamma) = \sum_{k=0}^{n} \binom{n}{k} \frac{\mathrm{d}^{n-k}}{\mathrm{d}\gamma^{n-k}} f(\gamma) \frac{\mathrm{d}^k}{\mathrm{d}\gamma^k} g(\gamma), \tag{67}$$

and using the relation

$$\sum_{k=0}^{\infty} \frac{(k+s)!}{k!s!} e^{-2\pi kp} = \frac{1}{(1-e^{-2\pi p})^{s+1}}, \tag{68}$$

which we proof in the appendix. We obtain

$$N_k^2 \sum_{n=0}^{\infty} e^{-2\pi nk} \frac{2^n}{n!} \frac{\delta^n}{\delta\xi_1(k)^n} \frac{\delta^n}{\delta\xi_2(k)^n} \exp\left(\frac{1}{2}\xi_1(k)\xi_2(k)\right) \exp\left(\frac{\mathrm{i}}{\sqrt{2}}\int \mathrm{d}^2x\, \mu(x)\left(\phi_k^R(x)\xi_1(k) + \overline{\phi_k^R(x)}\xi_2(k)\right)\right)\Bigg|_{\xi_1=\xi_2=0}$$
$$= N_k^2 \sum_{n=0}^{\infty} e^{-2\pi nk} \left(-\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x)\mu(x')\phi_k^R(x)\overline{\phi_k^R(x')}\right)^n \frac{1}{n!} \sum_{j=0}^{\infty} \frac{(j+n)!}{j!n!} e^{-2\pi kj}$$
$$= \exp\left(-\frac{e^{-\pi k}}{2\sinh(\pi k)} \int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x)\mu(x')\phi_k^R(x)\overline{\phi_k^R(x')}\right). \tag{69}$$

Hence, substituting in (66) we obtain after some rearrangements

$$\rho_R^F(D^h) = \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\, \mathrm{d}^2x'\, \mu(x)\left[\mathrm{i}\int \mathrm{d}k\, \phi_k^R(x)\overline{\phi_k^R(x')}\frac{e^{-\pi k}}{\sinh(\pi k)} + G_F^{\mathcal{R}}(x,x')\right]\mu(x')\right), \tag{70}$$

which coincides with expression (58). Consequently, (70) equals the observable map computed in Minkowski spacetime on the vacuum state, and we recover the local Unruh effect for quantum states in the holomorphic representation.

As already noticed the difference between the Berezin-Toeplitz quantization and the Feynman one amounts to the last terms in (41) and (42). These terms are independent of the representation chosen for the quantum states, and so we are reduced to the same situation as in Sec. (V A 2): no local Unruh effect appears adopting the Berezin-Toeplitz prescription for quantizing local observables.

VI. CONCLUSIONS AND OUTLOOK

We have applied the general boundary formulation of quantum field theory to quantize a massive scalar field in Minkowski and Rindler spacetimes. By comparing the two quantum theories we were able to study the Unruh effect from a GBF perspective. Our results are the following: (1) we recover the same difficulty to establish a direct relation between quantum states in Minkowski spacetime restricted to the right Rindler wedge and Rindler spacetime first obtained in [23]; indeed, the two quantum theories turn out to be unitary inequivalent because of the different boundary conditions the field has to satisfy; (2) nevertheless, we show that the expectation value of Weyl observables with compact spacetime support in the interior of the right Rindler wedge, computed in the Minkowski vacuum state coincides with the one calculated in a appropriately chosen mixed state in Rindler, if the observables are quantized according to the Feynman path integral prescription. Furthermore, we showed that this does not hold in the Berezin-Toeplitz quantization. Thus, for the Schrödinger-Feynman quantization scheme, these results suggest to distinguish between two notions of the Unruh effect indicated here with the adjectives global and local. The global Unruh effect can be summarized by equation (36) which turns out to be unacceptable because of the inequivalence of the theories mentioned above. In contrast, the local Unruh effect is concerned with expectation values of local observables for both theories.

The derivation of the local Unruh effect is of immediate relevance for the GBF program. It represents a concrete application of the quantization of observables and the opportunity to compare the Feynman and Berezin-Toeplitz schemes in a specific context. Moreover the computation of observable maps involved the use of mixed states for the first time within the GBF.

It should be noted that the spacetime regions considered for the evaluation of the observable maps are the standard ones bounded by two equal (Minkowski and Rindler) time hyperplanes. Of course the versatility of GBF enables to quantize the field and to compute expectation values in more general regions. Although the main focus is represented by compact spacetime regions, inspired by previous results obtained applying the GBF in Minkowski and curved spacetime, an interesting region is represented by the one bounded by one connected and timelike boundary. In particular, it is possible to apply the GBF for a field defined in a region of Rindler spacetime bounded by one hyperbola of constant Rindler spatial coordinate ρ. The origin of Minkowski spacetime lies outside this region and the comparison of the quantum field theory defined there and the one in Minkowski will then avoid the difficulty inherent with the behavior of the field in $(x,t)=(x,0)$.

Furthermore, the analysis of the properties of the Minkowski and Rindler quantum theories can be the basis for solving an open question within the GBF. The hyperplane $t=0$ in Minkowski spacetime is the union of the two semi-hyperplanes $\eta^R=0$ and $\eta^L=0$ in the right and left Rindler wedge respectively. However the Hilbert space associated to the hypersurface $t=0$ is not the tensor product of the Hilbert spaces associated with $\eta^R=0$ and $\eta^L=0$, due to the additional boundary condition at the origin. In order to compare the different Hilbert spaces one possibility would be to consider hypersurfaces with boundaries: in the present context the hyperplane $t=0$ for $x\geq 0$ ($x\leq 0$), namely with a boundary in the origin of Minkowski spacetime. However it is still not clear within the GBF which algebraic structure should be associated with an hypersurface with boundaries [40]. The solution of such a question is of paramount importance from the perspective of what we call here the global Unruh effect, as well as for more general contexts.

VII. APPENDIX

Here we prove the identity

$$\sum_{k=0}^{\infty}\frac{(k+n)!}{k!n!}e^{-2\pi kp/a}=\frac{1}{(1-e^{-2\pi p/a})^{n+1}}\,, \tag{71}$$

We start by remarking that with

$$f(n):=\sum_{k=0}^{\infty}\frac{(k+n)!}{k!n!}e^{-2\pi kp/a} \tag{72}$$

we have

$$f(n+1)=\left(1-\frac{1}{n+1}\frac{a}{2\pi}\frac{d}{dp}\right)f(n)\,. \tag{73}$$

For $s=0$ we find

$$f(0)=\frac{1}{1-e^{-2\pi p/a}}\,. \tag{74}$$

So we start the induction step with the ansatz

$$f(n)=\frac{1}{(1-e^{-2\pi p/a})^{n+1}} \tag{75}$$

and find

$$f(n+1)=\left(1-\frac{1}{n+1}\frac{a}{2\pi}\frac{d}{dp}\right)\frac{1}{(1-e^{-2\pi p/a})^{n+1}}=\frac{1}{(1-e^{-2\pi p/a})^{n+1}}+\frac{e^{-2\pi p/a}}{(1-e^{-2\pi p/a})^{n+2}}=\frac{1}{(1-e^{-2\pi p/a})^{n+2}} \tag{76}$$

which proves that the ansatz was correct. □

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[1] R. Oeckl, *General boundary quantum field theory: Foundations and probability interpretation*, Adv. Theor. Math. Phys. **12** (2008), 319–352, hep-th/0509122.
[2] R. Oeckl, *General boundary quantum field theory: Timelike hypersurfaces in Klein-Gordon theory*, Phys. Rev. **D 73** (2006), 065017, hep-th/0509123.
[3] R. Oeckl, *Holomorphic quantization of linear field theory in the general boundary formulation*, Preprint arXiv:1009.5615.
[4] R. Oeckl, *Affine holomorphic quantization*, J. Geom. Phys. **62** (2012), 1373–1396, arXiv:1104.5527.
[5] R. Oeckl, *A 'General boundary' formulation for quantum mechanics and quantum gravity*, Phys. Lett. **B575** (2003), 318–324, hep-th/0306025.
[6] D. Colosi, *The general boundary formulation of quantum theory and its relevance for the problem of quantum gravity*, Proceedings of the VIII Mexican School of the Gravitational and Mathematical Physics Division of the Mexican Physical Society, AIP Conf. Proc. **1396** (2011), 109–113.
[7] R. Oeckl, *States on timelike hypersurfaces in quantum field theory*, Phys. Lett. **B622** (2005), 172–177, hep-th/0505267.
[8] R. Oeckl, *Two-dimensional quantum YangâĂŞMills theory with corners*, J. Phys. A **41** (2008), no. 13, 135401, hep-th/0608218.
[9] D. Colosi and R. Oeckl, *S-matrix at spatial infinity*, Phys. Lett. **B665** (2008), 310–313, arXiv:0710.5203.
[10] D. Colosi and R. Oeckl, *Spatially asymptotic S-matrix from general boundary formulation*, Phys. Rev. **D 78** (2008), 025020, arXiv:0802.2274.
[11] D. Colosi and R. Oeckl, *States and amplitudes for finite regions in a two- dimensional Euclidean quantum field theory*, J. Geom. Phys. **59** (2009), 764–780, arXiv:0811.4166.
[12] D. Colosi, *On the structure of the vacuum state in general boundary quantum field theory*, Preprint arXiv:0903.2476.
[13] D. Colosi, *S-matrix in de Sitter spacetime from general boundary quantum field theory*, Preprint arXiv:0910.2756.
[14] D. Colosi, *General boundary quantum field theory in de Sitter spacetime*, Preprint arXiv:1010.1209.
[15] D. Colosi, M. Dohse and R. Oeckl, *S-Matrix for AdS from General Boundary QFT*, Preprint arXiv:1112.2225.
[16] W. G. Unruh, *Notes on black-hole evaporation*, Phys. Rev. **D 14** (1976), no. 4, 870–892.
[17] L. C. B. Crispino, A. Higuchi and G. E. A. Matsas, *The Unruh effect and its applications*, Rev. Mod. Phys. **80** (2008), 787–838, arXiv:0710.5373.
[18] W. G. Unruh and R. M. Wald, *What happens when an accelerating observer detects a Rindler particle*, Phys. Rev. **D 29** (1984), 1047–1056.
[19] D. W. Sciama, P. Candelas and D. Deutsch, *Quantum Field Theory, Horizons and Thermodynamics*, Adv. in Phys. **30** (1981), 327–366.
[20] S. Takagi, *Vacuum noise and stress induced by uniform acceleration*, Prog. of Theor. Phys. Suppl. **88** (1986), 1–142.
[21] V. L. Ginzburg and V. P. Frolov, *Vacuum in a homogeneous gravitational field and excitation of a uniformly accelerated detector*, Sov. Phys. Usp **30** (1987), 1073–1095.
[22] R. Brout, S. Massar, R. Parentani and P. Spindel, *A Primer for black hole quantum physics*, Phys. Rept. **260** (1995), 329–454, arXiv:0710.4345.
[23] V. Belinskii, B. Karnakov, V. Mur and N. Narozhnyi, *Does the Unruh effect exist?*, JETP Letters **65** (1997), 902–908, 10.1134/1.567447.
[24] A. Fedotov, V. Mur, N. Narozhnyi, V. Belinsky and B. Karnakov, *Quantum field aspect of Unruh problem*, Phys. Lett. **A 254** (1999), 126–132, hep-th/9902091.
[25] N. Narozhnyi, A. Fedotov, B. Karnakov, V. Mur and V. Belinsky, *Quantum fields in accelerated frames*, Annalen Phys. **9** (2000), 199–206.
[26] N. B. Narozhny, A. M. Fedotov, B. M. Karnakov, V. D. Mur and V. A. Belinskii, *Boundary conditions in the Unruh problem*, Phys. Rev. **D 65** (2001), 025004, hep-th/9906181v1.
[27] R. Oeckl, *Observables in the General Boundary Formulation*, Quantum Field Theory and Gravity (Regensburg, 2010), Birkhäuser, Basel, 2012, pp. 137–156, arXiv:1101.0367.
[28] R. Oeckl, *Schrödinger-Feynman quantization and composition of observables in general boundary quantum field theory*, Preprint arXiv:1201.1877.
[29] W. G. Unruh and N. Weiss, *Acceleration radiation in interacting field theories*, Phys. Rev. **D 29** (1984), 1656–1662.
[30] R. Oeckl, *The Schrödinger representation and its relation to the holomorphic representation in linear and affine field theory*, Preprint arXiv:1109.5215.
[31] R. Oeckl, *Probabilites in the general boundary formulation*, J. Phys. Conf. Ser. **67** (2007), 012049, hep-th/0612076.
[32] A. Ashtekar and A. Magnon, *Quantum fields in curved space-times*, Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **346** (1975), no. 1646, 375.
[33] S. A. Fulling, *Nonuniqueness of Canonical Field Quantization in Riemannian Space-Time*, Phys. Rev. **D 7** (1973), 2850–

2862.
[34] I. S. Gradshteyn and I. M. Ryzhik, *Tables of integrals, series, and products*, Academic Press, New York, 1980.
[35] V. A. Belinskii, *Quantum Fields in Black Hole Space-Time and in Accelerated Systems*, AIP Conf. Proc. **910** (2007), no. 1, 270–293.
[36] J. Fell, *The Dual Spaces of C^*-algebras*, Transactions of the American Mathematical Society **94** (1960), 365–403.
[37] R. Verch, *Local definiteness, primarity and quasiequivalence of quasifree Hadamard quantum states in curved space-time*, Commun. Math. Phys. **160**, 507–536.
[38] H. Bateman and A. Erdélyi, *Higher transcendental functions*, Higher Transcendental Functions, no. Bd. 1, McGraw-Hill, New York, 1955.
[39] D. Colosi and M. Dohse, *On the structure of the S-matrix in general boundary quantum field theory in curved space*, Preprint arXiv:1011.2243.
[40] R. Oeckl, private communication.