Algebras and non-geometric flux vacua

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ABSTRACT: In this work we classify the subalgebras satisfied by non-geometric Q-fluxes in type IIB orientifolds on $\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with three moduli (S, T, U). We find that there are five subalgebras compatible with the symmetries, each one leading to a characteristic flux-induced superpotential. Working in the 4-dimensional effective supergravity we obtain families of supersymmetric AdS$_4$ vacua with all moduli stabilized at small string coupling g_s. Our results are mostly analytic thanks to a judicious parametrization of the non-geometric, RR and NSNS fluxes. We are also able to leave the flux-induced C_4 and C_8 RR tadpoles as free variables, thereby enabling us to study which values are allowed for each Q-subalgebra. Another novel outcome is the appearance of multiple vacua for special sets of fluxes. However, they generically have $g_s > 1$ unless the net number of O3/D3 or O7/D7 sources needed to cancel the tadpoles is large. We also discuss briefly the issues of axionic shift symmetries and cancellation of Freed-Witten anomalies.

KEYWORDS: Flux compactifications, Superstring Vacua.

Contents

1. Introduction

The study of flux compactifications in string theory has been pursued intensively in recent years [1]. One important motivation is the possibility to stabilize the massless moduli at a minimum of the potential induced by the fluxes. The simplest scenarios for this mechanism are provided by type IIB and type IIA $\mathcal{N}{=}1$ orientifolds with p-form fluxes turned on [1]. In IIA compactifications the mixture of NSNS and RR fluxes generates a superpotential that depends on all closed string moduli allowing to stabilize them without invoking non-perturbative effects [2–6]. Moreover, in the IIA setup it is natural to add the so-called geometric f-fluxes that determine the isometry algebra of the internal space [3, 4, 6]. The case of nilpotent algebras was studied in [7–9] and an example with internal $\mathfrak{su}(2)^2$ was spelled out in [10].

To recover T-duality between IIA and IIB compactifications, it is necessary to introduce new parameters referred to as non-geometric fluxes [11–13]. The original observation is that performing a T-duality to NSNS $\bar{H}$-fluxes leads to geometric f-fluxes [14, 15]. Further T-dualities give rise to generalized Q and R-fluxes [11]. The Q's are called non-geometric because the emerging background after two T-dualities can be described locally but not globally. The third T-duality is formal, evidence for the R-fluxes comes rather from T-duality at the level of the effective superpotential [11]. Moreover, the Q and R-fluxes logically extend [11, 16] the set of structure constants of the gauge algebra, generated by isometries and shifts of the B field, that is known to contain the geometric and NSNS fluxes [17, 18].

In this article we consider type IIB orientifolds with O3/O7-planes in which only NSNS $\bar{H}$ and non-geometric Q-fluxes are invariant under the orientifold action. These fluxes together induce a superpotential that depends on all closed string moduli. One advantage of working with IIB is that the Q-fluxes by themselves appear as the structure constants of a subalgebra of the full gauge algebra. However, one must keep in mind that the $\bar{H}$ and Q in IIB map into all kinds of fluxes in type IIA with O6-planes, and into non-geometric R plus geometric f in IIB with O9/O5-planes. Similar examples with generalized fluxes have been considered by several authors [11, 12, 19–25].

Our guiding principle is precisely the classification of the subalgebras satisfied by the non-geometric Q-fluxes. We will discuss a simplified scheme with additional symmetries in order to reduce the number of fluxes. Concretely, we study compactification

on $(\mathrm{T}^2 \times \mathrm{T}^2 \times \mathrm{T}^2)/(\mathbb{Z}_2 \times \mathbb{Z}_2)$, and further impose invariance under exchange of the internal T^2's. In this way we obtain the same model with moduli (S, T, U) proposed in [11] and generalized in [19]. We have classified the allowed subalgebras of the Q-fluxes of the (S, T, U)-model. There are five inequivalent classes, namely $\mathfrak{so}(4)$, $\mathfrak{so}(3,1)$, $\mathfrak{su}(2) + \mathfrak{u}(1)^3$, $\mathfrak{iso}(3)$ and the nilpotent algebra denoted $n(3.5)$ in [7]. The non-semisimple solutions are contractions of $\mathfrak{so}(4)$ consistent with the symmetries. A compelling byproduct is that each subalgebra yields a characteristic flux-induced superpotential. The corresponding 12-dimensional gauge algebras can be easily identified after a convenient change of basis.

We are mostly interested in discovering supersymmetric flux backgrounds with non-geometric fluxes switched on, and all moduli stabilized. To this end we work exclusively with the $D=4$ effective action. We widen the search of vacua of [12] in several respects. A key difference is that in most cases we can solve the F-flatness conditions analytically and can therefore derive explicit expressions for the moduli vevs in terms of the fluxes. The computations are facilitated by using a transformed complex structure $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$, invariant under the modular group $\mathrm{SL}(2, \mathbb{Z})_U$. The independent non-geometric fluxes are precisely parametrized by $\Gamma = \left(\begin{smallmatrix} \alpha & \beta \\ \gamma & \delta \end{smallmatrix}\right)$. The parametrization of NSNS and RR fluxes is also dictated by Γ. By exploiting the variable $\mathcal{Z}$ we can effectively factor out the vacuum degeneracy due to modular transformations.

There is a further vacuum degeneracy originating from special constant translations in the axions $\mathrm{Re}\,S$ and $\mathrm{Re}\,T$. We argue that vacua connected by this type of translations are identical because the full background including the RR fluxes is invariant under such axionic shifts.

In our analysis the values of the flux-induced C_4 and C_8 RR tadpoles are treated as variables. To cancel these tadpoles in general requires to add D-branes besides the orientifold planes. These D-branes are also constrained by cancellation of Freed-Witten anomalies [6, 20]. In our concrete setup, D3-branes and unmagnetized D7-branes wrapping an internal T^4 are free of anomalies and can be included. However, such D-branes do not give rise to charged chiral matter.

By treating the flux tadpoles as variables we can deduce in particular that the vacua found in [12], having O3-planes and no O7/D7 sources, can only arise when the Q-subalgebra is the compact $\mathfrak{so}(4)$. For completeness we study the supersymmetric AdS$_4$ minima due to the fluxes of all compatible Q-subalgebras, including the non-compact $\mathfrak{so}(3,1)$. In general, such vacua exist in all cases but unusual types of sources might be needed to cancel the tadpoles. Interestingly, in models based on semisimple subalgebras we find that there can exist more than one vacuum for some combinations of fluxes.

It is well known that supersymmetric or no-scale Minkowski vacua in IIB orientifolds with RR and NSNS fluxes require sources of negative RR charge such as O3-planes or wrapped D7-branes [26]. However, working with the effective $D=4$ formalism we find that O3-planes and/or D7-branes can be bypassed in fully stabilized supersymmetric AdS$_4$ vacua, provided specific non-geometric fluxes are turned on. It is conceivable that such vacua only occur in the effective theory and will not survive after lifting to a full string background. Helpful hints in this direction can come from our results relating properties of the vacua with the gauge algebra. It might well be that only models built on certain algebras

can be lifted to full backgrounds. The newly proposed formulation of non-geometric fluxes based on compactification on doubled twisted tori suggests that the gauge algebra has to be compact or admit a discrete cocompact subgroup [27 – 29]. It is also feasible that the recent description of non-geometric fluxes in the context of generalized geometry [30] could be applied to deduce the generalized flux configurations which allow supersymmetric vacua. A discussion of these issues is beyond our present scope.

We now outline the paper. In section 2 we review the properties of the fluxes and write down the flux-induced effective quantities needed to investigate the vacua. The classification of the Q-subalgebras is carried out in section 3, where we also obtain the parametrization of the non-geometric and NSNS fluxes that is crucial in the subsequent analysis. In section 4 we introduce the transformed complex structure $\mathcal{Z}$ motivated by modular invariance. Using this variable then points to the efficient parametrization of the RR fluxes given in the appendix. In the end we are able to derive very compact expressions for the flux-induced superpotential and tadpoles according to the particular Q-subalgebra. In section 5 we solve the F-flatness conditions and collect the results that distinguish the vacua with moduli stabilized. The salient features of these vacua are discussed in section 6. Section 7 is devoted to some final comments.

2. Generalities

In this section we outline our notation to describe the non-geometric fluxes introduced in [11]. To be specific we will work in the context of toroidal orientifolds with O3/O7-planes. We will discuss the case of generic untwisted moduli, and also the simpler isotropic model considered in [11].

2.1 Fluxes

The starting point is a type IIB string compactification on a six-torus T^6 whose basis of 1-forms is denoted η^a. Moreover, we assume the factorized geometry

$$\mathrm{T}^6 = \mathrm{T}^2 \times \mathrm{T}^2 \times \mathrm{T}^2 \; : \; (\eta^1, \eta^2) \; \times \; (\eta^3, \eta^4) \; \times \; (\eta^5, \eta^6) \; . \tag{2.1}$$

As in [11], we will use greek indices α, β, γ for horizontal " $-$ " x-like directions (η^1, η^3, η^5) and latin indices i, j, k for vertical "|" y-like directions (η^2, η^4, η^6) in the 2-tori.

The $\mathbb{Z}_2$ orientifold involution denoted σ acts as

$$\sigma \; : \; (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) \; \rightarrow \; (-\eta^1, -\eta^2, -\eta^3, -\eta^4, -\eta^5, -\eta^6) \; . \tag{2.2}$$

There are 64 O3-planes located at the fixed points of σ. We further impose a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold symmetry with generators acting as

$$\theta_1 \; : \; (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) \; \rightarrow \; (\eta^1, \eta^2, -\eta^3, -\eta^4, -\eta^5, -\eta^6), \tag{2.3}$$
$$\theta_2 \; : \; (\eta^1, \eta^2, \eta^3, \eta^4, \eta^5, \eta^6) \; \rightarrow \; (-\eta^1, -\eta^2, \eta^3, \eta^4, -\eta^5, -\eta^6) \; .$$

Clearly, there is another order-two element $\theta_3 = \theta_1\theta_2$. Under this $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group, only 3-forms with one leg in each 2-torus survive. This also occurs in the compactification

with an extra $\mathbb{Z}_3$ cyclic permutation of the three 2-tori that was studied in [11, 12]. In that case there are only O3-planes and two geometric moduli, namely the overall Kähler and complex structure parameters. In contrast, in our setup, the full symmetry group $\mathbb{Z}_2^3$ includes additional orientifold actions $\sigma\theta_I$ that have fixed 4-tori and lead to O7$_I$-planes, $I = 1, 2, 3$. Another difference is that in principle we have one Kähler and one complex structure parameter for each 2-torus T_I^2.

The Kähler form and the holomorphic 3-form that encode the geometric moduli of the internal space can be written in a basis of invariant forms that also enters in the description of background fluxes. Under the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold action the invariant 3-forms are just

$$
\begin{aligned}
\alpha_0 &= \eta^{135} &;& \quad \alpha_1 = \eta^{235} &;& \quad \alpha_2 = \eta^{451} &;& \quad \alpha_3 = \eta^{613}, \\
\beta^0 &= \eta^{246} &;& \quad \beta^1 = \eta^{146} &;& \quad \beta^2 = \eta^{362} &;& \quad \beta^3 = \eta^{524}.
\end{aligned}
\tag{2.4}
$$

where, e.g. $\eta^{135} = \eta^1 \wedge \eta^3 \wedge \eta^5$. Clearly, these forms are all odd under the orientifold involution σ. On the other hand, the invariant 2-forms and their dual 4-forms are

$$
\begin{aligned}
\omega_1 &= \eta^{12} &;& \quad \omega_2 = \eta^{34} &;& \quad \omega_3 = \eta^{56}, \\
\tilde{\omega}^1 &= \eta^{3456} &;& \quad \tilde{\omega}^2 = \eta^{1256} &;& \quad \tilde{\omega}^3 = \eta^{1234}.
\end{aligned}
\tag{2.5}
$$

These forms are even under σ. We choose the orientation and normalization

$$
\int_{\mathrm{M}_6} \eta^{123456} = \mathcal{V}_6 .
\tag{2.6}
$$

The positive constant $\mathcal{V}_6$ gives the volume of the internal space that we generically denote M_6. Notice that the basis satisfies

$$
\int_{\mathrm{M}_6} \alpha_0 \wedge \beta^0 = -\mathcal{V}_6 \quad , \quad \int_{\mathrm{M}_6} \alpha_I \wedge \beta^J = \int_{\mathrm{M}_6} \omega_I \wedge \tilde{\omega}^J = \mathcal{V}_6 \delta_I^J \quad , \quad I, J = 1, 2, 3.
\tag{2.7}
$$

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold symmetry restricts the period matrix τ^{ij} to be diagonal. Then, up to normalization, the holomorphic 3-form is given by

$$
\Omega = (\eta^1 + \tau_1 \eta^2) \wedge (\eta^3 + \tau_2 \eta^4) \wedge (\eta^5 + \tau_3 \eta^6) = \alpha_0 + \tau_K \alpha_K + \beta^K \frac{\tau_1 \tau_2 \tau_3}{\tau_K} + \beta^0 \tau_1 \tau_2 \tau_3 , \tag{2.8}
$$

with the $H^3(\mathrm{M}_6, \mathbb{Z})$ basis displayed in (2.4).

The next step is to switch on background fluxes for the NSNS and RR 3-forms. Since both H_3 and F_3 are odd under the orientifold involution, the allowed background fluxes can be expanded as

$$
\bar{H}_3 = b_3 \, \alpha_0 + b_2^{(I)} \, \alpha_I + b_1^{(I)} \, \beta^I + b_0 \, \beta^0 ,
\tag{2.9}
$$

$$
\bar{F}_3 = a_3 \, \alpha_0 + a_2^{(I)} \, \alpha_I + a_1^{(I)} \, \beta^I + a_0 \, \beta^0 .
\tag{2.10}
$$

All flux coefficients are integers because the integrals of $\bar{H}_3$ and $\bar{F}_3$ over 3-cycles are quantized. To avoid subtleties with exotic orientifold planes we take all fluxes to be even [31, 32].

As argued originally in [14, 15], applying one T-duality transformation to the NSNS fluxes can give rise to geometric fluxes f_{bc}^a that correspond to structure constants of the

Type	Components	Fluxes
$Q_-^{--} \equiv Q_\alpha^{\beta\gamma}$	$Q_1^{35}, Q_3^{51}, Q_5^{13}$	$\tilde{c}_1^{(1)}, \tilde{c}_1^{(2)}, \tilde{c}_1^{(3)}$
$Q_|^{|-} \equiv Q_k^{i\beta}$	$Q_4^{61}, Q_6^{23}, Q_2^{45}$	$\hat{c}_1^{(1)}, \hat{c}_1^{(2)}, \hat{c}_1^{(3)}$
$Q_|^{-|} \equiv Q_k^{\alpha j}$	$Q_6^{14}, Q_2^{36}, Q_4^{52}$	$\check{c}_1^{(1)}, \check{c}_1^{(2)}, \check{c}_1^{(3)}$
$Q_|^{--} \equiv Q_k^{\alpha\beta}$	$Q_2^{35}, Q_4^{51}, Q_6^{13}$	$c_0^{(1)}, c_0^{(2)}, c_0^{(3)}$
$Q_-^{||} \equiv Q_\gamma^{ij}$	$Q_1^{46}, Q_3^{62}, Q_5^{24}$	$c_3^{(1)}, c_3^{(2)}, c_3^{(3)}$
$Q_-^{|-} \equiv Q_\gamma^{i\beta}$	$Q_5^{23}, Q_1^{45}, Q_3^{61}$	$\check{c}_2^{(1)}, \check{c}_2^{(2)}, \check{c}_2^{(3)}$
$Q_-^{-|} \equiv Q_\beta^{\gamma i}$	$Q_3^{52}, Q_5^{14}, Q_1^{36}$	$\hat{c}_2^{(1)}, \hat{c}_2^{(2)}, \hat{c}_2^{(3)}$
$Q_|^{||} \equiv Q_k^{ij}$	$Q_2^{46}, Q_4^{62}, Q_6^{24}$	$\tilde{c}_2^{(1)}, \tilde{c}_2^{(2)}, \tilde{c}_2^{(3)}$

Table 1: Non-geometric Q-fluxes.

isometry algebra of the internal space. Performing further T-dualities leads to generalized fluxes denoted Q_c^{ab} and R^{abc} [11]. The Q_c^{ab} are called non-geometric fluxes because the resulting metric after two T-dualities yields a background that is locally but not globally geometric [12, 13]. Compactifications with R^{abc} fluxes are not even locally geometric but these fluxes are necessary to maintain T-duality between type IIA and type IIB. The geometric and the R-fluxes must be even under the orientifold involution and are thus totally absent in type IIB with O3/O7-planes. On the other hand, the non-geometric fluxes must be odd and are fully permitted.

The main motivation of this work is to study supersymmetric vacua in toroidal type IIB orientifolds with NSNS, RR and non-geometric Q-fluxes turned on. In our construction, the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry only allows 24 components of the flux tensor Q_c^{ab}, namely those with one leg on each 2-torus. This set of non-geometric fluxes is displayed in table 1. All components of the tensor Q are integers that we take to be even.

2.2 Effective action

The NSNS, RR and non-geometric fluxes induce a potential for the closed string moduli. We will focus on the untwisted moduli of the toroidal orientifold. To write explicitly the effective action, recall first that the axiodilaton and the complex structure moduli are given by

$$S = C_0 + ie^{-\phi} \qquad ; \qquad U_I = \tau_I \quad ; \quad I = 1, 2, 3, \tag{2.11}$$

where C_0 is the RR 0-form, ϕ is the 10-dimensional dilaton and the τ_I are the components of the period matrix. The Kähler moduli T_I are instead extracted from the expansion of the complexified Kähler 4-form $\mathcal{J}$, i.e. $\mathcal{J} = -\sum T_I \tilde{\omega}^I$. In turn, the real (axionic) part of $\mathcal{J}$ arises from the RR 4-form C_4 whereas the imaginary part is $e^{-\phi} J \wedge J/2$, where J is the fundamental Kähler form. In fact, $\mathrm{Im}\, T_I$ is basically the area of the 4-cycle dual to the 4-form $\tilde{\omega}^I$.

We are interested in compactifications that preserve $\mathcal{N}=1$ supersymmetry in four dimensions. In this case we know that the scalar potential can be computed from the Kähler

potential and the superpotential. The Kähler potential for the moduli is given by the usual expression

$$K = -\sum_{K=1}^{3} \log\left(-i\left(U_K - \bar{U}_K\right)\right) - \log\left(-i\left(S - \bar{S}\right)\right) - \sum_{K=1}^{3} \log\left(-i\left(T_K - \bar{T}_K\right)\right), \quad (2.12)$$

which is valid to first order in the string and sigma model perturbative expansions. The NSNS and RR fluxes induce a superpotential only for S and the U_I. In absence of non-geometric fluxes Kähler moduli do not enter in the superpotential and non-perturbative effects such as gaugino condensation are required to get vacua with all moduli fixed. The Q-fluxes generate new couplings involving Kähler fields, thereby opening the possibility to stabilize all types of closed string moduli.

The general superpotential can be computed from [19]

$$W = \int_{\mathrm{M}_6} (G_3 + Q\,\mathcal{J}) \wedge \Omega, \quad (2.13)$$

where $G_3 = \bar{F}_3 - S\,\bar{H}_3$, and $Q\mathcal{J}$ is a 3-form with components defined by

$$(Q\,\mathcal{J})_{abc} = \frac{1}{2}\,Q_{[a}^{mn}\,\mathcal{J}_{bc]\,mn}\,. \quad (2.14)$$

Being a 3-form, $Q\mathcal{J}$ can be expanded in the basis (2.4). We obtain

$$Q\,\mathcal{J} = T_K\left(c_3^{(K)}\,\alpha_0 - \mathcal{C}_2^{(IK)}\,\alpha_I - \mathcal{C}_1^{(IK)}\,\beta^I + c_0^{(K)}\,\beta^0\right), \quad (2.15)$$

where $\mathcal{C}_1$ and $\mathcal{C}_2$ are the non-geometric flux matrices

$$\mathcal{C}_1 = \begin{pmatrix} -\tilde{c}_1^{(1)} & \check{c}_1^{(3)} & \hat{c}_1^{(2)} \\ \hat{c}_1^{(3)} & -\tilde{c}_1^{(2)} & \check{c}_1^{(1)} \\ \check{c}_1^{(2)} & \hat{c}_1^{(1)} & -\tilde{c}_1^{(3)} \end{pmatrix} \quad, \quad \mathcal{C}_2 = \begin{pmatrix} -\tilde{c}_2^{(1)} & \check{c}_2^{(3)} & \hat{c}_2^{(2)} \\ \hat{c}_2^{(3)} & -\tilde{c}_2^{(2)} & \check{c}_2^{(1)} \\ \check{c}_2^{(2)} & \hat{c}_2^{(1)} & -\tilde{c}_2^{(3)} \end{pmatrix}\,. \quad (2.16)$$

The expansion for the 3-form G_3 that combines the NSNS and the RR fluxes can be read off from (2.9) and (2.10). Substituting the expansions of the holomorphic 3-form and the background fluxes in (2.13) shows that the superpotential takes the form

$$W = P_1(U) + P_2(U)\,S + \sum_{K=1}^{3} P_3^{(K)}(U)\,T_K\,. \quad (2.17)$$

The P's are cubic polynomials in the complex structure moduli given by

$$P_1(U) = a_0 - \sum_{K=1}^{3} a_1^{(K)}\,U_K + \sum_{K=1}^{3} a_2^{(K)}\frac{U_1 U_2 U_3}{U_K} - a_3 U_1 U_2 U_3\,, \quad (2.18)$$

$$P_2(U) = -b_0 + \sum_{K=1}^{3} b_1^{(K)}\,U_K - \sum_{K=1}^{3} b_2^{(K)}\frac{U_1 U_2 U_3}{U_K} + b_3 U_1 U_2 U_3\,, \quad (2.19)$$

$$P_3^{(K)}(U) = c_0^{(K)} + \sum_{L=1}^{3} \mathcal{C}_1^{(LK)}\,U_L - \sum_{L=1}^{3} \mathcal{C}_2^{(LK)}\frac{U_1 U_2 U_3}{U_L} - c_3^{(K)} U_1 U_2 U_3\,. \quad (2.20)$$

$\bar{F}_{---}$	$\bar{F}_{|--}$	$\bar{F}_{-||}$	$\bar{F}_{|||}$	$\bar{H}_{---}$	$\bar{H}_{|--}$	$\bar{H}_{-||}$	$\bar{H}_{|||}$
a_3	a_2	a_1	a_0	b_3	b_2	b_1	b_0

Table 2: NS and RR isotropic fluxes.

$Q_|^{--}$	$Q_|^{|-}$	$Q_|^{-|}$	$Q_|^{--}$	$Q_-^{||}$	$Q_-^{|-}$	$Q_-^{-|}$	$Q_|^{||}$
$\tilde{c}_1$	$\hat{c}_1$	$\check{c}_1$	c_0	c_3	$\check{c}_2$	$\hat{c}_2$	$\tilde{c}_2$

Table 3: Non-geometric isotropic fluxes.

The main feature of the flux superpotential is that it depends on all untwisted closed string moduli.

At this point we have a model with seven moduli whose potential depends on forty flux parameters. Finding vacua in this generic setup is rather cumbersome. For this reason we consider a simpler configuration in which the fluxes are isotropic. Concretely, we make the Ansatz

$$\tilde{c}_1^{(I)} \equiv \tilde{c}_1 \quad ; \quad \hat{c}_1^{(I)} \equiv \hat{c}_1 \quad ; \quad \check{c}_1^{(I)} \equiv \check{c}_1 \quad ; \quad \tilde{c}_2^{(I)} \equiv \tilde{c}_2 \quad ; \quad \hat{c}_2^{(I)} \equiv \hat{c}_2 \quad ; \quad \check{c}_2^{(I)} \equiv \check{c}_2 \ ,$$
$$b_1^{(I)} \equiv b_1 \quad ; \quad b_2^{(I)} \equiv b_2 \quad ; \quad a_1^{(I)} \equiv a_1 \quad ; \quad a_2^{(I)} \equiv a_2 \ . \tag{2.21}$$

Isotropic fluxes are summarized in tables 2 and 3.

The Ansatz of isotropic fluxes is compatible with vacua in which the geometric moduli are also isotropic, namely

$$U_1 = U_2 = U_3 \equiv U \quad ; \quad T_1 = T_2 = T_3 \equiv T \ . \tag{2.22}$$

This means, that there is only one overall complex structure modulus U and one Kähler modulus T. The model also includes the axiodilaton. In this case, the Kähler potential and the superpotential reduce to

$$K = -3 \log \left(-i \left(U - \bar{U} \right) \right) - \log \left(-i \left(S - \bar{S} \right) \right) - 3 \log \left(-i \left(T - \bar{T} \right) \right)$$
$$W = P_1(U) + P_2(U) \, S + P_3(U) \, T \quad . \tag{2.23}$$

The P's are now cubic polynomials in the single complex structure moduli. They are given by

$$P_1(U) = a_0 - 3 \, a_1 \, U + 3 \, a_2 \, U^2 - a_3 \, U^3 \,, \tag{2.24}$$
$$P_2(U) = -b_0 + 3 \, b_1 \, U - 3 \, b_2 \, U^2 + b_3 \, U^3 \,, \tag{2.25}$$
$$P_3(U) = 3 \left(c_0 + (\hat{c}_1 + \check{c}_1 - \tilde{c}_1) \, U - (\hat{c}_2 + \check{c}_2 - \tilde{c}_2) \, U^2 - c_3 \, U^3 \right) \ . \tag{2.26}$$

This is the model considered in [11, 12].

2.3 Bianchi identities and tadpoles

The NSNS and generalized fluxes that follow from the T-duality chain can be regarded as structure constants of an extended symmetry algebra of the compactification [11, 16].

This algebra includes isometry generators Z_a as well as gauge symmetry generators X^a, $a = 1, \ldots, 6$, coming from the reduction of the B-field on T^6 with fluxes. We are interested in type IIB with O3/O7-planes where geometric and R-fluxes are forbidden. In this case the algebra is given by

$$
\begin{aligned}
\left[X^a, X^b \right] &= Q_c^{ab} \, X^c \,, \\
\left[Z_a, X^b \right] &= Q_a^{bc} \, Z_c \,, \\
[Z_a, Z_b] &= \bar{H}_{abc} \, X^c \,.
\end{aligned}
\tag{2.27}
$$

Notice that the X^a span a 6-dimensional subalgebra in which the non-geometric Q_c^{ab} are the structure constants.

Computing the Jacobi identities of the full 12-dimensional algebra we obtain the constraints

$$
\bar{H}_{x[bc} Q_{d]}^{ax} = 0 \qquad ; \qquad Q_x^{[ab} Q_d^{c]x} = 0 \,.
\tag{2.28}
$$

In the following we will refer to these identities in the shorthand notation $\bar{H}Q = 0$ and $QQ = 0$. The constraints on the fluxes can also be interpreted in terms of a nilpotency condition $\mathcal{D}^2 = 0$ on the operator $\mathcal{D} = H \wedge + Q\cdot$ introduced in [12].

The RR fluxes are also constrained by Bianchi identities of the type $\mathcal{D}\bar{F} = \mathcal{S}$, where $\mathcal{S}$ is a generalized form due to sources that are assumed smeared instead of localized. These Bianchi identities can be understood as tadpole cancellation conditions on the RR 4-form C_4 and C_8 that couple to the sources. The sources are just the orientifold O3/O7-planes and D3/D7-branes that can be present. In the IIB orientifold that we are considering there is a flux-induced C_4 tadpole due to the coupling

$$
\int_{M_4 \times M_6} C_4 \wedge \bar{H}_3 \wedge \bar{F}_3 \,.
\tag{2.29}
$$

There are further C_4 tadpoles due to O3-planes and to D3-branes that can also be added. The total orientifold charge is -32, equally distributed among 64 O3-planes located at the fixed points of the orientifold involution σ. Each D3-brane has charge $+1$ and if they are located in the bulk, as opposed to fixed points of $\mathbb{Z}_2^3$, images must be included. Adding the sources to the flux tadpole (2.29) leads to the cancellation condition

$$
a_0 \, b_3 - a_1^{(K)} \, b_2^{(K)} + a_2^{(K)} \, b_1^{(K)} - a_3 \, b_0 = N_3 \,,
\tag{2.30}
$$

where $N_3 = 32 - N_{D3}$ and N_{D3} is the total number of D3-branes.

The non-geometric and RR fluxes can also combine to produce a tadpole for the RR C_8 form. The contraction $Q\bar{F}_3$ is a 2-form and the flux-induced tadpole is due to the coupling

$$
\int_{M_4 \times M_6} C_8 \wedge (Q\bar{F}_3)
\tag{2.31}
$$

Expanding the 2-form $(Q\bar{F}_3)$ in the basis of 2-forms ω_I, $I = 1, 2, 3$, yields coefficients

$$
(Q\bar{F}_3)_I = a_0 \, c_3^{(I)} + a_1^{(K)} \, \mathcal{C}_2^{(KI)} - a_2^{(K)} \, \mathcal{C}_1^{(KI)} - a_3 \, c_0^{(I)} \quad ; \quad I = 1, 2, 3 \,.
\tag{2.32}
$$

This means that there are induced tadpoles for C_8 components of type $C_8 \sim d\text{vol}_4 \wedge \widetilde{\omega}^I$, where $d\text{vol}_4$ is the space-time volume 4-form and $\widetilde{\omega}^I$ is the 4-form dual to ω_I. On the other hand, there are also C_8 tadpoles due to $O7_I$-planes that have a total charge $+32$ for each I. As discussed before, due to the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$, there are $O7_I$-planes located at the 4 fixed tori of $\sigma\theta_I$, where θ_I are the three order-two elements of $\mathbb{Z}_2 \times \mathbb{Z}_2$. In the end we find the three tadpole cancellation conditions

$$a_0 \, c_3^{(I)} + a_1^{(K)} \, \mathcal{C}_2^{(KI)} - a_2^{(K)} \, \mathcal{C}_1^{(KI)} - a_3 \, c_0^{(I)} = N_{7_I} \quad ; \quad I = 1, 2, 3 \,, \tag{2.33}$$

where $N_{7_I} = -32 + N_{\text{D}7_I}$ and $N_{\text{D}7_I}$ is the number of $\text{D}7_I$-branes that are generically allowed.

In this work we mostly consider isotropic fluxes so that we will again make the Ansatz (2.21). Jacobi identities as well as tadpoles cancellation conditions become simpler. Computing $QQ = 0$ constraints from (2.28) leave us with

$$\hat{c}_2 \, \tilde{c}_1 - \tilde{c}_1 \, \check{c}_2 + \check{c}_1 \, \hat{c}_2 - c_0 \, c_3 = 0 \quad ; \quad c_3 \, \tilde{c}_1 - \check{c}_2^2 + \tilde{c}_2 \, \hat{c}_2 - \hat{c}_1 \, c_3 = 0 \,,$$
$$c_3 \, c_0 - \check{c}_2 \, \hat{c}_1 + \tilde{c}_2 \, \check{c}_1 - \hat{c}_1 \, \tilde{c}_2 = 0 \quad ; \quad c_0 \, \tilde{c}_2 - \check{c}_1^2 + \tilde{c}_1 \, \hat{c}_1 - \hat{c}_2 \, c_0 = 0 \,, \tag{2.34}$$

plus one additional copy of each condition with $\check{c}_i \leftrightarrow \hat{c}_i$. An important result is that saturating[1] this ideal with respect to the conditions $\check{c}_i \neq \hat{c}_i$ automatically implies that $\tilde{c}_i$ is complex. Therefore, it must be that

$$\check{c}_1 = \hat{c}_1 \equiv c_1 \qquad ; \qquad \check{c}_2 = \hat{c}_2 \equiv c_2 \,. \tag{2.35}$$

The cubic polynomial that couples the complex structure and Kähler moduli, c.f. (2.26), then reduces to

$$P_3(U) = 3 \left(c_0 + (2 \, c_1 - \tilde{c}_1) \, U - (2 \, c_2 - \tilde{c}_2) \, U^2 - c_3 \, U^3 \right) \,. \tag{2.36}$$

Recall that the non-geometric fluxes are integer parameters.

Upon using (2.35), the Jacobi constraints satisfied by the non-geometric fluxes become

$$c_0 \, (c_2 - \tilde{c}_2) + c_1 \, (c_1 - \tilde{c}_1) = 0 \,,$$
$$c_2 \, (c_2 - \tilde{c}_2) + c_3 \, (c_1 - \tilde{c}_1) = 0 \,, \tag{2.37}$$
$$c_0 c_3 - c_1 c_2 = 0 \,.$$

This system of equations is easy to solve explicitly. The solution variety has three disconnected pieces of different dimensions. The first piece has dimension four and it is characterized by fluxes

$$c_3 = \lambda_p \, k_2 \quad ; \quad c_2 = \lambda_p \, k_1 \quad ; \quad \tilde{c}_1 = \lambda_q \, k_2 + \lambda k_1 \quad ;$$
$$c_1 = \lambda_q \, k_2 \quad ; \quad c_0 = \lambda_q \, k_1 \quad ; \quad \tilde{c}_2 = \lambda_p \, k_1 - \lambda k_2 \quad . \tag{2.38}$$

---

[1]This can be done using a computational algebra program as Singular [33] and solving over the real field. In [11], an analogous result is obtained manipulating this set of polynomial constraints by hand.

Here $\lambda = 1$, (k_1, k_2) are two integers not zero simultaneously, and (λ_p, λ_q) are two rays given by

$$\lambda_p = 1 + \frac{p}{\mathrm{GCD}(k_1, k_2)} \quad ; \quad \lambda_q = 1 + \frac{q}{\mathrm{GCD}(k_1, k_2)} \,, \tag{2.39}$$

where p, q, $\in \mathbb{Z}$. By convention $\mathrm{GCD}(n, 0) = |n|$. With coefficients given by the fluxes (2.38) the polynomial $P_3(U)$ turns out to factorize as

$$P_3(U) = 3 \left(k_1 + k_2 U \right) \left(\lambda_q - \lambda U - \lambda_p U^2 \right) \,. \tag{2.40}$$

Notice that we have taken into account that the non-geometric fluxes are integers. The second piece of solutions is three dimensional, the set of fluxes can still be characterized by (2.38) and $P_3(U)$ by (2.40), but with $\lambda \equiv 0$ and $\lambda_p \equiv 1$. Finally, the third piece has only two dimensions with fluxes and $P_3(U)$ specified by setting $\lambda \equiv 0$, $\lambda_p \equiv 0$ and $\lambda_q \equiv 1$.

As a byproduct of the above analysis we have isolated the real root of $P_3(U)$ that always exist. In the next section we will explain how the nature of the remaining two roots is correlated with the type of algebra fulfilled by the X^a generators. For example, we will see that in the third piece of solutions with $k_2 = 0$, the algebra is nilpotent.

Let us now consider the constraints $\bar{H}Q = 0$ that mix non-geometric and NSNS fluxes. Inserting the isotropic fluxes in (2.28), and using (2.35), we find

$$\begin{aligned}
b_2 c_0 - b_0 c_2 + b_1(c_1 - \tilde{c}_1) &= 0 \,, \\
b_3 c_0 - b_1 c_2 + b_2(c_1 - \tilde{c}_1) &= 0 \,, \\
b_2 c_1 - b_0 c_3 - b_1(c_2 - \tilde{c}_2) &= 0 \,, \\
b_3 c_1 - b_1 c_3 - b_2(c_2 - \tilde{c}_2) &= 0 \,.
\end{aligned} \tag{2.41}$$

These conditions restrict the NSNS fluxes b_A that determine the coupling between the complex structure and the dilaton moduli through the polynomial $P_2(U)$ in (2.25). In the next section we will discuss solutions to the full set of constraints that will lead to specific forms for the polynomials $P_2(U)$ and $P_3(U)$.

The tadpole cancellation relations also become simpler in the isotropic case. In particular, the three constraints in (2.33), depending on I, reduce to just one condition. Substituting the isotropic Ansatz and (2.35) we obtain

$$a_0 b_3 - 3 a_1 b_2 + 3 a_2 b_1 - a_3 b_0 = N_3 \,, \tag{2.42}$$

$$a_0 c_3 + a_1 (2 c_2 - \tilde{c}_2) - a_2 (2 c_1 - \tilde{c}_1) - a_3 c_0 = N_7 \,. \tag{2.43}$$

These conditions constraint the RR fluxes. We consider the net O3/D3 and O7/D7 charges, N_3 and N_7, to be free parameters.

3. Algebras and fluxes

In this section we discuss solutions to the Jacobi identities satisfied by the NSNS and the non-geometric Q fluxes. The key idea is twofold. First, the generators X^a in (2.27) span a six-dimensional subalgebra whose structure constants are precisely the Q_c^{ab}. Second, when

these fluxes are invariant under the $\mathbb{Z}_2^3$ symmetry described in section 2.1, this subalgebra is rather constrained. We expect only a few subalgebras to be allowed and our strategy is to identify them. In this way we will manage to provide explicit parametrizations for non-geometric fluxes that satisfy the identity $QQ = 0$. Once this is achieved, we will also be able to find the corresponding NSNS fluxes that fulfill $\bar{H}Q = 0$.

We want to consider in detail the set of isotropic non-geometric fluxes given in table 3 plus the conditions $\check{c}_1 = \hat{c}_1 \equiv c_1$, $\check{c}_2 = \hat{c}_2 \equiv c_2$. In this case the subalgebra simplifies to

$$
\begin{aligned}
\left[X^{2I-1}, X^{2J-1}\right] &= \epsilon_{IJK}\left(\tilde{c}_1\, X^{2K-1} + c_0\, X^{2K}\right), \\
\left[X^{2I-1}, X^{2J}\right] &= \epsilon_{IJK}\left(c_2\, X^{2K-1} + c_1\, X^{2K}\right), \\
\left[X^{2I}, X^{2J}\right] &= \epsilon_{IJK}\left(c_3\, X^{2K-1} + \tilde{c}_2\, X^{2K}\right),
\end{aligned}
\tag{3.1}
$$

where $I, J, K = 1, 2, 3$. The Jacobi identities of this algebra are given in (2.37). To reveal further properties, it is instructive to compute the Cartan-Killing metric, denoted $\mathcal{M}$, with components

$$
\mathcal{M}^{ab} = Q_c^{ad}\, Q_d^{bc}.
\tag{3.2}
$$

For the above algebra of isotropic fluxes we find that the six-dimensional matrix $\mathcal{M}$ is block-diagonal, namely

$$
\mathcal{M} = \text{diag}\left(\mathcal{X}_2, \mathcal{X}_2, \mathcal{X}_2\right).
\tag{3.3}
$$

The 2×2 matrix $\mathcal{X}_2$ turns out to be

$$
\mathcal{X}_2 = -2 \begin{pmatrix} \tilde{c}_1^2 + 2c_0 c_2 + c_1^2 & \tilde{c}_1 c_2 + c_1 c_2 + c_0 c_3 + c_1 \tilde{c}_2 \\ \tilde{c}_1 c_2 + c_1 c_2 + c_0 c_3 + c_1 \tilde{c}_2 & \tilde{c}_2^2 + 2c_1 c_3 + c_2^2 \end{pmatrix}.
\tag{3.4}
$$

Since $\mathcal{X}_2$ is symmetric, we conclude that $\mathcal{M}$ can have up to two distinct real eigenvalues, each with multiplicity three.

The full 12-dimensional algebra also enjoys distinctive features. In the isotropic case the remaining algebra commutators involving NSNS fluxes are given by

$$
\begin{aligned}
\left[Z_{2I-1}, Z_{2J-1}\right] &= \epsilon_{IJK}\left(b_3\, X^{2K-1} + b_2\, X^{2K}\right), \\
\left[Z_{2I-1}, Z_{2J}\right] &= \epsilon_{IJK}\left(b_2\, X^{2K-1} + b_1\, X^{2K}\right), \\
\left[Z_{2I}, Z_{2J}\right] &= \epsilon_{IJK}\left(b_1\, X^{2K-1} + b_0\, X^{2K}\right).
\end{aligned}
\tag{3.5}
$$

The mixed piece of the algebra is determined by the non-geometric fluxes as

$$
\begin{aligned}
\left[Z_{2I-1}, X^{2J-1}\right] &= \epsilon_{IJK}\left(\tilde{c}_1\, Z_{2K-1} + c_2\, Z_{2K}\right), \\
\left[Z_{2I-1}, X^{2J}\right] &= \epsilon_{IJK}\left(c_2\, Z_{2K-1} + c_3\, Z_{2K}\right), \\
\left[Z_{2I}, X^{2J-1}\right] &= \epsilon_{IJK}\left(c_0\, Z_{2K-1} + c_1\, Z_{2K}\right), \\
\left[Z_{2I}, X^{2J}\right] &= \epsilon_{IJK}\left(c_1\, Z_{2K-1} + \tilde{c}_2\, Z_{2K}\right).
\end{aligned}
\tag{3.6}
$$

Besides the Jacobi identities purely involving non-geometric fluxes, there are the additional mixed constraints (2.41).

Computing the full Cartan-Killing metric, denoted $\mathcal{M}_{12}$, shows that there are no mixed XZ terms. In fact, the matrix is again block-diagonal

$$\mathcal{M}_{12} = \text{diag}\left(\mathcal{X}_2, \mathcal{X}_2, \mathcal{X}_2, \mathcal{Z}_2, \mathcal{Z}_2, \mathcal{Z}_2\right), \tag{3.7}$$

with $\mathcal{X}_2$ shown above. The new 2×2 matrix $\mathcal{Z}_2$ is found to be

$$\mathcal{Z}_2 = -4 \begin{pmatrix} b_3\tilde{c}_1 + 2b_2c_2 + b_1c_3 & b_2(c_1 + \tilde{c}_1) + b_1(c_2 + \tilde{c}_2) \\ b_2(c_1 + \tilde{c}_1) + b_1(c_2 + \tilde{c}_2) & b_0\tilde{c}_2 + 2b_1c_1 + b_2c_0 \end{pmatrix}. \tag{3.8}$$

Here we have simplified using the Jacobi identities (2.41). We conclude that the allowed 12-dimensional algebras are such that the Cartan-Killing matrix can have up to four distinct eigenvalues, each with multiplicity three.

Let us now return to the subalgebra spanned by the X generators and the task of solving the constraints (2.37) that arise from the Jacobi identities $QQ = 0$. The idea is to fulfill these constraints by choosing the non-geometric fluxes to be the structure constants of six-dimensional Lie algebras whose Cartan-Killing matrix has the simple block-diagonal form (3.3). To proceed it is convenient to distinguish whether $\mathcal{M}$ is non-degenerate or not, i.e. whether the algebra is semisimple or not. If $\det \mathcal{M} \neq 0$, and $\mathcal{M}$ is negative definite, the only possible algebra is the compact $\mathfrak{so}(4) \sim \mathfrak{su}(2)^2$. On the other hand, the only non-compact semisimple algebra with the required block structure is $\mathfrak{so}(3,1)$. When $\det \mathcal{M} = 0$, the algebra is non-semisimple. In this class to begin we find two compatible algebras, namely the direct sum $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ and the semi-direct sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$ that is isomorphic to the Euclidean algebra $\mathfrak{iso}(3)$. The remaining possibility is that the non-semisimple algebra be completely solvable. One example is the nilpotent $\mathfrak{u}(1)^6$ that we disregard because the non-geometric fluxes vanish identically. A second non-trivial solvable algebra, that is actually nilpotent, will be discussed shortly.

After classifying the allowed 6-dimensional subalgebras the next step is to find the set of corresponding non-geometric fluxes. Except for the nilpotent example, all other cases have an $\mathfrak{su}(2)$ factor. This suggests to make a change of basis from (X^{2I-1}, X^{2I}), $I = 1, 2, 3$, to new generators $(E^I, \widetilde{E}^I)$ such that basically one type, say E^I, spans $\mathfrak{su}(2)$. The $\mathbb{Z}_2^3$ symmetries of the fluxes require that we form combinations that transform in a definite way, For instance, E^I can only be a combination of X^{2I-1} and X^{2I} with the same I. Furthermore, for isotropic fluxes it is natural to make the same transformation for each I. We will then make the $\text{SL}(2,\mathbb{R})$ transformation

$$\begin{pmatrix} E^I \\ \widetilde{E}^I \end{pmatrix} = \frac{1}{|\Gamma|^2} \begin{pmatrix} -\alpha & \beta \\ -\gamma & \delta \end{pmatrix} \begin{pmatrix} X^{2I-1} \\ X^{2I} \end{pmatrix}, \tag{3.9}$$

for all $I = 1, 2, 3$. Here $|\Gamma| = \alpha\delta - \beta\gamma$, and it must be that $|\Gamma| \neq 0$. In the following we will refer to $(\alpha, \beta, \gamma, \delta)$ as the Γ parameters.

Substituting in (3.1) it is straightforward to obtain the algebra satisfied by the new generators E^I and $\widetilde{E}^J$. This algebra will depend on the non-geometric fluxes as well as on the parameters $(\alpha, \beta, \gamma, \delta)$. We can then prescribe the commutators to have the standard

form for the allowed algebras found previously. For instance, in the direct product examples we impose $\left[E^I, \widetilde{E}^J\right] = 0$.

In the following sections we will discuss each compatible 6-dimensional algebra in more detail. The goal is to parametrize the non-geometric fluxes in terms of $(\alpha, \beta, \gamma, \delta)$. By construction these fluxes will satisfy the Jacobi identities of the algebra. We will then solve the mixed constraints involving the NSNS fluxes. The main result will be an explicit factorization of the cubic polynomials $P_3(U)$ and $P_2(U)$ that dictate the couplings among the moduli.

3.1 Semisimple algebras

The algebra is semisimple when the Cartan-Killing metric is non-degenerate. This means $\det \mathcal{M} \neq 0$ and hence $\det \mathcal{X}_2 \neq 0$. Now, six-dimensional semisimple algebras are completely classified. If $\mathcal{M}$ is negative definite the algebra is compact so that it must be $\mathfrak{so}(4) \sim \mathfrak{su}(2) + \mathfrak{su}(2)$. When $\mathcal{M}$ has positive eigenvalues the algebra is non-compact and it could be $\mathfrak{so}(3, 1)$ or $\mathfrak{so}(2, 2)$ but the latter does not fit the required block-diagonal form (3.3).

3.1.1 $\mathfrak{so}(4) \sim \mathfrak{su}(2)^2$

The standard commutators of this algebra are

$$\left[E^I, E^J\right] = \epsilon_{IJK} E^K \quad ; \quad \left[\widetilde{E}^I, \widetilde{E}^J\right] = \epsilon_{IJK} \widetilde{E}^K \quad ; \quad \left[E^I, \widetilde{E}^J\right] = 0 . \tag{3.10}$$

After performing the change of basis in (3.1) we find that the non-geometric fluxes needed to describe this algebra can be parametrized as

$$\begin{array}{llll}
c_0 = \beta\,\delta\,(\beta + \delta) & ; & c_3 = -\alpha\,\gamma\,(\alpha + \gamma) & , \\
c_1 = \beta\,\delta\,(\alpha + \gamma) & ; & c_2 = -\alpha\,\gamma\,(\beta + \delta) & , \\
\tilde{c}_2 = \gamma^2\,\beta + \alpha^2\,\delta & ; & \tilde{c}_1 = -(\gamma\,\beta^2 + \alpha\,\delta^2) & ,
\end{array} \tag{3.11}$$

provided that $|\Gamma| = (\alpha\delta - \beta\gamma) \neq 0$. It is easy to show that these fluxes verify the Jacobi identities (2.37). What we have done is to trade the six non-geometric fluxes, constrained by two independent conditions, by the four independent parameters $(\alpha, \beta, \gamma, \delta)$. These parameters are real but the resulting non-geometric fluxes in (3.11) must be integers.

For future purposes we need to determine the cubic polynomial $P_3(U)$ that corresponds to the parametrized non-geometric fluxes. Substituting in (2.36) yields

$$P_3(U) = 3(\alpha U + \beta)(\gamma U + \delta)\left[(\alpha + \gamma)U + (\beta + \delta)\right] . \tag{3.12}$$

This clearly shows that in this case P_3 has three real roots. Moreover, the roots are all different because $|\Gamma| \neq 0$. We will prove that for other algebras P_3 has either complex roots or degenerate real roots. The remarkable conclusion is that P_3 has three different real roots if and only if the algebra of the non-geometric fluxes is the compact $\mathfrak{so}(4) \sim \mathfrak{su}(2) + \mathfrak{su}(2)$. Alternatively, we may start with the condition that the polynomial has three different real roots that we can choose to be at 0, -1 and ∞ without loss of generality. These roots can then be moved to arbitrary real locations by a linear fractional transformation

$$\mathcal{Z} = \frac{\alpha U + \beta}{\gamma U + \delta} . \tag{3.13}$$

with $(\alpha, \beta, \gamma, \delta) \in \mathbb{R}$ and $|\Gamma| \neq 0$. By comparing the roots of P_3 in terms of the fluxes with those in terms of the transformation parameters we rediscover the map (3.11) and the associated $\mathfrak{su}(2)^2$ algebra. In the next sections we will see that the variable $\mathcal{Z}$ introduced above plays a very important physical rôle.

We now turn to the Jacobi constraints (2.41) involving the NSNS fluxes. Inserting the non-geometric fluxes (3.11) we find that the b_A can be completely fixed by the Γ parameters plus two new real variables (ϵ_1, ϵ_2) as follows

$$
\begin{aligned}
b_0 &= -\left(\epsilon_1 \beta^3 + \epsilon_2 \delta^3\right), \\
b_1 &= \epsilon_1 \alpha \beta^2 + \epsilon_2 \gamma \delta^2, \\
b_2 &= -\left(\epsilon_1 \alpha^2 \beta + \epsilon_2 \gamma^2 \delta\right), \\
b_3 &= \epsilon_1 \alpha^3 + \epsilon_2 \gamma^3.
\end{aligned}
\tag{3.14}
$$

We also need to compute the polynomial $P_2(U)$ that depends on the NSNS fluxes. Substituting the above b_A in (2.25) yields

$$
P_2(U) = \epsilon_1(\alpha U + \beta)^3 + \epsilon_2(\gamma U + \delta)^3 .
\tag{3.15}
$$

It is easy to show that because $|\Gamma| \neq 0$, P_2 has complex roots whenever $\epsilon_1 \epsilon_2 \neq 0$. Contrariwise, P_2 has a triple real root if either ϵ_1 or ϵ_2 vanishes.

We may expect that the full 12-dimensional algebra has special properties when P_2 has a triple root. Indeed, inserting the fluxes in (3.8) yields $\det \mathcal{Z}_2 = 16\epsilon_1\epsilon_2|\Gamma|^6$. Hence, the full Cartan-Killing matrix $\mathcal{M}_{12}$ happens to be degenerate when $\epsilon_1\epsilon_2 = 0$. To learn more about the full algebra it is convenient to switch from the original Z_a generators to a new basis $(D_I, \widetilde{D}_I)$ defined by

$$
\begin{pmatrix} D_I \\ \widetilde{D}_I \end{pmatrix} = \frac{1}{|\Gamma|^2} \begin{pmatrix} \delta & \gamma \\ \beta & \alpha \end{pmatrix} \begin{pmatrix} Z_{2I-1} \\ Z_{2I} \end{pmatrix},
\tag{3.16}
$$

for $I = 1, 2, 3$. It is straightforward to compute the piece of the full algebra generated by the $(D_I, \widetilde{D}_I)$. Substituting the parametrized fluxes in (3.5) and (3.6) we obtain

$$
\begin{aligned}
[D_I, D_J] &= -\epsilon_1 \epsilon_{IJK} E^K &;& \quad [\widetilde{D}_I, \widetilde{D}_J] = -\epsilon_2 \epsilon_{IJK} \widetilde{E}^K, \\
[E^I, D_J] &= \epsilon_{IJK} D_K &;& \quad [\widetilde{E}^I, \widetilde{D}_J] = \epsilon_{IJK} \widetilde{D}_K.
\end{aligned}
\tag{3.17}
$$

All other commutators do vanish.

A quick inspection of the whole algebra encoded in (3.10) and (3.17) shows that when either ϵ_1, or ϵ_2, is zero, the D_I, or the $\widetilde{D}_I$, generate a 3-dimensional invariant Abelian subalgebra. Moreover, when say $\epsilon_1 = 0$ and $\epsilon_2 \neq 0$, the $\mathcal{Z}_2$ block of the full Cartan-Killing metric has one zero and one non-zero eigenvalue which is negative for $\epsilon_2 < 0$ and positive for $\epsilon_2 > 0$. The upshot is that when $\epsilon_1\epsilon_2 = 0$, the 12-dimensional algebra is $\mathfrak{iso}(3) + \mathfrak{g}$, where $\mathfrak{g}$ is either $\mathfrak{so}(4)$ or $\mathfrak{so}(3,1)$. On the other hand, when $\epsilon_1\epsilon_2 < 0$, the algebra is $\mathfrak{so}(4) + \mathfrak{so}(3,1)$, whereas for $\epsilon_1, \epsilon_2 < 0$ it is $\mathfrak{so}(4)^2$, and for $\epsilon_1, \epsilon_2 > 0$ it is $\mathfrak{so}(3,1)^2$.

The methods developed in this section will be applied shortly to other subalgebras. In summary, the non-geometric and NSNS fluxes can be parametrized using auxiliary variables

$(\alpha, \beta, \gamma, \delta)$ and (ϵ_1, ϵ_2) in such a way that the Jacobi identities are satisfied and flux-induced superpotential terms are explicitly factorized. The full 12-dimensional algebras can be simply characterized after the changes of basis (3.9) and (3.16) are performed.

The auxiliary variables are constrained by the condition that the resulting fluxes be integers. This issue deserves further explanation. There are two cases depending on whether the polynomial $P_2(U)$ has complex roots or not. If it does not, we can take $\epsilon_1 = 0$ to be concrete. From the structure of the NSNS fluxes in (3.14) it is then obvious that, for $\alpha \neq 0$, the quotient β/α is a rational number. Going back to the non-geometric fluxes it can be shown that the ratios γ/α and δ/α, as well as α^3 and ϵ_2 also belong to $\mathbb{Q}$. If $P_2(U)$ admits complex roots the generic result is that ϵ_2/ϵ_1, β/α, α^3, etc., involve square roots of rationals. However, it happens that when at least one of the non-geometric parameters $(\alpha, \beta, \gamma, \delta)$ is zero then all well defined quotients are again rational numbers.

3.1.2 $\mathfrak{so}(3, \mathbf{1})$

This is the Lorentz algebra. We can take E^I to be the angular momentum, and $\widetilde{E}^J$ to be the boost generators. Thus, the algebra can be written as

$$\left[E^I, E^J\right] = \epsilon_{IJK} E^K \quad ; \quad \left[\widetilde{E}^I, \widetilde{E}^J\right] = -\epsilon_{IJK} E^K \quad ; \quad \left[E^I, \widetilde{E}^J\right] = \epsilon_{IJK} \widetilde{E}^K . \tag{3.18}$$

In this case the non-geometric fluxes that produce the algebra are found to be

$$\begin{array}{llll}
c_0 = -\beta\left(\beta^2 + \delta^2\right) & ; & c_3 = \alpha\left(\alpha^2 + \gamma^2\right) & , \\
c_1 = -\alpha\left(\beta^2 + \delta^2\right) & ; & c_2 = \beta\left(\alpha^2 + \gamma^2\right) & , \\
\tilde{c}_2 = -\beta\left(\alpha^2 - \gamma^2\right) - 2\gamma\delta\alpha & ; & \tilde{c}_1 = \alpha\left(\beta^2 - \delta^2\right) + 2\beta\gamma\delta & ,
\end{array} \tag{3.19}$$

as long as $|\Gamma| \neq 0$.

Substituting the resulting non-geometric fluxes in (2.36) gives the $P_3(U)$ polynomial

$$P_3(U) = -3(\alpha U + \beta)\left[(\alpha U + \beta)^2 + (\gamma U + \delta)^2\right] . \tag{3.20}$$

Since $\Gamma \neq 0$, P_3 always has complex roots. We will see that for non-semisimple algebras all roots of P_3 are real, as for the compact $\mathfrak{so}(4)$. Hence, the important observation now is that P_3 has complex roots if and only if the algebra of the non-geometric fluxes is the non-compact $\mathfrak{so}(3, \mathbf{1})$.

The Jacobi constraints (2.41) for the NSNS fluxes can again be solved in terms of the Γ parameters plus two real constants that we again denote by (ϵ_1, ϵ_2). Concretely,

$$\begin{aligned}
b_0 &= -\beta\left(\beta^2 - 3\delta^2\right)\epsilon_1 - \delta\left(\delta^2 - 3\beta^2\right)\epsilon_2 , \\
b_1 &= (\alpha\beta^2 - 2\beta\gamma\delta - \alpha\delta^2)\epsilon_1 + \left(\gamma\delta^2 - 2\alpha\delta\beta - \gamma\beta^2\right)\epsilon_2 , \\
b_2 &= \left(\beta\gamma^2 + 2\gamma\delta\alpha - \beta\alpha^2\right)\epsilon_1 + \left(\delta\alpha^2 + 2\beta\gamma\alpha - \delta\gamma^2\right)\epsilon_2 , \\
b_3 &= \alpha\left(\alpha^2 - 3\gamma^2\right)\epsilon_1 + \gamma\left(\gamma^2 - 3\alpha^2\right)\epsilon_2 .
\end{aligned} \tag{3.21}$$

These fluxes give rise to

$$P_2(U) = (\gamma U + \delta)^3 (\epsilon_1 \mathcal{Z}^3 - 3\epsilon_2 \mathcal{Z}^2 - 3\epsilon_1 \mathcal{Z} + \epsilon_2), \tag{3.22}$$

where $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$ as before. The discriminant of this cubic polynomial is always negative. Therefore, P_2 has three different real roots.

3.2 Non-semisimple algebras

In this case the algebra is the semidirect sum of a semisimple algebra and a solvable invariant subalgebra. Lack of simplicity is detected imposing $\det \mathcal{M} = 0$ which requires $\det \mathcal{X}_2 = 0$, where $\mathcal{X}_2$ is shown in (3.4). Combining with the Jacobi identities (2.37) we deduce that up to isomorphisms there are only two solutions in which the solvable invariant subalgebra has dimension less than six. In practice this means that $\mathcal{X}_2$ has only one zero eigenvalue. As expected from the underlying symmetries, this invariant subalgebra can only have dimension three and be $\mathfrak{u}(1)^3$. The semisimple piece can only be $\mathfrak{su}(2)$. The two solutions are the direct and semidirect sum discussed below.

The remaining possibility consistent with the symmetries is for the solvable invariant subalgebra to have dimension six. The criterion for solvability is that the derived algebra $[\mathfrak{g}, \mathfrak{g}]$ be orthogonal to the whole algebra $\mathfrak{g}$ with respect to the Cartan-Killing metric. In our case this means $Q_c^{ab} \mathcal{M}^{dc} = 0$, $\forall a, b, d$. The non-geometric fluxes further satisfy the Jacobi identities $Q_x^{[ab} Q_d^{c]x} = 0$. On the other hand, the stronger condition for nilpotency is $\mathcal{M}^{dc} = 0$. For our algebra of isotropic fluxes given in (3.1), we find that all solvable flux configurations are necessarily nilpotent. The proof can be carried out using the algebraic package *Singular* to manipulate the various ideals. This result is consistent with the fact that in our model $\mathcal{M}$ is block-diagonal so that when $\det \mathcal{M} = 0$, it has three or six null eigenvalues and in the latter situation $\mathcal{M}$ is identically zero. One obvious nilpotent algebra is $\mathfrak{u}(1)^6$, but it is uninteresting because the associated fluxes vanish identically. There is a second solution described in more detail below.

The allowed non-semisimple subalgebras can all be obtained starting from $\mathfrak{su}(2)^2$ and performing contractions consistent with the underlying symmetries of the isotropic fluxes. For example, setting $E'^I = E^I$, $\widetilde{E}'^I = \lambda \widetilde{E}^I$ in (3.10) and then letting $\lambda \to 0$ obviously gives the direct sum $\mathfrak{su}(2) + \mathfrak{u}(1)^3$. More generically we can take $E'^I = \lambda^a (E^I + \widetilde{E}^I)$, $\widetilde{E}'^I = \lambda^b (E^I - \widetilde{E}^I)$, with $a \geq 0$, $b \geq 0$. The limit $a = 0$, $b > 0$, $\lambda \to 0$ yields the Euclidean algebra $\mathfrak{iso}(3)$. Letting instead $2b = a > 0$ and contracting gives the nilpotent algebra.

In the coming sections we present the explicit configurations of non-geometric fluxes associated to the non-semisimple subalgebras. The parametrization of NSNS fluxes is also computed. Evaluating the full 12-dimensional algebras in each case is straightforward.

3.2.1 $\mathfrak{su}(2) + \mathfrak{u}(1)^3$

Since the algebra is a direct sum and one factor is Abelian, the brackets take the simple form

$$\left[E^I, E^J \right] = \epsilon_{IJK} E^K \quad ; \quad \left[\widetilde{E}^I, \widetilde{E}^J \right] = 0 \quad ; \quad \left[E^I, \widetilde{E}^J \right] = 0 \ . \tag{3.23}$$

Requiring that upon the change of basis the algebra (3.1) is of this type returns the following non-geometric fluxes

$$\begin{aligned}
c_0 &= \beta \, \delta^2 & ; & \quad c_3 = -\alpha \, \gamma^2 & , \\
c_1 &= \beta \, \delta \, \gamma & ; & \quad c_2 = -\alpha \, \gamma \, \delta & , \\
\tilde{c}_2 &= \gamma^2 \beta & ; & \quad \tilde{c}_1 = -\alpha \, \delta^2 & ,
\end{aligned} \tag{3.24}$$

assuming $|\Gamma| \neq 0$. These fluxes automatically satisfy the Jacobi identities (2.37). They also satisfy the additional condition $c_0 c_2 = c_1 \tilde{c}_1$ arising from $\det \mathcal{X}_2 = 0$.

The non-geometric fluxes of the algebra $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ lead to the $P_3(U)$ polynomial

$$P_3(U) = 3(\alpha U + \beta)(\gamma U + \delta)^2 . \tag{3.25}$$

Evidently, P_3 has one single and one double real root.

The Jacobi identities $\bar{H}Q = 0$ again fix the NSNS fluxes as in the previous cases. The solution in terms of the free parameters is given by

$$
\begin{aligned}
b_0 &= -\left(\epsilon_1 \beta^3 + \epsilon_2 \delta^3\right), \\
b_1 &= \epsilon_1 \alpha \beta^2 + \epsilon_2 \gamma \delta^2, \\
b_2 &= -\left(\epsilon_1 \alpha^2 \beta + \epsilon_2 \gamma^2 \delta\right), \\
b_3 &= \epsilon_1 \alpha^3 + \epsilon_2 \gamma^3 .
\end{aligned}
\tag{3.26}
$$

For the associated polynomial $P_2(U)$ we then find

$$P_2(U) = \epsilon_1 (\alpha U + \beta)^3 + \epsilon_2 (\gamma U + \delta)^3 . \tag{3.27}$$

As in the compact case, this P_2 has complex roots whenever $\epsilon_1 \epsilon_2 \neq 0$.

3.2.2 $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3 \sim \mathfrak{iso}(3)$

According to Levi's theorem, in general this algebra can be characterized as

$$\left[E^I, E^J\right] = \epsilon_{IJK}\left(E^K + \widetilde{E}^K\right) \quad ; \quad \left[\widetilde{E}^I, \widetilde{E}^J\right] = 0 \quad ; \quad \left[E^I, \widetilde{E}^J\right] = \epsilon_{IJK}\widetilde{E}^K . \tag{3.28}$$

The typical form of the Euclidean algebra in three dimensions is recognized after the isomorphism $(E^I - \widetilde{E}^I) \to \widehat{E}^I$. The non-geometric fluxes needed to reproduce the above commutators turn out to be

$$
\begin{array}{llll}
c_0 = -\delta^2 (\beta - \delta) & ; & c_3 = \gamma^2 (\alpha - \gamma) & , \\
c_1 = -\delta^2 (\alpha - \gamma) & ; & c_2 = \gamma^2 (\beta - \delta) & , \\
\tilde{c}_2 = \gamma^2 (\beta + \delta) - 2\gamma\delta\alpha & ; & \tilde{c}_1 = -\delta^2 (\alpha + \gamma) + 2\gamma\delta\beta & ,
\end{array}
\tag{3.29}
$$

for $|\Gamma| \neq 0$. Besides the Jacobi identities these fluxes satisfy $4c_0 c_2 = -(c_1 - \tilde{c}_1)^2$, by virtue of $\det \mathcal{X}_2 = 0$.

For the flux configuration of this algebra the $P_3(U)$ polynomial becomes

$$P_3(U) = 3(\gamma U + \delta)^2 \left[(\gamma - \alpha)U + (\delta - \beta)\right] . \tag{3.30}$$

As in the direct sum $\mathfrak{su}(2) + \mathfrak{u}(1)^3$, P_3 has one single and one double real root.

The NSNS fluxes can be determined from the Jacobi identities (2.41). Introducing again parameters (ϵ_1, ϵ_2) leads to

$$
\begin{aligned}
b_0 &= -\delta^2 (\beta \epsilon_1 + \delta \epsilon_2), \\
b_1 &= \tfrac{1}{3}\delta(\alpha\delta + 2\beta\gamma)\epsilon_1 + \gamma\delta^2 \epsilon_2, \\
b_2 &= -\tfrac{1}{3}\gamma(\beta\gamma + 2\alpha\delta)\epsilon_1 - \gamma^2 \delta \epsilon_2, \\
b_3 &= \gamma^2 (\alpha \epsilon_1 + \gamma \epsilon_2),
\end{aligned}
\tag{3.31}
$$

The companion polynomial $P_2(U)$ of NSNS fluxes is fixed as

$$P_2(U) = (\gamma U + \delta)^2 \left[\epsilon_1(\alpha U + \beta) + \epsilon_2(\gamma U + \delta) \right] . \tag{3.32}$$

Analogous to the non-compact case, this P_2 has only real roots, but one of them is degenerate.

3.2.3 Nilpotent algebra

To search for flux configurations that generate a nilpotent algebra we impose that the Cartan-Killing metric vanishes. Now, in our model $\mathcal{M} = 0$ implies the much simpler conditions $\det \mathcal{X}_2 = 0$ and $\mathrm{Tr}\, \mathcal{X}_2 = 0$. Up to isomorphisms, we find only one non-trivial solution. This is the expected result based on the known classification of 6-dimensional nilpotent algebras.[2]

From the 34 isomorphism classes of nilpotent algebras, besides $\mathfrak{u}(1)^6$, only one is compatible with isotropic fluxes invariant under $\mathbb{Z}_2 \times \mathbb{Z}_2$. The algebra is 2-step nilpotent and its brackets can be written as

$$\left[E^I, E^J \right] = \epsilon_{IJK} \widetilde{E}^K \quad ; \quad \left[\widetilde{E}^I, \widetilde{E}^J \right] = 0 \quad ; \quad \left[E^I, \widetilde{E}^J \right] = 0 . \tag{3.33}$$

Up to isomorphisms this is the algebra labelled $n(3.5)$ in table 4 of [7].

The change of basis from the original (X^{2I-1}, X^{2I}) generators to the $(E^I, \widetilde{E}^I)$ is still given by (3.9). Starting from the X commutators in (3.1) we can then deduce fluxes such that the nilpotent algebra (3.33) is reproduced. In this way we obtain

$$\begin{aligned}
c_0 &= \delta^3 &;&\quad c_3 = -\gamma^3 \quad, \\
c_1 &= \delta^2\,\gamma &;&\quad c_2 = -\delta\,\gamma^2 \quad, \\
\tilde{c}_2 &= \delta\,\gamma^2 &;&\quad \tilde{c}_1 = -\delta^2\,\gamma \quad.
\end{aligned} \tag{3.34}$$

Notice that these fluxes only depend on two independent parameters. This occurs because besides the Jacobi constraints there are two more conditions $\det \mathcal{X}_2 = 0$ and $\mathrm{Tr}\, \mathcal{X}_2 = 0$. The non-geometric fluxes of the nilpotent algebra generate the $P_3(U)$ polynomial

$$P_3(U) = 3(\gamma U + \delta)^3 . \tag{3.35}$$

Clearly, P_3 always has one triple real root.

In analogy with all previous examples, the $\bar{H}Q = 0$ Jacobi identities determine the NSNS fluxes in terms of two additional parameters (ϵ_1, ϵ_2). Inserting the non-geometric fluxes of the nilpotent algebra in (2.41) readily yields

$$\begin{aligned}
b_0 &= -\delta^2\,(\delta\,\epsilon_2 + \gamma\,\epsilon_1)\,, \\
b_1 &= \gamma\,\delta^2\,\epsilon_2 - \tfrac{1}{3}\,\delta\left(\delta^2 - 2\,\gamma^2\right)\epsilon_1\,, \\
b_2 &= -\gamma^2\,\delta\,\epsilon_2 + \tfrac{1}{3}\gamma\left(2\,\delta^2 - \gamma^2\right)\epsilon_1\,, \\
b_3 &= \gamma^2\,(\gamma\,\epsilon_2 - \delta\,\epsilon_1)\,.
\end{aligned} \tag{3.36}$$

---

[2]A table and references to the original literature are given in [7].

Q-subalgebra	$\mathcal{P}_3(\mathcal{Z})/3$	$\mathcal{P}_2(\mathcal{Z})$	$\mathcal{P}_1(\mathcal{Z})$
$\mathfrak{so}(4)$	$\mathcal{Z}(\mathcal{Z}+1)$	$\epsilon_1\mathcal{Z}^3+\epsilon_2$	$\xi_3(\epsilon_1-\epsilon_2\mathcal{Z}^3)+3\xi_7\mathcal{Z}(1-\mathcal{Z})$
$\mathfrak{so}(3,1)$	$-\mathcal{Z}(\mathcal{Z}^2+1)$	$\epsilon_1\mathcal{Z}^3-3\epsilon_2\mathcal{Z}^2-3\epsilon_1\mathcal{Z}+\epsilon_2$	$\xi_3(\epsilon_1+3\epsilon_2\mathcal{Z}-3\epsilon_1\mathcal{Z}^2-\epsilon_2\mathcal{Z}^3)$ $+3\xi_7(\mathcal{Z}^2+1)$
$\mathfrak{su}(2)+\mathfrak{u}(1)^3$	$\mathcal{Z}$	$\epsilon_1\mathcal{Z}^3+\epsilon_2$	$\xi_3(\epsilon_1-\epsilon_2\mathcal{Z}^3)-3\xi_7\mathcal{Z}^2$
$\mathfrak{su}(2)\oplus\mathfrak{u}(1)^3$	$1-\mathcal{Z}$	$\epsilon_1\mathcal{Z}+\epsilon_2$	$3\lambda_1\mathcal{Z}+3\lambda_2\mathcal{Z}^2+\lambda_3\mathcal{Z}^3$
$\mathfrak{nil}$	1	$\epsilon_1\mathcal{Z}+\epsilon_2$	$3\lambda_1\mathcal{Z}+3\lambda_2\mathcal{Z}^2+\lambda_3\mathcal{Z}^3$

Table 4: Q-subalgebras and polynomials

Substituting in (2.25) we easily obtain the corresponding polynomial

$$P_2(U) = (\gamma U + \delta)^2 \left[\epsilon_2(\gamma U + \delta) + \epsilon_1(\gamma - \delta U)\right] . \tag{3.37}$$

As in $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$, this P_2 has one single and one double real root. Without loss of generality we can choose $\alpha = -\delta$ and $\beta = \gamma$ in order to write P_2 in terms of the variable $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$ as

$$P_2(U) = (\gamma U + \delta)^3(\epsilon_1\mathcal{Z} + \epsilon_2) . \tag{3.38}$$

The advantage of this choice of parameters will become evident when we perform a transformation from U to $\mathcal{Z}$ in the scalar potential.

4. New variables and RR fluxes

In type IIB orientifolds, the superpotential depends on the complex structure parameter U through the three cubic polynomials $P_1(U)$, $P_2(U)$ and $P_3(U)$ induced respectively by RR, NSNS and non-geometric Q-fluxes. Our results in last section show that the last two polynomials can be concisely written as

$$P_2(U) = (\gamma U + \delta)^3\mathcal{P}_2(\mathcal{Z}) \qquad ; \qquad P_3(U) = (\gamma U + \delta)^3\mathcal{P}_3(\mathcal{Z}) , \tag{4.1}$$

where $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$. The real parameters $(\alpha, \beta, \gamma, \delta)$, with $|\Gamma| = (\alpha\delta - \beta\gamma) \neq 0$, encode the non-geometric fluxes. For the NSNS fluxes two additional real constants (ϵ_1, ϵ_2) are needed. As summarized in table 4, $\mathcal{P}_2(\mathcal{Z})$ and $\mathcal{P}_3(\mathcal{Z})$ take very specific forms according to the subalgebra of the Q-fluxes.

A very nice property of the variable $\mathcal{Z}$ is its invariance under the $\mathrm{SL}(2,\mathbb{Z})_U$ modular transformations

$$U' = \frac{kU + \ell}{mU + n} \quad ; \quad k, \ell, m, n \in \mathbb{Z} \quad ; \quad kn - \ell m = 1 . \tag{4.2}$$

Since this is a symmetry of the compactification, the effective action must be invariant. The Kähler potential, $K = -3\log[-i(U - \bar{U})] + \cdots$, clearly transforms as

$$K' = K + 3\log|mU + n|^2 . \tag{4.3}$$

Therefore, the physically important quantity $e^K|W|^2$ is invariant as long as the superpotential satisfies

$$W' = \frac{W}{(mU + n)^3} \, . \tag{4.4}$$

In order for W to fulfill this property the fluxes must transform in definite patterns. In fact, it follows that (4.4) holds separately for each of the flux induced polynomial $P_i(U)$.

We claim that the fluxes transform under $\mathrm{SL}(2,\mathbb{Z})_U$ precisely in such a manner that $\mathcal{Z}' = \mathcal{Z}$. The proof begins by first finding how the Q-fluxes mix among themselves from the condition $P_3' = P_3/(mU + n)^3$. For example, under $U' = -1/U$, the non-geometric fluxes transform as

$$c_0' = -c_3 \quad , \quad c_1' = c_2 \quad , \quad c_2' = -c_1 \quad , \quad c_3' = c_0 \quad , \quad \tilde{c}_1' = \tilde{c}_2 \quad , \quad \tilde{c}_2' = -\tilde{c}_1 \, . \tag{4.5}$$

Next we read off the corresponding transformation of the parameters $(\alpha, \beta, \gamma, \delta)$ that are better thought of as the elements of a matrix Γ. The result is

$$\Gamma' = \begin{pmatrix} \alpha' \ \beta' \\ \gamma' \ \delta' \end{pmatrix} = \begin{pmatrix} \alpha \ \beta \\ \gamma \ \delta \end{pmatrix} \begin{pmatrix} n \ {-\ell} \\ {-m} \ k \end{pmatrix} \tag{4.6}$$

It easily follows that $\mathcal{Z}' = \mathcal{Z}$. Notice that $|\Gamma'| = |\Gamma|$.

For the NSNS fluxes we can study the transformation of P_2 with coefficients given by the b_A. Alternatively, we may start from P_2 written as function of $\mathcal{Z}$ as in (4.1). The conclusion is that the transformation of the b_A is also determined by Γ' together with $(\epsilon_1', \epsilon_2') = (\epsilon_1, \epsilon_2)$. This is valid for all Q-subalgebras.

At this point it must be evident that we want to change variables from U to $\mathcal{Z}$. It is also convenient to trade the axiodilaton S and the Kähler modulus T by new fields defined by

$$\mathcal{S} = S + \xi_s \quad ; \quad \mathcal{T} = T + \xi_t, \tag{4.7}$$

where the shifts ξ_s and ξ_t are some real parameters. The motivation is that such shifts in the axions $\mathrm{Re}\, S$ and $\mathrm{Re}\, T$ can be reabsorbed into RR fluxes as explained in the following.

4.1 Parametrization of RR fluxes

The systematic procedure is to express the RR fluxes a_A in such a way that their contribution to the superpotential is of the form

$$P_1(U) = (\gamma U + \delta)^3 \widehat{\mathcal{P}}_1(\mathcal{Z}) \, , \tag{4.8}$$

in complete analogy with (4.1). To arrive at this factorization we must relate the four RR fluxes a_A to the parameters $(\alpha, \beta, \gamma, \delta)$ that define $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$, and to four additional independent variables. Obviously, $\widehat{\mathcal{P}}_1(\mathcal{Z})$ can be expanded in the monomials $(1, \mathcal{Z}, \mathcal{Z}^2, \mathcal{Z}^3)$. However, a more convenient basis contains the already known polynomials $\mathcal{P}_3$ and $\mathcal{P}_2$ that are generically linearly independent. We still need two independent polynomials and these are taken to be the duals $\widetilde{\mathcal{P}}_3$ and $\widetilde{\mathcal{P}}_2$. The dual $\widetilde{\mathcal{P}}$ is such that $\mathcal{P} \to \widetilde{\mathcal{P}}/\mathcal{Z}^3$ when $\mathcal{Z} \to -1/\mathcal{Z}$. The last two subalgebras in table 4 must be treated slightly different

because linear independence of $\mathcal{P}_3$ and $\mathcal{P}_2$ fails for particular properties of the NSNS flux parameter ϵ_1.

We concretely make the expansion

$$\widehat{\mathcal{P}}_1(\mathcal{Z}) = \xi_s \mathcal{P}_2(\mathcal{Z}) + \xi_t \mathcal{P}_3(\mathcal{Z}) + \mathcal{P}_1(\mathcal{Z}) \ . \tag{4.9}$$

In the full superpotential the first two terms in $\widehat{\mathcal{P}}_1$ will precisely offset the axionic shifts in the new variables $\mathcal{S}$ and $\mathcal{T}$. Let us now discuss the remaining piece $\mathcal{P}_1(\mathcal{Z})$ that also depends on the Q-subalgebra and is displayed in table 4. As explained before, for the first three subalgebras in the table we can further choose

$$\mathcal{P}_1(\mathcal{Z}) = \xi_7 \widetilde{\mathcal{P}}_3(\mathcal{Z}) - \xi_3 \widetilde{\mathcal{P}}_2(\mathcal{Z}) \ . \tag{4.10}$$

A motivation for this choice is that the RR tadpoles turn out to depend on the RR fluxes only through the coefficients (ξ_3, ξ_7).

For the last two subalgebras in table 4, $\mathcal{P}_3$ and $\mathcal{P}_2$ are not independent when ϵ_1 takes a particular critical value. For $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$ this happens when $\epsilon_1 = -\epsilon_2$, whereas for the nilpotent algebra the critical value is $\epsilon_1 = 0$. To take into account these possibilities, compensating at the same time for the axionic shifts, we still make the decomposition (4.9) but with

$$\mathcal{P}_1(\mathcal{Z}) = 3\lambda_1 \mathcal{Z} + 3\lambda_2 \mathcal{Z}^2 + \lambda_3 \mathcal{Z}^3 \ . \tag{4.11}$$

Away from the critical values of ϵ_1 we can take $\lambda_1 = 0$ because ξ_s and ξ_t are independent parameters. At the critical value necessarily $\lambda_1 \neq 0$ but in this case ξ_s and ξ_t enter in the RR fluxes in only one linearly independent combination. The RR tadpoles happen to depend just on the parameters (λ_2, λ_3).

The next step is to compare the expansion of $P_1(U)$ in U with its factorized form, c.f. (4.8) and (2.24). In this way we can obtain an explicit parametrization of the RR fluxes a_A in terms of the variables that determine $\widehat{\mathcal{P}}_1(\mathcal{Z})$, namely (ξ_s, ξ_t) together with (ξ_3, ξ_7) or $(\lambda_1, \lambda_2, \lambda_3)$, depending on the Q-subalgebra. These results are collected in the appendix. We stress that the ξ's and λ's are real parameters but the emerging RR fluxes must be integers.

A vacuum solution in which the moduli $(\mathcal{Z}, \mathcal{S}, \mathcal{T})$ are fixed generically requires specific values of the non-geometric, NSNS and RR fluxes. These fluxes also generate RR tadpoles that must be balanced by adding orientifold planes or D-branes. To determine the type of sources that must be included we need to evaluate the RR tadpole cancellation conditions using all parametrized fluxes. Substituting in (2.42) and (2.43) we arrive at the very compact expressions for the number of sources N_3 and N_7 gathered in table 5. As advertised before, the RR fluxes only enter either through the parameters (ξ_3, ξ_7) or (λ_2, λ_3). The non-geometric and NSNS fluxes only contribute through $|\Gamma|^3$ and (ϵ_1, ϵ_2). We will see that there is also a clear correlation of the tadpoles with the vevs of the moduli.

Finally, let us remark that, just like (ϵ_1, ϵ_2), the ξ and λ variables are all invariant under modular transformations of the complex structure U. Indeed, from the explicit parametrization of the RR fluxes a_A we deduce that their correct behavior under $\mathrm{SL}(2, \mathbb{Z})_U$, analogous to (4.5), precisely follows from the transformation of $(\alpha, \beta, \gamma, \delta)$ in (4.6). This is

Q-subalgebra	$N_3/|\Gamma|^3$	$N_7/|\Gamma|^3$
$\mathfrak{so}(4)$	$(\epsilon_1^2 + \epsilon_2^2)\,\xi_3$	$2\,\xi_7$
$\mathfrak{so}(3,1)$	$4(\epsilon_1^2 + \epsilon_2^2)\,\xi_3$	$4\,\xi_7$
$\mathfrak{su}(2) + \mathfrak{u}(1)^3$	$(\epsilon_1^2 + \epsilon_2^2)\,\xi_3$	ξ_7
$\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$	$\lambda_2\,\epsilon_1 - \lambda_3\,\epsilon_2$	$\lambda_2 + \lambda_3$
$\mathfrak{nil}$	$\lambda_2\,\epsilon_1 - \lambda_3\,\epsilon_2$	λ_3

Table 5: Q-subalgebras and RR tadpoles

of course consistent with the fact that the number of sources N_3 and N_7 in the tadpoles are physical quantities that must be modular invariant.

4.2 Moduli potential in the new variables

We have just seen how a systematic parametrization of the fluxes has guided us to new moduli fields denoted $(\mathcal{Z}, \mathcal{S}, \mathcal{T})$. As we may expect, the effective action in the transformed variables also takes a form more suitable for finding vacua. The shifts in the axionic real parts of the axiodilaton and the Kähler field do not affect the Kähler potential K whereas in the superpotential W they can be reabsorbed in RR fluxes. On the other hand, the change from the complex structure U to $\mathcal{Z}$ is the SL$(2, \mathbb{R})$ transformation $U = (\beta - \delta\mathcal{Z})/(\gamma\mathcal{Z} - \alpha)$ whose effect on K and W is completely analogous to a modular transformation except for factors of $|\Gamma| = (\alpha\delta - \beta\gamma)$. Combining previous results we obtain $e^K|W|^2 \to e^{\mathcal{K}}|\mathcal{W}|^2$, where the transformed Kähler potential $\mathcal{K}$ and superpotential $\mathcal{W}$ are given by

$$\mathcal{K} = -3\log\left(-i\left(\mathcal{U} - \bar{\mathcal{U}}\right)\right) - \log\left(-i\left(\mathcal{S} - \bar{\mathcal{S}}\right)\right) - 3\log\left(-i\left(\mathcal{Z} - \bar{\mathcal{Z}}\right)\right), \qquad (4.12)$$

$$\mathcal{W} = |\Gamma|^{3/2}\left[\mathcal{T}\,\mathcal{P}_3(\mathcal{Z}) + \mathcal{S}\,\mathcal{P}_2(\mathcal{Z}) + \mathcal{P}_1(\mathcal{Z})\right]. \qquad (4.13)$$

The flux-induced polynomials $\mathcal{P}_i(\mathcal{Z})$ are displayed in table 4 for each Q-subalgebra. In the effective 4-dimensional action with $\mathcal{N}=1$ supergravity the functions $\mathcal{K}$ and $\mathcal{W}$ determine the scalar potential of the moduli according to

$$V = e^{\mathcal{K}}\left\{\sum_{\Phi = \mathcal{Z}, \mathcal{S}, \mathcal{T}} \mathcal{K}^{\Phi\bar{\Phi}}|D_\Phi\mathcal{W}|^2 - 3|\mathcal{W}|^2\right\}. \qquad (4.14)$$

We are interested in supersymmetric minima for which $D_\Phi\mathcal{W} = \partial_\Phi\mathcal{W} + \mathcal{W}\partial_\Phi\mathcal{K} = 0$, for all fields.

5. Supersymmetric vacua

This section is devoted to searching for supersymmetric vacua of the moduli potential induced by RR, NSNS and non-geometric fluxes together. We will show that by using our new variables the problem simplifies substantially and analytic solutions are feasible.

Supersymmetric vacua are characterized by the vanishing of the F-terms. In our setup the conditions are

$$
\begin{aligned}
D_{\mathcal{T}}\mathcal{W} &= \frac{\partial\mathcal{W}}{\partial\mathcal{T}} + \frac{3i\mathcal{W}}{2\mathrm{Im}\,\mathcal{T}} = 0\,, \\
D_{\mathcal{S}}\mathcal{W} &= \frac{\partial\mathcal{W}}{\partial\mathcal{S}} + \frac{i\mathcal{W}}{2\mathrm{Im}\,\mathcal{S}} = 0\,, \\
D_{\mathcal{Z}}\mathcal{W} &= \frac{\partial\mathcal{W}}{\partial\mathcal{Z}} + \frac{3i\mathcal{W}}{2\mathrm{Im}\,\mathcal{Z}} = 0\,.
\end{aligned}
\tag{5.1}
$$

The task is to determine whether there are solutions with moduli completely stabilized at vevs denoted

$$
\mathcal{Z}_0 = x_0 + iy_0 \quad ; \quad \mathcal{S}_0 = s_0 + i\sigma_0 \quad ; \quad \mathcal{T}_0 = t_0 + i\mu_0 \,.
\tag{5.2}
$$

The vacua are either Minkowski or AdS because the potential (4.14) at the minimum is given by $V_0 = -3e^{\mathcal{K}_0}|\mathcal{W}_0|^2 \leq 0$.

Besides stabilization, there are further physical requirements. At the minimum the imaginary part of the axiodilaton, σ_0, must be positive for the reason it is the inverse of the string coupling constant g_s. It can be argued that the geometric moduli are subject to similar conditions. The main assumption is that they arise from the metric of the internal space, which is T^6 in absence of fluxes. In particular, the Kähler modulus has $\mathrm{Im}\,\mathcal{T} = e^{-\phi}A$, where A is the area of a 4-dimensional subtorus. Hence, it must be $\mu_0 > 0$. Notice also that the internal volume is measured by $V_{\mathrm{int}} = (\mu_0/\sigma_0)^{3/2}$. For the transformed complex structure $\mathcal{Z}$ it happens that $\mathrm{Im}\,\mathcal{Z} = |\Gamma|\mathrm{Im}\,U/|\gamma U + \delta|^2$. Therefore, necessarily $\mathrm{Im}\,\mathcal{Z}_0 = y_0 \neq 0$ because for $\mathrm{Im}\,U_0 = 0$ the internal space is degenerate. Without loss of generality we choose that $\mathrm{Im}\,U_0$ is always positive.

Another physical issue is whether the moduli take values such that the effective supergravity action is a reliable approximation to string theory. Specifically, the string coupling $g_s = 1/\sigma_0$ is expected to be small to justify the exclusion of non-perturbative string effects. Conventionally, there is also a requirement of large internal volume to disregard corrections in α'. However, in presence of non-geometric fluxes the internal space might be a T-fold in which there can exist cycles with sizes related by T-duality [34, 16]. Thus, for large volume there could be tiny cycles whose associated winding modes would be light. To date these effects are not well understood. At any rate, in this work we limit ourselves to finding supersymmetric vacua of an effective field theory defined by a very precise Kähler potential and flux-induced superpotential. A more detailed discussion of the landscape of vacua is left for section 6. We will see that the moduli can be fixed at small string coupling and cosmological constant.

In the following we will first consider supersymmetric Minkowski vacua that have $\mathcal{W} = 0$ at the minimum. In our approach it is straightforward to show that for isotropic fluxes such vacua are disallowed. We then turn our attention to the richer class of AdS$_4$ vacua. Since superpotential terms adopt very specific forms depending on the particular subalgebra satisfied by the non-geometric fluxes, we will study the corresponding vacua case by case. We will mostly focus on the model associated to the non-geometric fluxes of the compact $\mathfrak{su}(2)^2$ but will also consider other allowed subalgebras to some extent.

5.1 Minkowski vacua

Minkowski solutions with zero cosmological constant require that the potential vanishes. Imposing supersymmetry further implies that the superpotential must be zero at the minimum $(\mathcal{Z}_0, \mathcal{S}_0, \mathcal{T}_0)$. A key property of the superpotential (4.13) is its linearity in $\mathcal{S}$ and $\mathcal{T}$. This implies in particular that the F-flatness conditions $D_{\mathcal{S}}\mathcal{W} = 0$ and $D_{\mathcal{T}}\mathcal{W} = 0$, together with $\mathcal{W} = 0$, reduce just to

$$\mathcal{P}_3(\mathcal{Z}_0) = \mathcal{P}_2(\mathcal{Z}_0) = \mathcal{P}_1(\mathcal{Z}_0) = 0 \ . \tag{5.3}$$

The third condition $D_{\mathcal{Z}}\mathcal{W} = 0$ yields a linear relation between $\mathcal{S}_0$ and $\mathcal{T}_0$ so that not all moduli can be stabilized. The situation is actually worse because (5.3) cannot be fulfilled appropriately. Indeed, for the specific polynomials for each subalgebra shown in table 4, it is evident that $\mathcal{P}_3$ and $\mathcal{P}_2$ can only have a common real root $\mathcal{Z}_0$. But then $\mathrm{Im}\, U_0 = \mathrm{Im}\, \mathcal{Z}_0 = 0$ and this is inconsistent with a well defined internal space.

It must be emphasized that we are assuming that non-geometric fluxes, and their induced $\mathcal{P}_3$, are non-trivial. Our motivation is to fix the Kähler modulus without invoking non-perturbative effects. If only RR and NSNS fluxes are turned on there do exist physical supersymmetric Minkowski vacua in which only the axiodilaton and the complex structure are stabilized [32, 35]. In such solutions the RR and NSNS fluxes must still satisfy a non-linear constraint [35, 36].

No-go results for supersymmetric Minkowski vacua in presence of non-geometric fluxes have been obtained previously [19, 22, 36].[3] In [19] the existence was disproved supposing special solutions for the Jacobi identities (2.37). We are now extending the proof to all possible non-trivial *isotropic* non-geometric fluxes solving these constraints.

5.2 AdS_4 vacua

We now want to solve the supersymmetry conditions when $\mathcal{W} \neq 0$. The three complex equations $D_\Phi \mathcal{W} = 0$, $\Phi = \mathcal{Z}, \mathcal{S}, \mathcal{T}$, in principle admit solutions with all moduli fixed at values $\mathcal{Z}_0 = x_0 + iy_0$, $\mathcal{S}_0 = s_0 + i\sigma_0$, and $\mathcal{T}_0 = t_0 + i\mu_0$. We will also impose the physical requirements $\sigma_0 > 0$, $\mu_0 > 0$ and $\mathrm{Im}\, U_0 > 0$ which implies $|\Gamma| y_0 > 0$. In general existence of such solutions demands that the fluxes satisfy some specific properties.

In the AdS_4 vacua, $\mathcal{P}_2$ and $\mathcal{P}_3$ are necessarily different from zero. Moreover, combining the equations $D_{\mathcal{S}}\mathcal{W} = 0$ and $\mathrm{D}_{\mathcal{T}}\mathcal{W} = 0$ shows that at the minimum $\mathrm{Im}\,(\mathcal{P}_3/\mathcal{P}_2) = 0$, or equivalently

$$(\mathcal{P}_3 \mathcal{P}_2^* - \mathcal{P}_3^* \mathcal{P}_2)|_0 = 0 \ . \tag{5.4}$$

From this condition we can quickly extract useful information. For example, for the polynomials of the nilpotent subalgebra we find that $\epsilon_1 = 0$. Similarly, for the semidirect product $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$, it follows that $\epsilon_1 = -\epsilon_2$. Thus, in these two cases $\mathcal{P}_2$ and $\mathcal{P}_3$ are forced to be parallel and equation (5.4) is inconsequential for the moduli. Having one equation less means that all moduli cannot be fixed. In fact, what happens is that only a linear combination of the axions s_0 and t_0 is determined [6].

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[3]In [22] it is further shown that Minkowski vacua with all moduli stabilized can exist in more general setups having more complex structure than Kähler moduli (in IIB language).

Another instructive example is that of the $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ subalgebra. With the polynomials provided in table 4 the condition (5.4) implies

$$\epsilon_2 - 2\epsilon_1 x_0 (x_0^2 + y_0^2) = 0 \,, \tag{5.5}$$

where we already used that $y_0 \neq 0$. Now we see that forcefully $\epsilon_1 \neq 0$ because otherwise ϵ_2, and thus $\mathcal{P}_2$ itself, would vanish. However, it could be $\epsilon_2 = 0$ and then $x_0 = 0$. If $\epsilon_2 \neq 0$ we will just have one equation that gives y_0 in terms of x_0.

In other examples with $\mathcal{P}_2$ and $\mathcal{P}_3$ not parallel there are analogous results. It can happen that (5.4) already fixes x_0 or it gives y_0 as function of x_0. The remaining five equations can be used to obtain $\mathcal{S}_0$ and $\mathcal{T}_0$ in terms of y_0 or x_0, and to find a polynomial equation that determines y_0 or x_0. This procedure can be efficiently carried out using the algebraic package *Singular*. The results are described below in more detail.

The superpotential for each Q-subalgebra is constructed with the flux-induced polynomials listed in table 4. The numbers of sources needed to cancel tadpoles are given in table 5. Recall that O3-planes (D3-branes) make a positive (negative) contribution to N_3, whereas O7-planes (D7-branes) yield negative (positive) values of N_7.

Each supersymmetric vacua can be distinguished by the modular invariant values of the string coupling constant g_s and the potential at the minimum V_0 that is equal to the cosmological constant up to normalization. In the models at hand these quantities are given by

$$V_0 = -\frac{3|\mathcal{W}_0|^2}{128\, y_0^3\, \mu_0^3\, \sigma_0} \qquad ; \qquad g_s = \frac{1}{\sigma_0} \,. \tag{5.6}$$

In all examples the vevs of the moduli y_0, σ_0, μ_0, as well as the value $\mathcal{W}_0$ of the superpotential at the minimum, can be completely determined and will be given explicitly. It is then straightforward to evaluate the characteristic data (V_0, g_s).

5.2.1 Nilpotent subalgebra

When $\epsilon_1 = 0$, the model based on the non-geometric fluxes of the nilpotent subalgebra is $U \leftrightarrow T$ dual to a IIA orientifold with only RR and NSNS fluxes already considered in the literature [5, 6]. Supersymmetry actually requires $\epsilon_1 = 0$. There are some salient features that are easily reproduced in our setup. For instance, a solution exists only if $\lambda_3 \neq 0$ and $(\lambda_1 \lambda_3 - \lambda_2^2) > 0$. The axions s_0 and t_0 can only be fixed in the linear combination

$$3t_0 + \epsilon_2 s_0 = \frac{\lambda_2}{\lambda_3^2}(3\lambda_1 \lambda_2 - 2\lambda_2^2) \,. \tag{5.7}$$

The rest of the moduli are determined as

$$x_0 = -\frac{\lambda_2}{\lambda_3} \quad ; \quad y_0^2 = \frac{5(\lambda_1 \lambda_3 - \lambda_2^2)}{3\lambda_3^2} \quad ; \quad \sigma_0 = -\frac{2(\lambda_1 \lambda_3 - \lambda_2^2)y_0}{3\epsilon_2 \lambda_3} \quad ; \quad \mu_0 = \epsilon_2 \sigma_0 \,. \tag{5.8}$$

The cosmological constant can be computed using $\mathcal{W}_0 = 2i\mu_0|\Gamma|^{3/2}$.

From the results we deduce that $\epsilon_2 > 0$, and $\lambda_3 > 0$ for $y_0 < 0$. Then $\mathrm{Im}\, U_0 > 0$ requires $|\Gamma| < 0$ as it happens for the nilpotent algebra. The tadpole conditions then verify $N_3 = -\lambda_3 \epsilon_2 |\Gamma|^3 > 0$ and $N_7 = \lambda_3 |\Gamma|^3 < 0$. The relevant conclusion is that the model necessarily requires O3-planes and O7-planes.

5.2.2 Semidirect sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$

The non-geometric fluxes of this subalgebra are $U \leftrightarrow T$ dual to NSNS plus *geometric* fluxes in a IIA orientifold. Models of this type have been studied previously [3, 4, 6]. For completeness we will briefly summarize our results that totally agree with the general solution presented in [6]. Existence of a supersymmetric minimum imposes the constraint $\epsilon_1 = -\epsilon_2$. In this case it occurs again that the axions s_0 and t_0 can only be determined in a linear combination given by

$$3t_0 + \epsilon_2 s_0 = 3\lambda_1 + 3\lambda_2(9 - 7x_0) + 3\lambda_3 x_0(9 - 8x_0) . \tag{5.9}$$

The imaginary parts of the axiodilaton and the Kähler field are stabilized at values

$$\mu_0 = \epsilon_2 \sigma_0 \quad ; \quad \epsilon_2 \sigma_0 = 6(\lambda_2 + \lambda_3 x_0)y_0 . \tag{5.10}$$

Notice that ϵ_2 must be positive. It also follows that $\mathcal{W}_0 = 2i\mu_0(1 - x_0 - iy_0)|\Gamma|^{3/2}$. The vevs of x_0 and y_0 depend on whether the RR flux parameter λ_3 is zero or not.

When $\lambda_3 = 0$ we obtain

$$x_0 = 1 \quad ; \quad 3\lambda_2 y_0^2 = -(\lambda_1 + \lambda_2) . \tag{5.11}$$

Notice that $\lambda_2 \neq 0$ to guarantee $\sigma_0 \neq 0$. In fact, choosing $y_0 > 0$ it must be $\lambda_2 > 0$. For the number of sources we find $N_3 = -\lambda_2 \epsilon_2 |\Gamma|^3 < 0$ and $N_7 = \lambda_2 |\Gamma|^3 > 0$. Therefore, D3 and D7-branes must be included.

When $\lambda_3 \neq 0$ we instead find

$$\lambda_3 y_0^2 = 15(x_0 - 1)(\lambda_2 + \lambda_3 x_0) , \tag{5.12}$$

whereas x_0 must be a root of the cubic equation

$$160(x_0-1)^3 + 294\left(1 + \frac{\lambda_2}{\lambda_3}\right)(x_0-1)^2 + 135\left(1 + \frac{\lambda_2}{\lambda_3}\right)^2(x_0-1) + \frac{1}{\lambda_3}(\lambda_3 + 3\lambda_2 + 3\lambda_1) = 0 . \tag{5.13}$$

The solution for x_0 must be real and such that $y_0^2 > 0$. For the tadpoles we now have $N_7 = |\Gamma|^3(\lambda_2 + \lambda_3)$ and $N_3 = -\epsilon_2 N_7$. Thus, in general N_3 and N_7 have opposite signs. The remarkable feature is that now they can be zero simultaneously. This occurs when the RR parameters satisfy $\lambda_2 = -\lambda_3$, in which case the cubic equation for x_0 can be solved exactly.

5.2.3 Direct sum $\mathfrak{su}(2) + \mathfrak{u}(1)^3$

As explained before, necessarily $\epsilon_1 \neq 0$. Let us consider $\epsilon_2 = 0$ which is the condition for $\mathcal{P}_2$ to have only real roots. Now it happens that all moduli can be determined. The axions are fixed at $x_0 = 0$, $s_0 = 0$ and $t_0 = 0$, whereas the real parts have vevs

$$y_0^2 = \frac{\epsilon_1 \xi_3}{\xi_7} \quad ; \quad \sigma_0 = -\frac{2\xi_7^2 y_0}{\epsilon_1^2 \xi_3} \quad ; \quad \mu_0 = 2\xi_7 y_0 . \tag{5.14}$$

The cosmological constant is easily found substituting $\mathcal{W}_0 = -2\mu_0 y_0 |\Gamma|^{3/2}$. Clearly, the solution exists only if $\xi_3 \neq 0$ and $\xi_7 \neq 0$. Moreover, $\epsilon_1 \xi_3 \xi_7 > 0$ and if we take $y_0 > 0$,

$\xi_3 < 0$, $\xi_7 > 0$ and $\epsilon_1 < 0$. The numbers of sources satisfy $N_3 < 0$ and $N_7 > 0$, so that D3 and D7-branes are needed.

Taking $\epsilon_2 \neq 0$ we deduce that there are no solutions at all when $\xi_7 = 0$ and $\xi_3 \neq 0$. However, there are minima that require $\epsilon_1 < 0$ and $N_7 > 0$ when $\xi_3 = 0$.

5.2.4 Non-compact $\mathfrak{so}(3,1)$

This is the only flux configuration for which $\mathcal{P}_3(\mathcal{Z})$ has complex roots. It also happens that $\mathcal{P}_2(\mathcal{Z})$ always has three different real roots. We will briefly discuss the vacua according to whether the NSNS flux parameter ϵ_2 vanishes or not.

- $\epsilon_2 = 0$

  In this setup the axions are determined to be $x_0 = 0$, $s_0 = 0$ and $t_0 = 0$. For the imaginary parts of the Kähler modulus and the axiodilaton we obtain

  $$\mu_0 = \frac{\epsilon_1 \sigma_0 (3 + y_0^2)}{(1 - y_0^2)} \quad ; \quad \epsilon_1 \sigma_0 = \frac{1}{2y_0(3 + y_0^2)} \left[3\xi_7(y_0^2 - 1) - \epsilon_1 \xi_3(3y_0^2 + 1) \right] . \quad (5.15)$$

  To evaluate the potential at the minimum we use $\mathcal{W}_0 = 2\mu_0 y_0 (1 - y_0^2)|\Gamma|^{3/2}$. Notice that ξ_3 and ξ_7 cannot be zero simultaneously and that $y_0^2 = 1$ is not allowed. Actually, the imaginary part of the transformed complex structure satisfies a third order polynomial equation in y_0^2 given by

  $$\epsilon_1 \xi_3 (5y_0^6 + 13y_0^4 + 15y_0^2 - 1) - \xi_7(y_0^2 - 1)(5y_0^4 + 6y_0^2 - 3) = 0 . \quad (5.16)$$

  We are interested in real roots $y_0 \neq 0$ and $y_0 \neq \pm 1$.

  Although we have not made an exhaustive analysis, it is clear that the solutions of (5.16) depend on the range of the ratio $\xi_7/\epsilon_1\xi_3$. For instance, there are values for which there is no real root at all, as it occurs e.g. for $2\xi_7 = -\epsilon_1\xi_3$. For other values there might be only one real positive solution for y_0^2. An special example happens when $\xi_3 = 0$ and the net O3/D3 charge N_3 is zero, while the net O7/D7 charge N_7 is negative as implied by the conditions $\mu_0 > 0$ and $|\Gamma|y_0 > 0$. Similarly, when $\xi_7 = 0$, there is only one solution in which $N_7 = 0$ while $N_3 < 0$.

  The third possibility is to have two allowed solutions. For instance, taking $\xi_7 = 2\epsilon_1\xi_3$ gives roots $y_0^2 = 1/5$ and $y_0^2 = 1 + 2\sqrt{2}$. However, in principle the corresponding vacua cannot be realized simultaneously because the net charges would have to jump. In fact, for $y_0^2 < 1$, it happens that $N_3 N_7 > 0$, whereas for $y_0^2 > 1$, it must be $N_3 N_7 < 0$. It can also arise that both solutions have $y_0^2 < 1$. For example, when $\xi_7 = -30\epsilon_1\xi_3$ each of the two vacua has $N_3 > 0$ and $N_7 < 0$. We will explore the phenomenon of multiple AdS vacua in more detail for the non-geometric fluxes of the $\mathfrak{su}(2)^2$ algebra.

- $\epsilon_2 \neq 0$

  We have only studied the special cases when one of the flux-tadpoles N_3 or N_7 is zero. We find that when $\epsilon_1 = 0$ the F-flatness conditions can not be solved but for $\epsilon_1 > 0$ there are consistent solutions for a particular range of $|\epsilon_2/\epsilon_1|$. Vacua with $\xi_3 = 0$

exist provided that $\xi_7 < 0$. Vacua with no O7/D7 flux-tadpoles, i.e. with $\xi_7 = 0$, require $\xi_3 < 0$. One important conclusion is that for the fluxes of the non-compact Q-subalgebra solutions with $N_7 = 0$ must have $N_3 < 0$.

5.2.5 Compact $\mathfrak{su}(2)^2$

This is the only situation in which the polynomial $P_3(U)$ induced by the non-geometric fluxes has three different real roots. The polynomial $P_2(U)$ generated by NSNS fluxes has complex roots whenever $\epsilon_1 \epsilon_2 \neq 0$, and one triple real root otherwise. We will study the vacua in both cases in some detail.

The full model based on the non-geometric fluxes of $\mathfrak{su}(2)^2$ has an interesting residual symmetry that exchanges the NSNS auxiliary parameters. It can be shown that the effective action is invariant under $\epsilon_1 \leftrightarrow \epsilon_2$, $\xi_3 \to \xi_3$ and $\xi_7 \to \xi_7$, together with the field transformations

$$\mathcal{Z} \to 1/\mathcal{Z}^* \quad ; \quad \mathcal{S} \to -\mathcal{S}^* \quad ; \quad \mathcal{T} \to -\mathcal{T}^* . \tag{5.17}$$

This symmetry leaves one of the $\mathcal{P}_3$ roots invariant while exchanging the other two.

5.2.6 $P_2(U)$ with triple real root

Due to the symmetry (5.17) it is enough to consider $\epsilon_1 = 0$ and $\epsilon_2 \neq 0$. In this model the axions are stabilized at vevs

$$x_0 = -\frac{1}{2} \quad ; \quad \epsilon_2 s_0 = 3\xi_7 - \frac{\epsilon_2 \xi_3}{2} \quad ; \quad t_0 = \xi_7 - \frac{\epsilon_2 \xi_3}{2} . \tag{5.18}$$

The imaginary parts of the Kähler modulus and the axiodilaton are fixed in terms of y_0 according to

$$\mu_0 = -\frac{4\epsilon_2 \sigma_0}{(1 + 4y_0^2)} \quad ; \quad \epsilon_2 \sigma_0 = -y_0 \big[3\xi_7 + \frac{\epsilon_2 \xi_3}{8}(4y_0^2 - 3)\big] . \tag{5.19}$$

At the minimum $\mathcal{W}_0 = 2i\epsilon_2 \sigma_0 |\Gamma|^{3/2}$. Clearly ξ_3 and ξ_7 cannot vanish simultaneously so that the model always requires additional sources to cancel tadpoles. Observe that necessarily $\epsilon_2 < 0$.

The modulus y_0 is determined by the fourth order polynomial equation

$$\epsilon_2 \xi_3 (4y_0^2 - 1)(4y_0^2 + 5) - 8\xi_7(4y_0^2 - 5) = 0 . \tag{5.20}$$

In the two special cases $\xi_7 = 0$ and $\xi_3 = 0$ an exact solution is easily found. When $\xi_3 \xi_7 \neq 0$ there can be two AdS solutions. The corresponding vacua, which can be characterized by the net tadpoles N_3 and N_7, are described more extensively in the following.

- $N_7 = 0$

  When $\xi_7 = 0$ the vevs have the very simple expressions

  $$y_0^2 = \frac{1}{4} \quad ; \quad \sigma_0 = \frac{\xi_3 y_0}{4} \quad ; \quad \mu_0 = -2\epsilon_2 \sigma_0 \quad ; \quad V_0 = \frac{12|\Gamma|^3 y_0}{\epsilon_2 \xi_3^2} . \tag{5.21}$$

  Since both μ_0 and σ_0 are positive, it must be $\epsilon_2 < 0$, and taking $y_0 > 0$, $\xi_3 > 0$. Therefore, $N_3 > 0$ and O3-planes must be included.

- $N_3 = 0$

  This is the case $\xi_3 = 0$. The moduli and the cosmological constant are fixed at values

  $$y_0^2 = \frac{5}{4} \quad ; \quad \epsilon_2 \sigma_0 = -3\xi_7 y_0 \quad ; \quad \mu_0 = -\frac{2}{3}\epsilon_2 \sigma_0 \quad ; \quad V_0 = \frac{9|\Gamma|^3 \epsilon_2 y_0}{500\,\xi_7^2}\,. \tag{5.22}$$

  Necessarily $\epsilon_2 < 0$, and choosing $y_0 > 0$, $\xi_7 > 0$. Hence, $N_7 > 0$ and D7-branes are required.

- $N_3 N_7 \neq 0$

  The solutions for y_0 depend on the ratio $\xi_7/\epsilon_2\xi_3$. A detailed analysis can be easily performed because the polynomial equation (5.20) is quadratic in y_0^2. We find that there are no real solutions in the interval $1/8 < \xi_7/\epsilon_2\xi_3 < (7 + 2\sqrt{10})/4$. On the other hand, when $0 < \xi_7/\epsilon_2\xi_3 < 1/8$, there is only one real positive solution for y_0^2 and it requires $N_3 > 0$ and $N_7 < 0$. For $\xi_7/\epsilon_2\xi_3 \leq 0$ there is only one acceptable root for y_0^2 and it leads to $N_3 > 0$ and $N_7 \geq 0$. A more interesting range of parameters is $\xi_7/\epsilon_2\xi_3 > (7 + 2\sqrt{10})/4$ because there are two allowed solutions for y_0^2 and for both it must be that $N_3 < 0$ and $N_7 > 0$. The upshot is that there can be metastable AdS vacua in the presence of D3 and D7-branes.

5.2.7 $P_2(U)$ with complex roots

The F-flatness conditions can be unfolded to obtain analytic expressions for the vevs of all moduli. However, for generic range of parameters, a higher order polynomial equation has to be solved to determine y_0 in the end. The main interesting feature is the appearance of multiple vacua even when $N_3 N_7 = 0$, i.e. when there are either no O7/D7 or no O3/D3 net charges present. We will first describe the overall picture and then present examples. For definiteness we always choose $y_0 > 0$ so that $|\Gamma| > 0$ is required to have $\operatorname{Im} U_0 > 0$ for the complex structure.

To obtain and examine the results it is useful to make some redefinitions. The idea is to leave as few free parameters as possible in the F-flatness equations. Since ϵ_1 is different from zero we can work with the ratio

$$\rho = \frac{\epsilon_2}{\epsilon_1}\,. \tag{5.23}$$

By virtue of the residual symmetry (5.17) there is an invariance under $\rho \to 1/\rho$. Therefore, we can restrict to the range $-1 \leq \rho \leq 1$, where the boundary corresponds to the fixed points of the inversion. Furthermore, as discussed at the end of section 3.1.1, the parameter ρ is either a rational number or involves at most square roots of rationals.

When $\xi_3 \neq 0$ it is also convenient to introduce new variables as

$$\mathcal{T} = \epsilon_1 \xi_3 \hat{\mathcal{T}} \quad ; \quad \mathcal{S} = \xi_3 \hat{\mathcal{S}} \quad ; \quad \xi_7 = \epsilon_1 \xi_3 (\rho^2 + 1)\eta\,. \tag{5.24}$$

The definition of the parameter η seems awkward but it simplifies the results. Notice that $\eta \to \eta\rho$ under (5.17). In the new variables the superpotential becomes

$$\mathcal{W} = |\Gamma|^{3/2}\epsilon_1 \xi_3 [3\hat{\mathcal{T}}\mathcal{Z}(\mathcal{Z}+1) + \hat{\mathcal{S}}(\mathcal{Z}^3 + \rho) + (1 - \rho\mathcal{Z}^3) + 3\eta(1 + \rho^2)\mathcal{Z}(1 - \mathcal{Z})]\,. \tag{5.25}$$

Since the F-flatness conditions are homogeneous in $\mathcal{W}$ the resulting equations will only depend on the parameters ρ and η. When $\xi_3 = 0$ we just make different field redefinitions, i.e. $\mathcal{T} = \epsilon_1 \xi_7 \hat{\mathcal{T}}$ and $\mathcal{S} = \xi_7 \hat{\mathcal{S}}$, so that the free parameters will be ρ and ξ_7/ϵ_1.

Manipulating the F-flatness conditions enables us to find the vevs $\mathcal{T}_0$ and $\mathcal{S}_0$ as functions of (x_0, y_0). The expressions are tractable but bulky so that we refrain from presenting them. The exception is the handy relation between the size and string coupling moduli

$$\mu_0 = \frac{\epsilon_1 \sigma_0 (3x_0^2 - y_0^2)}{1 + 2x_0}, \tag{5.26}$$

which is valid when $x_0 \neq -\frac{1}{2}$ and $y_0^2 \neq \frac{3}{4}$. There is a solution with $x_0 = -\frac{1}{2}$ and $y_0^2 = \frac{3}{4}$ but it has $\mu_0 = -\epsilon_1(1 + \rho)\sigma_0$, $\mu_0 = 3\xi_7 y_0$, and it requires $\eta = -(1 + \rho)/(\rho^2 - 7\rho + 1)$. There is another vacuum with $x_0 = -\frac{1}{2}$ that occurs when $\rho \to \infty$ ($\epsilon_1 = 0$) and was discussed in section 5.2.6. The case $x_0^2 = y_0^2$, which is better treated separately, requires $\xi_7 \neq 0$ unless $\rho = 0$.

The residual unknowns (x_0, y_0) are determined from the coupled system

$$y_0^4 + 2x_0(1 + x_0)y_0^2 - \rho(2x_0 + 1) + x_0^3(x_0 + 2) = 0, \tag{5.27}$$

$$y_0^6 \left(1 + 2\eta x_0 - 2\eta\right) + \left(1 + 30\eta x_0{}^3 - x_0{}^2 + 18\eta x_0{}^2 - 6\rho\eta\right) y_0^4$$
$$+ x_0 \left(54\eta x_0{}^4 + 11x_0{}^3 + 42\eta x_0{}^3 + 8x_0{}^2 + 12\rho\eta x_0 - 4x_0 - 6\rho\eta\right) y_0^2 \tag{5.28}$$
$$+ \left(2\rho + 4\rho x_0 + 11x_0{}^3 + 13x_0{}^4\right) \left(2\rho\eta + 2\eta x_0{}^3 + x_0{}^2 + x_0\right) = 0 .$$

The corresponding equations when $\xi_3 = 0$ can be obtained taking the limit $\eta \to \infty$. Eliminating y_0 for generic parameters gives a ninth-order polynomial equation for x_0.

For some range of parameters the above equations can admit several solutions for $\mathcal{Z}_0 = x_0 + iy_0$, which in turn yield consistent values for the remaining moduli. The existence of multiple vacua is most easily detected in the limiting cases in which one of the net tadpoles N_7 or N_3 vanishes, equivalently when $\xi_7 = 0$ ($\eta = 0$) or $\xi_3 = 0$ ($\eta \to \infty$). In either limit the NSNS parameter ρ can still be adjusted. We expect the results to be invariant under $\rho \to 1/\rho$ and this is indeed what happens.

We have mostly looked at models having no O7/D7 net charge, namely with $\eta = 0$. It turns out that the solutions require $\xi_3 > 0$ so that $N_3 > 0$ and O3-planes must be present. Below we list the main results.

1. For $\rho = 1$ there are no minima with moduli stabilized.

2. For $\rho = -1$ there is only one distinct vacuum with data

$$\mathcal{Z}_0 = -0.876 + 1.158\,i \; ; \quad \mathcal{S}_0 = \xi_3(-0.381 + 0.238\,i) \; ; \quad \mathcal{T}_0 = \epsilon_1 \xi_3(0.602 - 0.305\,i) \; ; \quad V_0 = \frac{2.38\,|\Gamma|^3}{\xi_3^2 \epsilon_1} \; . \tag{5.29}$$

Notice that necessarily $\xi_3 > 0$ and $\epsilon_1 < 0$. Actually, for $\rho = -1$, there is a second consistent solution but it is related to the above by the residual symmetry (5.17).

3. There can be only one solution when $\rho_c \leq \rho < 1$, where $\rho_c = -0.7267361874$. The critical value ρ_c is such that the discriminant of the polynomial equation that

$\mathcal{Z}_0$	$\mathcal{S}_0/\xi_3$	$\mathcal{T}_0/\xi_3\epsilon_1$	$V_0\,\xi_3^2\,\epsilon_1/|\Gamma|^3$
$-0.91105442 + 1.14050441\,i$	$-0.26002362 + 0.19059447\,i$	$0.53128071 - 0.27572497\,i$	3.353
$-0.43550654 + 0.73478523\,i$	$0.28605555 + 0.55017649\,i$	$0.60410811 + 0.12407321\,i$	-2.168
$-0.40368586 + 0.57866160\,i$	$0.49215445 + 0.33255331\,i$	$0.57101568 + 0.26593032\,i$	-1.880

Table 6: Degenerate vacua for $\xi_7 = 0$ and $\rho = -\frac{4}{5}$.

$\mathcal{Z}_0$	$\epsilon_1\mathcal{S}_0/\xi_7$	$\mathcal{T}_0/\xi_7$	$V_0\,\xi_7^2/\epsilon_1|\Gamma|^3$
$-0.88312113 + 0.74580943\,i$	$-6.1818994 - 1.6867660\,i$	$-4.20643209 + 3.92605399\,i$	0.026
$0.20646056 + 0.89488895\,i$	$0.03039439 - 2.49813344\,i$	$-0.06455485 + 1.18981502\,i$	0.084

Table 7: Vacua for $\xi_3 = 0$ and $\rho = \frac{3}{4}$.

determines x_0 is zero. Consistency requires $\epsilon_1 < 0$ and $\xi_3 > 0$ so that O3-planes are needed. For instance, when $\rho = 0$ the solution is exact and has

$$\mathcal{Z}_0 = -1 + i \quad ; \quad \mathcal{S}_0 = \frac{\xi_3}{8}(4 + i) \quad ; \quad \mathcal{T}_0 = \frac{\epsilon_1\xi_3}{4}(2 - i) \quad ; \quad V_0 = \frac{6\,|\Gamma|^3}{\xi_3^2\epsilon_1} \ . \qquad (5.30)$$

As expected, upon the transformation (5.17) this vacuum coincides with that having $\xi_7 = 0$ and $\epsilon_1 = 0$, given in (5.21). For other values of ρ the solution is numerical. For example, taking $\rho = \frac{1}{2}$ leads to the vevs

$$\mathcal{Z}_0 = -1.036 + 0.834\,i \ ; \quad \mathcal{S}_0 = \xi_3(1.561 + 0.192) \ ; \quad \mathcal{T}_0 = \xi_3\epsilon_1(1.055 - 0.453\,i) \ ; \quad V_0 = \frac{2.283|\Gamma|^3}{\xi_3^2\,\epsilon_1} \ . \tag{5.31}$$

4. The important upshot is that in the interval $-1 < \rho < \rho_c$ there can be two distinct solutions for the same set of fluxes. An example with $\rho = -\frac{4}{5}$ is shown in table 6. Notice that the last two solutions can exist for $\xi_3 > 0$ and $\epsilon_1 > 0$. The first solution can also occur but for $\xi_3 > 0$ and $\epsilon_1 < 0$.

For models having no O3/D3 net charge a detailed analysis is clearly feasible but we have only sampled narrow ranges of the adjustable parameter ρ. Consistent solutions must have $\epsilon_1 < 0$ and $\xi_7 > 0$. Hence, $N_7 > 0$ and D7-branes must be included. There are values of ρ, e.g. $\rho = -1$, for which there are no vacua with stabilized moduli. For $\rho = 1$ there is only one minimum which can be computed exactly. More interestingly, models of this type can also exhibit multiple vacua. In table 7 we show one example with $\rho = \frac{3}{4}$. Observe that both solutions exist for $\epsilon_1 < 0$ and $\xi_7 > 0$.

6. Aspects of the non-geometric landscape

In this section we discuss the main aspects of the AdS_4 vacua in our models that are standard examples of type IIB toroidal orientifolds with O3/O7-planes. Besides the axiodilaton S, after an isotropic Ansatz the massless scalars reduce to the overall complex structure U and the size modulus T. Fluxes of the RR and NSNS 3-forms generate a potential that

gives masses only to S and U. The new ingredient here are non-geometric Q-fluxes, that are required to restore T-duality between type IIA and type IIB, and that induce a super-potential for the Kähler field T. The various fluxes must satisfy certain constraints arising from Jacobi or Bianchi identities. The problem is then to minimize the scalar potential while solving the constraints. The question is whether there are solutions with all moduli stabilized. We have seen that the answer is affirmative and now we intend to analyze it in more detail.

It is instructive to begin by recounting the findings of the previous sections. The initial step is to classify the subalgebras whose structure constants are the Q's. With the isotropic Ansatz there are only five classes. For each type, the non-geometric fluxes can be written in terms of four auxiliary parameters $\left(\begin{smallmatrix} \alpha & \beta \\ \gamma & \delta \end{smallmatrix}\right) = \Gamma$, in such a way that the Jacobi identities are automatically satisfied. Other fluxes can also be parametrized using Γ plus additional vari-ables: (ϵ_1, ϵ_2) for NSNS, and $(\xi_3, \xi_7, \xi_s, \xi_t)$ or $(\lambda_1, \lambda_2, \lambda_3, \xi_s, \xi_t)$ for RR. The significance of Γ is that it defines a transformed complex structure $\mathcal{Z} = (\alpha U + \beta)/(\gamma U + \delta)$ that is invariant under the modular group $\mathrm{SL}(2, \mathbb{Z})_U$. The effective action can be expressed in terms of $\mathcal{Z}$ according to the Q-subalgebra. Once the subalgebra is chosen the vacua will depend only on the variables Γ, (ϵ_1, ϵ_2), and (ξ_3, ξ_7) or $(\lambda_1, \lambda_2, \lambda_3)$, that in turn determine the values of the cosmological constant and the string coupling (V_0, g_s), as well as the net tadpoles (N_3, N_7). In many examples, the vevs of the moduli can be determined in closed form.

Our approach to analyze the vacua in presence of non-geometric fluxes has the great advantage that the degeneracy due to modular transformations of the complex structure is already taken into account. Inequivalent vacua are just labelled by the vevs $(\mathcal{Z}_0, S_0, T_0)$ that are modular invariant. In practice this means that we can study families of modu-lar invariant vacua by choosing a particular structure for Γ. In section 6.2 we will give concrete examples.

There is an additional vacuum degeneracy because the characteristic data (V_0, g_s) hap-pen to be independent of the parameters (ξ_s, ξ_t). The explanation is that they correspond to shifts of the axions $\mathrm{Re}\, S$ and $\mathrm{Re}\, T$ which can be reabsorbed in the RR fluxes. The flux-induced RR tadpoles (N_3, N_7) are blind to (ξ_s, ξ_t) as well. Apparently, generic shifts in $\mathrm{Re}\, S$ and $\mathrm{Re}\, T$ are not symmetries of the compactification, so that two vacua differing only in the RR flux parameters (ξ_s, ξ_t) would be truly distinct. We argue below that the vacua are equivalent because the full background is symmetric under $S \to S - \xi_s$, and $T \to T - \xi_t$.

In absence of non-geometric fluxes the 3-form RR field strength that appears in the 10-dimensional action is given by $F_3 = dC_2 - H_3 \wedge C_0 + \bar{F}_3$, where $H_3 = dB_2 + \bar{H}_3$. The natural generalization to include non-geometric fluxes is

$$F_3 = dC_2 - H_3 \wedge C_0 + QC_4 + \bar{F}_3 \,, \tag{6.1}$$

where QC_4 is a 3-form that we can extract from (2.15) because $\mathrm{Re}\, \mathcal{J} = C_4$. In fact, $C_4 = -\mathrm{Re}\, T \sum_I \tilde{\omega}^I$, where $\tilde{\omega}^I$ are the basis 4-forms. Recall also that $C_0 = \mathrm{Re}\, S$. Notice then that F_3 involves the axions in question. The relevant result is that F_3 is invariant[4]

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[4]We thank P. Cámara for giving us this hint.

under the shifts $S \to S - \xi_s$, and $T \to T - \xi_t$. To show this we first compute the variation of $\bar{F}_3$ using the universal terms (A.1) in the parametrization of the RR fluxes and then substitute in (6.1). In the effective $D=4$ action the result is simply that the superpotential is invariant under these axionic shifts and the corresponding transformation of the RR fluxes. In turn this follows from (2.26) after substituting (A.1).

6.1 Overview

We now describe in order some prominent features of the AdS_4 vacua with non-geometric Q-fluxes switched on.

1. The explicit results of section 5.2 indicate that in all models the vevs $\sigma_0 = \operatorname{Im} S_0$ and $\mu_0 = \operatorname{Im} T_0$ are correlated. This generic property follows from the F-flatness conditions simply because the superpotential is linear in the axiodilaton and the Kähler modulus. Recall that the vevs in question determine physically important quantities, namely the string coupling $g_s = 1/\sigma_0$, and the overall internal volume $V_{\text{int}} = (\mu_0/\sigma_0)^{3/2}$. To trust the perturbative string approximation g_s must be small and we will shortly explain, as already shown in [12], that generically there are regions in flux space in which both g_s and the cosmological constant are small, while V_{int} is large. We stress again the caveat that even at large overall volume there could still exist light winding string states when non-geometric fluxes are in play. These effects are certainly important in trying to lift the solutions to full string vacua. In this paper we only claim to have found vacua of the effective field theory with a precise set of massless fields and interactions due to generalized fluxes.

2. Another common feature of all models is the relation between moduli vevs and net RR charges. In type IIB toroidal orientifolds it is known that in Minkowski supersymmetric vacua the contribution of RR and NSNS fluxes to the C_4 tadpole is positive ($N_3 > 0$) and this occurs if and only if $\operatorname{Im} S_0 > 0$ [32]. The interpretation is that to cancel the tadpole due to $\bar{F}_3$ and $\bar{H}_3$ it is mandatory to include O3-planes, whereas D3-branes can be added only as long as N_3 stays positive. This is also true for no-scale Minkowski vacua in which supersymmetry is broken by the F-term of the Kähler field. Turning on non-geometric fluxes enables to stabilize all moduli at a supersymmetric AdS_4 minimum. At the same time, the Q-fluxes induce a C_8 tadpole of magnitude N_7 that can be cancelled by adding O7-planes and/or D7-branes. We find in general that the vevs $\operatorname{Im} S_0$ and $\operatorname{Im} T_0$, that must be positive, are correlated to the tadpoles (N_3, N_7). According to the Q-subalgebra there are several possibilities for the type of sources that have to be included. For example, the models considered in [12], having $N_3 > 0$ and $N_7 = 0$, proceed only with the fluxes of the compact $\mathfrak{su}(2)^2$.

   For the Q-fluxes of the nilalgebra, and the semidirect sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$, there is a relation $N_3 = -\epsilon_2 N_7$, with $\epsilon_2 > 0$. Only in the latter case it is allowed to have $N_3 = N_7 = 0$, and the sources can be avoided altogether. For the fluxes of $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ it turns out that orientifold planes are unnecessary to cancel tadpoles, but both D3 and D7-branes must be added ($N_3 < 0$, $N_7 > 0$).

The fluxes of the semisimple subalgebras are more flexible. In particular, it can happen that one flux-tadpole vanishes while the other must have a definite sign. Moreover, the sign is opposite for the compact and non-compact cases. For instance, when $N_7 = 0$, $N_3 > 0$ and O3-planes are obligatory for the $\mathfrak{su}(2)^2$ fluxes, while for $\mathfrak{so}(3,1)$ $N_3 < 0$ and D3-branes are required.

The magnitudes of the vevs are also proportional to the net tadpoles. This then implies that the string coupling typically decreases when N_3 and/or N_7 increase. However, the number of D-branes cannot be increased arbitrarily without taking into account their backreaction.

3. Consistency of the vacua can in fact be related to the full 12-dimensional algebra in which the $\bar{H}$ and Q-fluxes are the structure constants. The reason is that the conditions $\mathrm{Im}\,S_0 > 0$ and $\mathrm{Im}\,T_0 > 0$ also impose restrictions on the signs of the NSNS parameters (ϵ_1, ϵ_2). For instance, in section 5.2.5 we have seen that for Q-fluxes of the compact $\mathfrak{so}(4) \sim \mathfrak{su}(2)^2$, the solutions with $\epsilon_1 = 0$ require $\epsilon_2 < 0$. This in turn implies, as explained in section 3.1.1, that the full gauge algebra is $\mathfrak{so}(4) + \mathfrak{iso}(3)$. Another simple example is the model based on the $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ Q-subalgebra. The vacua of 5.2.3 with $\epsilon_2 = 0$ require $\epsilon_1 < 0$ and it can then be shown that the full gauge algebra is $\mathfrak{so}(4) + \mathfrak{u}(1)^6$. A more detailed study of the 12-dimensional algebras is left for future work [38].

4. We defer to section 6.2 a more thorough discussion of the landscape of values attained by the string coupling g_s and the cosmological constant V_0, for the fluxes of the compact $\mathfrak{su}(2)^2$ Q-subalgebra. The situation for $\mathfrak{so}(3,1)$ is similar and can be analyzed using the results of section 5.2.4. The model based on the direct product $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ is different because both N_3 and N_7 must be non-zero, but it can still be shown that there exist vacua with small g_s and V_0. The models built using the nilpotent and semidirect Q-subalgebras have been studied in their T-dual IIA formulation in refs. [5, 6], where it was found that there are infinite families of vacua within the perturbative region.

5. A peculiar result is the appearance of multiple vacua for certain combination of fluxes. These events occur only in models based on the semisimple Q-subalgebras. They can have $N_3 N_7 = 0$ or $N_3 N_7 \neq 0$, but in the former case both NSNS parameters (ϵ_1, ϵ_2) must be non-zero. Reaching small string coupling and cosmological constant typically requires that N_3 and/or N_7 be sufficiently large.

6. To cancel RR tadpoles it might be necessary to add stacks of D3 and/or D7-branes. These additional D-branes could also generate a charged chiral spectrum but more generally a different sector of D-branes will serve this purpose. In any case, the D-branes that can be included are constrained by cancellation of Freed-Witten anomalies [6, 20]. In absence of non-geometric fluxes the condition amounts to the vanishing of $\bar{H}_3$ when integrated over any internal 3-cycle wrapped by the D-branes. For unmagnetized D7-branes in $\mathrm{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$, with $\bar{H}_3$ given in (2.9), it is easy to see that the

condition is met, whereas for D3-branes it is trivial. When Q-fluxes are switched on the modified condition [20] is still satisfied basically because the 3-form $Q\mathcal{J}$, defined in (2.15), can be expanded in the same basis as $\bar{H}_3$.

D3-branes and unmagnetized D7-branes in $\mathrm{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ do not give rise to charged chiral matter. Therefore the models will not have U(1) chiral anomalies. This is consistent with the fact that the axions $\mathrm{Re}\,S$ and $\mathrm{Re}\,T$ are generically stabilized by the fluxes and having acquired a mass they could not participate in the Green-Schwarz mechanism to cancel the chiral anomalies.[5]

To construct a more phenomenologically viable scenario one could introduce magnetized D9-branes as in the $\mathrm{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ type IIB orientifolds with NSNS and RR fluxes that were considered some time ago [37]. Now, care has to be taken because magnetized D9-branes suffer from Freed-Witten anomalies. They are actually forbidden in absence of non-geometric fluxes when $\bar{H}_3 \neq 0$.

The effect of the Q-fluxes can be studied as explained in [20]. Cancellation of Freed-Witten anomalies translates into invariance of the superpotential under shifts $S \to S + q_s\nu$ and $T \to T + q_t\nu$, where the real charges (q_s, q_t) depend on the U(1) gauged by the D-brane. Applying this prescription we conclude that in our setup with isotropic fluxes magnetized D9-branes could be introduced only in models based on the nilpotent and semidirect sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$ Q-subalgebras. The reason is that only in these cases the flux-induced polynomials $P_2(U)$ and $P_3(U)$ can be chosen parallel and then W can remain invariant under the axionic shifts. Equivalently, only in these cases the axions $\mathrm{Re}\,S$ and $\mathrm{Re}\,T$ are not fully determined and the residual massless linear combination can give mass to an anomalous U(1). For other Q-subalgebras the polynomials $P_2(U)$ and $P_3(U)$ are linearly independent and both axions are completely stabilized.

It would be interesting to study the consistency conditions on magnetized D9-branes in models with non-isotropic fluxes. In principle there could exist configuration of fluxes such that the general superpotential (2.17), (2.20) is invariant under axionic shifts of S and the Kähler moduli T_I.

6.2 Families of modular invariant vacua

To generate specific families of vacua we first choose the Q-subalgebra and then select the parameters in Γ. In general Γ can be chosen so that the non-geometric fluxes are even integers. The NSNS fluxes turn out to be even integers by picking (ϵ_1, ϵ_2) appropriately. One can also start from given non-geometric and NSNS even integer fluxes and deduce the corresponding Γ and (ϵ_1, ϵ_2). Similar remarks apply to the RR fluxes. We will illustrate the procedure for the compact $\mathfrak{su}(2)^2$.

If one of the parameters vanishes, say $\gamma = 0$, it can be shown from (3.11) that the ratios δ/α and β/α are rational numbers (recall that $|\Gamma| \neq 0$ so that $\alpha, \delta \neq 0$). It then

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[5]We thank L. Ibáñez for discussions on this point.

follows that by a modular transformation, c.f. (4.6), we can go to a canonical gauge in which also $\beta = 0$.

The canonical diagonal gauge $\gamma = \beta = 0$ is completely generic when $\epsilon_2 = 0$ ($\epsilon_1 \neq 0$). In this case we find that β/α and γ/δ are rational because they are given respectively by quotients of NSNS and non-geometric fluxes. Therefore, β and γ can be gauged away by modular transformations. If instead $\epsilon_1 = 0$, but $\epsilon_2 \neq 0$, we can take $\alpha = \delta = 0$.

When $\epsilon_1 \epsilon_2 \neq 0$ we can still use the canonical gauge but it will not give the most general results that are obtained simply by considering $\alpha, \beta, \gamma, \delta \neq 0$.

6.2.1 Canonical families for $\mathfrak{su}(2)^2$ fluxes

For each subalgebra we can obtain families of vacua starting from the canonical gauge defined by $\gamma = \beta = 0$. In the $\mathfrak{su}(2)^2$ case only the non-geometric fluxes $\tilde{c}_1$ and $\tilde{c}_2$ are different from zero and can be written as

$$\tilde{c}_1 = -2m \quad ; \quad \tilde{c}_2 = 2n \quad ; \quad m, n \in \mathbb{Z} . \tag{6.2}$$

From (3.11) we easily find $\alpha/\delta = n/m$, $\delta^3 = 2m^2/n$, so that $|\Gamma|^3 = 4nm$. The non-zero NSNS and RR fluxes are easily found to be

$$b_0 = -\frac{2m^2}{n}\epsilon_2 \quad ; \quad b_3 = \frac{2n^2}{m}\epsilon_1 \quad ; \quad a_0 = \frac{2m^2}{n}(\epsilon_1\xi_3 + \epsilon_2\xi_s), \tag{6.3}$$

$$a_1 = -2m(\xi_t + \xi_7) \quad ; \quad a_2 = 2n(\xi_t - \xi_7) \quad ; \quad a_3 = -\frac{2n^2}{m}(\epsilon_1\xi_s - \epsilon_2\xi_3) .$$

Since the b's and a's are (even) integers, it is obvious that (ϵ_1, ϵ_2) and $(\xi_3, \xi_7, \xi_s, \xi_t)$ are all rational numbers.

The moduli vevs depend on (ξ_3, ξ_7) and (ϵ_1, ϵ_2). For concreteness, and to compare with the results of [12], we focus on the case $\xi_7 = 0$. Other cases can be studied using the results of section 5.2.5. When $\xi_7 = 0$ the RR fluxes a_1 and a_2 are spurious, they can be eliminated by setting $\xi_t = 0$, i.e. by a shift in $\mathrm{Re}\,T$.

To continue we have to distinguish whether one of the NSNS parameters ϵ_1 or ϵ_2 is zero. Recall that in this case the flux induced polynomial P_2 does not have complex roots.

- $\epsilon_1\epsilon_2 = 0$

  Let us consider $\epsilon_2 = 0$. Then, also a_3, or ξ_s, is irrelevant and can be set to zero by a shift in $\mathrm{Re}\,S$. The important physical parameters are ϵ_1 and ξ_3, they can be deduced from b_3 and a_0. Notice also that at this point $N_3 = a_0 b_3$. Using (5.30) we obtain the values of the cosmological constant and the string coupling

  $$V_0 = \frac{48\,m^6 b_3^3}{n^3 N_3^2} \quad ; \quad g_s = \frac{8\,m^3 b_3^2}{n^3 N_3} . \tag{6.4}$$

  Consistency requires $\epsilon_1 < 0$ and $\xi_3 > 0$, or equivalently $V_0 < 0$ and $g_s > 0$. For the purpose of counting distinct vacua we can safely assume $b_3 > 0$ and then $m, n < 0$.

  As noticed in [12], the important outcome is that g_s and V_0 can be made arbitrarily small by keeping b_3 and m fixed while letting $n \to \infty$.

In our approach it is also easy to see that (V_0, g_s) always take values of the form (6.4) whenever P_2 has only real roots. This follows because all vacua are related by modular transformations plus axionic shifts. However, if as in [12] we want to count the vacua with fluxes bounded by an upper limit L, it does not suffice to just consider the canonical gauge. The reason is that by performing modular transformations and axionic shifts we can reach larger effective values of b_3 that seem to violate the tadpole condition. Rather than an elaborate argument we will just provide a simple example. We can go to a non-canonical gauge with $\gamma = 0$ but $\beta \neq 0$ and also take $\xi_t = 0$ but $\xi_s \neq 0$. With these choices it is straightforward to show that $N_3 = a_0 b_3 - a_3 b_0$, which would allow to take e.g. $b_3 = N_3$ that is forbidden when $b_0 = 0$ ($\beta = 0$), or $a_3 = 0$ ($\xi_s = 0$), because a_0 must be even. To do detailed vacua statistics it is necessary to use generic gauge and axionic shifts.

- $\epsilon_1 \epsilon_2 \neq 0$

  As in section 5.2.7 we set $\epsilon_2 = \rho \epsilon_1$. In the canonical gauge the parameter ρ is a rational number that we assume to be given. We choose to vary the NSNS flux b_3 that determines

  $$\epsilon_1 = \frac{m b_3}{2n^2} \quad ; \quad b_0 = -\frac{\rho \, m^3 b_3}{n^3} \,, \tag{6.5}$$

  where m, n are the integers coming from the non-geometric fluxes. The vacuum data have been found to be

  $$V_0 = \frac{4 F_V \, nm}{\epsilon_1 \xi_3^2} \qquad ; \qquad g_s = \frac{1}{F_g \xi_3} \,, \tag{6.6}$$

  where we used $|\Gamma|^3 = 4nm$. The numerical factors F_V and F_g depend on ρ. For instance, for $\rho = 0$, $F_V = 6$ and $F_g = 1/8$. Other examples are given in section 5.2.7. We remark that for ρ in a particular range there can be multiple vacua, meaning that for some ρ the above numerical factors might take different values (e.g. table 6).

  It is most convenient to extract ξ_3 from the tadpole relation $N_3 = 4mn\epsilon_1^2(1 + \rho^2)\xi_3$, which in terms of the integer fluxes reads $N_3 = a_0 b_3 - a_3 b_0$. Combining all the information we readily find

  $$V_0 = \frac{8 F_V \, m^6 b_3^3 (1 + \rho^2)^2}{n^3 N_3^2} \qquad ; \qquad g_s = \frac{m^3 b_3^2 (1 + \rho^2)}{F_g \, n^3 N_3} \,. \tag{6.7}$$

  Unlike the case when $\rho = 0$, in general we cannot keep m and b_3 fixed while letting $n \to \infty$. The reason is that the NSNS flux b_0 in (6.5) must be an integer.

  The main conclusion is that it is not always possible to obtain small string coupling and cosmological constant. In fact, when $\rho \neq 0$, there are no vacua with $g_s < 1$ unless the tadpole N_3 is sufficiently big. To prove this, notice first that the string coupling can be rewritten as $g_s = -b_3 b_0 (1 + \rho^2)/(F_s \rho N_3)$. The most favorable situation occurs when $\rho = -1$ for which $F_s = 0.238$. The smallest allowed NSNS fluxes are $b_0 = b_3 = 2$ (compatible with $\rho = -1$). Hence, the minimum value of the coupling is $g_s^{\min} = 8/(F_s N_3)$ and $g_s^{\min} < 1$ would require $N_3 > 33$. The situation is worse for

values of ρ such that multiple vacua can appear. The problem is that since such ρ's are rational, b_3 must be largish for b_0 to be integer. Going to a more general gauge does not change the conclusion.

We have just provided a quantitative, almost analytic, explanation of why there are no perturbative vacua when the flux polynomial P_2 has complex roots and N_3 is not large enough. This observation was first made in [12] based on a purely numerical analysis.

7. Final remarks

In this paper we have investigated supersymmetric flux vacua in a type IIB orientifold with RR, NSNS and non-geometric Q-fluxes turned on. We enlarged the related analysis of [12] by considering the most general fluxes solving the Jacobi identities, and by including variable numbers of O3/D3 and O7/D7 sources to cancel the flux-induced RR tadpoles.

Our approach is based on the classification of the subalgebras satisfied by the non-geometric fluxes. A convenient parametrization of the Q-fluxes leads to an auxiliary complex structure that turns out to be invariant under modular transformations. Writing the superpotential in terms of this invariant field simplifies solving the F-flatness conditions and enables us to obtain analytic expressions for the moduli vevs. We have found families of supersymmetric AdS$_4$ vacua in all models defined by the inequivalent Q-subalgebras. General properties of the solutions were discussed in section 6. The vacua typically exist in all cases, provided that arbitrary values of the flux-induced RR tadpoles are allowed.

In type IIB orientifolds with only RR and NSNS fluxes there is a non-trivial induced tadpole that must be cancelled by O3-planes or wrapped D7-branes. But including non-geometric fluxes can require other types of sources. For instance, similar to well understood AdS$_4$ models in type IIA, the induced flux-tadpoles might vanish implying that sources can be avoided. There are also examples in which sources of positive RR charge are sufficient to cancel the tadpoles. As one might expect, these latter exotic vacua occur in models built using Q-fluxes satisfying the non-compact $\mathfrak{so}(3,1)$ subalgebra. Such solutions might be ruled out once a deeper understanding of non-geometric fluxes has been developed.

We discussed a simplified set of fluxes but our methods could be used to study other configurations. The starting point would be the classification of the Q-subalgebras consistent with the underlying symmetries.

Although our main goal was to explore supersymmetric vacua with moduli stabilized, our results could have further applications. We have succeeded in connecting properties of the vacua to the underlying gauge algebra and this can help towards extending the description of non-geometric fluxes beyond the effective action limit. At present one of the most challenging problems in need of new insights is precisely to formulate string theory on general backgrounds at the microscopic level.

Acknowledgments

A. Parametrized RR fluxes

In this appendix we give the explicit expressions for the original RR fluxes a_A in terms of the axionic shifts (ξ_s, ξ_t) and the tadpole parameters (ξ_3, ξ_7) or (λ_2, λ_3), depending on the Q-subalgebra. For the semidirect sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$ and the nilpotent algebra there is another auxiliary variable λ_1 as explained in 4.1. In all cases there is a non-singular rotation matrix from the a_A's to the new variables.

In principle the ξ's and λ's are just real constants but the resulting a_A fluxes must be integers. The exact nature of these parameters can be elucidated starting with the non-geometric fluxes of each subalgebra. For example, following the discussion at the end of section 3.1.1, for $\mathfrak{su}(2)^2$ when $\epsilon_1\epsilon_2 = 0$ it transpires that $(\xi_3, \xi_7, \xi_s, \xi_t) \in \mathbb{Q}$.

There is a universal structure in the RR fluxes that is worth noticing. For all Q-subalgebras the dependence on the axionic shift parameters (ξ_s, ξ_t) is of the form

$$
\begin{aligned}
a_0 &= -b_0\xi_s + 3c_0\xi_t + \cdots \\
a_1 &= -b_1\xi_s - (2c_1 - \tilde{c}_1)\xi_t + \cdots \\
a_2 &= -b_2\xi_s - (2c_2 - \tilde{c}_2)\xi_t + \cdots \\
a_3 &= -b_3\xi_s + 3c_3\xi_t + \cdots
\end{aligned}
\tag{A.1}
$$

where $\cdots$ stands for extra terms depending on the tadpole parameters.

A.1 Compact $\mathfrak{su}(2)^2$ background

$a_0 = \delta^3(\epsilon_1\xi_3 + \epsilon_2\xi_s) + \beta^3(\epsilon_1\xi_s - \epsilon_2\xi_3) + 3\delta\beta^2(\xi_t - \xi_7) + 3\beta\delta^2(\xi_t + \xi_7)$

$a_1 = -\gamma\delta^2(\epsilon_1\xi_3 + \epsilon_2\xi_s) - \alpha\beta^2(\epsilon_1\xi_s - \epsilon_2\xi_3) - \beta(\beta\gamma + 2\alpha\delta)(\xi_t - \xi_7) - \delta(\alpha\delta + 2\beta\gamma)(\xi_t + \xi_7)$

$a_2 = \delta\gamma^2(\epsilon_1\xi_3 + \epsilon_2\xi_s) + \beta\alpha^2(\epsilon_1\xi_s - \epsilon_2\xi_3) + \alpha(\alpha\delta + 2\beta\gamma)(\xi_t - \xi_7) + \gamma(\beta\gamma + 2\alpha\delta)(\xi_t + \xi_7)$

$a_3 = -\gamma^3(\epsilon_1\xi_3 + \epsilon_2\xi_s) - \alpha^3(\epsilon_1\xi_s - \epsilon_2\xi_3) - 3\gamma\alpha^2(\xi_t - \xi_7) - 3\alpha\gamma^2(\xi_t + \xi_7)$

A.2 Non-compact $\mathfrak{so}(3,1)$ background

$a_0 = \delta(\delta^2 - 3\beta^2)(\epsilon_1\xi_3 + \epsilon_2\xi_s) + \beta(\beta^2 - 3\delta^2)(\epsilon_1\xi_s - \epsilon_2\xi_3) - 3(\beta^2 + \delta^2)(\beta\xi_t - \delta\xi_7)$

$a_1 = (\gamma\beta^2 + 2\alpha\beta\delta - \gamma\delta^2)(\epsilon_1\xi_3 + \epsilon_2\xi_s) + (\alpha\delta^2 + 2\beta\gamma\delta - \alpha\beta^2)(\epsilon_1\xi_s - \epsilon_2\xi_3)$
$\qquad + (\beta^2 + \delta^2)(\alpha\xi_t - \gamma\xi_7) + 2(\alpha\beta + \gamma\delta)(\beta\xi_t - \delta\xi_7)$

$a_2 = (\delta\gamma^2 - 2\alpha\beta\gamma - \delta\alpha^2)(\epsilon_1\xi_3 + \epsilon_2\xi_s) + (\beta\alpha^2 - 2\alpha\gamma\delta - \beta\gamma^2)(\epsilon_1\xi_s - \epsilon_2\xi_3)$
$\qquad - 2(\alpha\beta + \gamma\delta)(\alpha\xi_t - \gamma\xi_7) - (\alpha^2 + \gamma^2)(\beta\xi_t - \delta\xi_7)$

$a_3 = -\gamma(\gamma^2 - 3\alpha^2)(\epsilon_1\xi_3 + \epsilon_2\xi_s) - \alpha(\alpha^2 - 3\gamma^2)(\epsilon_1\xi_s - \epsilon_2\xi_3) + 3(\alpha^2 + \gamma^2)(\alpha\xi_t - \gamma\xi_7)$

A.3 Direct sum $\mathfrak{su}(2) + \mathfrak{u}(1)^3$ background

$$a_0 = \delta^3(\epsilon_1\xi_3 + \epsilon_2\xi_s) + \beta^3(\epsilon_1\xi_s - \epsilon_2\xi_3) + 3\beta\delta^2\xi_t - 3\delta\beta^2\xi_7$$
$$a_1 = -\gamma\delta^2(\epsilon_1\xi_3 + \epsilon_2\xi_s) - \alpha\beta^2(\epsilon_1\xi_s - \epsilon_2\xi_3) - \delta(\alpha\delta + 2\beta\gamma)\xi_t + \beta(\beta\gamma + 2\alpha\delta)\xi_7$$
$$a_2 = \delta\gamma^2(\epsilon_1\xi_3 + \epsilon_2\xi_s) + \beta\alpha^2(\epsilon_1\xi_s - \epsilon_2\xi_3) + \gamma(\beta\gamma + 2\alpha\delta)\xi_t - \alpha(\alpha\delta + 2\beta\gamma)\xi_7$$
$$a_3 = -\gamma^3(\epsilon_1\xi_3 + \epsilon_2\xi_s) - \alpha^3(\epsilon_1\xi_s - \epsilon_2\xi_3) - 3\alpha\gamma^2\xi_t + 3\gamma\alpha^2\xi_7$$

A.4 Semidirect sum $\mathfrak{su}(2) \oplus \mathfrak{u}(1)^3$ background

$$a_0 = \delta^3(\epsilon_2\xi_s + 3\xi_t) + \beta\delta^2(\epsilon_1\xi_s - 3\xi_t + 3\lambda_1) + 3\delta\beta^2\lambda_2 + \beta^3\lambda_3$$
$$a_1 = -\gamma\delta^2(\epsilon_2\xi_s + 3\xi_t) - \tfrac{1}{3}\delta(\alpha\delta + 2\beta\gamma)(\epsilon_1\xi_s - 3\xi_t + 3\lambda_1) - \beta(\beta\gamma + 2\alpha\delta)\lambda_2 - \alpha\beta^2\lambda_3$$
$$a_2 = \delta\gamma^2(\epsilon_2\xi_s + 3\xi_t) + \tfrac{1}{3}\gamma(\beta\gamma + 2\alpha\delta)(\epsilon_1\xi_s - 3\xi_t + 3\lambda_1) + \alpha(\alpha\delta + 2\beta\gamma)\lambda_2 + \beta\alpha^2\lambda_3$$
$$a_3 = -\gamma^3(\epsilon_2\xi_s + 3\xi_t) - \alpha\gamma^2(\epsilon_1\xi_s - 3\xi_t + 3\lambda_1) - 3\gamma\alpha^2\lambda_2 - \alpha^3\lambda_3$$

A.5 Nilpotent $\mathfrak{nil}$ background

$$a_0 = \delta^3(\epsilon_2\xi_s + 3\xi_t) + \gamma\delta^2(\epsilon_1\xi_s + 3\lambda_1) + 3\delta\gamma^2\lambda_2 + \gamma^3\lambda_3$$
$$a_1 = -\gamma\delta^2(\epsilon_2\xi_s + 3\xi_t) + \tfrac{1}{3}\delta(\delta^2 - 2\gamma^2)(\epsilon_1\xi_s + 3\lambda_1) - \gamma(\gamma^2 - 2\delta^2)\lambda_2 + \delta\gamma^2\lambda_3$$
$$a_2 = \delta\gamma^2(\epsilon_2\xi_s + 3\xi_t) + \tfrac{1}{3}\gamma(\gamma^2 - 2\delta^2)(\epsilon_1\xi_s + 3\lambda_1) + \delta(\delta^2 - 2\gamma^2)\lambda_2 + \gamma\delta^2\lambda_3$$
$$a_3 = -\gamma^3(\epsilon_2\xi_s + 3\xi_t) + \delta\gamma^2(\epsilon_1\xi_s + 3\lambda_1) - 3\gamma\delta^2\lambda_2 + \delta^3\lambda_3$$

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