

Deterministic spin-wave interferometer based on Rydberg blockade

Ran Wei,¹ Bo Zhao,^{2,3,*} Youjin Deng,^{1,†} Yu-Ao Chen,^{4,5} and Jian-Wei Pan¹

¹*Hefei National Laboratory for Physical Sciences at Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei, Anhui 230026, China*

²*Institute for Theoretical physics, University of Innsbruck, A-6020 Innsbruck, Austria*

³*Institute for Quantum Optics and Quantum Information of the Austrian Academy of Science, A-6020 Innsbruck, Austria*

⁴*Fakultät für Physik, Ludwig-Maximilian-Universität, Schellingstrasse 4, 80798 München, Germany*

⁵*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, 85748 Garching, Germany*

The spin-wave (SW) NOON state is an N -particle Fock state with two atomic spin-wave modes maximally entangled. Attributed to the property that the phase is sensitive to collective atomic motion, the SW NOON state can be utilized as a novel atomic interferometer and has promising application in quantum enhanced measurement. In this paper we propose an efficient protocol to deterministically produce the atomic SW NOON state by employing Rydberg blockade. Possible errors in practical manipulations are analyzed. A feasible experimental scheme is suggested. Our scheme is far more efficient than the recent experimentally demonstrated one, which only creates a heralded second-order SW NOON state.

PACS numbers: 42.50.-p, 42.50.Dv, 32.80.Ee, 32.80.Qk, 37.25.+k, 03.75.Dg

I. INTRODUCTION

The NOON state, an N -particle Fock state with two modes maximally entangled, has attracted many interests since it has the potential to enhance the measurement precision by employing quantum entanglement [1]. Attributed to the property of superresolution and super-sensitivity, the NOON state has been experimentally realized in various photonic systems [2–5]. Recently, a new type of NOON state - the atomic spin wave (SW) NOON state - was proposed, and a heralded second-order SW NOON state as well, was experimentally demonstrated [6]. The scheme [6] employs Raman transitions to generate the atom-photon entanglement and the SW NOON state is created in a herald way by detecting the photons. The SW NOON state can be used as an atomic SW interferometer and can in principle be implemented in a scalable way. However, owing to the probabilistic nature, this SW interferometer works in a very low efficiency and thus cannot be put into practical measurement.

In recent years, the Rydberg atom draw extensive concern in quantum information processing [7]. It has large size and can exhibit large electric dipole moment. This property introduces strong interactions between two Rydberg atoms. Consequently, in a small volume, when an atom is excited to the Rydberg state $|r\rangle$, the energy level of state $|r\rangle$ for other atoms will be shifted by Δ_e . Therefore, the probability for other atoms being excited to $|r\rangle$ is suppressed by a factor of $1/\Delta_e^2$. In the limit $\Delta_e \rightarrow \infty$, only one atom is excited to $|r\rangle$. This is the so-called Rydberg blockade mechanism. The Rydberg blockade has been proposed to deterministically implement quantum computer and quantum repeater [8–16].

In this paper, we propose an efficient way to implement the SW interferometer by deterministically generating the SW NOON state with Rydberg blockade. An elaborate error analysis shows that the 20th-order SW NOON

state can be generated with 91% fidelity under realistic parameters, and accordingly a high fidelity SW interferometer with $F \approx 82\%$ can be realized. This Rydberg-based SW interferometer is much more efficient than the one based on photon detection and might be used as an inertial sensor, for measuring position and displacement, or further, for measuring acceleration and platform rotation. The remaining of this paper is organized as follows. Sec. II describes an envisioned setup and presents the scheme to generate and measure the SW NOON state. Error analysis in practical implementations is given in Sec. III. Experimental realization is suggested in Sec. IV, and finally we conclude in Sec. V.

II. PROTOCOL

We envision a setup as illustrated in Fig. 1(a). An ensemble of N atoms is confined in a volume V , where the blockade mechanism is effective. In other words, the scale of V is smaller than the blockade radius. The working atomic energy levels are chosen to be of the double- Λ type, as shown in Fig. 1(b). They are labeled as the ground state $|g\rangle$, the Rydberg state $|r_a\rangle$, $|r_b\rangle$, and the metastable state $|s_a\rangle$, $|s_b\rangle$. The atoms are coupled by four types of classic light pulses propagating along two spatial modes a, b , whose wave vectors are denoted as \mathbf{k}_{gr_a} , $\mathbf{k}_{r_a s_a}$, \mathbf{k}_{gr_b} and $\mathbf{k}_{r_b s_b}$ respectively. They will also be used to denote the corresponding light pulses if no ambiguity arises. These light pulses couple $|g\rangle$ and $|r_a\rangle$, $|r_a\rangle$ and $|s_a\rangle$, $|g\rangle$ and $|r_b\rangle$, and $|r_b\rangle$ and $|s_b\rangle$ respectively, as illustrated in Fig. 1(b).

Before giving the detailed scheme, we shall first introduce some definitions. We define a collective ground state $|\mathbf{0}\rangle \equiv |g\dots g\rangle$, a collective operator $M_{\mathbf{k},\epsilon}^\dagger \equiv \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\cdot\mathbf{r}_j} |\epsilon_j\rangle \langle g|$, and $|\mathbf{1}, \mathbf{k}\rangle_\epsilon$ ($\epsilon = r_a, r_b, s_a, s_b$) to de-

scribe a collective state with wave vector \mathbf{k} ,

$$|\mathbf{1}, \mathbf{k}\rangle_\epsilon \equiv \frac{1}{\sqrt{N}} \sum_j e^{i\mathbf{k}\cdot\mathbf{r}_j} |g\dots\epsilon_j\dots g\rangle = M_{\mathbf{k},\epsilon}^\dagger |0\rangle. \quad (1)$$

Namely, state $|\mathbf{1}, \mathbf{k}\rangle_\epsilon$ is a coherent superposition of states which have a specific atom at $|\epsilon\rangle$ with the position-dependent phase under the wave vector \mathbf{k} . The same applies to the higher-order collective state $|\ell, \mathbf{k}\rangle_\epsilon \equiv \frac{1}{\sqrt{\ell!}} (M_{\mathbf{k},\epsilon}^\dagger)^\ell |0\rangle_\epsilon$, with ℓ a positive integer. On this basis, a ℓ th-order SW NOON state can be written as

$$|\text{NOON}\rangle_\ell = \frac{1}{\sqrt{2}} (|\ell, \mathbf{k}\rangle_{s_a} + |\ell, \mathbf{k}\rangle_{s_b}). \quad (2)$$

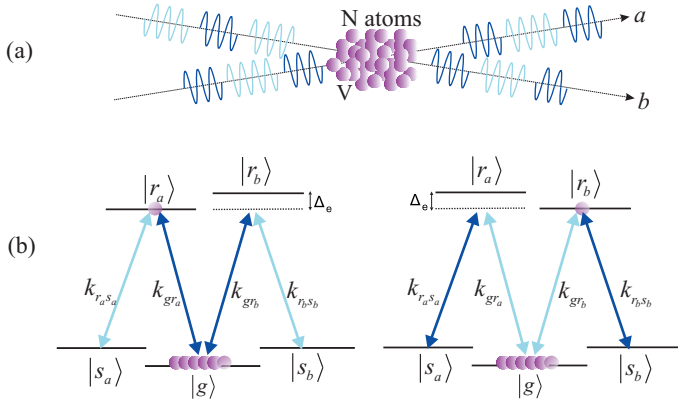


FIG. 1: (Color Online) (a) An ensemble of N atoms trapped in volume V . The atoms are coupled by four types of light pulses, propagating along two spatial modes a, b . (b) The double- Λ type energy levels. An effective energy shift Δ_e is introduced because of the strong interaction between the atoms at the Rydberg states.

We first consider the ideal case by making the following assumptions. (1), the atom number is exactly known, i.e., $\Delta N = 0$; (2), the Rydberg blockade mechanism is perfect, i.e., $\Delta_e \rightarrow \infty$; (3), the lifetime of the Rydberg state is infinite and thus no spontaneous decay occurs; (4), the atomic cloud remains still during the whole process. On this basis, our scheme to generate a ℓ th-order SW NOON state can be described as

1. Prepare an ensemble at the ground state $|0\rangle$.
2. Apply sequentially a collective π pulse \mathbf{k}_{gr_a} and a single-atomic $\pi/2$ pulse $\mathbf{k}_{r_a s_a}$. The former flips one of the N atoms from $|0\rangle$ to the Rydberg state $|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}$ and the latter flips $|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}$ to the equal superposition of the first-order SW state $|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a}$ and $|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}$, where $\mathbf{k}_{\epsilon_1 \epsilon_2 \epsilon_3} \equiv \mathbf{k}_{\epsilon_1 \epsilon_2} - \mathbf{k}_{\epsilon_2 \epsilon_3}$ ($\epsilon_1, \epsilon_2, \epsilon_3 = g, r_a, r_b, s_a, s_b$). Accordingly, one obtains

$$i|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} + |\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a},$$

where a relative phase shift $\pi/2$ is introduced.

3. Apply successively three collective π pulses \mathbf{k}_{gr_b} , \mathbf{k}_{gr_a} and \mathbf{k}_{gr_b} , which leads to

$$\begin{aligned} & |\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b} - |\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a} \\ & \quad \downarrow \\ & i|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b} + |0\rangle \\ & \quad \downarrow \\ & i|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} + |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b}. \end{aligned}$$

4. Apply in order a collective π pulse \mathbf{k}_{gr_a} and a single-atomic π pulse $\mathbf{k}_{r_b s_b}$, and a collective π pulse \mathbf{k}_{gr_b} and a single-atomic π pulse $\mathbf{k}_{r_a s_a}$, which results in

$$\begin{aligned} & |\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_a} - |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b} \\ & \quad \downarrow \\ & |\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_a} - i|\mathbf{1}, \mathbf{k}_{gr_b s_b}\rangle_{s_b} \\ & \quad \downarrow \\ & |\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_a} + |\mathbf{1}, \mathbf{k}_{gr_b s_b}\rangle_{s_b} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b} \\ & \quad \downarrow \\ & i|\mathbf{2}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} + |\mathbf{1}, \mathbf{k}_{gr_b s_b}\rangle_{s_b} |\mathbf{1}, \mathbf{k}_{gr_b}\rangle_{r_b}. \end{aligned}$$

5. Repeatedly apply a sequence of four collective π pulses \mathbf{k}_{gr_a} , $\mathbf{k}_{r_b s_b}$, \mathbf{k}_{gr_b} , $\mathbf{k}_{r_a s_a}$ for $\ell - 2$ times, and one obtains

$$|\ell, \mathbf{k}_{gr_a s_a}\rangle_{s_a} + |\ell - 1, \mathbf{k}_{gr_b s_b}\rangle_{s_b} |\mathbf{1}, \mathbf{k}_{r_b s_b}\rangle_{r_b}.$$

6. Apply a collective π pulse to flip the atom from $|\ell - 1, \mathbf{k}_{gr_b s_b}\rangle_{s_b} |\mathbf{1}, \mathbf{k}_{r_b s_b}\rangle_{r_b}$ to $|\ell, \mathbf{k}_{gr_b s_b}\rangle_{s_b}$ and take into account the normalized factor, and one obtains a ℓ th-order SW NOON state

$$|\Psi\rangle_\ell = (|\ell, \mathbf{k}_{gr_a s_a}\rangle_{s_a} + |\ell, \mathbf{k}_{gr_b s_b}\rangle_{s_b}) / \sqrt{2}. \quad (3)$$

According to the above procedure, the generation of a ℓ th-order SW NOON state needs totally $4\ell + 2$ light pulses, the number of which is linear to ℓ . Note that one needs two π pulses \mathbf{k}_{gr_a} and $\mathbf{k}_{r_a s_a}$ to produce a first-order SW state $|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a}$. Accordingly, two ℓ th-order SW states $|\ell, \mathbf{k}_{gr_a s_a}\rangle_{s_a}$ and $|\ell, \mathbf{k}_{gr_b s_b}\rangle_{s_b}$ would consume 4ℓ light pulses. The ℓ th-order SW NOON state is the superposition of two ℓ th-order SW states at the a and b modes. Thus, we consider the above protocol close to being optimal, albeit the possibility of further improvement is not entirely excluded.

Here we demonstrate how the SW NOON state can be utilized as an atomic interferometer. Let's assume that, after the ℓ th-order SW NOON state is prepared, the atomic cloud moves to a new position with a displacement $\Delta\mathbf{x}$. To measure $\Delta\mathbf{x}$, we apply a sequence of operations reverse to the generation procedure, until the last operation, i.e., the collective π pulse \mathbf{k}_{gr_a} . Detailed calculations show that we obtain the superposition state

$$\begin{aligned} |\Psi'\rangle_\ell &= (ie^{i(\mathbf{k}_{gr_a s_a})\cdot\Delta\mathbf{x}}(1 + e^{i\ell\Delta\mathbf{k}\cdot\Delta\mathbf{x}})|\mathbf{1}, \mathbf{k}_{gr_a s_a}\rangle_{s_a} \\ & \quad + e^{i\mathbf{k}_{gr_b}\cdot\Delta\mathbf{x}}(1 - e^{i\ell\Delta\mathbf{k}\cdot\Delta\mathbf{x}})|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}) / \sqrt{2}, \quad (4) \end{aligned}$$

where $\Delta\mathbf{k} \equiv -\mathbf{k}_{gr_a s_a} - \mathbf{k}_{gr_b s_b}$. Note that, by applying an ionizing electric field, the Rydberg state $|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}$ will be ionized and a free electron will fly out of the atomic ensemble. Thus, the state (4) can be measured onto the $|\mathbf{1}, \mathbf{k}_{gr_a}\rangle_{r_a}$ basis, and the average result will reflect the phase shift $\ell\Delta\mathbf{k} \cdot \Delta\mathbf{x}$. Since the wave vectors of the light pulses are known, this gives the displacement $\Delta\mathbf{x}$. The phase shift is proportional to the order ℓ , and thus the larger ℓ would bring $\Delta\mathbf{x}$ the better precision.

III. ERROR ANALYSIS

In actual implementations, errors can always occur. For instance, the precise number N of atoms in the ensemble is normally unknown, and the atom number N also varies for different experimental trials. This leads to an uncertainty ΔN of the atom number, which is $\Delta N \simeq \sqrt{N}$ for large N . Since the collective Rabi frequency Ω_c of the π pulse \mathbf{k}_{gr_λ} [20] is related to the atom number N as $\Omega_c \propto \sqrt{N}$, ΔN would induce an imprecision in Ω_c as $\Delta\Omega_c/\Omega_c \simeq 1/(2\sqrt{N})$. This means that, when a collective π pulse \mathbf{k}_{gr_λ} is applied to flip one of the atoms from $|\mathbf{0}\rangle$ to $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$, there exists a probability $p \simeq \pi^2/(16N)$ that the flip fails. To generate a ℓ th-order SW NOON state, the total error introduced by ΔN is about $\pi^2\ell/(8N)$. In lab, one can prepare an ensemble of $N \approx 400$ atoms, and thus the error is about $\pi^2\ell/(8N) \approx 6\%$ for order $\ell = 20$.

Aside from the error induced by the uncertainty of the atom number, the imperfect blockade mechanism and the finite lifetime of the Rydberg state also introduces errors. Attributed to these factors, each operation in our scheme is implemented with a non-unity probability. We step by step analyze all the operations from Step 1 to Step 6, and find that, these non-unity probabilities can be categorized into five types, denoted as P^I , P^{II} , P^{III} , P_q^{IV} , P_q^V , and the generated ℓ th-order SW NOON state should be rewritten approximately as

$$\sqrt{\mathcal{P}_\ell(P^I P^{II} P^{III})^\ell} |\Psi\rangle_\ell, \quad (5)$$

where $\mathcal{P}_\ell = \prod_{q=1}^\ell P_q^{IV} P_q^V$. Symbol q stands for the order of the SW state during the generation process, and it increases from 1 to ℓ as one produces the ℓ th-order SW NOON state. Accordingly, the probability for preparing the ℓ th-order SW NOON state is

$$P(\ell) = \mathcal{P}_\ell(P^I P^{II} P^{III})^\ell. \quad (6)$$

The total error accumulated by these operations is the probability that one fails to generate the ℓ th-order SW NOON state, thus it reads $E(\ell) = 1 - P(\ell)$. (The error induced by the uncertainty of atom number is not included in $E(\ell)$.) Before evaluating $E(\ell)$, we shall first analyze the origins of these probabilities.

The probability P^I is introduced by the imperfect blockade that occurs between the atoms of the same

mode when the pulse \mathbf{k}_{gr_λ} flips one of the atoms from $|\mathbf{0}\rangle$ to $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$. In other words, there is an error that two atoms are excited to the Rydberg state $|\mathbf{2}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$ due to the non-infinite energy shift. This mechanics is described by the following equations,

$$i\dot{c}_0 = -\frac{\sqrt{N}\Omega}{2}c_1, \quad (7)$$

$$i\dot{c}_1 = -i\frac{\gamma}{2}c_1 - \frac{\sqrt{N}\Omega}{2}c_0 - \frac{\sqrt{2N}\Omega}{2}c_2, \quad (8)$$

$$i\dot{c}_2 = (\Delta_e - i\gamma)c_2 - \frac{\sqrt{2N}\Omega}{2}c_1, \quad (9)$$

where c_0, c_1, c_2 stand for the amplitudes of $|\mathbf{0}\rangle$, $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$, $|\mathbf{2}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$. Symbols $\gamma/2$ and γ are the decay rates of $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$ and $|\mathbf{2}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$. Symbol Δ_e is the effective finite energy shift, and $\sqrt{N}\Omega, \sqrt{2N}\Omega$ are the corresponding two collective Rabi frequencies, which have been assumed to be real. Since the amplitudes for the states of more than two atoms being excited are significantly suppressed due to the Rydberg blockade, we have neglected them here and in the following. Besides, we have assumed the number of atoms $N \gg 1$ and the coupling light pulses are all in resonance. The initial condition describing this mechanics is $c_0(0) = 1, c_1(0) = 0, c_2(0) = 0$. After applying the collective π pulse \mathbf{k}_{gr_λ} with the operation time $\Delta t = \pi/(\sqrt{N}\Omega)$, one can express the probability for generating $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$ from $|\mathbf{0}\rangle$, as $P^I = |c_1(\Delta t)|^2$.

The probability P^{II} characterizes the imperfect blockade that takes place between the atoms of the different modes during Δt . That is to say, there is an error that the pulse \mathbf{k}_{gr_λ} would flip one of the atoms from $|\mathbf{0}\rangle$ to $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$ when another atom has already been excited to $|\mathbf{1}, \mathbf{k}_{gr_\lambda}\rangle_{r_\lambda}$. Accordingly, this mechanics is governed by the following equations,

$$i\dot{c}_0 = -\frac{\sqrt{N}\Omega}{2}c_1, \quad (10)$$

$$i\dot{c}_1 = (\Delta_e - i\frac{\gamma}{2})c_1 - \frac{\sqrt{N}\Omega}{2}c_0. \quad (11)$$

The initial condition describing this mechanics is $c_0(0) = 1, c_1(0) = 0$, and one can express the probability for holding the atoms at the ground state, as $P^{II} = |c_0(\Delta t)|^2$.

The probability P^{III} is contributed by the decay rate of the Rydberg state. The finite lifetime will inevitably cause some loss when the atom is still at the Rydberg state during Δt , thus the probability for the atom remaining at the Rydberg state is $P^{III} = e^{-\gamma\Delta t}$.

These three types (P^I, P^{II}, P^{III}) are all determined by a shared Rabi frequency Ω or a shared operation time Δt . Note that there is tradeoff between the imperfect Rydberg blockade and the loss caused by the decay, and a simple argument is that if we enhance the the magnitude of the Rabi frequency to shorten the operation time, which reduces the loss from the Rydberg state, it will be associated with more errors from the imperfect blockade.

Therefore, there is an optimal Rabi frequency to maximize the value of $P^I P^{II} P^{III}$. By numerically solving Eqs. (7-9) and Eqs. (10-11), one can easily obtain this maximal value.

The probability P_q^{IV} reflects an error that one of the atoms at $|\mathbf{q} - \mathbf{1}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda} |\mathbf{1}, \mathbf{k}_{gr\lambda}\rangle_{r_\lambda}$ would be flipped back to $|\mathbf{q} - \mathbf{2}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda} |\mathbf{2}, \mathbf{k}_{gr\lambda}\rangle_{r_\lambda}$ when the pulse $\mathbf{k}_{r_\lambda s_\lambda}$ is applied to flip the atom from $|\mathbf{q} - \mathbf{1}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda} |\mathbf{1}, \mathbf{k}_{gr\lambda}\rangle_{r_\lambda}$ to $|\mathbf{q}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda}$. This mechanics is described by the following equations,

$$i\dot{\tilde{c}}_0 = -\frac{\sqrt{q}\tilde{\Omega}}{2}\tilde{c}_1, \quad (12)$$

$$i\dot{\tilde{c}}_1 = -i\frac{\gamma}{2}\tilde{c}_1 - \frac{\sqrt{q}\tilde{\Omega}}{2}\tilde{c}_0 - \frac{\sqrt{2(q-1)}\tilde{\Omega}}{2}\tilde{c}_2, \quad (13)$$

$$i\dot{\tilde{c}}_2 = (\Delta_e - i\gamma)\tilde{c}_2 - \frac{\sqrt{2(q-1)}\tilde{\Omega}}{2}\tilde{c}_1, \quad (14)$$

where $\tilde{c}_0, \tilde{c}_1, \tilde{c}_2$ are the amplitudes of $|\mathbf{q}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda}, |\mathbf{q} - \mathbf{1}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda} |\mathbf{1}, \mathbf{k}_{gr\lambda}\rangle_{r_\lambda}, |\mathbf{q} - \mathbf{2}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda} |\mathbf{2}, \mathbf{k}_{gr\lambda}\rangle_{r_\lambda}$. Symbols $\sqrt{q}\tilde{\Omega}, \sqrt{2(q-1)}\tilde{\Omega}$ are the corresponding two collective Rabi frequencies, which have also been assumed to be real. The initial condition describing this mechanics is $\tilde{c}_0(0) = 0, \tilde{c}_1(0) = 1, \tilde{c}_2(0) = 0$. After applying the collective π pulse $\mathbf{k}_{r_\lambda s_\lambda}$ with the operation time $\Delta\tilde{t}_q = \pi/(\sqrt{q}\tilde{\Omega})$, one can express the probability for producing the q th-order SW state $|\mathbf{q}, \mathbf{k}_{gr\lambda s_\lambda}\rangle_{s_\lambda}$, as $P_q^{IV} = |\tilde{c}_0(\Delta\tilde{t}_q)|^2$.

The origin of P_q^V is similar to P^{III} , it reflects the probability that the atom remains at the Rydberg state during $\Delta\tilde{t}_q$, and thus $P_q^V = e^{-\gamma\Delta\tilde{t}_q}$. The value of $P_q^{IV} P_q^V$ is determined by a shared Rabi frequency $\tilde{\Omega}$ or a shared operation time $\Delta\tilde{t}_q$. Likewise, one can calculate the maximal value of $P_q^{IV} P_q^V$ by numerically solving Eqs. (12-14) with q from 1 to ℓ .

To evaluate $E(\ell)$, we choose the parameters as, the atom number $N = 400$, the lifetime of the Rydberg state $\tau = 1/(2\pi\gamma) = 300 \mu s$ and $400 \mu s$, and the energy shift Δ_e varying from $20 MHz$ to $400 MHz$. Accordingly, Eq.(6) can be calculated in a numerical way. We obtain the error $E(\ell)$ versus the energy shift Δ_e , shown in Fig.2.

From the figure, we see that the larger the energy shift, the smaller the error, and the error vanishes as Δ_e tends to infinity. This is an anticipated result since the error $E \sim \Omega^2/\Delta_e^2$. However, in actual experiment, Δ_e cannot be unlimitedly large. An intrinsic limitation originates from the average distance of two Rydberg atoms, which should be larger than the radius of each Rydberg atom. In the limit of high density where the Rydberg atoms remarkably overlap, our blockade model is inappropriate, and a more elaborate mechanism should be taken into account. This mechanism goes beyond the extent of our paper and will not be discussed. Besides, as one readily expects, the figure shows that the error is suppressed as the lifetime of the Rydberg state becomes longer, and is intensified when the order ℓ of the SW NOON state

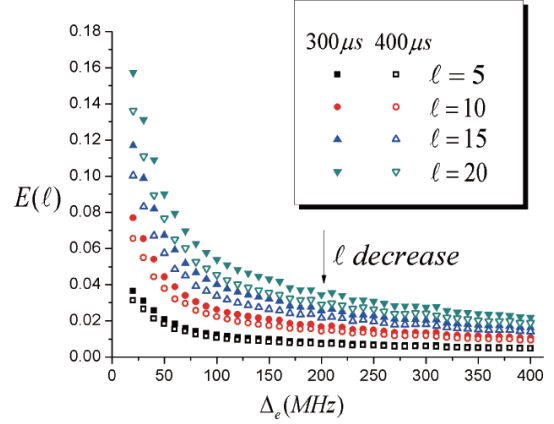


FIG. 2: (Color Online) The figure demonstrates the error $E(\ell)$ versus the energy shift Δ_e under various order ℓ after generating the SW NOON state. The solid data and the open one respectively denote the lifetime of the Rydberg state with $\tau = 300 \mu s$ and $\tau = 400 \mu s$.

increases.

IV. EXPERIMENTAL REALIZATION

To design an atomic interferometer with sufficiently high precision and relatively high fidelity, we use the 20th-order SW NOON state for the practical application. The interferometer can be implemented by cold alkali atoms. By choosing the suitable laser polarization, the two spacial modes a and b can be individually addressed. The energy shift is isotropic due to the property of repulsive van der Waals interaction. The lifetime of the Rydberg state with $\tau = 300 \sim 400 \mu s$ is achievable by exciting the atoms to the Rydberg s state with a principal quantum number $n = 100$ [10]. In our scheme, the energy shift Δ_e of the Rydberg state can be expressed as $\Delta_e = -n^{11}(c_0 + c_1 n + c_2 n^2)/r^6$ [17], where the terms $1/r^8$ and $1/r^{10}$ are neglected due to the dominating long-range property. For Rubidium, $c_0 = 13, c_1 = -0.85, c_2 = 0.0034$ [17], and thus an ensemble of atoms with the radius $R = 3.8 \mu m$ enables the energy shift $\Delta_e \geq 300 MHz$, which ensures the error $E(20) < 3\%$, as illustrated in Fig.2. In a volume of $4\pi/3R^3$, a density of $1.7 \times 10^{12} cm^{-3}$ allows $N \approx 400$ atoms in an ensemble. Based on these estimated parameters above, we suggest to employ the one-dimensional optical lattice as the experimental setup, where the size of the ensemble can be controlled by tuning the angle between the trapping light fields [18]. Finally, we should point out that, to detect the displacement of atomic cloud by the interferometer, the reverse operations to those in the generation procedure should be considered, and thus the total error is doubled. Fortunately, the field ionization can be implemented with near-unity detection efficiency [19]. Therefore, taking into account the error in-

duced by the uncertainty of atom number, our proposed atomic SW interferometer with a high precision ($\ell = 20$) can reach a high fidelity as $F \approx 1 - 2 \times (6\% + 3\%) = 82\%$.

V. SUMMARY

By employing Rydberg blockade, we have demonstrated an efficient scheme to deterministically produce the atomic SW NOON state, of which, a direct application is the atomic SW interferometer. Possible errors in practical manipulations are analyzed, and the experimen-

tal realization also is suggested. Our proposed atomic SW interferometer is far more efficient than the recent experimentally demonstrated one, and holds promise in the practical application.

VI. ACKNOWLEDGEMENT

This work is supported by the NNSFC, the NNSFC of Anhui (under Grant No. 090416224), the CAS, the National Fundamental Research Program (under Grant No. 2011CB921304), and the SFB FOQUS of FWF.

* Electronic address: bo.zhao@uibk.ac.at

† Electronic address: yjdeng@ustc.edu.cn

- [1] H. Lee, P. Kok and J. P. Dowling, *J. Mod. Opt.* **49**, 2325 (2002).
- [2] P. Walther, J.-W. Pan, M. Aspelmeyer, R. Ursin, S. Gasparoni and A. Zeilinger, *Nature (London)* **429**, 158 (2004).
- [3] M. Mitchell, J. Lundeen, and A. Steinberg, *Nature (London)* **429**, 161 (2004).
- [4] T. Nagata, R. Okamoto, J. L. O'Brien, K. Sasaki and S. Takeuchi, *Science* **316**, 726 (2007).
- [5] K. J. Resch, K. L. Pregnell, R. Prevedel, A. Gilchrist, G. J. Pryde, J. L. O'Brien, and A. G. White, *Phys. Rev. Lett.* **98**, 223601 (2007).
- [6] Y.-A. Chen, X.-H. Bao, Z.-S. Yuan, S. Chen, B. Zhao, and J.-W. Pan, *Phys. Rev. Lett.* **104**, 043601 (2010).
- [7] M. Saffman, T. G. Walker and K. Mølmer, *Rev. Mod. Phys.* **82**, 2313 (2010).
- [8] D. Jaksch, J. I. Cirac, P. Zoller, S. L. Rolston, R. Côté, and M. D. Lukin, *Phys. Rev. Lett.* **85**, 2208 (2000).
- [9] M. D. Lukin, M. Fleischhauer, R. Côté, L. M. Duan, D. Jaksch, J. I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **87**, 037901 (2001).
- [10] M. Saffman and T. G. Walker, *Phys. Rev. A* **72**, 022347 (2005).
- [11] M. Saffman and T. G. Walker, *Phys. Rev. A* **72**, 042302 (2005).
- [12] M. Saffman and K. Mølmer, *Phys. Rev. Lett.* **102**, 240502(2009).
- [13] M. Müller, I. Lesanovsky, H. Weimer, H. P. Büchler, and P. Zoller, *Phys. Rev. Lett.* **102**, 170502 (2009).
- [14] B. Zhao, M. Müller, K. Hammerer, and P. Zoller, *Phys. Rev. A* **81**, 052329 (2010).
- [15] Y. Han, B. He, K. Heshami, C.-Z. Li and C. Simon, *Phys. Rev. A* **81**, 052311 (2010).
- [16] L. Isenhower, E. Urban, X. L. Zhang, A. T. Gill, T. Henage, T. A. Johnson, T. G. Walker and M. Saffman, *Phys. Rev. Lett.* **104**, 010503 (2010).
- [17] K. Singer, J. Stanojevic, M. Weidemüller and R. Côté, *J. Phys. B* **38**, S295 (2005).
- [18] L. Fallani, C. Fort, J. E. Lye and M. Inguscio, *Opt. Exp.* **13**, 4303 (2005).
- [19] C. Guerlin, J. Bernu, S. Deléglise, Clément Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J.-M. Raimond and S. Haroche *Nature (London)* **448**, 889 (2007).
- [20] For the compactness, we define the notation $\{\lambda, \bar{\lambda}\} \equiv \{a, b\}, \{b, a\}$.