Jumping Through Loops:
On Soft Terms from Large Volume Compactifications

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Abstract

We subject the phenomenologically successful large volume scenario of hep-th/0502058 to a first consistency check in string theory. In particular, we consider whether the expansion of the string effective action is consistent in the presence of D-branes and O-planes. Due to the no-scale structure at tree-level, the scenario is surprisingly robust. We compute the modification of soft supersymmetry breaking terms, and find only subleading corrections. We also comment that for large-volume limits of toroidal orientifolds and fibered Calabi-Yau manifolds the corrections can be more important, and we discuss further checks that need to be performed.

Contents

1 Introduction

The KKLT strategy [1,2] for producing stabilized string vacua that can serve as a starting point for phenomenology has been a source of great interest for the last few years. The "large-volume scenario" (LVS) [3,4] is an extension of KKLT where string corrections to the tree-level supergravity effective action computed in [5] play a significant role, and where the compactification volume can be as large as 10^{15} in string units. In LVS, work has been done on soft supersymmetry breaking [4,6,7], the QCD axion [8,9], neutrino masses [10], inflation [11–14], and even first attempts at LHC phenomenology [15].

Although tantalizing, the models discussed in the aforementioned papers (nominally "string compactifications") raise many questions. It remains an open problem to construct complete KKLT models in string theory, as opposed to supergravity. Problems one faces include things like the description of RR fluxes in string theory, showing that the necessary nonperturbative effects actually can and do appear in a way consistent with other contributions to the potential (for progress in this direction, see [16–32]), and verifying that one can uplift to a Minkowski or deSitter vacuum without ruining

stabilization [26,33–36]. In LVS, since string corrections play a crucial role, striving for actual string constructions seems quite important. In the end, the restrictiveness this entails may greatly improve predictivity, or kill the models completely as string compactifications.

In this paper, we will not improve on the consistency of KKLT or LVS in general, but rather assume the existence of LVS models in string theory, and then perform self-consistency checks. This is a modest step on the way towards reconciling phenomeno-logically promising scenarios with underlying string models. We will see that although a priori the situation looks very bleak, and one might have hastily concluded that even our modest consistency check would put very strong constraints on LVS, things are more interesting. It turns out that LVS jumps through every hoop we present it with, and instead of broad qualitative changes, we find only small quantitative changes.

The main difference between KKLT and LVS is that LVS includes a specific string α' correction $\Delta K_{\alpha'}$ in the Kähler potential K of the four-dimensional $\mathcal{N} = 1$ effective supergravity. Naturally, the four-dimensional string effective action also contains other string corrections. Here, we will focus on g_s corrections due to sources (D-branes and O-planes). For some $\mathcal{N} = 1$ and $\mathcal{N} = 2$ toroidal orientifolds, these corrections were computed in [37] (see also [38]; for a comprehensive introduction to orientifolds, see [39]). Compared to the α' correction $\Delta K_{\alpha'}$ considered in LVS, the g_s corrections to the Kähler potential ΔK_{g_s} will scale as

$$\left(\Delta K_{\alpha'} : \Delta K_{g_s}\right) \sim \left(\mathcal{O}(\alpha'^3) : \mathcal{O}(g_s^2 \alpha'^2)\right) \qquad \text{(string frame)} . \qquad (1)$$

By naive dimensional analysis, one would expect that in a $1/\mathcal{V}$ expansion, where $\mathcal{V}$ is the overall volume in the Einstein frame, eq. (1) implies

$$\Delta K_{\alpha'} \sim \mathcal{O}(g_s^{-3/2} \mathcal{V}^{-1}) \quad , \quad \Delta K_{g_s} \sim \mathcal{O}(g_s \mathcal{V}^{-2/3}) \qquad \text{(Einstein frame)} . \qquad (2)$$

If there is more than one Kähler modulus, as is usually the case, various combinations of Kähler moduli may appear in ΔK_{g_s} in eq.(2), and a priori this could lead to even *weaker* suppression in $1/\mathcal{V}$ than that shown. However, we will argue that (2) is actually correct as far as the suppression factors in the $1/\mathcal{V}$ expansion go. Nevertheless, even the suppression displayed in (2) seems to be a challenge for LVS, if indeed $\mathcal{V} \sim 10^{15}$. For $\mathcal{V}$ this large, ΔK_{g_s} would dominate $\Delta K_{\alpha'}$, since we do not expect the string coupling g_s to be stabilized extremely small. On the other hand, if we are interested in the effects g_s corrections may have on the existence of the large volume minima, the relevant quantity to look at is the *scalar potential V*, rather than the Kähler potential K. It turns out that certain cancellations in the expression for the scalar potential leave us

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with leading correction terms to V that scale as

$$\Delta V_{\alpha'} \sim \mathcal{O}(g_{\mathrm{s}}^{-1/2}\mathcal{V}^{-3}) \quad , \quad \Delta V_{g_{\mathrm{s}}} \sim \mathcal{O}(g_{\mathrm{s}}\mathcal{V}^{-3}) \ . \tag{3}$$

This is already much better news for LVS. However, restoring numerical factors in (3), and with g_{s} typically not stabilized extremely small, it would seem that $\Delta K_{g_{\mathrm{s}}}$ could still have a significant effect both on stabilization and on the resultant phenomenology (like soft supersymmetry breaking terms, which also depend on the Kähler potential). We will see that although this is indeed so in principle, in practice the models we consider are surprisingly robust against the inclusion of $\Delta K_{g_{\mathrm{s}}}$. The clearest example of this is the calculation of gaugino masses in sec. 4. The result is that for the "11169 model" (analyzed in [6]), the correction to the gaugino masses due to $\Delta K_{g_{\mathrm{s}}}$ is negligible. Thus, for the most part, LVS survives our onslaught unscathed.

We consider this a sign that scenarios such as LVS deserve to be taken seriously as *goals* to be studied in detail in string theory, even as the caveats above (that apply to any KKLT-like setup) serve to remind us that there is much work left to be done to really understand phenomenologically viable stabilized flux compactifications in string theory.

2 Review

Let us begin by a quick review of the KKLT and large volume scenarios. For reasons that will become clear, we will want to allow for more than a single Kähler modulus.

2.1 KKLT

The KKLT setup [1,2] is a warped Type IIB flux compactification on a Calabi-Yau (or more generally, F-theory) orientifold, with all moduli stabilized. In this paper, we will neglect warping. For progress towards taking warping into account in phenomenological contexts, see [40,41].

In the four-dimensional $\mathcal{N} = 1$ effective supergravity, the Kähler potential and superpotential read

$$\begin{aligned} K &= -\ln(S + \bar{S}) - 2\ln(\mathcal{V}) + K_{\mathrm{cs}}(U, \bar{U}) \ , \\ W &= W_{\mathrm{tree}} + W_{\mathrm{np}} = W(S, U) + \sum_i A_i(S, U)e^{-a_i T_i} \ , \end{aligned} \tag{4}$$

where the volume $\mathcal{V}$ is a function of the Kähler moduli $T_i = \tau_i + ib_i$ whose real parts are 4-cycle volumes and whose imaginary parts are axions b_i, arising from the integral of the RR 4-form over the corresponding 4-cycles. In particular, the volume $\mathcal{V}$ depends on the T_i only through the real parts τ_i,

$$\mathcal{V} = \mathcal{V}(T_i + \bar{T}_i) = \mathcal{V}(\tau_i) \; , \tag{5}$$

and the nonperturbative superpotential W_{np} a priori depends on the complexified dilaton S and the complex structure moduli U. After stabilization of S and U by demanding $D_U W = 0 = D_S W$, we have

$$\begin{aligned}
K &= -\ln(S + \bar{S}) - 2\ln(\mathcal{V}) + K_{\mathrm{cs}}(U, \bar{U}) \; , \\
W &= W_{\mathrm{tree}} + W_{\mathrm{np}} = W_0 + \sum_i A_i e^{-a_i T_i} \; .
\end{aligned} \tag{6}$$

We keep the dependence on the complexified dilaton S and the complex structure moduli U in the Kähler potential for now, since the Kähler metric in the F-term potential

$$V = e^K \left(G^{\bar{J}I} D_{\bar{J}}\bar{W} D_I W - 3|W|^2 \right) \tag{7}$$

is to be calculated with the full Kähler potential K, including the dependence on S and U. (In eq. (7), the index I a priori runs over all moduli, but after fixing the complex structure moduli and the dilaton, only the sum over the Kähler moduli remains). The scalar potential V has a supersymmetric AdS minimum at a radius that is barely large enough to make the use of a large-radius effective supergravity self-consistent, typically $\tau \sim 100$ (recall that τ has units of (length)^4).[1] In addition, to obtain a supersymmetric minimum at all, one needs to tune the flux superpotential W_0 to very small values. That is, the stabilization only works for a small parameter range. This is easy to understand, since we are balancing a nonperturbative term against a tree-level term. Let us briefly digress on the reasons for and implications of this balancing.

2.2 Consistency of KKLT

In the previous section we only considered the lowest-order supergravity effective action. As was already noted in the original KKLT paper, α' corrections and g_{s} corrections (string loops) that appear in addition to the tree-level effective action could in principle

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[1]This minimum then has to be uplifted to dS or Minkowski by an additional contribution to the potential. Various mechanisms were suggested in [2,36,42–45].

affect stabilization. Oftentimes, the logic of string effective actions is that if one such correction matters, they all do, so no reliable physics can be learned from considering the first few corrections. If this is true, one can only consider regimes in which all corrections are suppressed. This is not necessarily so if some symmetry prevents the tree-level contribution to the effective action from appearing, so that the first correction (be it α' or g_s) constitutes lowest order. This indeed happens for type IIB flux compactifications; given the tree level Kähler potential (6), if we were to set $W_{\rm np} = 0$, the remaining K and W in (6) produce a no-scale potential, i.e. the scalar potential for the Kähler moduli then vanishes [46]. In KKLT, this no-scale structure is only broken by the nonperturbative contribution to the superpotential $W_{\rm np}$. Since each term in $W_{\rm np}$ is exponentially suppressed in some Kähler modulus, the resulting terms in the potential are also exponentially suppressed. For instance, for the simpler example of a single modulus τ, the potential (after already fixing the axionic partner along the lines of appendix A.2) reads

$$\frac{V}{e^K} = \left[4|A|^2 a\tau e^{-a\tau} \left(\frac{1}{3} a\tau + 1 \right) - 4a\tau |AW_0| \right] e^{-a\tau} , \tag{8}$$

meaning that even for moderate values of the Kähler modulus τ, all these terms are numerically very small. Corrections in α' and g_s, however, are expected to go as powers of Kähler moduli τ, so will *dominate* the scalar potential for most of parameter space. In particular, it was argued in [3,4] that only for very small values of W_0 can perturbative corrections to the Kähler potential be neglected. It was the insight of [3] that even if W_0 is $\mathcal{O}(1)$ (which is more generic than the tiny value for W_0 required in KKLT), there can still be a competition between the perturbative and nonperturbative corrections to the potential in regions of the Kähler cone where large hierarchies between the Kähler moduli are present. We now review this scenario.

2.3   Large volume scenario (LVS)

As was shown in [5], the no-scale structure (and factorization of moduli space) is broken by perturbative α' corrections to the Kähler potential, such as

$$K = -\ln(2S_1) - 2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) + K_{\rm cs}(U, \bar{U}) , \tag{9}$$

where[2] $\xi = -\zeta(3)\chi/2(2\pi)^3$ and $S_1 = \mathrm{Re}\, S$. For large volume $\mathcal{V}$, we see that the perturbative correction goes as a power in the volume,

$$-2\ln(\mathcal{V} + \tfrac{1}{2}\xi S_1^{3/2}) = -2\ln\mathcal{V} - \frac{\xi S_1^{3/2}}{\mathcal{V}} + \dots , \tag{10}$$

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[2]Here ξ differs by a factor $(2\pi)^{-3}$ from [5] because we use the string length $l_s = 2\pi\sqrt{\alpha'}$.

which by the discussion in the previous subsection will dominate in the scalar potential if all Kähler moduli are even moderately large. Using the superpotential

$$W = W_0 + W_{\mathrm{np}} = W_0 + \sum_i A_i e^{-a_i T_i} \ , \tag{11}$$

the scalar potential has the structure

$$
\begin{aligned}
V &= V_{\mathrm{np1}} + V_{\mathrm{np2}} + V_3 \\
&= e^K \left\{ G^{\bar{j}i} \partial_{\bar{j}} \bar{W}_{\mathrm{np}} \partial_i W_{\mathrm{np}} + \left[G^{\bar{j}i} K_{\bar{j}} (\bar{W}_0 + \bar{W}_{\mathrm{np}}) \partial_i W_{\mathrm{np}} + c.c. \right] \right. \\
&\quad \left. + \left(G^{\bar{j}i} K_{\bar{j}} K_i - 3 \right) |W|^2 \right\} \ .
\end{aligned}
\tag{12}
$$

For concrete calculations we will use the model based on the hypersurface of degree 18 in $\mathbb{P}^4_{[1,1,1,6,9]}$ (see [16,47,48] for background information on its topology. Some comments about generalizations to other models with arbitrary numbers of Kähler moduli are given in appendix A.2). The defining equation is

$$z_1^{18} + z_2^{18} + z_3^{18} + z_4^3 + z_5^2 - 18\psi z_1 z_2 z_3 z_4 z_5 - 3\phi z_1^6 z_2^6 z_3^6 = 0 \tag{13}$$

and it has the Hodge numbers $h^{1,1} = 2$ and $h^{2,1} = 272$ (only two of the complex structure moduli ψ and ϕ have been made explicit in (13); moreover, not all of the 272 survive orientifolding). We denote the two Kähler moduli by $T_b = \tau_b + i b_b$ and $T_s = \tau_s + i b_s$, where τ_b and τ_s are the volumes of 4-cycles, and the subscripts "b" and "s" are chosen in anticipation of the fact that one of the Kähler moduli (τ_b) will be stabilized big, and the other one (τ_s) will be stabilized small. An interesting property of this model is that it allows expressing the 2-cycle volumes t_i explicitly as functions of the 4-cycle volumes τ_j, so that the total volume of the manifold can be written directly in terms of 4-cycle volumes, yielding

$$\mathcal{V} = \frac{1}{9\sqrt{2}} \left(\tau_b^{3/2} - \tau_s^{3/2} \right) \ , \tag{14}$$

$$\tau_b = \frac{(t_s + 6t_b)^2}{2} \ , \quad \tau_s = \frac{t_s^2}{2} \ .$$

Following [4], we are interested in minima of the potential with the peculiar property that one Kähler modulus $\tau_b \sim \mathcal{V}^{2/3}$ is stabilized large and the rest are relatively small (but still large compared to the string scale),

$$a\tau_s \sim \ln \mathcal{V} \sim \frac{3}{2} \ln \tau_b \tag{15}$$

in the case at hand. Thus, we expand the potential around large volume, treating $e^{-a\tau_s}$ as being of the same order as $\mathcal{V}^{-1}$. In the end one has to check that the resulting

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potential indeed leads to a minimum consistent with the exponential hierarchy $a\tau_s \sim \ln \mathcal{V}$, so that the procedure is self-consistent. Applying this strategy, the scalar potential at leading order in $1/\mathcal{V}$ becomes[3]

$$V_{\mathcal{O}(1/\mathcal{V}^3)} = \left(\frac{12\sqrt{2}|A|^2 a^2 \sqrt{\tau_s} e^{-2a\tau_s}}{\mathcal{V}S_1} - \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{\mathcal{V}^2 S_1} + \xi \frac{3|W_0|^2 \sqrt{S_1}}{8\mathcal{V}^3} \right) e^{K_{cs}} . \quad (16)$$

From here one can see the existence of the large volume minima rather generally. By the Dine-Seiberg argument [49], the scalar potential goes to zero asymptotically in every direction. Along the direction (15), for large volume the leading term in (16) is

$$V \sim V_{\rm np2} \propto -\frac{\ln \mathcal{V}}{\mathcal{V}^3} , \quad (17)$$

which is negative, so the potential V approaches zero from below. For moderately small values of the volume, V is positive (this is guaranteed if the Euler number χ is negative, hence ξ positive), so in between there is a minimum. This minimum is typically nonsupersymmetric, and because we are no longer balancing a tree-level versus a nonperturbative term, we can find minima at large volume — hence the name large volume scenario (LVS).[4] To be precise, in flux compactifications we move in parameter space by the choice of discrete fluxes, but since $\mathcal{V}$ is exponentially sensitive to parameters like S_1, large volume minima appear easy to achieve also by small changes in flux parameters. If we allow for very small values of W_0 (so that KKLT minima exist at all), the above minimum can coexist with the KKLT minimum [4,50]. Here, we will allow W_0 to take generic values of order one.

The astute reader will have noticed that this argument for the existence of the LVS minimum is "one dimensional", as it only takes into account the behavior of the potential along the direction (15). One must of course check minimization with respect to all Kähler moduli. In [3] a plausibility argument to this effect was given, and the existence of the minimum was explicitly checked in the case of the $\mathbb{P}^4_{[1,1,1,6,9]}$ model by explicitly minimizing the potential (16) with respect to the Kähler moduli. In doing so, it is convenient to trade the two independent variables $\{\tau_b, \tau_s\}$ for $\{\mathcal{V}, \tau_s\}$ so that

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[3]Here we have already stabilized the axion b_s, i.e. solved $\partial V/\partial b_s = 0$, which produces the minus sign in the second term; this is also true with many small moduli τ_i. See appendix A.2 for details. Also note that solving $D_U W = 0 = D_S W$ causes the values of U and S at the minimum to depend on the Kähler moduli. However, this dependence arises either from the nonperturbative terms in the superpotential or from the α'-correction to the Kähler potential. Thus it would only modify the potential at subleading order in the $1/\mathcal{V}$ expansion.

[4]By "tree-level" we intend "tree-level supergravity", i.e. for the purposes of this paper we call both α' and g_s corrections "quantum corrections".

$\partial_{\tau_s} \mathcal{V} = 0$, as then the last term in (16) is independent of τ_s (this will be different when we include loop corrections). Extremizing with respect to τ_s, and defining

$$X \quad \equiv \quad A e^{-a\tau_s} \, , \tag{18}$$

one obtains a quadratic equation for X,

$$0 = \frac{\partial V}{\partial \tau_s} = \left(-\frac{6\sqrt{2}a^2}{\sqrt{\tau_s}S_1\mathcal{V}} \left(4a\tau_s - 1\right) X^2 + \frac{2a|W_0|}{S_1\mathcal{V}^2} \left(a\tau_s - 1\right) X \right) e^{K_{\mathrm{cs}}} \, . \tag{19}$$

In (18), we chose A to be real as a potential phase can be absorbed into a shift of the axion b and disappears after minimization with respect to b (see section A.2). Two comments are in order. The quadratic equation (19) has just one meaningful solution ($X = 0$ corresponds to $\tau_s = \infty$). Moreover, when expanding (19) in $1/(a\tau_s)$, the leading terms arise from derivatives of the exponential.

Formula (19) is an implicit equation determining τ_s. However, one can easily solve (19) for X and obtains

$$X = A e^{-a\tau_s} = \frac{\sqrt{2}|W_0|}{24a\mathcal{V}} \sqrt{\tau_s} \left(1 - \frac{3}{4a\tau_s}\right) + \mathcal{O}\left(\frac{1}{(a\tau_s)^2}\right) \, . \tag{20}$$

The hierarchy (15) is obvious in this solution, rendering the procedure self-consistent. One also notices that reasonably large values of τ_s (e.g. 35) are not difficult to obtain, if $\mathcal{V}$ is stabilized large enough; for example, simply set $a \sim 1$, $A \sim 1$, $W_0 \sim 1$. We fill in the numerical details, following [3], in appendix A.1 (including some further observations).

3   String loop corrections to LVS

As already emphasized, the α' correction proportional to ξ is only one among many corrections in the string effective action. We now consider the effect of string loop corrections on this scenario and what the regime of validity is for including or neglecting those corrections. Volume stabilization with string loop corrections but without nonperturbative effects was considered in [51].

To be precise, the corrections considered in [51] were those of [37], that were computed for toroidal $\mathcal{N} = 1$ and $\mathcal{N} = 2$ orientifolds. Here, we would need the analogous corrections for smooth Calabi-Yau orientifolds. Needless to say, these are not known. Faced with the fact that the string coupling g_{s} is stabilized at a finite (and typically

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not terribly small) value, we propose that attempting to estimate the corrections based on experience with the toroidal case is better than arbitrarily discarding them. As we will see, if our estimates are correct, typically the loop corrections can be neglected, though there may at least be some regions of parameter space where they must be taken into account (see figure 4). (In section 5, we will briefly consider "cousins" of LVS where they cannot be neglected anywhere in parameter space.) Improvement on our guesswork would of course be very desirable.

3.1 From toroidal orientifolds to Calabi-Yau manifolds

We would like to make an educated guess for the possible form of one-loop corrections in a general Calabi-Yau orientifold. All we can hope to guess is the scaling of these corrections with the Kähler moduli T and the dilaton S. The dependence on other moduli, like the complex structure moduli U, cannot be determined by the following arguments (even in the toroidal orientifolds this dependence was quite complicated).

In order to generalize the results of [37] to the case of smooth Calabi-Yau manifolds, we should first review them and in particular remind ourselves where the various corrections come from in the case of toroidal orientifolds. There, the Kähler potential looks as follows (we will explain the notation as we go along):

$$K = -\ln(2S_1) - 2\ln(\mathcal{V}) + K_{cs}(U,\bar{U}) - \frac{\xi S_1^{3/2}}{\mathcal{V}} \qquad (21)$$
$$+ \sum_{i=1}^{3} \frac{\mathcal{E}_i^{(K)}(U,\bar{U})}{4\tau_i S_1} + \sum_{i\neq j\neq k}^{3} \frac{\mathcal{E}_k^{(W)}(U,\bar{U})}{4\tau_i\tau_j} \ .$$

There are two kinds of corrections. One comes from the exchange of Kaluza-Klein (KK) modes between D7-branes (or O7-planes) and D3-branes (or O3-planes, both localized in the internal space), which are usually needed for tadpole cancellation, cf. fig. 1. This leads to the first kind of corrections in (21), proportional to $\mathcal{E}_i^{(K)}$ where the superscript (K) reminds us that these terms originate from KK modes. In the toroidal orientifold case, this type of correction is suppressed by the dilaton and a single Kähler modulus τ_i, related to the volume of the 4-cycle wrapped by the D7-branes (or O7-planes, respectively).[5] We expect an analog of these terms to arise more

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[5]We should mention that there was no additional correction of this kind coming from KK exchange between (parallel) D7-branes in [37] (actually that paper considered the T-dual version with D5-branes, but here we directly translate the result to the D7-brane language). This was due to the fact that in [37] the D7-brane scalars were set to zero. In general we would also expect a correction coming
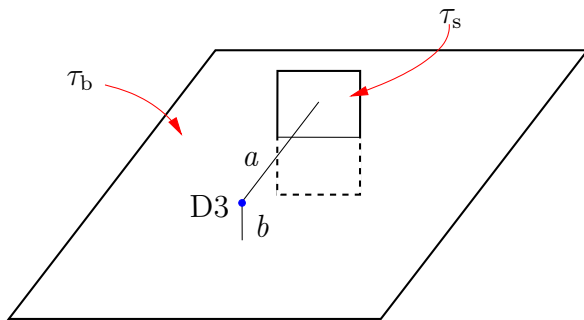
FIGURE 1: The loop correction $\mathcal{E}^{(K)}$ comes from the exchange of closed strings, or equivalently an open-string one-loop diagram, between the D3-brane and D7-branes (or O7-planes) wrapped on either the small 4-cycle τ_s (as in a) or the large 4-cycle τ_b (as in b). The exchanged closed strings carry Kaluza-Klein momentum.

generally, given that they originate from the exchange of KK states which are present in all compactifications.

The second type of correction comes from the exchange of winding strings between intersecting stacks of D7-branes (or between intersecting D7-branes and O7-planes). The exchanged strings are wound around non-contractible 1-cycles within the intersection locus of the D7-branes (and O7-planes, respectively), cf. fig. 2. This leads to the
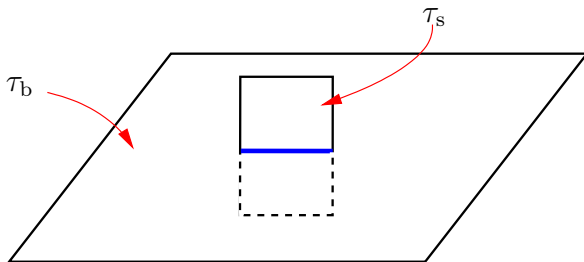
FIGURE 2: The loop correction $\mathcal{E}^{(W)}$ comes from the exchange of winding strings on the intersection between the small 4-cycle τ_s and the large 4-cycle τ_b. If this intersection is empty, there are no terms with $\mathcal{E}^{(W)}$.

second kind of correction in (21) proportional to $\mathcal{E}_i^{(W)}$. The superscript (W) reminds us that these terms arise from the exchange of winding strings. In toroidal orientifolds, this type of correction is suppressed by the two Kähler moduli measuring the volumes

from parallel (or more generally, non-intersecting) D7-branes by exchange of KK-states. These should scale in the same way with the Kähler moduli as those arising from the KK exchange between D3- and D7-branes.

of the 4-cycles wrapped by the D7-branes (and O7-planes). One might a priori think that this kind of correction does not generalize easily to a smooth Calabi-Yau which has vanishing first Betti number (and therefore at most torsional 1-cycles). However, the exchanged winding strings are, from the open string point of view, Dirichlet strings with their endpoints stuck on the D7-branes. Thus, the topological condition is on the cycle over which the two D7-brane stacks (or one D7-brane stack and an O7-plane) intersect, as in figure 3. Thus, it depends on the topology of specific cycles within cycles whether winding open strings exist in a given model.[6]
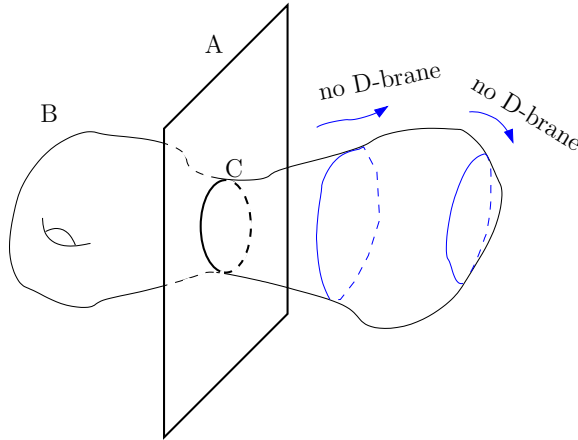

FIGURE 3: A D7-brane is wrapped on a 4-cycle A, which intersects the 4-cycle B on a 2-cycle C. For Dirichlet strings, the relevant topological condition (the existence of nontrivial 1-cycles) is on the intersection locus C, not on cycle B or on the whole Calabi-Yau. In other words, without the D-brane, the string on cycle C could have been unwound by sliding it along cycle B (as shown in the figure). With the D-brane, the string on cycle C is stuck.

Given the expressions in [37] and the subset reproduced in (21) above, it is tempting to conjecture that some terms at one loop might be suppressed only by powers of single Kähler moduli like the τ_i (and the dilaton):

$$\text{Calabi-Yau:} \quad \Delta K_{g_s} \overset{?}{=} \frac{\mathcal{E}}{S_1 \tau_i} \tag{22}$$

for some function $\mathcal{E}$ of the complex structure and open string moduli. If this were the case, the one-loop corrections would typically dominate the α' correction in (21)

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[6]The toroidal orientifold case seems to be a bit degenerate. Two stacks of D7-branes intersect along a 2-cycle with the topology of $\mathbb{P}^1$. However, there are point-like curvature singularities along the $\mathbb{P}^1$ at the orbifold point and strings winding around these singular points cannot be contracted without crossing the singularities. This seems to allow for stability of winding strings (at least classically).

(which is suppressed by the overall volume $\mathcal{V}$) in the Kähler potential, if there are large hierarchies among the Kähler moduli. However, one should keep in mind that toroidal orientifolds are rather special in that they have very simple intersection numbers. In particular, the overall volume can be written as $\mathcal{V} \sim \tau_i t_i$, where there is no summation over i implied. Thus, it is not obvious whether a generalization to the case of a general Calabi-Yau really contains terms suppressed by single Kähler moduli instead of the overall volume. Even though we cannot exclude the presence of such terms, we deem it more likely that the scaling of one-loop corrections to the Kähler potential is not (22) but

$$\text{Calabi-Yau:} \quad \Delta K_{g_\mathrm{s}} \stackrel{!}{\sim} \frac{\sum_{\mathrm{KK}} m_{\mathrm{KK}}^{-2}}{S_1 \mathcal{V}} \sim \sum_{\mathfrak{a}} \frac{g_\mathrm{K}^{\mathfrak{a}}(t, S_1) \mathcal{E}_{\mathfrak{a}}^{(K)}}{S_1 \mathcal{V}}$$

$$\text{and} \quad \frac{\sum_{\mathrm{W}} m_{\mathrm{W}}^{-2}}{\mathcal{V}} \sim \sum_{\mathfrak{q}} \frac{g_\mathrm{W}^{\mathfrak{q}}(t, S_1) \mathcal{E}_{\mathfrak{q}}^{(W)}}{\mathcal{V}} \,, \tag{23}$$

where the sums run over KK and winding states, respectively. Also, $\mathcal{E}^{(K)}$ and $\mathcal{E}^{(W)}$ are again unknown functions of the complex structure and open string moduli, t stands for the 2-cycle volumes (in the Einstein frame; see appendix C.1) and the functions $g_\mathrm{K}(t, S_1)$ and $g_\mathrm{W}(t, S_1)$ determine the scaling of the KK and winding mode masses with the Kähler moduli and the dilaton.[7] As we review in appendix E, in the toroidal orientifold case the suppression by the overall volume arises naturally through the Weyl rescaling to the 4-dimensional Einstein frame.

Starting with the ansatz (23) for smooth Calabi-Yau manifolds, the known form (22) for toroidal orientifolds follows simply by substituting g_K, g_W and the intersection numbers for the toroidal orientifold case. In particular, $g_\mathrm{K} \sim t_i$ for the 2-cycle transverse to the relevant D7-brane, while $g_\mathrm{W} \sim t_i^{-1}$ for the 2-cycle along which the two D7-branes intersect. Then, the first of the terms in (23) reduces to $\mathcal{E}_i^{(K)}/(S_1 \tau_i)$ for toroidal orientifolds, the second to $\mathcal{E}_i^{(W)}/(\tau_j \tau_k)$ with $j \neq i \neq k$, cf (50). Our strategy in the following chapters will therefore be to assume a scaling like (23) for the 1-loop corrections to the Kähler potential for general Calabi-Yau spaces.

As already mentioned, the dependence on the complex structure and open string moduli cannot be inferred by analogy to the orientifold case. We parameterize our igno-

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[7]In rewriting the sums over KK and winding states in terms of the functions g and $\mathcal{E}$, we assume that the dependence of the corresponding spectra on the complex structure and Kähler moduli factorizes. In the known examples of toroidal orientifolds (with or without world-volume fluxes), this is always the case, cf. [52]. Moreover, in general there can appear several contributions (denoted by $\mathfrak{a}$ and $\mathfrak{q}$) depending on which tower of KK or winding states are exchanged in a given process. We will see explicit examples of this in the following.

rance by keeping the expressions $\mathcal{E}$ in (23) as unknown functions of the corresponding moduli. Then we investigate the consequences of the one-loop terms, depending on the size of these unknown functions at the minimum of the potential for the complex structure and open string moduli. Some further comments on the form of ΔK_{g_s} will appear in section 5.2.

3.2   LVS with loop corrections

Thus, allowing for string loop corrections of the form (23) in (9), and expanding the α' correction as in (10), we can write

$$
\begin{aligned}
K &= -\ln(2S_1) - 2\ln(\mathcal{V}) + K_{\mathrm{cs}}(U,\bar{U}) - \frac{\tilde{\xi}S_1^{3/2}}{\mathcal{V}} + \sum_{\mathfrak{a}} \frac{g_{\mathrm{K}}^{\mathfrak{a}} \mathcal{E}_{\mathfrak{a}}^{(K)}}{S_1 \mathcal{V}} + \sum_{\mathfrak{q}} \frac{g_{\mathrm{W}}^{\mathfrak{q}} \mathcal{E}_{\mathfrak{q}}^{(W)}}{\mathcal{V}} \,, \\
W &= W_0 + \sum_i A_i e^{-a_i T_i} \,,
\end{aligned}
\tag{24}
$$

where as explained in the previous section, we have not specified the explicit form of the loop corrections $\mathcal{E}$, that are allowed to be functions of U (and in general of the open string moduli, that we neglect in our analysis, assuming that they can be stabilized by fluxes). The Kähler potential for the complex structure moduli $K_{\mathrm{cs}}(U,\bar{U})$ is left unspecified in (24), indeed we will not need its explicit form. For consistency, we have also included loop corrections to the α' correction.[8] This changes ξ to $\tilde{\xi}$, which is a small change; for $S_1 = 10$, numerically $\tilde{\xi} \approx 1.02\,\xi$.

Neglecting fluxes, the functions $g_{\mathrm{K}}^{\mathfrak{a}}$ and $g_{\mathrm{W}}^{\mathfrak{q}}$ are proportional and inversely proportional to some 2-cycle volume, respectively. (We will come back to corrections from fluxes in appendix D.) When using a particular basis of 2-cycles (with volumes t_i as in appendix C.1), the 2-cycle volume appearing in $g_{\mathrm{K}}^{\mathfrak{a}}$ or $g_{\mathrm{W}}^{\mathfrak{q}}$ might be given by a linear combination $t_{\mathfrak{a}} = \sum_i c_i t_i$ of the basis cycles t_i (and similarly for $t_{\mathfrak{q}}$). Depending on which 2-cycle is the relevant one, this linear combination might or might not contain the large 2-cycle $t_b \sim \mathcal{V}^{1/3}$, which always exists in LVS. If it is present in the linear combination, one can neglect the contribution of the small 2-cycles to leading order in a large volume expansion and obtains possible terms proportional to $\mathcal{E}_b^{(K)} S_1^{-1} \mathcal{V}^{-2/3}$ or $\mathcal{E}_b^{(W)} \mathcal{V}^{-4/3}$, where the subscript b refers to the large 4-cycle τ_b.

---

[8]We remind the reader that the α' correction arises from the R^4 term in 10 dimensions whose coefficient receives corrections at 1-loop (and from D-instantons). The 1-loop correction amounts to a shift of the prefactor from ξ to $\tilde{\xi} = \xi\left(1 + \frac{\pi^2}{3\zeta(3)S_1^2}\right)$, see for instance [53] for a review.

Before getting into the details, it is hard to resist trying to anticipate what might happen. For those terms that are *more* suppressed in volume than the $\tilde{\xi}$ term (e.g $\mathcal{E}_b^{(W)}$), one would expect the loop corrections to have little effect on stabilization. They could still represent a small but interesting correction to physical quantities in LVS. For those that are *less* suppressed in volume than the $\tilde{\xi}$ term (e.g. $\mathcal{E}_b^{(K)}$), one would expect the loop correction to have a huge effect on stabilization, and severely constrain the allowed values for the complex structure moduli and the dilaton in LVS (in particular, constrain them to a region in moduli space where the function $\mathcal{E}_b^{(K)}$ takes very small values). We will find, however, that this expectation is sometimes too naive. For example, there can be cancellations in the scalar potential that are not obvious from just looking at the Kähler potential.

Let us now get into more detail on what happens in the LVS model with loop corrections.

3.3 The $\mathbb{P}^4_{[1,1,1,6,9]}$ model

We would now like to specify the general form of the Kähler- and superpotential (24) to the case of the $\mathbb{P}^4_{[1,1,1,6,9]}$ model. In this space, the divisors that produce nonperturbative superpotentials when D7- (or D3-) branes are wrapped around them do not intersect, as reviewed for instance in [48]. Therefore, we do not expect any correction of the $\mathcal{E}^{(W)}$ type in this model (for the generalization to models where there are such intersections, see appendix D). Moreover, we neglect flux corrections to the KK mass spectrum in the main text. It is shown in appendix D that, for small fluxes, this correctly captures all the qualitative features we are interested in, and it leads to much clearer formulas. Thus, we now consider the scalar potential resulting from

$$
\begin{aligned}
K &= -\ln(2S_1) - 2\ln(\mathcal{V}) + K_{\mathrm{cs}}(U,\bar{U}) - \frac{\tilde{\xi}S_1^{3/2}}{\mathcal{V}} + \frac{\sqrt{\tau_b}\mathcal{E}_b^{(K)}}{S_1\mathcal{V}} + \frac{\sqrt{\tau_s}\mathcal{E}_s^{(K)}}{S_1\mathcal{V}} \ , \\
W &= W_0 + Ae^{-aT_s}.
\end{aligned}
\tag{25}
$$

As τ_b is very large the corresponding non-perturbative term in the superpotential of (24) can be neglected, which allowed us to simplify the notation by setting $A_s = A$ and $a_s = a$.

The general structure of the scalar potential was already given in (12). The three

16

contributions at leading order ($\mathcal{O}(\mathcal{V}^{-3})$) in the large volume expansion are

$$V_{\mathrm{np1}} = e^{K_{\mathrm{cs}}}\frac{24a^2|A|^2\tau_s^{3/2}e^{-2a\tau_s}}{\mathcal{V}\Delta} \ , \tag{26}$$

$$V_{\mathrm{np2}} = -e^{K_{\mathrm{cs}}}\frac{2a|AW_0|\tau_s e^{-a\tau_s}}{S_1\mathcal{V}^2}\left[1+\frac{6\mathcal{E}_s^{(K)}}{\Delta}\right] \ , \tag{27}$$

$$V_3 = \frac{3e^{K_{\mathrm{cs}}}|W_0|^2}{8\mathcal{V}^3}\left[S_1^{1/2}\tilde{\xi}+\frac{4(\mathcal{E}_s^{(K)})^2\sqrt{\tau_s}}{S_1^2\Delta}\right] \ , \tag{28}$$

where the axion has already been minimized for, as discussed in section A.2, and

$$\Delta \equiv \sqrt{2}S_1\tau_s - 3\mathcal{E}_s^{(K)} \ . \tag{29}$$

The leading α'-correction is the $\tilde{\xi}$ term in V_3 above. We now see that it scales with the volume and the string coupling $g_{\mathrm{s}} = 1/S_1$ as claimed in the Introduction, in eq. (3). Also the volume dependence of the loop correction ($\mathcal{E}_s^{(K)}$ term) in V_3 is as announced in (3). The g_{s} factors seem to differ from (3); we see g_s^2, g_s^2 and g_s^3 for V_{np1}, V_{np2} and V_3, respectively. This is because the g_{s} dependence advertised in (3) arises in models where, unlike in $\mathbb{P}^4_{[1,1,1,6,9]}$, the $\mathcal{E}^{(W)}$ correction is present as well, cf. appendix D.[9] It is also worth mentioning that the loop correction proportional to $\mathcal{E}_s^{(K)}$ modifies V_{np1} and V_{np2} at leading order in the $\mathcal{V}$-expansion. whereas the α' correction does not; it only appears in V_3. This is so even though both corrections are equally suppressed in the Kähler potential (i.e. $\sim \mathcal{V}^{-1}$). The reason for this can be traced back to the fact that the loop-correction explicitly depends on τ_s and not only on the overall volume, cf. the discussion in appendix C.4 and C.5.

As anticipated, $\mathcal{E}_b^{(K)}$ and its first derivatives appear only at the next order, $\mathcal{O}(\mathcal{V}^{-10/3})$:

$$V_{10/3} = 2\frac{6^{1/3}|W_0|^2e^{K_{\mathrm{cs}}}}{S_1^3\mathcal{V}^{10/3}}\left[(\mathcal{E}_b^{(K)})^2+\frac{3}{4}\partial_\alpha\mathcal{E}_b^{(K)}\partial_{\bar{\alpha}}\mathcal{E}_b^{(K)}K_{\mathrm{cs}}^{\alpha\bar{\alpha}}\right], \tag{30}$$

where $\partial_\alpha = \partial/\partial U^\alpha$ and $\partial_{\bar{\alpha}} = \partial/\partial\bar{U}^{\bar{\alpha}}$ and α enumerates the complex structure moduli. For $\mathcal{E}_b^{(K)} = \mathcal{E}_s^{(K)} = 0$, the potential terms at leading order coincide with the original case discussed in (56), cf. appendix A.1. The singularity from zeros of the denominator is an artifact of the expansion as discussed in appendix B. The range of validity is

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[9]There, it is shown that including the effect of fluxes on the KK spectrum might also produce this behavior.

limited to the range in moduli space where the denominator Δ does not become too small. It is also apparent that the loop terms are subleading in a large τ_s, large S_1 expansion. However, depending on the relative values of the parameters $\{\mathcal{E}_s^{(K)}, \tau_s, S_1\}$, a truncation to the first terms in such an expansion may or may not be valid. We perform a numerical comparison of the two contributions to V_3 in figure 4.
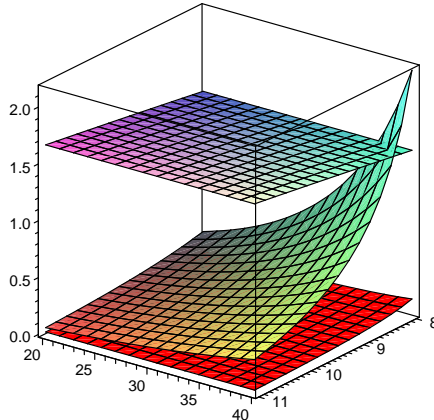

FIGURE 4: The top surface is the α' correction, the second is the g_s correction, and the "red carpet" is $10/\Delta$ (we used the values $A = 1, W_0 = 1, a = 2\pi/8$). We see that for most of the parameter range, the α' correction dominates, and only for large $\mathcal{E}_s^{(K)}$, with the string coupling $g_\mathrm{s} = 1/S_1$ not too small, do the contributions become comparable.

We can understand the volume dependence of the terms (26)-(30) as follows. The common prefactor e^K gives an overall suppression $\tau_b^{-3} \simeq \mathcal{V}^{-2}$. The quantum corrections obey the rule that a term proportional to $1/\tau_b^\lambda$ in K appears in V_3 at order $1/\tau_b^{\lambda+3}$ (where the $+3$ comes from the overall e^K factor) for all values of λ *except* for $\lambda = 1$. When it does appear, it is generated by the term $(K^i K_i - 3)$ and breaks the no-scale structure. For $\lambda = 1$ there is a cancellation at leading order, so it appears only at order $1/\tau_b^{2+3}$ (see appendix C.3 and C.4). This rule can explicitly be verified in our calculation: the α' and the $\mathcal{E}_s^{(K)}$ corrections are suppressed by $1/\tau_b^{3/2}$ in K, and therefore they appear with the suppression $1/\tau_b^{9/2}$ in V_3. On the other hand, for the $\mathcal{E}_b^{(K)}$ term a cancellation takes place to leading order ($\lambda = 1$). It appears neither in V_{np1} nor in V_{np2} at leading order (which can be understood more generally, cf. appendix C.5). Thus, it only appears subleading in the potential, at $\mathcal{O}(\mathcal{V}^{-10/3})$.[10]

---

[10]This cancellation for $\lambda = 1$ was already noticed in [51], albeit in the case without nonperturbative superpotential. In [54] it was argued that this cancellation can be understood from a field redefinition

We now proceed to minimize the potential (26)-(28), using the same strategy as in the case without loop corrections, cf. section 2.3 and appendix A.1. The equations $\partial_{\mathcal{V}} V = 0 = \partial_{\tau_s} V$ are of course more complicated now, but it is easy to solve them numerically. Doing so we find that the volume $\mathcal{V}$ and the small 4-cycle volume τ_s, viewed as functions of S_1 and $\mathcal{E}_s^{(K)}$, are well fit by linear functions when restricted to a sufficiently limited range in parameter space. For example,

range: $S_1 = [8, 11]$, $\mathcal{E}_s^{(K)} = [20, 40]$

$$
\begin{aligned}
\log_{10} \mathcal{V} &= 1.720\, S_1 - 0.1208\, \mathcal{E}_s^{(K)} - 3.437 \ , \\
\tau_s &= 5.000\, S_1 - 0.3581\, \mathcal{E}_s^{(K)} - 8.638 \ .
\end{aligned}
\tag{31}
$$

The fits are quite good; the error is no greater than ± 0.3 for τ_s and ± 0.1 for $\log_{10} \mathcal{V}$ in this range, for an $\{S_1, \mathcal{E}_s^{(K)}\}$ grid of 40^2 points.

From (31) we see an interesting difference to the case without loop corrections. The value of τ_s at the minimum depends on the complex structure moduli U, through $\mathcal{E}_s^{(K)}$. This is in contrast to the case without loop corrections, where the value of τ_s is only determined by the value of the Euler number ξ and the dilaton S_1, cf. (57) below. It is analogous to the perturbative stabilization in [51] where the volume at the minimum of the potential also depends on U.

The result (26)-(28) was derived in a particular model, but we expect the appearance of loop corrections in V to be more general. This opens up the possibility that in principle, one might obtain large volume minima even for manifolds of vanishing (or even positive) Euler number, where LVS is not applicable, as LVS-style stabilization only holds for one sign of ξ. In practice it might be difficult to get large enough values for the 1-loop corrections to stabilize τ_s at a value sufficiently bigger than the string scale. This deserves further study.

We also note that the special structure of (28) and (30), i.e. the appearance of $\mathcal{E}_s$ only in (28) and of $\mathcal{E}_b$ only in (30), offers additional flexibility in tuning the relative size of these terms in a purely perturbative stabilization of the Kähler moduli along the lines of [51,55]. Also this point deserves further study.

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argument combined with the no-scale structure of the tree-level Kähler potential. That argument holds for the case of a single Kähler modulus T with tree-level Kähler potential $-3\ln(T + \bar{T})$ and under the assumption that the coefficient of the loop correction to the Kähler potential $\sim (T + \bar{T})^{-1}$ is independent of the complex structure moduli and the dilaton. Here, these assumptions do not hold, but we showed that the term $\sim (T_b + \bar{T}_b)^{-1}$ in the Kähler potential nevertheless only appears at subleading order in the *potential* in LVS, cf. (26)-(30).

4 Gaugino masses

Now that we know how the stabilization of the (Kähler) moduli is modified by loop corrections, it is natural to extend our analysis to the soft supersymmetry breaking Lagrangian (For a review see for instance [56,57].) In LVS, supersymmetry breaking is mostly due to F-terms: $F_s \neq 0$, $F_b \neq 0$. These determine the soft supersymmetry breaking terms which can be present in the low energy effective action without spoiling the hierarchy between the electroweak and the Planck scale,

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{MSSM}} + \mathcal{L}_{\text{soft}} \ . \tag{32}$$

The soft Lagrangian contains gaugino masses M, scalar masses m, further scalar bilinear terms B and trilinear terms A. (For explicit expressions, see the aforementioned reviews, or e.g. [6].)

Let us start considering gaugino masses. In [6] it was shown that in LVS, gaugino masses M_a are generically suppressed with respect to the gravitino mass $m_{3/2}$:

$$|M_a| \simeq \frac{m_{3/2}}{\ln(1/m_{3/2})} \left[1 + \mathcal{O}\left(\frac{1}{\ln(1/m_{3/2})} \right) \right] \tag{33}$$

(we use units in which $M_{\text{Pl}} = 1$). This suppression results from a cancellation of the leading order F-term contribution to gaugino masses. We briefly review this calculation. Given the F-terms

$$F^I = e^{K/2} G^{\bar{J}I} D_{\bar{J}} \bar{W} \ , \tag{34}$$

gaugino masses are given by [56]

$$M_a = \frac{1}{2} \frac{1}{\text{Re} f_a} \sum_I F^I \partial_I f_a \ , \tag{35}$$

where f_a are the gauge kinetic functions and a labels the different gauge group factors. In LVS the Standard Model (SM) gauge groups arise from D7-branes wrapped around small 4-cycles. We do not try to go into the details of how to embed the SM concretely, but we mention that different gauge group factors might arise from brane stacks wrapping the same 4-cycle if world volume fluxes are present on the branes. In that case the gauge kinetic functions are given by[11]

$$f_a = \frac{T_a}{4\pi} + h_a(\mathcal{F})S + f_a^{(1)}(U) \ , \tag{36}$$

---

[11]We use the "phenomenology" normalization of the gauge generators, in the language of [58]; that explains the relative factor of 4π in (36).

where h_a depends on the world volume fluxes and we also included a possible 1-loop correction to the gauge kinetic function which depends on the complex structure (and possibly open string) moduli. If several gauge groups arise from branes wrapped around the same cycle, the same Kähler modulus T would appear in all of them. From (36) it is clear why the D7-branes of the SM have to wrap small 4-cycles, because otherwise the gauge coupling would come out too small (unless there is an unnatural cancellation between the different contributions to f_a).

As is also apparent from (36), the gauge kinetic function in general depends not only on the Kähler moduli but also on the dilaton and the complex structure. Thus, according to (35) we need to know F^U, F^S and F^i for the small Kähler moduli.[12] From the definition (34), it is clear that F^U and F^S might be non-vanishing even though we assume $D_U W = 0 = D_S W$, provided the inverse metric $G^{\bar{J}I}$ contains mixed components between Kähler moduli on the one hand and complex structure moduli and dilaton on the other hand. Without loop corrections (i.e. considering only the leading α' correction) there is no mixing between the Kähler and complex structure moduli, and one finds

$$F^U = 0 , \quad F^S \sim \mathcal{O}(\mathcal{V}^{-2}) \quad \text{and} \quad F^i \sim \mathcal{O}(\mathcal{V}^{-1}) \quad \text{(without loop corrections)} . \quad (37)$$

Thus, at leading order in the large volume expansion, the sum in (35) only runs over the Kähler moduli. Moreover, taking into account the linear dependence of the gauge kinetic functions (36) on the (small) Kähler moduli, the sum effectively only involves a single term, i.e.

$$M_a = \frac{1}{8\pi} \frac{1}{\mathrm{Re} f_a} F^a + \mathcal{O}(\mathcal{V}^{-2}) , \quad (38)$$

where F^a is the F-term of the (small) Kähler modulus appearing in f_a.

As a concrete example we consider again the $\mathbb{P}^4_{[1,1,1,6,9]}$ model with only one small Kähler modulus τ_s. The corresponding F-term is given by

$$
\begin{aligned}
F^s &= e^{K/2} \left(G^{\bar{s}s} \partial_{\bar{s}} \bar{W} + (G^{\bar{s}s} K_{\bar{s}} + G^{\bar{b}s} K_{\bar{b}}) \bar{W} \right) \\
&= 2\tau_s e^{K/2} \bar{W}_0 \left(\left(1 - \frac{3}{4a\tau_s}\right) - 1 + \mathcal{O}((a\tau_s)^{-2}) \right) + \mathcal{O}(\mathcal{V}^{-2}) ,
\end{aligned}
\quad (39)
$$

where we used (20) and (61) for the first term and (64) for the second.

---

[12]With a slight abuse of notation, we denote the F-terms of the Kähler moduli by the index i, but the F-terms of the other moduli are identified by the symbol for the corresponding modulus, like F^S. This is to avoid introducing too many indices.

Now the leading order cancellation is obvious in (39). Determining the gaugino masses requires dividing by $\mathrm{Re}f_s$, cf. (35). In order to further evaluate this, [6,7] assumed that the dilute flux approximation $f_s = (4\pi)^{-1}T_s$ is valid, i.e. they neglected the contributions from world-volume fluxes and one-loop terms compared to the tree-level term. This puts some constraints on the allowed discrete flux values determining h_s. We want to stress that the cancellation appearing in (39) is independent of this approximation. We are mainly interested in the fate of this cancellation when including loop corrections, and do not have anything to add concerning phenomenological constraints that may arise from imposing the dilute flux approximation. Using it, the gaugino masses simplify to

$$
\begin{aligned}
|M_s| &= \left|\frac{F^s}{2\tau_s}\right| = e^{K/2}|W_0|\left(\frac{3}{4a\tau_s} + \mathcal{O}((a\tau_s)^{-2})\right) \\
&\sim \frac{m_{3/2}}{\ln(1/m_{3/2})}\left[1 + \mathcal{O}\left(\frac{1}{\ln(1/m_{3/2})}\right)\right] ,
\end{aligned}
\tag{40}
$$

which is the announced result. In (40) we used

$$
m_{3/2} \sim |W_0|/\mathcal{V} \qquad \text{and} \qquad a\tau_s \sim \ln(\mathcal{V}/|W_0|) ,
\tag{41}
$$

where the second relation holds in LVS due to (20).

4.1 Including loop corrections

The previous section was a review of the results found in [6]. Now we ask what changes if one considers the loop corrected Kähler potential (25). A priori, as (40) results from a leading order cancellation, one might wonder whether loop corrections might spoil this small hierarchy between the gaugino and gravitino masses. To address this concern we start by observing that the gaugino masses are still determined by the F-terms of the small Kähler moduli (in the large volume limit). More precisely, the scaling of the F-terms (37) now becomes

$$
F^U = \mathcal{O}(\mathcal{V}^{-2}), \quad F^S \sim \mathcal{O}(\mathcal{V}^{-2}) \quad \text{and} \quad F^i \sim \mathcal{O}(\mathcal{V}^{-1}) \quad \text{(with loop corrections)} ,
\tag{42}
$$

i.e. F^U no longer vanishes, but it is just as suppressed as F^S.

We again focus on the $\mathbb{P}^4_{[1,1,1,6,9]}$ model and ask how (39) is modified by loop corrections. Amongst other things, we need to generalize equation (20) to include loop corrections, since we need it to calculate the first term in (39). Thus, we need to

extremize the potential again with respect to τ_s by setting

$$
\begin{aligned}
\partial_{\tau_s} V \;=\; \Bigg\{ &-\frac{12\sqrt{\tau_s}a^2}{\mathcal{V}\Delta^2}\Big[\Big(4a\tau_s-1\Big)\Delta+6\mathcal{E}_s^{(K)}\Big]X^2 \\
&+\frac{2a|W_0|}{\mathcal{V}^2 S_1\Delta^2}\Big[\Big(a\tau_s-1\Big)\Big(\Delta^2-18(\mathcal{E}_s^{(K)})^2\Big)+6\sqrt{2}aS_1\tau_s^2\mathcal{E}_s^{(K)}\Big]X \\
&-\frac{3|W_0|^2(\mathcal{E}_s^{(K)})^2}{4S_1^2\mathcal{V}^3\Delta\sqrt{\tau_s}} \Bigg\} \, e^{K_{\mathrm{cs}}}
\end{aligned}
\tag{43}
$$

to zero. Obviously, $X=0$ is no longer a solution. Instead, there are now two non-trivial solutions, one of which goes to zero in the limit $\mathcal{E}_s^{(K)}\to 0$. This solution corresponds to a maximum of the potential, so it is of no use to us here. We can expand the other solution for large $a\tau_s$, as in the case without loop corrections, yielding

$$
X = Ae^{-a\tau_s} = \frac{\sqrt{2}|W_0|}{24a\mathcal{V}}\sqrt{\tau_s}\left(1-\frac{3}{4a\tau_s}\left(1-\frac{2\sqrt{2}a\mathcal{E}_s^{(K)}}{S_1}\right)\right)+\mathcal{O}\left(\frac{1}{(a\tau_s)^2}\right) .
\tag{44}
$$

Another ingredient we need is the quantity $G^{\bar{\imath}s}K_{\bar{\imath}}$, in order to evaluate the second term in (39). Using equation (65) we obtain

$$
\begin{aligned}
G^{\bar{\imath}s}K_{\bar{\imath}} \;=\; &-2\tau_s\left(1+\frac{6\mathcal{E}_s^{(K)}}{\Delta}\right)+\dots \\
\;=\; &-2\tau_s-\frac{6\sqrt{2}\mathcal{E}_s^{(K)}}{S_1}-\frac{18(\mathcal{E}_s^{(K)})^2}{S_1^2\tau_s}-\frac{27\sqrt{2}(\mathcal{E}_s^{(K)})^3}{\tau_s^2 S_1^3}+\mathcal{O}\left(\frac{1}{\tau_s^3}\right)+\dots ,
\end{aligned}
\tag{45}
$$

where the ellipsis represents terms that are more suppressed in $\mathcal{V}^{-1}$.

Now we see from (44), (45) and (65) that at leading order in an expansion in $a\tau_s$, the quantities relevant to evaluate (39) are not affected by the loop corrections. Thus, the leading order cancellation in the gaugino masses survives the inclusion of loop effects.[13] At first glance, though, equations (44), (45) and (65) seem to suggest a correction to the subleading term, that could potentially give a significant contribution to the gaugino masses after the leading-order cancellation, cf. (39).

In the actual calculation, this contribution drops out. Putting all the ingredients together (and employing the dilute flux approximation again), the gaugino mass turns

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[13]One might argue that this result was to be expected, because the main assumption of [6] is that the stabilization is due to nonperturbative effects, i.e. the dominant effect in $\partial_{\tau_s} V$ should arise from the nonperturbative superpotential. However, in view of (43), it is no longer obvious that the nonperturbative terms dominate when loop corrections are included.

out to be

$$|M_s| = \left|\frac{F^s}{2\tau_s}\right| = 3e^{K/2}|W_0|\left|\frac{1}{4a\tau_s} + \frac{1}{16a^2\tau_s^2} + \frac{S_1 - 12\sqrt{2}a\mathcal{E}_s^{(K)}}{64S_1a^3\tau_s^3} + \cdots\right|$$
$$\sim \frac{m_{3/2}}{\ln(1/m_{3/2})}\left[1 + \mathcal{O}\left(\frac{1}{\ln(1/m_{3/2})}\right)\right] . \tag{46}$$

The result of [6] is therefore very robust. Unexpectedly, the correction to (40) due to $\mathcal{E}_s^{(K)}$ only appears at sub-sub-leading order in the $1/\ln(1/m_{3/2})$ expansion.

4.2 Other soft terms

In [7] all other soft terms were calculated for LVS. The main result (see p. 15 of [7]) is that roughly speaking, all the soft parameters are determined by F^s and by the power with which the chiral matter metrics scale with τ_s. As we saw in the previous section, F^s gets modified by loop corrections only at sub-sub-leading order in a $1/\tau_s$ expansion (see (46)). Therefore, the calculation of all the soft terms in [7] appears to be quite robust against including loop effects.

One of the key assumptions in [7] is that all the Yukawa couplings Y are already present in perturbation theory, i.e. they have the schematic form $Y = Y^{\text{pert}}(U) + Y^{\text{np}}(e^{-T})$. This requirement featured prominently already in the derivation of the volume dependence of the chiral matter metrics in [59] by scaling arguments. In [7] the same schematic form is essential for simplifying the trilinear soft terms A. In general these terms receive a contribution of the schematic form

$$F^T\partial_T\log Y = F^T\frac{\partial_T(Y^{\text{pert}}(U) + Y^{\text{np}}(e^{-T}))}{Y^{\text{pert}}(U) + Y^{\text{np}}(e^{-T})} \sim \frac{\mathcal{O}(e^{-T})}{\mathcal{O}(T^0) + \mathcal{O}(e^{-T})} , \tag{47}$$

which is exponentially suppressed if and only if $Y^{\text{pert}}(U)$ is non-vanishing. However, in many examples the Yukawa couplings are actually only generated nonperturbatively, see for instance the discussion in [60], and [61] for some examples. This poses a constraint on the way the Standard Model is realized in LVS, if one wants to ensure flavor universality of the soft breaking terms as advertised in [7].

One more comment about the important issue of flavor universality. In [7], section 3.4., it was argued that in LVS, approximate flavor universality is a natural consequence of the zeroth-order factorization of Kähler and complex structure moduli spaces. We provide some more details on the factorized approximation in appendix F.

5 LVS for other classes of Calabi-Yau manifolds?

In section 3.3 and 4 we saw that the 1-loop corrections to the moduli Kähler potential only have relatively small effects on the large volume scenario based on the $\mathbb{P}^4_{[1,1,1,6,9]}$ model of [3]. In this chapter, we would like to ask the question how generic the "Swiss cheese" form is for a Calabi-Yau manifold and if there are other models in which the one-loop corrections discussed above might become more important. This is indeed to be expected if the Calabi-Yau under consideration has a fibered structure, as we explain in the following. If $g_{\rm s}$ corrections do dominate α' corrections, they could ruin the volume expansion of LVS.

5.1 Abundance of "Swiss cheese" Calabi-Yau manifolds

In the LVS examples discussed in [4] the volume in terms of the Kähler moduli takes the "Swiss cheese" form

$$\mathcal{V} = \left(\tau_b + \sum a_i \tau_i\right)^{3/2} - \left(\sum b_i \tau_i\right)^{3/2} - \ldots - \left(\sum c_i \tau_i\right)^{3/2} , \qquad (48)$$

where the coefficients $a_i, ..., c_i$ are only non-vanishing for the small Kähler moduli. The LVS limit consists in scaling the overall volume of the Calabi-Yau more or less isotropically while having small holes inside the manifold. The τ's are linear combinations of $\partial_{t_i}\mathcal{V}$, where now $\mathcal{V}$ is considered as a (cubic) function of the 2-cycle volumes t_i. For the effective field theory analysis to be valid one should not only demand that the 4-cycle volumes τ_i are large compared to the string scale, but also that the 2-cycle volumes t_i are large. In the cases discussed in [4], the linear combinations $\partial_{t_i}\mathcal{V}$ are indeed such that one can have one of them exponentially large and the others small (but still sufficiently larger than the string scale), without taking any of the t_i to be exponentially small. This is obvious for the $\mathbb{P}^4_{[1,1,1,6,9]}$ example where the 2-cycle volume t_b only appears in the definition of one of the τ's, cf. (14), but it is also true for the second example of [4], cf. their formulas (84).

However, the $\mathcal{F}_{18}$ model of [16] does not seem to allow its volume to be written in the form (48) with one Kähler modulus τ_b that can become large while keeping all the others small (again, demanding that the t_i stay larger than 1 in string units). Thus, it is an interesting question how generic or non-generic the "Swiss cheese" Calabi-Yau manifolds are. We do not attempt to give a general answer; instead, we turn to two examples in which the form of the volume differs from (48).

5.2 Toroidal orientifolds

The reason loop corrections may be more important in toroidal orientifolds than in compactifications on "Swiss cheese" Calabi-Yau manifolds is the following. As we already discussed in section 3.1, the conjectured form of 1-loop corrections (23) simplifies in the case of toroidal orientifolds, because they have very special and simple intersection numbers. More concretely, using the definition $\tau_i = \partial_{t_i}\mathcal{V}$, together with the special form of the intersection numbers in the toroidal case, i.e. $\mathcal{V} = t_1 t_2 t_3$, the volume can alternatively be expressed as

$$\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3} = t_i \tau_i \quad \text{(no summation; } i = 1, 2 \text{ or } 3) \ . \tag{49}$$

Thus, formula (23) simplifies and the 1-loop corrections proportional to $\mathcal{E}_i^{(K)}$ are only suppressed by single Kähler moduli instead of by the overall volume. Also the terms proportional to $\mathcal{E}_i^{(W)}$ can be rewritten in the toroidal orientifold case and the Kähler potential takes the form (for the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ example)

$$
\begin{aligned}
K \ = \ & -\ln(2S_1) - 2\ln(\mathcal{V}) + K_{\text{cs}}(U, \bar{U}) - \frac{\xi S_1^{3/2}}{\mathcal{V}} \\
& + \sum_{i=1}^{3} \frac{\mathcal{E}_i^{(K)}(U, \bar{U})}{4\tau_i S_1} + \sum_{i \neq j \neq k}^{3} \frac{\mathcal{E}_k^{(W)}(U, \bar{U})}{4\tau_i \tau_j} \ ,
\end{aligned}
\tag{50}
$$

where the functions $\mathcal{E}$ are non-holomorphic Eisenstein series in this case [37]. It is easy to see that the origin of this simplification is the fact that there is just a single non-vanishing intersection number in the toroidal orientifold case and all Kähler moduli appear linearly in the cubic expression for the volume.

The difference of the toroidal orientifold to the "Swiss cheese" case of LVS can also be seen in the different forms of the functional dependence of the volume on the Kähler moduli. In the toroidal orientifold case one has the relations

$$\partial_{t_1}\mathcal{V} = t_2 t_3 \quad , \quad \partial_{t_2}\mathcal{V} = t_1 t_3 \quad , \quad \partial_{t_3}\mathcal{V} = t_1 t_2 \ , \tag{51}$$

so that two of them will always become large if one takes one of the t_i to be large and demands that the other two stay larger than 1. This also holds for any linear combinations of them. The difference is also obvious from the fact that the 2-cycle volume t_b that is responsible for τ_b becoming large in the LVS examples of [4] always appears cubically in the volume. This is related to the fact that the term $(\tau_b + \sum a_i \tau_i)$ should be the square of a linear combination of the t_i, in order for (48) to be expressible

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as a cubic polynomial in the t_i. In contrast, any (untwisted) 2-cycle volume in the toroidal orientifold case only appears *linearly* in the cubic volume polynomial.

To illustrate the effect of terms in the Kähler potential that are suppressed only by single Kähler moduli instead of the overall volume, we take the Kähler potential (50) and expand V_3 in the region of the Kähler cone where $\tau_1 = \tau_2 = \tau_b \gg \tau_3 = \tau_s$ (as we explained above, at least two of the Kähler moduli have to become large simultaneously, if one wants to avoid any of the 2-cycle volumes becoming very small). This leads to (for simplicity setting all $U_i = U$, all $\mathcal{E}_i^{(K)} = \mathcal{E}^{(K)}$ and all $\mathcal{E}_i^{(W)} = \mathcal{E}^{(W)}$):

$$
V_3 = \frac{|W_0|^2 e^{K_{cs}}}{2 S_1 \mathcal{V}^2} \Bigg\{ \left[\frac{(\mathcal{E}^{(K)})^2 + \frac{1}{2}(\partial_{U\bar{U}} K_{cs})^{-1} \partial_U \mathcal{E}^{(K)} \partial_{\bar{U}} \mathcal{E}^{(K)}}{8 \tau_s^2 S_1^2} + \mathcal{O}\left(\frac{1}{\tau_s^3}\right) \right]
$$

$$
+ \frac{1}{\tau_b} \left[\frac{3 \tilde{\xi} S_1^{3/2}}{4 \sqrt{\tau_s}} + \frac{\mathcal{E}^{(W)}}{\tau_s} + \frac{(\mathcal{E}^{(K)})^2 + (\partial_{U\bar{U}} K_{cs})^{-1} \partial_U \mathcal{E}^{(K)} \partial_{\bar{U}} \mathcal{E}^{(K)}}{4 S_1^2 \tau_s} + \mathcal{O}\left(\frac{1}{\tau_s^{3/2}}\right) \right]
$$

$$
+ \mathcal{O}\left(\frac{1}{\tau_b^2}\right) \Bigg\} . \tag{52}
$$

Obviously, the leading term in the large τ_b expansion now comes from the loop correction and not from the α' term (which term really dominates depends on the values of S_1 and U as well, of course). Thus, an expansion of the potential as in LVS, cf. (16), would not be realized in this case, even if one found a way to lift enough zero modes by fluxes for τ_s to appear in a nonperturbative superpotential.

This toy example was meant to show that for a consistent large volume expansion in models with large hierarchies in the Kähler moduli, it is important to make sure that there are no correction terms in the Kähler potential (from loop or α'-corrections) that are suppressed only by some of the small Kähler moduli. We should stress again that also terms suppressed by the large volume can be dangerous if the suppression is less than for the α' term, i.e. if it is $\tau_b^{-\lambda}$ with $\lambda < 3/2$. The only exception to this rule is the case $\lambda = 1$ as we showed above (and as is shown more generally in appendices C.4 and C.5). In this respect it would be important to know if the conjecture (25) really bears out. If it turns out that the actual form of the 1-loop corrections also contains terms like

$$
\Delta K_{g_s} \stackrel{?}{=} \frac{t_b^{\lambda_1} t_i^{\lambda_2} \mathcal{E}_i^{(K)}}{S_1 \mathcal{V}} , \tag{53}
$$

with $\lambda_1 + \lambda_2 = 1$ but $0 < \lambda_1 \neq 1$ or 0, such a 1-loop correction would spoil the large volume expansion performed in (16).[14]

---

[14]In principle, one would also need an argument that no such terms arise at higher loop order, which

5.3 Fibered Calabi-Yau manifolds

The feature of orientifolds that all Kähler moduli appear linearly in the cubic expression for the volume shows that a similar simplification can occur in the case of $(K3)$ fibered Calabi-Yau manifolds, which also have the property that one Kähler modulus (the one corresponding to the volume of the base) only appears linearly in the cubic expression for the volume. This takes the form

$$\mathcal{V} = t_b \eta_{ij} t_i t_j + d_{ijk} t_i t_j t_k \ , \tag{54}$$

where η_{ij} are the intersection numbers of the $(K3)$ fiber, and neither they nor the triple intersection numbers d_{ijk} contain the index b, which is chosen to denote "base", but it is also suggestively the same index as the one we used for the large Kähler modulus in the $\mathbb{P}^4_{[1,1,1,6,9]}$ model. Two-parameter examples of this type appear in e.g. [47,62]. In a region of the Kähler moduli space where the base t_b is rather large but all the other t_i stay relatively small, the volume is approximately $\mathcal{V} = t_b \eta_{ij} t_i t_j$. Thus, if the Kähler potential has a 1-loop correction $\sim \mathcal{E}_b^{(K)} t_b / \mathcal{V}$, it could be approximated in this region by

$$\frac{\mathcal{E}_b^{(K)} t_b}{\mathcal{V}} \quad \sim \quad \frac{\mathcal{E}_b^{(K)}}{\tau_f} + \mathcal{O}\left(t_b^{-1}\right) \ , \tag{55}$$

where $\tau_f = \eta_{ij} t_i t_j$ is the volume of the $(K3)$ fiber (which is small compared to t_b^2). Obviously, this would lead to a correction to the Kähler potential that is only suppressed by a single (small) 4-cycle volume, similar to the toroidal orientifold example we discussed in the last section.

We should note that this limit (large base and small fiber for $(K3)$ fibered Calabi-Yau manifolds), is quite different from the one performed in the usual LVS of [3], even though both cases involve hierarchical limits of the Kähler moduli. As explained in section 5.1, the LVS limit consists in scaling the overall volume of the Calabi-Yau more or less isotropically while keeping holes in the bulk of the manifold small. In contrast, the limit of large base and small fiber is anisotropic. At the moment we have nothing to add about whether such anisotropic configurations with all moduli stabilized actually exist. We merely wanted to point out that if they do exist, that would be an example of smooth Calabi-Yau compactifications where the g_s corrections we consider dominate over the α' corrections considered in the large volume limit, as in the toroidal orientifold case.

would, however, have to be further suppressed in the dilaton S_1.

6 Further corrections

In [4], further α' corrections to the string effective action beyond the one in (9) were considered. In the case of bulk α' corrections (i.e. those already present in type IIB bulk theory without D-branes, arising from sphere level) scaling arguments were given as to why they are suppressed in the large volume limit. Although that discussion was surprisingly powerful in its simplicity, we do not consider it completely conclusive, if large hierarchies between the Kähler moduli exist. After all, dimensional analysis alone does not guarantee that the other α'-corrections are always suppressed by additional powers in the overall volume, instead of powers of some of the small Kähler moduli. Moreover, in addition to the bulk α' corrections that appear at order $\mathcal{O}(\alpha'^3)$, in the models of interest for LVS further α'-corrections arise on the worldvolume of D-branes and O-planes, cf. [63–70]. These corrections begin already at order $\mathcal{O}(\alpha')$ and scaling arguments of the kind used for the bulk corrections do not seem to be sufficient to neglect them.

Indeed, there are correction terms involving two powers of the Riemann tensor which do modify the effective D3-brane charge and tension, if the D7-branes are wrapped over 4-cycles with non-vanishing Euler number. These terms were already taken into account in [1]. However, there are further contributions to the DBI action at the same order in α', like $F_3^2 R$ or F_3^4, where F_3 stands for the RR 3-form field strength, R for the Riemann tensor and we left index contractions unspecified. If the D7-branes do not break supersymmetry and remain BPS, it seems unlikely that these terms could contribute to the potential for the closed string moduli, i.e. induce some effective D3-brane tension. The reason is that there does not seem to be a corresponding term in the Chern-Simons action that could lead to the necessary modification of the effective D3-brane charge at the same time. This could be checked in more detail.

In general, we think that the question of additional corrections to the moduli (Kähler) potential deserves further attention. Here we only outlined some steps in that direction.

7 Conclusions

In this paper, we have investigated whether string loop corrections may impact *a)* stabilization in the large volume compactification scenario (LVS), and *b)* the phenomenology of those scenarios, as manifested in the soft supersymmetry breaking terms. The result

is that for the specific class of compactification manifolds considered in LVS, so-called "Swiss cheese" Calabi-Yau manifolds, changes are minuscule. Only if the loop corrections become abnormally large (in the toroidal orientifold case, this can happen if the complex structure is stabilized very large) do they affect LVS. For other classes of manifolds, the corrections may be important. We hasten to add that the detailed expressions for the loop corrections in LVS remain unknown; we have merely tried to infer their scaling with the Kähler moduli from experience in the toroidal orientifold limit. We think it is important to attempt to address this issue, as the string coupling is stabilized at a nonzero value, so the corrections cannot be turned off.

We also stress the (to some readers obvious) fact that there remain a host of issues that must ultimately be dealt with if one wishes to claim that these are "string compactifications".

- We cannot be sure that fluxes do not alter the corrections, since backgrounds with RR and NSNS fluxes are not well understood in string perturbation theory.

- Additional corrections may appear (see section 6) that could be equally threatening to LVS as the loop corrections, or worse.

- In [37] only the corrections to the Kähler potential coming from $\mathcal{N} = 2$ sectors were determined and we based our generalization on those results. However, there might be interesting corrections coming from the $\mathcal{N} = 1$ sectors as well.

- It has not yet been shown that a local Standard Model-like construction can be embedded in the simplest examples like the $\mathbb{P}^4_{[1,1,1,6,9]}$ model. If more general models turn out to be needed, one needs to reconsider whether the requisite nonperturbative superpotentials are generated.

- We have largely ignored open string moduli, under the proviso that they are stabilized heavy, as are the dilaton and complex structure moduli.

- The coefficient $A(S, U)$ in the nonperturbative superpotential is generally assumed to be of order 1. It is not known how generic this is.

- All string computations we have discussed were performed in a supersymmetric context. In LVS supersymmetry is broken already before uplift, in the AdS minimum. Supersymmetry breaking directly in string theory is not very well understood [39,71].

Faced with all these caveats, a pessimist might be inclined to give up. We think we have shown that it is worth considering these issues in detail. Sometimes, an effect one would have thought to be devastating turns out to be as gentle as a summer breeze.

Acknowledgments

A   Some details on LVS

In this appendix we collect some details on the minimization of the potential in LVS, mainly reviewing the results of [3,8], but filling in some details. The minimization with respect to the axions (i.e. the imaginary parts of the Kähler moduli) is performed for an arbitrary number of Kähler moduli, while for the minimization of the real parts, we restrict to the example of the hypersurface in $\mathbb{P}^4_{[1,1,1,6,9]}$ discussed throughout the main text.

A.1   LVS for $\mathbb{P}^4_{[1,1,1,6,9]}$

Here we give some more numerical details on large-volume stabilization in the $\mathbb{P}^4_{[1,1,1,6,9]}$ orientifold. The relevant features of this Calabi-Yau have been described in chapter 2.3. The leading terms of the scalar potential are

$$V\, e^{-K_{cs}} = \frac{\lambda\sqrt{\tau}e^{-2a\tau}}{\mathcal{V}} - \frac{\mu}{\mathcal{V}^2}\tau e^{-a\tau} + \frac{\nu}{\mathcal{V}^3} \ , \tag{56}$$

where we use $\tau = \tau_s$ and $\mathcal{V}$ as the independent variables and for the expansion we have in mind the limit (15). The minimum of this potential under the assumption that $a\tau \gg 1$ is given by

$$\tau \;=\; \left(\frac{4\nu\lambda}{\mu^2}\right)^{2/3} \ ,$$

$$\mathcal{V} \;=\; \frac{\mu}{2\lambda}\left(\frac{4\nu\lambda}{\mu^2}\right)^{1/3} e^{a\tau} \ . \tag{57}$$

In the $\mathbb{P}^4_{[1,1,1,6,9]}$ orientifold the coefficients λ, μ and ν can be calculated explicitly, yielding

$$\lambda = \frac{12\sqrt{2}a^2|A|^2}{S_1} \ , \quad \mu = \frac{2a|AW_0|}{S_1} \ \ \text{and} \ \ \nu = \xi\frac{3}{8}\sqrt{S_1}|W_0|^2 \ . \tag{58}$$

We notice that the value of τ at the minimum is determined only by the Euler number $\tau \propto \chi^{2/3}$ and the value of the dilaton S_1 at its minimum. An example of a set of possible parameters (using $a = 2\pi/10$, $A = 1$, $S_1 = 10$ and $W_0 = 10$) is

$$\xi = -\frac{\zeta(3)\chi}{2(2\pi)^3} \simeq 1.31 \quad \longrightarrow \quad \nu \simeq 155 \ ,$$

$$\lambda \simeq 0.67 \ , \qquad \mu = \frac{4\pi}{10} \ . \tag{59}$$

There is an unknown overall factor $e^{K_{cs}}$ that does not change the shape of the potential and so leaves the position of the minima unchanged. For the parameters given in equation (59), the minimum is at $\tau \simeq 41.1$ and $\mathcal{V} \simeq 9.96 \cdot 10^{11}$. These values come from equation (57) which is just approximated using the assumption $a\tau \gg 1$. This solution has the shortcoming that, if one is interested in the value of the potential at the minimum, after substitution of (57) into (56), one finds $V = 0$. If instead one solves the exact equation for the minimum of the potential numerically, the result is $V \simeq -6.6 \cdot 10^{-37}$ at the point $\tau \simeq 41.7$ and $\mathcal{V} \simeq 1.38\,10^{12}$. From this one checks that, apart from the shortcoming that $V = 0$, the approximate solution gives the position of the minimum with a good precision.

A.2 Many Kähler moduli

The simple picture of $\mathbb{P}^4_{[1,1,1,6,9]}$, gets slightly more involved in models with more than two Kähler moduli, but some general statements can still be made. For a single small Kähler modulus, among the leading contributions to the potential only the one from V_{np2} is axion dependent, while the leading terms in V_3 and V_{np1} are axion independent. For several small Kähler moduli, all three terms are axion dependent. However, the argument that the leading term in V_{np2} only receives a sign change due to axion stabilization generalizes (and holds also for the regular KKLT scenario with relatively small volume, see e.g. [8], section 3.2).

Indeed, with the superpotential (11) one obtains

$$
V_{\mathrm{np1}} \;=\; e^K\, G^{\bar{j}i}\left[a_i a_j |A_i A_j| e^{-a_i\tau_i - a_j\tau_j}\cos(-a_i b_i + a_j b_j + \beta_i - \beta_j)\right]\;,
$$

$$
\begin{aligned}
V_{\mathrm{np2}} \;=\; -2e^K\, a_i G^{\bar{k}i} K_{\bar{k}}\Big[&|A_i W_0| e^{-a_i\tau_i}\cos(-a_i b_i + \beta_i - \beta_{W_0}) \\
&+|A_i A_j| e^{-a_i\tau_i - a_j\tau_j}\cos(-a_i b_i + a_j b_j + \beta_i - \beta_j)\Big]\;,
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
V_3 \;=\; e^K\,(G^{\bar{k}l}K_{\bar{k}}K_l - 3)\Big[&|W_0|^2 + 2\,|W_0 A_i| e^{-a_i\tau_i}\cos(-a_i b_i + \beta_i - \beta_{W_0}) \\
&+|A_i A_j| e^{-a_i\tau_i - a_j\tau_j}\cos(-a_i b_i + a_j b_j + \beta_i - \beta_j)\Big]\;,
\end{aligned}
$$

where $A_i = |A_i|e^{i\beta_i}$, $W_0 = |W_0|e^{i\beta_{W_0}}$ and a sum over repeated indices is understood throughout. As the only dependence on the axions is in form of cosines, one can easily see that this potential has a minimum for

$$
a_i b_i = -\beta_{W_0} + \beta_i + n_i\pi \quad,\quad n_i \in 2\mathbb{Z}+1\;. \tag{61}
$$

We notice that the minimum of the b_i depends on the (already fixed) complex structure moduli, but it is independent of the Kähler moduli.

In the regime (15) the scalar potential again contains three terms at leading order,

$$
\begin{aligned}
V_{\mathrm{np1}} &\sim 2e^{K_{\mathrm{cs}}}\frac{a_i a_j |A_i A_j| e^{-a_i\tau_i - a_j\tau_j} M^l_j M^k_i(-\mathcal{V}\mathcal{V}_{lk} + \mathcal{V}_l\mathcal{V}_k)}{S_1\mathcal{V}^2} + \dots\;, \\
V_{\mathrm{np2}} &\sim -2e^{K_{\mathrm{cs}}}\frac{a_i |A_i W_0| e^{-a_i\tau_i}\tau_i}{S_1\mathcal{V}^2} + \dots\;, \\
V_3 &\sim e^{K_{\mathrm{cs}}}\frac{3\xi S_1^{1/2}}{8\mathcal{V}^3}|W_0|^2 + \dots\;,
\end{aligned} \tag{62}
$$

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where the sum over i and j effectively only picks up terms from the small moduli because of the exponential suppression of V_{np1} and V_{np2}. Moreover, for V_{np1} we used the form (89) for the inverse of the moduli metric with respect to the basis (83). The Kähler moduli appearing in the nonperturbative superpotential are linear combinations of these, which we account for by a basis-changing matrix M_i^k, i.e.

$$T_i = M_i^k \tilde{T}_k \ , \tag{63}$$

where $\tilde{T}_k$ are the fields defined in (83) and T_i are the Kähler moduli appearing in the nonperturbative superpotential. (Another way of saying this is that the real parts of T_i measure the volumes of a basis of divisors that have the right properties to contribute to the nonperturbative superpotential.) In the second term we used

$$G^{\bar{k}i} K_{\bar{k}} = -2\tau_i + \ldots = -2\operatorname{Re}T_i + \ldots \ . \tag{64}$$

In the basis (83), this would follow straightforwardly from (87), (89) and the relations (80), but it holds equally well after a change of basis, because both sides of (64) transform linearly under a change of basis (63).

Note finally that the ellipsis in (62) and (64) stand for subleading corrections in the large volume limit (assuming also (15)).

B Loop corrected inverse Kähler metric for $\mathbb{P}^4_{[1,1,1,6,9]}$

We now have a closer look at the inverse metric from the Kähler potential in equation (25). We invert the 4×4 matrix and focus on the four terms that appear in the scalar potential for the Kähler moduli,

$$
\begin{aligned}
G^{\bar{b}b} &= \frac{4}{3}\tau_b^2 + \mathcal{O}\left(\tau_b\right) \ , \\
G^{\bar{b}s} = G^{\bar{s}b} &= 4\tau_b\tau_s \left(1 + \frac{6\mathcal{E}_s^{(K)}}{\Delta}\right) + \mathcal{O}\left(\tau_b^0\right) \ , \\
G^{\bar{s}s} &= \frac{8}{3}\tau_b^{3/2}\sqrt{\tau_s}\,\frac{\sqrt{2}S_1\tau_s}{\Delta} + \mathcal{O}\left(\tau_b^{1/2}\right) \ ,
\end{aligned}
\tag{65}
$$

where we have performed an expansion in $\tau_b \simeq \mathcal{V}^{2/3}$ and the quantity Δ was introduced in (29). We notice that only $G^{\bar{b}b}$ is not corrected at leading order. The apparent divergence from zeros of the denominator Δ is an artifact of the expansion. In fact, the determinant of the (entire) Kähler metric behaves as

$$\det G \sim A\tau_b^{-7/2} + B\tau_b^{-9/2} + \ldots \tag{66}$$

for some expressions[15] A and B, which depend on the moduli τ_s, U and S_1. In particular, one finds $A \sim \Delta$, but B does not vanish at a zero of Δ. Thus, in general the expansion in large τ_b picks up the factor A, which is responsible for the apparent divergence in (65). However, this is fictitious because when $\Delta = 0$, the next term proportional to B is non-vanishing and the determinant stays away from zero. Indeed, we do not expect to find any zero of the determinant in the range of validity of the parameters.

If $\mathcal{E}_s^{(K)} \ll (S_1 \tau_s)$, one can further expand (65) with respect to $\mathcal{E}_s^{(K)}/(S_1 \tau_s)$, yielding

$$
\begin{aligned}
G^{\bar{b}b} &= \frac{4}{3}\tau_b^2 + \mathcal{O}\left(\tau_b\right), \\
G^{\bar{b}s} = G^{\bar{s}b} &= 4\tau_b\tau_s\left(1 + \frac{3\sqrt{2}\mathcal{E}_s^{(K)}}{S_1\tau_s} + \mathcal{O}\left(\frac{(\mathcal{E}_s^{(K)})^2}{S_1^2\tau_s^2}\right)\right), \\
G^{\bar{s}s} &= \frac{8}{3}\tau_b^{3/2}\sqrt{\tau_s}\left(1 + \frac{3\mathcal{E}_s^{(K)}}{\sqrt{2}S_1\tau_s} + \mathcal{O}\left(\frac{(\mathcal{E}_s^{(K)})^2}{S_1^2\tau_s^2}\right)\right).
\end{aligned}
\tag{67}
$$

Depending on the values of the moduli (τ_s and S_1), this expansion may or may not be useful. In general, only the expansion in τ_b makes sense and one has to deal with the full expressions (65). That is what we did in section 3.3.

C No-scale Kähler potential in type II string theory

In this appendix we review why compactification of type IIA and type IIB theory on general Calabi-Yau manifolds, or orientifolds thereof, lead to no-scale (F-term) potentials if

$$\text{i) the superpotential does not depend on the Kähler moduli} \tag{68}$$

and if

$$\text{ii) one uses the tree-level form of the Kähler potential.} \tag{69}$$

(Of course, in LVS neither i) nor ii) holds, but one can think of jointly imposing i) and ii) as a zeroth-order approximation, that we will successively move away from in later subsections of this appendix.)

---

[15]This A has nothing to do with the A in W_{np}.

If the moduli spaces of Kähler and complex structure moduli factorize (see appendix F for more details on this), and under assumption *i)*, the F-term potential takes the form

$$V \;=\; e^K \left(G^{\bar{I}J} D_{\bar{I}} \bar{W} D_J W - 3|W|^2 \right) \tag{70}$$

$$\;=\; e^K \left(G^{\bar{a}b} D_{\bar{a}} \bar{W} D_b W + (G^{\bar{i}j} K_{\bar{i}} K_j - 3)|W|^2 \right) \; . \tag{71}$$

The indices a and b run over the complex structure moduli and the dilaton, i, j over the Kähler moduli and I and J refer to all moduli.

The condition for a no-scale potential ($V = 0$ for the Kähler moduli) is then

$$G^{\bar{i}j} K_{\bar{i}} K_j = 3 \; , \tag{72}$$

and we will verify in turn that this is fulfilled in both type IIA and type IIB Calabi-Yau compactifications, if one uses the tree-level Kähler potential, as in assumption *ii)*. In that case, the moduli spaces of Kähler and complex structure moduli do factorize exactly.

C.1   No-scale structure in type IIA

The tree level Kähler potential for the Kähler moduli is

$$K = -\ln\left[\frac{1}{48} d_{ijk}(\sigma + \bar{\sigma})_i(\sigma + \bar{\sigma})_j(\sigma + \bar{\sigma})_k\right] = -\ln\left[\frac{1}{6} d_{ijk} t_i t_j t_k\right] = -\ln(\mathcal{V}) \; , \tag{73}$$

where d_{ijk} are the intersection numbers of the Calabi-Yau,

$$d_{ijk} = \int_{CY} \omega_i \wedge \omega_j \wedge \omega_k \; , \tag{74}$$

and

$$\sigma_i = t_i + i c_i \tag{75}$$

are the complexified Kähler moduli whose real parts t_i represent the volumes of 2-cycles and whose imaginary parts originate from the expansion of the NSNS 2-form. Using the Kähler form

$$J = t_i \omega_i \tag{76}$$

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of the Calabi-Yau, it is useful to introduce the notation

$$\begin{aligned}
\mathcal{V} &= \frac{1}{6}\int J \wedge J \wedge J = \frac{1}{6}d_{ijk}t_i t_j t_k \ , \\
\mathcal{V}_i &= \frac{1}{2}\int \omega_i \wedge J \wedge J = \frac{1}{2}d_{ijk}t_j t_k \ , \\
\mathcal{V}_{ij} &= \int \omega_i \wedge \omega_j \wedge J = d_{ijk}t_k \ .
\end{aligned} \tag{77}$$

Note that here the index i does not denote a derivative with respect to the Kähler variables (in contrast to subscripts on the Kähler potential K). Instead, one has the relations $\mathcal{V}_i = 2\partial_{\sigma_i}\mathcal{V}$ and $\mathcal{V}_{ij} = 4\partial_{\sigma_i}\partial_{\bar{\sigma}_j}\mathcal{V}$. It is straightforward to calculate

$$K_i = -\frac{\mathcal{V}_i}{2\mathcal{V}} = K_{\bar{\imath}} \quad G_{i\bar{\jmath}} = K_{i\bar{\jmath}} = -\frac{1}{4}\left(\frac{\mathcal{V}_{ij}}{\mathcal{V}} - \frac{\mathcal{V}_i\mathcal{V}_j}{\mathcal{V}^2}\right) \ . \tag{78}$$

Then one can show that the inverse Kähler metric is

$$G^{\bar{\jmath}i} = -4\mathcal{V}^{ji}\mathcal{V} + 2t_j t_i \ . \tag{79}$$

To verify this, one has to use

$$\mathcal{V}^{ij}\mathcal{V}_j = \frac{1}{2}t_i \ , \quad \mathcal{V}_{ij}t_j = 2\mathcal{V}_i \ , \quad \mathcal{V}_i t_i = 3\mathcal{V} \ . \tag{80}$$

Putting everything together, one arrives at

$$G^{\bar{\imath}j}K_{\bar{\imath}}K_j = \left[-4\mathcal{V}^{ij}\mathcal{V} + 2t_i t_j\right]\frac{1}{4}\frac{\mathcal{V}_i}{\mathcal{V}}\frac{\mathcal{V}_j}{\mathcal{V}} = 3 \ , \tag{81}$$

i.e. (72) is fulfilled under assumption (69).

C.2 No-scale structure in type IIB

In the type IIB case, the tree-level Kähler potential for the Kähler moduli is

$$K = -2\ln\left[\frac{1}{6}d_{ijk}t_i t_j t_k\right] = -2\ln(\mathcal{V}) \ . \tag{82}$$

The difference to the IIA case is that, even if K in (82) is expressed in terms of the 2-cycle volumes t_i, the real parts of the good Kähler moduli, $\tilde{T}_i$, are now the 4-cycle volumes $\tilde{\tau}_i$ (the imaginary parts, on the other hand, arise from the RR 4-form). The relation between them depends on the particular Calabi-Yau:

$$\operatorname{Re}\tilde{T}_i = \tilde{\tau}_i = \frac{1}{2}d_{ijk}t_j t_k = \mathcal{V}_i \ , \tag{83}$$

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which cannot be inverted in general.[16] In order to calculate $K_i = \partial_{\tilde{T}_i} K$ we note that

$$
\begin{aligned}
\partial_{t_i} &= (\partial_{t_i} \tilde{T}_j)\partial_{\tilde{T}_j} + (\partial_{t_i} \bar{\tilde{T}}_{\bar{j}})\partial_{\bar{\tilde{T}}_{\bar{j}}} \\
&= \mathcal{V}_{ij}\left(\partial_{\tilde{T}_j} + \partial_{\bar{\tilde{T}}_{\bar{j}}}\right) \; .
\end{aligned}
\tag{84}
$$

If acting on a function F that only depends on $\tilde{T} + \bar{\tilde{T}}$, as is the case for K, (84) simplifies to

$$
\partial_{t_i} F(\tilde{T} + \bar{\tilde{T}}) = 2\mathcal{V}_{ij}\partial_{\tilde{T}_j} F(\tilde{T} + \bar{\tilde{T}}) \; ,
\tag{85}
$$

where on the left hand side $\tilde{T}$ is understood as a function of t. Alternatively, one has

$$
\partial_{\tilde{T}_i} F(\tilde{T} + \bar{\tilde{T}}) = \frac{1}{2}\mathcal{V}^{ij}\partial_{t_j} F(\tilde{T} + \bar{\tilde{T}}) = \partial_{\bar{\tilde{T}}_{\bar{i}}} F(\tilde{T} + \bar{\tilde{T}}) \; .
\tag{86}
$$

Using this, one can calculate

$$
K_i = -\frac{2}{\mathcal{V}}\partial_{\tilde{T}_i}\mathcal{V} = -\frac{\mathcal{V}^{ij}\mathcal{V}_j}{\mathcal{V}} = -\frac{t_i}{2\mathcal{V}} = K_{\bar{i}} \; ,
\tag{87}
$$

where in the last step we used (80). In the same way one can calculate

$$
G_{i\bar{j}} = \frac{1}{4}\left(-\frac{\mathcal{V}^{ij}}{\mathcal{V}} + \frac{1}{2}\frac{t_i t_j}{\mathcal{V}^2}\right) \; .
\tag{88}
$$

Using this formula one can check that the inverse Kähler metric is given by

$$
G^{\bar{i}j} = 4\left(-\mathcal{V}\mathcal{V}_{ij} + \mathcal{V}_i\mathcal{V}_j\right) \; .
\tag{89}
$$

Putting everything together, no-scale structure holds also for type IIB:

$$
G^{\bar{i}j} K_{\bar{i}} K_j = (-\mathcal{V}\mathcal{V}_{ij} + \mathcal{V}_i\mathcal{V}_j)\frac{t_i}{\mathcal{V}}\frac{t_j}{\mathcal{V}} = 3 \; ,
\tag{90}
$$

again under the assumption (69).

---

[16]Note that the Kähler moduli appearing in the non-perturbative superpotentials in the examples of [16] are related to the ones in (83) by a linear field redefinition. However, this does not play any role in verifying the no-scale structure at leading order, as (90) below is invariant under field redefinitions. We chose to make the distinction clear by using tildes for the Kähler moduli defined by (83).

C.3 Cancellation with just the volume modulus

Now we relax assumption (69). For simplicity, let us first consider the Kähler potential

$$K = -3\ln(T + \bar{T}) + \frac{\Xi}{(T + \bar{T})^{\lambda}} \; , \qquad (91)$$

which corresponds to the case of a single Kähler modulus and the complex structure moduli and the dilaton are neglected. A generic quantum correction was added to the tree level term, which could be an α' or a loop correction, depending on the value of λ. Focusing on V_3, i.e.

$$\frac{V_3}{e^K \, |W|^2} = G^{\bar{j}i} K_{\bar{j}} K_i - 3 \; , \qquad (92)$$

one calculates

$$
\begin{aligned}
\frac{V_3}{e^K \, |W|^2} &= \frac{(3(2\tau)^{\lambda} + \xi\lambda)^2}{3(2\tau)^{2\lambda} + \Xi(2\tau)^{\lambda}\lambda(\lambda+1)} - 3 \\
&= 3 - 3 + \frac{(\lambda - \lambda^2)\Xi}{(2\tau)^{\lambda}} + \frac{\Xi^2\lambda^4}{3(2\tau)^{2\lambda}} + \mathcal{O}\left(\frac{1}{\tau^{3\lambda}}\right) .
\end{aligned} \qquad (93)
$$

This simplified calculation gives an intuition of why the $\mathcal{E}_b^{(K)}$-term does not appear in V_3 of (28) whereas the α'- and $\mathcal{E}_s^{(K)}$-terms contribute. When the exponent of the quantum correction is exactly 1, there is a cancellation at leading order in the scalar potential (compare also the discussion in footnote 10). Note that since we focused on V_3 in this subsection, it did not matter whether assumption (68) holds or not.

C.4 Cancellation with many Kähler moduli

We would now like to see how the previous result is changed when we have an arbitrary number of moduli. We do not make *any* assumption on the dependence of the volume on the Kähler moduli ("Swiss cheese" or fibered manifolds are special cases). Due to its relevance for LVS, we consider a single correction to the Kähler potential which only depends on the large Kähler modulus T_b (an example would be the α'-correction or the loop term proportional to $\mathcal{E}_b^{(K)}$, considering the moduli other than the Kähler moduli as fixed; this is allowed at leading order in a τ_b-expansion, as we argue in appendix F). Thus, we take the Kähler potential to be of the form

$$K = K^{(0)} + \delta K = -2\ln(\mathcal{V}) + \delta K(T_b, \bar{T}_b) \equiv -2\ln(\mathcal{V}) + \frac{\Xi_b}{(T_b + \bar{T}_b)^{\lambda}} \; . \qquad (94)$$

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Again focusing on V_3, we obtain

$$\frac{V_3}{e^K|W|^2} = G^{\bar{j}i}K_{\bar{j}}K_i - 3 = (G_0^{\bar{j}i} + \delta G^{\bar{j}i})\left[K_{\bar{j}}^{(0)} + \delta K_{\bar{j}}\right]\left[K_i^{(0)} + \delta K_i\right] - 3 \,,$$

(95)

where $\delta K_i \equiv \partial_{T_i}\delta K$ and $G_0^{\bar{j}i}$ is the inverse metric of appendix C.2; finally $\delta G^{\bar{j}i}$ is the modification of the inverse metric coming from considering the modified Kähler potential (94). Explicitly one has

$$G^{\bar{j}i} = (G_{i\bar{j}}^0 + \delta K_{i\bar{j}})^{-1} \simeq G_0^{\bar{j}i} - G_0^{\bar{j}h}\delta K_{h\bar{k}}G_0^{\bar{k}i} + \dots \,,$$

$$\delta K_i = -\frac{\lambda\,\Xi_b}{(2\tau_b)^{\lambda+1}}\delta_{ib} \,, \qquad \delta K_{i\bar{j}} = \frac{(\lambda^2 + \lambda)\,\Xi_b}{(2\tau_b)^{\lambda+2}}\delta_{ib}\delta_{\bar{j}b} \,.$$

(96)

We now put everything together and use the results of appendix C.2 and formula (64) (which, for the unperturbed metric and Kähler potential, is an exact equality) to arrive at

$$\frac{V_3}{e^K|W|^2} = \left[G_0^{\bar{j}i}K_{\bar{j}}^{(0)}K_i^{(0)} - 3\right] + 2\left[G_0^{\bar{j}i}K_{\bar{j}}^{(0)}\delta K_i\right] - \left(G_0^{\bar{j}k}\delta K_{k\bar{h}}G_0^{\bar{h}i}\right)K_{\bar{j}}^{(0)}K_i^{(0)} + \dots$$

$$= 0 + \frac{4\lambda\Xi_b}{(2\tau_b)^{\lambda+1}}\tau_b - \frac{4(\lambda^2 + \lambda)\Xi_b}{(2\tau_b)^{\lambda+2}}\tau_b\tau_b + \dots$$

$$= \frac{(\lambda - \lambda^2)\Xi_b}{(2\tau_b)^\lambda} + \dots \,.$$

(97)

We notice that the term $1/\tau_b^\lambda$ vanishes exactly for $\lambda = 1$, independently of the explicit form of the volume in terms of the Kähler moduli. In particular, the loop correction proportional to $\mathcal{E}_b^{(K)}$ experiences a cancellation at leading order in V_3 (and it is not difficult to see that the subleading order is suppressed by $\tau_b^{-2\lambda}$). Therefore, the loop correction is subleading in the potential compared to the α' correction, even though it is leading in the Kähler potential. Next, we would like to extend this analysis to the other parts of the potential, i.e. V_{np1} and V_{np2}.

C.5 Perturbative corrections to V_{np1} and V_{np2}

We now introduce the nonpertubative superpotential into the game, i.e. relax assumption (68), and look at the other terms of the scalar potential, V_{np1} and V_{np2} (see eq. (12)). For this, we restrict to the $\mathbb{P}_{[1,1,1,6,9]}^4$ model again. The contribution V_{np1} is proportional to $G^{\bar{s}s}\bar{W}_{,\bar{s}}W_{,s}$. From (65) we see that no $\mathcal{E}_b^{(K)}$ appears at leading order. This

can be understood as follows. Consider the Kähler potential (94) where now $\mathcal{V}$ is the volume of $\mathbb{P}^4_{[1,1,1,6,9]}$, given in (14). Then the scaling with the large Kähler modulus τ_b is schematically given by

$$
\begin{aligned}
G^{\bar{s}s} &\simeq G_0^{\bar{s}s} - G_0^{\bar{s}b}\delta K_{b\bar{b}}G_0^{\bar{b}s} + \dots \\
&\sim \tau_b^{3/2} + \tau_b^2 \frac{\Xi_b}{\tau_b^{\lambda+2}} \sim \tau_b^{3/2} + \frac{\Xi_b}{\tau_b^\lambda} + \dots \,,
\end{aligned}
\tag{98}
$$

which shows that any loop correction to the Kähler potential of the form Ξ/τ_b^λ leads to a subleading contribution to V_{np1} in the large volume expansion. As usual, the ellipsis stands for terms that are even more subleading in the τ_b expansion.

To understand the $\mathcal{E}_s^{(K)}$ correction to $G^{\bar{s}s}$ we need to consider

$$
K = -2\ln(\mathcal{V}) + \tilde{\delta}K(T,\bar{T}) \equiv -2\ln(\mathcal{V}) + \frac{\Xi_b \, g(T_s,\bar{T}_s)}{(T_b + \bar{T}_b)^\lambda}
\tag{99}
$$

for some function $g(T_s,\bar{T}_s)$ of the small Kähler modulus and we assume $\lambda \geq 3/2$ in the following. Then, again very schematically, the scaling behavior is given by[17]

$$
\begin{aligned}
G^{\bar{s}s} &\simeq G_0^{\bar{s}s} - G_0^{\bar{s}i}\tilde{\delta}K_{i\bar{j}}G_0^{\bar{j}s} + \dots \\
&\sim \tau_b^{3/2} + \Xi_b\left(\tau_b, \tau_b^{3/2}\right)\begin{pmatrix} \tau_b^{-\lambda-2} & \tau_b^{-\lambda-1} \\ \tau_b^{-\lambda-1} & \tau_b^{-\lambda} \end{pmatrix}\begin{pmatrix} \tau_b \\ \tau_b^{3/2} \end{pmatrix} + \dots \\
&\sim \tau_b^{3/2} + \tau_b^{-\lambda} + \tau_b^{-\lambda+3/2} + \tau_b^{-\lambda+3} + \dots \, .
\end{aligned}
\tag{100}
$$

One sees that $\lambda = 3/2$ indeed contributes at the same order as $G_0^{\bar{s}s}$. This is confirmed by the dependence of $G^{\bar{s}s}$ in (65) on $\mathcal{E}_s^{(K)}$ through Δ, cf. (29).

We now consider V_{np2}. This is proportional to $G^{\bar{j}s}K_{\bar{j}}$. Again we start by considering a correction to the Kähler potential whose only dependence on the Kähler moduli is via τ_b, as in (94). Schematically, we find

$$
\begin{aligned}
G^{\bar{j}s}K_{\bar{j}} &\simeq (G_0^{\bar{j}s} - G_0^{\bar{j}b}\delta K_{b\bar{b}}G_0^{\bar{b}s})\left[K_{\bar{j}}^{(0)} + \delta K_{\bar{j}}\right] + \dots \\
&\sim \tau_s + \frac{G_0^{\bar{s}b}\,\Xi_b}{\tau_b^{\lambda+1}} + \dots \sim \tau_b^0 + \frac{\Xi_b}{\tau_b^\lambda} + \dots \, .
\end{aligned}
\tag{101}
$$

This result is confirmed by the absence of $\mathcal{E}_b^{(K)}$ in the leading term of V_{np2}. A calculation very similar to the one in (100) shows, however, that V_{np2} *is* modified by a correction to the Kähler potential of the form (99) for $\lambda = 3/2$. It is straightforward to generalize this analysis to a more general form of the "Swiss cheese" volume, with more than one small Kähler modulus.

---

[17]For $\lambda = 3/2$ we can still use the expansion of the inverse metric (96), because the correction term would also be further suppressed e.g. in the dilaton.

D KK spectrum with fluxes

In this section we would like to develop some intuition on how the analysis of sections 3.3 and 4 might change in the presence of fluxes. We will restrict the discussion to one possible effect of the fluxes, namely their influence on the KK spectrum. It is not known explicitly how closed string fluxes, which are present in LVS, would change the mass spectrum. We will consider a toy example, using an analogy to the correction arising from world volume fluxes (cf. [52]), in order to get a feeling for what kind of effects one might expect. In particular, for the purposes of this appendix we assume a modified KK mass spectrum of the form

$$m_{\rm KK}^2 \sim \frac{1}{t_{\rm str}\left(1 + \frac{F^2}{t_{\rm str}^2}\right)} = \frac{\sqrt{S_1}}{t\left(1 + \frac{F^2 S_1}{t^2}\right)} \ , \tag{102}$$

where F represents any of the fluxes that may be present, and in the second equality the factors of S_1 appeared when expressing the 2-cycle volumes in Einstein frame as compared to the string frame ($t \sim e^{-\Phi/2} t_{\rm str}$). Note that expanding (102) for large values of t would lead to a correction $\Delta m^2 \sim F^2/t^3$, whose scaling with the flux and with t is reminiscent of the moduli masses induced by closed string 3-form flux [72,73]. In that case, the suppression would be by the overall volume (which would lead to only mild effects in LVS), but in (102) we allow for a suppression by single 2-cycle volumes (which might be the small 2-cycle in the $\mathbb{P}^4_{[1,1,1,6,9]}$ model).

Substituting (102) in (23), we now consider the scalar potential resulting from

$$K = -\ln(2S_1) - 2\ln(\mathcal{V}) + K_{\rm cs}(U,\bar{U}) - \frac{\tilde{\xi} S_1^{3/2}}{\mathcal{V}} + \frac{\sqrt{\tau_b}\mathcal{E}_b^{(K)}}{S_1\mathcal{V}} + \frac{\sqrt{\tau_s}\mathcal{E}_s^{(K)}}{S_1\mathcal{V}}\left(1 + \frac{F^2 S_1}{\tau_s}\right) \ ,$$

$$W = W_0 + Ae^{-aT_s} \ . \tag{103}$$

We have not included any flux correction to the term proportional to $\mathcal{E}_b^{(K)}$ because we expect such corrections to be subleading in a large volume expansion.[18] Note that the F-dependent correction term we did include is of the same form as the winding string

---

[18]Even though we think it is unlikely, we cannot exclude that the correction to KK masses that scale like t_b^{-1} without fluxes is only suppressed by F^2/τ_s instead of F^2/τ_b. In that case, one would have to redo the analysis of appendix C.5, using (99) with $\lambda = 1$. This would prohibit the use of the expansion (96), because in the large volume limit the leading contribution to $G_{s\bar{s}}$ would arise from the loop correction (it would scale as τ_b^{-1} as opposed to the scaling of the tree level contribution $\sim \tau_b^{-3/2}$). In that case the leading terms in $V_{\rm np1}$ and $V_{\rm np2}$ would be suppressed compared to V_3 and only arise at order $\mathcal{V}^{-10/3}$, thus invalidating the volume expansion of LVS.

correction $\sim \mathcal{E}_s^{(W)}$, when one neglects any potential flux corrections to the winding string spectrum, cf. (24) (remember that g_W^s would just be proportional to $1/\sqrt{\tau_s}$ without fluxes). Thus, by considering (103) we implicitly also analyze in the following the effect of corrections from winding strings (recall from section 3.3 that this correction is not present in the $\mathbb{P}^4_{[1,1,1,6,9]}$ model, but may be present in general).

We now give the generalization of (26)-(28) when using the modified Kähler potential (103). The three contributions at leading order ($\mathcal{O}(\mathcal{V}^{-3})$) in the large volume expansion are

$$V_{\mathrm{np1}} = e^{K_{\mathrm{cs}}} \frac{24a^2|A|^2\tau_s^{3/2}e^{-2a\tau_s}}{\mathcal{V}\Delta} \ , \tag{104}$$

$$V_{\mathrm{np2}} = -e^{K_{\mathrm{cs}}} \frac{2a|AW_0|\tau_s e^{-a\tau_s}}{S_1\mathcal{V}^2} \left[1 + \frac{6\mathcal{E}_s^{(K)}}{\Delta} \left(1 - 2\frac{F^2 S_1}{\tau_s} \right) \right] \ , \tag{105}$$

$$V_3 = \frac{3e^{K_{\mathrm{cs}}}|W_0|^2}{8\mathcal{V}^3} \left[S_1^{1/2}\tilde{\xi} + \right. \tag{106}$$
$$\left. + \frac{\sqrt{\tau_s}\left(4(\mathcal{E}_s^{(K)})^2 - 8(\mathcal{E}_s^{(K)})^2 F^2 S_1\tau_s^{-1}(1 + F^2 S_1\tau_s^{-1}) - \frac{8\sqrt{2}}{3}F^2 S_1^2\mathcal{E}_s^{(K)} \right)}{S_1^2\Delta} \right] \ ,$$

where the axion has already been minimized for, as discussed in section A.2, and now Δ is generalized to

$$\Delta \equiv \sqrt{2}S_1\tau_s - 3\mathcal{E}_s^{(K)} \left(1 - 3\frac{F^2 S_1}{\tau_s} \right) \ . \tag{107}$$

Plots for $F = 1$ and $F = 3$ are given in fig. 5, and they look quite similar to the plot without flux, fig. 4. Qualitatively, the conclusion is the same; only for nongeneric values of the g_s corrections do they compete with the α' correction. Note, however, that the amount of fine-tuning seems to depend on the value of the flux, cf. fig. 5. The same is true for the dependence of the values of $\mathcal{V}$ and τ_s at the minimum on S_1 and $\mathcal{E}_s^{(K)}$. For $F = 3$, for instance, this dependence becomes more complicated than what we found in (31). For the parameter range shown in fig. 5, the values of τ_s and $\mathcal{V}$ in the minimum vary in the ranges $\tau_s \in [14.6, 46.3]$ and $\log_{10}\mathcal{V} \in [3.7, 15.5]$, where the smallest value for both of them is reached in the corner where the two corrections become comparable.

Also the cancellation that we found for the gaugino masses survives the inclusion of the flux factor in (103). The correction still only appears at sub-sub-leading order
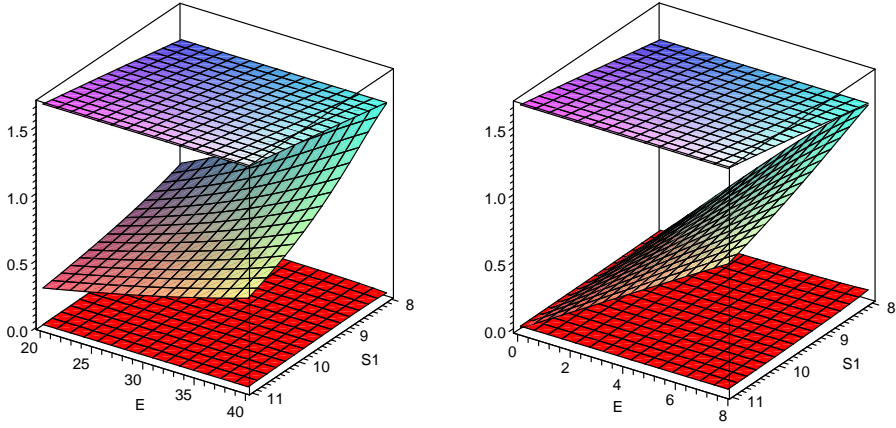
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FIGURE 5: Similarly to figure 4, the top surface is the α' correction, the second is the g_s correction (with $F = 1$ in the left graph and $F = 3$ in the right), and the "red carpet" is $10/\Delta$, with Δ from (107), using the same values as in fig. 4. The result is qualitatively the same as before. Note, however, that the range for $\mathcal{E}_s^{(K)}$ differs. For larger values of F one does not need to fine-tune $\mathcal{E}_s^{(K)}$ as much in order for the two corrections to become of similar order.

in an expansion in $\ln(1/m_{3/2})$ and we find (again using the dilute flux approximation for the prefactor $(\text{Re}f)^{-1}$):

$$
|M_s| = \left| \frac{F^s}{2\tau_s} \right| = 3e^{K/2}|W_0| \left| \frac{1}{4a\tau_s} + \frac{1}{16a^2\tau_s^2} + \frac{S_1 - 12\sqrt{2}a(1 - 2F^2 S_1\tau_s^{-1})\mathcal{E}_s^{(K)}}{64 S_1 a^3 \tau_s^3} + \ldots \right|
$$

$$
\sim \frac{m_{3/2}}{\ln(1/m_{3/2})} \left[1 + \mathcal{O}\left(\frac{1}{\ln(1/m_{3/2})} \right) \right] . \tag{108}
$$

This concludes our brief study of the direct effects of fluxes on the loop corrections.

E   The orientifold calculation

In the main text, we are interested in how ΔK_{g_s}, the one-loop correction to the Kähler potential, scales with the Kähler moduli T_i. Our argument in section 3.1 is based on the known result for ΔK_{g_s} in the case of $\mathcal{N} = 2$ supersymmetric $K3 \times T^2$ orientifolds and $\mathcal{N} = 1$ supersymmetric $T^6/\mathbb{Z}_N$ (or $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$) orientifolds, from [37] (see also [74]). Here we review this computation for the case of $K3 \times T^2$, and take this opportunity to adapt it to our case of D3-branes and D7-branes from the beginning. (One can also

obtain them by T-duality on the final D9/D5 results of [37], e.g. as in the appendix of [75], but as we shall see, the direct computation is enlightening in its own right.) We will leave out details that are essentially identical to [37], and only emphasize the differences.

As shown in [37] using "Kähler adapted" vertex operators, the easiest way to compute ΔK_{g_s} is by considering the 2-point function of the complex structure modulus U of T^2, with vanishing Wilson line moduli, i.e.

$$\langle V_U V_{\bar{U}} \rangle = -\sum_\sigma \frac{4g_c^2 \alpha'^{-4}}{(U - \bar{U})^2} \langle V_{ZZ}^{(0,0)} V_{\bar{Z}\bar{Z}}^{(0,0)} \rangle_\sigma \ . \tag{109}$$

Here, we use the notation of [37],

$$V_U^{(0,0)} = -g_c \alpha'^{-2} \frac{2}{U - \bar{U}} V_{ZZ}^{(0,0)} \ ,$$

$$V_{\bar{U}}^{(0,0)} = g_c \alpha'^{-2} \frac{2}{U - \bar{U}} V_{\bar{Z}\bar{Z}}^{(0,0)} \tag{110}$$

and

$$V_{ZZ}^{(0,0)} = -\frac{2}{\alpha'} \int_\Sigma d^2z \left[i\partial Z + \frac{1}{2}\alpha'(p \cdot \psi)\Psi \right] \left[i\bar{\partial}Z + \frac{1}{2}\alpha'(p \cdot \tilde{\psi})\tilde{\Psi} \right] e^{ipX} \ ,$$

$$V_{\bar{Z}\bar{Z}}^{(0,0)} = -\frac{2}{\alpha'} \int_\Sigma d^2z \left[i\partial\bar{Z} + \frac{1}{2}\alpha'(p \cdot \psi)\bar{\Psi} \right] \left[i\bar{\partial}\bar{Z} + \frac{1}{2}\alpha'(p \cdot \tilde{\psi})\tilde{\bar{\Psi}} \right] e^{ipX} \ . \tag{111}$$

As in [37], and [76] before that, we find these complex worldsheet variables particularly convenient:

$$Z = \sqrt{\frac{\sqrt{G_{\text{str}}}}{2U_2}}(X^4 + \bar{U}X^5) \quad , \quad \bar{Z} = \sqrt{\frac{\sqrt{G_{\text{str}}}}{2U_2}}(X^4 + UX^5) \ ,$$

$$\Psi = \sqrt{\frac{\sqrt{G_{\text{str}}}}{2U_2}}(\psi^4 + \bar{U}\psi^5) \quad , \quad \bar{\Psi} = \sqrt{\frac{\sqrt{G_{\text{str}}}}{2U_2}}(\psi^4 + U\psi^5) \ , \tag{112}$$

where $\sqrt{G_{\text{str}}}$ is the volume of T^2 measured in string frame. The 2-point function (109)

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can be expanded for small momenta, $p_1 \cdot p_2 \ll 1$, and we obtain

$$
\begin{aligned}
\langle V_{ZZ}^{(0,0)} V_{\bar{Z}\bar{Z}}^{(0,0)} \rangle_\sigma &= -V_4 \frac{(p_1 \cdot p_2)\sqrt{G_{\mathrm{str}}}^{-1}}{16(4\pi^2\alpha')^2} \int_0^\infty \frac{dt}{t^4} \int_{\mathcal{F}_\sigma} d^2\nu_1 d^2\nu_2 \\
&\quad \times \sum_{k=0,1} \sum_{\vec{n}=(n,m)^T} \mathrm{tr}\Bigg[e^{-\pi\vec{n}^T G_{\mathrm{str}}^{-1}\vec{n}t^{-1}} \sum_{\substack{\alpha\beta \\ \mathrm{even}}} \frac{\vartheta[{}^\alpha_\beta](0,\tau)}{\eta^3(\tau)} \gamma_{\sigma,k} \mathcal{Z}_{\sigma,k}^{\mathrm{int}}[{}^\alpha_\beta] \\
&\quad \times \Big[\langle \bar{\partial}Z(\bar{\nu}_1)\bar{\partial}\bar{Z}(\bar{\nu}_2)\rangle_\sigma \langle \Psi(\nu_1)\bar{\Psi}(\nu_2)\rangle_\sigma^{\alpha,\beta} \langle \psi(\nu_1)\psi(\nu_2)\rangle_\sigma^{\alpha,\beta} \\
&\quad + \langle \bar{\partial}Z(\bar{\nu}_1)\partial\bar{Z}(\nu_2)\rangle_\sigma \langle \Psi(\nu_1)\bar{\tilde{\Psi}}(\bar{\nu}_2)\rangle_\sigma^{\alpha,\beta} \langle \psi(\nu_1)\tilde{\psi}(\bar{\nu}_2)\rangle_\sigma^{\alpha,\beta} + \mathrm{c.c.}\Big]\Bigg] + \mathcal{O}((p_1 \cdot p_2)^2) \, .
\end{aligned}
\tag{113}
$$

For the details we refer to [37]. The main difference to the corresponding formula (C.3) in [37] is the appearance of the inverse metric G_{str}^{-1} in the exponent arising from the zero mode sum, and in the prefactor. This is due to the fact that the D3 and D7 branes are localized along the T^2, and so the closed string channel involves a Kaluza-Klein momentum sum instead of a winding sum. The sum over bosonic zero modes has been made explicit, since there is also an implicit dependence on m, n in the bosonic correlators: this arises from the classical piece in the split into zero modes and fluctuations. That is, $Z(\nu) = Z_{\mathrm{class}}(\nu) + Z_{\mathrm{qu}}(\nu)$, where the classical part is given by

$$
Z_{\mathrm{class}} = \sqrt{\alpha'}\sqrt{\frac{\sqrt{G_{\mathrm{str}}}}{2U_2}}\Big(n + m\bar{U}\Big)\mathrm{Re}(\nu)\,\tilde{c}_\sigma \, , \quad \tilde{c}_\sigma = \left\{ \begin{array}{ll} 1 & \text{for } \mathcal{K} \\ 2 & \text{for } \mathcal{A}\,, \mathcal{M} \end{array} \right. \, .
\tag{114}
$$

These zero modes have the right periodicity under $\mathrm{Re}(\nu) \to \mathrm{Re}(\nu) + \pi$ (for $\mathcal{A}, \mathcal{M}$) or $\mathrm{Re}(\nu) \to \mathrm{Re}(\nu) + 2\pi$ (for $\mathcal{K}$), i.e. $X^4 \to X^4 + 2\pi n\sqrt{\alpha'}$ and $X^5 \to X^5 + 2\pi m\sqrt{\alpha'}$. In contrast to [37] they involve the real part of ν. The reason is again that in the D3/D7 case the branes are localized along T^2 and thus the winding appears in the open string channel as opposed to the closed string channel (as was the case for D9/D5 branes).

The sum over spin structures is performed using Riemann identities. This leaves the correlators of the bosonic fields as the only piece that depends on the positions ν_i of the vertex operators. The ν_i integral can then be evaluated. As the zero modes (114) involve the real part of ν in the case of D3/D7-branes, in contrast to the D9/D5-case studied in [37], the zero mode contribution in the Z-correlators drops out. The quantum part is evaluated using the method of images on the worldsheet [38,77,78]. To evaluate the KK sum in (113), it is useful to regularize the integral over t by a UV

cutoff Λ. With this we obtain

$$\int_{1/(e_\sigma \Lambda^2)}^{\infty} \frac{dt}{t^4} \sum_{\vec{n}=(n,m)^T}' \left[\pi^3 c_\sigma^2 t\alpha' e^{-\pi \vec{n}^T G_{\text{str}}^{-1} \vec{n} t^{-1}} \right]$$

$$= \frac{1}{2}\pi^3 \alpha' c_\sigma^2 e_\sigma^2 \Lambda^4 + \pi \alpha' c_\sigma^2 \sqrt{G_{\text{str}}}^2 E_2(0,U) + \ldots , \tag{115}$$

where the prime at the sum indicates that the $(n,m) = (0,0)$ term is left out, and c_σ, e_σ are constants whose precise values will not be important in the following (but can be found in [37]). Terms that go to zero in the limit $\Lambda \to \infty$ have been dropped, as indicated by the ellipsis. The nonholomorphic Eisenstein series $E_2(0,U)$ is the $s = 2$ special case of

$$E_s(0,U) = \sum_{\vec{n}=(n,m)^T}' \frac{U_2^s}{|n+mU|^{2s}} . \tag{116}$$

The terms involving the UV cutoff Λ drop out after summing over all diagrams, due to tadpole cancellation. We have then reduced (113) to

$$\langle V_{ZZ}^{(0,0)} V_{\bar{Z}\bar{Z}}^{(0,0)} \rangle_\sigma = -(p_1 \cdot p_2)\alpha' \frac{V_4}{(4\pi^2\alpha')^2} \frac{c_\sigma^2 \pi \sqrt{G_{\text{str}}}}{8} \sum_k \text{tr}\left[E_2(0,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k} \right]$$

$$+ \mathcal{O}((p_1 \cdot p_2)^2) . \tag{117}$$

The quantities $\mathcal{Q}_{\sigma,k}$ come from the sum over spin structures and are defined in [37]. Introducing the notation

$$\mathcal{E}_2(0,U) = \sum_\sigma c_\sigma^2 \sum_{k=0,1} \text{tr}\left[E_2(0,U)\gamma_{\sigma,k}\mathcal{Q}_{\sigma,k} \right] , \tag{118}$$

we end up with (neglecting some irrelevant factors of g_c, α', terms subleading in the low-energy expansion, and constants of order 1)

$$\langle V_U V_{\bar{U}} \rangle \sim -i(p_1 \cdot p_2)\frac{V_4}{(4\pi^2\alpha')^2} \frac{\sqrt{G_{\text{str}}}}{(U-\bar{U})^2}\mathcal{E}_2(0,U) . \tag{119}$$

To read off the one-loop correction to the kinetic term of U we need to perform a Weyl rescaling to the Einstein frame. In the one-loop term (119) this just leads to

$$\text{Weyl rescaling:} \quad \times \frac{e^{2\Phi}}{\mathcal{V}^{\text{str}}} , \tag{120}$$

where

$$\mathcal{V}^{\text{str}} = \mathcal{V}_{\text{K3}}^{\text{str}}\sqrt{G_{\text{str}}} \tag{121}$$

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is the overall volume in string frame. The Kähler potential can then be read off from the kinetic term by use of the identity

$$\partial_U \partial_{\bar{U}} E_2(0, U) = -\frac{2}{(U - \bar{U})^2} E_2(0, U) \ , \tag{122}$$

producing the final result

$$\Delta K_{g_s} \ \sim \ \frac{\sqrt{G_{\mathrm{str}}} e^{\Phi}}{\mathcal{V}^{\mathrm{str}}(S + \bar{S})} \mathcal{E}_2(0, U) \ , \tag{123}$$

where $\sqrt{G_{\mathrm{str}}} e^{\Phi} / \mathcal{V}^{\mathrm{str}}$ is to be interpreted as a function of the Kähler variables. In the $K3 \times T^2$ orientifold case, using (121), this is just proportional to $e^{\Phi} / \mathcal{V}^{\mathrm{str}}_{K3} \sim (T + \bar{T})^{-1}$ (with Re T the volume of $K3$ measured in Einstein frame), giving a result T-dual to [37] (note that we switched the real and imaginary parts in the definition of T and S as compared to [37], to conform with the rest of this paper). As we argue in section 3.1, in general the dependence on the Kähler moduli will be more complicated than this, because there is no analog to the relation (121). It is still clear that the inverse suppression in the overall volume will appear as in (123), given that it is a direct consequence of the Weyl rescaling.

F   Factorized approximation

As mentioned in section 4.2, it is an important issue to what extent the moduli spaces of Kähler and complex structure moduli factorize. In this appendix, we give further details on the factorized approximation.

A common starting point in the analysis of the potential arising in type IIB theory with 3-form fluxes is to assume that all complex structure moduli U^{α} and the dilaton S are stabilized by demanding

$$D_{U^{\alpha}} W = 0 = D_S W \ . \tag{124}$$

In this case the F-term potential for the moduli (7) reduces to

$$V = e^K \left(G^{\bar{j}i} D_{\bar{j}} \bar{W} D_i W - 3|W|^2 \right) \ , \tag{125}$$

where as in the main text, the indices i and j refer only to the Kähler moduli and thus run from 1 to $h^{1,1}$. Note that even though the complex structure moduli and the dilaton are assumed to be stabilized by (124), the inverse metric $G^{\bar{j}i}$ is part of

the inverse of the whole moduli space metric. More precisely, if we denote the Kähler moduli by T_i, as before, and all other moduli (i.e. the complex structure moduli and the dilaton) collectively as Z^a, the moduli space metric is given by

$$G_{I\bar{J}} \sim \begin{pmatrix} K_{i\bar{j}} & K_{i\bar{b}} \\ K_{a\bar{j}} & K_{a\bar{b}} \end{pmatrix} . \tag{126}$$

We denote the inverse of this (whole) metric by $G^{\bar{J}I}$. In general

$$G^{\bar{j}i} \neq (K_{i\bar{j}})^{-1} . \tag{127}$$

Equality only holds if $G_{i\bar{b}} = 0$, i.e. if the moduli space of the Kähler moduli is factorized from the rest, as it is the case without loop and α' corrections.

In this appendix, we would like to investigate at which order in a large volume expansion the two matrices in (127) start to deviate from each other. For this analysis we assume a volume of the "Swiss cheese" form as in (48) and a Kähler potential of the form (24) (without taking possible effects of fluxes on the KK and winding mode spectra into account as was done in appendix D; thus, $g_{\mathrm{K}}^{\mathfrak{a}} \sim t_{\mathfrak{a}}$ and $g_{\mathrm{K}}^{\mathfrak{q}} \sim t_{\mathfrak{q}}^{-1}$ for some 2-cycle volumes). To avoid cumbersome notation we will indicate all the small moduli collectively as τ_s. We then use the formula

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1}(1 + BP^{-1}CA^{-1}) & -A^{-1}BP^{-1} \\ -P^{-1}CA^{-1} & P^{-1} \end{pmatrix} , \tag{128}$$

where P is the Schur complement of A, defined as

$$P = D - CA^{-1}B . \tag{129}$$

In our case P is the Schur complement of $K_{i\bar{j}}$. From (24) we read off that

$$G_{I\bar{J}} \sim \begin{pmatrix} \tau_b^{-2} & \tau_b^{\beta} & \tau_b^{-2} & \tau_b^{-2} \\ \tau_b^{\beta} & \tau_b^{-3/2} & \tau_b^{-3/2} & \tau_b^{-3/2} \\ \tau_b^{-2} & \tau_b^{-3/2} & \tau_b^{0} & \tau_b^{-1} \\ \tau_b^{-2} & \tau_b^{-3/2} & \tau_b^{-1} & \tau_b^{0} \end{pmatrix} , \tag{130}$$

where we only indicate the τ_b dependence and the indices run over $I, J = \{T_b, T_s, U, S\}$. Here $\beta = -2$ for those τ_i with a nonvanishing a_i in (48) (so β has an implicit index i), otherwise $\beta = -5/2$ (which is in particular the value in the $\mathbb{P}_{[1,1,1,6,9]}$ case). We

decompose $G_{I\bar{J}}$ as in (128)

$$A \sim \begin{pmatrix} \tau_b^{-2} & \tau_b^{\beta} \\ \tau_b^{\beta} & \tau_b^{-3/2} \end{pmatrix}, \qquad A^{-1} \sim \begin{pmatrix} \tau_b^{2} & \tau_b^{7/2+\beta} \\ \tau_b^{7/2+\beta} & \tau_b^{3/2} \end{pmatrix},$$

$$B = C^T \sim \begin{pmatrix} \tau_b^{-2} & \tau_b^{-2} \\ \tau_b^{-3/2} & \tau_b^{-3/2} \end{pmatrix}, \qquad D \sim \begin{pmatrix} \tau_b^{0} & \tau_b^{-1} \\ \tau_b^{-1} & \tau_b^{0} \end{pmatrix},$$

$$P \sim \begin{pmatrix} \tau_b^{0} & \tau_b^{-1} \\ \tau_b^{-1} & \tau_b^{0} \end{pmatrix}, \qquad P^{-1} \sim \begin{pmatrix} \tau_b^{0} & \tau_b^{-1} \\ \tau_b^{-1} & \tau_b^{0} \end{pmatrix}. \tag{131}$$

Using (128) one easily obtains the scaling of the inverse:

$$G^{\bar{J}I} \sim \begin{pmatrix} \tau_b^{2} & \tau_b^{7/2+\beta} & \tau_b^{0} & \tau_b^{0} \\ \tau_b^{7/2+\beta} & \tau_b^{3/2} & \tau_b^{0} & \tau_b^{0} \\ \tau_b^{0} & \tau_b^{0} & \tau_b^{0} & \tau_b^{-1} \\ \tau_b^{0} & \tau_b^{0} & \tau_b^{-1} & \tau_b^{0} \end{pmatrix}. \tag{132}$$

Now, from (128), $G^{\bar{J}i}$ receives two contributions. The first is $K^{\bar{J}i}$, that would be the only term in the case of a factorized metric; the second is $K_{h\bar{j}}^{-1} K_{h\bar{b}} P_{a\bar{b}}^{-1} K_{a\bar{l}} K_{i\bar{l}}^{-1}$, that breaks factorization. Let us compare their τ_b scaling:

$$\begin{aligned} G^{\bar{j}i} &= A^{-1} + A^{-1} B P^{-1} C A^{-1} \tag{133} \\ &\sim \begin{pmatrix} \tau_b^{2} & \tau_b^{7/2+\beta} \\ \tau_b^{7/2+\beta} & \tau_b^{3/2} \end{pmatrix} + \begin{pmatrix} \tau_b^{0} & \tau_b^{0} \\ \tau_b^{0} & \tau_b^{0} \end{pmatrix}. \end{aligned}$$

Thus the corrections coming from non-vanishing off-diagonal metric elements in (126) set in with a suppression by τ_b^{-2}, $\tau_b^{-7/2-\beta}$ and $\tau_b^{-3/2}$ in $G^{\bar{b}b}$, $G^{\bar{b}s}$ and $G^{\bar{s}s}$, respectively. In the explicit example based on $\mathbb{P}_{[1,1,1,6,9]}$, $\beta = -5/2$, and we checked this result by comparing to the subleading terms in (65).

F.1   Factorized approximation of the scalar potential

What we are really interested in is not the (inverse) metric itself, but the scalar potential, to which we now turn. For the nonperturbative terms V_{np1} and V_{np2}, the suppression of the off-diagonal terms in (133) is inherited by the scalar potential, as they are proportional to $G^{\bar{s}s}\bar{W}_{,\bar{s}}W_{,s}$ and $G^{\bar{j}s}K_{\bar{j}}$, respectively. For V_3 things are not as simple, due to the no-scale structure at leading order. Let us neglect for a moment all

the quantum corrections, then the no-scale structure implies

$$\left[G^{\bar{\imath}j} K_{\bar{\imath}} K_j - 3 \right]_{\text{no-scale}} \sim (\tau_b^{-1}, \tau_b^{\beta+1}) \begin{pmatrix} \tau_b^2 & \tau_b^{7/2+\beta} \\ \tau_b^{7/2+\beta} & \tau_b^{3/2} \end{pmatrix} \begin{pmatrix} \tau_b^{-1} \\ \tau_b^{\beta+1} \end{pmatrix} - 3$$

$$\sim \tau_b^0 + \tau_b^{2\beta+7/2} = 0 \ . \tag{134}$$

The two terms have to vanish independently. Now let us add corrections that break no-scale structure. Because of the cancellation described in appendix C.3, the leading contribution can be seen to come at order $\tau_b^{-3/2}$ (from the α', $\mathcal{E}_s^{(K)}$ and $\mathcal{E}_s^{(W)}$ corrections). On the other hand, the off-diagonal terms appear at order

$$\left[G^{\bar{\imath}j} K_{\bar{\imath}} K_j \right]_{\text{off-diagonal}} \sim (\tau_b^{-1}, \tau_b^{\beta+1}) \begin{pmatrix} \tau_b^0 & \tau_b^0 \\ \tau_b^0 & \tau_b^0 \end{pmatrix} \begin{pmatrix} \tau_b^{-1} \\ \tau_b^{\beta+1} \end{pmatrix}$$

$$\sim \tau_b^{-2} + \tau_b^{\beta} + \tau_b^{2\beta+2}$$

$$\sim \tau_b^{-2} + \dots \ , \tag{135}$$

for both $\beta = -2$ and $\beta = -5/2$. Therefore, the off-diagonal terms of the moduli space metric appear in the scalar potential with a suppression of at least $\tau_b^{-1/2}$ (as is confirmed by the explicit example of section 3.3, cf. formulas (26)-(30)). The suppression can be even stronger if some corrections are absent and the leading term in (135) vanishes.

To summarize: if one is only interested in the leading term of the scalar potential in the large volume (i.e. large τ_b) expansion, then one can use the *factorized approximation*, i.e.

$$G^{\bar{\jmath}i} = K^{\bar{\jmath}i} + \mathcal{O}\left(\tau_b^0\right) . \tag{136}$$

This provides a useful tool to simplify the calculations.

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