Published by Institute of Physics Publishing for SISSA

Received: *January 31, 2007*
Accepted: *March 19, 2007*
Published: *April 3, 2007*

Holography of D-brane reconnection

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Abstract: Gukov, Martinec, Moore and Strominger found that the D1-D5-D5′ system with the D5-D5′ angle at 45 degrees admits a deformation ρ preserving supersymmetry. Under this deformation, the D5-branes and D5'-branes reconnect along a single special Lagrangian manifold. We construct the near-horizon limit of this brane setup (for which no supergravity solution is currently known), imposing the requisite symmetries perturbatively in the deformation ρ. Reducing to the three-dimensional effective gauged supergravity, we compute the scalar potential and verify the presence of a deformation with the expected properties. We compute the conformal dimensions as functions of ρ. This spectrum naturally organizes into $\mathcal{N} = 3$ supermultiplets, corresponding to the 3/16 preserved by the brane system. We give some remarks on the symmetric orbifold CFT for $Q_{\mathrm{D5}} = Q_{\mathrm{D5}'}$, outline the computation of ρ-deformed correlators in this theory, and probe computations in our ρ-deformed background.

Keywords: Conformal Field Models in String Theory, AdS-CFT Correspondence, Gauge-gravity correspondence, D-branes.

JHEP04(2007)013

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1. Introduction

The D1-D5 system has been a target of much interest ever since the classic Strominger-Vafa paper on its relation to black hole entropy [1], and its later inception in the AdS/CFT correspondence. The worldsheet theory on the D1-brane — the boundary theory in AdS/CFT — is an $\mathcal{N} = (4, 4)$ superconformal theory. It is textbook knowledge [2, Ch. 11.1] that in addition to the $\mathcal{N} = 4$ superconformal algebra, that has an $SU(2)$ bosonic subalgebra, there is the "large" $\mathcal{N} = 4$ algebra, with an $SU(2) \times SU(2)$ subalgebra. The place of this enlarged symmetry in the AdS/CFT correspondence, and how to break it, is the subject of this paper. Breaking this enlarged symmetry may ultimately help understanding deformations of the original D1-D5 system broken down to $\mathcal{N} = (3, 3)$ supersymmetry.

Already in 1999, de Boer, Pasquinucci and Skenderis [3] (based on earlier work [4, 5]) initiated the study of AdS/CFT dual pairs with large $\mathcal{N} = (4, 4)$ symmetry, and found the requisite solution of Type IIB supergravity: two orthogonal D5-brane stacks, intersecting over a D-string. This is the D1-D5-D5′ system (figure 1b). Compared to the D1-D5 system, the $SO(4)$ symmetry of transverse rotations of the single D5-brane stack is doubled to $SO(4) \times SO(4)$ to include independent transverse rotations of both D5-brane stacks. In

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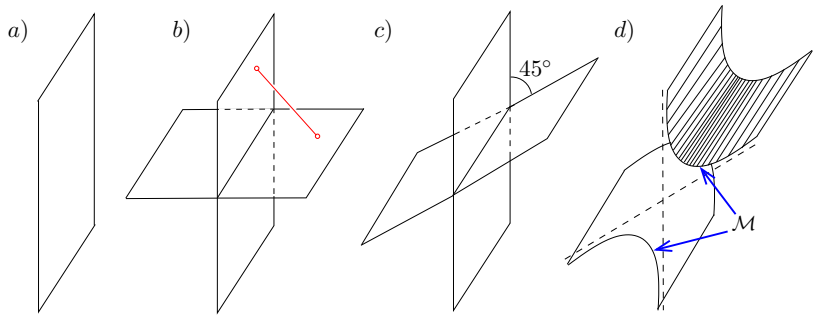

Figure 1: *a*) D1-D5 system, for comparison. The D1-brane is delocalized on the D5-brane. *b*) Orthogonal D1-D5-D5′ system with string inducing nonlocal interactions on D1-brane. *c*) Tilted D1-D5-D5′ system. *d*) ρ-deformed D1-D5-D5′ system on the curved manifold $\mathcal{M}$ times the 1+1 intersection. As will be explained later, $\mathcal{M}$ is simply connected, despite appearances.

the near-horizon limit, the boundary theory on the D1-branes should be the large $\mathcal{N} = (4,4)$ theory. The D1-D5-D5′ system received some further attention [6, 7], but it remains mysterious; simply decreasing the ratio of D5′ to D5 charge, one does not arrive at the D1-D5 system, and the worldsheet theory is nonlocal [3], due to D5-D5′ interactions (the open string shown in figure 1b).

Tilting the branes gave a new perspective on this. In the first of a series of three papers [8 – 10], Gukov, Moore, Martinec and Strominger (GMMS) found[1] that if the D5′-branes are tilted at 45° relative to the D5-branes as shown in figure 1c (breaking supersymmetry to 3/16), a new possibility appears; there is a supersymmetric noncompact manifold $\mathcal{M}$ that the branes can reconnect along, as in figure 1d. GMMS identified a deformation called ρ that would describe this reconnection, and pointed out that it would break the rotational symmetry to the diagonal.

Although the deformed system is less symmetric, the complications of the D1-D5-D5′ system that come from nonlocal interactions should not arise when D5-branes and D5′-branes are joined on $\mathcal{M}$, and in fact the ρ-deformed system may have more in common with the single D1-D5 system (e.g. its Higgs branch [8]) than the undeformed system did.

The corresponding deformed 10-dimensional solution with the branes extended along the curved manifold $\mathcal{M}$ is not known explicitly. In section 3 we construct its near-horizon limit (eq. (3.16)) by imposing the requisite symmetries perturbatively in the deformation parameter ρ. Identifying the field ρ in the Kaluza-Klein spectrum of fluctuations around the undeformed $AdS_3 \times S^3 \times S^3 \times S^1$ background, we reduce to the three-dimensional effective supergravity with $SO(4) \times SO(4) \to SO(4)_{\rm diag}$ gauge symmetry breaking. In section 5, we compute the scalar potential and show the (from a supergravity point of view somewhat surprising) presence of a flat "valley", i.e. a deformation marginal to all orders, shown in figure 4. We verify that evolution of the scalars along the valley reproduces all the properties expected for the ρ-deformation and in particular breaks supersymmetry to $\mathcal{N} = (3,3)$. We compute the conformal dimensions Δ in the theory as functions of ρ,

[1] Earlier related work on intersecting branes in M-theory includes [11 – 13].

by computing the spectrum along the flat valley. This spectrum naturally organizes into $\mathcal{N} = 3$ supermultiplets. In section 6, we give some remarks on the symmetric orbifold CFT that was conjectured in [3, 8] to describe at least some aspects of the deformed boundary theory for $Q_{\mathrm{D5}} = Q_{\mathrm{D5}'}$. We outline the computation of ρ-deformed correlators in this theory, and probe computations in the ρ-deformed background.

To some readers, D-brane reconnection will be more familiar [2, Ch. 13.4] in the context of D-branes at angles, where below a certain critical angle, a tachyonic mode develops and the branes move apart. Clearly this is quite different from the marginal deformation we are interested in, where the reconnected branes can be disconnected again by sending $\rho \to 0$, at no cost in energy (see also [14]). A more closely related kind of brane reconnection along special Lagrangian manifolds has been extensively studied in the literature in other contexts, like [15, 16] for M-theory on G_2 manifolds.

2. The D1-D5-D5′ system intersecting at angles

We begin with a review of the D1-D5-D5′ system with $SO(4) \times SO(4)'$ symmetry and explain how its near-horizon limit $AdS_3 \times S^3 \times S^3 \times S^1$ arises. The boundary theory of the AdS_3 factor has large $\mathcal{N} = (4,4)$ supersymmetry in $1+1$ dimensions. The supergravity solution for this system was studied for orthogonal intersection in [3], for D5-D5′ intersecting at angles in [11], and for D1-D5-D5′ intersecting at angles in [8]. Here we summarize the results without derivation.

We denote the 10-dimensional coordinates by $\{t, z, x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4\}$, and number them by $0, 1, \ldots, 9$. We will often use the notation $x^2 = x_1^2 + \ldots + x_4^2$, $y^2 = y_1^2 + \ldots + y_4^2$. The angles between D5 and D5′-branes are denoted θ_{26} for the angle in the $x_1 - y_1$ plane, and so on. For D5-branes intersecting at equal angles in all four planes $\theta_{26} = \theta_{37} = \theta_{48} = \theta_{59} =: \theta$, we have the supersymmetry conditions [11]

$$\Gamma_{012345}\epsilon_{\mathrm{R}} = \epsilon_{\mathrm{L}} \ , \quad \exp(\theta(\Gamma_{26} + \Gamma_{37} + \Gamma_{48} + \Gamma_{59}))\epsilon_{\mathrm{L,R}} = \epsilon_{\mathrm{L,R}} \ , \tag{2.1}$$

that together preserve 6 Killing spinors, i.e. 3/16 supersymmetry. Including D1-branes delocalized in the $\vec{x}$ and $\vec{y}$ directions adds the condition

$$\Gamma_{01}\epsilon_{\mathrm{R}} = \epsilon_{\mathrm{L}} \ , \tag{2.2}$$

that further breaks supersymmetry from 3/16 to 1/16, again at nonzero θ. The Type IIB supergravity solution for angle θ is

$$\begin{aligned} ds^2 = {} & (H_1^{(+)} H_1^{(-)} \det U)^{-1/2}(-dt^2 + dz^2) + \sqrt{H_1^{(+)} H_1^{(-)}}\frac{U_{11}}{\sqrt{\det U}}(d\vec{x})^2 \\ & + \sqrt{H_1^{(+)} H_1^{(-)}}\frac{U_{22}}{\sqrt{\det U}}(d\vec{y})^2 + \frac{2U_{12}}{\sqrt{\det U}} d\vec{x} \cdot d\vec{y} \ , \end{aligned} \tag{2.3}$$

with RR 3-form field strength

$$F_3 = dt \wedge dz \wedge d(H_1^{(+)} H_1^{(-)})^{-1} + *_x dU_{11} + *_y dU_{22} \ , \tag{2.4}$$

and dilaton

$$e^{-2\phi} = \frac{1}{g_s^2}\frac{\det U}{H_1^{(+)}H_1^{(-)}} \,, \tag{2.5}$$

where the harmonic functions $H_1^{(+)}$, $H_1^{(-)}$ are

$$H_1^{(+)} = 1 + \frac{g_s Q_1}{x^2} \,, \qquad H_1^{(-)} = 1 + \frac{g_s Q_1}{y^2} \,, \tag{2.6}$$

and the matrix U is given by

$$U = \begin{pmatrix} \csc\theta + \frac{g_s Q_5^+}{x^2} & -\cot\theta \\ -\cot\theta & \csc\theta + \frac{g_s Q_5^-}{y^2} \end{pmatrix} \,. \tag{2.7}$$

Here Q_1 is the D1-brane charge, Q_5^+ is the D5-brane charge, and Q_5^- is the D5′-brane charge. We set $Q_5^+ = Q_5^- =: Q$ for reasons discussed in [3, 8]. The D1-brane charge Q_1 is fixed in terms of the D5-brane charges as $Q_1 = \frac{L}{4\pi^2}Q_5$.

The near-horizon limit of this solution was studied in the above papers [3, 8, 13]. Somewhat counterintuitively, the near-horizon limit is the same regardless of the rotation angle θ. It is, setting $g_s = 1$ and $L = 4\pi^2\tilde{L}$,

$$ds^2 = \frac{x^2y^2}{\tilde{L}Q^2}(-dt^2 + dz^2) + Q\tilde{L}\left(\frac{dx^2}{x^2} + d\Omega_+^2\right) + Q\tilde{L}\left(\frac{dy^2}{y^2} + d\Omega_-^2\right) \,. \tag{2.8}$$

After a change of coordinates

$$r = (\sqrt{2}Q^{-1/2})xy \,, \qquad \phi = \frac{1}{\sqrt{2}}\log\frac{y}{x} \,, \tag{2.9}$$

using that $dx^2/x^2 + dy^2/y^2 = a\,dr^2/r^2 + b\,d\phi^2$ for some constants a and b, and identifying[2] ϕ to make an S^1, the near-horizon metric becomes

$$ds^2 = \frac{r^2}{2R^2}(-dt^2 + dz^2) + \frac{R^2}{2r^2}dr^2 + R^2 d\Omega_+^2 + R^2 d\Omega_-^2 + R^2 d\phi^2 \,, \tag{2.10}$$

with $R^2 = Q\tilde{L}$, which we recognize as $AdS_3 \times S^3 \times S^3 \times S^1$. The matter fields are

$$F_3 = \mathrm{vol}(AdS_3) + \mathrm{vol}(S_+^3) + \mathrm{vol}(S_-^3) \,, \qquad e^{\phi} = 1/\tilde{L}^2 \,. \tag{2.11}$$

Here, we have $\mathcal{N} = (4,4)$ supersymmetry. With the solution at hand, it is fairly clear from (2.9) that the new coordinate system (r, ϕ) moves along with θ. Also, θ only occurs in the constant part of the matrix U, which is neglected in the near-horizon limit. We see that the dependence on the angle θ will be lost in the near-horizon limit.

Going back to the full solution, it is interesting to consider whether instead of flat worldvolumes, there are any natural supersymmetric manifolds the branes could extend along. One suitable manifold was found in [8], as we now discuss.

[2]This identification was performed in [3] and criticized in [8]. We will have nothing new to say about this.

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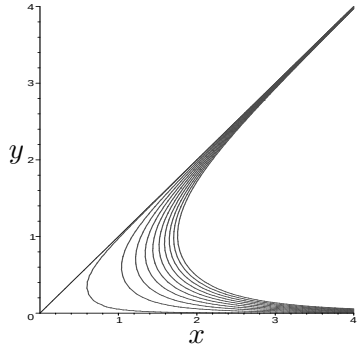

Figure 2: $\rho = xy(x^2 - y^2)$ for various values of ρ.

2.1 Brane reconnection

Let $\mathbf{C}^m$ have complex coordinates $z_k = x_k + iy_k$, $k = 1, \ldots, m$. Consider the submanifold given by

$$\operatorname{Im} z^n = \text{const} \ , \tag{2.12}$$

where $z = |x_k| + i|y_k|$. It is special Lagrangian for $m = n$ [17]. For the system described in the previous section, the nonintersecting part of the D5-branes is $\mathbf{R}^4 \times \mathbf{R}^4 \simeq \mathbf{C}^4$, so combining the two lengths $x^2 = x_1^2 + \ldots + x_4^2$ and $y^2 = y_1^2 + \ldots + y_4^2$ into a complex number[3] $z = x + iy$, we could consider letting the D5-branes extend along on the manifold (2.12) for $n = 4$. In particular, let

$$\rho := \frac{1}{4} \operatorname{Im} z^4 = xy(x^2 - y^2) \ , \tag{2.13}$$

then we can define the noncompact special Lagrangian manifold $\mathcal{M}$ as [8]

$$\mathcal{M} = \left\{ \rho = \text{constant} \ , \ \frac{x_i}{x} = \frac{y_i}{y} \right\} \quad i = 1, \ldots, 4 \ . \tag{2.14}$$

We plot (2.13) in figure 2 for a few values of ρ.

Note that the first quadrant is all there is, since x and y are lengths. Defining $\sigma = (1/4) \operatorname{Re} z^4$, and noting that

$$\rho^2 + \sigma^2 = \frac{(x^2 + y^2)^4}{16} =: r^2 \ , \tag{2.15}$$

we can go to cylindrical coordinates $[r, \theta, \sigma]$ with σ as the vertical axis. A plot in these coordinates appears in figure 3.

For $\rho > 0$, the full manifold has topology $\mathbf{R} \times \mathbf{S}^3$, and this is represented by $\mathbf{R} \times \mathbf{S}^1$ in figure 3. We see clearly that for $\rho = 0$, the manifold degenerates to two separate branches $\mathbf{R}^4 \times \mathbf{R}^4$, represented by $\mathbf{R}^2 \times \mathbf{R}^2$ in the figure (the conical singularity is an artifact of the embedding). We do not know of a supergravity solution for D5-branes extended along the noncompact special Lagrangian manifold $\mathcal{M}$.

[3] Note that this complex z has nothing to do with the coordinate z in the previous section.

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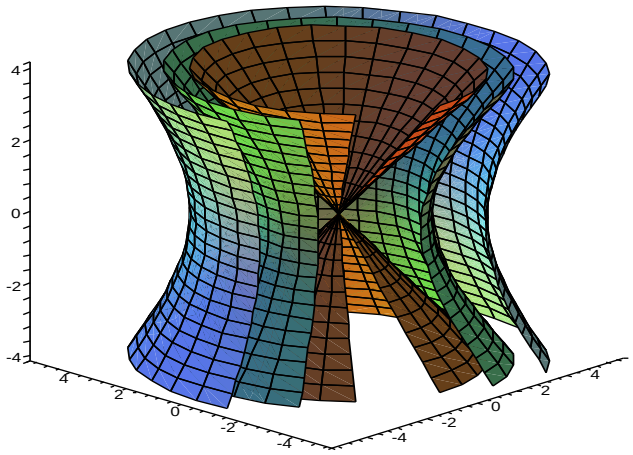

Figure 3: Cylindrical plot $[r, \theta, \sigma]$ from eq. (2.15) for various values of ρ. The surface at the center represents $\rho = 0$. The apparent disconnectedness in figure 1 can now be understood as a hyperbolic "conic section" of the connected $\rho > 0$ manifold.

Importantly, if we take the near-horizon limit of the solutions with the branes wrapped on (2.13), we expect the symmetry to be reduced. In the undeformed brane system, the $SO(4) \times SO(4)'$ symmetry corresponds to independent transverse rotations of the two sets of D5-branes. After deformation, we see in figure 3 that transverse rotations are no longer independent; the two sets of branes are now wrapping a single connected manifold, with the symmetry broken as [8]

$$SO(4) \times SO(4)' \quad \stackrel{\rho>0}{\longrightarrow} \quad SO(4)_{\text{diag}} \ . \tag{2.16}$$

This is intriguing; through this deformation, one could hope to make progress in connecting the D1-D5-D5$'$ system to the system of (presently) greater physical interest, the single D1-D5 system. Since the deformation is only an $\mathcal{N} = 3$ modulus and not an $\mathcal{N} = 4$ modulus — the deformation only exists in the tilted system, not the orthogonal system — this will primarily apply to a version of the D1-D5 system broken to $\mathcal{N} = 3$ supersymmetry. We will comment on this in the Conclusions.

A valuable clue for understanding the ρ deformation by holography is parity. In (2.13), ρ is odd under interchange of the two three-spheres ($x \leftrightarrow y$). This will be important in each of the following sections.[4]

Even though we do not have a full deformed supergravity solution, we can attempt to model its near-horizon limit by imposing the symmetry reduction (2.16). If the boundary theory is to remain a CFT, we can try to look for deformations that leave the AdS_3 part of the dual geometry untouched. As we will see in the next section, such a deformation exists and it breaks supersymmetry to $\mathcal{N} = (3, 3)$.

[4]The paper [18] even studied an orbifold under this parity. That orbifold preserves different symmetries than we are interested in here.

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3. The deformed near-horizon limit

In this section we construct the near-horizon limit of the (currently unknown) ρ-deformed brane solution perturbatively in the deformation parameter ρ by exploiting the symmetries preserved by the deformation.

Let us first consider what simplifications we can impose on the Type IIB field equations. If we want a deformed solution where the dilaton is still constant, the source $F^{MNK}F_{MNK}$ in the dilaton equation of motion will have to stay zero. In the undeformed case, this was ensured by tuning the three-form fluxes (2.4) as

$$F_{mnk}F^{mnk} + F_{\bar{m}\bar{n}\bar{k}}F^{\bar{m}\bar{n}\bar{k}} = \frac{8}{g^2} \,, \tag{3.1}$$

so this must still hold for the deformed flux. The remaining nontrivial field equations are

$$R_{AB} = \frac{1}{4}F_A{}^{CD}F_{BCD} \,, \qquad D_A F^{ABC} = 0 \,. \tag{3.2}$$

Then we make the ten-dimensional metric ansatz

$$ds^2 = ds^2_{AdS_3} + ds^2_6 + R^2 d\phi^2 \,, \tag{3.3}$$

where ds^2_6 denotes the deformation of $S^3 \times S^3$ that we will now construct. (The answer is given in (3.16) below). Note that as previously stated, these equations are nontrivial only along the sphere coordinates but leave the AdS_3 part intact.

We now summarize the symmetries we want to impose. Recall that the isometry group of the undeformed background $AdS_3 \times S^3 \times S^3$ is given by

$$G_{\text{iso}} = SO(2,2) \times SO(3)_L \times SO(3)'_L \times SO(3)_R \times SO(3)'_R \,. \tag{3.4}$$

Here $SO(2,2) = SL(2)_L \times SL(2)_R$ describes the AdS_3 isometries and $SO(4) \equiv SO(3)_L \times SO(3)_R$ and $SO(4)' \equiv SO(3)'_L \times SO(3)'_R$ constitute the isometry groups of the two spheres. The subscripts L,R, on the other hand, correspond to the splitting into left- and right-movers in the two-dimensional boundary CFT. Out of the $SO(3)$ factors in (3.4), the deformation preserves only the diagonal subgroup

$$\begin{aligned} SO(4)^{(D)} &= SO(3)^{(D)}_L \times SO(3)^{(D)}_R \\ &\equiv \text{diag}\Big(SO(3)_L \times SO(3)'_L\Big) \times \text{diag}\Big(SO(3)_R \times SO(3)'_R\Big) \,. \end{aligned} \tag{3.5}$$

The solution can thus be constructed in terms of those S^3 sphere harmonics that are left invariant by the corresponding diagonal combinations of Killing vector fields. Moreover, as pointed out in the previous section, the deformation parameter ρ is odd under exchange of the two spheres. Together with the invariance requirements this puts very strong restrictions on the deformed solution.

To make this manifest, we need to introduce a little more notation. We parametrize the upper hemispheres of the spheres by coordinates x^m and $y^{\bar{m}}$, which are simply the projections of the Cartesian coordinates of the embedding space $\mathbf{R}^4$,

$$X^{\hat{A}} = (x^m, \sqrt{1-x^2}) \,, \qquad Y^{\hat{A}} = (y^{\bar{m}}, \sqrt{1-y^2}) \,, \tag{3.6}$$

with $x^2 = \sum_i (x^i)^2$, $y^2 = \sum_i (y^i)^2$. The sphere metrics in these coordinates are given by

$$g_{mn} = \delta_{mn} + \frac{x_m x_n}{1-x^2} , \qquad g_{\bar{m}\bar{n}} = \delta_{\bar{m}\bar{n}} + \frac{y_{\bar{m}} y_{\bar{n}}}{1-y^2} . \tag{3.7}$$

The $SO(4)$ isometries on the first sphere are generated by 6 Killing vectors $K_{L\,(k)}^{\;\;i}$, $K_{R\,(k)}^{\;\;i}$, which read

$$K_{L,R\,(k)}^{\;\;i} = -\frac{1}{2}\left(\varepsilon^i{}_{km} x^m \pm \delta^i_k \sqrt{1-x^2}\right) , \tag{3.8}$$

and similarly for $SO(4)'$. The normalisation is chosen such that the Lie brackets close according to the standard $SO(3)$ algebra:

$$[K_{L\,(a)}, K_{L\,(b)}] = \varepsilon_{ab}{}^c K_{L\,(c)} , \qquad \text{etc.} \tag{3.9}$$

The computations are significantly simplified by use of the vielbein formalism. A convenient $SO(3)$ frame is given by either half of the Killing vectors themselves, e.g. the K_L:

$$e_a^{\;\;m}(x) := 2K_{L\,(a)}^{\;\;m}(x) , \qquad e_{\bar{a}}^{\;\;\bar{m}}(y) := 2K_{L\,(\bar{a})}^{\;\;\bar{m}}(y) . \tag{3.10}$$

Because of the algebra (3.9), invariance under the diagonal combinations (3.5) of Killing vector fields reduces to invariance under the three combinations

$$K_{D\,(k)} \equiv \sqrt{1-x^2}\frac{\partial}{\partial x^k} + \sqrt{1-y^2}\frac{\partial}{\partial y^{\bar{k}}} , \tag{3.11}$$

which upon commutation generate the full $SO(4)^{(D)}$. We will now construct these invariant sphere harmonics.

Let us start with a ten-dimensional scalar field and consider its full Kaluza-Klein expansion (A.3) in terms of $S^3 \times S^3$ sphere functions $X^{[j,j]}(x)$, $Y^{[j',j']}(y)$ labeled by their spins j, j'. Under the diagonal $SO(4)_D$ this expansion contains an infinite number of singlet excitations, namely one in each product $X^{[j,j]}Y^{[j',j']}$ for $j = j'$, corresponding to the decomposition $[j,j;j,j] \to [0,0] + \ldots$ under the diagonal $SO(4)_D$. Explicitly, this corresponds to a truncation of (A.3) to an expansion

$$\Phi(z,x,y) = \sum_j \varphi_j(z)\, u^{2j} , \tag{3.12}$$

where u is the inner product of (3.6) in the embedding space:

$$u \;\equiv\; X^{\hat{A}} Y^{\hat{A}} = \sum_m x^m y^{\bar{m}} + \sqrt{1-x^2}\sqrt{1-y^2} . \tag{3.13}$$

One immediately verifies that u and thus the entire series (3.12) is indeed invariant under (3.11) and thus under the full diagonal $SO(4)^{(D)}$.

From these scalar invariants we can construct the invariant vector harmonics as

$$\begin{aligned} \mathcal{X}_a &= e_a^{\;\;m}\,\partial_m u = e_a^{\;\;m}\left(y_{\bar{m}} - \sqrt{\frac{1-y^2}{1-x^2}}\,x_m\right) , \qquad \mathcal{X}_{\bar{a}} = 0 , \\ \mathcal{Y}_{\bar{a}} &= e_{\bar{a}}^{\;\;\bar{m}}\,\partial_{\bar{m}} u = e_{\bar{a}}^{\;\;\bar{m}}\left(x_m - \sqrt{\frac{1-x^2}{1-y^2}}\,y_{\bar{m}}\right) , \qquad \mathcal{Y}_a = 0 , \end{aligned} \tag{3.14}$$

in flat indices $a, \bar{a}$ on $S^3 \times S^3$. Under (3.11) they transform as under a Lorentz transformation. We note the useful relations $\mathcal{X}_a \mathcal{X}^a = \mathcal{Y}_{\bar{a}} \mathcal{Y}^{\bar{a}} = 1 - u^2$.

The invariant tensor harmonics $\mathcal{Z}_{a\bar{b}}$ can be constructed along the same lines. Begin with a bivector $(1,1)$. It follows from (A.4) (and its analogue on the second S^3) that invariant tensors in the Kaluza-Klein tower on top of a bivector $(1,1)$ can arise from either of the series of representations $[j,j;j,j]$, $[j{+}1,j{+}1;j,j]$, and $[j,j;j{+}1,j{+}1]$. Indeed, there are three independent tensor harmonics $\mathcal{Z}^0_{a\bar{b}}$, $\mathcal{Z}^{\pm}_{a\bar{b}}$ which can explicitly be constructed as

$$\mathcal{Z}^0_{a\bar{b}} \equiv e_a{}^m \partial_m \mathcal{Y}_{\bar{b}} = e_{\bar{b}}{}^{\bar{m}} \partial_{\bar{m}} \mathcal{X}_a \,, \qquad \mathcal{Z}^{\pm}_{a\bar{b}} \equiv u \mathcal{Z}^0_{a\bar{b}} - \mathcal{X}_a \mathcal{Y}_{\bar{b}} \pm \epsilon_a{}^{cd} \mathcal{X}_c \mathcal{Z}_{d\bar{b}} \,. \tag{3.15}$$

The most general deformation of the metric (3.7) on $S^3 \times S^3$ preserving the diagonal isometries (3.11) can then be described by the six-dimensional vielbein (in triangular gauge)

$$\begin{aligned} E_m{}^a &= R\, e_m{}^b \left(a(u) \delta_b{}^a + c_1(u) \mathcal{X}_b \mathcal{X}^a\right) \,, \\ E_{\bar{m}}{}^{\bar{a}} &= R\, e_{\bar{m}}{}^{\bar{b}} \left(b(u) \delta_{\bar{b}}{}^{\bar{a}} + c_2(u) \mathcal{Y}_{\bar{b}} \mathcal{Y}^{\bar{a}}\right) \,, \\ E_m{}^{\bar{a}} &= R\, e_m{}^b \left(d(u) \mathcal{Z}^{0\,\bar{a}}_b + d_+(u) \mathcal{Z}^{+\,\bar{a}}_b + d_-(u) \mathcal{Z}^{-\,\bar{a}}_b\right) \,, \\ E_{\bar{m}}{}^a &= 0 \,, \end{aligned} \tag{3.16}$$

with a priori seven undetermined functions of u, and R is the S^3 radius (the two radii being equal). By fixing part of the diffeomorphism symmetry some of the free functions can be set to zero. Namely, employing a diffeomorphism

$$\xi^m = f(u) \mathcal{X}^m \,, \qquad \xi^{\bar{m}} = g(u) \mathcal{Y}^{\bar{m}} \,, \tag{3.17}$$

the functions $f(u)$ and $g(u)$ can be chosen such that $c_1(u) = c_2(u) = 0$ in (3.16).

Similarly the most general ansatz for the 3-form flux compatible with the diagonal isometries can be constructed. To this end, we write

$$\begin{aligned} &F_{mnk} = \kappa\, \omega_{mnk} + 3 \partial_{[m} c_{nk]} \,, \quad F_{\bar{m}\bar{n}\bar{k}} = \kappa\, \omega_{\bar{m}\bar{n}\bar{k}} + 3 \partial_{[\bar{m}} c_{\bar{n}\bar{k}]} \,, \\ &F_{mn\bar{k}} = 3 \partial_{[m} c_{n\bar{k}]} \,, \quad \text{etc.} \,, \end{aligned} \tag{3.18}$$

with

$$c_{mn} = b_1(u) \omega_{mnk} \mathcal{X}^k \,, \qquad c_{\bar{m}\bar{n}} = b_2(u) \omega_{\bar{m}\bar{n}\bar{k}} \mathcal{Y}^{\bar{k}} \,, \tag{3.19}$$

$$c_{m\bar{n}} = b_3(u) \mathcal{Z}_{m\bar{n}} + b_+(u) \mathcal{Z}^+_{m\bar{n}} + b_-(u) \mathcal{Z}^-_{m\bar{n}} \,. \tag{3.20}$$

Here $\kappa = 2R^2$ and ω_{mnk} and $\omega_{\bar{m}\bar{n}\bar{k}}$ denote the volume forms on the undeformed spheres S^3, respectively. The tensor gauge symmetry can be used to set the component $b_3(u)$ to zero.

With the most general ansatz compatible with the symmetry (3.5) at hand, we can now solve the IIB field equations (3.2), with the deformed flux satisfying (3.1). This leads to a highly complicated nonlinear system of differential equations for the functions $a(u)$, $b(u)$, $d(u)$, $d_\pm(u)$, $b_{1,2}(u)$, $b_\pm(u)$. Rather than attempting a solution in closed form we expand the system in the deformation parameter ρ and solve it order by order in ρ (using

Mathematica). Further imposing antisymmetry of ρ under exchange of the two spheres, we find for the metric

$$a(u) = 1 + u\rho + \tfrac{1}{2}u^2\rho^2 + \tfrac{1}{2}u^3\rho^3 + \mathcal{O}(\rho^4) \,, \tag{3.21}$$

$$b(u) = 1 - u\rho + \tfrac{1}{2}u^2\rho^2 - \tfrac{1}{2}u^3\rho^3 + \mathcal{O}(\rho^4) \,, \tag{3.22}$$

$$d(u) = -2u\rho^2\,(1+u\rho) + O(\rho^4) \,, \qquad d_\pm(u) = \mathcal{O}(\rho^4) \,, \tag{3.23}$$

while for the 3-form solution we obtain

$$b_1(u) = -2\rho\left(1 - u\rho + (u^2 + \tfrac{2}{3})\,\rho^2\right) + \mathcal{O}(\rho^4) \,, \tag{3.24}$$

$$b_2(u) = 2\rho\left(1 + u\rho + (u^2 + \tfrac{2}{3})\,\rho^2\right) + \mathcal{O}(\rho^4) \,, \qquad b_\pm(u) = \pm\tfrac{4}{3}\,u\rho^3 + \mathcal{O}(\rho^4) \,. \tag{3.25}$$

In particular, we see that to lowest order in ρ the deformation just corresponds to a relative warping between the two spheres. At higher orders, also off-diagonal components of the metric are excited.

4. The Kaluza-Klein spectrum

Before discussing the Kaluza-Klein spectrum of fluctuations around the deformed near-horizon limit constructed in the previous section, we first have to review the spectrum on the undeformed background (2.10). Its isometry supergroup under which the spectrum is organized is the direct product of two $\mathcal{N} = 4$ supergroups

$$D^1(2,1;\alpha)_L \times D^1(2,1;\alpha)_R \,, \tag{4.1}$$

in which each factor combines a bosonic $SO(3) \times SO(3) \times SL(2,\mathbb{R})$ with eight real supercharges (see [19] for definitions). More precisely, the noncompact factors $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R = SO(2,2)$ join into the isometry group of AdS_3 while the compact factors build up the isometry groups $SO(4) \times SO(4)'$ of the two spheres. The parameter α of (4.1) describes the ratio of the radii of the two spheres S^3, i.e. the ratio of D5 brane charges, which we have set to one. We note that $D^1(2,1;1) = OSp(4|2,\mathbb{R})$.

The massive Kaluza-Klein spectrum of maximal nine-dimensional supergravity on the $AdS_3 \times S^3 \times S^3$ background has been computed in [3]. We give a short review of the computation in appendix A. The resulting three-dimensional spectrum can be summarized as

$$\bigoplus_{\ell\geq 0,\ell'\geq 1/2} (\ell,\ell';\ell,\ell')_{\rm S} \;\oplus \bigoplus_{\ell\geq 1/2,\ell'\geq 0} (\ell,\ell';\ell,\ell')_{\rm S} \oplus \bigoplus_{\ell,\ell'\geq 0} \left((\ell,\ell';\ell+\tfrac{1}{2},\ell'+\tfrac{1}{2})_{\rm S} \oplus (\ell+\tfrac{1}{2},\ell'+\tfrac{1}{2};\ell,\ell')_{\rm S}\right) \,, \tag{4.2}$$

in terms of supermultiplets built from left-right tensor products of the short supermultiplets $(\ell,\ell')_{\rm S}$ of $OSp(4|2,\mathbb{R})$ [20], summarized in table 1. Note that the resulting multiplets

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h			
h_0		(ℓ,ℓ')	
$h_0+\frac{1}{2}$	$(\ell-\frac{1}{2},\ell'-\frac{1}{2})$	$(\ell+\frac{1}{2},\ell'-\frac{1}{2})$	$(\ell-\frac{1}{2},\ell'+\frac{1}{2})$
h_0+1	$(\ell,\ell'-1)$	$(\ell-1,\ell')$	(ℓ,ℓ')
$h_0+\frac{3}{2}$		$(\ell-\frac{1}{2},\ell'-\frac{1}{2})$	

Table 1: The generic short supermultiplet $(\ell,\ell')_{\rm S}$ of $OSp(4|2,\mathbb{R})$, with $h_0=\frac{1}{2}(\ell+\ell')$.

h_L \ h_R	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{4}$	$(0,\frac{1}{2};0,\frac{1}{2})$	$(0,\frac{1}{2};\frac{1}{2},0)$
$\frac{3}{4}$	$(\frac{1}{2},0;0,\frac{1}{2})$	$(\frac{1}{2},0;\frac{1}{2},0)$

h_L \ h_R	$\frac{1}{2}$	1	$\frac{3}{2}$
$\frac{1}{2}$	$(0,1;0,1)$	$(0,1;\frac{1}{2},\frac{1}{2})$	$(0,1;0,0)$
1	$(\frac{1}{2},\frac{1}{2};0,1)$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};0,0)$
$\frac{3}{2}$	$(0,0;0,1)$	$(0,0;\frac{1}{2},\frac{1}{2})$	$(0,0,0,0)$

Table 2: The spin-$\frac{1}{2}$ multiplet $(0,\frac{1}{2};0,\frac{1}{2})_{\rm S}$, and the spin-1 multiplet $(0,1;0,1)_{\rm S}$.

$(\ell,\ell';\ell,\ell')_{\rm S}$ generically contain massive fields with spin running from 0 to $\frac{3}{2}$, whereas multiplets of the type $(\ell,\ell';\ell+\frac{1}{2},\ell'+\frac{1}{2})_{\rm S}$ represent massive spin-2 multiplets. The lowest massive multiplets in the spectrum (4.2) are somewhat degenerate and collected in table 2; we will refer to these as the spin-$\frac{1}{2}$ matter multiplet and the (massive) YM multiplet, respectively.

Included in (4.2) is the massless supergravity multiplet $(\frac{1}{2},\frac{1}{2};0,0)_{\rm S}\oplus(0,0;\frac{1}{2},\frac{1}{2})_{\rm S}$ which contains no propagating degrees of freedom and consists of the vielbein, eight gravitinos transforming as

$$\psi^I_\mu:\quad (\tfrac{1}{2},\tfrac{1}{2};0,0)\oplus(0,0;\tfrac{1}{2},\tfrac{1}{2}) \tag{4.3}$$

under (3.4), and topological gauge vectors, corresponding to the $SO(4)_L\times SO(4)_R$ gauge group of the effective three-dimensional theory. The effective three-dimensional theories describing the coupling of the supergravity multiplet to the lowest massive supermultiplets from (4.2) have been constructed in [21 – 23].

In order to study the deformation of the spectrum and the associated effective theory, we need to identify the field corresponding to the deformation parameter ρ within (4.2). Since the deformation breaks supersymmetry $\mathcal{N}=(4,4)\to\mathcal{N}=(3,3)$ and the isometry group $SO(4)\times SO(4)'$ down to the diagonal (3.5), it should be contained in a scalar representation of the type $(\ell_L,\ell_L;\ell_R,\ell_R)$, with ℓ_R and ℓ_L not both equal to zero. Moreover, since the deformation preserves the AdS_3 factor, the corresponding field should have no AdS mass, i.e. come with boundary conformal dimension $\Delta=2$. From table 7 we identify four possible candidates with $\Delta=2$: two in the $(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ representation and sitting in the spin-1 (YM) multiplets of table 2, and two in the $(1,1;1,1)$ representation that originate from higher supermultiplets. Note that these fields come with $\Delta=2$ only for $\alpha=1$, i.e. coinciding D5 brane charges, in accordance with the above remarks about the existence of the brane reconnection.

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h	
h_0	(ℓ)
$h_0+\frac{1}{2}$	$(\ell)+(\ell-1)$
h_0+1	$(\ell-1)$

Table 3: The generic short supermultiplet $(\ell)_{\rm S}$ of $OSp(3|2,\mathbb{R})$, with $h_0=\ell/2$.

To narrow down which of the representations $(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ and $(1,1;1,1)$ actually contain the deformation, we make use of the fact that according to its definition (2.13) ρ should be odd under exchange of the two spheres. The two $(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ descend from chiral multiplets ($(0,1;0,1)_{\rm S}$ and $(1,0;1,0)_{\rm S}$), so will have one odd and one even combination, whereas the two $(1,1;1,1)$ come from nonchiral multiplets. Thus, parity suggests that the only combination of fields odd under exchange of the two spheres is a combination of the two $(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ scalars. In order to study the ρ deformation in the effective three-dimensional theory we will thus have to consider the coupling of two YM multiplets. This is the goal of the next section. Indeed, in this effective theory we find a potential with a flat direction (figure 4 below) for the aforementioned combination, along which supersymmetry is broken from $\mathcal{N}=(4,4)$ down to $\mathcal{N}=(3,3)$. We take this as strong support of our claim that the deformation in fact arises from the $(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$ representation and not the $(1,1;1,1)$.

Let us close this section by a few general remarks on $\mathcal{N}=(4,4)\to\mathcal{N}=(3,3)$ supersymmetry breaking. In terms of supergroups this corresponds to the natural embedding $OSp(3|2,\mathbb{R})\subset OSp(4|2,\mathbb{R})$. A short $OSp(3|2,\mathbb{R})$ supermultiplet $(\ell)_{\rm S}$ is defined by its highest weight state $(\ell)^{h_0}$, where ℓ labels the $SO(3)$ spin and $h=h_0=\ell/2$ is the charge under the Cartan subgroup $SO(1,1)\subset SL(2,\mathbb{R})$. The short supermultiplet is generated from the highest weight state by the action of two out of the three supercharges and carries 8ℓ degrees of freedom [24]. Its $SO(3)^{\pm}$ representation content is summarized in table 3.

The generic long multiplet $(\ell)_{\rm long}$ is instead built from the action of all three supercharges on the highest weight state and correspondingly carries $8(2\ell+1)$ degrees of freedom. Its highest weight state satisfies the unitarity bound

$$h\geq\ell/2\ . \tag{4.4}$$

Here it is worthwhile to pause and contrast the simplicity of this unitarity bound with the *nonlinear* bound for the unbroken large $\mathcal{N}=4$ algebra. This nonlinearity was responsible for many of the complications in constructing holographic dual pairs for the large $\mathcal{N}=4$ theory [3, 8]. For example, states that are classically BPS can receive quantum corrections in the large $\mathcal{N}=4$ theory, an unusual situation. By comparison, a bound as simple as (4.4) seems a compelling reason for studying $\mathcal{N}=3$ theories in further detail, both in their own right and for their connection with $\mathcal{N}=4$ theories. (A nice summary is given in [25]).

When the bound (4.4) is saturated, the long multiplet decomposes into two short multiplets according to

$$(\ell)_{\rm long}=(\ell)_{\rm S}\oplus(\ell{+}1)_{\rm S}\ , \tag{4.5}$$

from which one may read off the $SO(3)$ content of $(\ell)_{\rm long}$. A semishort $\mathcal{N}=4$ multiplet $(\ell^+,\ell^-)_{\rm S}$ breaks according to

$$(\ell,\ell')_{\rm S} = (\ell+\ell')_{\rm S} \oplus (\ell+\ell'-1)_{\rm long} \oplus \ \ldots \ \oplus (|\ell-\ell'|)_{\rm long} \ , \tag{4.6}$$

into semishort and genuine long $\mathcal{N}=3$ multiplets. From (4.6) one may read off the decomposition of the spectrum (4.2) after turning on the deformation. The masses of the long multiplets are not protected and may acquire ρ-dependent deformation contributions. In principle, even semi-short multiplets originating from different $\mathcal{N}=4$ multiplets may recombine according to (4.5) into long $\mathcal{N}=3$ multiplets and lift off from the mass bound along the deformation. We will see an example of this in the next section.

5. The effective action in $D=3$

In this section we discuss the effective supergravity action in three dimensions, which describes the YM (spin-1) multiplets $(1,0;1,0)_{\rm S}\oplus(0,1;0,1)_{\rm S}$ discussed above. In particular, we compute the scalar potential. At the origin of scalar field space we have the undeformed background $AdS_3\times S^3\times S^3$. We show that there is a flat direction along which supersymmetry is broken down to $\mathcal{N}=(3,3)$ and compute the deformation of the mass spectrum along the valley.

5.1 Effective action for the YM multiplets

To start with, we note that in accordance with the amount of supersymmetry preserved by the undeformed background, the relevant three-dimensional supergravity will be a gauged $\mathcal{N}=8$ theory with gauge group $SO(4)\times SO(4)$. Here, we briefly review the construction of the effective theory based on [21 – 23] to which we refer for details.

The field content of the two YM multiplets $(1,0;1,0)_{\rm S}\oplus(0,1;0,1)_{\rm S}$ is given in table 2, in particular they contain 32 bosonic degrees of freedom each. Together with $\mathcal{N}=8$ supersymmetry this implies that the scalar degrees of freedom of the three-dimensional theory are described by a coset space $SO(8,8)/(SO(8)\times SO(8))$. (The massive vector degrees of freedom appear through their Goldstone scalars). This in turn requires an embedding of the gauge group $SO(4)\times SO(4)$ into $SO(8)\times SO(8)$, such that the corresponding branching of the $(\mathbf{8},\mathbf{8})$ representation of the latter reproduces the correct $SO(4)\times SO(4)$ representations of table 2. The explicit embedding was given in [22, 23], and is described by a constant $SO(8,8)$ tensor $\Theta_{\mathcal{M}\mathcal{N}}=\Theta_{(\mathcal{M}\mathcal{N})}$ in the symmetric product of two adjoint representations whose explicit form determines the entire Lagrangian.

Explicitly, the action is given by

$$\mathcal{L} = -\frac{1}{4}\sqrt{g}R + \frac{1}{4}\sqrt{g}\mathcal{P}^{Ir}_\mu\mathcal{P}^{\mu\,Ir} + \mathcal{L}_{\rm CS} + \mathcal{L}_{\rm ferm} - \sqrt{g}V \ , \tag{5.1}$$

where the individual ingredients are as follows. The scalar fields are described by an $SO(8,8)$ valued matrix and its current

$$\mathcal{S}^{-1}(\partial_\mu + \Theta_{\mathcal{M}\mathcal{N}}A^{\mathcal{M}}_\mu t^{\mathcal{N}})\mathcal{S} = \tfrac{1}{2}\mathcal{Q}^{IJ}_\mu X^{[IJ]} + \tfrac{1}{2}\mathcal{Q}^{rs}_\mu X^{[rs]} + \mathcal{P}^{Ir}_\mu X^{Ir} \ , \tag{5.2}$$

decomposed into compact ($\mathcal{Q}_\mu$) and noncompact ($\mathcal{P}_\mu$) contributions. Indices $I, J, \ldots$ and $r, s, \ldots$ are vector indices of the two $SO(8)$ subgroups; adjoint $SO(8,8)$ indices $\mathcal{M}, \mathcal{N}$ thus split into pairs $([IJ], [rs], Ir)$. The vector fields couple by a Chern-Simons term

$$\mathcal{L}_{\rm CS} = -\tfrac{1}{4}\varepsilon^{\mu\nu\rho} A^{\mathcal{M}}_{\mu}\, \Theta_{\mathcal{M}\mathcal{N}} \left(\partial_\nu A^{\mathcal{N}}_{\rho} + \tfrac{1}{3}\, f^{\mathcal{N}\mathcal{P}}{}_{\mathcal{L}}\, \Theta_{\mathcal{P}\mathcal{K}}\, A^{\mathcal{K}}_{\nu} A^{\mathcal{L}}_{\rho}\right) , \tag{5.3}$$

with the SO(8,8) structure constants $f^{\mathcal{N}\mathcal{P}}{}_{\mathcal{L}}$. The potential V is given as a function of the scalar fields as

$$V = -\tfrac{1}{48}\left(T_{[IJ,KL]} T_{[IJ,KL]} + \tfrac{1}{4!}\, \epsilon^{IJKLMNPQ}\, T_{IJ,KL} T_{MN,PQ} - 2\, T_{IJ,Kr} T_{IJ,Kr}\right) , \tag{5.4}$$

in terms of the so-called T-tensor

$$T_{IJ,KL} = \mathcal{V}^{\mathcal{M}}{}_{IJ} \mathcal{V}^{\mathcal{N}}{}_{KL}\, \Theta_{\mathcal{M}\mathcal{N}} , \qquad T_{IJ,Kr} \;=\; \mathcal{V}^{\mathcal{M}}{}_{IJ} \mathcal{V}^{\mathcal{N}}{}_{Kr}\, \Theta_{\mathcal{M}\mathcal{N}} , \tag{5.5}$$

where $\mathcal{V}$ defines the group matrix $\mathcal{S}$ in the adjoint representation:

$$\mathcal{S}^{-1} t^{\mathcal{M}} \mathcal{S} \equiv \tfrac{1}{2}\, \mathcal{V}^{\mathcal{M}}{}_{IJ}\, X^{IJ} + \tfrac{1}{2}\, \mathcal{V}^{\mathcal{M}}{}_{rs}\, X^{rs} + \mathcal{V}^{\mathcal{M}}{}_{Ir}\, Y^{Ir} . \tag{5.6}$$

For the fermionic contributions $\mathcal{L}_{\rm ferm}$ we refer to [21].

5.2 The marginal $\mathcal{N} = (3,3)$ deformation

We are mainly interested in the scalar potential (5.4). A ρ-dependent deformation that preserves AdS_3 while deforming the two spheres and breaking the symmetry according to (3.5) should manifest itself in the existence of a corresponding flat direction of the potential. The full potential (5.4), being a rather complicated function of the 64 scalar fields, is not needed. For our purposes it will be sufficient to consider its truncation to $SO(4)^{(D)}$ (defined in (3.5)) singlets. Indeed, extremal points in this truncated potential will lift to extremal points of the full potential [26].

Under $SO(4)^{(D)}$, each YM multiplet contains two scalar singlets, i.e. we have a four-dimensional manifold of scalars invariant under $SO(3)^{(D)}_L \times SO(3)^{(D)}_R$. At the origin, these scalars come in two pairs with square masses 0 and 3, i.e. they correspond to operators of conformal dimensions $\Delta = 2$ and $\Delta = 3$. In particular, there are two marginal operators, in accordance with table 2, above. In order to describe the truncation of the Lagrangian to this four-dimensional target space manifold, we parametrize the $SO(8,8)$ matrix $\mathcal{S}$ as

$$\mathcal{S} = \exp \begin{pmatrix} 0 & 0 & v_1 & w_2 \\ 0 & 0 & w_1 & v_2 \\ v_1 & w_1 & 0 & 0 \\ w_2 & v_2 & 0 & 0 \end{pmatrix} , \tag{5.7}$$

where each entry represents a multiple of the 4×4 unit matrix. Note that v_1, v_2 parametrize the two $SO(4) \times SO(4)'$ singlets. (Truncation to a single YM multiplet would correspond to setting $w_1 = v_2 = 0$.) Unfortunately, even the truncation of the potential (5.4) to the four-dimensional subspace (5.7) is a highly complicated function. We computed it using

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Mathematica but refrain from giving it here.[5] Instead, we further truncate to the two-dimensional subspace defined by $v_1 = v_2$, $w_1 = -w_2$. This again is a consistent truncation as it corresponds to the fixed points of an inner automorphism that leaves $\Theta_{\mathcal{MN}}$ invariant.

In terms of the variables

$$z^2 = v_1^2 + w_1^2 \,, \quad \phi = \arctan(w_1/v_1) \,, \tag{5.8}$$

this gives rise to a Lagrangian

$$\frac{1}{\sqrt{g}}\mathcal{L} = \partial_\mu z\, \partial^\mu z + \sinh^2 z\, \partial_\mu \phi\, \partial^\mu \phi - V \,, \tag{5.9}$$

with the scalar potential

$$V = -2 + 8\sinh^2 z\, (\sinh z - \cos\phi \cosh z)^2\, (1 + 2\cosh 2z - 2\cos\phi \sinh 2z) \,. \tag{5.10}$$

Obviously, this potential is bounded from below ($V \geq -2$). Further transforming to coordinates

$$\tau = \sin\phi \sinh z \,, \; \zeta = \cos\phi \sinh z \,, \tag{5.11}$$

we find that the minimum $V = -2$ is actually taken along a curve

$$\zeta = \frac{\tau}{\sqrt{1-\tau^2}} \,, \tag{5.12}$$

with τ running from 0 to 1, which thus constitutes a flat direction in the potential, depicted in figure 4. We have verified by explicit computation that this extends to a flat direction in the full four-dimensional target space (5.7) and thus of the full scalar potential. In terms of the coordinates τ, ζ, exchange of the two spheres corresponds to $\tau \to -\tau$, so the graph shows that infinitesimally, the valley indeed points into an odd direction in accordance with the odd parity of the deformation ρ discussed in earlier sections. In other words, we can identify $\tau = \rho$ to lowest order.

By construction, any nonvanishing expectation value of the scalar fields corresponding to (5.12) yields an AdS_3 solution of the three-dimensional theory which breaks the original $SO(4) \times SO(4)'$ symmetry down to the diagonal and supersymmetry down to $\mathcal{N} = (3,3)$ as we shall see below. From a purely supergravity point of view the existence of this flat direction is surprising, but it finds a natural interpretation in terms of the brane reconnection described in earlier sections. Note further that in the three-dimensional theory the deformation (5.12) is very simple (once the scalar potential (5.10) has been computed) and exact to all orders in the deformation, whereas in 10 dimensions we have only been able to perturbatively compute the corresponding solution. This shows how nontrivially the effective theory (5.1) must be embedded within the IIB theory. Techniques such as those employed in [27, 28] might prove useful to obtain the corresponding IIB solution in closed form.

[5] It can be found at www.aei.mpg.de/∼mberg/physics/potential.

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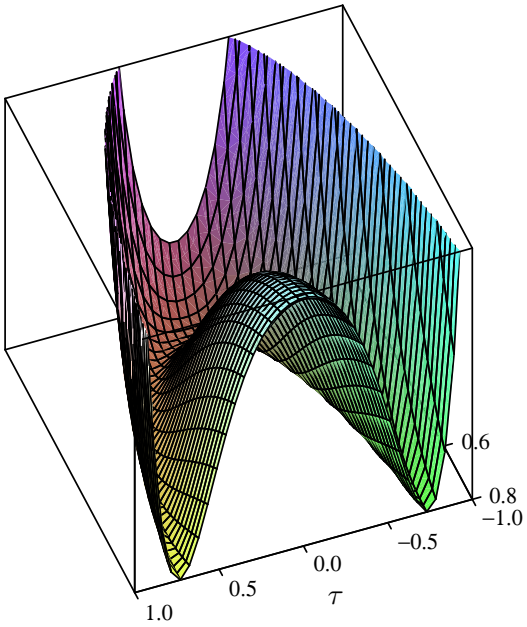

Figure 4: Flat valley in the potential, the $\mathcal{N} = (4,4)$ origin is located at $(0,0)$. Coordinates here are (τ, ζ) as in (5.11).

5.3 Deformed spectrum

Having established a one-parameter class of solutions of the three-dimensional effective theory, we can now study how the mass spectrum changes upon moving along the valley (5.12), that we can parameterize by τ. As τ is odd under exchange of the two spheres, the spectrum should be even in τ.

As the deformation preserves the diagonal subgroup (3.5), the deformed spectrum organizes under $SO(4)^{(D)}$. Of particular interest is the remaining supersymmetry. This is found by calculating the deformation of the gravitino masses as eigenvalues of [21]

$$A_1^{AB} = -\delta^{AB}\theta - \frac{1}{48}\Gamma_{AB}^{IJKL} T_{IJ|KL} \,. \tag{5.13}$$

As a result we find

$$m_i = \pm\tfrac{1}{2} \quad (\times 3) \,, \qquad m_i = \pm\tfrac{1}{2} f(\tau) \quad (\times 1) \,, \tag{5.14}$$

where $f(\tau)$ is given in terms of the coordinate τ in figure 4 by

$$f(\tau) = \sqrt{\frac{1+15\tau^2}{1-\tau^2}} \,. \tag{5.15}$$

Thus, we see that when we move away from the origin along the valley ($\tau > 0$), supersymmetry is broken from $\mathcal{N} = (4,4)$ down to $\mathcal{N} = (3,3)$, confirming the arguments in

h_L \ h_R	$\frac{1}{2}$	1	$\frac{3}{2}$
$\frac{1}{2}$	$(1;1)$	$(1;0)+(1;1)$	$(1;0)$
1	$(0;1)+(1;1)$	$(0;0)+(0;1)+(1;0)+(1;1)$	$(0;0)+(1;0)$
$\frac{3}{2}$	$(0;1)$	$(0;0)+(0;1)$	$(0;0)$

Table 4: The short $\mathcal{N}=(3,3)$ multiplet $(1;1)_{\rm S}$.

earlier sections. From the point of view of the 3-dimensional effective theory, this was by no means guaranteed.

By linearizing the full scalar potential (5.4) around the deformed solution, we obtain the deformed scalar masses, most conveniently expressed in terms of the associated conformal dimensions $\Delta = 1+\sqrt{1+m^2}$:

$$\Delta_i = \begin{cases} 1 & (\times 9) \\ 2 & (\times 34) \\ 3 & (\times 1) \\ \frac{1}{2}\left(-1+f(\tau)\right) & (\times 1) \\ \frac{1}{2}\left(1+f(\tau)\right) & (\times 9) \\ \frac{1}{2}\left(3+f(\tau)\right) & (\times 9) \\ \frac{1}{2}\left(5+f(\tau)\right) & (\times 1) \end{cases} \tag{5.16}$$

From these values we can infer the entire spectrum in terms of $\mathcal{N}=(3,3)$ supermultiplets. Comparing (5.16) to table 3 we conclude that the $\mathcal{N}=(3,3)$ spectrum along the deformation is given by

$$\mathcal{H}_\rho = (1;1)_{\rm S} \oplus (0;0)^h_{\rm long}\ , \tag{5.17}$$

where the mass of the long multiplet is given by $h=\frac{1}{4}\left(-1+f(\tau)\right)$. We see that the entire spectrum is indeed even in τ.

For convenience, we have collected the field content of these two multiplets in tables 4, 5. As τ tends to one, the long multiplet $(0;0)^h_{\rm long}$ becomes infinitely massive, and we are left with the semi-short multiplet $(1;1)_{\rm S}$ whose coupling to $\mathcal{N}=(3,3)$ supergravity is described by a gauged theory with target space $SU(4,4)/S(U(4)\times U(4))$. Comparing this multiplet to the original field content (table 2) one recognizes a diagonal combination of the two $\mathcal{N}=4$ YM multiplets.

Now that we have the deformed spectrum, it is instructive to turn around and study the behavior of (5.17) as the deformation is switched off ($\tau\to 0$). At this point, the long $\mathcal{N}=(3,3)$ multiplet hits the unitarity bound $h=0$ and falls apart according to (4.6):

$$(0;0)^0_{\rm long} \to (1;1)_{\rm S} \oplus (1;0)_{\rm S} \oplus (0;1)_{\rm S} \oplus (0;0)_{\rm S}\ . \tag{5.18}$$

Simple counting of states shows that for these low spins the formulae degenerate such that $(0;0)_{\rm S}$, $(1;0)_{\rm S}$, and $(0;1)_{\rm S}$ denote unphysical multiplets without propagating degrees of

h_L \ h_R	h	$h+\frac{1}{2}$	$h+1$	$h+\frac{3}{2}$
h	$(0;0)$	$(0;1)$	$(0;1)$	$(0;0)$
$h+\frac{1}{2}$	$(1;0)$	$(1;1)$	$(1;1)$	$(1;0)$
$h+1$	$(1;0)$	$(1;1)$	$(1;1)$	$(1;0)$
$h+\frac{3}{2}$	$(0;0)$	$(0;1)$	$(0;1)$	$(0;0)$

Table 5: The long $\mathcal{N}=(3,3)$ multiplet $(0;0)^h_{\rm long}$.

h_L \ h_R	0	$\frac{1}{2}$	1
0	$(0;0)$		$-(0;0)$
$\frac{1}{2}$			
1	$-(0;0)$		$(0;0)$

h_L \ h_R	0	$\frac{1}{2}$	1
$\frac{1}{2}$	$(1;0)$		$-(1;0)$
1	$(0;0)+(1;0)$		$-(0;0)-(1;0)$
$\frac{3}{2}$	$(0;0)$		$-(0;0)$

Table 6: Unphysical $\mathcal{N}=(3,3)$ multiplets $(0;0)_{\rm S}$, $(1;0)_{\rm S}$.

freedom, given in table 6. The negative multiplicities should be understood as (first order differential) constraints that eliminate the physical degrees of freedom. To understand the role of these unphysical multiplets at $\tau=0$ we have to also consider the (non-propagating) supergravity multiplet. Applying (4.6) to the (unphysical) $\mathcal{N}=(4,4)$ supergravity multiplet $(\frac{1}{2},\frac{1}{2};0,0)_{\rm S}\oplus(0,0;\frac{1}{2},\frac{1}{2})_{\rm S}$ shows that under $\mathcal{N}=(3,3)$ it decomposes as

$$(\tfrac{1}{2},\tfrac{1}{2};0,0)_{\rm S}\oplus(0,0;\tfrac{1}{2},\tfrac{1}{2})_{\rm S}\to(0_{\rm long};0_{\rm S})\oplus(0_{\rm S};0_{\rm long})\oplus(1;0)_{\rm S}\oplus(0;1)_{\rm S}\ , \qquad (5.19)$$

where the first two terms represent the $\mathcal{N}=(3,3)$ supergravity multiplet and in the second two terms one recognizes the unphysical part of (5.18). Put together, at $\tau=0$ the long $\mathcal{N}=(3,3)$ multiplet splits according to (5.18), of which the first term coincides with an $\mathcal{N}=(4,4)$ YM multiplet (tables 2, 4), whereas the unphysical multiplets $(1;0)_{\rm S}\oplus(0;1)_{\rm S}$ combine with the supergravity multiplet in order to reconstitute the $\mathcal{N}=(4,4)$ supergravity multiplet.

Having understood how things combine when we switch *off* the deformation τ, we can now go back to the $\mathcal{N}=4$ theory and summarize in $\mathcal{N}=3$ language what happens when we switch on the deformation. Then, $\mathcal{N}=3$ semi-short multiplets originating from different $\mathcal{N}=4$ ancestors (gravity and YM multiplet) combine to form a long $\mathcal{N}=3$ multiplet and lift off the mass bound.[6]

[6] An analogous situation is encountered in the $AdS_5/{\rm CFT}_4$ correspondence upon switching on the 't Hooft coupling λ. This breaks the higher spin symmetry present at $\lambda=0$ down to $PSU(2,2|4)$. In the process, semi-short multiplets originating from different higher-spin multiplets then combine into long multiplets of $PSU(2,2|4)$ [29].

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6. D1-brane CFT

Using the holographic correspondence, we should be able to compare the spectrum (5.16) computed from supergravity to those in the CFT on the D-brane intersection. Although a full comparison is beyond the scope of this paper, we give some initial steps towards this general goal.

First, we consider the symmetric product CFT $Sym^N(\mathbf{S}^1 \times \mathbf{S}^3)$, which consists of N copies of $\mathbf{S}^1 \times \mathbf{S}^3$ orbifolded by the symmetric group S_N. For equal D5-brane and D5′-brane charge, this was conjectured in [3, 8] to be the CFT dual of Type IIB string theory on $AdS_3 \times S^3 \times S^3 \times S^1$. Orbifolds of this type have been extensively studied in the literature, and we used results from [30, 31].

The bosonic part of the worldsheet action of a D1-brane on $\mathbf{S}^1 \times \mathbf{S}^3$ is

$$S_{\rm ws} = \frac{1}{2\pi} \int_{\rm D1} d^2z \, G_{ab} \partial X^a \bar{\partial} X^b + \int_{\rm D1} C_{(2)} \, , \tag{6.1}$$

where G_{ab} is the induced metric on $\mathbf{S}^1 \times \mathbf{S}^3$, $C_{(2)}$ is the RR 2-form potential, and we have suppressed labels of the N copies. This is the *undeformed* theory. Rather than considering a sigma model $Sym^N(\mathcal{M})$ on the full complicated ρ-deformed solution directly, we represent the deformation by an operator $\mathcal{O}_\rho$, that we obtain by expanding the worldsheet action (6.1) in the deformation parameter ρ in the full solution. As probe of the deformation, we then consider an untwisted probe operator $\mathcal{O}_3$, one of the $\Delta = 3$ operators in the spectrum, coupling to the deformation. We should then compute

$$\langle \mathcal{O}_3 \mathcal{O}_3 \rangle_\rho = \langle \mathcal{O}_3 \mathcal{O}_3 \rangle_0 + \langle \mathcal{O}_3 \mathcal{O}_3 \mathcal{O}_\rho \rangle_0 + \dots \, , \tag{6.2}$$

where the subscript 0 refers to the undeformed theory. Since the bulk theory has an AdS_3 factor also after deformation, the boundary theory will remain conformal, and we should be able to compute corrections to Δ this way:

$$\Delta(\rho) = \Delta_0 + \Delta_1 \rho^2 + \dots \, . \tag{6.3}$$

Since we have already computed this deformation for all ρ on the supergravity side in (5.16), we could then compare results. For now, we will content ourselves with computing the first term in the series (6.2), since for our case of Lorentzian AdS, we did not find this done explicitly in the literature.

The worldsheet coordinate is z. The symmetric orbifold ground state has twist insertions $\sigma_{(1\dots n)}(z)$ at $z = 0, \infty$, and the coordinates of the $n \leq N$ copies cyclically permute as $X^a(z)$ encircles these points (see e.g. [30]). On the covering space $z \sim t^n$, however, all fields are single-valued, so we have the ordinary 2-point function:

$$\langle \mathcal{O}_3(t_1) \mathcal{O}_3(\bar{t_2}) \rangle = \frac{1}{(t_1 - t_2)^3 (\bar{t}_1 - \bar{t}_2)^3} \, . \tag{6.4}$$

Going back to the z variable, we obtain n correlators, one for each branch of the multiple covering, that must be summed over. This is simpler in cylinder coordinates $z = e^{-iw}$,

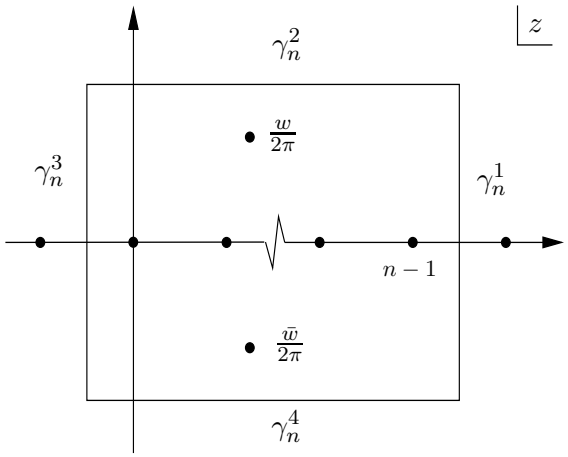

Figure 5: Jordan curve for the sum (6.5).

where the sum is over shifts in w:

$$\langle \mathcal{O}_3(w_1)\mathcal{O}_3(\bar{w_2})\rangle = \sum_{k=0}^{n-1} \frac{1}{(2n\sin\frac{w-2\pi k}{2n})^3(2n\sin\frac{\bar{w}-2\pi k}{2n})^3}\,, \tag{6.5}$$

where $w = w_1 - w_2$. This finite sum can be performed by contour integration. To be precise, it is performed by integrating an analytic function $f(z)$ that has poles at $z = 0, 1, \ldots, n-1$ (and possibly elsewhere) around a suitable contour. We pick the analytic function

$$f(z) = \frac{\pi\cot\pi z}{(2n\sin\frac{w-2\pi z}{2n})^3(2n\sin\frac{\bar{w}-2\pi z}{2n})^3}\,, \tag{6.6}$$

and choose a square contour as in figure 5, where the integrals over γ_n^1 and γ_n^3 cancel by periodicity under $z \to z+n$, and the γ_n^2 and γ_n^4 can be moved off to infinity, where $|f(z)|$ vanishes exponentially. Hence the sum is the negative of the contribution from the two remaining poles, which yields

$$\begin{aligned}\langle \mathcal{O}_3(w_1)\mathcal{O}_3(\bar{w_2})\rangle = -\frac{1}{64n^3\sin^3\frac{w-\bar{w}}{2n}}\Bigg[&\left(\frac{\cos\frac{w}{2}}{\sin^3\frac{w}{2}} - \frac{\cos\frac{\bar{w}}{2}}{\sin^3\frac{\bar{w}}{2}}\right) \\ &-\frac{3}{n\cot\frac{w-\bar{w}}{2n}}\left(\frac{1}{\sin^2\frac{w}{2}} - \frac{1}{\sin^2\frac{\bar{w}}{2}}\right) - \frac{6\sin\frac{w-\bar{w}}{2}}{n^2\sin^2\frac{w-\bar{w}}{2n}\sin\frac{w}{2}\sin\frac{\bar{w}}{2}} + \frac{4}{n^2}\frac{\sin\frac{w-\bar{w}}{2}}{\sin\frac{w}{2}\sin\frac{\bar{w}}{2}}\Bigg]\,.\end{aligned} \tag{6.7}$$

We note that all terms in the bracket except for the last persist in the large n limit, which

yields[7]

$$\langle \mathcal{O}_3(w_1)\mathcal{O}_3(\bar{w}_2)\rangle = -\frac{1}{8(w-\bar{w})^3}\left[\left(\frac{\cos\frac{w}{2}}{\sin^3\frac{w}{2}}-\frac{\cos\frac{\bar{w}}{2}}{\sin^3\frac{\bar{w}}{2}}\right)\right.$$
$$\left.-6(w-\bar{w})\left(\frac{1}{\sin^2\frac{w}{2}}-\frac{1}{\sin^2\frac{\bar{w}}{2}}\right)-\frac{24\sin\frac{w-\bar{w}}{2}}{(w-\bar{w})^2\sin\frac{w}{2}\sin\frac{\bar{w}}{2}}\right] . \quad (6.8)$$

The task to compute the first deformed correlator is clearly more formidable, and we will not perform it here.

6.1 Probe approximation

For now, a less ambitious computation would be to probe the ρ-deformed background by a "long" D1-brane probe in AdS_3, and quantize it semiclassically along the lines of how it is done in AdS_5 (e.g. [32], that considers both static and conformal gauge). In fact, for massive fluctuations in the warped region, we could use the simpler "effective string wavefunction" argument, as for instance in [33]. We would not expect to be able to reproduce the full contributions to the spectrum this way, of course. For the related NS5-brane configuration in [34], D1-brane probes have been studied in [35 – 37].

For this purpose, we can consider the bosonic part of the D-brane action for a D1-brane probe in the ρ-deformed background in section 3, with open-string vectors turned off:

$$S_{\rm D1} = \frac{1}{2\pi}\int_{\rm D1} d^2\sigma\sqrt{-\det(h_{ab})} + \int_{\rm D1} C_{(2)} , \quad (6.9)$$

where h_{ab} is the 2-dimensional metric induced by our ρ-deformed $AdS_3\times S^3\times S^3$ background:

$$h_{ab} = g^{\rm AdS}_{\mu\nu}\partial_a x^\mu\partial_b x^\nu + G_{ij}(X)\partial_a X^i\partial_b X^j . \quad (6.10)$$

Here $i,j=1,\ldots,6$ label the $S^3\times S^3$ coordinates, collectively denoted by X^i, and $g^{\rm AdS}_{\mu\nu}$ is the AdS_3 metric. We consider a static configuration $x^0=\sigma^0$, $x^1=\sigma^1$, $x^2=$ constant, and allow for arbitrary fluctuations of X^i around the origin. To zeroth order, there is no nontrivial static potential. To second order in fluctuations, the induced metric (6.10) becomes[8]

$$h_{ab} = \frac{R^2}{2r^2}\eta_{ab} + G_{ij}(0)\partial_a X^i\partial_b X^j + O(X^3) , \quad (6.11)$$

with η_{ab} denoting the flat 2d metric and X^i now meaning fluctuations. We define $\bar{\eta}_{ab} = \frac{R^2}{2r^2}\eta_{ab}$. Expanding the square root of the determinant to linear order (which is then second order in the fluctuations), we find

$$\sqrt{-\det(h_{ab})} = \sqrt{-\det(\bar{\eta}_{ab})}\left(1+\frac{1}{2}\bar{\eta}^{ab}G_{ij}(0)\partial_a X^i\partial_b X^j\right) + O(X^3) . \quad (6.12)$$

[7] Unlike in [31], the correlator in the large-n theory seems to have a higher-order pole than that in the covering space.

[8] Here we used AdS_3 coordinates $ds^2 = (R^2/(2r^2))(-dt^2+dr^2+dz^2)$, unlike in (2.10).

Next let us evaluate $G_{ij}(0)$ explicitly, using the ρ-deformed near-horizon limit in section 3. In the notation of that section, at $X^i = (x^m, y^{\bar{m}}) = 0$ we have $u = 1$, $\mathcal{Z}^0_{a\bar{b}} = \mathcal{Z}^{\pm}_{a\bar{b}} = \delta_{a\bar{b}}$, while the other harmonics $\mathcal{X}_a$, $\mathcal{Y}_{\bar{a}}$, etc. vanish. The 6-dimensional part of the target space metric then simply reduces to

$$G_{ij}(0) = \begin{pmatrix} (a^2(1) + d^2(1))\delta_{mn} & b(1)d(1)\delta_{m\bar{n}} \\ b(1)d(1)\delta_{\bar{m}n} & b^2(1)\delta_{\bar{m}\bar{n}} \end{pmatrix} . \tag{6.13}$$

Thus, using the explicit expressions for a, b and d at $u = 1$ to the given order in ρ, the quadratic fluctuation action is

$$\begin{aligned} S_{\mathrm{D1},\mathcal{O}(X^2)} = \frac{g^2}{4\pi} \int d^2\sigma \big[& (1 + 2(\rho + \rho^2 + \rho^3))\partial^a x^m \partial_a x^m \\ & + (1 - 2(\rho - \rho^2 + \rho^3))\partial^a y^{\bar{m}} \partial_a y^{\bar{m}} - 4\rho^2 \partial^a x^m \partial_a y^{\bar{m}} \big] . \end{aligned} \tag{6.14}$$

Here everything is contracted with the flat 2d metric, i.e. we have used the fact that the conformal factor differing between $\bar{\eta}$ and η cancels in two dimensions.

We find that these fluctuations are massless. To see a mass term we would have to start from a non-constant background such that the derivative term can give a quadratic background term, which after expanding the scalar metric $G_{ij}(X)$ up to second order in X^i supports a mass term. One would like to expand in normal coordinates, i.e. choose normal coordinates on our deformed $S^3 \times S^3$ (with $G_{ij}(X_0) = \delta_{ij}$) and then write

$$G_{ij}(X) = \delta_{ij} - \frac{1}{3} R_{ikjl}(X_0) \delta X^k \delta X^l + O(\delta X^3) . \tag{6.15}$$

To compute this explicitly in ρ we would therefore need an explicit non-trivial (non-constant) background solution and the Riemann tensor in the corresponding normal coordinates, which are different from ours. For massless fluctuations, the "effective string wavefunction" argument is not applicable. There is clearly much left to do here, but we leave this for future work.

7. Conclusion

In this paper, we initiated a detailed study of the ρ-deformed D1-D5-D5$'$ system. We computed the near-horizon limit of the deformed brane configuration perturbatively in the deformation parameter ρ. Within the three-dimensional effective gauged supergravity, we verified the existence of a flat direction (valley) in the potential (figure 4) that corresponds to the deformation, and computed the deformed mass spectrum along the valley.

There appeared many new questions along the way. The most glaring omission seems to be the construction of the complete ρ-deformed brane solution, of which we have only constructed the near-horizon limit. Given the technology that already exists for related cases (e.g. [11 – 13]), one could hope that this would be accomplished relatively soon. From that vantage point one could easily answer geometrical questions left unanswered by our solution in section 3, such as the details of the variable transformation generalizing (2.9) to $\rho > 0$.

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With some more work along the lines of what we presented here, one should be able to nail down the precise couplings between the ρ deformation and the supergravity fluctuations in ten dimensions. This would pave the way for computing the deformed correlator in (6.2) in the CFT, leading to a highly nontrivial comparison with the deformed $\Delta(\rho)$ in (5.16). If successful — and there are some pitfalls, when using the boundary CFT at the orbifold point — this would constitute one of the most detailed tests that have ever been performed of the AdS/CFT correspondence. It is made more feasible than most deformed correspondences by the great simplification of the deformation being marginal, ensuring that the boundary theory is conformal at all scales. (Indeed, for RG flows, general Δ functions can only at best make approximate, scheme-dependent (cf. [7]) sense away from conformal fixed points). Using the techniques developed in [38 – 41] for generic RG flows one might be able to extend the analysis to compute deformed (but conformal) higher-point correlation functions.

The $\mathcal{N} = 3$ theory is interesting in its own right, not the least because of the simpler BPS bound (4.4). The super-Higgs dynamics seems quite rich in this case (cf. eq. (5.18)), and we would expect a closer study of this dynamics could shed light on the $\mathcal{N} = 3$ single D1-D5 system, as outlined in section 2.1.

By analogy with the breaking $\mathcal{N} = 4 \to \mathcal{N} = 3$ in 4 dimensions [42 – 45], one should be able to think of this as adding judiciously chosen flux. A related topic of interest would be a study of deformations of the Chern-Simons theory with flux discussed in [8, Section 3], and [34, 46].

It would also be interesting to understand how our results fit into the bigger picture, if any, of marginal deformations in AdS/CFT along the lines of [47 – 49].

Acknowledgments

We wish to thank D. Berenstein, M. Cederwall, S. Cherkis, S. Fredenhagen, J. Gauntlett, S. Gukov, G. Horowitz, A. Kleinschmidt, J. Maldacena, D. Marolf, J. Polchinski, M. Shigemori and K. Skenderis, for helpful and stimulating discussions and comments. This work was supported in part by the European RTN Program MRTN-CT-2004-503369 and the DFG grant SA 1336/1-1. O.H. is supported by the stichting FOM and the RTN network MRTN-CT-2004-005104. M.B. is supported by The European Superstring Theory Network, MRTN-CT-2004-512194, and would like to thank the KITP in Santa Barbara for hospitality. This research was supported in part by the National Science Foundation under Grant No. PHY99-07949.

A. Kaluza-Klein spectrum on $AdS_3 \times S^3 \times S^3$

In this appendix we give a brief review of the group-theoretical analysis of the Kaluza-Klein spectrum on the $AdS_3 \times S^3 \times S^3$ background, following [50] (see also [51, 3, 52, 53]). Starting from maximal nine-dimensional supergravity, the physical fields can be classified under the $SO(1,2) \times SO(3) \times SO(3)'$ subgroup of the nine-dimensional Lorentz group $SO(1,8)$ with the different factors corresponding to the AdS_3 and the two sets of S^3 coordinates, which we

$\mathcal{R}$ \ Δ	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4	$\frac{9}{2}$	5	$\frac{11}{2}$	6
$[0,0;0,0]$				2		2		2				
$[\frac{1}{2},0;\frac{1}{2},0]$	1		1		2		3		2			
$[1,0;1,0]$		1		2		2		3		2		
$[\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}]$		2		2		6		4		2		
$[1,\frac{1}{2};1,\frac{1}{2}]$			2		3		6		4		2	
$[1,1;1,1]$				2		4		6		4		2

Table 7: The lowest scalars and their masses.

denote collectively by z, x, and y, respectively. Labeling the corresponding representations by K, J, and J', respectively, the fields can be expanded in terms of S^3 sphere functions according to

$$\Phi_{[K;J,J']}(z,x,y) = \sum_{L,L'} \phi_{[K;L,L']}(z)\, X_J^{(L)}(x)\, Y_{J'}^{(L')}(y)\ . \tag{A.1}$$

The sphere functions $X_J^{(L)}(x), Y_{J'}^{(L')}(y)$ are labeled by representations L, L' of the isometry group $SO(4)\times SO(4)$. The coefficients $\phi_{[K;L,L']}(z)$ describe the complete three-dimensional Kaluza-Klein spectrum. The structure of the spectrum is thus encoded in the range of representations L, L' over which the sum (A.1) is taken. This has been determined in [50]: the sum in (A.1) is running precisely over those representations L, which contain the representations J upon breaking of the isometry groups $SO(4)$ down to the Lorentz groups $SO(3)$, and similarly for $SO(4)'$.

For illustration let us consider a scalar field, i.e. a singlet under the Lorentz group. The above algorithm gives rise to a Kaluza-Klein tower

$$(J,J') \ = \ (0,0) \longrightarrow \sum_{j,j'} [j,j';j,j']\ , \tag{A.2}$$

built from $SO(4)$ representations which we label by their spins $[j_L,j'_L;j_R,j'_R]$ according to (3.4). Explicitly, this corresponds to an expansion

$$\Phi(z,x,y) = \sum_{j,j'} \phi_{[j,j';j,j']}(z)\, X^{[j,j]}(x)\, Y^{[j',j']}(y)\ , \tag{A.3}$$

where the sphere functions $X^{[j,j]}(x)$, $Y^{[j',j']}(y)$ are explicitly given as symmetric traceless products of (3.6).

Similarly, a vector say on the first S^3 gives rise to the Kaluza-Klein towers

$$(J,J') = (1,0) \longrightarrow \sum_{j>0,\,j'} [j,j';j,j'] + \sum_{j,j'} [j{+}1,j';j,j'] + \sum_{j,j'} [j,j';j{+}1,j']\ , \tag{A.4}$$

	(0,0; 0,0)	$(\frac{1}{2},0;\frac{1}{2},0)$	$(0,\frac{1}{2};0,\frac{1}{2})$	$(\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2})$	(1,0; 1,0)	(0,1; 0,1)	$(\frac{1}{2},1;\frac{1}{2},1)$	$(1,\frac{1}{2};1,\frac{1}{2})$	(1,1; 1,1)
$h_{\mu\nu}$		+	+	+	+	+	+	+	+
h_{mn}					+			+	+
h^m_m	+	+	+	+	+	+	+	+	+
$h_{\bar{m}\bar{n}}$						+	+		+
$h^{\bar{m}}_{\bar{m}}$	+	+	+	+	+	+	+	+	+
$h_{\mu m}$		+		+	+		+	+	+
$h_{\mu\bar{m}}$			+	+		+	+	+	+
$h_{m\bar{n}}$				+			+	+	+
$h_{\mu 9}$	+	+	+	+	+	+	+	+	+
h_{m9}		+		+	+		+	+	+
$h_{\bar{m}9}$			+	+		+	+	+	+
h_{99}	+	+	+	+	+	+	+	+	+
ϕ, c_0	+	+	+	+	+	+	+	+	+
$c_{\mu m}$		+		+	+		+	+	+
$c_{\mu\bar{m}}$			+	+		+	+	+	+
c_{mn}		+		+	+		+	+	+
$c_{\bar{m}\bar{n}}$			+	+		+	+	+	+
$c_{m\bar{n}}$				+			+	+	+
$c_{\mu 9}$	+	+	+	+	+	+	+	+	+
c_{m9}		+		+	+		+	+	+
$c_{\bar{m}9}$			+	+		+	+	+	+
$c_{\mu mnk}$	+	+	+	+	+	+	+	+	+
$c_{\mu\bar{m}\bar{n}\bar{k}}$	+	+	+	+	+	+	+	+	+
$c_{\mu mn\bar{k}}$				+			+	+	+
$c_{\mu m\bar{n}\bar{k}}$				+			+	+	+
$c_{mnk\bar{l}}$			+	+		+	+	+	+
$c_{mn\bar{k}\bar{l}}$				+			+	+	+
$c_{m\bar{n}\bar{k}\bar{l}}$		+		+	+		+	+	+

Table 8: The lowest states in the KK decomposition of the IIB fields.

and so on. Applying the algorithm to the full spectrum of maximal nine-dimensional supergravity leads to the final result [3]

$$\bigoplus_{\ell\geq 0,\,\ell'\geq 1/2} (\ell,\ell';\ell,\ell')_{\rm S} \;\oplus\; \bigoplus_{\ell\geq 1/2,\,\ell'\geq 0} (\ell,\ell';\ell,\ell')_{\rm S} \\ \oplus \bigoplus_{\ell,\,\ell'\geq 0} \big((\ell,\ell';\ell+\tfrac{1}{2},\ell'+\tfrac{1}{2})_{\rm S} \oplus (\ell+\tfrac{1}{2},\ell'+\tfrac{1}{2};\ell,\ell')_{\rm S}\big)\,, \qquad \text{(A.5)}$$

where the fields have already been assembled into supermultiplets of the supergroup $D^1(2,1;\alpha)_L \times D^1(2,1;\alpha)_R$ as discussed in section 4.

As an illustration we collect in table 8 for all the ten-dimensional bosonic degrees of freedom the lowest $SO(4)\times SO(4)'$ KK states that appear in their KK decomposition (A.1). Here h, (ϕ, c_0), $c_{(2)}$, and $c_{(4)}$ denote the fluctuations of the metric, the scalars, the 2-forms and the 4-form, respectively. Indices $\mu,\nu,\ldots$ label AdS_3, $m,n,\ldots$ and $\bar{m},\bar{n},\ldots$ label the

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two spheres. We have omitted all components which do not give rise to propagating degrees of freedom on AdS_3, in particular half of the self-dual 4-form.

Another interesting piece of information is gathered in table 7. Comparing the field content of table 8 with the supermultiplet structure from (A.5), we have identified the multiplicites of the lowest scalar representations together with their AdS masses (expressed in terms of the boundary conformal dimensions $\Delta = 1 + \sqrt{1 + m^2}$).

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