

# Conformal symmetry and the Standard Model

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## Abstract

We re-examine the question of radiative symmetry breaking in the Standard Model in the presence of right-chiral neutrinos and a minimally enlarged scalar sector. We demonstrate that, with these extra ingredients, the hypothesis of classically unbroken conformal symmetry, besides naturally introducing and stabilizing a hierarchy, is compatible with all available data; in particular, there exists a set of parameters for which the model may remain viable even up to the Planck scale. The decay modes of the extra scalar field provide a unique signature of this model which can be tested at LHC.

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## 1. Introduction

A striking property of the Standard Model (SM) of elementary particles (see e.g. [1,2] for an introduction and bibliography) is its ‘near conformal’ invariance. Conformal invariance is only broken by the explicit mass term for the scalar fields, which induces spontaneous breaking of the  $SU(2)_w \times U(1)_Y$  symmetry and gives mass to the  $W$  and  $Z$  bosons [3] as well as to the fermions. This tree level mass term is also at the root of the so-called hierarchy problem, namely the need to cancel quadratically divergent terms  $\propto \Lambda^2$  to exceedingly high precision (where  $\Lambda$  is the UV cutoff, or the scale at which the SM is replaced by another theory). The desire to avoid such unnatural fine tuning, or at least to stabilize such a seemingly unnatural hierarchy, constitutes the main motivation for various proposals to extend the SM (see e.g. [4] for a nice summary), and has in particular led to the development of supersymmetric extensions of the SM.

Nevertheless, it has been known for a long time that radiative corrections in an initially conformally invariant scalar field theory may also induce spontaneous breaking of symmetry, such

that the introduction of explicit mass terms can in principle be avoided [5]. However, in spite of its aesthetical appeal, the concrete implementation of the Coleman–Weinberg (CW) mechanism in the context of the SM so far has not met with much success for a variety of reasons (but see [6,7] for more recent work). In particular, we now know that the Higgs mass must be larger than 115 GeV, much in excess of the original prediction ( $\sim 10$  GeV) of [5], thereby forcing the scalar self-couplings to be so large that a nearby Landau pole seems unavoidable and the one-loop approximation may no longer be valid. As a further constraint, the unexpectedly large Yukawa couplings of the top quark require the scalar self-coupling to be sufficiently large in order to prevent the de-stabilization of the effective potential.

In this Letter we re-examine the question of radiative symmetry breaking for the SM in a slightly more general context than done before. While the main ingredients that underlie the present work have been available for a long time, the following three key features are new.<sup>1</sup>

- We proceed from the hypothesis that classically unbroken *conformal symmetry* is the basic reason for the existence

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<sup>1</sup> A similar model but with explicit scalar mass term, and thus *without* radiative symmetry breaking, was recently proposed and studied in [8].

of small mass scales in nature, whose emergence should thus be viewed as a manifestation of a *conformal anomaly* (other sources of violations of conformal invariance in the SM, like gluon and quark condensates, are non-perturbative and concern much lower scales than we are interested in here).

- *Dimensional regularization* is crucial in that it provides a self-consistent and economical way to define the renormalized effective action to any order in perturbation theory, ensuring that conformal invariance is broken ‘in the least possible way’ in the quantum theory. However, we stress that its preferred status with regard to a Planck scale theory of quantum gravity remains an *assumption*, see remarks at the end.
- In contrast to previous work on the effective potential, we incorporate the right-chiral neutrinos and the associated Yukawa (Dirac and Majorana-like) couplings from the outset; this leads us to introduce a concomitant scalar field, implementing the standard see-saw mechanism [9]. The decay of this scalar fields provides a distinctive and unique signature of the present model that can be tested at LHC.

Our aim is to compute the effective potential for this combined classically conformally invariant theory, and to derive all known mass scales from this effective potential. Due to the mixing of the scalar fields and the presence of logarithmic terms in the effective potential it now becomes possible to reproduce all the observed features without the need to introduce unduly large mass hierarchies ‘by hand’.

Our proposal is ‘minimalistic’ in the sense that we do not invoke grand unification (GUTs) nor any other ‘beyond the SM’ scenario, but rely only on those ingredients that are known to be there. Indeed, the very idea of grand unification, or any other scheme involving the introduction of a large intermediate scale between the weak scale and the Planck scale, is evidently at odds with our basic hypothesis of nearly unbroken conformal invariance—which is presumably the reason why the CW potential has not played any role in GUT scenarios, or softly broken supersymmetric theories (in fact, as shown in [10] the effective potential vanishes identically to all orders in an exactly supersymmetric theory). On the other hand, as we will show here, for a very reasonable range of parameters these minimal ingredients suffice to reproduce, via the CW mechanism, all observed features of the SM, including small neutrino masses, in such a way that Landau poles and instabilities can be pushed above the Planck scale. Contrary to the usual reasoning, the smallness of neutrino masses does not necessarily require a very large ‘new physics’ scale, but can be explained by the respective neutrino Yukawa couplings if these are taken to be of the same order as the electron Yukawa coupling ( $10^{-5}$ ). As our proposal allows for some range of Higgs masses, the (perhaps sobering) conclusion is that the model proposed here may remain perfectly viable in all respects well beyond the range of energies accessible to LHC. In particular, while supersymmetry is expected to be part of any scheme unifying the basic forces with gravity, there is no need for *low energy* supersymmetry in the present scheme.

## 2. Lagrangian and effective potential

Omitting kinetic terms the Lagrangian reads (see also [11], where (1) was considered in the different context of local Weyl invariance)

$$\begin{aligned} \mathcal{L}' = & (\bar{L}^i \Phi Y_{ij}^E E^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^D D^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^U U^j \\ & + \bar{L}^i \epsilon \Phi^* Y_{ij}^V v_R^j + \varphi v_R^{iT} C Y_{ij}^M v_R^j + \text{h.c.}) \\ & - \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \varphi^2 (\Phi^\dagger \Phi) - \frac{\lambda_3}{4} \varphi^4 \end{aligned} \quad (1)$$

with standard notation:  $Q^i$  and  $L^i$  are the left-chiral quark and lepton doublets of  $SU(2)_w$ ,  $U^i$  and  $D^i$  are the right-chiral up- and down-like quarks,  $E^i$  the right-chiral ‘electron-like’ leptons, and  $v_R^i$  the right-chiral neutrinos. We have suppressed all  $SU(2)_w$  and color  $SU(3)_c$  indices, but explicitly indicate family indices  $i, j = 1, 2, 3$ . The real diagonal matrices  $Y_{ij}^U, Y_{ij}^E, Y_{ij}^M$  and the complex matrices  $Y_{ij}^D, Y_{ij}^V$  contain all the relevant Yukawa couplings and parameterize the most general ‘family mixing’. Finally, besides the standard  $SU(2)_w$  Higgs doublet  $\Phi$ , the spectrum contains an additional real scalar  $\varphi$ .<sup>2</sup> Because of the assumed conformal invariance, no scalar self-couplings other than those appearing in (1) are allowed. The above Lagrangian (together with the kinetic terms which we have not written) is the most general compatible with (classical) conformal invariance, and in particular contains no explicit mass terms (it is also automatically renormalizable).

We next wish to compute the one loop effective (CW) potential. Using the standard formulas [12], writing  $H^2 \equiv \Phi^\dagger \Phi$  for the usual Higgs doublet, and defining

$$\begin{aligned} F_{\pm}(H, \varphi) & := \frac{3\lambda_1 + \lambda_2}{4} H^2 + \frac{3\lambda_3 + \lambda_2}{4} \varphi^2 \\ & \pm \sqrt{\left[ \frac{3\lambda_1 - \lambda_2}{4} H^2 - \frac{3\lambda_3 - \lambda_2}{4} \varphi^2 \right]^2 + \lambda_2^2 \varphi^2 H^2} \end{aligned} \quad (2)$$

the one-loop contributions from the scalar fields to the effective potential are (in the  $\overline{\text{MS}}$ -scheme)

$$\begin{aligned} V_{\text{eff}}^{(1)}(H, \varphi) & = \frac{N-1}{256\pi^2} (\lambda_1 H^2 + \lambda_2 \varphi^2)^2 \ln \left( \frac{\lambda_1 H^2 + \lambda_2 \varphi^2}{v^2} \right) \\ & + \frac{M-1}{256\pi^2} (\lambda_2 H^2 + \lambda_3 \varphi^2)^2 \ln \left( \frac{\lambda_2 H^2 + \lambda_3 \varphi^2}{v^2} \right) \\ & + \frac{1}{64\pi^2} F_+^2 \ln \left( \frac{F_+}{v^2} \right) + \frac{1}{64\pi^2} F_-^2 \ln \left( \frac{F_-}{v^2} \right), \end{aligned} \quad (3)$$

where  $v$  is some scale (see below). The formula is valid for  $(N+M)$  scalar fields, with  $O(N) \times O(M)$  invariant quartic

<sup>2</sup> In principle, the field  $\varphi$  could be taken complex or even to transform in a non-trivial representation of a family symmetry (in which case  $M > 1$  in formula (3)). The phase of  $\varphi$  would then be a Goldstone or pseudo-Goldstone boson (sometimes called ‘Majoron’), which couples to observable matter only via the right-chiral neutrinos, and might thus be useful for other purposes.

interactions; in the case at hand, we thus take  $N = 4$  (complex doublet) and  $M = 1$  (real scalar).

With the assumption of classical conformal invariance, it is crucial to use a regularization for the computation of quantum corrections that violates this invariance in the least possible way. Unlike other schemes<sup>3</sup> dimensional regularization satisfies this requirement (as it does for ordinary gauge invariance). More explicitly, all divergent integrals are regulated by the replacement

$$\int \frac{d^4k}{(2\pi)^4} \rightarrow \frac{1}{(2\pi v^2)^\epsilon} \int \frac{d^{4+2\epsilon}k}{(2\pi)^{4+2\epsilon}} \quad (4)$$

where  $v$  is some mass scale (which breaks conformal invariance explicitly). Because  $v$  comes with an ‘evanescent’ exponent, the scale parameter  $v$  always appears under a logarithm. Consequently, the singular part, and hence the required infinite counterterms are of the same form as the tree level Lagrangian (1), and thus at any order in perturbation theory, the effective action contains neither mass terms nor a cosmological constant (which would have to depend *polynomially* on  $v$ ). The one-loop result (3) is then obtained by analytic continuation of the formula

$$\int_0^\infty d\xi \xi^{\nu-1} \ln(1 + b/\xi) = \frac{\pi b^\nu}{\nu \sin(\pi\nu)} \quad (5)$$

to  $\nu = 2 + \epsilon$  (the integral converges for  $0 < \text{Re } \nu < 1$ ).

The computation of the fermionic contribution is more involved due to family mixing, and cannot be done in closed form without resorting to some approximations. First of all, inspection of (1) shows that in the one-loop approximation we can separate the calculation into a part involving only the quark fields, and one involving only the leptons. The contribution of the quarks is clearly dominated by the top quark (i.e. the largest Yukawa coupling  $g_t \equiv Y_{33}^U \approx 1.0$ ) and gives the standard result

$$V_{\text{eff}}^{(2)}(H) = -\frac{6}{32\pi^2} g_t^4 (H^2)^2 \ln(H^2/v^2). \quad (6)$$

The leptonic contribution, on the other hand, cannot be reduced so easily as it involves a matrix linking ( $L^i, E^i, \nu_R^i$ ) and their charge conjugates. To simplify the calculation, we neglect all terms involving  $Y_{ij}^E$  (whose largest entry comes from the  $\tau$ -lepton with  $g_\tau$  of order 0.01). The remaining matrix only couples the doublets  $L^i$  and the right-chiral neutrinos  $\nu_R^i$ ; before renormalization the relevant expression can be reduced to the integral

$$\begin{aligned} & -\frac{1}{16\pi^2 (4\pi v^2)^\epsilon \Gamma(2+\epsilon)} \\ & \times \int d\xi \xi^{1+\epsilon} \ln \det[1 + (Y_M Y_M \cdot \varphi^2 \\ & + Y_M (Y_\nu \bar{Y}_\nu + \bar{Y}_\nu Y_\nu) Y_M^{-1} \cdot H^2)/\xi \\ & + Y_M \bar{Y}_\nu Y_\nu Y_M^{-1} Y_\nu \bar{Y}_\nu \cdot (H^2)^2/\xi^2], \end{aligned} \quad (7)$$

<sup>3</sup> E.g. with a momentum cutoff  $\Lambda$ , the conformal invariance is ‘more and more badly broken’ as  $\Lambda \rightarrow \infty$ .

where the remaining determinant under the integral is to be taken w.r.t. a Hermitean 3-by-3 matrix in the family indices. Further evaluation of this expression would thus require the factorization of a sixth order polynomial in  $\xi$  which again is in general not possible in closed form, especially if there is ‘maximal mixing’ in the Yukawa matrices (meaning that  $Y_{ij}^v$  is far away from a diagonal matrix). For this reason, we resort to yet another approximation by assuming  $Y_\nu(H) \ll Y_M(\varphi)$ , in agreement with the observed smallness of neutrino masses. Then the above expression can be calculated exactly and the full effective potential becomes, in this approximation,

$$\begin{aligned} V_{\text{eff}}(H, \varphi) & = \frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2 \varphi^2}{2} + \frac{\lambda_3 \varphi^4}{4} \\ & + \frac{3}{256\pi^2} (\lambda_1 H^2 + \lambda_2 \varphi^2)^2 \ln \left[ \frac{\lambda_1 H^2 + \lambda_2 \varphi^2}{v^2} \right] \\ & + \frac{1}{64\pi^2} F_+^2 \ln \left[ \frac{F_+}{v^2} \right] + \frac{1}{64\pi^2} F_-^2 \ln \left[ \frac{F_-}{v^2} \right] \\ & - \frac{6}{32\pi^2} g_t^4 (H^2)^2 \ln \left[ \frac{H^2}{v^2} \right] - \frac{1}{32\pi^2} g_M^4 \varphi^4 \ln \left[ \frac{\varphi^2}{v^2} \right], \end{aligned} \quad (8)$$

where  $g_M^4 := \text{Tr } Y_M^4$ . We do not include here the terms from  $SU(2)_w \times U(1)_Y$  gauge fields because the respective gauge couplings are small, nor from  $SU(3)_c$  gauge fields because it is a two-loop effect (although numerically it can be important and is included in the RG analysis described below).

### 3. Minimization of effective potential

As we cannot minimize this potential in closed form, we now search for minima numerically. Since the problem is highly non-linear we have to use a trial-and-error method in order to arrive at a set of ‘reasonable’ values satisfying the following requirements: the standard Higgs mass  $m_H$  must be bigger than 115 GeV, and the effective coupling constants  $\lambda_i^{\text{eff}}$  (see (15) below) should be such that there are no Landau poles or instabilities up to some large scale. The numerical search shows that the ‘window’ left open by these requirements is not very large, but in particular allows for the following set of values:

$$\begin{aligned} \lambda_1 = 3.4, \quad \lambda_2 = 2.6, \quad \lambda_3 = 3.3, \\ g_t = 1, \quad g_M^2 = 0.4. \end{aligned} \quad (9)$$

For these values, the minimum lies at  $\langle H \rangle = 4.15 \times 10^{-6} v$ ,  $\langle \varphi \rangle = 25.06 \times 10^{-6} v$ . We emphasize that these numbers are merely chosen to illustrate the possible viability of the proposed scenario up to very large scales, and by no means constitute a definitive prediction of our model (*idem* for the mass values (13) below).

Next, we must choose one mass scale which sets the scale for all other quantities. This we do by imposing  $\langle H \rangle = 174$  GeV. Hence,

$$\begin{aligned} \langle H \rangle = 174 \text{ GeV}, \quad \langle \varphi \rangle = 1050 \text{ GeV}, \\ v = 2.41 \times 10^5 \langle H \rangle. \end{aligned} \quad (10)$$

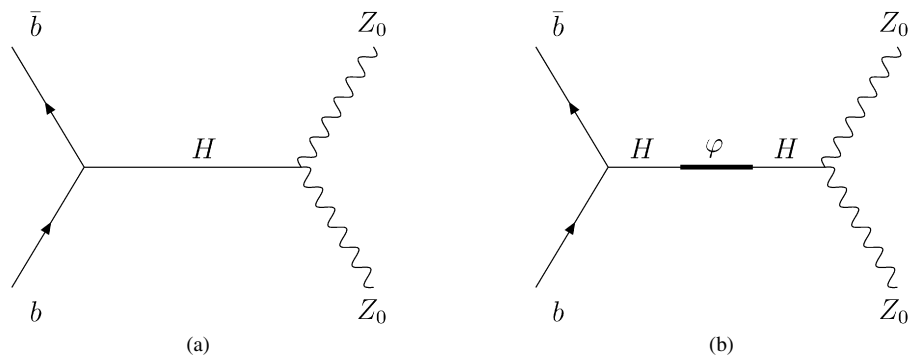


Fig. 1. Production of  $Z_0$  pairs in (a) Higgs and (b) new scalar channels (resonant production is possible if  $m_H > 2m_Z$  or  $m_\phi > 2m_Z$ ).

Assuming  $|Y_\nu| < 10^{-5}$ , so the neutrino Yukawa couplings are of the same order as the electron Yukawa coupling, we can arrive at very small neutrino masses:

$$m_\nu \approx \frac{(Y_\nu(H))^2}{Y_M(\varphi)} < 1 \text{ eV}. \quad (11)$$

After symmetry breaking three degrees of freedom of  $\Phi$  are converted into longitudinal components of  $W^\pm$  and  $Z^0$ , so we are left with two real scalar fields  $H$  and  $\varphi$ , and the potential  $V(\Phi, \varphi)$  should be understood from now as  $V(H, \varphi)$  (with non-canonical normalization). Calculating second derivatives at the minimum and defining

$$H' = H \cos \beta + \varphi \sin \beta, \quad \varphi' = -H \sin \beta + \varphi \cos \beta \quad (12)$$

we obtain the mass values:

$$m_{H'} = 217 \text{ GeV}, \quad m_{\varphi'} = 439 \text{ GeV} \quad (13)$$

with mixing angle

$$\sin \beta = 0.119. \quad (14)$$

Note that only the components along  $H$  of the mass eigenstates couple to the usual SM particles. The *effective coupling constants* are calculated as respective fourth-order derivatives of the effective potential, that is,  $\lambda_1^{\text{eff}} = (1/6)\partial^4 V_{\text{eff}}/\partial H^4$ , etc.; at the minimum we obtain the values

$$\lambda_1^{\text{eff}} = 1.463, \quad \lambda_2^{\text{eff}} = 0.348, \quad \lambda_3^{\text{eff}} = 0.626. \quad (15)$$

#### 4. Renormalization group analysis

The next task is to check for the presence of Landau poles or instabilities (negative coupling constants) as a function of the scale, and to ascertain the validity of the one-loop approximation (see e.g. [13] for a discussion of the subtleties involved). Ideally, this would require calculation of the full resummed and renormalization group invariant effective action, where the Landau pole should manifest itself as a singularity of the effective momentum dependent terms. However, it appears difficult to proceed analytically in this way (see [7]) so we content ourselves here with the conventional procedure, according to which one should evolve the coupling constants with the renormalization group equations. Although the initial values are potentially subject to modification at higher order, we take (15) as the most natural choice. Defining

$$y_1 = \frac{\lambda_1^{\text{eff}}}{4\pi^2}, \quad y_2 = \frac{\lambda_2^{\text{eff}}}{4\pi^2}, \quad y_3 = \frac{\lambda_3^{\text{eff}}}{4\pi^2},$$

$$x = \frac{g_t^2}{4\pi^2}, \quad u = \frac{g_M^2}{4\pi^2}$$

we have the renormalization group equations

$$\mu \frac{dy_1}{d\mu} = \frac{3}{2}y_1^2 + \frac{1}{8}y_2^2 - 6x^2,$$

$$\mu \frac{dy_2}{d\mu} = \frac{3}{8}y_2 \left( 2y_1 + y_3 + \frac{4}{3}y_2 \right),$$

$$\mu \frac{dy_3}{d\mu} = \frac{9}{8}y_3^2 + \frac{1}{2}y_2^2 - u^2, \quad \mu \frac{du}{d\mu} = \frac{3}{4}u^2,$$

$$\mu \frac{dx}{d\mu} = \frac{9}{4}x^2 - 4xz, \quad \mu \frac{dz}{d\mu} = -\frac{7}{2}z^2, \quad (16)$$

where we added the strong coupling contribution  $z = \alpha_s/\pi$ . As dictated by (8) we use one-particle-irreducible (and not the full)  $\beta$ -functions, since in the effective action the renormalized external fields are used. With the initial values at 174 GeV given by  $g_t = 1$ ,  $\alpha_s = 0.1$  and (15) one obtains the evolution curves displayed in Fig. 2, from which it is evident that there are neither Landau poles nor instabilities below the Planck scale (the instability occurs at  $10^{21}$  GeV). Because of its non-linearity, the system of coupled evolution equations (16) is rather delicate and highly sensitive to small changes in the initial values. Nevertheless, the numerical scan over the range of parameters satisfying the requirements shows that the standard Higgs in all cases comes out to be rather light [ $\sim \mathcal{O}(200 \text{ GeV})$ ], while  $m_{\varphi'}$  can vary over a larger range. However, the determination of the allowed range of values in the  $(m_{H'}, m_{\varphi'})$  parameter plane, which are compatible with all our requirements and which might lead to more definite predictions, will be left to future work.

#### 5. Discussion

Phenomenologically, and for low energies, the proposed scenario is largely indistinguishable from the SM with massive neutrinos, but for large energies differs significantly from SM extensions like the MSSM. Apart from the obvious lack of superpartners, the Higgs couplings are very different; for instance, the standard Higgs can now couple to right-chiral neutrinos via mixing with the new scalar. In fact, the Higgs mixing provides

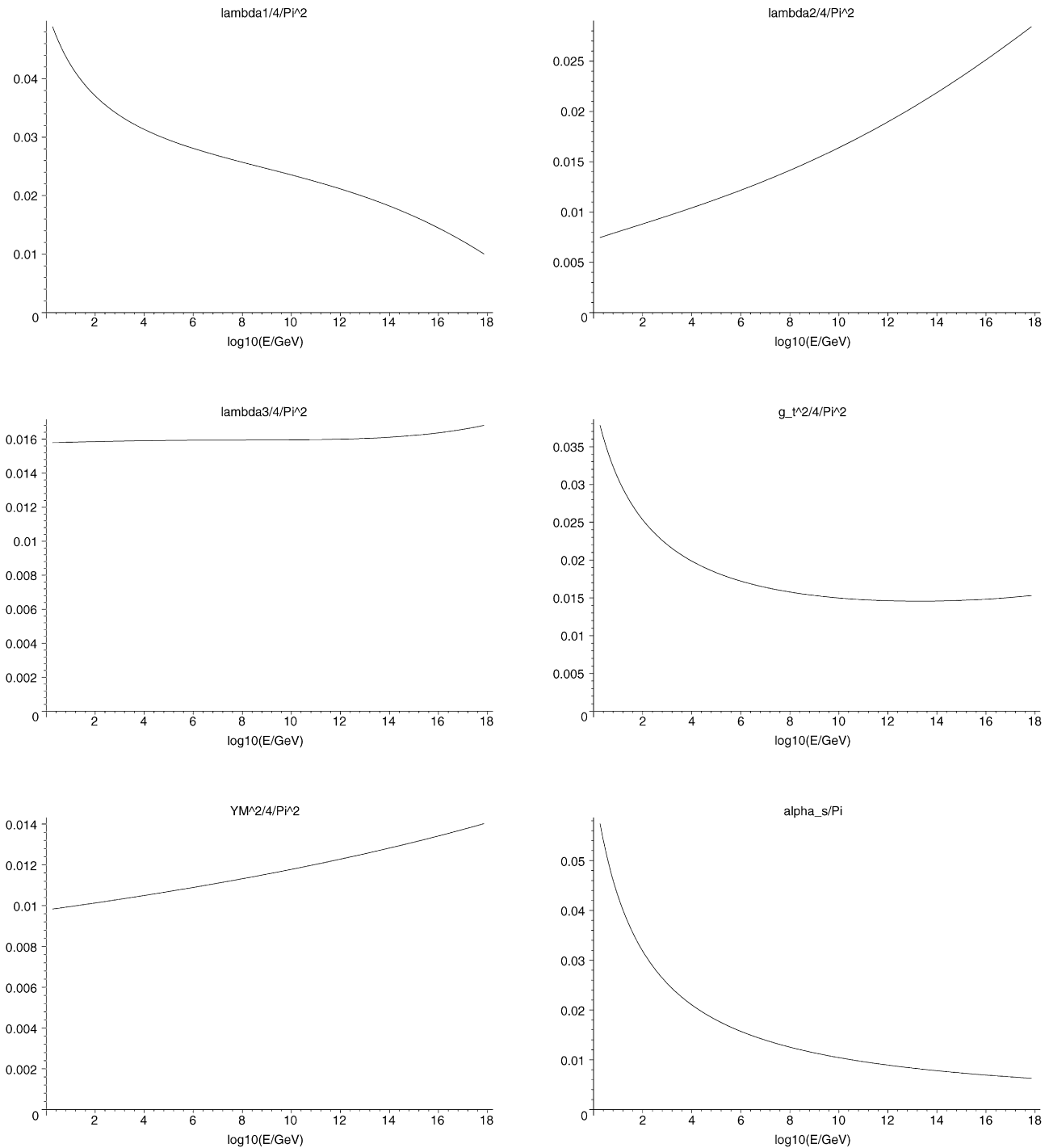


Fig. 2. Evolution of coupling constants (from left to right, top to bottom):  $y_1(\mu)$ ,  $y_2(\mu)$ ,  $y_3(\mu)$ ,  $x(\mu)$ ,  $u(\mu)$  and  $z(\mu)$ .

a rather striking (and unique) signature of the present model, which would set it apart from other ‘beyond the SM’ scenarios, and should be testable at LHC.<sup>4</sup> Namely, besides decaying via the usual SM decay modes, the standard Higgs field can now oscillate into the new scalar  $\varphi$ , which after reconversion into  $H$  leads to a second resonance at  $m_{\varphi'}$  with the same branching ratios—thus casting a ‘shadow’ of the standard Higgs parti-

cle, whose size depends on the mixing angle  $\beta$ , cf. (14). This process is illustrated in Fig. 1. Unlike for the standard Higgs, this second resonance can be narrow even for larger values of  $m_{\varphi'}$  if the mixing angle  $\beta$  remains sufficiently small. However, a more detailed discussion of these more phenomenological aspects is outside the scope of this Letter and will be given elsewhere.

It is worthwhile to note that the usual hierarchy problem is addressed here in a way which is very different from the solution proposed in the context of the MSSM (see e.g. [14]). The

<sup>4</sup> We are indebted to W. Buchmüller for a discussion on this point.

latter relies mainly on the fact that supersymmetry forces the Higgs self-coupling to be a function of the gauge couplings, which themselves are kept under control by gauge invariance, so the Landau pole effectively gets shifted beyond the Planck scale. The hierarchy itself is explained in the MSSM by certain soft supersymmetry breaking terms extremely finely tuned at the GUT scale that run slowly in such a way that  $m_H^2$  eventually becomes negative around 1 TeV. While the absence of quadratic and quartic divergencies in supersymmetric theories is due to boson fermion cancellation, there are, by contrast, neither mass terms nor a cosmological constant in our proposal by virtue of the assumed conformal invariance. The theory contains only dimensionless parameters to start with, and the dimensional regularization ensures that the conformal symmetry is broken in the radiative corrections not by powers of the cutoff, but only by the unavoidable choice of scale  $v$  under the logarithms. In this sense the hierarchy of scales emerges more ‘naturally’ than it would with explicit mass terms. Although the Landau pole or instability problems are in principle there, they can be avoided without excessive fine-tuning as we showed.

The key question is therefore how a classically conformally invariant action at low energies can emerge from gravity, which is *not* conformally invariant due to the presence of a dimensional parameter, the Planck mass  $M_{\text{Pl}}$  (with gravity and explicit scalar mass terms, classical dilatational invariance—achieved by means of a dilaton—is likewise broken by quantum effects, see [15]). More precisely, can the privileged status of dimensional regularization be explained by the fact that a finite theory of quantum gravity must act as a universal regulator for matter interactions? If so, quantum gravity effects may *dynamically* suppress explicit breaking of conformal invariance by power-like counterterms in this way, allowing only for logarithmic terms or non-local terms with *inverse* powers of  $M_{\text{Pl}}$  in the effective action.<sup>5</sup> A possible analog for such a phenomenon is noncritical string (Liouville) theory [16], a theory of matter-coupled quantum gravity in two space–time dimensions, which does not possess classical conformal invariance, but where conformal invariance is restored at the quantum level via the quantum mechanical decoupling of an infinite tower of null states (as explained e.g. in [17]).

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<sup>5</sup> The suppression of powerlike terms is also suggested by the fact that, whatever the correct theory of quantum gravity will turn out to be, it must be such that gravity smoothly decouples in the limit  $M_{\text{Pl}} \rightarrow \infty$  so as to leave a flat space quantum field theory.