Conformal Symmetry and the Standard Model

Krzysztof A. Meissner[1,2] and Hermann Nicolai[1]
[1] *Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut), Mühlenberg 1, D-14476 Potsdam, Germany*
[2] *Institute of Theoretical Physics, Warsaw University, Hoża 69, 00-681 Warsaw, Poland*

We re-examine the question of radiative symmetry breaking in the standard model in the presence of right-chiral neutrinos and a minimally enlarged scalar sector. We demonstrate that, with these extra ingredients, the hypothesis of classically unbroken conformal symmetry, besides naturally introducing and stabilizing a hierarchy, is compatible with all available data; in particular, there exists a set of parameters for which the model may remain viable even up to the Planck scale.

arXiv:hep-th/0612165v2 16 Dec 2006

A striking property of the standard model (SM) of elementary particles (see e.g. [1] for an introduction and bibliography) is its 'near conformal' invariance. Conformal invariance is a symmetry of all kinetic terms, Yukawa couplings, as well as the gauge field interactions and the quartic self-interaction of the scalar fields, and is only broken by the explicit mass term for the scalar fields, which induces spontaneous breaking of the $SU(2)_w \times U(1)_Y$ symmetry and gives mass to the W and Z bosons [2] as well as to the fermions. This tree level mass term is also at the root of the so-called hierarchy problem, namely the need to cancel quadratically divergent terms $\propto \Lambda^2$ to exceedingly high precision (where Λ is the UV cutoff, or the scale at which the SM is replaced by another theory). The desire to avoid such unnatural fine tuning, or at least to stabilize such a seemingly unnatural hierarchy, constitutes the main motivation for various proposals to extend the SM (see e.g. [3] for a nice summary), and has in particular led to the development of supersymmetric extensions of the SM.

Nevertheless, it has been known for a long time that radiative corrections in an initially conformally invariant scalar field theory may also induce spontaneous breaking of symmetry, such that the introduction of explicit mass terms can in principle be avoided [4]. However, in spite of its aesthetical appeal, the concrete implementation of the Coleman Weinberg (CW) mechanism in the context of the SM so far has not met with much success for a variety of reasons (but see [5, 6] for more recent work). In particular, we now know that the Higgs mass must be larger than 115 GeV, much in excess of the original prediction ($\sim$ 10 GeV) of [4], thereby forcing the scalar self-couplings to be so large that a nearby Landau pole seems unavoidable and the one-loop approximation may no longer be valid. As a further constraint, the unexpectedly large Yukawa couplings of the top quark require the scalar self-coupling to be sufficiently large in order to prevent the de-stabilization of the effective potential.

In this letter we re-examine the question of radiative symmetry breaking for the SM in a slightly more general context than done before. While the main ingredients that underlie the present work have been available for a long time, there are the following two new key features.

- We proceed from the hypothesis that classically unbroken *conformal symmetry* is the basic reason for the existence of small mass scales in nature, whose emergence should thus be viewed as a manifestation of a *conformal anomaly*.

- In contrast to previous work on the effective potential, we incorporate the right-chiral neutrinos and the associated Yukawa (Dirac and Majorana-like) couplings from the outset; this leads us to introduce a concomitant (new in this context) Higgs field, implementing the standard see-saw mechanism [7], and leading to a new type of Higgs mixing.

Our aim is to compute the effective potential for this combined theory, and to derive all known mass scales from this effective potential without introducing any scalar mass terms in the tree level Lagrangian. Due to the mixing of the Higgs fields and the presence of logarithmic terms in the effective potential it now becomes possible to reproduce all the observed features without the need to introduce unduly large mass hierarchies 'by hand'.

Our proposal is 'minimalistic' in the sense that we do not invoke grand unification (GUTs) nor any other 'beyond the SM' scenario, but rely only on those ingredients that are known to be there. Indeed, the very idea of grand unification, or any other scheme involving the introduction of a large intermediate scale between the weak scale and the Planck scale, is evidently at odds with our basic hypothesis of nearly unbroken conformal invariance (which is presumably the reason why the CW potential has not played any role in GUT scenarios, or softly broken supersymmetric theories [8]). On the other hand, as we will show here, for a very reasonable range of parameters these minimal ingredients suffice to reproduce, via the CW mechanism, all observed features of the SM, including small neutrino masses, in such a way that the Landau pole can be pushed above the Planck scale. Contrary to the usual reasoning, the smallness of neutrino masses thus does not necessarily require a very large 'new physics' scale, but can be explained by the respective neutrino Yukawa couplings if these are taken to be of the same order as the electron Yukawa couplings, which is known to be of order 10^{-5}. As our proposal allows for some range of Higgs masses, the (perhaps sobering) conclusion is that the model proposed here may remain perfectly viable in all respects well beyond the range of energies accessible to LHC. In particular, while supersymmetry is expected to be part of any scheme unifying the basic forces with gravity, there is no need for *low energy* supersymmetry in the present scheme.

Omitting kinetic terms, the relevant parts of the Lagrangian are

$$\begin{aligned}\mathcal{L}' \;=\; &\Big(\bar{L}^i \Phi Y^E_{ij} E^j + \bar{Q}^i \epsilon \Phi^* Y^D_{ij} D^j + \bar{Q}^i \epsilon \Phi^* Y^U_{ij} U^j + \\ &+ \bar{L}^i \epsilon \Phi^* Y^\nu_{ij} \nu^j_R + \varphi \nu^i_R \mathcal{C} Y^M_{ij} \nu^j_R + \text{h.c.}\Big) - \\ &- \frac{\lambda_1}{4}(\Phi^\dagger\Phi)^2 - \frac{\lambda_2}{2}\varphi^2(\Phi^\dagger\Phi) - \frac{\lambda_3}{4}\varphi^4 \qquad (1)\end{aligned}$$

with standard notation: Q^i and L^i are the left-chiral quark and lepton doublets of $SU(2)_w$, U^i and D^i are the right-chiral up- and down-like quarks, E^i the right-chiral 'electron-like' leptons, and ν^i_R the right-chiral neutrinos. We have suppressed all $SU(2)_w$ and color $SU(3)_c$ indices, but explicitly indicate family indices $i, j = 1, 2, 3$. The (complex) matrices Y^U_{ij} and Y^D_{ij} for the quarks, and Y^E_{ij}, Y^ν_{ij} and Y^M_{ij} for the leptons, contain all the relevant Yukawa couplings and parameterize the most general 'family mixing'. Finally, besides the standard $SU(2)_w$ Higgs doublet Φ, the scalar sector contains an additional real Higgs field φ [9]; because of the assumed conformal invariance, no scalar self-couplings other than those appearing in (1) are allowed. With these assumptions, the above Lagrangian (together with the kinetic and Yang-Mills terms which we have not written) is the most general compatible with (classical) conformal invariance, and in particular contains no explicit mass terms (it is also automatically renormalizable).

We next wish to compute the one loop effective (CW) potential that follows from the above Lagrangian. This is done by means of the general formula [10]

$$\Gamma[\phi_a] = S[\phi_a] \pm \frac{1}{2}\ln\det\left(\frac{\delta^2 S[\phi_a]}{\delta\phi_a(x)\delta\phi_b(y)}\right) \qquad (2)$$

where $\{\phi_a\}$ is a collection of bosonic or fermionic fields, with plus or minus signs for bosons and fermions, respectively, and where 'det' stands for the full (functional) determinant. Writing $H^2 \equiv \Phi^\dagger\Phi$ for the usual Higgs doublet, and defining

$$\begin{aligned}F_\pm(H,\varphi) \;:=\; &\frac{3\lambda_1+\lambda_2}{4}H^2 + \frac{3\lambda_3+\lambda_2}{4}\varphi^2 \pm \\ &\pm\sqrt{\left[\frac{3\lambda_1-\lambda_2}{4}H^2 - \frac{3\lambda_3-\lambda_2}{4}\varphi^2\right]^2 + \lambda_2^2\varphi^2 H^2} \qquad (3)\end{aligned}$$

the contributions from the scalar fields (without fermionic contributions) to the one loop effective potential are

$$\begin{aligned}V^{(1)}_{\text{eff}}(H,\varphi) \;=\; &\frac{N-1}{256\pi^2}\big(\lambda_1 H^2 + \lambda_2\varphi^2\big)^2 \ln(\lambda_1 H^2 + \lambda_2\varphi^2) \\ &+ \frac{M-1}{256\pi^2}\big(\lambda_2 H^2 + \lambda_3\varphi^2\big)^2 \ln(\lambda_2 H^2 + \lambda_3\varphi^2) \\ &+ \frac{1}{64\pi^2}F_+^2 \ln F_+ + \frac{1}{64\pi^2}F_-^2 \ln F_- \\ &\quad + \alpha(H^2)^2 + \beta\varphi^2 H^2 + \gamma\varphi^4 \qquad (4)\end{aligned}$$

In this form, the formula is actually valid for $(N+M)$ scalar fields, with $O(N)\times O(M)$ invariant quartic interactions; in the case at hand, we thus take $N=4$ (complex doublet) and $M=1$ (real scalar). The last three terms correspond to finite counterterms whose coefficients α, β, γ can be freely adjusted. It is important here that despite the square roots in (3), the required infinite counterterms are of the same form as the tree level Lagrangian. Furthermore, in keeping with our basic hypothesis, all mass terms are set to zero by imposing conformal invariance.

The computation of the fermionic contribution is more involved due to family mixing, and cannot be done in closed form without resorting to some approximations. First of all, inspection of (1) shows that in the one-loop approximation we can separate the calculation into a part involving only the quark fields, and one involving only the leptons. The contribution of the quarks is clearly dominated by the top quark (i.e. the largest Yukawa coupling $g_t \equiv Y^U_{33} \approx 1.0$) and gives the standard result

$$V^{(2)}_{\text{eff}}(H) = -\frac{6}{32\pi^2} g_t^4 (H^2)^2 \ln H^2 \qquad (5)$$

The leptonic contribution, on the other hand, cannot be reduced so easily as it involves a matrix linking (L^i, E^i, ν^i_R) and their charge conjugates. To simplify the calculation, we now neglect the terms involving Y^E_{ij} (whose largest entry comes from the τ-lepton with g_τ of order 0.01). The remaining matrix only couples the doublets L^i and the right-chiral neutrinos ν^i_R; before renormalization the relevant expression can be reduced to the integral (with UV cutoff Λ)

$$\begin{aligned}&-\frac{1}{16\pi^2}\int_0^{\Lambda^2} d\xi\, \xi\, \ln\det\Big[\xi^2 + \\ &\Big(Y_M\bar{Y}_M\cdot\varphi^2 + Y_M(Y_\nu\bar{Y}_\nu + \bar{Y}_\nu Y_\nu)Y_M^{-1}\cdot H^2\Big)\cdot\xi \\ &\qquad + Y_M\bar{Y}_\nu Y_\nu Y_M^{-1} Y_\nu\bar{Y}_\nu\cdot(H^2)^2\Big] \qquad (6)\end{aligned}$$

where the remaining determinant under the integral is to be taken w.r.t. a complex 3-by-3 matrix in the family indices. Further evaluation of this expression would thus require the factorization of a sixth order polynomial in ξ which again is in general not possible in closed form, especially if there is 'maximal mixing' in the Yukawa matrices (meaning that Y^ν_{ij} is far away from a diagonal matrix).

For this reason, we resort to yet another approximation by assuming $Y_\nu \langle H\rangle \ll Y_M\langle\varphi\rangle$, in agreement with the observed smallness of neutrino masses. Then the above expression can be calculated exactly and the full effective potential becomes, in this approximation,

$$\begin{aligned} V_{\rm eff}(H,\varphi) &= \frac{3}{256\pi^2}\left(\lambda_1 H^2+\lambda_2\varphi^2\right)^2 \ln\left[\frac{\lambda_1 H^2+\lambda_2\varphi^2}{v_1^2}\right] \\ &+\frac{1}{64\pi^2}F_+^2\ln\left[\frac{F_+}{v_2^2}\right]+\frac{1}{64\pi^2}F_-^2\ln\left[\frac{F_-}{v_2^2}\right] \\ &-\frac{6}{32\pi^2}g_t^4(H^2)^2\ln\left[\frac{H^2}{v_3^2}\right]-\frac{1}{32\pi^2}{\rm Tr}(Y_M\bar{Y}_M)^2\varphi^4\ln\left[\frac{\varphi^2}{v_4^2}\right] \end{aligned} \quad (7)$$

where the constants $v_1,\ldots,v_4$ can be freely adjusted as their choice corresponds to a choice of the counterterm coefficients α,β,γ in (4). We do not include here the terms from $SU(2)_w\times U(1)_Y$ gauge fields because the respective gauge couplings are small nor from $SU(3)_c$ gauge fields because it is a two-loop effect (although numerically it can be important and is included in the RG analysis described below). The one-loop contribution to the effective action is of the same order as the tree level one (and not much bigger as usually argued in the context of CW mechanism).

As we cannot minimize this potential in closed form, we now search for minima numerically. Since the problem is highly non-linear we have to use a trial-and-error method to arrive at a set of 'reasonable' values satisfying the following requirements: the standard Higgs mass m_H must be bigger than 115 GeV, the effective coupling constants $\lambda_i^{\rm eff}$ (defined as appropriate fourth derivatives of the potential at the minimum) should be such that there are no Landau poles or instabilities up to some large scale, and finally some hierarchy between the expectation values of $\langle H\rangle$ and $\langle\varphi\rangle$ for the see-saw mechanism to operate. The numerical search shows that the 'window' left open by these requirements is not very large, but in particular allows for the following set of values:

$$\begin{aligned} &\lambda_1=9.75, \quad \lambda_2=3.59, \quad \lambda_3=3.16, \\ &v_1^2=1.9, \quad v_2^2=23, \quad v_3^2=1, \quad v_4^2=0.85, \\ &g_t=1, \quad g_M^4:={\rm Tr}(Y_M\bar{Y}_M)^2=1.1. \end{aligned} \quad (8)$$

For these values, the minimum lies at $\langle H\rangle=0.198$, $\langle\varphi\rangle=1.805$. Up to this point, we have effectively calculated with dimensionless quantities. The appearance of the logarithms in (7) now dictates that we choose one mass scale which sets the scale for all other quantities. This we do by imposing $\langle H\rangle=174$ GeV. Hence,

$$\langle H\rangle=174\ {\rm GeV}, \quad \langle\varphi\rangle=1582\ {\rm GeV} \quad (9)$$

Assuming $|Y_\nu|<10^{-5}$, so the neutrino Yukawa couplings are of the same order as the electron Yukawa coupling, we can arrive at very small neutrino masses:

$$m_\nu\approx\frac{(Y_\nu\langle H\rangle)^2}{Y_M\langle\varphi\rangle}<1\ {\rm eV} \quad (10)$$

After symmetry breaking three degrees of freedom of Φ are converted into longitudinal components of $W^\pm$ and Z^0, so we are left with two real scalar fields H and φ, and the potential $V(\Phi,\varphi)$ should be understood from now as $V(H,\varphi)$ (with noncanonical normalization). Calculating second derivatives at the minimum and defining

$$H'=H\cos\beta+\varphi\sin\beta\ , \quad \varphi'=-H\sin\beta+\varphi\cos\beta \quad (11)$$

we obtain the mass values:

$$m_{H'}=117\ {\rm GeV}, \quad m_{\varphi'}=658\ {\rm GeV} \quad (12)$$

with mixing angle

$$\sin\beta=0.233 \quad (13)$$

Note that only the components along H of the mass eigenstates couple to the usual SM particles.

The effective coupling constants are found to be

$$\lambda_1^{\rm eff}=1.370, \quad \lambda_2^{\rm eff}=0.969, \quad \lambda_3^{\rm eff}=0.596 \quad (14)$$

According to the conventional procedure one should now evolve these values with the renormalization group equations in order to check for the presence of Landau poles or instabilities (negative coupling constants). Defining

$$y_1=\frac{\lambda_1^{\rm eff}}{4\pi^2}, \quad y_2=\frac{\lambda_2^{\rm eff}}{4\pi^2}, \quad y_3=\frac{\lambda_3^{\rm eff}}{4\pi^2}, \quad x=\frac{g_t^2}{4\pi^2}, \quad u=\frac{g_M^2}{4\pi^2}$$

we have the renormalization group equations

$$\begin{aligned} \mu\frac{{\rm d}y_1}{{\rm d}\mu} &= \frac{3}{2}y_1^2+\frac{1}{8}y_2^2-6x^2, \\ \mu\frac{{\rm d}y_2}{{\rm d}\mu} &= \frac{3}{8}y_2(2y_1+y_3+\frac{4}{3}y_2), \\ \mu\frac{{\rm d}y_3}{{\rm d}\mu} &= \frac{9}{8}y_3^2+\frac{1}{2}y_2^2-u^2, \quad \mu\frac{{\rm d}u}{{\rm d}\mu}=\frac{3}{4}u^2 \\ \mu\frac{{\rm d}x}{{\rm d}\mu} &= \frac{9}{4}x^2-4xz, \quad \mu\frac{{\rm d}z}{{\rm d}\mu}=-\frac{7}{2}z^2. \end{aligned} \quad (15)$$

where we added the strong coupling contribution $z=\alpha_s/\pi$. As dictated by (7) we use 1PI-irreducible (and not the full) β-functions since in the effective action the renormalized external fields are used. With the initial values at 174 GeV given by $g_t=1$, $\alpha_s=0.1$ and (14) it turns out that there are no Landau poles or instabilities below the Planck scale. We should emphasize that the numbers in (8) are merely chosen to illustrate the possible viability of the proposed scenario up to the Planck scale. Because of its non-linearity, the system of coupled evolution equations (15) is rather delicate and highly sensitive to small changes in the initial values. The numerical scan over the range of parameters satisfying the previously mentioned constraints shows that the standard Higgs in all cases comes out to be rather light.

Let us also point out that a complete treatment would require calculation of the full resummed effective action (which is renormalization group invariant!) and to plug in the renormalized values of the coupling constants, rather than simply evolving the couplings as above. The problem with the Landau pole should then manifest itself as a singularity of the effective action at some value of the momentum dependent terms. However, it appears difficult to proceed analytically in this way (see [6]) so we content ourselves here with the conventional procedure.

Phenomenologically, and for low energies, the proposed scenario is virtually indistinguishable from the SM with massive neutrinos, but for large energies differs significantly from SM extensions like the MSSM. Apart from the obvious lack of superpartners, the Higgs couplings are very different – for instance, the decay of the standard Higgs can proceed (via mixing with the new Higgs) into right-chiral neutrinos, i.e. missing energy. Discussion of such phenomenological implications is, however, outside the scope of this letter.

It is worthwhile to note that the usual hierarchy problem is addressed here in a way which is very different from the solution proposed in the context of the MSSM (see e.g. [11]). The latter relies on two features, namely (*i*) the fact that the coupling constants run logarithmically (avoiding large corrections to the mass terms, thereby stabilizing the hierarchy) and (*ii*) the fact that supersymmetry forces the Higgs self-coupling to be a function of the gauge couplings, which themselves are kept under control by gauge invariance (this helps to avoid the Landau pole problem). The hierarchy itself is explained in the MSSM by certain soft supersymmetry breaking terms extremely finely tuned at the GUT scale that run slowly in such a way that m_H^2 eventually becomes negative around 1 TeV.

By contrast, there are no explicit mass terms in our proposal by virtue of the assumed conformal invariance. With this assumption, the theory contains only dimensionless parameters to start with, and the symmetry can be broken only by anomalies, that is, the unavoidable choice of scale under the logarithms. In this sense the hierarchy of scales emerges 'naturally', and moreover the conformal symmetry ensures that it is not broken by powers of the cutoff in the radiative corrections. Although the Landau pole problem is in principle there, it can be avoided without excessive fine-tuning as we showed (for the values (8) it occurs around 10^{20} GeV).

The key question is therefore how a conformally invariant action at low energies, for which one would expect the underlying theory at large scales to be also conformally invariant (up to logarithms) can emerge from gravity, which is *not* conformally invariant due to the presence of a dimensionful parameter, the Planck mass (there are also other sources of conformal noninvariance in the SM, like gluon and quark condensates, but they concern much lower scales than we are interested in here). Although this might appear implausible at first sight, it is quite conceivable that quantum gravity effects may *dynamically* suppress explicit breaking of conformal invariance by powerlike counterterms and allow only for logarithmic terms in the effective action. The prime example for such a phenomenon is noncritical string (Liouville) theory [12], a theory of matter-coupled quantum gravity in two space-time dimensions, which does not possess classical conformal invariance, but where conformal invariance is restored at the quantum level via the quantum mechanical decoupling of an infinite tower of null states (as explained e.g. in [13]).

A more detailed account of the results presented in this letter will be given elsewhere.

Acknowledgments: K.A. Meissner thanks the Albert-Einstein-Institut for hospitality and financial support.

[1] O. Nachtmann, *Elementary Particle Physics: Concepts and Phenomena*, Springer Verlag (1999);
S. Pokorski *Gauge Field Theories*, Cambridge Univ. Press, 2nd edition (2000)
[2] R. Brout and F. Englert, Phys. Rev. Lett. **13** (1964) 321;
P.W. Higgs, Phys. Lett. **12** (1964) 132.
[3] P. Ramond, *Journeys beyond the Standard Model*, Perseus Books Group (1999).
[4] S. Coleman and E. Weinberg, Phys. Rev. **D7** (1973) 1888.
[5] M. Sher, Phys. Rep. **179** (1989) 273.
[6] F.A. Chishtie, V. Elias, R.B. Mann, D.G.C. McKeon and T.G. Steele, Nucl.Phys. **B743** (2006) 104.
[7] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors And Unified Theories*, in Supergravity, P. van Nieuwenhuizen and D.Z. Freedman (eds.) (North-Holland) (1979) 315;
T. Yanagida, Prog.Theor.Phys. **64** (1980) 1103.
[8] In fact, as shown in P.C. West, Nucl. Phys. **B106** (1976) 219, the effective potential vanishes identically to all orders in an exactly supersymmetric theory.
[9] In principle, the field φ could be taken complex or even to transform in a non-trival representation of a family symmetry (in which case $M > 1$ in formula (4)). The phase of φ would then be a Goldstone or pseudo-Goldstone boson, which couples to observable matter only via the right-chiral neutrinos, and might thus be useful for some other purposes.
[10] C. Itzykson and J.B. Zuber, *Quantum Field Theory*, Univ. Press (2000).
[11] S. Weinberg, *The Quantum Theory of Fields, III: Supersymmetry*, Cambridge Univ. Press, 2000.
[12] T.L. Curtright and C.B. Thorn, Phys.Rev.Lett. **48** (1982) 1309;
J.-L. Gervais and A.Neveu, Nucl.Phys. **B209** (1982) 125;
R. Marnelius, Nucl.Phys. **B211** (1983) 14;
A.M. Polyakov, Mod.Phys.Lett. **A2** (1987) 893;
V.G. Knizhnik, A.M. Polyakov and A.B. Zamolodchikov, Mod.Phys.Lett. **A3** (1988) 819;
J. Distler and H. Kawai, Nucl.Phys. **B321** (1989) 509.
[13] Z. Jaskolski and K.A. Meissner, Nucl.Phys. **B428** (1994) 331-373.