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Lovelock type gravity and small black holes in heterotic string theory

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ABSTRACT: We analyze near horizon behavior of small D-dimensional 2-charge black holes by modifying tree level effective action of heterotic string with all extended Gauss-Bonnet densities. We show that there is a nontrivial and unique choice of parameters, independent of D, for which the black hole entropy in any dimension is given by $4\pi\sqrt{nw}$, which is exactly the statistical entropy of 1/2-BPS states of heterotic string compactified on $T^{9-D} \times S^1$ with momentum n and winding w. This extends the results of Sen [*JHEP* **0507** (2005) 073] to all dimensions. We also show that our Lovelock type action belongs to the more general class of actions sharing the simmilar behaviour on the $AdS_2 \times S^{D-2}$ near horizon geometry.

Contents

1. Introduction

Recently black holes in heterotic string theory had attracted a lot of attention[1]. Special class are 2-charge small black holes. On the string side these black holes should correspond to perturbative half-BPS states of heterotic string compactified on $T^{9-D} \times S^1$, with momentum and winding on S^1 equal to n and w, respectively, for which one can easily calculate asymptotic expression ($n, w \gg 1$) for the number of states [6, 7]. Logarithm (which is the entropy in microcanonical ensamble) is in the leading order given by

$$S = 4\pi\sqrt{nw} \tag{1.1}$$

This result, obtained for a free string, due to supersymmetry remains to be valid after switching on the string coupling g_s. Now, as the string coupling is increased, at one point de Broglie-Compton wavelength $1/M$ becomes smaller then the corresponding Schwarzschild radius $\ell_P^2 M \sim g_s^2 \alpha' M$, which should lead to formation of (extremal) black hole. This is a one way to argue that elementary string states with mass large enough should describe black holes [2, 3, 4, 5].

Indeed, exact black hole solutions of the low energy effective action of heterotic string theory in the leading order in α' were found which decribe D-dimensional extremal black holes with "correct" quantum numbers (e.g., they have two electric charges proportional to n and w) [8, 9]. They are in some sense pathological having null singularities and zero

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[1]A overview of recent results for black holes in string theory is given in [1].

horizon area[2]. This implies vanishing Bekenstein-Hawking entropy which is obviously in disagreement with the string result (1.1).

To understand what is happening, one should go back to the derivation of (1.1) – and to see that although it is perturbative in string coupling, it is *nonperturbative* in α'. This means that on the gravity side one should start from the complete tree-level (in string coupling) effective action which contains all α' higher-derivative corrections. This is also visible from the structure of the solution in the leading order – singularity of the horizon implies that one cannot neglect higher curvature terms (or treat them as perturbation) in the efective action near the horizon, as it is usually done for large black holes. In fact, a priory all terms should be of the same importance. The remarkable property of small black holes is that they give us some information on the *complete* tree-level (in string coupling) effective action.

In [10, 11, 12, 13, 14, 15] it was shown in $D = 4$ that adding to the action just one type of the higher-derivative terms, obtained by supersymmetrizing square of the Weyl tensor [16, 17, 18], one obtains that corrected black holes have regular horizon of $AdS_2 \times S^2$ type, for which generalised Wald entropy formula[3] [19, 20, 21] gives a desired result (1.1). This result is at the same time exciting and mysteriuos, because there is no apparent reason why should only terms quadratic in curvature contribute to the entropy, with all higher-order terms somehow cancelling.[4] It is important to note that for the entropy one only needs behaviour of the solution near the horizon, so this cancelation could just appear there (as a consequence of the $AdS_2 \times S^2$ geometry). Indeed, numerical extrapolations to the far-away region show that solution does not aproach to Schwarzschild solution but has oscillating behaviour connected with spurious degrees of freedom typicaly present in higher order gravity theories [12, 13]. This could suggest that other higher order terms become important away from horizon.

A natural question is what is happening in $D > 4$? Unfortunately, it is imposible to perform the same analysis, as it is not known how to supersymmetrize R^2-terms in the action. In lack of this, Sen [24] took as a "toy-model" just the gravitational part, which is proportional to Gauss-Bonnet density[5], and analysed near-horizon behaviour of the solution (for which he assumed $AdS_2 \times S^{D-2}$ geometry). Although this action is not supersymmetric, surprisingly, Wald entropy formula again gave (1.1), now in $D = 4$ and $D = 5$ (but not for $D \geq 6$). Even more surprisingly, in the recent paper [26], it was shown that for the same type of the action, applied to the large class of 8-charge black holes in $D = 4$, entropy, near horizon metric, gauge field strengths and the axion-dilaton field are identical to those obtained in [27, 28] from a supersymmetric version of the theory based on squared Weyl tensor.

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[2]This is the reason why they are called small or microscopic.

[3]Note that although Wald derivation demands existence of the bifurkate Killing horizon, and so does not apply to extremal black holes, one can formally take the limit of extremality in the final formula.

[4]In [22, 23] an explanation was presented based on anomalies induced by particular Chern-Simons terms. However, it is not clear to us why only those terms should contribute.

[5]There is also a term proportional to the Pontryagin density, but it vanishes identicaly in $AdS_2 \times S^n$ background.

In this paper we extend Sen's analysis of two-charge black holes to any number of dimensions $D \geq 4$. For the effective action near the horizon we take obvious generalisation, i.e., we use extended Gauss-Bonnet densities as higher-order terms in curvature [29, 30]. These "Lovelock type" actions have several appealing properties, e.g, they are of the first order (no ghosts or spurious states [31, 32]), have good boundary value problem, and contain only finite number of terms. We perform near horizon analysis assuming $AdS_2 \times S^{D-2}$ geometry and, using Wald formula, calculate entropy, which has a complicated dependance on D and[6] $[D/2]$ coupling constants[7] λ_m. We show that there is a unique choice for λ_m (independent of D) which gives exactly the expression (1.1) in *any* D. It should be emphasized that this is a nontrivial result, in the sence that to fix the entropy for D black holes one has only $[D/2]$ free parameters to play with (or, in other words, for each couple of dimensions enters only one parameter). This result trivially extends to black holes with more electric charges, connected with heterotic string compactifications on $M_D \times T^{10-D-k} \times (S^1)^k$.

2. Effective action with extended Gauss-Bonnet terms

We are interested in heterotic string compactified on $T^{9-D} \times S^1$, for which effective low energy action in the leading order in string coupling can be written in the form

$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g}\, S \sum_{m=1} \alpha'^{m-1} \mathcal{L}_m \tag{2.1}$$

where S is the dilaton field, which is connected to the effective closed string coupling constant g by $S = 1/g^2$.

Leading order term in α' is given by [24]

$$\mathcal{L}_1 = R + S^{-2}(\nabla S)^2 - T^{-2}(\nabla T)^2 - T^2 \left(F^{(1)}_{\mu\nu}\right)^2 - T^{-2}\left(F^{(2)}_{\mu\nu}\right)^2 \tag{2.2}$$

where we assumed that all other fields are vanishing. In this order exact half-BPS electricaly charged extremal black hole solutions in any D were found [9] which have the same quantum numbers as perturbative half-BPS string states (where two electric charges are proportional to momentum and winding of the string along S^1). These solutions have singular horizon (null singularity) with a vanishing area, on which effective string coupling also vanishes. This properties are in contrast with what one expects from string theory, which for example gives the nonvanishing result for the entropy (1.1).

It is obvious what is wrong in the above analysis. As the horizon is singular, the curvature invariants (and some other fields like S) are also, which means that in the effective action (2.1) one cannot neglect higher-order terms which typicaly contain higher powers and/or derivatives of the Riemann tensor. In $D = 4$ dimensions it was shown in [10, 11, 12, 13] that if one adds a particular class of higher-derivative terms (obtained by supersymmetrization of the square of the Weyl tensor), corrections completely change the nature of singularity - one gets timelike singularity hidden behind a horizon with the finite

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[6] $[x]$ denote integer part of x.

[7] $[D/2]$ is the number of extended Gauss-Bonnet terms in D dimensions, including the Einstein term.

area. Also, the dilaton field S becomes finite on the horizon, which means that effective string coupling is nonvanishing. Using Wald formula it was shown that the entropy is equal to the string result (1.1). Now, the mystery is why other terms, which are known to be present in the effective action (especially ones containing higher powers of the Riemann tensor), are appearing to be irrelevant for the entropy calculation.

One way to understand what is happening would be to make the same analysis in higher dimensions. Unfortunately, for $D > 4$ supersymmeric version of the action containing curvature squared terms is not known. In lack of this, in [24] Sen took as a toy model an action obtained by adding just the Gauss-Bonnet term. Although this action is not supersymmetric, from the near horizon analysis he obtained that the entropy is again given by (1.1), but only in $D = 4, 5$. Now, the interesting thing is that in $D = 6$ a next extended Gauss-Bonnet term is present, so the natural question to ask is what is happening if we include in the action all extended Gauss-Bonnet terms. That is the main subject of this paper.

We propose to analyse the actions of the Lovelock type where higher order terms in α' in (2.1) are given by the extended Gauss-Bonnet densities [29, 30]

$$\mathcal{L}_m = \lambda_m \mathcal{L}_m^{GB} = \frac{\lambda_m}{2^m} \delta^{\rho_1 \sigma_1 \ldots \rho_m \sigma_m}_{\mu_1 \nu_1 \ldots \mu_m \nu_m} R^{\mu_1 \nu_1}{}_{\rho_1 \sigma_1} \cdots R^{\mu_m \nu_m}{}_{\rho_m \sigma_m} , \qquad m = 2, \ldots, [D/2] \quad (2.3)$$

where λ_m are some (at the moment free) dimensionless parameters, $\delta^{\beta_1 \ldots \beta_k}_{\alpha_1 \ldots \alpha_k}$ is totally anti-symmetric product of k Kronecker deltas, normalized to take values 0 and ± 1, $[x]$ denote integer part of x, and all greek indeces are running from 0 to $D - 1$. Extended Gauss-Bonnet densities $\mathcal{L}_m^{GB}$ are in many respects generalisation of the Einstein term (note that $\mathcal{L}_1^{GB} = R$). Especially, m-th term is topological in $D = 2m$ dimensions. Also note that they identicaly vanish for $m > [D/2]$, so for any D there is a finite number of terms in the action.

3. Near horizon analysis

We want to study solutions of the action given by (2.1–2.3) which should be deformations of the exact small black hole solutions obtained in lowest order in α'. We do not know how to exactly solve equations of motion, but we are primarily interested in the entropy which is given by the Wald formula [19, 20, 21]

$$S = 2\pi \int_{\mathcal{H}} \hat{\epsilon} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\rho\sigma}} \eta_{\mu\nu} \eta_{\rho\sigma} \tag{3.1}$$

Important here is to notice that integration is done on the cross section of the horizon H, so to calculate the entropy one only needs to know a solution near the horizon.

Now, in [25] it was shown that symmetries of the horizon can enormously simplify calculation of the entropy. In $D = 4$ case it was shown that near horizon geometry is of $AdS_2 \times S^2$ type, where effect of α' corrections was to make radius of horizon nonvanishing. Following [24] we conjecture that the same happens in $D > 4$ so the near horizon geometry

should be $AdS_2 \times S^{D-2}$. This implies that near the horizon fields have the following form

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = v_1 \left(-x^2 dt^2 + \frac{dx^2}{x^2} \right) + v_2\, d\Omega_{D-2}^2$$
$$S = u_S$$
$$T = u_T$$
$$F_{rt}^{(i)} = e_i\,, \qquad i = 1,2 \tag{3.2}$$

where v_i, u_S, u_T, e_i are constants, and moreover that the covariant derivatives of the scalar fields S and T, the gauge fields $F_{\mu\nu}^{(i)}$ and the Riemann tensor $R_{\mu\nu\rho\sigma}$ vanish on the horizon $x = 0$. This makes solving the equations of motions (EOM's) near the horizon (i.e., finding v_i, u_S, u_T and e_i) very easy. One first defines

$$f(\vec{u}, \vec{v}, \vec{e}) = \int_{S^{D-2}} \sqrt{-g}\, \mathcal{L} \tag{3.3}$$

where the integration is over S^{D-2}, and one uses (3.2). Equations of motion are near the horizon given by

$$\frac{\partial f}{\partial u_S} = 0\,, \qquad \frac{\partial f}{\partial u_T} = 0\,, \qquad \frac{\partial f}{\partial v_1} = 0\,, \qquad \frac{\partial f}{\partial v_2} = 0 \tag{3.4}$$

Notice that configuration (3.2) solves EOM's for gauge fields identicaly on the horizon for any e_i. We also need to know electric charges q_i. In [25] it was shown that they are given by

$$q_i = \frac{\partial f}{\partial e_i}\,, \qquad i = 1,2 \tag{3.5}$$

We would also like to connect conserved charges (3.5) with corresponding quantum numbers of half-BPS states of heterotic string, which are momentum n and winding w around S^1. This is given by [26]

$$q_1 = \frac{2\,n}{\sqrt{\alpha'}}\,, \qquad q_2 = \frac{2\,w}{\sqrt{\alpha'}} \tag{3.6}$$

It was shown in [25] that the entropy for the configuration (3.2) is given by

$$S = 2\pi \left(\sum_{i=1}^{2} e_i\, q_i - f \right) \tag{3.7}$$

For the actions of the type (2.1) EOM for dilaton S implies that f vanishes on-shell near the horizon, so we have just

$$S = 2\pi \sum_{i=1}^{2} e_i\, q_i \tag{3.8}$$

4. Entropy of small black holes

We now apply procedure from the previous section to analyse extremal small black hole solutions in D dimensions, with the $AdS_2 \times S^{D-2}$ horizon geometry, when the action is

given by (2.1–2.3). First we need to calculate function f (3.3) using (3.2). It was shown [33] that for the metrics of the type

$$ds^2 = \gamma_{ab}(x)dx^a dx^b + r(x)^2 d\Omega_{D-2} , \qquad a, b = 1, 2 \tag{4.1}$$

the Gauss-Bonnet densities, integrated over the unit sphere S^{D-2}, give

$$\int_{S^{D-2}} \sqrt{-g}\, \mathcal{L}_m = -\Omega_{D-2}\lambda_m \frac{(D-2)!}{(D-2m)!}\sqrt{-\gamma}\, r^{D-2m-2} \left[1 - (\nabla r)^2\right]^{m-2}$$

$$\times \left\{ 2m(m-1)r^2 \left[(\nabla_a \nabla_b r)^2 - (\nabla^2 r)^2\right] \right.$$

$$+ 2m(D-2m)r\nabla^2 r \left[1 - (\nabla r)^2\right] - m\mathcal{R}r^2 \left[1 - (\nabla r)^2\right]$$

$$\left. - (D-2m)(D-2m-1)\left[1 - (\nabla r)^2\right]^2\right\} . \tag{4.2}$$

where $\mathcal{R}$ is a two-dimensional Ricci scalar calculated from γ_{ab}. Specializing further to $AdS_2 \times S^{D-2}$ metric (3.2) all terms having covariant derivatives vanish on the horizon and using this and (3.2) one obtains the following expression for the function f

$$f = \frac{\Omega_{D-2}}{16\pi G_N} u_S v_1 v_2^{(D-2)/2} \left\{ \frac{2\, u_T^2\, e_1^2}{v_1^2} + \frac{2\, e_2^2}{u_T^2\, v_1^2} \right.$$

$$\left. + \sum_{m=1}^{[D/2]} \alpha'^{m-1}\lambda_m \frac{(D-2)!}{(D-2m)!}\, v_2^{-m} \left[(D-2m)(D-2m-1) - 2m\frac{v_2}{v_1}\right] \right\} \tag{4.3}$$

where $\lambda_1 = 1$.

Now we can use (3.4–3.8) to calculate entropy. For better understanding we specialize first to $D \le 7$ and then take the general case.

4.1 $D = 4, 5$

In this case we have only $m = 1, 2$ terms in (4.3). Although the analysis was already done in [24], for completeness we shall repeat it here. From (4.3) we get

$$f = \frac{\Omega_{D-2}}{16\pi G_N} u_S v_1 v_2^{(D-2)/2} \left[\frac{2\, u_T^2\, e_1^2}{v_1^2} + \frac{2\, e_2^2}{u_T^2\, v_1^2} - \frac{2}{v_1} + \frac{(D-2)(D-3)}{v_2}\left(1 - \frac{4\,\alpha'\lambda_2}{v_1}\right)\right] \tag{4.4}$$

Now we impose EOM's (3.4), and use (3.5,3.6) to express results in terms of n and w. One obtains a unique solution

$$v_1 = 4\,\alpha'\lambda_2 \tag{4.5}$$

$$v_2 = 2(D-2)(D-3)\alpha'\lambda_2 \tag{4.6}$$

$$u_T = \sqrt{\frac{n}{w}} \tag{4.7}$$

$$u_S = \frac{4\pi G_N}{\Omega_{D-2}} \frac{v_1}{v_2^{(D-2)/2}} \frac{q_1}{e_2} = \frac{4\pi G_N}{\Omega_{D-2}} \frac{v_1}{v_2^{(D-2)/2}} \frac{\sqrt{2nw}}{\alpha'\sqrt{\lambda_2}} \tag{4.8}$$

$$e_1 = \sqrt{2\,\alpha'\lambda_2 \frac{w}{n}} , \qquad e_2 = \sqrt{2\,\alpha'\lambda_2 \frac{n}{w}} \tag{4.9}$$

Using (4.5-4.9) and (3.6) in (3.8) we obtain the entropy

$$S = 4\pi\sqrt{8\,\lambda_2}\sqrt{nw} \tag{4.10}$$

We now see that to match the statistical entropy of string states (1.1) one has to take

$$\lambda_2 = \frac{1}{8} \tag{4.11}$$

As noticed in [24] this is exactly the value which appears in front of the Gauss-Bonnet term in the low energy effective action of heterotic strings. Observe also that by fixing only one parameter λ_2 one obtains (1.1) for both $D = 4$ and $D = 5$.

Notice here some aspects of solution which we shall show to be common for all D. First, dilaton field $u_S \propto \sqrt{nw}$, so for the effective string coupling on the horizon $g^2 = 1/u_S \propto 1/\sqrt{nw} \ll 1$ for $n, w \gg 1$. So, tree level in string coupling is a good approximation. Second, $v_i \propto \alpha'$, which means that all terms in our effective action are of the same order in α'. All higher curvature terms ar a priori important.

4.2 $D = 6, 7$

When we go up to $D = 6$ and $D = 7$, we see from (4.3) that the function f receives additional contribution (comparing to (4.4)), given by

$$\Delta f_{6,7} = \frac{\Omega_{D-2}}{16\pi G_N}\, u_S\, v_1\, v_2^{(D-2)/2}(D-2)(D-3)(D-4)(D-5)\frac{\alpha'}{v_2^2}\left(\lambda_2 - \frac{6\,\alpha'\lambda_3}{v_1}\right) \tag{4.12}$$

We saw in the previous subsection that $\lambda_2 = 1/8$.

Now we solve the EOM's. It is obvious that we again obtain (4.7) and the first equality in (4.8). Solving EOM's for v_1 and v_2 we obtain

$$t_1 = \frac{t_2^2 + a(t_2 + 48b\lambda_3)}{a(t_2 - 8b)} \tag{4.13}$$

where t_2 is a solution of the cubic equation

$$t_2^3 - (a - b)t_2^2 - 144ab\lambda_3 t_2 - 48ab^2\lambda_3 = 0 \tag{4.14}$$

In the above formulae we have used the notation

$$t_i \equiv \frac{4v_i}{\alpha'}\,, \qquad a \equiv (D-2)(D-3)\,, \qquad b \equiv (D-4)(D-5) \tag{4.15}$$

For any given λ_3 we have generally three solutions for $v_{1,2}$, but it can be shown that there is only one physicaly interesting for which both v_1, v_2 are real and positive. Using this solution one can proceed further and as in $D = 4, 5$ solve all EOM's and calculate the entropy. As the corresponding expressions are cumbersome and noniluminating functions of λ_3, we shall not write them explicitly.

The entropy (3.8) has the form

$$S = \omega(\lambda_3, D)\sqrt{nw} \tag{4.16}$$

where ω is some complicated function of λ_3 and D. Now, we search for such λ_3 for which in $D = 6$ and $D = 7$ we obtain (1.1). One way to fix λ_3 is to demand[8] that entropy is the same in both dimensions

$$\omega(\lambda_3, D = 6) = \omega(\lambda_3, D = 7) \tag{4.17}$$

It is easy to show that the only solution is

$$\lambda_3 = \frac{1}{96} \tag{4.18}$$

Now we use this value for λ_3 in (4.16) and obtain that the entropy is given by

$$S = 4\pi\sqrt{nw} \tag{4.19}$$

which is again exactly the string result (1.1). For the choice (4.18) solution is given by

$$v_1 = \frac{\alpha'}{2} \tag{4.20}$$

$$v_2 = \frac{\alpha'}{8}(D-2)(D-3)\left[1 + \sqrt{1 + \frac{2(D-4)(D-5)}{(D-2)(D-3)}}\right] \tag{4.21}$$

$$u_T = \sqrt{\frac{n}{w}} \tag{4.22}$$

$$u_S = \frac{4\pi G_N}{\Omega_{D-2}}\frac{v_1}{v_2^{(D-2)/2}}\frac{q_1}{e_2} = \frac{8\pi G_N}{\Omega_{D-2}}\frac{\sqrt{nw}}{v_2^{(D-2)/2}} \tag{4.23}$$

$$e_1 = \sqrt{\frac{\alpha'}{4}\frac{w}{n}}, \qquad e_2 = \sqrt{\frac{\alpha'}{4}\frac{n}{w}} \tag{4.24}$$

4.3 General dimensions

We now pass to general number of dimensions D recursively. From (4.3) we see that passing from (odd) dimension $D = 2m - 1$ to $D = 2m$ and $D = 2m + 1$ the function f gets additional contribution

$$\Delta f = \frac{\Omega_{D-2}}{16\pi G_N} u_S v_1 v_2^{(D-2)/2} \alpha'^{m-2} \frac{(D-2)!}{(D-2m)!} v_2^{-m+1}\left(\lambda_{m-1} - \frac{2m\alpha'}{v_1}\lambda_m\right) \tag{4.25}$$

We assume that all λ_k, $k = 1, \ldots, m-1$ are determined from lower-dimensional analyses, so the only free parameter at the moment is λ_m.

In principle we could apply the same analysis as in previos subsections, i.e., solve the EOM's, calculate the entropy for general λ_m and then look is there a value of λ_m for which the entropy is equal to (1.1). The problem is that for this one has to solve polinomial equation, like (4.14), which is now of the order $(2m-3)$ and so for $m \geq 4$ cannot be solved analyticaly for general λ_m.

However, closer inspection of the solution (4.20-4.24) for $D \leq 7$ reveals the shortcut. We notice that only v_2 depends on D, and that v_1, u_T, e_i are depending just on n and w.

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[8]Equivalently, we could ask that $\omega = 4\pi$ for $D = 6$, and then check do we obtain the same result for $D = 7$.

From (3.6) and (3.8) we see that to obtain for the entropy string result (1.1) it is necessary that e_i are given by (4.24). One obvious way to have this is to fix $m\lambda_m/\lambda_{m-1}$ to be the same for all m. Then

$$v_1 = 2m\alpha'\frac{\lambda_m}{\lambda_{m-1}} \tag{4.26}$$

is one solution of EOM. Then, to have (4.24) we see that v_1 has to be given by (4.20), which combined with (4.26) gives the coupling constants

$$\lambda_m = \frac{\lambda_{m-1}}{4m} = \frac{4}{4^m m!} \tag{4.27}$$

where we have used $\lambda_1 = 1$.

To summarise, for the choice of coupling constants given in (4.27) there is a solution[9] of EOM for any D given by (4.20), (4.22-4.24), and with $v_2 = \alpha' y(D)$, where $y(D)$ is some complicated function of D (which is a real and positive root of $(m-1)$-th order polynomial), for which the Wald entropy formula gives

$$S = 4\pi\sqrt{nw} . \tag{4.28}$$

And this is exactly the statistical entropy of half-BPS states of heterotic string given in (1.1).

5. Some remarks

Before discussing our results, let us make two remarks. First, we would like to note that the gravitational part of the Lovelock type action with coefficients given by (4.27) apparently can be written in the exponential form

$$S_{grav} = \frac{1}{4\pi G_N \alpha'} \int d^D x \sqrt{-g}\, S \left[\exp\left(\sum_{m=1}^{} \frac{\alpha'^m}{4}\lambda_m \tilde{\mathcal{L}}_m^{GB}\right) - 1\right] \tag{5.1}$$

where $\tilde{\mathcal{L}}_m^{GB}$ are obtained from the extended Gauss-Bonnet densities $\mathcal{L}_m^{GB}$ given in (2.3) by throwing away all terms which are products of two or more scalars (like e.g., R^2, $R(R_{\mu\nu})^2$, etc.). We do not have a proof of this, but we have checked it explicitely for terms up to α'^3 order, and also confirmed that terms of the type $R^k X$ are in agreement with the known recursion relation

$$\frac{\partial \mathcal{L}_m^{GB}}{\partial R} = m\mathcal{L}_{m-1}^{GB} . \tag{5.2}$$

This makes us believe that (5.1) is correct. As far as we know, the Lovelock action with the particular choice of parameters given in (4.27) was not mentioned in the literature before.

For a second remark, notice that from (4.3) and (4.27) follows that f function can be put in the form

$$f = \frac{\Omega_{D-2}}{16\pi G_N} u_S v_1 v_2^{(D-2)/2} \left[\frac{2\,u_T^2\,e_1^2}{v_1^2} + \frac{2\,e_2^2}{u_T^2\,v_1^2} - \frac{2}{v_1} - \left(\frac{1}{v_1} - \frac{2}{\alpha'}\right)A\right] \tag{5.3}$$

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[9]We have checked that for $D \leq 9$ this is a unique solution with both v_1, v_2 real and positive.

where the function A is given by

$$A = A(v_2) = \sum_{m=1}^{[D/2]} \alpha'^m \lambda_{m+1} \frac{2m(D-2)!}{(D-2m-2)!} \frac{1}{v_2^m} \tag{5.4}$$

Equation for v_2 ($\partial f/\partial v_2 = 0$) gives directly a solution $v_1 = \alpha'/2$, which substituted back into f leaves just the term

$$-\frac{2}{v_1} = R_{AdS_2} \tag{5.5}$$

plus the terms with gauge fields. In equation for dilaton u_S (equivalent to $f = 0$) all dependence on v_2 vanishes and we obtain

$$e_1 e_2 = \frac{\alpha'}{4} \tag{5.6}$$

from which, using (3.8), we obtain result (1.1) for the entropy without ever needing to solve for v_2.

It is obvious that in the arguments above a precise form of the function A was completely arbitrary, moreover it could depend also on v_1 and e_i. One always gets (4.20,4.22-4.24,1.1) where the exact form of $A(v_1, v_2)$ only affects the solution for v_2 (which affects also dilaton u_S through (4.23.). As a consequence, any action which for the $AdS_2 \times S^{D-2}$ near horizon geometry has the form (5.3) will give the same result for the entropy of 2-charged black holes, i.e., (1.1).

The same conclusion does not hold for 4-charged and 8-charged black holes in $D = 4$. In these cases there is additional term inside the square brackets in (5.3) proportional to v_2^{-2} [26] and only for some special choices of the function A one would get the entropy equal to statistical entropy of string states.

6. Discussion

We have analysed solutions with $AdS_2 \times S^{D-2}$ geometry in the theories with actions of the Lovelock type which contain all extended Gauss-Bonnet densities. We expect that these solutions describe D-dimensional asymptoticaly flat two-charge black holes near the horizon. The idea was to check could Sen's results for $D = 4, 5$ [24] be generalized to all dimensions.

In the lowest order in α' and string these actions are equal to truncated tree level (in string coupling *and* tension α') low energy effective actions of the heterotic string compactified on $T^{9-D} \times S^1$, for which analytic black hole solutions having singular horizon with vanishing area, and thus also the entropy, were found [8, 9]. They are believed to correspond to perturbative half-BPS states of heterotic string, for which the statistical entropy (i.e., logarithm of the number of states) is asymptoticaly given by (1.1) [6, 7]. A reason for the discrepancy in the results for the entropy is that these black holes are small, in fact singular, with the curvature diverging on the horizon. This suggests that higher curvature terms in the action are important. On the other hand, dilaton field near

the horizon is large, which means that string coupling is small. One concludes that it is necessary to consider effective action which is tree level in string coupling, but *not* in α'.

Now, the small black holes we have analysed in this paper are obviously some deformations of these singular black hole solutions, but of course the question is have they anything at all with the black holes of heterotic string. We have shown that parameters which appear in the Lovelock type action can be uniquely chosen such that the black hole entropy matches statistical entropy of heterotic string states for *all D*. Moreover, this choice is nontrivial, in the sence that there is "one parameter for every couple of dimensions". Certainly, this matching could be just a coincidence. But, recently it was shown [26] that the same type of the action applied to 4-charge and 8-charge black holes in $D = 4$ produced the same results for the entropy, gauge field strengths and the axion-dilaton field as in the analyses based on supersymmetric action obtained by supersymmetrizing square of the Weyl tensor [27, 28]. Unfortunately, as corresponding supersymmetric formulations in $D > 4$ are unknown it is impossible to make simmilar comparison in our case. In spite of this, these results are hinting that there could be some connection between the Lovelock type action we used and the heterotic string on the tree level in the string coupling. If true, then our analysis shows how increasing the dimension D naturally introduces terms of higher and higher order in curvature ($[D/2]$-order in D dimensions).

Obviously, the action we used differs from the low energy effective action of heterotic string on $M_D \times T^{9-D} \times S^1$ background. Alhough we do not know the exact form of the latter, we do know that it should be supersymmetric and to contain additional higher curvature terms beside extended Gauss-Bonnet ones, and also higher derivative terms including gauge fields. Moreover, it is known that $\mathcal{L}_3^{GB}$ term is not present in the low energy effective action, and that some of the terms on the $m = 4$ level are proportional to the transcedental number $\zeta(3)$. This is in contrast to our results $\lambda_3 = 1/96$ and $\lambda_4 = 1/3 \cdot 2^9$. On the other hand, as noted in [24], the result $\lambda_2 = 1/8$ is exactly the value which appears in the low energy effective action of heterotic string [34, 35]. Curiously, $\lambda_3 = 1/96$ is exactly the value which appears in the case of the bosonic string. Here the following observation is important. Any term which is obtained by multiplying and contracting m Riemann and field strength tensors evaluated on $AdS_2 \times S^{D-2}$ background (3.2) gives just a linear combination of terms $v_1^{-k} v_2^{k-m}$, $k = 1, \ldots, m$ with some coefficients generally depending on D. Now, there is an infinite set of actions which are equivalent to ours when evaluated on this background, and even bigger one consisting of actions which lead to the more general form (5.3). It can be explicitly shown that one can use above this freedom to avoid disagreement with cubic and quartic higher curvature terms mentioned above. The question can supersymmetry be accomodated is opened. We shall present details elsewhere ([36]).

It is clear that the sole results from this paper and from [24, 26] are insufficient for making any strong claims. One can construct other actions leading to same results. As an illustration, let us consider an action obtained by adding higher curvature correction

$$\mathcal{L}_2 = \frac{1}{8} \left[(R_{\mu\nu\rho\sigma})^2 - (R_{\mu\nu})^2 \right] \tag{6.1}$$

to the leading term given by (2.2). This action does not belong to the type (5.3). It can be shown that it gives the same result for the entropy (1.1) as Lovelock type action for

2-charged black holes in $D = 4$ and $D = 5$, and for 4-charge and 8-charge black holes in $D = 4$. In fact, we could with this action repeat the analysis in [26] and obtain exactly the same solutions, including the atractor equations (4.11). Adding apropriate higher derivative terms (with coefficients not depending on D) it is posible to match the entropy of 2-charge black holes in any dimension D.

To conclude, the results in this paper support and extend to all dimensions Sen's suggestion of a possible role of Gauss-Bonnet densities in description of black holes in heterotic string theory. It would be interesting to relate our results to the anomaly cancelation arguments of [22, 23], especially concidering the topological origin of the extended Gauss-Bonnet densities. In any case, further analyses, including more examples, could either clarify this role, or to show that obtained agreement is accidental.

Acknowledgments

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