# A note on gauge fixing in Supergravity/Kac–Moody correspondences

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**Abstract:** We explain how to achieve the traceless gauge for the spatial part of the spin connection in the framework of the recently proposed correspondence between the (appropriately truncated) bosonic sectors of maximal supergravities and the 'geodesic' $\sigma$-model over $E_{10}/K(E_{10})$ at low levels. After making this gauge choice, the residual symmetries on both sides of this correspondence match precisely. The gauge choice also allows us to give a physical interpretation to the multiplicity of certain primitive affine null roots of $E_{10}$.

Recent work has established intriguing evidence for the realization of indefinite (sometimes hyperbolic) Kac–Moody algebras in supergravity and M-theory. In particular, for maximal $D = 11$ supergravity [1], there are now several proposals on how to realize these symmetries. The approach of [2] seeks a covariant implementation of the 'very-extended' Kac–Moody algebra $E_{11}$ via a non-linear realization directly in eleven dimensions (possibly augmented by further central charge coordinates [3]). By contrast, the approach of [4, 5], based on the hyperbolic Kac–Moody algebra $E_{10}$, has its roots in the classic BKL analysis of Einstein's equations in the vicinity of a space-like (cosmological) singularity [6], according to which the theory near the singularity is effectively described by a one-dimensional reduction, in which spatial gradients are neglected in comparison with time derivatives (for a

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recent review with many references, see [7]). A 'hybrid' approach, combining some of the features of [2, 4] has been developed in [8, 9, 10].

In spite of important conceptual differences between these approaches, a common feature is that they all require the tracelessness of the anholonomicity coefficients (or, equivalently, the spin connection) in order to match the (appropriately truncated) degrees of freedom between supergravity and the Kac–Moody $\sigma$-model. For $E_{11}$, the issue has been discussed in [11]. In this note, we explain how to realize this gauge in the $E_{10}$-based approach of [4], by making joint use of diffeomorphisms and local Lorentz transformations in such a way that, at the end of the gauge fixing procedure, the residual symmetries on both sides of the correspondence match precisely. Our arguments underline a point already made in [12] concerning the importance of gauge fixing *before* making the identification between the supergravity theory and the 'geodesic' Kac–Moody $\sigma$-model, both at the kinematical and the dynamical level. The traceless gauge choice also resolves a puzzle concerning the multiplicity of the affine null root (=8 for $E_{10}$) and its images under permutations of the spatial coordinates; namely, we will show that this multiplicity indeed coincides with the number of physically relevant degrees of freedom for each choice of null root. In the final section, we comment on related issues in the context of the $E_{11}$ proposal of [2], and on the extension of the present results to the fermionic sector.

Let us first summarize the basic conjecture and results of [4]. As shown in [7], the relevant equations of motion simplify near a space-like singularity in the sense that the degrees of freedom can be divided into 'active' ones (the diagonal metric components), and 'passive' ones (off-diagonal metric and various matter degrees of freedom) which freeze near the singularity. The resulting dynamics is thus described by a one-dimensional reduction of the higher dimensional field equations (i.e. purely time-dependent equations at a fixed, but arbitrary spatial point $\mathbf{x}_0$) which receives effective corrections from the passive degrees of freedom (in lowest order in the form of 'walls' leading

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to a cosmological billiards).[1] In the context of supergravity, the possible relevance of a reduction to one dimension, and the possible appearance of $E_{10}$ in this reduction, had already been foreseen in [13], but one crucial difference here is that the dependence on the spatial coordinates is conjectured to re-emerge via a gradient expansion, which gets linked to a level expansion (or height expansion) on the $\sigma$-model side. More precisely, the correspondence is made between the purely $t$-dependent $\sigma$-model degrees of freedom of the Kac–Moody $\sigma$-model, and the time-dependent supergravity fields and their (so far only first order) spatial gradients *at a fixed spatial point* $\mathbf{x}_0$.

We now explain the successive gauge choices required for the correspondence of [4], stressing the residual symmetries at every step.

**Pseudo-Gaussian gauge**: The analysis of [4] proceeds from a space-time metric in the zero shift (or pseudo-Gaussian) gauge[2]

$$ds^2 = -N^2 dt^2 + g_{mn}dx^m dx^n \quad , \qquad N(t,\mathbf{x}) = n(t)\sqrt{g(t,\mathbf{x})} \tag{1}$$

where indices $m, n, \cdots = 1, \ldots, 10$ label the spatial coordinates, and $g$ denotes the determinant of the spatial metric, and where the purely time-dependent lapse $n(t)$ is to be identified with the one of the geodesic Kac–Moody $\sigma$-model, and hence left free. The above gauge is supposed to be valid in a tubular neighborhood of the worldline parametrized by $\{(t,\mathbf{x}_0) \,|\, t > 0\}$ (in comoving coordinates). After making this choice, the metric (1) is left invariant by *separate* reparametrizations of the time- and space coordinates, respectively, that is, $t \to t'(t)$ and $\mathbf{x} \to \mathbf{x}'(\mathbf{x})$, but coordinate changes mixing space- and time coordinates are disallowed. The pure space reparametrizations are assumed to leave the point $\mathbf{x}_0$ invariant (and hence the worldline).

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[1]It has already been noted before that this mechanism offers new possibilities for 'emergent spacetime' scenarios, as the dependence on the spatial degrees of freedom here is thought to 'emerge out of' (or 'disappear into') the spacelike singularity.

[2]For clarity, we will stick mostly to $D = 11$ supergravity, but the argument remains the same for other models of interest in various space-time dimensions $D \leq 11$.

**Vielbein gauge**: Next we make partial use of the local Lorentz group to bring the elfbein which gives rise to (1) into block-diagonal form. With a (1+10) split of the indices we demand the form:

$$E_M{}^A = \left( \begin{array}{c|c} N & 0 \\ \hline 0 & e_m{}^a \end{array} \right). \tag{2}$$

The local space-time Lorentz group $SO(1,10)$ is thereby broken to its rotation subgroup $SO(10)$; that is, (2) still admits *space-time dependent* spatial rotations $\Lambda_{ab}(t,\mathbf{x})$ as a residual symmetry.

**Traceless spin connection gauge**: We now wish to exploit this remaining rotation symmetry to set

$$\Omega_{ab\,b}(t,\mathbf{x}) = 0 \quad \Leftrightarrow \quad \omega_{b\,ba}(t,\mathbf{x}) = 0 \tag{3}$$

where

$$\begin{aligned}
\Omega_{ab\,c} &:= e_a{}^m e_b{}^n (\partial_m e_{nc} - \partial_n e_{mc}) = -\Omega_{ba\,c} \\
\omega_{a\,bc} &:= \frac{1}{2}\left(\Omega_{ab\,c} + \Omega_{ca\,b} - \Omega_{bc\,a}\right) = -\omega_{a\,cb}
\end{aligned} \tag{4}$$

are the spatial components of the coefficients of anholonomicity, and the spin connection, respectively. Relation (3) is supposed to hold in the same tubular neighborhood as (1), and implies the vanishing of the trace and all its spatial gradients along the the world line $(t,\mathbf{x}_0)$. The necessity of the tracelessness condition arises from the appearance of a representation for the magnetic dual of the graviton [14, 15, 16, 2, 17, 11] at level $\ell = 3$ in a level decomposition of $E_{10}$ under its $A_9 = SL(10)$ subgroup [4]. The associated tensor of mixed symmetry is related via the correspondence of ref. [4] to this dual graviton by

$$P_{a_0|a_1...a_8} = \frac{3}{2}N\epsilon_{a_1...a_8bc}\Omega_{bc\,a_0}. \tag{5}$$

However, from the level decomposition it follows that this representation is subject to the irreducibility constraint

$$P_{[a_0|a_1...a_8]} = 0 \quad \Longleftrightarrow \quad \Omega_{ab\,b} = \omega_{b\,ba} = 0, \tag{6}$$

which, as indicated, is equivalent under the dictionary to the traceless gauge (3). Inspection of the available tables of higher level representations [18] reveals the absence of such a trace representation at low levels; the relevant representation (000000001) appears only at level $\ell = 13$, with outer multiplicity equal to 22. Similar comments apply to representations corresponding to the spatial gradients of the trace.

Because both $\Omega_{abc}$ and $\omega_{abc}$ transform as scalars under coordinate transformations, it is clear that diffeomorphisms are of no further use at this point; in particular, a spatially constant $\Omega_{abc}$ (with or without trace, e.g. Bianchi cosmologies) remains invariant under relabeling of the coordinates. This is analogous to the traceless gauge $\Gamma^n{}_{nm} = 0$ for the Christoffel symbol, which transforms as a scalar under local Lorentz transformations, whence the role of diffeomorphisms and the local Lorentz group is interchanged. Therefore, given a spatial spin connection $\omega_{abc}$, the problem reduces to solving the equation

$$\omega'_{bba} = \partial_b U_{ab} + U_{ab}\omega_{ccb} = 0 \tag{7}$$

in terms of the spatial rotation matrix $U_{ab}(t, \mathbf{x}) \in SO(10)$. In infinitesimal form (with $V_a \equiv \omega_{bba}$ small, and $\partial_b U_{ab} = \partial_b \Lambda_{ab}$), this equation becomes

$$\partial_b \Lambda_{ba} = V_a. \tag{8}$$

Making the ansatz $\Lambda_{ab} = \partial_a v_b - \partial_b v_a$, and noticing that $v_a$ can be chosen divergence-free by shifting $v_a \to v_a + \partial_a v$ with a suitable $v = v(t, \mathbf{x})$, we arrive at a continuous set of Poisson equations (one for each $t$)

$$\triangle v_a(t, \mathbf{x}) = V_a(t, \mathbf{x}) \ ; \tag{9}$$

where $\triangle \equiv \partial_a \partial_a$ is the 10-dimensional spatial Laplacian. The set of equations (9) are to be solved in some tubular neighborhood of the worldline $(t, \mathbf{x}_0)$ with appropriate boundary conditions. The known local existence of solutions to the Poisson equation guarantees that the gauge (3) can be chosen; moreover the required $SO(10)$ rotation only fixes the *space-dependent*

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part of the $SO(10)$ transformations since it follows from (7) that $\omega_{bba} = 0$ is not changed by purely *time-dependent $SO(10)$* rotations.

**Summary of residual symmetries**: Having achieved the gauge choices (1), (2) and (3) we are left with the following three residual symmetries on the supergravity side, which can now be directly identified with the residual symmetries of the $E_{10}/K(E_{10})$ $\sigma$-model in the level decomposition under $A_9$:

($i$) Reparametrizations of the time parameter $t \rightarrow t'(t)$, where the time-dependent lapse $n(t)$ in (1) is identified with the lapse function of the $E_{10}/K(E_{10})$ $\sigma$-model.

($ii$) Purely space-dependent coordinate transformations (leaving $\mathbf{x_0}$ inert) that can be expanded around $\mathbf{x}_0$ according to

$$\xi^m(\mathbf{x}) = \xi^m{}_n(x^n - x_0^n) + \ldots \tag{10}$$

The first order term $\xi^m{}_n$ realizes the $GL(10)$ subgroup of the (global) $E_{10}$. The higher order terms in this expansion are related to higher order spatial gradients of the various fields, which are expected to correspond to higher level representations in the decomposition of $E_{10}$ under its $A_9$ subalgebra.[3]

($iii$) Eq. (3) is left invariant by purely time-dependent spatial rotations $\Lambda_{ab} = \Lambda_{ab}(t)$. The resulting group $SO(10)$ can be identified with the subgroup of $t$-dependent $SO(10)$ rotations within the local 'R symmetry' group $K(E_{10})$ on the $\sigma$-model side, which is the finite dimensional residual invariance left by fixing the triangular gauge for all fields except in the level $\ell = 0$ sector.

In summary, *we have a precise matching not only of the degrees of freedom and equations of motion up to level $\ell = 3$, but also of the residual symmetries*

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[3]The relevant $E_{10}$ transformations in the $\sigma$-model will be accompanied by local (in time) compensating $K(E_{10})$ transformations. This is analogous to fixing a triangular gauge of the spatial vielbein $e_m{}^a$ in (2).

*on both sides of the correspondence.*

Analogous results hold for the $D_9$ and $A_8 \times A_1$ decompositions [19, 20] of $E_{10}$: one similarly finds no trace representations at low levels. For the $A_8 \times A_1$ decomposition (corresponding to IIB, see [20]) this is straightforward since one deals with the dual of the graviton over $A_8 = SL(9)$ instead of $SL(10)$, and the irreducibility constraint (6) still implies that one has to fix the space-dependent rotations to arrive at the traceless gauge.

For $D_9 = SO(9,9)$ (related to massive IIA supergravity in [19]) the situation is slightly more involved since the relevant tensor containing the dual of the graviton is now contained in an antisymmetric three-form representation of $SO(9,9)$ (at $D_9$ level $\ell = 2$), which we denote by $P_{IJK}$ (with $I, J, K = 1, \ldots, 18$). Seen from the compact subgroup $SO(9) \times \overline{SO(9)} \subset SO(9,9)$ there are four different components that need to be distinguished ($i, j = 1, \ldots, 9 \, ; \, \bar{\imath}, \bar{\jmath} = 10, \ldots, 18$, cf. [19])

$$
\begin{array}{ccccc}
 & P_{ijk} & P_{\bar{\imath}jk} & P_{\bar{\imath}\bar{\jmath}k} & P_{\bar{\imath}\bar{\jmath}\bar{k}} \\
SO(9)_{\mathrm{diag}} & \square\!\square\!\square \text{ (column)} & \square\square\,\square \oplus \square & \square\square\,\square \oplus \square & \square\!\square\!\square \text{ (column)}
\end{array}
\tag{11}
$$

We have indicated the structure of these four tensors under the diagonal rotation group $SO(9)_{\mathrm{diag}} \subset SO(9) \times \overline{SO(9)}$. We see that those tensors which allow for the mixed symmetry which is required for (part of) the dual graviton also allow for the presence of a vector representation. The nine-dimensional trace $\sum_{b=1}^{9} \omega_{bba}$ transforms in a vector representation of $SO(9)_{\mathrm{diag}}$ and therefore it would seem unnecessary to choose a gauge for it. However, this reasoning overlooks the dual field for the type IIA dilaton gradient $\partial_a \phi$ which also transforms as a vector[4]. Now the appropriate gauge condition relates the two vectors

$$
\frac{1}{2}\partial_a \phi + \sum_{b=1}^{9} \omega_{bba} = 0
\tag{12}
$$

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[4]$\phi$ is the scalar field defined in [19] and not strictly identical to the standard IIA dilaton.

Interestingly, this is precisely what the original gauge condition summed over ten space directions

$$0 = \sum_{b=1}^{10} \omega_{b\,ba} = \omega_{10\,10a} + \sum_{b=1}^{9} \omega_{b\,ba} \tag{13}$$

translates into if one follows through the redefinitions of [19]. This will be discussed in more detail in [21].

In both cases we see that the matching between supergravity and the $E_{10}/K(E_{10})$ $\sigma$-model is possible only if (3) is satisfied and all gauges are fixed so that the residual symmetries agree.

**Interpretation of root multiplicity**: The significance and proper physical interpretation of the imaginary roots of $E_{10}$ and their multiplicities in the present context is far from understood[5] (recall that, generically, imaginary roots $\alpha$ are degenerate with exponentially growing multiplicities $\text{mult}(\alpha) > 1$). The above choice of gauge now allows us to extend the matching (and hence the 'dictionary') beyond real roots, and to give a physical interpretation at least for the fact that lightlike (null) roots are associated with root multiplicity $> 1$. Namely, the roots associated with latter fall into two classes [7]. First, there are the gravitational roots (giving rise to 'gravitational walls') associated with those components $\Omega_{bc\,a}$, for which the indices $a, b, c$ are all different: these correspond to level-3 roots $\alpha_{abc}$ defined by the wall forms (cf. [7], section 6.2)

$$\alpha_{abc}(\beta) = 2\beta^a + \sum_{e \neq b,c} \beta^e \tag{14}$$

and are real: $\alpha_{abc}^2 = 2$. The corresponding components of the dual field $P_{a_0|a_1\ldots a_8}$ are the ones where $a_0$ is equal to one of the indices $a_1, \ldots, a_8$.

In addition, [7] identified ten subleading gravitational walls associated with ten null roots, designated as $\mu_a$ for $a = 1, \ldots, 10$, cf. eqn. (6.16) there,

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[5]Other ideas on the physical rôle of imaginary roots can be found in [22].

and defined by the wall forms

$$\mu_a(\beta) = \sum_{e \neq a} \beta^e \tag{15}$$

These ten null roots (for $a = 1, \ldots, 10$) can all be obtained by $\mathfrak{sl}(10)$ Weyl reflections (or, equivalently, by permuting the spatial coordinates) from the primitive (i.e. lowest height) null root at height 30, which has $\delta^2 = 0$, $\text{mult}(\delta) = 8$ and is identical to the null root of the affine subalgebra $\mathfrak{e}_9 \subset \mathfrak{e}_{10}$ (in the notation of [7], we have $\delta = \mu_1$). This null root, and its images under the $\mathfrak{sl}(10)$ Weyl group, are the only imaginary roots appearing on levels $\ell \leq 3$ in the $A_9$ decomposition. The associated components of the dual field $P_{a_0|a_1\ldots a_8}$ belonging to these null roots are the ones for which the indices $a_0, \ldots, a_8$ are all distinct. Using the correspondence we can now give a physical interpretation to the multiplicity $\text{mult}(\delta)$. Since the indices on $P_{a_0|a_1\ldots a_8}$ are all different, two indices on the dual coefficient of anholonomicity $\Omega_{abc}$ must be equal, i.e. we must consider the components[6] $\Omega_{abb}$. As shown in [7], these components are then all associated with the null root $\mu_a$, and it would thus appear that we have nine possible values for $b$. However, thanks to our gauge choice (3), there is now one linear relation $\sum_b \Omega_{abb} = 0$, whence the number of independent field components associated to each null root $\mu_a$ is only eight — in agreement with the root multiplicity $\text{mult}(\delta) = 8$!

How are these statements mirrored in $E_{11}$ [2, 23, 24, 11]? At least locally, the traceless gauge $\Omega_{AB}{}^B = 0$ (contractions now to be taken with the Minkowski metric in eleven dimensions) can be reached by exploiting the full local Lorentz group $SO(1,10)$ [11]. The difference is now that, after gauge fixing, the local Lorentz group has been 'used up' completely, and there remains no symmetry to identify with the $SO(1,10)$ subgroup of the local group $K(E_{11})$, while the traceless gauge is still compatible with full 11-dimensional diffeomorphism invariance. A second difference is that a counting argument analogous to the one given above would suggest that there are now *nine*

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[6]We temporarily suspend the summation convention for this discussion, *i.e.* there is *no* summation on $b$ here!

independent components in $\Omega_{AB}{}^B = 0$ for each $A$ (*no* summation on $B$), whereas the multiplicity of the associated null root $\delta$ remains the same ($= 8$) when $\delta$ is considered as a root of $E_{11}$. As also mentioned in [11], instead of discarding the trace (in order to retain full Lorentz invariance), one might look for a trace representation at higher levels. Inspection of the tables [18] reveals that the relevant representation (0000000010) does appear in the $A_{10}$ decomposition of $E_{11}$, but only at level $\ell = 14$, and with outer multiplicity 491.

**Supersymmetric generalization**: Similar considerations apply to the supersymmetric version of the $E_{10}$ $\sigma$-model [25, 26]. The Kac–Moody model allows only a local supersymmetry with parameter $\epsilon(t)$ depending only on time.[7] Therefore, we should require on the supergravity side, a similar gauge conditon on the supergravity fermions involving spatial gradients, which reduces $\epsilon(t, \mathbf{x})$ to purely time-dependent supersymmetry transformations with parameter $\epsilon(t)$. The precise form of this condition is presently unknown but will be schematically of the form $\partial^m \Psi_m = 0$ (where $\Psi_m$ denotes the spatial components of the gravitino). We note also that one can consider a completely gauge-fixed version of the model where one chooses the lapse $n(t) = 1$ which is reflected in the supersymmetric partner constraint $\psi_0 - \Gamma_0 \Gamma^a \psi_a = 0$ [25].

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[7]Of course, one envisages an analogue of the gradient conjecture where the space dependence of the $\epsilon(t, \mathbf{x})$ is encoded in some 'higher level' components of an infinite-dimensional spinor of $K(E_{10})$. Here, we discuss only the truncation to the unfaithful spinor representation studied in [25, 26].

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