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General $\mathcal{N} = 1$ Supersymmetric Flux Vacua of (Massive) Type IIA String Theory

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We derive conditions for the existence of four-dimensional $\mathcal{N}=1$ supersymmetric flux vacua of massive type IIA string theory with general supergravity fluxes turned on. For an SU(3) singlet Killing spinor, we show that such flux vacua exist only when the internal geometry is nearly-Kähler. The geometry is not warped, all the allowed fluxes are proportional to the mass parameter and the dilaton is fixed by a ratio of (quantized) fluxes. The four-dimensional cosmological constant, while negative, becomes small in the vacuum with the weak string coupling.

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Insights into four-dimensional $\mathcal{N} = 1$ supersymmetric vacua of M- and string theory with non-Abelian gauge sectors and chiral matter are of phenomenological interest: they may provide an important link between the M-theory and particle physics, describing the Standard Model and/or Grand Unified models. In particular, intersecting D6-brane constructions [1, 2] on Type IIA orientifolds provide a intriguing avenue to construct semi-realistic particle physics. [For a recent review see [3] and references therein; for first examples of $\mathcal{N}=1$ supersymmetric quasi-realistic models see [4, 5]]

On the other hand more general compactifications of string theory in the presence of supergravity fluxes can lift the continuous moduli space of string vacua, while still preserving some supersymmetries and hence might be an essential input in realistic compactifications of string/M-theory. These fluxes generate a back reaction onto the geometry, which in the simplest case results in a non-trivial warping, but in general the internal space ceases to be Calabi-Yau. Unfortunately, the explicit metric and fluxes are known only for very few examples.

One of the important phenomenological goals in the constructions of general N=1 supersymmetric vacua is the implementation of (supergravity) flux compactifications, which would yield the moduli stabilization, along with the (probe) D-brane configurations, which would yield the non-Abelian gauge group structure and chiral matter in four-dimensions. On one hand, a better understanding of the resulting internal geometry can be achieved by relating the fluxes to specific non-trivial torsion components, which can be classified with respect to the structure group of the internal manifold, for a recent review see [6]. The Killing spinor has to be a singlet under the structure group which for a six-dimensional space is at most SU(3). [However non-zero fluxes often require a reduction of the structure group down to SU(2).] It is this maximal SU(3) structure group that plays an important role in the construction of D-brane configurations with chiral matter. In particular, within Type IIA string theory the chiral supermultiplets appear at the intersections of two D6-branes, whose three-cycles are related to each other by an SU(3) rotations [7] and supersymmetry is preserved if the Killing spinor is a singlet under the SU(3) rotations.

The goal of this letter is to derive within massive type IIA string theory explicit constraints for the most general four-dimensional N=1 supersymmetric flux compactification whose internal space maintains SU(3) structure. These results thus provide an important stepping stone toward implementation of chiral theories from D-brane configurations along with the moduli stabilization within Type IIA theory. In particular we show that such vacua exist for massive Type IIA string theory only when the internal geometry is nearly-Kähler. The geometry is not warped, the allowed flux components are related to the mass parameter and the dilaton is fixed by a ratio of supergravity fluxes. The four-dimensional cosmological constant is negative, however in the weak coupling limit it becomes arbitrarily small. We also give explicit examples with nearly-Kähler internal geometry. In this letter we primarily summarize results, technical details as well as the analysis of SU(2) structures shall be presented in [8], see [9].

In massive IIA string theory [10], the NS-NS 2-form and the RR 1-form potential combine into a gauge invariant (massive) 2-form F which fixes also the Chern-Simons part of the RR 4-form G

$$F = mB + dA \ , \ G = dC + \frac{3}{m} F \wedge F \tag{1}$$

were C is the RR 3-form potential. These forms are not closed but

$$dF = mH \ , \ dG = 6F \wedge H \ . \tag{2}$$

In the massless case, F and G are independent fields, where G is still not closed but $F = F^{(0)} = dA$ is exact. Unbroken supersymmetry requires the existence of

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at least one Killing spinor ϵ, which is fixed by the vanishing of the fermionic supersymmetry transformations. These variations have been first setup for massive type IIA supergravity in Einstein frame [10], but we will use the string frame where the fermionic variations read

$$\begin{aligned}\delta\psi_M &= \big\{D_M - \tfrac{1}{4}H_M\Gamma_{11} - \tfrac{1}{4}\,e^{\phi}m\,\Gamma_M \\ &\quad -\tfrac{1}{8}e^{\phi}\big[(\Gamma_M F - 4F_M)\Gamma_{11} - \tfrac{1}{12}(\Gamma_M G - 12G_M)\big]\big\}\epsilon \\ \delta\lambda &= \big\{\tfrac{1}{2}\partial\phi - \tfrac{1}{12}H\Gamma_{11} + e^{\phi}\big[\tfrac{5}{4}m - \tfrac{3}{8}F\Gamma_{11} - \tfrac{1}{96}G\big]\big\}\epsilon\end{aligned} \tag{3}$$

$[\partial \equiv \Gamma^M\partial_M,\ F \equiv F_{MN}\Gamma^{MN}]$. Apart from the differential forms that we introduced already, the mass parameter is denoted by m and ϕ is the dilaton. In type IIA string theory, the Killing spinor ϵ is Majorana and can be decomposed into two Majorana-Weyl spinors of opposite chirality. The massless case can be lifted to M-theory and this Majorana spinor becomes the 11-dimensional Killing spinor.

We are interested in the compactifications to a 4-d spacetime that is either flat or anti deSitter, i.e. up to warping the 10-d space time factorizes $M_{10} = X_{1,3}\times Y_6$ and we write the metric as

$$ds^2 = e^{-2V(y)}\,ds_4^{(AdS)} + e^{2U(y)}\,h_{mn}(y)\,dy^m dy^n\ . \tag{4}$$

where h_{mn} is the metric on Y_6 and the warp factors may depend only on the coordinates of the internal space. Consistent with this metric Ansatz is the assumption that the fluxes associated with the forms F and H have non-zero components only in the internal space Y_6 whereas G may have in addition a Freud-Rubin parameter λ:

$$\begin{aligned}F &= F_{mn}dy^m\wedge dy^n\ ,\\ H &= H_{mnp}dy^m\wedge dy^n\wedge dy^p\ ,\\ G &= \lambda\,dx^0\wedge dx^1\wedge dx^2\wedge dx^3\\ &\quad +G_{mnpq}\,dy^m\wedge dy^n\wedge dy^p\wedge dy^q\ .\end{aligned} \tag{5}$$

Note, all forms as well as the warp factor and the dilaton are in general functions of internal coordinates y^m.

Keeping four supercharges unbroken, the 4-d vacuum should allow for a single Majorana or Weyl (Killing) spinor and the geometry of the 6-d internal space for a single (Weyl) Killing spinor. This internal spinor has to be a singlet of the structure group $G\subseteq \mathrm{SU}(3)\subset \mathrm{SO}(6)$: for the case $G=$SU(3), only a single 6-dimensional Weyl Killing spinor can exist whereas for $G\subset \mathrm{SU}(3)$ more singlet spinors are possible. Actually, a reduced structure group is equivalent to the existence of vector field(s) on Y_6 and puts severe constraints on the geometry of the internal space. If there are no fluxes, the Killing spinors are covariantly constant and the structure group determines the holonomy of the space. On the other hand, non-trivial fluxes, which decompose into representations of the structure group, act as (intrinsic) torsion components and the holonomy group of Y_6 is in general unrestricted.

We decompose the 10-d spinor as

$$\epsilon = \theta\otimes\eta + \theta^\star\otimes\eta^\star \tag{6}$$

where θ is the 4-d spinor and the 6-d spinor reads

$$\eta = \frac{1}{\sqrt{2}}e^{\alpha+i\beta}\,(\mathbb{I}-\gamma^7)\,\eta_0 \tag{7}$$

with α and β as real functions and η_0 being a constant spinor. Since we want to keep this spinor as an SU(3) singlet, it has to obey the projector conditions

$$\begin{aligned}(\gamma_m - iJ_{mn}\gamma^n)\,\eta &= 0\\ (\gamma_{mn} + i\,J_{mn})\,\eta &= \tfrac{i}{2}\,e^{2i\beta}\Omega_{mnp}\gamma^p\,\eta^\star\ ,\\ (\gamma_{mnp} + 3iJ_{[mn}\gamma_{p]})\,\eta &= i\,e^{2i\beta}\Omega_{mnp}\eta^\star\end{aligned} \tag{8}$$

with the almost complex structure and holomorphic 3-form defined by

$$\eta\,\gamma_{mn}\eta^\star = i\,e^{2\alpha}J_{mn}\ ,\ \ \eta\gamma_{mnp}\eta = i\,e^{2(\alpha+i\beta)}\Omega_{mnp} \tag{9}$$

so that $e^{2\alpha} = \eta^\star\eta$. The phase β comes always together with Ω and to keep the notation simple, we will drop this phase and will only comment on it where necessary. Note, these are the only differential forms that can be constructed from a single chiral spinor and for non-zero fluxes they are in general *not* covariantly constant nor closed and this failure is related to non-vanishing intrinsic torsion components. Following the literature, see [11, 12, 13, 14, 15], one introduces five classes $\mathcal{W}^i$ by

$$\begin{aligned}dJ &= \tfrac{3i}{4}\,(\mathcal{W}_1\bar\Omega - \bar{\mathcal{W}}_1\Omega) + \mathcal{W}_3 + J\wedge\mathcal{W}_4\ ,\\ d\Omega &= \mathcal{W}_1 J\wedge J + J\wedge\mathcal{W}_2 + \Omega\wedge\mathcal{W}_5\end{aligned} \tag{10}$$

with the constraints: $J\wedge J\wedge\mathcal{W}_2 = J\wedge\mathcal{W}_3 = \Omega\wedge\mathcal{W}_3 = 0$. Depending on which torsion components are non-zero, one can classify the geometry of the internal space. E.g., if only $\mathcal{W}_1\neq 0$ the space is called nearly Kähler, for $\mathcal{W}_2\neq 0$ almost Kähler, the space is complex if $\mathcal{W}_1 = \mathcal{W}_2 = 0$ and it is Kähler if only $\mathcal{W}_5\neq 0$. Although it is not possible to introduce complex coordinates globally, one can nevertheless employ the holomorphic projector: $\frac{1}{2}(\mathbb{I}\pm iJ)$ to distinguish between holomorphic and anti-holomorphic indices locally. This is useful to decide whether flux components can cancel or not.

The 4-d spinor can be Weyl or Majorana. A detailed analysis shows for the Weyl case [8], that the mass and all RR-fields have to vanish and only the H-flux can be non-zero; see also [14, 16]. So, we shall consider a 4-d Majorana spinor implying that the 10-d spinor is not chiral, which is generic for an type IIA vacuum. Hence,

$$e^{i\beta}\theta \equiv \hat\theta = \hat\theta^\star\ , \tag{11}$$

where the phase β describes the mixing of the two chiralities of the 10-d spinor (6); for $\beta = 0$ both chiralities are on equal footing[1]. Since we allow for an external AdS-space, this spinor obeys

$$\nabla_\mu\,\theta = \hat\gamma_\mu(W_1 + i\hat\gamma^5 W_2)\,\theta \tag{12}$$

[1] In order to solve the Killing spinor equations we had to identified β with the phase factor for the holomorphic 3-form in (9).

where the 4-d γ-matrices are hatted. Obviously, upon dimensional reduction $W_{1/2}$ (not to be confused with the torsion classes $\mathcal{W}_i$) will fix the real and imaginary part of the superpotential and its absolute value acts as cosmological constant yielding an anti deSitter vacuum.

Now, we have to separate all terms containing θ from the terms proportional to $\hat\gamma^5\theta$. The gravitino variation (3) is spited into an external and internal part and together with dilatino variation, each supersymmetry variation yields a constraint equation on the fluxes and one differential equation. Collecting all terms of the same 6-d chirality, the constraint equations can be written as

$$\begin{aligned} \tfrac{1}{4}e^{\phi}(m - \tfrac{1}{24}e^{-4U}G)\eta &= e^{V}\,W_1\eta \\ e^{\phi}(15m - \tfrac{1}{8}e^{-4U}G)\eta &= e^{-3U}\,H\,\eta^\star \\ 4\,e^{U+V}W_1\gamma_m\eta + \tfrac{1}{2}e^{\phi-3U}G_m\eta &= e^{-2U}\,H_m\;\eta^\star \end{aligned} \tag{13}$$

whereas for the differential equations we derive

$$\begin{aligned} &e^{-U}\partial V\eta = \tfrac{1}{4}(e^{\phi}[e^{-2U}F + \tfrac{i}{6}e^{4V}\lambda] + 2iW_2e^{V})\eta^\star \\ &e^{-U}\partial\phi\,\eta = -\tfrac{3}{4}e^{\phi}(e^{-2U}F + \tfrac{i}{36}e^{4V}\,\lambda)\,\eta^\star \\ &e^{-U}\hat\nabla_m\eta = \tfrac{1}{2}e^{\phi}(e^{-2U}F_m + \tfrac{i}{16}e^{4V}\,\lambda\,\gamma_m\,)\eta^\star \\ &\qquad\qquad + ie^{V}\,W_2\,\gamma_m\eta^\star \end{aligned} \tag{14}$$

where: $\hat\nabla_m \equiv \nabla_m + \frac{1}{2}\gamma_m{}^n\partial_n(U+V) + \frac{1}{2}\partial_m U$. Recall, the indices of G, F etc., are contracted with the γ-matrices, but using the relations (8) it follows from (13) that the only non-zero components of the fluxes are given by

$$G = G_0\,J\wedge J\ ,\ \ H = H_0\,\mathrm{Im}\Omega \tag{15}$$

which are the singlet components under an SU(3) decomposition. In addition, we infer

$$W_1 = 0\ ,\ \ G_0 = -8\,e^{4U}m\ ,\ \ H_0 = 12\,e^{3U+\phi}m \tag{16}$$

Eq. (14) yield

$$F_0 = -\frac{1}{36}\lambda e^{2U+4V}\ ,\ \ W_2 = -\frac{5}{72}\lambda\,e^{\phi+3V} \tag{17}$$

and moreover

$$\begin{aligned} 2\nabla_m\hat\eta &= -\gamma_m{}^n\partial_n(U+V)\,\eta + e^{\phi-U}F_{mn}\gamma^n\,\hat\eta^\star \\ e^{U}\partial_m V &= -\tfrac{i}{8}e^{\phi}\,\Omega_{mpq}F^{pq} \end{aligned} \tag{18}$$

with $\hat\eta = e^{\frac{U}{2}}\eta$. Next, the 10-d equations of motion for G and H imply

$$d(^\star G) \sim G\wedge H\ ,\quad d(e^{-2\phi}\,{}^\star H) \sim G\wedge G\ . \tag{19}$$

The rhs is non-zero only if the Freud-Rubin parameter is non-zero and when projected onto the internal components we find from (15): ${}^\star G \sim J$, ${}^\star H \sim \mathrm{Re}\Omega$. Going back to our torsion classes as introduced in (10), the equations of motion can be solved only if

$$\mathcal{W}_2 = \mathcal{W}_3 = 0\ .$$

Next, by a proper conformal rescaling of the internal manifold, one can eliminate $\mathcal{W}_4$ (see [12]), which amounts to a proper choice for the warp factor U. Using the differential equations (18) to calculate dJ and $d\Omega$ we find that this is consistent if: $U = V$. This requirement would also be consistent with the Bianchi identity $dG = 6F\wedge G$, but does not solve the equations of motion (19) (which would require $U = -2V$). The only solution that we found requires

$$d\phi = dV = dU = 0\ ,\quad F = F_0 J \tag{20}$$

with constant G_0, H_0 given by (16) and F_0 by (17). Note, U and V can be eliminated by an rescaling of the external and internal coordinates combined by a rescaling of the cosmological constant $W_2 \sim \lambda$. Hence, setting $U = V = 0$ from the very beginning, we find that the dilaton is fixed by the ratio of the (quantized) fluxes

$$e^{\phi} = -\frac{2\,H_0}{3\,G_0}\ . \tag{21}$$

The differential equation for the spinor becomes finally

$$\nabla_m\eta = -\frac{i}{72}e^{\phi}\lambda\gamma_m\eta^\star \tag{22}$$

which yields (with $\beta = \alpha = 0$)

$$\mathcal{W}_1 = \frac{\lambda}{54}e^{\phi}\ ,\quad \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0\ .$$

This identifies the internal space as a nearly Kähler manifold, which is Einstein but neither complex nor Kähler! In fact, $dJ = -\frac{\lambda}{36}e^{\phi}\mathrm{Im}\Omega$, $d\,\mathrm{Re}\Omega \sim J\wedge J$ which ensures $dF = mH$, $dH = 0$ and $d^\star H \sim G\wedge G$. This also fixes the Freud-Rubin parameter in terms of the mass:

$$\frac{\lambda}{72} = \sqrt{3}m\ .$$

In the limit of vanishing mass, our solution becomes trivial, i.e. all fluxes vanish and the internal space becomes Calabi-Yau. There is however no direct limit to massless configurations related to intersecting D6-branes, which have $\mathcal{W}_1 = \mathcal{W}_3 = 0$, but $\mathcal{W}_2, \mathcal{W}_4, \mathcal{W}_5 \neq 0$ [17]. Note, the external space is anti deSitter with the cosmological constant given by the Freud-Rubin parameter, which in turn is related to the mass. The differential equation of the spinor can be solved by a constant spinor if one imposes first order differential equations on the Vielbeine e^n

$$\omega^{pq}J_{pq} = 0\ ,\quad \omega^{pq}\Omega_{pq}{}^n = -\frac{\lambda}{9}e^{\phi}e^n \tag{23}$$

where $\omega^{pq} \equiv \omega^{pq}_m dy^m$ are the spin-connection 1-forms. Therefore, 6-dimensional nearly Kähler spaces can be seen as a weak SU(3)-holonomy space, which as Calabi-Yau spaces have, e.g., a vanishing first Chern class. Their close relationship to special holonomy spaces comes also due to the fact that the cone over nearly Kähler 6-manifolds become a G_2-holonomy spaces [18], defined by a covariantly constant spinor. This can be verified by multiplying (23) with J from the right and identifying the rhs as the spin connection ω^{7n}. Note, the spin connection 1-form of G_2 holonomy spaces satisfy $\omega^{MN}\varphi_{MNP} = 0$, where φ_{MNP} is the G_2-invariant 3-form. It is hence straightforward to construct nearly Kähler spaces starting from G_2 holonomy spaces and the almost Kähler form, that defines our vacuum completely, is then given by $J_{mn} = \varphi_{mn7}$. Let us end with a discussion of some coset examples; for more details we refer to [19, 20].

(*i*) $\frac{G_2}{SU(3)} \simeq S_6$ This is a standard example of a nearly Kähler space, where the cone becomes the flat 7-d space. Note, one can express the 6-sphere also by the coset $SO(7)/SO(6)$ which however breaks supersymmetry.

(*ii*) $\frac{Sp(2)}{Sp(1)\times U(1)} \simeq \frac{S_7}{U(1)} \simeq \mathbb{C}P_3$ The corresponding G_2-holonomy space is an $\mathbb{R}_3$ bundle over S_4 and hence it is the $SO(5)$ invariant metric of $\mathbb{C}P_3$ appearing here and not the $SU(4)$-invariant, which is Kähler (instead of nearly Kähler) and hence would break supersymmetry.

(*iii*) $\frac{SU(3)}{U(1)\times U(1)}$ The cone over this space gives the G_2-holonomy space related to an $\mathbb{R}_3$ bundle over $\mathbb{C}P_2$ and therefore the 6-dimensional metric is $SU(3)$-invariant. This space is isomorphic to the flag manifold, which again allows for another metric which is Kähler and would break supersymmetry.

(*iv*) $\frac{SU(2)^3}{SU(2)} \simeq S_3 \times S_3$ There are different possibilities of modding out the $SU(2)$ and the nearly Kähler space appearing in our context is obtained by a diagonal embedding yielding as G_2 manifold an $\mathbb{R}_4$ bundle over S_3.

Having identified the 7-dimensional space with G_2-holonomy it is straightforward to obtain the metric and the almost complex structure J of the nearly Kähler 6-manifold; see [21, 22]. Actually there is also a whole class of known non-homogeneous (singular) examples, which are obtained from G_2 manifold given by an $\mathbb{R}_3$ bundle over any 4-d selfdual Einstein space, where the nearly Kähler space becomes an S_2 bundle over the 4-d Einstein space, which is also known as the twistor space; (*i*) and (*ii*) are just the simplest (regular) examples, see also [23, 24]. Having related the nearly Kähler space to G_2-manifolds, it is tempting to identify the 7^{th} direction with the radial direction of the AdS space so that the 10-d metric can thus be interpreted as the near-horizon geometry of a massive D2-brane. The transversal space is a (conical) G_2-manifold accommodating the NS-NS and RR-fluxes (15) and it breaks 1/8 of supersymmetry.

It is obvious to ask also for brane sources consistent with this background. This question deserves of course a detailed analysis, but the topology of the spaces suggest already a number of interesting candidates. E.g. 4- and 6-branes on $\mathbb{C}P_3$ might be wrapped on 2- and 4-cycles yielding a domain wall in the external space. But more interesting from the field theory perspective are branes that extend along the whole external space time, which would be the case if one can wrap 6-branes around each S_3 of the coset $\frac{SU(2)^3}{SU(2)}$. In either case the mass parameter can be related to 8-branes wrapping the 6-manifold and extending in two external directions yielding the AdS_4 geometry.

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