HU-EP-04/09
AEI-2004-21

arXiv:hep-th/0405127 v2 5 Oct 2004

Holography and the Higgs branch of $\mathcal{N}$=2 SYM theories

Zachary Guralnik[a], Stefano Kovacs[b] and Bogdan Kulik[b] *

[a] *Institut für Physik*
Humboldt-Universität zu Berlin
Newtonstraße 15
12489 Berlin, Germany

[b] *Max-Planck-Institut für Gravitationsphysik*
Albert-Einstein-Institut
Am Mühlenberg 1, D-14476 Golm, Germany

Abstract

We present a proposal for the description of the Higgs branch of four-dimensional $\mathcal{N}$=2 supersymmetric Yang–Mills theories in the context of the AdS/CFT correspondence. We focus on a finite Sp(N) $\mathcal{N}$=2 theory arising as dual of a configuration of N D3-branes in the vicinity of four D7-branes and an orientifold 7-plane in type I′ string theory. The field theory contains hypermultiplets in the second rank anti-symmetric and in the fundamental representations. The Higgs branch has a dual description in terms of gauge field configurations with non-zero instanton number on the world-volume of the D7-branes. In this setting the non-renormalisation of the metric on the Higgs branch implies constraints on the α' corrections to the D7-brane effective action, including couplings to the curvature and five-form field strength. In the second part of the paper we discuss non-renormalisation properties of BPS Wilson lines, which are closely related to the physics of the Higgs branch. Using a formulation of the four-dimensional $\mathcal{N}$=2 theory in terms of a three-dimensional $\mathcal{N}$=2 superspace we show that the expectation value of certain Wilson-line operators with hypermultiplets at the end points is independent of the length and thus coincides with the expectation value of the local operators parametrising the Higgs branch.

*zack@physik.hu-berlin.de, stefano.kovacs@aei.mpg.de, bogdan.kulik@aei.mpg.de

1 Introduction

The holographic string/gauge theory equivalence known as AdS/CFT duality [1–3] has provided new insights into numerous aspects of both string theory and strongly coupled gauge theory, and great effort has gone into building the AdS/CFT dictionary. In this article we formulate a proposal for the description of the Higgs branch of $\mathcal{N}$=2 supersymmetric Yang–Mills (SYM) theories. This will allow us to obtain an AdS realization of a well known non-renormalization theorem in four dimensional $\mathcal{N} = 2$ theories, namely the non-renormalization of the metric on the Higgs branch. Besides adding this entry to the AdS/CFT dictionary, we shall discuss how, reversing the point of view, the non-renormalisation theorems known in field theory can be used in this set-up to obtain constraints on the non-abelian Dirac-Born-Infeld (DBI) action in a curved background with Ramond-Ramond flux.

To discuss the AdS description of the Higgs branch, we will focus on a $\mathcal{N} = 2$ Sp(N) gauge theory [1] dual to a Z_2 orientifold of $\mathrm{AdS}_5 \times S^5$ [4, 5]. This background is the near horizon limit of N D3-branes which are coincident with a tadpole cancelling configuration of D7-branes and a negative charge O7-plane. In the near horizon limit on the D3-branes the O7-plane and D7-branes fill AdS_5 and wrap the S^3 inside S^5, which is a fixed surface of the orientifold. The field content of the dual $\mathcal{N}$=2 theory consists of a vector multiplet, one hypermultiplet in the second rank antisymmetric representation and four hypermultiplets in the fundamental representation, resulting in a vanishing β-function. In the limit of vanishing expectation values for the scalars the theory is believed to be exactly conformal. We shall instead construct an AdS description of the Higgs branch of this model. On the field theory side the Higgs branch corresponds to vacua with non-zero expectation values for the scalars in the fundamental hypermultiplets. The bulk dual description involves turning on a SO(8) field strength with non-zero instanton number in the D7-brane world volume. Such instantons can in turn be viewed as D3-branes dissolved into the world volume of the D7-branes.

The world-volume effective action of the D7-branes is a non-abelian gauge theory whose form in flat space is known only to low orders in an expansion in α' [6, 8–12]. The leading term in the D7-brane action is quadratic in field strengths but, due to the embedding in $\mathrm{AdS}_5\times \mathrm{S}^5/Z_2$, differs from flat space Yang–Mills theory. Nevertheless, we shall find that this term admits solutions corresponding to field strengths which are anti-self-dual with respect to a *flat* metric ($F^+ = 0$), namely the usual instanton solutions in $\mathbb{R}^4$.

For the $\mathcal{N} = 2$ theory which we consider there is an exact equivalence between the the Higgs branch and the moduli space of Yang–Mills instantons [14–16] (see [17, 18] for a review). Realizing this correspondence in the dual AdS description requires that instanton configurations be exact solutions of the full D7-brane action in the AdS background, including α' corrections. Note that, using the AdS/CFT dictionary, the α' expansion of the D7-brane effective action is converted into an expansion in $1/\sqrt{\lambda}$, where λ is the 't Hooft coupling of the dual four-dimensional $\mathcal{N} = 2$ super Yang–Mills theory.

The above requirement implies independent conditions that must be satisfied separately by terms in the effective action involving only gauge fields or terms which involve

[1] In our conventions the algebra sp(N) corresponds to C_N, *i.e.* we define Sp(N) so that Sp(1)=SU(2).

couplings to bulk fields with non-zero background value, namely the curvature and R⊗R five-form. We shall first consider terms of the first type, involving only powers of the gauge field strength. There are no F^3 terms at order α' and thus the leading corrections come from F^4 couplings that arise at order α'^2. We will show that taking such terms into account a self-dual field strength, $F^+ = 0$, remains a solution of the field equations.

While the F^4 terms are known exactly (at least in flat space) [6, 8–10], little is known about other relevant terms that might modify the field equations for the gauge potential on the D7-brane world-volume. These include couplings to the curvature, R⊗R five-form and their derivatives which are non-vanishing in the AdS background we consider. Among the terms that need to be considered are those of the form $D^r \mathcal{R}^m D^s \mathcal{F}_5^n F^2$ arising at order $(\alpha')^{m+\frac{1}{2}(n+r+s)}$. Here $\mathcal{R}$ generically denotes the curvature and $\mathcal{F}_5$ the self-dual R⊗R five-form. In order for the self-dual field configurations to remain solutions of the complete field equations the sum of these terms must vanish in the AdS background. This constraint holds at each order in α'.

Some of the terms of order α'^2 involving two powers of $\mathcal{R}$ can be deduced from flat-space calculations of two-graviton disk amplitudes [19–21]. We shall verify that these known terms do not vanish in the AdS background and therefore we shall argue that other terms must be present in the D7-brane effective action at the same order. These may include couplings to the R⊗R five-form, but also couplings to pull-backs of the bulk Ricci tensor which are not fixed by the disk amplitudes computed in a background with vanishing Ramond-Ramond flux.

In addition to preserving the instanton solutions, the D7-brane action in the AdS background must give the correct metric on the Higgs branch. The hyper-Kähler metric on the Higgs branch of four-dimensional $\mathcal{N} = 2$ gauge theories is exactly given by the tree level result [23]. For the particular $\mathcal{N} = 2$ theory we consider, this metric is also given by the moduli space approximation for the dynamics of slowly varying instantons in eight-dimensional super Yang–Mills theory [14–17]. The dynamics of slowly varying instantons on a D7-brane wrapping $\mathrm{AdS}_5 \times S^3$ must be described by the same metric. We will find that the leading term in the α' (or $1/\sqrt{\lambda}$) expansion of the D7 action gives the exact metric on the Higgs branch. All the sub-leading terms must therefore give a vanishing contribution. We will verify that this is the case to order $1/\lambda$, assuming the sum of the five-form and curvature couplings to F^2 vanish in the AdS background.

Although we have not analysed this aspect here, using our proposal for the description of the Higgs branch in the AdS/CFT context it should be possible to compute correlation functions of composite operators in the gauge theory utilising the same prescription as in the conformal phase. Correlation functions should be obtained from supergravity amplitudes involving appropriately modified bulk-to-boundary propagators, which encode the information about the non-trivial field strength on the world volume of the D7-branes.

In the second half of this article we shall discuss non-renormalization properties for a class of Wilson line operators in theories with eight supercharges. This part of our discussion is a sequel to [24], in which a non-renormalization theorem for certain BPS Wilson loops in maximally supersymmetric Yang–Mills theories was proven using field theoretic methods. The eight supercharge variant of this non-renormalization theorem is closely related to the physics of the Higgs branch. We will find that the vacuum expectation value (vev) of certain straight BPS Wilson lines with hypermultiplet fields at the end points is independent of the length and thus coincides with the expectation value of local operators. These vev's of bilinear local operators made of scalar components of the hypermultiplets are the quantities which parametrise the Higgs branch.

In the case of four-dimensional $\mathcal{N}=2$ SYM theories, we will obtain this result by using a three-dimensional $\mathcal{N}=2$ superspace which makes a very useful subgroup of the four-dimensional $\mathcal{N}=2$ supersymmetry manifest. Our result then follows from equations satisfied by the $\mathcal{N}=2$, $d=3$ chiral ring. The same results apply to other eight-supercharge theories obtained by dimensional reduction. In five dimensions, the result is modified by a generalized Konishi anomaly which is crucial for the validity of proposals relating effective superpotentials to bosonic matrix models [25–29].

The organization of this paper is as follows. In section 2, we review the AdS construction of an $\mathcal{N}=2$ theory with fundamental hypermultiplets. In section 3, we show how to construct the AdS description of the Higgs branch, working to leading order in α'. In section 4 we discuss the α' corrections and the constraints imposed on curvature and five-form couplings by the correspondence between the Higgs branch and Yang–Mills instantons. In section 5, we verify to order α'^2 that the metric on the Higgs branch is correctly given by the dynamics of slowly moving instantons on a D7-brane wrapping $\mathrm{AdS}_5 \times S^3$, assuming that couplings of the curvature and five-form to F^2 sum to zero in the AdS background. In section 6, we present a new non-renormalization theorem for a class of straight open Wilson lines in eight-supercharge SYM theories.

2 AdS/CFT duality for $\mathcal{N}=2$ theories with fundamental representations

We will consider a finite $\mathcal{N}=2$ gauge theory with gauge group $\mathrm{Sp}(N)$ and matter content comprising one hypermultiplet in the second rank anti-symmetric representation [2] and four hypermultiplets in the fundamental representation. The conformal phase of this theory was first studied in the AdS/CFT context in [4, 5] at the perturbative level. The correspondence was subsequently generalised to include instanton effects in [30–32] and the Penrose limit for this system was studied in [33]. The AdS dual is obtained from the type IIB orientifold $T^2/(-1)^F\Omega I$, where Ω is the world sheet parity and I the inversion, $z \to -z$, on the torus T^2 [34]. At each of the four fixed points of I there are an orientifold 7-plane and four D7-branes. The resulting type I′ theory has $\mathrm{SO}(8)^4$ gauge symmetry, with one SO(8) factor associated with each group of D7-branes. Considering a stack of N D3-branes probing the region near one of the O7-planes and taking the near horizon limit leads to the $\mathrm{AdS}_5 \times S^5/Z_2$ geometry with four D7-branes filling AdS_5 and wrapping the S^3 inside S^5 which is fixed under the orbifold action. The gravity dual of the $\mathrm{Sp}(N)$ gauge theory is therefore a theory of closed and open strings. In the duality closed string states correspond to gauge invariant composite operators written as traces of fields in the adjoint representation, whereas open strings propagating in the bulk are associated with meson-like operators containing fields in the fundamental and anti-symmetric representations.

[2] The anti-symmetric representation of $\mathrm{Sp}(N)$ is reducible, containing the singlet plus a $N(2N-1)-1$ dimensional irreducible representation. Throughout this paper we will refer to the latter as the second rank anti-symmetric representation.

The isometries and SO(8) gauge symmetry of the string theory correspond to conformal and global symmetries in the gauge theory.

The near horizon geometry on the D3-branes is $\mathrm{AdS}_5 \times S^5/Z_2$, with metric

$$\mathrm{d}s^2 = \frac{r^2}{L^2}\left(-\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2\right) + \frac{L^2}{r^2}\left(\mathrm{d}r^2 + r^2 \mathrm{d}\hat{\Omega}_5^2\right) , \tag{2.1}$$

where L is the radius of both the AdS_5 and the S^5 factors and as usual $L^4 = 4\pi g_s N \alpha'^2$. In (2.1) we have denoted with x_μ, $\mu = 0,1,2,3$, the coordinates on the AdS_5 boundary and with r the radial coordinate transverse to the D3-branes, $r^2 = X_4^2 + \cdots + X_9^2$. In (2.1) $\mathrm{d}\hat{\Omega}_5^2$ denotes the metric on S^5/Z_2 given by

$$\mathrm{d}\hat{\Omega}_5^2 = \mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2 + \cos^2\theta\, \mathrm{d}\Omega_3^2 \,, \tag{2.2}$$

where the range of ϕ is $[0, \pi]$ instead of $[0, 2\pi]$ as for an ordinary S^5.

The D7-branes are at a fixed point of the orientifold, $X_8 = X_9 = 0$. After taking the near horizon limit they fill AdS_5 and wrap the S^3 corresponding to $\theta = 0$ in (2.2), which is fixed under Z_2. The induced metric on the D7-branes is

$$\mathrm{d}s^2 = \frac{U^2}{L^2}\,\mathrm{d}x_\parallel^2 + \frac{L^2}{U^2}\left(\mathrm{d}U^2 + U^2 \mathrm{d}\Omega_3^2\right) = \upsilon^2\, \mathrm{d}x_\parallel^2 + \frac{1}{\upsilon^2}\,\mathrm{d}X_\perp^2 \,, \tag{2.3}$$

where

$$U^2 = r^2\big|_{X_8 = X_9 = 0} = X_4^2 + X_5^2 + X_6^2 + X_7^2$$

and

$$\mathrm{d}x_\parallel^2 = -\mathrm{d}x_0^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 \,, \qquad \mathrm{d}X_\perp^2 = \mathrm{d}X_4^2 + \mathrm{d}X_5^2 + \mathrm{d}X_6^2 + \mathrm{d}X_7^2 \,.$$

For convenience of notation in (2.3) we have also defined the dimensionless variable υ related to U by $\upsilon^2 = U^2/L^2$.

The isometry group of the background is thus SO(2,4)×SO(4)×SO(2), where the SO(2,4) factor corresponds to the isometries of AdS_5 and the remaining two factors to rotations in the S^3 directions and in the X^8 and X^9 directions transverse to the D7-branes. Moreover there is a SO(8) gauge symmetry associated with the open strings on the D7-branes. In the dual field theory these become conformal and global symmetries and it will be convenient to rewrite them as $\mathrm{SO}(2,4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_R\times\mathrm{SO}(8)$ in order to classify the various fields according to their transformation. Here SO(2,4) is the conformal group of the four-dimensional theory, $\mathrm{SU}(2)_R\times\mathrm{U}(1)_R$ is the $\mathcal{N}$=2 R-symmetry and $\mathrm{SU}(2)_L$ together with SO(8) form a global 'flavour' symmetry. The fields in the $\mathcal{N}$=2 vector multiplet transforming in the adjoint of Sp(N), that we denote by $(A_\mu, \varphi, \lambda_\alpha, \bar{\lambda}_\alpha)$, are singlets of SO(8) and $\mathrm{SU}(2)_L$. The gauge field A_μ is also a singlet of $\mathrm{SU}(2)_R$ and is not charged under $\mathrm{U}(1)_R$. The complex scalars, φ and $\varphi^\dagger$, are $\mathrm{SU}(2)_R$ singlets and have $\mathrm{U}(1)_R$ charge ± 2. The fermions, λ_α and $\bar{\lambda}_{\dot{\alpha}}$, transform in the **2** of $\mathrm{SU}(2)_R$ and have $\mathrm{U}(1)_R$ charge ± 1. The hypermultiplet in the second rank anti-symmetric tensor representation of Sp(N) contains scalars, ϕ and $\widetilde{\phi}$, in the $(\mathbf{2},\mathbf{2})$ of $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_L$ with zero $\mathrm{U}(1)_R$ charge and fermions, $(\psi_\alpha, \widetilde{\psi}_\alpha)$ and $(\bar{\psi}_{\dot{\alpha}}, \widetilde{\bar{\psi}}_{\dot{\alpha}})$, which are singlets of $\mathrm{SU}(2)_R$, transform

		$SU(2)_L$	$SU(2)_R$	$U(1)_R$	$SO(8)$	$Sp(N)$
$\mathscr{V}_{\rm adj}$	A_μ	$\mathbf{1}$	$\mathbf{1}$	0	$\mathbf{1}$	$\boldsymbol{N}(\mathbf{2}\boldsymbol{N}+\mathbf{1})$
	λ_α	$\mathbf{1}$	$\mathbf{2}$	+1	$\mathbf{1}$	$\boldsymbol{N}(\mathbf{2}\boldsymbol{N}+\mathbf{1})$
	φ	$\mathbf{1}$	$\mathbf{1}$	+2	$\mathbf{1}$	$\boldsymbol{N}(\mathbf{2}\boldsymbol{N}+\mathbf{1})$
$\mathscr{H}_{\rm a.s.}$	ϕ	$\mathbf{2}$	$\mathbf{2}$	0	$\mathbf{1}$	$\boldsymbol{N}(\mathbf{2}\boldsymbol{N}-\mathbf{1})$
	ψ_α	$\mathbf{2}$	$\mathbf{1}$	−1	$\mathbf{1}$	$\boldsymbol{N}(\mathbf{2}\boldsymbol{N}-\mathbf{1})$
$\mathscr{H}_{\rm f}$	q	$\mathbf{1}$	$\mathbf{2}$	0	$\mathbf{8}_{\rm v}$	$\mathbf{2}\boldsymbol{N}$
	η_α	$\mathbf{1}$	$\mathbf{1}$	−1	$\mathbf{8}_{\rm v}$	$\mathbf{2}\boldsymbol{N}$

Table 1: The $SU(2)_L\times SU(2)_R\times U(1)_R\times SO(8)$ and $Sp(N)$ quantum numbers of the elementary fields in the $\mathcal{N}$=2 SYM theory.

in the $\mathbf{2}$ of $SU(2)_L$ and have $U(1)_R$ charge ∓ 1 respectively. All these fields are also SO(8) singlets. Finally the hypermultiplets in the fundamental of $Sp(N)$ contain complex scalars, q and $\widetilde{q}$, which are in the $\mathbf{2}$ of $SU(2)_R$ with zero $U(1)_R$ charge and are singlets of $SU(2)_L$ and fermions, $(\eta_\alpha, \widetilde{\eta}_\alpha)$ and $(\bar{\eta}_{\dot\alpha}, \bar{\widetilde{\eta}}_{\dot\alpha})$, which are singlets of $SU(2)_L\times SU(2)_R$ and have $U(1)_R$ charge ± 1. The fields in the fundamental hypermultiplets transform in the $\mathbf{8}_{\rm v}$ of SO(8). The $SU(2)_L\times SU(2)_R\times U(1)_R\times SO(8)$ quantum numbers of the elementary fields are summarised in table 1.

The combination of $\mathcal{N}$=2 supersymmetry and the above global symmetries completely determines the form of the action of the theory, which is given explicitly in section 6.

AdS/CFT constructions for different $\mathcal{N}$=2 theories have also been proposed. In particular a $U(N)$ $\mathcal{N}$=2 SYM theory with hypermultiplets in the fundamental representation has been considered in [36]. The set-up involves N_f D7-branes wrapping $AdS_5\times S^3$ in the $AdS_5\times S^5$ background obtained as near horizon geometry of a stack of N D3-branes. The resulting dual gauge theory has $U(N)$ gauge group and the field content of $\mathcal{N}$=4 SYM plus N_f hypermultiplets in the fundamental representation. In [36] it was argued that this configuration of D3- and D7-branes is stable in spite of the fact that the S^3 contained in S^5 wrapped by the D7-branes is contractible. At the level of supergravity this follows from the observation that the scalar mode corresponding to the D7-branes slipping off the S^3 although 'tachionic' does not violate the Breitenlohner–Freedman bound. At leading order in α' our analysis of the Higgs branch can be repeated for this $U(N)$ theory essentially without modifications. However in the following we shall be interested in including α' corrections in the discussion and it is not clear whether the above stability argument would still apply at higher orders. This is because unlike the configuration dual to the $Sp(N)$ SYM theory, the D3-D7 system without the orientifold plane cannot arise as a consistent string background. Moreover the $Sp(N)$ gauge theory discussed above is finite for any N whereas the $U(N)$ theory with N_f hypermultiplets has a β-function (for the 't Hooft coupling) which vanishes in the strict $N\to\infty$ limit, but is positive for any finite N. We hope to generalise our discussion of the Higgs branch to other models in the future.

3 The Higgs branch

The Higgs branch of $\mathcal{N} = 2$ theories consists of the vacua for which the scalar components of the $\mathcal{N} = 2$ fundamental hypermultiplets get expectation values. In the context of the above D3-D7-O7 system, vacuum expectation values for the fundamental hypermultiplets correspond to D3-branes which are "dissolved" in the D7-branes. Dissolved D3-branes can be viewed as instantons inside the SO(8) world-volume gauge theory of the D7-branes [14]. This is due to the Wess-Zumino term in the D7-brane action,

$$\mu_7 \int C^{(4)}_{\rm pb} \wedge {\rm tr}(F \wedge F)\,, \tag{3.1}$$

where F is the world volume field strength on the D7-branes, and $C^{(4)}_{\rm pb}$ is the pull-back of the Ramond-Ramond four-form which couples to D3-brane charge. A non-zero instanton number K associated with field strengths in the $4, 5, 6$ and 7 directions corresponds to K D3-branes extended in the $0, 1, 2$ and 3 directions and bound to the D7-branes. It is well known that F- and D-flatness conditions describing the Higgs branch can be mapped [15,16] to the ADHM constraints [37] determining the moduli space of instantons.

We will implement this general construction in the context of the AdS/CFT correspondence in order to provide a holographic description of the Higgs branch in the large N limit of the above $\mathcal{N}$=2 gauge theory. More precisely we will take as a starting point a stack of $N = N' + M$ (with $M \ll N'$) D3-branes in the orientifold geometry and we will consider the portion of the Higgs branch corresponding to dissolving M D3-branes in the world volume of the D7-branes. In this way we can assume that the near horizon geometry of the remaining N' D3-branes is not modified [3] and remains $\mathrm{AdS}_5 \times S^5/Z_2$. The resulting AdS/CFT construction describes a part of the Higgs branch in which a $\mathrm{Sp}(N')$ subgroup of the original $\mathrm{Sp}(N)$ gauge group is unbroken.

We will argue that the usual instantons, namely SO(8) field strengths with support in the $X_\perp$ directions which are anti-self-dual with respect to the flat metric ${\rm d}X_\perp^2$, remain solutions of the D7-brane equations of motion in the near horizon limit where the D7 geometry becomes (2.3). Such configurations provide a holographic description of (part of) the Higgs branch of the boundary field theory.

The $\mathcal{N}$=2 SYM theory we are studying has a rich structure of global symmetries as discussed in the previous section. It is interesting to analyse the pattern of global symmetry breaking in the portion of the Higgs branch captured by our AdS construction. As already remarked this corresponds to points where a residual $\mathrm{Sp}(N')$ gauge invariance is preserved. The moduli identifying points on the Higgs branch correspond to the moduli of the SO(8) M-instanton configuration on the D7-brane world volume. Analysing the ADHM construction of instantons for SO(n) gauge theories [17] we can characterise the corresponding points on the Higgs branch of the dual theory. In particular it is possible to identify the specific instanton configurations associated with field theory vacua in which different subgroups of the global symmetry are broken.

The logic here is the following. A generic instanton configuration breaks some of the isometries of $\mathrm{AdS}_5 \times S^5/Z_2$ background as well as the SO(8) gauge symmetry. In

[3] The background we are considering, although not maximally supersymmetric, may be an exact solution in the full string theory (see also [35]), in which case the construction we are considering should not require the above approximation.

the dual boundary $\mathcal{N}$=2 SYM theory this corresponds to giving vev's to scalars that break the same part of the conformal and global symmetries. By matching patterns of symmetry breaking on the two sides one can determine a precise correspondence between moduli of instantons and points on the Higgs branch. Clearly in the presence of vev's for the scalar fields conformal invariance is broken. This effect corresponds to the fact that the instantons we consider break the SO(2,4) symmetry of the AdS_5 space, *e.g.* via the moduli associated with instanton locations in the radial direction. Among the moduli on the Higgs branch there are vev's for the scalars in the fundamental of $\mathrm{Sp}(N)$ giving rise to non-vanishing values for $\mathrm{Sp}(N')$ invariant operators of the type $\mathscr{Q}^{mn} = \widetilde{q}^m q^n$ (where gauge indices have been suppressed). The quantities $\mathscr{Q}^{mn}$ are in the $\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}$ of SO(8). We can thus distinguish points on the Higgs branch where the global SO(8) is unbroken, corresponding to $\mathscr{Q}$'s in the singlet, or broken by vev's in either the **28** or the **35**. The vev's $\langle q^n \rangle$ for the fundamental scalars can be identified with the instanton moduli, w^n, associated with global SO(8) gauge orientations and the $\mathscr{Q}^{mn}$'s correspond to combinations of such moduli. In the one-instanton sector the SO(8) singlet modulus corresponds to the combination $w^n w_n$ which is related to the instanton size ρ. The (relative) positions of the M instantons in the D7 world-volume theory generically break not only the SO(2,4) symmetry of AdS_5, but also the SO(4) symmetry on S^3 and in particular its $\mathrm{SU}(2)_L$ subgroup. Since only fields belonging to the hypermultiplet in the anti-symmetric second rank representation of $\mathrm{Sp}(N)$ are charged with respect to $\mathrm{SU}(2)_L$ (see table 1) we conclude that the AdS configuration we are considering captures regions of the phase space in which vev's for the anti-symmetric hypermultiplet are also turned on. More precisely, because of the unbroken $\mathrm{Sp}(N')$, we can have non-vanishing vev's for composite operators of the form $\mathscr{S} = \widetilde{\phi}\phi$, where again gauge indices have not been indicated, but it is understood that the operator is $\mathrm{Sp}(N')$ invariant. In this sense the phase of the theory that we are describing can be considered as a generalised Higgs branch in that it is characterised not only by fundamental hypermultiplet vev's, but by non-zero vev's for the hypermultiplets in the anti-symmetric representation as well. Analogously a generic instanton configuration breaks the $\mathrm{SU}(2)_R$ part of the R-symmetry. Notice however that the SO(8) instantons cannot break the symmetry under SO(2) rotations in the (X^8, X^9) directions. This agrees with the fact that the vev's of combinations of hypermultiplet scalars are not charged under $\mathrm{U}(1)_R$.

A more detailed analysis of the relevant multi-instanton ADHM construction and of how it allows to precisely identify different points on Higgs branch and characterise the corresponding pattern of symmetry breaking is beyond our purposes here. We shall however return to some of these aspects in section 6, where we shall consider non-local Wilson-line operators and show that their expectation values coincide with the vev's parametrising the Higgs branch.

3.1 Instantons from D7 equations of motion at leading order in α'

The D7-brane action is $S_{\rm D7} = S_{\rm DBI} + S_{\rm WZ}$ where the terms which involve only the field-strengths are

$$S_{\rm DBI} = \frac{1}{(2\pi)^7 g_{\rm s}\alpha'^4}\int {\rm d}^8x\,\sqrt{-g}\,(2\pi\alpha')^2\,\frac{1}{4}\,{\rm tr}(F_{ab}F^{ab})\,+\cdots \tag{3.2}$$

$$S_{\rm WZ} = \frac{1}{(2\pi)^7 g_{\rm s}\alpha'^4}\int \sum_q C^{(q)}\wedge\,{\rm tr}\,({\rm e}^{2\pi\alpha' F})\,. \tag{3.3}$$

The "$\cdots$" in (3.2) represents terms of higher order in α'. In the AdS background, the α' expansion is effectively converted into an expansion in $1/\sqrt{\lambda}$, where $\lambda = 4\pi g_{\rm s}N$ is the 't Hooft coupling of the dual gauge theory. The self-dual five-form field strength in $\mathrm{AdS}_5\times S^5$ corresponds to a Ramond-Ramond four-form

$$C^{(4)}_{0123} = \frac{U^4}{L^4}\,. \tag{3.4}$$

Taking the $\mathrm{AdS}_5\times S^3$ D7-brane embedding (2.3) and turning on field strengths with support in the $X_\perp$ directions, $X_4,\ldots,X_7$, one has

$$S_{\rm DBI} = \frac{N}{(2\pi)^4\lambda L^4}\int {\rm d}^4x_\parallel\,{\rm d}^4X_\perp\,\upsilon^4\,\frac{1}{2}\,{\rm tr}(F_{mn}F_{mn})+\cdots \tag{3.5}$$

$$S_{\rm WZ} = \frac{N}{(2\pi)^4\lambda L^4}\int {\rm d}^4x_\parallel\,{\rm d}^4X_\perp\,\upsilon^4\,\frac{1}{4}\,\varepsilon_{mnrs}{\rm tr}(F_{mn}F_{rs})\,, \tag{3.6}$$

where roman indices indicate the directions $m = 4,5,6,7$ and υ was defined after equation (2.3). Summing (3.5) and (3.6) gives

$$S_{\rm D7} = \frac{N}{(2\pi)^4\lambda L^4}\int {\rm d}^4x_\parallel\,{\rm d}^4X_\perp\,\upsilon^4\,{\rm tr}(F^+)^2\,, \tag{3.7}$$

where $F^+_{mn} \equiv \frac{1}{2}\left(F_{mn}+\frac{1}{2}\varepsilon_{mnrs}F_{rs}\right)$. Therefore, to at least leading order, a field strength satisfying $F^+ = 0$ gives also a solution for the D7-branes embedded on $\mathrm{AdS}_5\times S^3$. Such a configuration corresponds to an ordinary flat space instanton solution. This is because, despite the curved background, F_{mn} is anti-self-dual with respect to a *flat* metric in the $X^{4,5,6,7}$ directions and moreover with our choice of variables the range of these coordinates is $(-\infty,+\infty)$. Note that the D7-brane action is the same (*i.e.* vanishing) for all values of the instanton number. This is expected, since the action of the string backgrounds dual to Higgs branch vacua should be the same for all such vacua.

4 Higher order terms

The correspondence between the moduli space of Yang–Mills instantons and the Higgs branch of $\mathcal{N}=2$ theories [14–16] suggests that connections which are anti self-dual with respect to a flat metric should be exact solutions of the non-abelian D7-brane action in the AdS background, even when all $1/\sqrt{\lambda}$ (or α') corrections are taken into account.

Little is known about the exact form of the non-abelian D7-brane action. The terms that are relevant for our analysis are those involving powers of the gauge field strength and fields which have a non vanishing background value in $\mathrm{AdS}_5 \times S^5/Z_2$, namely the curvature, $\mathcal{R}$, and R⊗R five-form, $\mathcal{F}_5$. We are interested in verifying that the anti self-dual configurations solving the field equations at leading order remain solutions after including the α' corrections. For this purpose we can consider separately the terms involving only gauge fields and those involving couplings to $\mathcal{R}$ and $\mathcal{F}_5$, since the $F = F^-$ solution must be unaltered regardless of the magnitude of F_{mn} with respect to the background values of $\mathcal{R}$ and $\mathcal{F}_5$.

The first corrections from terms involving only powers of the gauge field strength are couplings of the form F^4 arising at order α'^2, since the cubic terms of order α' are known to vanish. These F^4 terms are known exactly [6, 8–10]. Little is known about curvature couplings and essentially nothing is known about the couplings to the R⊗R five-form. Some of the $\mathcal{R}^2 F^2$ are known [21], based on T-duality arguments and a comparison with the $\sqrt{-g}\mathcal{R}^2$ terms computed in [19, 20]. The term in the action of the form $\upsilon^4 F_{mn}{}^* F_{mn}$ is exactly given by the CP-odd Wess-Zumino term (see 3.6), which is un-modified by higher order terms in the α' expansion. For connections with $F^+ = 0$ to solve the equations of motion, the CP-even terms quadratic in F must be of the form $\upsilon^4 F_{mn}F_{mn}$, with exactly the same coefficient. This term is already present with the correct coefficient in the absence of curvature and five-form couplings to F^2. Thus the sum of the additional couplings of F^2 to $\mathcal{R}$ and $\mathcal{F}_5$ must vanish when the curvature and five-form are set to their AdS background values. The non-trivial order α'^2 term in the AdS background is then just the F^4 term, $\mathcal{L}_{F^4}$. To preserve the $F^+ = 0$ solutions to this order requires $\delta\mathcal{L}_{F^4}|_{F^+=0} = 0$. We will verify that this is the case in the next section, and then turn our attention to the curvature and five-form couplings.

4.1 Terms of higher order in the field strength

We first analyse higher order terms in the α' expansion of the D7-brane action which only involve powers of the field strength. Curvature and five-form couplings to F^2 in the AdS background will be discussed in the next subsection. There are no cubic terms at order α' and therefore the next to leading terms in the effective action are of the form F^4 [6–8, 11]. More precisely,

$$
\begin{aligned}
S_{F^4} \;=\; & -\frac{1}{(2\pi)^5 g_s \alpha'^2} \int \sqrt{-g}\,\mathrm{tr}\left[\alpha'^2 \left(\frac{1}{24} F_{AB}F^{BC}F_{CD}F^{DA} + \frac{1}{12} F_{AB}F^{BC}F^{DA}F_{CD}\right.\right. \\
& \left.\left. - \frac{1}{48} F_{AB}F^{BA}F_{CD}F^{DC} - \frac{1}{96} F_{AB}F_{CD}F^{BA}F^{DC}\right)\right] , \qquad (4.1)
\end{aligned}
$$

where capital Roman indices indicate the real coordinates $0, 1, \ldots, 7$. Taking the induced metric to be (2.3), turning on field strengths only in the $X_\perp$ directions, and including the Wess-Zumino term, the D7-brane action to order α'^2 can be written as

$$
\begin{aligned}
S_{\mathrm{D7}} = & \frac{N}{(2\pi)^4 \lambda L^4} \int \mathrm{d}^4 x_\parallel \mathrm{d}^4 X_\perp \,\mathrm{tr}\left[\upsilon^4\, F^+_{mn} F^+_{mn}\right. \\
& \left. - \frac{L^4}{12\lambda}\,\upsilon^8 \left(2F^+_{mn}F^+_{mn}F^-_{rs}F^-_{rs} + F^+_{mn}F^-_{rs}F^+_{mn}F^-_{rs}\right)\right] , \qquad (4.2)
\end{aligned}
$$

where lower case Roman indices indicate the directions $4, 5, 6$ and 7, and $F^{\pm}_{mn} \equiv \frac{1}{2}\left(F_{mn} \pm \frac{1}{2}\varepsilon_{mnrs}F_{rs}\right)$. All terms (4.2) involve two factors of F^+, therefore the associated equations of motion are proportional to F^+ and thus are manifestly solved by anti-self-dual connections.

The instanton solutions $F^+ = 0$ can be viewed as stable holomorphic bundles in two complex dimensions. In fact, a requirement which has been used to constrain the form of the non-abelian DBI action (modulo curvature and p-form couplings) is that there should be BPS solutions which are holomorphic vector bundles satisfying a *deformed* stability condition [11,12], which reduces to the usual condition in the $\alpha' \to 0$ limit. This approach has recently been used to propose the form of the $O(\alpha'^4)$ terms [12], while checks of this approach have been made to order α'^3 [13]. In terms of complex coordinates, x^α, a holomorphic bundle satisfies

$$F_{\alpha\beta} = F_{\bar{\alpha}\bar{\beta}} = 0\,, \tag{4.3}$$

while a stable holomorphic bundle also satisfies, in flat space,

$$\sum_\alpha F_{\alpha\bar{\alpha}} = 0\,. \tag{4.4}$$

To order α'^2, the equations of motion of a D-brane in a flat background are solved by holomorphic bundles satisfying the deformed stability condition

$$F_{\alpha\bar{\alpha}} - \alpha'^2 \frac{1}{6}\left(F_{\alpha\bar{\beta}}F_{\beta\bar{\gamma}}F_{\gamma\bar{\alpha}} + F_{\alpha\bar{\beta}}F_{\gamma\bar{\alpha}}F_{\beta\bar{\gamma}}\right) = 0\,. \tag{4.5}$$

In four real dimensions, with complex coordinates $x^1 + ix^2$ and $x^3 + ix^4$, the holomorphic condition is $F_{13} = F_{24}$ and $F_{23} = -F_{14}$, while the stability condition is $F_{12} = -F_{34}$. The deformed stability condition (4.5) can be written as

$$\begin{aligned} iF^+_{12} + \alpha'^2 \frac{i}{12}\Big[&F^{-\,2}_{12}F^+_{12} + F^-_{12}F^+_{12}F^-_{12} + F^+_{12}F^{-\,2}_{12} + F^{+\,3}_{12} \\ &+F^+_{12}(F^{-\,2}_{13} + F^{-\,2}_{14}) + F^-_{13}\{F^-_{13}, F^+_{12}\} + F^-_{14}\{F^-_{14}, F^+_{12}\}\Big] = 0\,, \end{aligned} \tag{4.6}$$

which is clearly satisfied when $F^+ = 0$. Note that, despite the deformation of the stability condition, connections with $F^+ = 0$ are solutions to at least order α'^2 and presumably to all orders in α'. As we have discussed and verified to order α'^2, the $F^+ = 0$ solutions should also be exact in the AdS background.

4.2 Curvature and five-form couplings

We now wish to consider relevant couplings of the D7-brane field strength to the bulk fields. In the AdS background, the couplings which could modify the field equations for the field strength on the D7-branes involve the curvature tensors $\mathcal{R}$, five-form $\mathcal{F}_5$ and their derivatives. These include terms quadratic in the D7-brane field strength which are of the schematic form

$$\mathcal{L} \sim D^r \mathcal{R}^m D^s \mathcal{F}_5^n F^2\,, \qquad m, n, r, s = 1, 2, \ldots\,, \tag{4.7}$$

arising at order $(\alpha')^{m+\frac{1}{2}(n+r+s)}$. Terms of this type of order α' are expected to vanish. The absence of terms of this order involving couplings of scalars to the curvature in the D-brane effective action was verified in [50], where terms of the form $\mathcal{R}X^2$, with X a scalar field, were computed. As noted in the previous section, the couplings of the form (4.7) must sum to zero when the five-form and curvature tensors are set to their AdS values. As we will see shortly, this is not true of the *known* couplings of this type, which are at order α'^2.

Among the known couplings of D-branes to curvature are those contained in the Wess-Zumino term [38],

$$S_{\rm WZ} = \mu_p \int C \wedge {\rm tr}\left({\rm e}^{2\pi\alpha' F}\right) \wedge \left(\frac{\hat{\mathcal{A}}(4\pi^2\alpha' R_T)}{\hat{\mathcal{A}}(4\pi^2\alpha' R_N)}\right)^{1/2} , \tag{4.8}$$

where $\hat{\mathcal{A}}$ is the "A-roof genus", which has an expansion in terms of even powers of the curvature two-form. In (4.8) R_T and R_N are the tangent and normal bundle components of the curvature respectively. The Wess-Zumino couplings of an orientifold plane have a form similar to (4.8), but with F set to zero and the genus polynomials, $\hat{\mathcal{A}}(R_T)$ and $\hat{\mathcal{A}}(R_N)$, replaced by the Hirzebruch polynomials, $\hat{\mathcal{L}}(R_T/4)$ and $\hat{\mathcal{L}}(R_N/4)$, respectively [39–41]. The O-plane charge, μ'_p, is related to the D-brane charge, μ_p, by $\mu'_p = -2^{p-4}\mu_p$. In the following we shall only be interested in vertices involving the field strength and therefore we shall not consider the O-plane couplings. In the AdS background, the only non-zero Ramond-Ramond form is $C^{(4)}$, so that there are no couplings of field strengths to curvature in the Wess-Zumino component of the D7-brane action.

The $\mathcal{R}^2$ terms in the D-brane effective action fixed by two-graviton disk amplitudes were first obtained in [19]. These terms are

$$\begin{aligned} S = \mu_p \int \sqrt{-g}\,\Big[1 - \frac{1}{24}\frac{(4\pi^2\alpha')^2}{32\pi^2}\big((R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} \\ -2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - (R_N)_{ab\alpha\beta}(R_N)^{ab\alpha\beta} + 2\bar{R}_{ab}\bar{R}^{ab}\big)\Big] \,, \end{aligned} \tag{4.9}$$

where the various curvature tensors appearing here are defined in the appendix. For the special case of an embedding with vanishing second fundamental form, the tensors $(R_T)_{\alpha\beta\gamma\delta}$ and $(R_N)_{ab\alpha\beta}$ are pull-backs of the bulk Riemann tensor to the tangent and normal bundle, indicated by Greek and Roman indices respectively (we emphasize that this is a change of notation from the previous sections). The tensors $(R_T)_{\alpha\beta}$ and $\bar{R}_{ab}$ are not pull-backs of the bulk Ricci tensor, but are obtained from contractions of tangent indices in the pull-backs of the Riemann tensor. Specifically, for vanishing second fundamental form,

$$\bar{R}_{ab} \equiv g^{\alpha\beta} R_{\alpha a b \beta}, \qquad (R_T)_{\alpha\beta} \equiv g^{\lambda\mu} R_{\lambda\alpha\mu\beta} \,, \tag{4.10}$$

where $g_{\alpha\beta}$ is the induced metric on the D-brane. For the $\mathrm{AdS}_5 \times S^3$ embedding in $\mathrm{AdS}_5 \times S^5$, the second fundamental form vanishes (see the appendix).

Consistency of the couplings (4.9) with T-duality implies the existence of $\mathcal{R}^2F^2$ terms of the form [21]

$$S = \mu_p (2\pi\alpha')^2 \int \sqrt{-g}\, \frac{1}{4} \mathrm{tr}(F_{\alpha\beta}F^{\alpha\beta}) \left[1 - \frac{1}{24}\frac{(4\pi^2\alpha')^2}{32\pi^2} \Big((R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} -2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - (R_N)_{ab\alpha\beta}(R_N)^{ab\alpha\beta} + 2\bar{R}_{ab}\bar{R}^{ab} \Big) \right] . \quad (4.11)$$

For D7-branes wrapping $\mathrm{AdS}_5 \times S^3$ inside $\mathrm{AdS}_5 \times S^5$, the curvature couplings in (4.11) are non-vanishing

$$(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} - 2(\hat{R}_T)_{\alpha\beta}(\hat{R}_T)^{\alpha\beta} - (R_N)_{ab\alpha\beta}(R_N)^{ab\alpha\beta} + 2\bar{R}_{ab}\bar{R}^{ab} = -\frac{6}{25}L^2 \, , \quad (4.12)$$

where $L^2 = \sqrt{\lambda}\alpha'$ is the square of the AdS_5 (or S^5) curvature scalar. In light of (4.12), there must be extra couplings to the background at order α'^2, besides those in (4.11). There are many other possible terms of this order which could be non-zero, but in order to determine them one would need to compute the relevant string amplitudes in the AdS background. There may be couplings involving mixed contractions between curvature tensors and world-volume field strengths, which cannot be determined from the known R^2 couplings [21]). Furthermore couplings of D-branes to pull-backs of the bulk Ricci tensor are also not fixed by the disk amplitudes computed in [19]. The bulk Ricci-tensor vanishes via the leading order equations of motion in an expansion about a background without flux. However, both the Ramond-Ramond five-form flux and Ricci tensor are non-vanishing in the AdS background. Finally the contributions of $\mathcal{F}_5$ couplings must be taken into account. No couplings of the R⊗R five-form to world volume gauge fields have been computed explicitly. The set of possible terms in involving $\mathcal{F}_5$ can be restricted and in principle determined using supersymmetry arguments [22].

The correspondence between the Higgs branch and Yang–Mills instantons requires that the sum of all the above terms vanish for the $\mathrm{AdS}_5 \times S^3$ embedding in $\mathrm{AdS}_5 \times S^5$ and therefore provides constraints order by order in α' on the D-brane effective action.

5 Non-renormalization of the Higgs Branch metric

Thus far we have shown (to order α'^2) that connections with $F^+ = 0$ solve the equations of motion of D7-branes wrapping $\mathrm{AdS}_5 \times S^3$, provided that certain constraints on the curvature and five-form couplings are satisfied. To complete the story, we should also show that the moduli-space approximation [52] for the dynamics of slowly moving instantons on a D7-brane in this background reproduces the correct metric on the Higgs branch. This metric is known to be equivalent to the metric describing the dynamics of slowly varying instantons in eight-dimensional flat space super Yang–Mills theory (see [17] for a review). To order α'^2, we will find the correct metric, assuming the cancellation of curvature and five-form couplings to F^2 in the AdS background.

The gauge field part of the leading order action for a D7-brane with the induced metric (2.3) is given by

$$S = \frac{N}{(2\pi)^4 L^4 \lambda} \int \mathrm{d}^4 x_\parallel \mathrm{d}^4 X_\perp \, \mathrm{tr}\left(\frac{1}{2\upsilon^4} F_{\mu\nu}F_{\mu\nu} + F_{m\mu}F_{m\mu} + \upsilon^4 F^+_{mn}F^+_{mn} \right) + \cdots . \quad (5.1)$$

Greek indices now indicate the $x_\parallel$ directions $0,1,2,3$ while Roman indices indicate the $X_\perp$ directions $4,5,6,7$. To this order, the equations of motion are

$$D_m\left(\upsilon^4 F^+_{mn}\right) + D_\mu F_{\mu n} = 0\,, \qquad D_m F_{m\nu} + \frac{1}{\upsilon^4} D_\mu F_{\mu\nu} = 0\,. \tag{5.2}$$

Let us write the instanton solutions as $A_m = \bar{A}_m(X_\perp, \mathcal{M}^i)$, $A_\mu = 0$, where $\mathcal{M}^i$ are the instanton moduli. These solutions are exact provided the $\mathcal{M}^i$'s have no dependence on $x_\parallel$. Slowly varying instantons may be studied by considering the gauge field configuration

$$A_m = \bar{A}_m(X_\perp, \mathcal{M}^i(x_\parallel)), \qquad A_\mu = \Omega_i \partial_\mu \mathcal{M}^i\,, \tag{5.3}$$

where Ω is Lie algebra valued. Choosing Ω_i such that the equations of motion (5.2) are satisfied to linear order in derivatives with respect to $x_\parallel$ requires

$$D_m\left(\frac{\partial \bar{A}_m}{\partial \mathcal{M}^i} - D_m\Omega_i\right) = 0\,. \tag{5.4}$$

Inserting (5.3) into the (5.1) gives the following effective action for the collective coordinates to quadratic order in $\partial_\mu \mathcal{M}^i$

$$\begin{aligned} S \;&=\; \frac{N}{(2\pi)^4 L^4 \lambda}\int \mathrm{d}^4x_\parallel \mathrm{d}^4 X_\perp\, \frac{1}{4}\,\delta_i\bar{A}_m \delta_j \bar{A}_m \partial_\mu\mathcal{M}^i\partial_\mu\mathcal{M}^j \\ &= -\frac{N}{(2\pi)^4 L^4\lambda}\int \mathrm{d}^4 x_\parallel\, \frac{1}{2}\, G_{ij}(\mathcal{M})\,\partial_\mu\mathcal{M}^i\partial_\mu\mathcal{M}^j\,, \end{aligned} \tag{5.5}$$

where

$$\delta_i \bar{A}_m \equiv \frac{\partial \bar{A}_m}{\partial \mathcal{M}^i} - D_m\Omega_i \tag{5.6}$$

and we have introduced the metric on the moduli space, $G_{ij}(\mathcal{M})$, which, at leading order in the $1/\sqrt{\lambda}$ expansion, is therefore

$$G_{ij}(\mathcal{M}) = -\frac{1}{2}\int \mathrm{d}^4 X_\perp\, \delta_i\bar{A}_m(X_\perp,\mathcal{M})\,\delta_j\bar{A}_m(X_\perp,\mathcal{M})\,. \tag{5.7}$$

This is equivalent to the metric for Yang–Mills instantons in flat space, which captures the exact metric on the Higgs branch, so we do not expect corrections at higher orders.

The metric on the Higgs branch of $\mathcal{N}=2$ gauge theories is known to be given exactly by the tree level result [23]. This result is obtained by pulling back the flat metric associated with the tree level hypermultiplet kinetic terms to the space determined by the F- and D-flatness constraints, subject to a quotient by the gauge symmetry. Therefore the only dependence on the gauge coupling should be an overall $1/g^2_{\rm YM} = N/\lambda$ in front of the moduli space action. The tree level result should be the leading (and only non-trivial) term in the strong coupling expansion which is obtained from the AdS/CFT duality.

The next to leading term in the strong coupling expansion of the metric is obtained from the order α'^2 terms in the DBI action. To this order,

$$\begin{aligned} \partial_\mu\mathcal{M}^i\partial_\mu\mathcal{M}^j G_{ij}(\mathcal{M}) = &-\int \mathrm{d}^4 X_\perp\,\frac{1}{2}\mathrm{tr}(F_{\mu m}F_{\mu m}) \\ &+\frac{L^4}{\lambda}\int \mathrm{d}^4X_\perp\,\frac{\upsilon^4}{6}\mathrm{tr}\left[F_{s\mu}F_{\mu n}\left(\{F_{nr},F_{rs}\} - \frac{1}{2}\delta_{sn}F_{tu}F_{ut}\right)\right.\\ &\left.+\frac{1}{2}\left(F_{\mu n}F_{nr}F_{s\mu}F_{rs} + F_{\mu n}F_{rs}F_{s\mu}F_{nr} - \frac{1}{2}F_{\mu n}F_{rs}F_{n\mu}F_{sr}\right)\right]\,. \end{aligned} \tag{5.8}$$

Rewriting the field strengths with all components in the $X_\perp$ directions in terms of self-dual and anti-self-dual components, (5.8) becomes

$$\begin{aligned}
\partial_\mu \mathcal{M}^i \partial_\mu \mathcal{M}^j G_{ij}(\mathcal{M}) = & -\int \mathrm{d}^4 X_\perp \, \frac{1}{2} \operatorname{tr}(F_{\mu m} F_{\mu m}) \\
& + \frac{L^4}{\lambda} \int \mathrm{d}^4 x_\perp \, \frac{\upsilon^4}{12} \operatorname{tr} \left[F_{s\mu} F_{\mu n} \left(\{F^+_{nr}, F^-_{rs}\} + \{F^-_{nr}, F^+_{rs}\} \right) \right. \\
& \left. + \frac{1}{2} \left(F_{\mu n} F^+_{nr} F_{s\mu} F^-_{rs} + F_{\mu n} F^-_{nr} F_{s\mu} F^+_{rs} + F_{\mu n} F^+_{rs} F_{s\mu} F^-_{nr} + F_{\mu n} F^-_{rs} F_{s\mu} F^+_{nr} \right) \right] .
\end{aligned} \tag{5.9}$$

To order α'^2, the equations of motion are still solved by (5.3) to linear order in derivatives with respect to $x_\parallel$ provided that (5.4) is satisfied. Furthermore the additional contributions to the metric arising from the order $1/\lambda^2$ terms in (5.9) vanish when $F^+ = 0$. The non-renormalization of the metric on the Higgs branch implies the absence of corrections to (5.7) at any order.

6 Non-renormalization of chiral Wilson lines

In this section we shall present a proof a new non-renormalization property for eight-supercharge Yang-Mills theories in dimension $d \leq 4$. Specifically, we will show that certain straight BPS Wilson lines with scalar components of hypermultiplets at the end-points have length independent expectation values. These expectation values are the same as those of local operators parameterizing the Higgs branch. The non-renormalization theorem we will find is very similar to one conjectured in [42] and demonstrated in [24] for a class of BPS Wilson loops in maximally supersymmetric Yang-Mills theories. Our result is also closely related to the non-renormalization of the metric on the Higgs branch, for reasons that will become clear shortly.

We will obtain our result by making a particularly useful sub-group of the full supersymmetry manifest. In the context of $\mathcal{N} = 2$ gauge theories in four-dimensions, we will write the action using $\mathcal{N} = 2$ *three dimensional* superspace [4]. The $\mathcal{N} = 2$ four-dimensional supersymmetry algebra generated by $Q_{i\alpha}$, $\bar{Q}^i_{\dot{\alpha}}$ is given (in the absence of central charge) by

$$\begin{aligned}
&\{Q_{i\alpha}, \bar{Q}^j_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta^j_i \\
&\{Q_{i\alpha}, Q_{j\beta}\} = \{\bar{Q}^i_{\dot{\alpha}}, \bar{Q}^j_{\dot{\beta}}\} = 0
\end{aligned} \tag{6.1}$$

where $i = 1, 2$ is the SU(2) R-symmetry index and α=1,2 is a spinor index. This algebra contains two copies of an $\mathcal{N} = 2$, $d = 3$ supersymmetry algebra. Defining

$$\begin{aligned}
Q_\alpha &\equiv \frac{1}{2}(Q_{1\alpha} + \bar{Q}^1_{\dot{\alpha}}) + \frac{i}{2}(Q_{2\alpha} + \bar{Q}^2_{\dot{\alpha}}) \\
\hat{Q}_\alpha &\equiv \frac{i}{2}(Q_{1\alpha} - \bar{Q}^1_{\dot{\alpha}}) - \frac{1}{2}(Q_{2\alpha} - \bar{Q}^2_{\dot{\alpha}}) ,
\end{aligned} \tag{6.2}$$

[4] The idea of writing the action for a supersymmetric theory in terms of a lower dimensional superspace has been discussed and applied in a variety of other situations [27, 43–48].

The $\mathcal{N}=2$, $d=4$ algebra becomes

$$\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^M_{\alpha\beta} P_M\,, \qquad M=0,1,3$$
$$\{\hat{Q}_\alpha, \bar{\hat{Q}}_\beta\} = 2\sigma^M_{\alpha\beta} P_M \tag{6.3}$$
$$\{Q_\alpha, \bar{\hat{Q}}_\beta\} = \{\hat{Q}_\alpha, \bar{Q}_\beta\} = -2i\sigma^2_{\alpha\beta} P_2\,, \tag{6.4}$$

with all other commutators vanishing. The supercharges Q_α and $\bar{Q}_\beta$ (or $\hat{Q}_\alpha$ and $\bar{\hat{Q}}_\beta$) generate an $\mathcal{N}=2$, $d=3$ supersymmetry. In the next section we re-write the $\mathcal{N}=2$ theory in a superspace which makes an $\mathcal{N}=2$, $d=3$ supersymmetry and not an $\mathcal{N}=1$, $d=4$ supersymmetry manifest. A remarkable feature of this superspace is that a class of straight Wilson lines can be written as bottom components of chiral superfields. The constraints on the chiral ring can then be used to make exact statements about expectations values of these Wilson lines.

6.1 $\mathcal{N}=2$, $d=4$ SYM in $\mathcal{N}=2$, $d=3$ superspace

We wish to write the action for the four-dimensional $\mathcal{N}=2$ Sp(N) SYM theory that we have considered in the previous sections in terms of three-dimensional $\mathcal{N}=2$ superspace. It is straightforward to generalize the analysis in this section to other $\mathcal{N}=2$ theories. Similar non-renormalisation properties for straight Wilson-line operators can be proven for a large class of eight supercharge Yang-Mills theories with hypermultiplets.

The action of the $\mathcal{N}$=2 Sp(N) SYM theory can be written in terms of conventional four-dimensional $\mathcal{N}$=1 superfields as

$$\begin{aligned} S \;=\; \frac{1}{g^2_{\rm YM}} \int \mathrm{d}^4x \Bigg\{ & \frac{1}{16}\left[\int \mathrm{d}^2\theta\, \mathrm{tr}\left(\mathcal{W}^\alpha \mathcal{W}_\alpha\right) + \text{ h.c.}\right] + \int \mathrm{d}^2\theta \mathrm{d}^2\bar\theta\, \mathrm{tr}\left(\mathrm{e}^{-V}\Phi^\dagger \mathrm{e}^V \Phi\right) \\ & + \int \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \left[\Psi^\dagger_r \left(\mathrm{e}^V\right)^i \left(T^i_{\rm a.s.}\right)^{rs} \Psi_s + \widetilde{\Psi}^\dagger_r \left(\mathrm{e}^{-V}\right)^i \left(T^i_{\rm a.s.}\right)^{rs} \widetilde{\Psi}_s\right] \\ & + \int \mathrm{d}^2\theta \mathrm{d}^2\bar\theta \left[(\mathcal{Q}^{9-n})^\dagger_a \left(\mathrm{e}^V\right)^i \left(T^i_{\rm f}\right)^{ab} \mathcal{Q}^n_b + \widetilde{\mathcal{Q}}^{n\,\dagger}_a \left(\mathrm{e}^{-V}\right)^i \left(T^i_{\rm f}\right)^{ab} \widetilde{\mathcal{Q}}^{9-n}_b\right] \\ & + \left[\int \mathrm{d}^2\theta \left(\widetilde{\Psi}_r \Phi^i \left(T^i_{\rm a.s.}\right)^{rs} \Psi_s + \widetilde{\mathcal{Q}}^{9-n}_a \Phi^i \left(T^i_{\rm f}\right)^{ab} \mathcal{Q}^n_b\right) + \text{ h.c.}\right]\Bigg\}\,, \end{aligned} \tag{6.5}$$

where we have denoted by V and Φ the $\mathcal{N}$=1 vector and chiral multiplets in the adjoint of Sp(N) which form the $\mathcal{N}$=2 vector multiplet, by Ψ and $\widetilde{\Psi}$ the two $\mathcal{N}$=1 chiral multiplets in the second rank anti-symmetric hypermultiplet and by $\mathcal{Q}$ and $\widetilde{\mathcal{Q}}$ the chiral multiplets forming the hypermultiplets in the fundamental. The traces are over matrices in the fundamental used to represent the fields in the $\mathcal{N}$=2 vector multiplet. The $T^i_{\rm a.s.}$'s, $i = 1,\ldots,N(2N+1)$ are generators in the anti-symmetric and the indices r and s run from 1 to $N(2N-1)$, whereas the $T^i_{\rm f}$'s in the fundamental have indices a and b running from 1 to $2N$. The index $n=1,\ldots,4$ is used to label the fundamental of SO(8). The hypermultiplets $\mathcal{H}^u_{\rm f}$, $u=1,\ldots,8$, are decomposed into $\mathcal{N}$=1 chiral multiplets $\mathcal{Q}^1,\ldots,\mathcal{Q}^4$ and $\widetilde{\mathcal{Q}}^5,\ldots,\widetilde{\mathcal{Q}}^8$ and similarly their conjugates $\mathcal{H}^{u\,\dagger}_{\rm f}$ into $\widetilde{\mathcal{Q}}^{1\,\dagger},\ldots,\widetilde{\mathcal{Q}}^{4\,\dagger}$ and $\mathcal{Q}^{5\,\dagger},\ldots,\mathcal{Q}^{8\,\dagger}$. The field strength superfield $\mathcal{W}_\alpha$ is defined as usual as

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}\bar{D}\mathrm{e}^{-V} D_\alpha \mathrm{e}^V\,. \tag{6.6}$$

In (6.5) a gauge fixing term has not been indicated explicitly.

Note that the dimensional reduction of $\mathcal{N}=1$, $d=4$ superspace to three dimensions gives $\mathcal{N}=2$, $d=3$ superspace. As a starting point for obtaining the above action in an $\mathcal{N}=2$, $d=3$ superspace, one can first dimensionally reduce (6.5) on a circle (say the x^3 direction), giving a three dimensional eight supercharge theory written in $\mathcal{N}=2$, $d=3$ superspace (which is thus manifestly invariant under four real supersymmetries). One can then re-introduce the non-zero modes in the x^3 direction, while keeping the dimension of the superspace fixed. The three-dimensional $\mathcal{N}=2$ superfields used to describe a four-dimensional $\mathcal{N}=2$ theory have the general form $F(x^0,x^1,x^2,\theta,\bar\theta|X^3)$, where the superspace is spanned by $x^0,x^1,x^2,\theta,\bar\theta$ and X^3 is to be viewed as a continuous label. An $\mathcal{N}=2$, $d=4$ vector multiplet corresponds to a continuous set of $\mathcal{N}=2$, $d=3$ vector superfields $V(X^3)$ together with a continuous set of $\mathcal{N}=2$, $d=3$ chiral superfields $\Phi(X^3)$. An $\mathcal{N}=2$, $d=4$ hypermultiplet corresponds to a continuous set of doublets of $\mathcal{N}=2$, $d=3$ chiral multiplets, $\mathcal{Q}(X^3)$ and $\widetilde{\mathcal{Q}}(X^3)$.

Aside from the continuous index, the superfield content of an $\mathcal{N}=2$, $d=4$ theory in $\mathcal{N}=2$, $d=3$ superspace is basically the same as in $\mathcal{N}=1$, $d=4$ superspace. However the component fields are distributed amongst the superfields differently. For the $\mathrm{Sp}(N)$ theory we have been considering, the requisite $\mathcal{N}=2$, $d=3$ superfields are again a vector superfield V, adjoint chiral Φ, anti-symmetric chirals Ψ and $\widetilde{\Psi}$, and fundamental chirals $\mathcal{Q}^m$ and $\widetilde{\mathcal{Q}}^{9-m}$. The $\mathcal{N}=2$, $d=4$ vector multiplet contains the gauge connections $A_{0,1,2,3}$ and adjoint Hermitian scalars X^1, X^2, which are distributed amongst the $\mathcal{N}=2$, $d=3$ superfields V and Φ as follows

$$\begin{aligned} &V \to A_{0,1,2}\,,\ X^1\,,\\ &\Phi \to A_3\,,\ X^2\,. \end{aligned} \tag{6.7}$$

The bottom component of Φ is A_3+iX^2. The remarkable fact that a gauge connection belongs to a chiral superfield will be used to obtain exact results for expectation values of Wilson lines. Henceforward, all superfields we write are in $\mathcal{N}=2, d=3$ superspace.

An example of an $\mathcal{N}=2$, $d=4$ Yang-Mills action in $\mathcal{N}=2$, $d=3$ superspace was written in [45]. For the case which we consider the structure is essentially the same. For our purposes the most important term in the $\mathcal{N}=2$, $d=3$ superspace representation of the action is the superpotential, which is given by

$$\begin{aligned} \mathrm{W} \;=\; \int \mathrm{d}X^3\,\mathrm{d}^3x\,\mathrm{d}^2\theta\,\Big[&\widetilde{\mathcal{Q}}^{9-n}_a\left(i\delta^{ab}\partial_{X^3}-\Phi^i\left(T^i_{\mathrm{f}}\right)^{ab}\right)\mathcal{Q}^n_b\\ &+\widetilde{\Psi}_r\left(i\delta^{rs}\partial_{X^3}-\Phi^i\left(T^i_{\mathrm{a.s.}}\right)^{rs}\right)\Psi_s+\mathrm{tr}\left(\mathcal{W}^\alpha\mathcal{W}_\alpha\right)\Big]\,. \end{aligned} \tag{6.8}$$

Although the details of the Kähler potential will not be be important in the subsequent discussion, we record it below for completeness

$$\begin{aligned} \mathrm{K} \;=\; \frac{1}{g^2_{\mathrm{YM}}}\int \mathrm{d}X^3\,\mathrm{d}^3x\,\Big\{&\int \mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,\mathrm{tr}\left(\mathrm{e}^{-V}\Phi'^{\dagger}\mathrm{e}^{V}\Phi'\right)\\ &+\int \mathrm{d}^2\theta\mathrm{d}^2\bar\theta\left[\Psi^\dagger_r\left(\mathrm{e}^V\right)^i\left(T^i_{\mathrm{a.s.}}\right)^{rs}\Psi_s+\widetilde{\Psi}^\dagger_r\left(\mathrm{e}^{-V}\right)^i\left(T^i_{\mathrm{a.s.}}\right)^{rs}\widetilde{\Psi}_s\right]\\ &+\int \mathrm{d}^2\theta\mathrm{d}^2\bar\theta\left[\left(\mathcal{Q}^{9-n}\right)^\dagger_a\left(\mathrm{e}^V\right)^i\left(T^i_{\mathrm{f}}\right)^{ab}\mathcal{Q}^n_b+\widetilde{\mathcal{Q}}^{n\,\dagger}_a\left(\mathrm{e}^{-V}\right)^i\left(T^i_{\mathrm{f}}\right)^{ab}\widetilde{\mathcal{Q}}^{9-n}_b\right]\Big\}\,, \end{aligned} \tag{6.9}$$

where

$$\Phi' \equiv \Phi + e^{-V}(i\partial_{X^3} - \bar{\Phi})e^V \tag{6.10}$$

Note that the kinetic terms for fields arising from $\mathcal{N}=2$, $d=4$ hypermultiplets involving derivatives with respect to X^3 arise from the superpotential in $\mathcal{N}=2$, $d=3$ superspace rather than the Kähler potential. This fact, together with four-dimensional Lorentz invariance, can be used to show the absence of radiative corrections to the metric on the Higgs branch. We will use the fact that a kinetic term is contained in the superpotential to show that the expectation values of a class of straight BPS Wilson lines are the same as the expectation values of local operators parameterizing the Higgs branch.

6.2 Chiral Wilson lines

One can define Wilson lines in the four-dimensional $\mathcal{N}=2$ theory which are chiral with respect to $\mathcal{N}=2$, $d=3$ supersymmetry. This is possible because the bottom component of the adjoint $\mathcal{N}=2$, $d=3$ chiral superfield contains a gauge connection, $\Phi = A_3 + iX^2 + \cdots$. Under gauge transformations parameterized by $\mathcal{N}=2$, $d=3$ superfields $\Lambda(X^3)$, Φ transforms as

$$\Phi \to \mathrm{e}^{i\Lambda}\Phi\mathrm{e}^{-i\Lambda} - i\,\mathrm{e}^{i\Lambda}\frac{\partial}{\partial x^3}\mathrm{e}^{-i\Lambda}\,. \tag{6.11}$$

Thus a gauge invariant chiral superfield whose components are Wilson lines is given by

$$\mathscr{W}_{\mathrm{f}}^{mn}(X^3) \equiv \widetilde{\mathcal{Q}}_a^{9-m}(0)\,\mathcal{P}\exp\left(i\int_0^{X^3}\mathrm{d}X'^3\,\Phi\right)^{ab}\mathcal{Q}_b^n(X^3)\,. \tag{6.12}$$

Note that this Wilson line is straight, extending only in the X^3 direction transverse to the superspace. The chiral structure is lost if one considers Wilson lines which are not straight.

$\mathscr{W}^{mn}(X^3)$ in equation (6.12) decomposes into $\mathbf{1}\oplus\mathbf{28}\oplus\mathbf{35}$ with respect to the SO(8) global symmetry. The expectation value of the bottom component,

$$W_{\mathrm{f}}^{mn} = \mathscr{W}_{\mathrm{f}}^{mn}\Big|_{\theta=\bar{\theta}=0}\,,$$

satisfies

$$\begin{aligned}\partial_{X^3}\langle W_{\mathrm{f}}^{mn}(X^3)\rangle &= \langle\,\widetilde{\mathcal{Q}}_a^{9-m}(0)\mathcal{P}\exp\left(i\int_0^{X^3}\mathrm{d}X'^3\,\Phi\right)^{ac}\left(\partial_{X^3}+i\Phi(X^3)\right)^{cb}\mathcal{Q}_b^n(X^3)\Big|_{\theta=\bar{\theta}=0}\rangle \\ &= -i\langle\,\widetilde{\mathcal{Q}}_a^{9-m}(0)\mathcal{P}\exp\left(i\int_0^{X^3}\mathrm{d}X'^3\,\Phi\right)^{ac}\frac{\delta\mathrm{W}}{\delta\widetilde{\mathcal{Q}}_c^{9-n}(X^3)}\Bigg|_{\theta=\bar{\theta}=0}\rangle\,,\end{aligned} \tag{6.13}$$

where W is the superpotential (6.8).

Consider an infinitesimal variation of the form

$$\widetilde{\mathcal{Q}}_c^{9-n}(X^3) \to \widetilde{\mathcal{Q}}_c^{9-n}(X^3) + \epsilon f_c^n(X^3) \tag{6.14}$$

where $f_c(X^3)$ is a functional of the chiral superfields. Specifically, we choose

$$f_c^n(X^3) = \widetilde{\mathcal{Q}}_b^{9-n}(0)\mathcal{P}\exp\left(i\int_0^{X^3} \mathrm{d}X'^3\, \Phi(X'^3)\right)^{bc} . \tag{6.15}$$

The classical equation derived from this variation is

$$\bar{D}^2\left[f_a^m \left(\mathrm{e}^{-V}\right)^{ab} \widetilde{\mathcal{Q}}_b^{n\dagger}\right] = f_a^m \frac{\delta \mathrm{W}}{\delta \widetilde{\mathcal{Q}}_a^{9-n}} . \tag{6.16}$$

Classical equations of this form are often modified quantum mechanically, giving rise to what is known as a generalized Konishi anomaly [28]. In appendix B we show that there is no Konishi anomaly in this case.

In a supersymmetric vacuum $\langle \bar{D}^2(\cdots)|_{\theta=\bar{\theta}=0}\rangle = 0$. Hence equation (6.16) implies $\langle f_a^m \frac{\delta \mathrm{W}}{\delta \widetilde{\mathcal{Q}}_a^{9-n}}\rangle = 0$, so that (6.13) becomes

$$\partial_{X^3}\langle W_{\mathrm{f}}^{mn}(X^3)\rangle = 0 . \tag{6.17}$$

Therefore the expectation value of these Wilson lines is the same as that of a local operator parameterizing the Higgs branch,

$$\langle W_{\mathrm{f}}^{mn}(L)\rangle = \langle W_{\mathrm{f}}^{mn}(0)\rangle = \langle \widetilde{q}_a^m q_a^n\rangle , \tag{6.18}$$

where $\widetilde{q}$ and q are the bottom components of $\widetilde{\mathcal{Q}}$ and $\mathcal{Q}$ respectively.

As already observed, the operators W_{f}^{mn} decompose with respect to the SO(8) global symmetry into

$$\begin{aligned} W_{\mathrm{f};\mathbf{1}} &= W_{\mathrm{f}}^{mm} \\ W_{\mathrm{f};\mathbf{28}}^{[mn]} &= \frac{1}{2}\left(W_{\mathrm{f}}^{mn} - W_{\mathrm{f}}^{nm}\right) \\ W_{\mathrm{f};\mathbf{35}}^{\{mn\}} &= \frac{1}{2}\left(W_{\mathrm{f}}^{mn} + W_{\mathrm{f}}^{nm}\right) - \frac{1}{8}\delta^{mn}W_{\mathrm{f}}^{kk} . \end{aligned} \tag{6.19}$$

At different points on the Higgs branch some or all of the combinations (6.19) can be non-zero. As a result the SO(8) global symmetry is unbroken if only $W_{\mathrm{f};\mathbf{1}}$ is non-vanishing, or broken if either $W_{\mathrm{f};\mathbf{28}}^{[mn]}$ or $W_{\mathrm{f};\mathbf{35}}^{\{mn\}}$ have a non-vanishing expectation value.

The same analysis can be repeated without any modification for straight Wilson lines constructed from the hypermultiplets in the anti-symmetric representation. In this case one defines

$$\mathscr{W}_{\mathrm{a.s.}}(X^3) \equiv \widetilde{\Psi}_r(0)\mathcal{P}\exp\left(i\int_0^{X^3} \mathrm{d}X'^3\, \Phi\right)^{rs} \Psi_s(X^3) \tag{6.20}$$

and

$$W_{\mathrm{a.s.}} = \mathscr{W}_{\mathrm{a.s.}}\Big|_{\theta=\bar{\theta}=0} . \tag{6.21}$$

By the same steps described above one can show that

$$W_{\mathrm{a.s.}}(X^3) = W_{\mathrm{a.s.}}(0) = \langle \widetilde{\phi}_r \phi_r\rangle , \tag{6.22}$$

where $\widetilde{\phi}$ and ϕ are the bottom components of $\widetilde{\Psi}$ and Ψ respectively.

Analogous arguments can be applied in other $\mathcal{N}$=2 SYM theories, *e.g.* in the case of the U(N) theory with N_f hypermultiplets in fundamental considered in the AdS/CFT context in [36]. In these theories one can define similar BPS Wilson-line operators whose expectation values are independent of the length and parameterize the Higgs branch. Furthermore the arguments leading to equation (6.17) are unmodified upon dimensionally reducing in directions belonging to the $\mathcal{N}=2$, $d=3$ superspace. Thus (6.17) also applies to eight-supercharge Yang–Mills theories in $3, 2$ and 1 dimension [5]. However the situation is different for five dimensional $\mathcal{N}=1$ theories, for which there is a non-trivial generalized Konishi anomaly (see appendix B).

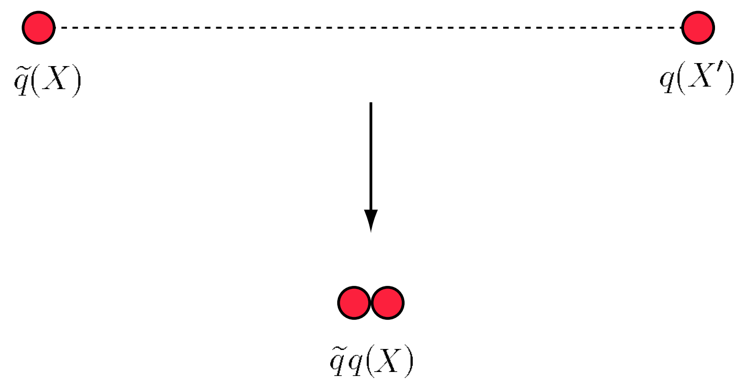

Figure 1: Chiral Wilson lines with the same expectation values. The circles indicate scalar components of fundamental hypermultiplets. Wilson lines with anti-symmetric hypermultiplets at the end points have the same property.

In the case of five-dimensional theories one uses a four-dimensional $\mathcal{N}$=1 superspace and it is possible to derive a classical equation like (6.16). However in this case there is a quantum anomaly, as is shown in appendix B.

7 Conclusions and future directions

In this article we have presented a proposal for the dual AdS description of the Higgs branch of a finite four-dimensional $\mathcal{N}=2$ Sp(N) gauge theory with one hypermultiplet in the second rank anti-symmetric representation and four hypermultiplets in the fundamental representation. The theory has a SO(8) flavour symmetry which corresponds to a bulk gauge symmetry associated with the presence of D7-branes. There is a one to one map between instantons in the eight-dimensional SO(8) super Yang-Mills theory on the world-volume of the D7-branes and the Higgs branch of the $\mathcal{N}=2$ Sp(N) gauge theory. Although the AdS description involves D7-branes in a curved background with Ramond-Ramond flux, the equations of motion at leading order in α' admit solutions corresponding to field strengths which are anti-self-dual with respect to a flat four dimensional metric, *i.e.* ordinary SO(8) instantons. Furthermore the metric describing

[5]Notice that in 1 dimension, the superspace one uses to show (6.17) is actually zero dimensional.

the dynamics of slowly varying instantons on the D7-brane world-volume reproduces the correct metric on the Higgs branch.

To order α'^2, we have checked that terms in the D7-action which only involve field strengths are of the right form to preserve the anti-self-dual solutions and give the correct metric on the Higgs branch. Little is known about bulk couplings to the D7-brane beyond leading order in α'. The correspondence with the Higgs branch of the dual gauge theory in the set-up that we have considered, together with known non-renormalisation theorems for $\mathcal{N}$=2 SYM theories, leads to constraints on the higher-derivative terms in the D-brane effective action in the AdS background. The absence of corrections to the metric on the Higgs branch requires that the anti-self-dual solutions of the D7-brane field equations are not modified by the inclusion of α' corrections. The known couplings at order α'^2 do not satisfy this requirement and therefore additional, yet to be determined, couplings to the curvature and/or R⊗R five-form must be present.

The α' corrections to bulk couplings of D-branes are very important in the context of brane-world physics (see [57] for example) and for obtaining the strong coupling expansion of gauge theories with fields in the fundamental representation using AdS/CFT duality. The supergravity realization of the Higgs branch non-renormalization is not sufficient to determine the bulk-brane couplings, but does provide an important constraint. It would be interesting to consider the supergravity description of the Higgs branch of other eight supercharge Yang-Mills theories, which should yield additional insights into this issue.

We have also presented a new non-renormalization theorem for straight BPS Wilson lines in eight-supercharge Yang–Mills theories. This non-renormalization theorem is closely related to the physics of the Higgs branch. We have shown that open Wilson lines with hypermultiplet insertions at the end points have length independent expectation values. This result was shown using a lower-dimensional superspace which makes a particularly useful subgroup of the full extended supersymmetry manifest. In this superspace, the Wilson lines are bottom components of chiral superfields and the non-renormalisation result follows from equations satisfied by the corresponding chiral ring. It would be interesting to study whether similar superspace methods can be used to obtain other non-renormalization theorems.

We expect that the non-renormalisation properties of the Wilson-line operators be preserved when instanton effects are included. Adapting the methods developed in [56] to the case of $\mathcal{N}$=2 theories and using the BPS nature of these Wilson lines, the analysis of fermion zero-modes in the instanton background should allow to prove the absence of instanton corrections to their expectation values.

Another interesting open question is whether the expectation values of BPS open Wilson lines in the conformal Sp(N) $\mathcal{N} = 2$ theory can be computed using the AdS/CFT duality, in a way analogous to the AdS computation of the expectation values of closed Wilson loops in $\mathcal{N} = 4$ super Yang-Mills theory [53–55]. One might expect the open Wilson line expectation value to be captured by a semiclassical approximation to an open string partition function, where the open string has endpoints on D7-branes. The result should depend on the point on the Higgs branch through holonomies $\exp(i \int A)$

involving the gauge field of an SO(8) instanton background on the D7-branes.

Acknowledgments

We wish to thank M. Bianchi, G. Cardoso, J. Erdmenger, M. Green, A. Hanany, D. Lüst, J. Maldacena, D. Mateos, H. Nastase, K. Peeters, S. Ramgoolam, H. Schnitzer, W. Skiba, S. Stieberger and M. Zamaklar for useful discussions.

A Embedding geometry

Here we very briefly summarize some of the facts about embedding geometry relevant to this article. A more detailed discussion of the geometry of embeddings may be found in [19] and references therein. Consider a d-dimensional submanifold of a D-dimensional space described by $Y^A(\zeta^\alpha)$, where $A = 1, \ldots, D$ and $\alpha = 1, \ldots, d$. Pull-backs of bulk tensors to the tangent bundle are defined by contractions with $\partial_\alpha Y^A$, while pull backs to the normal bundle are defined by contractions with ξ_a^A, where lower case roman indices correspond to the normal bundle and

$$\xi_a^A \xi_b^B G_{AB} = \delta_{ab}, \qquad \xi_a^A \partial_\alpha Y^B G_{AB} = 0\,. \tag{A.1}$$

Raising and lowering of indices in the tangent bundle is defined with respect to the induced metric,

$$g_{\alpha\beta} = \partial_\alpha Y^A \partial_\beta Y^B G_{AB}\,, \tag{A.2}$$

while indices in the normal bundle are raised or lowered with respect to δ_{ab}.

The second fundamental form is defined as the covariant derivative of the tangent frame $\partial_\alpha Y^A$

$$\Omega^A_{\alpha\beta} = \Omega^A_{\beta\alpha} \equiv \partial_\alpha\partial_\beta Y^A - (\Gamma_T)^\gamma_{\alpha\beta}\partial_\gamma Y^A + \Gamma^A_{BC}\partial_\alpha Y^B \partial_\beta Y^C\,, \tag{A.3}$$

where Γ_T is the Levi-Civita connection constructed from $g_{\alpha\beta} = \partial_\alpha Y^A \partial_\beta Y^B G_{AB}$. In terms of the second fundamental form and pull-backs of the bulk Riemann tensor, the curvature tensors appearing in (4.9) are defined as follows;

$$(R_T)_{\alpha\beta\gamma\delta} \equiv R_{\alpha\beta\gamma\delta} + \delta_{ab}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^a_{\alpha\delta}\Omega^b_{\beta\gamma}) \tag{A.4}$$

$$(R_N)_{\alpha\beta}{}^{ab} \equiv -R^{ab}{}_{\alpha\beta} + g^{\gamma\delta}(\Omega^a_{\alpha\gamma}\Omega^b_{\beta\delta} - \Omega^b_{\alpha\gamma}\Omega^a_{\beta\delta}) \tag{A.5}$$

$$(\hat{R}_T)_{\alpha\beta} \equiv (R_T)^\gamma{}_{\alpha\gamma\beta} \tag{A.6}$$

$$\bar{R}_{ab} \equiv R^\alpha{}_{ab\alpha} + g^{\alpha\gamma}g^{\beta\delta}\Omega_{a|\alpha\beta}\Omega_{b|\gamma\delta} \tag{A.7}$$

The various curvature tensors appearing in (4.9) are

$$(R_T)_{\alpha\beta\gamma\delta} = \begin{cases} -\dfrac{L}{20}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) & \text{for AdS}_5 \text{ indices} \\ \dfrac{L}{20}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) & \text{for } S^3 \text{ indices} \\ 0 & \text{for mixed indices} \end{cases}$$

$$(R_T)_{\alpha\beta} = \begin{cases} -\dfrac{L}{5} g_{\alpha\beta} & \text{for AdS}_5 \text{ indices} \\ \dfrac{L}{10} g_{\alpha\beta} & \text{for } S^3 \text{ indices} \\ 0 & \text{for mixed indices} \end{cases} \tag{A.8}$$

$$(R_N)_{ab\alpha\beta} = 0\,, \tag{A.9}$$

$$\bar{R}_{ab} = -\frac{3}{20} L \delta_{ab}\,, \tag{A.10}$$

where $L = (\lambda\alpha'^2)^{1/4}$ is the scalar curvature of S^5, or minus the scalar curvature of AdS_5.

B Variation of the functional measure

In this appendix we investigate if there is a non-zero anomalous modification of the Ward identities derived in (6.16). We will compute the variation of the functional measure under (6.14) using methods discussed in [28,49]. Our discussion will closely parallel that of [24].

To be somewhat more general, we consider an eight supercharge theory in $d+1$ dimensions written in a d-dimensional four-supercharge superspace,

$$\begin{aligned} S \;=\; & \int \mathrm{d}X\, \mathrm{d}^d y\, \mathrm{d}^2\theta\, \Big(\widetilde{\mathcal{Q}}_m (i\partial_X - \Phi)\mathcal{Q}_m + \mathrm{tr}\, \mathcal{W}_\alpha \mathcal{W}^\alpha\Big) \\ & + \int \mathrm{d}X \mathrm{d}^d y\, \mathrm{d}^4\theta\, \Big(\mathcal{Q}_m^\dagger \mathrm{e}^V \mathcal{Q}_m + \widetilde{\mathcal{Q}}_m \mathrm{e}^{-V} \widetilde{\mathcal{Q}}_m^\dagger + \mathrm{tr}(\Phi'^\dagger \mathrm{e}^V \Phi' \mathrm{e}^{-V})\Big)\,, \end{aligned} \tag{B.1}$$

where V and Φ are the d-dimensional $\mathcal{N}=1$ vector and scalar multiplets and $\mathcal{Q}$, $\widetilde{\mathcal{Q}}$ are $\mathcal{N}=1$ chiral multiplets in any representation of the gauge group and m is a flavour index.

Under the infinitesimal transformation

$$\widetilde{\mathcal{Q}}_n(X) \to \widetilde{\mathcal{Q}}_n(X) + \epsilon f_{n,X}[\mathcal{Q}, \widetilde{\mathcal{Q}}, \Phi, \mathcal{W}_\alpha]\,, \tag{B.2}$$

the Jacobian is

$$J = 1 + \mathrm{tr}_c\, \epsilon \bar{D}^2 \frac{\delta f_{m,X}}{\delta \widetilde{\mathcal{Q}}_m(X)} = 1 + \int \mathrm{d}Z_c \sum_i \epsilon(Z) \langle Z, i | \bar{D}^2 \frac{\delta f^m}{\delta \widetilde{\mathcal{Q}}^m} | Z, i\rangle\,, \tag{B.3}$$

where Z collectively denotes the superspace coordinates, $z = (\vec{y}, \theta, \bar{\theta})$, and the transverse coordinate X, $\mathrm{d}Z_c$ is the chiral measure $\mathrm{d}^d y\, \mathrm{d}^2\theta\, \mathrm{d}X$, and we have defined a Hilbert space spanned by $|Z, i\rangle$, which are eigenstates of the coordinate operator Z and belong to the same representation of the gauge group under which the matter fields transform

$$\hat{Z}|Z, i\rangle = Z|Z, i\rangle, \qquad \hat{T}^A |Z, i\rangle = T^A_{ij} |Z, j\rangle\,. \tag{B.4}$$

such that

$$\frac{\delta \widetilde{\mathcal{Q}}_{mi}(Z')}{\delta \widetilde{\mathcal{Q}}_{nj}(Z)} = \delta_{mn}\, \delta_{ij}\, \delta(X - X')\, \delta^d(y - y')\, \delta^2(\theta - \theta') = \delta_{mn}\, \langle Z, i | \bar{D}^2 | Z', j\rangle\,. \tag{B.5}$$

The matrix

$$\langle Z,i|\frac{\delta f_{m,X}}{\delta\widetilde{\mathcal{Q}}_m(X)}(-\frac{1}{4}\bar{D}^2)|Z',i\rangle \tag{B.6}$$

is proportional to $\delta^2(\theta-\theta')=(\theta-\theta')^2$ and so has vanishing diagonal entries. Thus naively $J=1$. However this is not necessarily true upon regularizing the trace.

To obtain the Jacobian for the transformation $\widetilde{\mathcal{Q}}_m \to \widetilde{\mathcal{Q}}_m+\epsilon f^m$, we need to compute a regularized version of the diagonal matrix element,

$$\mathcal{M}_{X,z,i} \equiv \langle X,z,i|\bar{D}^2\frac{\delta f^m}{\delta\widetilde{\mathcal{Q}}^m}|X,z,i\rangle\,. \tag{B.7}$$

The regularization used in [49] in the more familiar context a four-dimensional $\mathcal{N}=1$ gauge theory involves insertion of an operator $\exp(-\hat{L}/M^2)$, where

$$\hat{L}\equiv -\frac{1}{16}\bar{D}^2\mathrm{e}^{-V}D^2\mathrm{e}^{V}\,. \tag{B.8}$$

Note that this operator is gauge covariant and chiral. However, the insertion of $\exp(-\hat{L}/M^2)$ will not suffice in our case, since this only cuts off large momenta in directions belonging to the superspace. The regularized version of (B.7) which we will consider is

$$\mathcal{M}_{X,z,i} \equiv \langle X,z,i|\exp(-\hat{\mathcal{L}}/M^2)\,\bar{D}^2\,\frac{\delta f^m}{\delta\widetilde{\mathcal{Q}}^m}|X,z,i\rangle\,, \tag{B.9}$$

where

$$\hat{\mathcal{L}}=\hat{L}+\left(\frac{\partial}{\partial X}+i\Phi\right)^2\,. \tag{B.10}$$

To evaluate (B.9), note that

$$\hat{L}(\bar{D}^2\cdots)=(\partial_t^2-1/2\mathcal{W}^\alpha D_\alpha+C\partial_t+F)(\bar{D}^2\cdots)\,, \tag{B.11}$$

where

$$C\equiv\frac{1}{2}\bar{D}_\alpha\mathrm{e}^{-V}\bar{D}_\alpha\mathrm{e}^{V}$$
$$F\equiv\frac{1}{16}\bar{D}^2\mathrm{e}^{-V}D^2\mathrm{e}^{V}\,. \tag{B.12}$$

We can write

$$\mathrm{e}^{-\mathcal{L}/M^2}=\mathrm{e}^{-\nabla^2/M^2}\hat{\mathcal{S}}\,, \tag{B.13}$$

where ∇^2 is the Laplacian in the space including all bosonic coordinates, $\nabla^2\equiv\partial_X^2+\nabla_y^2$. The factor $\hat{\mathcal{S}}=1+\cdots$ must contain a term with two D_α operators for $\exp(\hat{\mathcal{L}}/M^2)\bar{D}^2$ to give a non-zero contribution to (B.9). To illustrate this property, note that

$$\begin{aligned}\langle z'|D^2\bar{D}^2|z\rangle &= \delta^d(y-y')\,D^2\bar{D}^2\,\delta^2(\theta-\theta')\,\delta^2(\bar\theta-\bar\theta')\\ &= \delta^d(y-y')\,D^2\bar{D}^2(\theta-\theta')^2\,(\bar\theta-\bar\theta')^2=\delta^d(y-y')\,.\end{aligned} \tag{B.14}$$

If the D^2 were removed, the diagonal matrix element would vanish. Thus, (B.11) implies that, in a large M^2 expansion, the leading non-zero contribution to (B.9) is

$$\mathcal{M}_{X,z,i} = \langle X, z, i| \exp(-\nabla^2/M^2) \frac{W_\alpha W^\alpha}{M^4} D^2 \bar{D}^2 |X', z', i'\rangle\langle X', z', i'| \frac{\delta f^m}{\delta \widetilde{\mathcal{Q}}^m} |X, z, i\rangle + \cdots \quad \text{(B.15)}$$

where

$$\langle X', z', i'| \frac{\delta f^m}{\delta \widetilde{\mathcal{Q}}^m} |X, z, i\rangle \equiv \frac{\delta f^m_{X',i'}}{\delta \widetilde{\mathcal{Q}}^m_{X,i}}\, \delta^d(\vec{y}' - \vec{y})\, \delta^2(\theta - \theta')\, \delta^2(\bar{\theta}' - \bar{\theta})\,. \quad \text{(B.16)}$$

Equation (B.15) can be evaluated by inserting the identity $|k_X, \vec{k}_y, j\rangle\langle k_X, \vec{k}_y, j|$ after $\exp(-\nabla^2/M^2)$, where $|k_X, \vec{k}_y, j\rangle$ is an eigenvector of the momentum operators in the transverse direction X and the bosonic part of the superspace z. The result is

$$\begin{aligned}\mathcal{M}_{X,z,i} &= \int \mathrm{d}k_X\, \mathrm{d}^d k_y\, \mathrm{d}X'\, \mathrm{d}^d y'\, \exp\left(-\frac{k_X^2 + \vec{k}_y^2}{M^2} + i k_X (X - X') + i \vec{k}_y \cdot (\vec{y} - \vec{y}') \right) \\ &\quad \frac{1}{M^4}\, W^D_\alpha(X', z')\, W^{\alpha E}(X', z')\, \langle i|\hat{T}^D \hat{T}^E|i'\rangle\, \frac{\delta f^m_{X',i'}}{\delta \widetilde{\mathcal{Q}}^m_{X,i}}\, \delta^d(\vec{y}' - \vec{y}) \\ &= M^{d-4}\, W^D_\alpha(X, z)\, W^{\alpha E}(X, z)\, \langle i|\hat{T}^D \hat{T}^E|i'\rangle \frac{\delta f^m_{X,i'}}{\delta \widetilde{\mathcal{Q}}^m_{X,i}}\,. \end{aligned} \quad \text{(B.17)}$$

The anomaly vanishes if the superspace is less than four-dimensional ($d < 4$), but it is non-trivial when $d = 4$, corresponding to a five-dimensional $\mathcal{N} = 1$ theory. This anomaly is crucial for the validity of Dijkgraff-Vafa conjectures relating effective superpotentials to auxiliary matrix models (or an auxiliary matrix quantum mechanics in this case) [27–29]. It does not make sense to discuss $d > 4$, since there is no four supercharge superspace in more than four dimensions.

References

[1] J. M. Maldacena, "The large N limit of superconformal field theories and supergravity", *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory", *Phys. Lett.* **B428** (1998) 105 [hep-th/9802109].

[3] E. Witten, "Anti-de Sitter space and holography", *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] A. Fayyazuddin and M. Spalinski, "Large N superconformal gauge theories and supergravity orientifolds", *Nucl. Phys.* **B535** (1998) 219 [hep-th/9805096].

[5] O. Aharony, A. Fayyazuddin and J. M. Maldacena, "The large N limit of N = 2,1 field theories from three-branes in F-theory", *J. High Energy Phys.* **07** (1998) 013 [hep-th/9806159].

[6] D. J. Gross and E. Witten, "Superstring Modifications Of Einstein's Equations", *Nucl. Phys.* **B277** (1986) 1.

[7] E. Bergshoeff, M. Rakowski and E. Sezgin, "Higher Derivative Super-Yang-Mills Theories", *Phys. Lett.* **B185** (1987) 371.

[8] A. A. Tseytlin, "Vector Field Effective Action In The Open Superstring Theory", *Nucl. Phys.* **B276** (1986) 391 [Erratum-ibid. **B291** (1987) 876].

[9] A. A. Tseytlin, "On non-abelian generalisation of the Born-Infeld action in string theory", *Nucl. Phys.* **B501** (1997) 41 [hep-th/9701125].

[10] E. A. Bergshoeff, A. Bilal, M. de Roo and A. Sevrin, "Supersymmetric non-abelian Born-Infeld revisited", *J. High Energy Phys.* **07** (2001) 029 [hep-th/0105274].

[11] P. Koerber and A. Sevrin, "The non-Abelian Born-Infeld action through order α'^3", *J. High Energy Phys.* **0110** (2001) 003 [hep-th/0108169].

[12] P. Koerber and A. Sevrin, "The non-abelian D-brane effective action through order α'^4", *J. High Energy Phys.* **10** (2002) 046 [hep-th/0208044].

[13] A. Collinucci, M. De Roo and M. G. C. Eenink, "Supersymmetric Yang–Mills theory at order α'^3", *J. High Energy Phys.* **06** (2002) 024 [hep-th/0205150].

[14] M. R. Douglas, "Branes within branes", hep-th/9512077.

[15] M. R. Douglas, "Gauge Fields and D-branes", *J. Geom. Phys.* **28** (1998) 255 [hep-th/9604198].

[16] E. Witten, "Small Instantons in String Theory", *Nucl. Phys.* **B460** (1996) 541 [hep-th/9511030].

[17] N. Dorey, T. J. Hollowood, V. V. Khoze and M. P. Mattis, "The calculus of many instantons", *Phys. Rept.* **371** (2002) 231 [hep-th/0206063].

[18] A. Giveon and D. Kutasov, "Brane dynamics and gauge theory", *Rev. Mod. Phys.* **71** (1999) 983 [hep-th/9802067].

[19] C. P. Bachas, P. Bain and M. B. Green, "Curvature terms in D-brane actions and their M-theory origin", *J. High Energy Phys.* **05** (1999) 011 [hep-th/9903210].

[20] A. Fotopoulos, "On α'^2 corrections to the D-brane action for non-geodesic world-volume embeddings", *J. High Energy Phys.* **09** (2001) 005 [hep-th/0104146].

[21] M. Wijnholt, "On curvature-squared corrections for D-brane actions", hep-th/0301029.

[22] M. B. Green and C. Stahn, "D3-branes on the Coulomb branch and instantons", *J. High Energy Phys.* **09** (2003) 052 [hep-th/0308061].

[23] P. C. Argyres, M. R. Plesser and N. Seiberg, “The Moduli Space of N=2 SUSY QCD and Duality in N=1 SUSY QCD”, *Nucl. Phys.* **B471** (1996) 159 [hep-th/9603042].

[24] Z. Guralnik and B. Kulik, “Properties of chiral Wilson loops”, hep-th/0309118.

[25] R. Dijkgraaf and C. Vafa, “Matrix models, topological strings, and supersymmetric gauge theories”, *Nucl. Phys.* **B644** (2002) 3 [hep-th/0206255].

[26] R. Dijkgraaf and C. Vafa, “A perturbative window into non-perturbative physics”, hep-th/0208048.

[27] R. Dijkgraaf and C. Vafa, “N = 1 supersymmetry, deconstruction, and bosonic gauge theories”, hep-th/0302011.

[28] F. Cachazo, N. Seiberg and E. Witten, “Chiral Rings and Phases of Supersymmetric Gauge Theories”, *J. High Energy Phys.* **04** (2003) 018 [hep-th/0303207].

[29] I. Bena and R. Roiban, “N = 1* in 5 dimensions: Dijkgraaf-Vafa meets Polchinski-Strassler”, *J. High Energy Phys.* **11** (2003) 001 [hep-th/0308013].

[30] M. Gutperle, “Heterotic/type I duality, D-instantons and a $\mathcal{N} = 2$ AdS/CFT correspondence”, *Phys. Rev.* **D60** (1999) 126001 [hep-th/9905173].

[31] E. Gava, K. S. Narain and M. H. Sarmadi, “Instantons in $\mathcal{N}$=2 Sp(N) superconformal gauge theories and the AdS/CFT correspondence”, *Nucl. Phys.* **B569** (2000) 183 [hep-th/9908125].

[32] T. J. Hollowood, “Instantons, finite $\mathcal{N}$=2 Sp(N) theories and the AdS/CFT correspondence”, *J. High Energy Phys.* **11** (1999) 012 [hep-th/9908201].

[33] D. Berenstein, E. Gava, J. M. Maldacena, K. S. Narain and H. Nastase, “Open strings on plane waves and their Yang-Mills duals”, hep-th/0203249.

[34] A. Sen, “F-theory and Orientifolds”, *Nucl. Phys.* **B475** (1996) 562 [hep-th/9605150].

[35] H. J. Schnitzer and N. Wyllard, “An orientifold of $\mathrm{AdS}_5 \times T^{11}$ with D7-branes, the associated α'^2 corrections and their role in the dual $\mathcal{N}$=1 Sp($2N$+$2M$)×Sp($2N$) gauge theory”, *J. High Energy Phys.* **08** (2002) 012 [hep-th/0206071].

[36] A. Karch and E. Katz, “Adding flavor to AdS/CFT”, *J. High Energy Phys.* **06** (2002) 043 [hep-th/0205236].

[37] M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld and Y. I. Manin, “Construction Of Instantons”, *Phys. Lett.* **A65** (1978) 185.

[38] M. B. Green, J. A. Harvey and G. W. Moore, “I-brane inflow and anomalous couplings on D-branes”, *Class. and Quant. Grav.* **14** (1997) 47 [hep-th/9605033].

[39] K. Dasgupta, D. P. Jatkar and S. Mukhi, “Gravitational couplings and Z(2) orientifolds”, *Nucl. Phys.* **B523** (1998) 465 [hep-th/9707224].

[40] J. F. Morales, C. A. Scrucca and M. Serone, "Anomalous couplings for D-branes and O-planes", *Nucl. Phys.* **B552** (1999) 291 [hep-th/9812071].

[41] B. Stefanski, "Gravitational couplings of D-branes and O-planes", *Nucl. Phys.* **B548** (1999) 275 [hep-th/9812088].

[42] K. Zarembo, "Supersymmetric Wilson loops", *Nucl. Phys.* **B643** (2002) 157 [hep-th/0205160].

[43] N. Marcus, A. Sagnotti and W. Siegel, "Ten-Dimensional Supersymmetric Yang–Mills Theory In Terms Of Four-Dimensional Superfields", *Nucl. Phys.* **B224** (1983) 159.

[44] N. Arkani-Hamed, T. Gregoire and J. Wacker, "Higher dimensional supersymmetry in 4D superspace", *J. High Energy Phys.* **03** (2002) 055 [hep-th/0101233].

[45] J. Erdmenger, Z. Guralnik and I. Kirsch, "Four-dimensional superconformal theories with interacting boundaries or defects", *Phys. Rev.* **D66** (2002) 025020 [hep-th/0203020].

[46] N. R. Constable, J. Erdmenger, Z. Guralnik and I. Kirsch, "Intersecting D3-branes and holography", *Phys. Rev.* **D68** (2003) 106007 [hep-th/0211222].

[47] N. R. Constable, J. Erdmenger, Z. Guralnik and I. Kirsch, "(De)constructing intersecting M5-branes", *Phys. Rev.* **D67** (2003) 106005 [hep-th/0212136].

[48] J. Erdmenger, Z. Guralnik, R. Helling and I. Kirsch, "A world-volume perspective on the recombination of intersecting branes", hep-th/0309043.

[49] K. I. Konishi and K. I. Shizuya, "Functional Integral Approach To Chiral Anomalies In Supersymmetric Gauge Theories", *Nuovo Cim.* **A90** (1985) 111.

[50] A. Fotopoulos and A. A. Tseytlin, "On gravitational couplings in D-brane action", *J. High Energy Phys.* **12** (2002) 001 [hep-th/0211101].

[51] T. Banks, M. R. Douglas and N. Seiberg, "Probing F-theory with branes", *Phys. Lett.* **B387** (1996) 278 [hep-th/9605199].

[52] N. S. Manton, "A Remark On The Scattering Of BPS Monopoles", *Phys. Lett.* **B110** (1982) 54.

[53] J. M. Maldacena, "Wilson loops in large N field theories", *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002].

[54] Soo-Jong Rey and Jung-Tay Yee, "Macroscopic strings as heavy quarks: Large-N gauge theory and anti-de Sitter supergravity", *Eur. Phys. J.* **C22** (2001) 379 [hep-th/9803001].

[55] N. Drukker, D. J. Gross and H. Ooguri, "Wilson loops and minimal surfaces", *Phys. Rev.* **D60** (1999) 125006 [hep-th/9904191].

[56] M. Bianchi, M.B. Green and S. Kovacs, "Instanton corrections to circular Wilson loops in $\mathcal{N}=4$ supersymmetric Yang–Mills", *J. High Energy Phys.* **04** (2002) 040 [hep-th/0202003]; "Instantons and BPS Wilson loops", hep-th/0107028.

[57] A. R. Frey, "String theoretic bounds on Lorentz-violating warped compactification", *J. High Energy Phys.* **04** (2003) 012 [hep-th/0301189].

[58] A. M. Polyakov, "Fine Structure Of Strings", *Nucl. Phys.* **B268** (1986) 406.

[59] H. Kleinert, "The Membrane Properties Of Condensing Strings", *Phys. Lett.* **B174** (1986) 335.