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Complete adiabatic waveform templates for a test-mass in the Schwarzschild spacetime: VIRGO and Advanced LIGO studies

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Post-Newtonian expansions of the binding energy and gravitational wave flux truncated at the *same relative* post-Newtonian order form the basis of the *standard adiabatic* approximation to the phasing of gravitational waves from inspiralling compact binaries. Viewed in terms of the dynamics of the binary, the standard approximation is equivalent to neglecting certain conservative post-Newtonian terms in the acceleration. A new *complete adiabatic* approximant constructed from the energy and flux functions is proposed. At the leading order it employs the 2PN energy function rather than the 0PN one in the standard approximation, so that, effectively the approximation corresponds to the dynamics where there are no missing post-Newtonian terms in the acceleration. We compare the overlaps of the standard and complete adiabatic approximant templates with the exact waveform (in the adiabatic approximation) for a test-particle orbiting a Schwarzschild black hole. Overlaps are computed for the VIRGO and the Advanced LIGO noise spectra.

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I. INTRODUCTION

Coalescing compact binaries consisting of black holes and/or neutron stars are among the most promising sources for ground-based interferometric gravitational wave (GW) detectors. One of the main challenges of the data analysis for this kind of sources is to create a template bank with which the detector output may be optimally filtered. This, in turn, requires an accurate description of the time-evolution of the GW phase. Binary coalescences are the end state of a long period of adiabatic dynamics in which the orbital frequency of the system changes as a result of gravitational radiation reaction but the change in frequency per orbit is negligible compared to the orbital frequency itself. Then the inspiral orbit can be thought of as an adiabatic perturbation of a number of circular orbits (with a specific *conserved* energy associated with each of them). Given the binding energy $E(v)$ and gravitational wave luminosity $\mathcal{F}(v)$ of the binary, the phasing $\varphi(t)$ of the waves can be calculated in the *adiabatic* approximation using the following ordinary, coupled differential equations:

$$\frac{d\varphi}{dt} = \frac{2v^3}{m}, \qquad \frac{dv}{dt} = -\frac{\mathcal{F}(v)}{mE'(v)}, \tag{1.1}$$

where $E'(v) = dE(v)/dv$, and $m = m_1 + m_2$ is the total mass of the binary. The binding energy $E(v)$ and gravitational wave luminosity $\mathcal{F}(v)$ are calculated, in general, as post-

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Newtonian (PN) expansions in terms of an invariantly defined velocity parameter v [1]. The phasing of GWs obtained by numerically solving the above phasing formula is called *Taylor T1* approximant [1].

A. Standard and complete approximants

The standard approach to the GW phasing is based on the PN expansions of the binding energy (energy function) and GW luminosity (flux function) truncated at the *same relative* PN order [2]. At the lowest order, it uses only the leading terms in the energy (Newtonian) and flux (quadrupolar) functions. For higher order phasing, the energy and flux functions are retained to the same relative PN orders. We refer to this usual physical treatment of the phasing of GWs computed in the adiabatic approximation as the *standard adiabatic* approximation.

With a view to going beyond the adiabatic approximation, we must think of this in terms of the dynamics of the binary under conservative relativistic forces and gravitational radiation damping. In the conservative dynamics of the binary, wherein there is no dissipation, the energy is expressed as a post-Newtonian expansion in v^2, with the dynamics involving only even powers of v. When radiation reaction is added to the dynamics, then the equation of motion will have terms of both odd and even powers of v. The radiation reaction is a correction to the dynamics that first appears at the 2.5PN (i.e. v^5) order and not at 1PN or 1.5PN order. It is possible to construct the phasing of GWs from the direct integration of the equations of motion, without relying on the adiabatic approximation. At leading order, the *standard non-adiabatic* approximant uses the 0PN and 2.5PN terms in the acceleration (equivalent to using Newtonian conserved energy and quadrupolar flux), neglecting the intervening 1PN and 2PN terms. But a complete treatment of the acceleration at leading order should include *all* terms up to 2.5PN, without any gaps. We define the phasing of the GWs constructed in this approximation as the *complete non-adiabatic* approximant.

In the *adiabatic* approximation, the energy/flux functions can be thought of as carrying the information of the conservative/radiative dynamics of the system. So it is possible to construct adiabatic variants of *standard* and *complete non-adiabatic* approximants entirely based on energy and flux functions. In this sense *standard adiabatic* approximation is equivalent to neglecting certain conservative terms in the acceleration and thus resulting in gaps in the dynamics. In Ref. [3], we have proposed a new *complete adiabatic* approximant based on energy and flux functions. At the leading order, it uses the 2PN energy function instead of the 0PN energy function so that, heuristically, no intermediate post-Newtonian terms in the acceleration are missed. Following the notation used in Ref. [3], we denote the *standard adiabatic* approximant at nPN order as $T(E_{[n]}, \mathcal{F}_n)$ and the corresponding *complete adiabatic* approximant as $T(E_{[n+2.5]}, \mathcal{F}_n)$, where $[p]$ denotes the integer part of p.

In Ref. [3], the performance of the standard and complete approximants was evaluated in terms of their *effectualness* and *faithfulness* [4]. We first investigated the problem as a general mathematical question concerning the nature of PN templates assuming a flat power spectrum for the detector noise. We then repeated the study using the initial LIGO noise spectrum. It was found that, at low (<3PN) PN orders the complete approximants bring about a remarkable improvement over the standard approximants for the construction of *effectual* templates. However, the standard approximants are nearly as good as the complete approximants at higher orders. In terms of the faithfulness, it

[1] Throughout this paper, we use geometrical units in which $G = c = 1$.

was again found that the complete approximants generally performed better than the standard approximants.

In the present study, we assess the relative performances of the standard and complete adiabatic approximants for the VIRGO and Advanced LIGO noise spectra, as characterized by effectualness and faithfulness. We first perform this for the case of test-mass templates by comparing them with an exact waveform calculated in the adiabatic approximation of a test mass falling in to a Schwarzschild black hole. We then explore the extension of the results to the comparable mass case, where the approximants are compared with a fiducial exact waveform. We restrict our study to the inspiral phase of the coalescing binary, neglecting the plunge and quasi-normal ring down phases.

B. Noise spectra of the interferometers

The one-sided noise power spectral density (PSD) of VIRGO is given in terms of a dimensionless frequency $x = f/f_0$ by [1]

$$S_h(f) = 3.24 \times 10^{-46} \left[(6.23x)^{-5} + 2x^{-1} + 1 + x^2\right] \tag{1.2}$$

where $f_0 = 500$ Hz; while the same for the Advanced LIGO reads [5, 6]

$$S_h(f) = 10^{-49} \left[x^{-4.14} - 5x^{-2} + 111\Big(\frac{1 - x^2 + x^4/2}{1 + x^2/2}\Big)\right] \tag{1.3}$$

where $f_0 = 215$ Hz.

II. TEST MASS WAVEFORMS IN THE ADIABATIC APPROXIMATION

In the case of a test-particle orbiting a Schwarzschild black hole, the energy function $E(v)$ is exactly calculable analytically, while the flux function $\mathcal{F}(v)$ is exactly calculable numerically [7]. We use these functions to construct the exact waveform in the adiabatic approximation. The analytical exact energy function can be Taylor-expanded to get the approximants of the energy function. In addition, $\mathcal{F}(v)$ has been calculated analytically to 5.5PN order [8] by black hole perturbation theory. These approximants of energy and flux function are used to construct the approximate templates. In this study, we restrict to TaylorT1 approximants, since they do not involve any further re-expansion in the phasing formula. Hence there is no ambiguity in constructing the phasing of the waveforms using approximants with unequal orders of the energy and flux functions as required for the complete adiabatic approximant[2]. The exact waveform is also constructed by the TaylorT1 method. The waveforms (both the exact and approximate) are all terminated at $v_{\rm lso} = 1/\sqrt{6}$, which corresponds to $F_{\rm lso} \simeq 86$ Hz for the $(1M_\odot, 50M_\odot)$ binary and $F_{\rm lso} \simeq 399$ Hz for the $(1M_\odot, 10M_\odot)$ binary. The lower frequency cut-off of the waveforms is chosen to be $F_{\rm low} = 20$ Hz.

A. Comparison of standard and complete adiabatic approximants

The effectualness and faithfulness of various PN templates for two archetypical binaries with component masses $(1M_\odot, 10M_\odot)$ and $(1M_\odot, 50M_\odot)$ are tabulated in Tables I and

[2] See Sec. I.B of Ref. [3] for a detailed discussion.

TABLE I: Effectualness of *standard(S) adiabatic* $T(E_{[n]}, \mathcal{F}_n)$ and *complete(C) adiabatic* $T(E_{[n+2.5]}, \mathcal{F}_n)$ approximants in the test mass limit. Percentage biases σ_m and σ_η in determining parameters m and $\eta = m_1 m_2/m^2$ are given in brackets.

	$(1M_\odot, 10M_\odot)$		$(1M_\odot, 50M_\odot)$	
PN Order (n)	S	C	S	C
Advanced LIGO				
0PN	0.4281 (9.2, 2.6)	0.8960 (32, 42)	0.6461 (27, 22)	0.8099 (48, 54)
1PN	0.3498 (28, 8.9)	0.7258 (156, 75)	0.6200 (25, 123)	0.7093 (27, 13)
1.5PN	0.9010 (48, 49)	0.9653 (11, 21)	0.6919 (27, 20)	0.9532 (2.0, 8.7)
2PN	0.9266 (14, 20)	0.9814 (2.6, 4.2)	0.8835 (31, 39)	0.9833 (6.3, 13)
2.5PN	0.8917 (89, 66)	0.9913 (26, 31.7)	0.6720 (26, 6.2)	0.9194 (17, 21)
3PN	0.9913 (0.7, 1.6)	0.9989 (3.9, 7.3)	0.9645 (8.4, 16)	0.9740 (1.4, 1.4)
3.5PN	0.9816 (4.5, 7.4)	0.9994 (0.4, 0.3)	0.9875 (14, 23)	0.9987 (2.0, 3.9)
4PN	0.9895 (4.2, 7.1)	0.9970 (3.0, 5.3)	0.9967 (9.5, 16)	0.9973 (4.4, 6.9)
4.5PN	0.9965 (2.1, 3.6)	0.9999 (0.8, 1.6)	0.9932 (6.1, 11)	1.0000 (0.9, 1.9)
5PN	0.9954 (2.9, 5.2)	0.9977 (2.6, 4.0)	0.9986 (5.7, 9.9)	0.9960 (3.6, 6.2)
5.5PN	0.9963 (2.8, 4.2)	0.9983 (2.4, 3.9)	0.9989 (5.3, 8.7)	0.9951 (2.7, 4.5)
VIRGO				
0PN	0.3894 (42, 41)	0.7256 (0.8, 3.8)	0.6004 (50, 25)	0.8689 (51, 56)
1PN	0.2956 (11, 6.5)	0.6876 (187, 80)	0.5498 (51, 30)	0.7217 (52, 28)
1.5PN	0.8474 (31, 37)	0.9487 (12, 22)	0.7308 (56, 53)	0.9619 (1.1, 6.9)
2PN	0.8933 (9.9, 15)	0.9711 (3.0, 4.6)	0.9291 (34, 43)	0.9854 (5.4, 12)
2.5PN	0.8179 (69, 59)	0.9864 (26, 32)	0.6579 (49, 41)	0.9446 (19, 23)
3PN	0.9845 (0.6, 1.5)	0.9970 (3.8, 7.3)	0.9697 (7.4, 14)	0.9818 (1.5, 1.5)
3.5PN	0.9722 (4.3, 7.2)	0.9991 (0.4, 0.3)	0.9885 (14, 22)	0.9980 (1.9, 3.8)
4PN	0.9829 (4.1, 7.1)	0.9955 (2.9, 5.2)	0.9971 (9.5, 16)	0.9973 (4.4, 6.9)
4.5PN	0.9937 (2.0, 3.5)	0.9999 (0.8, 1.6)	0.9926 (6.0, 11)	1.0000 (0.9, 1.9)
5PN	0.9920 (3.0, 5.3)	0.9967 (2.6, 4.1)	0.9987 (5.7, 10)	0.9960 (3.5, 6.2)
5.5PN	0.9932 (2.8, 4.2)	0.9976 (2.3, 3.8)	0.9991 (5.4, 9.7)	0.9948 (2.8, 4.6)

II. All the results are in perfect agreement with the results of our earlier study [3] using the initial LIGO noise spectrum. Complete adiabatic approximants bring about a remarkable improvement in the effectualness for all systems at low PN orders (< 3PN). On the other hand, the difference in effectualness between the standard and complete adiabatic approximants at orders greater than 3PN is very small. Thus, we conclude that *standard adiabatic approximants of order* $\geq$ *3PN provides a good lower bound to the complete adiabatic approximants in the construction of effectual templates.* This study also confirms our earlier observation [3] that complete adiabatic approximants are generally less 'biased' in estimating the parameters of the binary.

Faithfulness of complete adiabatic approximants is generally better at all PN orders (even at very high orders) studied, which suggests that the complete approximants are closer to the exact solution than the corresponding standard approximants. But there are some cases of anomalous behavior. In the next subsection we will try to understand the reason for these anomalous cases where the complete approximants perform worse than the standard.

TABLE II: Faithfulness of *standard(S) adiabatic* $T(E_{[n]}, \mathcal{F}_n)$ and *complete(C) adiabatic* $T(E_{[n+2.5]}, \mathcal{F}_n)$ approximants in the test mass limit.

	Advanced LIGO				VIRGO			
	$(1M_\odot, 10M_\odot)$		$(1M_\odot, 50M_\odot)$		$(1M_\odot, 10M_\odot)$		$(1M_\odot, 50M_\odot)$	
PN Order (n)	S	C	S	C	S	C	S	C
0PN	0.1456	0.4915	0.1608	0.2955	0.1384	0.3644	0.1265	0.3881
1PN	0.0853	0.1041	0.1159	0.1609	0.0682	0.0818	0.0887	0.1205
1.5PN	0.2711	0.3063	0.2187	0.6735	0.2524	0.2348	0.1859	0.5783
2PN	0.6998	0.6140	0.2765	0.8403	0.7451	0.4617	0.2514	0.8597
2.5PN	0.2143	0.2710	0.1961	0.3094	0.2003	0.2496	0.1612	0.2420
3PN	0.8889	0.5791	0.7252	0.6971	0.8339	0.5745	0.7978	0.6210
3.5PN	0.7476	0.9985	0.3852	0.9087	0.7684	0.9968	0.3821	0.9259
4PN	0.7314	0.8144	0.4404	0.5761	0.7501	0.7892	0.4024	0.5306
4.5PN	0.9001	0.9718	0.5714	0.9078	0.8753	0.9595	0.5298	0.9132
5PN	0.8273	0.8518	0.5303	0.6166	0.8033	0.8232	0.4968	0.5617
5.5PN	0.8376	0.8640	0.5563	0.6460	0.8124	0.8340	0.5147	0.5862

B. Understanding the results

Table III summarizes the PN orders showing the anomalous behavior (i.e. the complete approximants being less faithful than the standard approximants) for the different noise spectra studied by us. The best-sensitivity bandwidth of each detector is shown in brackets [3]. The left-most column in the table shows the flattest noise spectrum and the right-most column shows the narrowest one.

In order to understand the anomalous behavior shown at certain PN orders, we compare the approximants of the $E'(v)/\mathcal{F}(v)$ function with the corresponding exact function. Fig. 1 shows the standard and complete approximants of $E'(v)/\mathcal{F}(v)$ in the case of the $(1M_\odot, 10M_\odot)$ and $(1M_\odot, 50M_\odot)$ binaries along with the corresponding exact functions. These results show that, while the complete approximants are far superior to the standard approximants in modelling the late-inspiral, the early inspiral is better modelled by the standard approximants at these PN orders. In the case of the $(1M_\odot, 10M_\odot)$ binary, the 0PN standard approximant is closer to the exact function than the corresponding complete approximant in the frequency region 20-50 Hz. But, since none of the detectors is sensitive in this frequency band, this effect shows up in the white-noise case only. Similarly the 1.5PN and 2PN standard approximants are closer to the exact function in the frequency regions 20-60 Hz and 20-80 Hz, respectively. But the 1.5PN approximant does not show the anomalous behavior in the case of the Advanced LIGO and Initial LIGO because the 20-60 Hz region does not fall in the best-sensitivity bandwidth of these detectors. The anomalous behavior exhibited by the 3PN approximant can be understood in a similar way. It should be noted that the final stages of the inspiral is much better modelled by the complete approximants (see the top panel of Fig. 1). But, since the binary spends more cycles in the early inspiral phase, the overlaps are heavily influenced by the efficiency of the modelling of the early inspiral.

[3] It should be noted that there is no rigorous definition for the 'best-sensitivity' bandwidth. We define it as the bandwidth where the detector's effective noise amplitude $h = \sqrt{f S_h(f)}$ is within a factor of two of its lowest value.

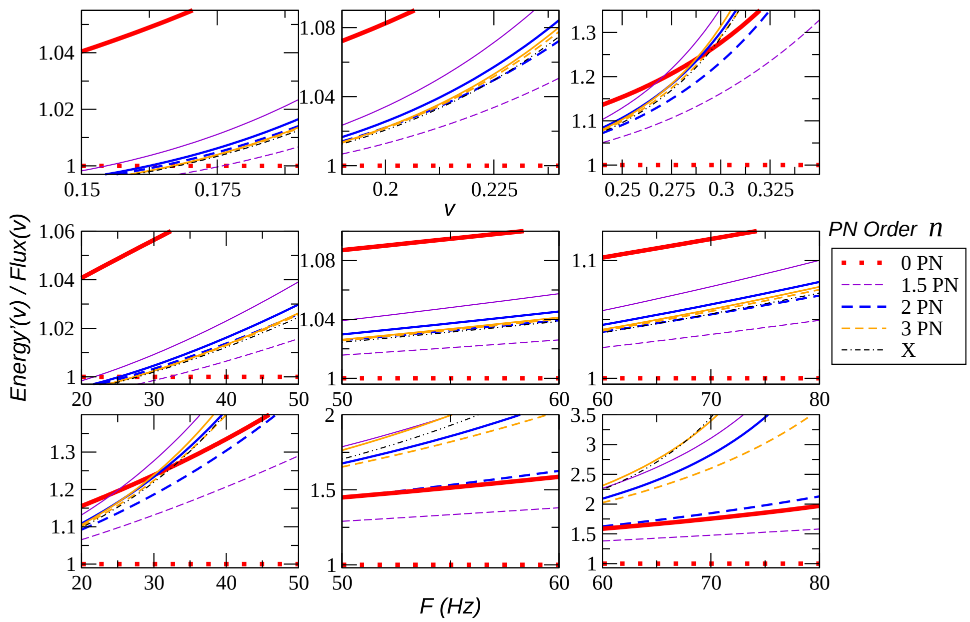

FIG. 1: Top panel shows the approximants of $E'(v)/\mathcal{F}(v)$ plotted as a function of v. Dashed lines indicate standard approximants $E'_{[n]}(v)/\mathcal{F}_n(v)$ and solid lines indicate the corresponding complete approximants $E'_{[n+2.5]}(v)/\mathcal{F}_n(v)$. Middle and bottom panels show the same approximants plotted as a function of the GW frequency $F = v^3/\pi m$ in the case of the $(1M_\odot, 10M_\odot)$ binary and the $(1M_\odot, 50M_\odot)$ binary, respectively.

TABLE III: PN orders showing anomalous behavior in the context of different noise spectra. The best-sensitivity bandwidth of each detector is shown in brackets.

White-noise		VIRGO (50-400 Hz)		Advanced LIGO (60-300 Hz)		Initial LIGO (80-200 Hz)	
$(1,10)M_\odot$	$(1,50)M_\odot$	$(1,10)M_\odot$	$(1,50)M_\odot$	$(1,10)M_\odot$	$(1,50)M_\odot$	$(1,10)M_\odot$	$(1,50)M_\odot$
0PN							
1.5PN		1.5PN					
2PN		2PN		2PN			
3PN	3PN	3PN	3PN	3PN	3PN	3PN	

III. COMPARABLE MASS WAVEFORMS IN THE ADIABATIC APPROXIMATION

In the case of comparable mass binaries there is no *exact* waveform available and the best we can do is to compare the performance of the standard adiabatic and complete adiabatic templates by studying their overlaps with some plausible fiducial exact waveform. The required energy and flux functions have been calculated by supplementing the exact functions in the test mass limit by all the *known* η-dependent corrections (from

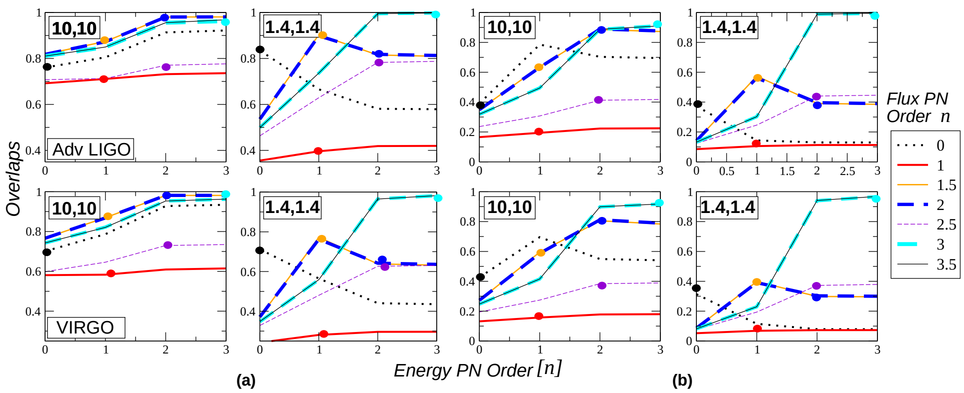

FIG. 2: Effectualness (four panels in the left) and faithfulness (four panels in the right) of various TaylorT1 templates in detecting two prototype binaries with component masses $(10M_\odot,\, 10M_\odot)$ and $(1.4M_\odot,\, 1.4M_\odot)$. Top panels show overlaps calculated using the Advanced LIGO noise spectrum and the bottom panels show overlaps calculated using the VIRGO noise spectrum. Different lines in the panels correspond to different orders of the flux function. Each line shows how the overlaps are evolving as a function of the accuracy of the energy function. Standard adiabatic approximants $T(E_{[n]}, \mathcal{F}_n)$ are marked with thick dots.

post-Newtonian theory) in the comparable mass case[4]. In the case of comparable mass binaries, the energy function is currently known up to 3PN order [9, 10, 11, 12, 13, 14] and the flux function up to 3.5PN order [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. The waveforms ('exact' and approximate) are constructed by the TaylorT1 method and are terminated at $v_{lso} = 1/\sqrt{6}$, which corresponds to $F_{\rm lso} \simeq 1570$ Hz for a $(1.4M_\odot,\, 1.4M_\odot)$ binary and $F_{\rm lso} \simeq 220$ Hz for a $(10M_\odot,\, 10M_\odot)$ binary. The lower frequency cut-off of the waveforms is chosen to be $F_{\rm low} = 20$ Hz.

A. Comparable mass results in the adiabatic approximation

Effectualness and faithfulness of various TaylorT1 templates in detecting two prototype binaries with component masses $(10M_\odot,\, 10M_\odot)$ and $(1.4M_\odot,\, 1.4M_\odot)$ are shown in Fig. 2 and are tabulated in Tables IV and V. It should be noted that the complete adiabatic approximants are presently calculated only up to 1PN order. Thus, it is not possible to make strong statements of general trends. Heuristically one can conclude from Fig. 2 that standard adiabatic approximants of order ≥ 1.5PN provide a good lower-bound to the complete adiabatic approximants for the construction of both effectual and faithful templates. We also note from Table IV that standard adiabatic approximants of order 2PN/3PN produce the target value 0.965 in effectualness (corresponding to 10% event-loss) in the case of the BH-BH/NS-NS binaries.

IV. NON-ADIABATIC APPROXIMANTS

In [3], we also looked at the relative performance of *standard* and *complete non-adiabatic approximants*, using the Lagrangian templates described by Buonanno, Chen

[4] Sec. IV of Ref. [3] discusses the precise method used in this construction.

TABLE IV: Effectualness of *standard(S) adiabatic* $T(E_{[n]}, \mathcal{F}_n)$ and *complete(C) adiabatic* $T(E_{[n+2.5]}, \mathcal{F}_n)$ approximants in the comparable-mass case. Percentage biases σ_m and σ_η in determining parameters m and $\eta = m_1 m_2/m^2$ are given in brackets.

	$(10M_\odot, 10M_\odot)$		$(1.4M_\odot, 1.4M_\odot)$	
PN Order (n)	S	C	S	C
Advanced LIGO				
0PN	0.7606 (8.5, 0.1)	0.9132 (5.0, 0.3)	0.8347 (1.4, 0.1)	0.5809 (4.3, 0.1)
1PN	0.7110 (57, 0.6)	0.7360 (39, 0.6)	0.3959 (6.1, 0.0)	0.4194 (5.0, 0.1)
1.5PN	0.8741 (2.4, 0.1)		0.9034 (0.0, 0.2)	
2PN	0.9803 (0.8, 0.2)		0.8179 (0.4, 0.0)	
2.5PN	0.7705 (7.3, 0.1)		0.7826 (0.4, 0.0)	
3PN	0.9626 (0.5, 0.0)		0.9981 (0.4, 0.3)	
3.5PN	0.9683 (1.3, 1.5)		0.9977 (0.4, 0.3)	
VIRGO				
0PN	0.7009 (5.3, 0.0)	0.9280 (4.7, 0.9)	0.7119 (1.4, 0.1)	0.4405 (5.0, 0.0)
1PN	0.5834 (56, 0.3)	0.6148 (30, 0.2)	0.2808 (3.9, 0.1)	0.2968 (2.9, 0.0)
1.5PN	0.8698 (1.3, 0.0)		0.7724 (0.4, 0.7)	
2PN	0.9815 (0.8, 0.2)		0.6420 (0.0, 0.0)	
2.5PN	0.7299 (4.8, 0.0)		0.6266 (0.0, 0.1)	
3PN	0.9624 (0.5, 0.1)		0.9822 (0.0, 0.3)	
3.5PN	0.9627 (0.5, 0.1)		0.9823 (0.0, 0.3)	

TABLE V: Faithfulness of the *standard(S) adiabatic* $T(E_{[n]}, \mathcal{F}_n)$ and *complete(C) adiabatic* $T(E_{[n+2.5]}, \mathcal{F}_n)$ approximants in the comparable-mass case.

	Advanced LIGO				VIRGO			
	$(10M_\odot, 10M_\odot)$		$(1.4M_\odot, 1.4M_\odot)$		$(10M_\odot, 10M_\odot)$		$(1.4M_\odot, 1.4M_\odot)$	
PN Order (n)	S	C	S	C	S	C	S	C
0PN	0.3902	0.7030	0.3731	0.1300	0.4262	0.5490	0.3138	0.0794
1PN	0.1944	0.2248	0.1054	0.1128	0.1574	0.1798	0.0686	0.0732
1.5PN	0.6362		0.5735		0.5950		0.3986	
2PN	0.8895		0.3964		0.8120		0.3027	
2.5PN	0.4125		0.4407		0.3842		0.3726	
3PN	0.9117		0.9947		0.9169		0.9668	
3.5PN	0.9106		0.9952		0.9177		0.9686	

and Vallisneri [27], comparing them with the exact waveform [5] constructed in the adiabatic approximation. The overlaps were computed for the white noise spectrum and the initial LIGO noise spectrum. We had found that, as for the adiabatic approximants, the complete approximants were far better than the standard approximants for the construction of effectual templates. However, in marked contrast to the adiabatic case, we

[5] In the comparable mass case, the approximants were compared with a fiducial exact waveform which is discussed in Sec. III.

TABLE VI: Effectualness and faithfulness of the *standard(S)* and *complete(C) non-adiabatic approximants* in the test mass case.

	Effectualness				Faithfulness			
	$(1M_\odot, 10M_\odot)$		$(1M_\odot, 50M_\odot)$		$(1M_\odot, 10M_\odot)$		$(1M_\odot, 50M_\odot)$	
PN Order (n)	S	C	S	C	S	C	S	C
Advanced LIGO								
0PN	0.4259	0.8682	0.6384	0.9360	0.2636	0.0754	0.4857	0.0999
1PN	0.5256	0.8280	0.6080	0.9211	0.3419	0.1368	0.4493	0.2038
VIRGO								
0PN	0.3720	0.7631	0.5985	0.9618	0.1991	0.0570	0.4947	0.1057
1PN	0.3599	0.7386	0.5777	0.9525	0.2499	0.0911	0.5259	0.1954

TABLE VII: Effectualness and faithfulness of the *standard(S)* and *complete(C) non-adiabatic approximants* in the comparable mass case.

	Effectualness				Faithfulness			
	$(10M_\odot, 10M_\odot)$		$(1.4M_\odot, 1.4M_\odot)$		$(10M_\odot, 10M_\odot)$		$(1.4M_\odot, 1.4M_\odot)$	
PN Order (n)	S	C	S	C	S	C	S	C
Advanced LIGO								
0PN	0.9147	0.4338	0.7417	0.8322	0.0637	0.0546	0.4662	0.2192
1PN	0.3937	0.5132	0.7443	0.8158	0.0794	0.0703	0.6594	0.4788
VIRGO								
0PN	0.8142	0.3113	0.6895	0.7341	0.0414	0.0334	0.3439	0.1661
1PN	0.2880	0.3944	0.6807	0.7420	0.0803	0.0463	0.5709	0.3704

found that the use of complete templates led to a decrease in faithfulness compared to the standard templates.

In this study, we look at the relative performance of the standard and complete approximants for the case of the Advanced LIGO and VIRGO noise spectra. The results are tabulated in Tables VI and VII. It should be noted that, at present, results are available at too few PN orders to make statements about general trends in effectualness and faithfulness. However, we find that, as in [3], the complete approximants are generally better than the standard for the construction of effectual templates, but the faithfulness of the complete approximants is less than that of the standard.

V. SUMMARY

The *standard adiabatic* approximation to the phasing of GWs from inspiralling compact binaries is based on PN expansions of the binding energy and GW flux truncated at the *same relative* PN order. Viewed in terms of the dynamics of the binary, this standard treatment is equivalent to neglecting certain conservative terms in the acceleration. In an earlier work [3], we have proposed a new *complete adiabatic* approximant which, in spirit, corresponds to a complete treatment of the acceleration accurate up to the respective PN order. In this study we have investigated the performance of the *standard* and *complete adiabatic* approximants in the cases of the VIRGO and Advanced LIGO noise spectra. This has been done by measuring their *effectualness* (i.e. larger maximum overlap with the exact waveform), and *faithfulness* (i.e. smaller biases in parameter estimation). In the test-mass case, the approximants were compared with the exact waveform; while in the comparable mass case, the approximants were compared with a fiducial 'exact' waveform. We have considered only the inspiral phase of the binary, neglecting the plunge and quasi-normal ring down phases. We summarize the results of our study as follows:

- In the test-mass case, effectualness of the templates improves significantly in the complete adiabatic approximation at lower ($<$ 3PN) PN orders. But standard adiabatic approximants of order $\geq$ 3PN are nearly as good as the complete approximants.
- Faithfulness of complete adiabatic approximants is generally better at all PN orders studied. But there are some cases of anomalous behavior. We have shown that, at these PN orders (0PN, 1.5PN, 2PN and 3PN) the early inspiral is better modelled by the standard approximants than the corresponding complete approximants, which explains the better faithfulness exhibited by the standard approximants at these orders. But complete adiabatic approximants are far superior to the standard adiabatic approximants in modelling the final inspiral.
- Complete adiabatic approximants are generally less 'biased' in estimating the parameters of the binary.
- In the case of comparable-mass binaries, standard adiabatic approximants of order $\geq$ 1.5PN provides a good lower-bound to the complete adiabatic approximants for the construction of both effectual and faithful templates.
- In the comparable-mass case, standard adiabatic approximants of order 2PN/3PN produce the target value 0.965 in effectualness (corresponding to 10% event-loss) in the case of the BH-BH/NS-NS binaries.

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[1] T. Damour, B. R. Iyer and B. S. Sathyaprakash, Phys. Rev. D **63**, 044023 (2001).
[2] C. Cutler and E. E. Flanagan, Phys. Rev. D **49**, 2658 (1994).
[3] P. Ajith, B. R. Iyer, C. A. K. Robinson and B. S. Sathyaprakash, Phys. Rev. D **71**, 044029 (2005).
[4] T. Damour, B. R. Iyer and B. S. Sathyaprakash, Phys. Rev. D **57**, 885 (1998).
[5] C. Cutler and K.S. Thorne, Proceedings of 16th international conference on General relativity and Gravitation (Eds N.T. Bishop and S.D. Maharaj) (2002); `gr-qc/0204090`.
[6] K. G. Arun, B. R. Iyer, B. S. Sathyaprakash and P. A. Sundararajan, `gr-qc/0411146`.
[7] C. Cutler, L.S. Finn, E. Poisson, and G.J. Sussman, Phys. Rev. D **47**, 1511 (1993); E. Poisson, Phys. Rev. D **52**, 5719 (1995).
[8] T. Tanaka, H. Tagoshi and M. Sasaki, Prog. Theor. Phys. **96,** 1087 (1996).
[9] T. Damour, P. Jaranowski, G. Schäfer, Phys. Rev. D. **62**, 044024 (2000); **62**, 084011 (2000); **62**, 021501 (2000); **63** 029903 (E) (2001); and **63**, 044021 (2001); **66** 029901 (E) (2002).
[10] L. Blanchet and G. Faye, Phys. Lett. **A271**, 58 (2000); Phys. Rev. D **63**, 062005 (2001); V. C. de Andrade, L. Blanchet and G. Faye, Class. Quant. Grav. **18**, 753 (2001).
[11] T. Damour, P. Jaranowski and G. Schäfer, Phys. Lett. B **513**, 147 (2001).
[12] L. Blanchet, T. Damour and G. Esposito-Farèse, Phys. Rev. D **69**, 124007 (2004).
[13] Y. Itoh and T. Futamase, Phys. Rev. D **68**, 121501 (2003).
[14] Y. Itoh, Phys. Rev. D **69** , 064018 (2004).
[15] L. Blanchet, T. Damour, B. R. Iyer, C. M. Will and A. G. Wiseman, Phys. Rev. Lett. **74**, 3515 (1995).
[16] L. Blanchet, T. Damour and B. R. Iyer, Phys. Rev. D **51**, 5360 (1995).
[17] C. M. Will and A. G. Wiseman, Phys. Rev. D**54**, 4813 (1996).

[18] L. Blanchet, B. R. Iyer, C. M. Will and A. G. Wiseman, Class. Quantum. Gr. **13**, 575, (1996).
[19] L. Blanchet, Phys. Rev. D **54**, 1417 (1996).
[20] L. Blanchet, B. R. Iyer, B. Joguet, Phys. Rev. D **65**, 064005 (2002).
[21] L. Blanchet, G. Faye, B. R. Iyer, B. Joguet, Phys. Rev. D **65**, 061501(R) (2002).
[22] K. G. Arun, L. Blanchet, B. R. Iyer and M. S. S. Qusailah, Class. Quantum. Gr. **21**, 3771 (2004).
[23] L. Blanchet, T. Damour, G. Esposito-Farèse and B. R. Iyer, Phys. Rev. Lett. **93**, 091101 (2004).
[24] L. Blanchet and B. R. Iyer, Phys. Rev. D **71**, 024004 (2005).
[25] L. Blanchet, T. Damour and B. R. Iyer, Class. Quant. Grav. **22**, 155-182 (2005).
[26] L. Blanchet, T. Damour, G. Esposito-Farèse and B. R. Iyer, Phys. Rev. D (Submitted) (2005); gr-qc/0503044.
[27] A. Buonanno, Y. Chen, M. Vallisneri, Phys. Rev. D **67**, 024016 (2003).