

# Less is More: Non-renormalization Theorems from Lower Dimensional Superspace

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## Abstract

We discuss a new class of non-renormalization theorems in  $\mathcal{N} = 4$  and  $\mathcal{N} = 2$  Super-Yang-Mills theory, obtained by using a superspace which makes a lower dimensional subgroup of the full supersymmetry manifest. Certain Wilson loops (and Wilson lines) belong to the chiral ring of the lower dimensional supersymmetry algebra, and their expectation values can be computed exactly.

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# 1 Introduction

In four dimensional theories with extended supersymmetry, it is simple to make an  $\mathcal{N} = 1$  subgroup of the supersymmetries manifest and possible, albeit more complicated, to make  $\mathcal{N} = 2$  manifest using harmonic superspace. The advantages of an off-shell formulation which make as much supersymmetry as possible manifest include demonstrations of non-renormalization theorems as well as calculational efficiency.

In the following, we will make supersymmetries other than  $\mathcal{N} = 1$  manifest, but not by using a harmonic superspace. Instead, we will make use of the fact that four-dimensional extended supersymmetry algebras have lower-dimensional subalgebras with four supercharges and an associated superspace which is a dimensional reduction of the familiar four-dimensional  $\mathcal{N} = 1$  superspace. By a lower dimensional subalgebra, we mean a subalgebra which does not include momenta in all four directions. Despite the simplicity of the lower dimensional superspace, a useful part of the full supersymmetry which does not belong to four dimensional  $\mathcal{N} = 1$  is realized off-shell.

One of the general features of such lower dimensional superspaces is that certain kinetic terms (in directions transverse to the superspace) appear in a superpotential rather than a Kähler potential. Furthermore gauge connections in these directions are bottom components of chiral superfields. As usual, supersymmetry leads to strong constraints on the holomorphic sector involving chiral superfields, which does not include gauge connections and kinetic terms in the usual four-dimensional  $\mathcal{N} = 1$  formalism.

In the case of  $\mathcal{N} = 4$  super Yang-Mills theory, the lower dimensional superspace formalism which we will use makes certain supersymmetries manifest which are not accessible to either  $\mathcal{N} = 1$  superspace or even  $\mathcal{N} = 2$  harmonic superspace. Specifically, we will express the  $\mathcal{N} = 4$  theory in terms of  $\mathcal{N} = 4$ ,  $d = 1$  superspace. This will allow us to prove and extend a non-renormalization theorem for BPS Wilson loops which was conjectured in [1]. A more detailed discussion appeared in [2].

In the case of four dimensional  $\mathcal{N} = 2$  super Yang-Mills theories, we will use  $\mathcal{N} = 2$ ,  $d = 3$  superspace to obtain exact results for expectation values of straight BPS Wilson lines with scalar components of hypermultiplets at the endpoints. These expectation values are non-trivial on the Higgs branch, and can be expressed exactly in terms of expectation values of local operators parameterizing the Higgs branch [3].

## 2 Warming up: A free $\mathcal{N} = 2$ , $d = 4$ hypermultiplet in $\mathcal{N} = 2$ , $d = 3$ superspace

The first example of supersymmetric action written using lower dimensional superspace appeared in [4]. Since then, this formalism has been applied to many supersymmetric theories in various different contexts [5, 6, 7, 8, 9, 10, 11]. We now describe a very simple example, in which a free  $\mathcal{N} = 2$  hypermultiplet in four dimensions is described by an action in  $\mathcal{N} = 2$ ,  $d = 3$  superspace.

In the absence of central extensions, the  $\mathcal{N} = 2$ ,  $d = 4$  supersymmetry algebra is

$$\{Q_{i\alpha}, \bar{Q}^j_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \delta_i^j, \quad \{Q_{i\alpha}, Q_{j\beta}\} = \{\bar{Q}^i_{\dot{\alpha}}, \bar{Q}^j_{\dot{\beta}}\} = 0, \quad (2.1)$$

where  $i = 1, 2$ . Defining

$$Q_\alpha \equiv \frac{1}{2}(Q_{1\alpha} + \bar{Q}^1_{\dot{\alpha}}) + i\frac{1}{2}(Q_{2\alpha} + \bar{Q}^2_{\dot{\alpha}}), \quad (2.2)$$

one finds an  $\mathcal{N} = 2$ ,  $d = 3$  subalgebra,

$$\begin{aligned} \{Q_\alpha, \bar{Q}_\beta\} &= 2\sigma_{\alpha\beta}^M P_M \quad M = 0, 1, 3 \\ \{Q_\alpha, Q_\beta\} &= \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0. \end{aligned} \quad (2.3)$$

A four-dimensional  $\mathcal{N} = 2$  theory can be written in terms of the associated  $\mathcal{N} = 2$ ,  $d = 3$  superspace. This superspace is equivalent to the dimensional reduction of the familiar  $\mathcal{N} = 1$ ,  $d = 4$ , and is spanned by  $x^0, x^1, x^3, \theta, \bar{\theta}$ . The  $\mathcal{N} = 2$ ,  $d = 3$  superfields necessary to describe a  $\mathcal{N} = 2$ ,  $d = 4$  theory have the general form  $F(x^0, x^1, x^3, \theta, \bar{\theta}|x^2)$ , where the spatial coordinate  $x^2$  transverse to the superspace should be thought of as a continuous index labeling an infinite number of  $\mathcal{N} = 2$ ,  $d = 3$  superfields.

A free massless  $\mathcal{N} = 2$ ,  $d = 4$  hypermultiplet can be built from two  $\mathcal{N} = 2$ ,  $d = 3$  chiral multiplets  $\Phi_1(x^2)$  and  $\Phi_2(x^2)$ , which are annihilated by the superspace derivative

$$\bar{D}_\alpha \equiv -\frac{\partial}{\partial\theta^\alpha} - i\theta^\beta\sigma_{\beta\alpha}^\mu\partial_\mu, \quad \mu = 0, 1, 3. \quad (2.4)$$

The action is

$$S = \int dx^2 \int dx^0 dx^1 dx^3 \left[ \int d^4\theta (\bar{\Phi}_1\Phi_1 + \bar{\Phi}_2\Phi_2) + \int d^2\theta \Phi_1 \frac{\partial}{\partial X^2} \Phi_2 + \text{c.c.} \right]. \quad (2.5)$$

Note that the Kähler potential by itself only gives rise to kinetic terms in the 0, 1, 3 directions belonging to the superspace. Remarkably, the kinetic terms in the  $x^2$  direction arise from the superpotential rather than the Kähler potential. Herein lies the advantage of a lower dimensional superspace over the familiar  $\mathcal{N} = 1$ ,  $d = 4$  superspace: one can exchange Kähler terms for superpotential terms. This is a strong hint that the lower-dimensional superspace formalism can be used to find new non-renormalization theorems. We will now discuss two examples of non-renormalization theorems which have been found using the lower-dimensional superspace formalism.

### 3 Chiral Wilson loops in $\mathcal{N} = 4$ SYM

The four-dimensional  $\mathcal{N} = 4$  supersymmetry algebra contains a one dimensional  $\mathcal{N} = 4$  subalgebra containing four real supercharges. The superspace associated with this subalgebra is equivalent to a dimensional reduction of familiar four dimensional  $\mathcal{N} = 1$  superspace, and is spanned by  $t, \theta, \bar{\theta}$ . When writing the four-dimensional  $\mathcal{N} = 4$  theory in this superspace, a generic superfield has the form  $F(t, \theta, \bar{\theta}|\vec{x})$ , where the spatial coordinates  $\vec{x} = x^{1,2,3}$  are continuous indices labeling an infinite number of  $\mathcal{N} = 4$ ,  $d = 1$  superfields. The superfield content of the theory consists of chiral superfields  $\Phi_i(\vec{x})$  for  $i = 1, 2, 3$  and vector superfields  $V(\vec{x})$ . Although this is similar to the superfield content in the more familiar  $\mathcal{N} = 1$ ,  $d = 4$  superspace, the way component fields are distributed

among the  $\mathcal{N} = 4$ ,  $d = 1$  superfields is very different, as we will see shortly. The action in  $\mathcal{N} = 4$ ,  $d = 1$  superspace is

$$S = \frac{1}{g^2} \int d^3x \int dt \left\{ \int d^2\theta \operatorname{tr} \left[ \mathcal{W}_\alpha \mathcal{W}^\alpha + \epsilon_{ijk} \left( \Phi_i \frac{\partial}{\partial x^j} \Phi_k + \frac{2}{3} \Phi_i \Phi_j \Phi_k \right) \right] + \text{c.c.} \right. \\ \left. + \int d^4\theta \operatorname{tr} \bar{\Omega}_i e^V \Omega_i e^{-V} \right\}, \quad (3.1)$$

where

$$\Omega_i \equiv \Phi_i + e^{-V} (i\partial_i - \bar{\Phi}_i) e^V. \quad (3.2)$$

Note that the index  $i = 1, 2, 3$  labeling chiral superfields has been “identified” with the spatial index (similar to the identification of spatial and lie algebra indices in the ’t Hooft Polyakov monopole). The bottom components of the chiral superfields are

$$\Phi_i|_{\theta=\bar{\theta}=0} = A_i + iX^i, \quad (3.3)$$

where  $A_i$  are gauge connections in the spatial directions transverse to the superspace, and  $X^i$  are three (out of the six) adjoint hermitian scalars. The rest of the adjoint scalars are contained in the vector superfield. Under gauge transformations parameterized by chiral superfields  $\Lambda(\vec{x})$ ,

$$e^V \rightarrow e^{i\Lambda^\dagger} e^V e^{-i\Lambda}, \quad \Phi_i \rightarrow e^{i\Lambda} \Phi_i e^{-i\Lambda} - e^{i\Lambda} i \frac{\partial}{\partial x^i} e^{-i\Lambda}. \quad (3.4)$$

A special class of Wilson loops are chiral superfields in  $\mathcal{N} = 4$ ,  $d = 1$  superspace,

$$\mathcal{W}(C) \equiv \operatorname{tr} \mathcal{P} \left( e^{i \oint_C \Phi_i dx^i} \right), \quad (3.5)$$

which have bottom components

$$\mathcal{W}(C)|_{\theta=\bar{\theta}=0} = \operatorname{tr} \mathcal{P} \left( e^{i \oint_C (A_i + iX^i) dx^i} \right). \quad (3.6)$$

The latter belong to a class of BPS Wilson loops in the  $\mathcal{N} = 4$  theory originally discussed in [1]. These in turn belong to a class of “locally BPS” Wilson loops containing adjoint scalars which were introduced in the context of AdS/CFT duality [12, 13, 14, 15]. The loops (3.6) preserve 1/4 or 1/8 of the 16 supersymmetries of the theory, depending on whether the path  $C$  lies in a two or three-dimensional subspace. In [1], it was conjectured that the expectation values of 1/4 BPS loops are un-renormalized in the large  $N$  limit. This conjecture was based on perturbative calculations as well as strong coupling results obtained using the AdS/CFT duality. In fact one can show that both the 1/4 and 1/8 BPS loops are not renormalized, even at finite  $N$ , by using the fact that they belong to a chiral ring with respect to  $\mathcal{N} = 4$ ,  $d = 1$  supersymmetry. The chiral ring is constrained by the superpotential, which in this case resembles a Chern-Simons action and has a three-dimensional diffeomorphism invariance.

Consider a variation

$$\Phi_i(\vec{x}) \rightarrow \Phi_i(\vec{x}) + \epsilon g_{i,\vec{x}}[\Phi], \quad (3.7)$$

where the small parameter  $\epsilon$  is a chiral superfield and  $g_{I,\vec{x}}$  is an arbitrary functional of chiral superfields. The equations of motion,  $\delta S = 0$ , which follow from this variation are

$$\bar{D}^2 \text{tr} \left( g_{i,\vec{x}}[\Phi] (e^{-V(\vec{x})} \bar{\Omega}_i(\vec{x}) e^{V(\vec{x})} - \Omega_i(\vec{x})) \right) + \text{tr} g_{i,\vec{x}}[\Phi] \frac{\delta W}{\delta \Phi_i(\vec{x})} = \mathcal{A}, \quad (3.8)$$

where  $\mathcal{A}$  is a possible anomalous term which vanishes classically. We will choose  $g_{i,\vec{x}}[\Phi]$  to be a spatial Wilson line on a contour  $\mathcal{C}_{\vec{x}}$  which begins and ends at the point  $\vec{x}$ ,

$$g_{i,\vec{x}}[\Phi] = W(C, \vec{x}) = P \exp \left( i \oint_{\mathcal{C}_{\vec{x}}} \vec{\Phi} \cdot d\vec{y} \right). \quad (3.9)$$

Equation (3.8) then becomes

$$\bar{D}^2(\dots) = \text{tr} (W(C, \vec{x}) \epsilon_{ijk} \mathcal{F}_{jk}(\vec{x})) + \mathcal{A}, \quad (3.10)$$

where

$$\mathcal{F}_{jk} = \partial_j \Phi_k - \partial_k \Phi_j + i[\Phi_j, \Phi_k]. \quad (3.11)$$

The relation  $(\bar{D}^2 J)|_{\theta=\bar{\theta}=0} = [\bar{Q}, [\bar{Q}, J|_{\theta=\bar{\theta}=0}]]$  implies that  $\langle \bar{D}^2 J|_{\theta=\bar{\theta}=0} \rangle = 0$  in a supersymmetric vacuum. Therefore (3.10) implies

$$\langle \text{tr} (W(C, \vec{x}) \epsilon_{ijk} \mathcal{F}_{jk}(\vec{x}))|_{\theta=\bar{\theta}=0} \rangle = \langle \mathcal{A}|_{\theta=\bar{\theta}=0} \rangle. \quad (3.12)$$

It can be shown that the anomalous term vanishes [2]. This may seem surprising to the reader expecting a non-trivial Konishi anomaly [16]. The absence of an anomaly is related to the fact that the fermionic components of the  $\mathcal{N} = 4$ ,  $d = 1$  chiral superfields  $\Phi_i(\vec{x})$  are not of definite chirality from the point of view of the four-dimensional Lorentz group, unlike the fermionic components of  $\mathcal{N} = 1$ ,  $d = 4$  superfields. For  $\mathcal{A} = 0$ , (3.12) implies shape invariance<sup>1</sup> of the Wilson loop expectation value, since the insertion of the field strength  $\mathcal{F}_{jk}$  generates an infinitesimal deformation of the contour  $\mathcal{C}$  in the  $jk$  plane.

Shape invariance of the Wilson loop expectation value leads to the conclusion

$$\left\langle \frac{1}{N} \text{tr} P \exp \left( i \oint \phi \right) \right\rangle = 1. \quad (3.13)$$

Note that because of conformal invariance, this can not be shown simply by shrinking the Wilson loop. Instead we use the following argument. Given a Wilson-loop  $\frac{1}{N} \text{tr} W$  associated with a path  $C$  in  $\mathbb{R}^3$  one can smoothly deform the path within  $\mathbb{R}^3$  such that it goes around  $C$  multiple times and the Wilson loop becomes  $\frac{1}{N} \text{tr} W^n$  for any  $n > 1^2$ . Shape independence implies that the expectation value is unchanged,

$$\left\langle \frac{1}{N} \text{tr} W^n \right\rangle = \left\langle \frac{1}{N} \text{tr} W \right\rangle. \quad (3.14)$$

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<sup>1</sup>This is actually a larger symmetry than diffeomorphism invariance, which does not relate knots of different topology. Breaking shape invariance to diffeomorphism invariance would require a non-zero anomaly.

<sup>2</sup>Deforming to  $n=0$  can not be done in a smooth way without introducing a cusp.

Furthermore, there are relations amongst the variables  $\text{tr}W^n$  of the form

$$\text{tr}W^n(C) = \sum \text{tr}W^q(C)\text{tr}W^p(C)\cdots, \quad (3.15)$$

such that  $\text{tr}W^n$  for  $n = 1, \dots, 2N^2$  form a complete independent set. This follows from the fact that  $W$  is an  $N \times N$  matrix with complex entries and no constraints. Note that, unlike the usual Wilson loop, the chiral Wilson loop is not the trace of a unitary matrix because the exponent involves both hermitian and anti-hermitian parts. Since these Wilson loops belong to a chiral ring, the expectation values factorize,

$$\left\langle \frac{1}{N}\text{tr}W^n \frac{1}{N}\text{tr}W^m \right\rangle = \left\langle \frac{1}{N}\text{tr}W^n \right\rangle \left\langle \frac{1}{N}\text{tr}W^m \right\rangle. \quad (3.16)$$

The relations (3.15) amongst the  $\frac{1}{N}\text{tr}W^n$ , together with factorization (3.16) and shape independence (3.14) are solved by  $\langle \frac{1}{N}\text{tr}W^n \rangle = \frac{1}{N}\text{tr}M^n$ , where  $M$  is an  $N \times N$  matrix satisfying  $\text{tr}M^n = \text{tr}M$ . One set of solutions are the projection matrices which satisfy  $M^n = M$ . However the only solution of  $\text{tr}M^n = \text{tr}M$  which is smoothly related to the weak coupling result,  $\langle \frac{1}{N}\text{tr}W^n \rangle|_{g=0} = 1$ , corresponds to  $M = I$  such that  $\langle \frac{1}{N}\text{tr}W^n \rangle = 1$ .

These results can be trivially extended to maximally supersymmetric Yang-Mills in 3, 5 and 6 dimensions, by using a four supercharge superspace with dimension 0, 2 and 3 respectively. In these cases the gauge coupling is dimensionful and the result (3.13) can be extracted from shape invariance either by shrinking the Wilson loop in 3 dimensions, or expanding it in 5 and 6 dimensions. In 7-dimensional maximally supersymmetric Yang-Mills, one encounters a non-trivial generalized Konishi anomaly<sup>3</sup>, while in higher dimensions  $d > 7$ , there is no four-supercharge subalgebra of dimension  $d - 3$  so our formalism is not applicable.

## 4 Chiral Wilson lines in $\mathcal{N} = 2$ SYM

A similar non-renormalization theorem applies to Wilson lines in four dimensional  $\mathcal{N} = 2$  Yang-Mills theories, and is obtained by using  $\mathcal{N} = 2$ ,  $d = 3$  superspace. In terms of this superspace, the  $\mathcal{N} = 2$ ,  $d = 4$  vector multiplet is comprised of vector superfields  $V(x^2)$  and adjoint chiral superfields  $\Phi(x^2)$ , where the superspace spans  $x^0, x^1, x^3, \theta, \bar{\theta}$ . However unlike the usual  $\mathcal{N} = 1$ ,  $d = 4$  superspace formulation,  $V$  contains only the components  $A_{0,1,3}$  of the gauge connections while  $A_2$  is contained in  $\Phi$ . The two adjoint hermitian scalars  $X, Y$  are split between  $V$  and  $\Phi$ . The bottom component of  $\Phi$  is  $A_2 + iX$ . As discussed previously, hypermultiplets of  $\mathcal{N} = 2$ ,  $d = 4$  are comprised of pairs  $\mathcal{N} = 2$ ,  $d = 3$  chiral superfields  $Q^1(x^2)$  and  $Q_2(x^2)$ .

If there are hypermultiplets in the fundamental representation, we can define a chiral Wilson lines extended in the  $x^2$  direction, of the form

$$Q_1^m(0) \mathcal{P} \exp \left( i \int_0^{X^2} dx^2 \Phi \right) Q_2^n(X^2), \quad (4.1)$$

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<sup>3</sup>This anomaly is important for a Dijkgraaf-Vafa conjecture proposed in [10] via the arguments of [17].

where  $m$  and  $n$  are flavor indices. The bottom component belongs to the  $\mathcal{N} = 2, d = 3$  chiral ring. The properties of the chiral ring are determined by the superpotential, which is

$$W = \int dx^2 \tilde{Q}_i \left( \frac{\partial}{\partial x^2} - i\Phi \right) Q_i. \quad (4.2)$$

In the absence of a Konishi anomaly,

$$\begin{aligned} & \frac{\partial}{\partial X^2} \left\langle Q_1^m(0) \mathcal{P} \exp \left( i \int_0^{X^2} dx^2 \Phi \right) Q_2^n(X^2) \right\rangle_{\theta=\bar{\theta}=0} \\ &= \left\langle Q_1^m(0) \mathcal{P} \exp \left( i \int_0^{X^2} dx^2 \Phi \right) \frac{\delta W}{\delta Q_1^n(X^2)} \right\rangle_{\theta=\bar{\theta}=0} = 0. \end{aligned} \quad (4.3)$$

Expectation values of this particular class of Wilson line have no dependence on the length of the Wilson line. Since the scalar components of  $Q_i^m$  belong to  $\mathcal{N} = 2, d = 4$  hypermultiplets, these expectation values are equivalent to expectation values of local operators parameterizing the Higgs branch. These results are readily generalized to eight supercharge Yang-Mills theories in 1, 2 and 3 dimensions. In five dimensions one encounters a Konishi anomaly, while for dimension  $d > 5$  a four supercharge  $d - 1$  dimensional subalgebra does not exist.

## References

- [1] K. Zarembo, “Supersymmetric Wilson loops”, *Nucl. Phys.* **B643** (2002) 157 [[hep-th/0205160](#)].
- [2] Z. Guralnik and B. Kulik, “Properties of chiral Wilson loops,” *JHEP* **0401**, 065 (2004) [[arXiv:hep-th/0309118](#)].
- [3] Z. Guralnik, S. Kovacs and B. Kulik, “Holography and the Higgs branch of  $N = 2$  SYM theories,” [arXiv:hep-th/0405127](#).
- [4] N. Marcus, A. Sagnotti and W. Siegel, “Ten-Dimensional Supersymmetric Yang–Mills Theory In Terms Of Four-Dimensional Superfields”, *Nucl. Phys.* **B224** (1983) 159.
- [5] N. Arkani-Hamed, T. Gregoire and J. Wacker, “Higher dimensional supersymmetry in 4D superspace”, *J. High Energy Phys.* **03** (2002) 055 [[hep-th/0101233](#)].
- [6] J. Erdmenger, Z. Guralnik and I. Kirsch, “Four-dimensional superconformal theories with interacting boundaries or defects”, *Phys. Rev.* **D66** (2002) 025020 [[hep-th/0203020](#)].
- [7] J. Erdmenger, Z. Guralnik, R. Helling and I. Kirsch, “A world-volume perspective on the recombination of intersecting branes”, [hep-th/0309043](#).

- [8] N. R. Constable, J. Erdmenger, Z. Guralnik and I. Kirsch, “(De)constructing intersecting M5-branes”, *Phys. Rev.* **D67** (2003) 106005 [[hep-th/0212136](#)].
- [9] N. R. Constable, J. Erdmenger, Z. Guralnik and I. Kirsch, “Intersecting D3-branes and holography”, *Phys. Rev.* **D68** (2003) 106007 [[hep-th/0211222](#)].
- [10] R. Dijkgraaf and C. Vafa, “N = 1 supersymmetry, deconstruction, and bosonic gauge theories”, [hep-th/0302011](#).
- [11] I. Bena and R. Roiban, “N = 1\* in 5 dimensions: Dijkgraaf-Vafa meets Polchinski-Strassler”, *J. High Energy Phys.* **11** (2003) 001 [[hep-th/0308013](#)].
- [12] J. Maldacena, “Wilson loops in large N field theories,” *Phys. Rev. Lett.* **80** (1998) 4859 [[hep-th/9803002](#)].
- [13] S. J. Rey and J. T. Yee, “Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity,” *Eur. Phys. J. C* **22**, 379 (2001) [[arXiv:hep-th/9803001](#)].
- [14] D. Berenstein, R. Corrado, W. Fischler and J. M. Maldacena, “The operator product expansion for Wilson loops and surfaces in the large N limit,” *Phys. Rev. D* **59**, 105023 (1999) [[arXiv:hep-th/9809188](#)].
- [15] N. Drukker, D. J. Gross and H. Ooguri, “Wilson loops and minimal surfaces,” *Phys. Rev. D* **60**, 125006 (1999) [[arXiv:hep-th/9904191](#)].
- [16] K. i. Konishi and K. i. Shizuya, “Functional Integral Approach To Chiral Anomalies In Supersymmetric Gauge Theories”, *Nuovo Cim.* **A90** (1985) 111.
- [17] F. Cachazo, M. R. Douglas, N. Seiberg and E. Witten, “Chiral rings and anomalies in supersymmetric gauge theory,” *JHEP* **0212**, 071 (2002) [[arXiv:hep-th/0211170](#)].