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Non-semisimple and Complex Gaugings of $N = 16$ Supergravity

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Abstract

Maximal and non-maximal supergravities in three dimensions allow for a large variety of semisimple (Chern-Simons) gauge groups. In this paper, we analyze non-semisimple and complex gauge groups that satisfy the pertinent consistency relations for a maximal ($N = 16$) gauged supergravity to exist. We give a general procedure how to generate non-semisimple gauge groups from known admissible semisimple gauge groups by a singular boost within $E_{8(8)}$. Examples include the theories with gauge group $SO(8) \times T_{28}$ that describe the reduction of IIA/IIB supergravity on the seven-sphere. In addition, we exhibit two ‘strange embeddings’ of the complex gauge group $SO(8, \mathbb{C})$ into (real) $E_{8(8)}$ and prove that both can be consistently gauged. We discuss the structure of the associated scalar potentials as well as their relation to those of $D \geq 4$ gauged supergravities.

1 Introduction

Locally supersymmetric theories in three space time dimensions have at most $N \leq 16$ supersymmetries [22, 25, 10]. For $N > 4$, the scalar sectors of these theories are governed by non-linear σ -models over coset spaces G/H , where H is always the maximal compact subgroup of G (the associated Lie algebras will be denoted as $\mathfrak{g} \equiv \text{Lie } G$ and $\mathfrak{h} \equiv \text{Lie } H$ throughout this paper). As shown only relatively recently, these theories admit extensions where a subgroup $G_0 \subset G$ is promoted to a local symmetry [27, 28, 29, 6]. In contrast to higher dimensional gauged supergravities, the vector fields in general appear via a Chern-Simons (CS) rather than a Yang-Mills (YM) term. As it turns out, there is a surprisingly rich structure and variety of possible gauge groups, which have no analogs in higher dimensional ($D \geq 4$) gauged supergravities. In particular, for the maximal $N = 16$ theory, the following semisimple subgroups of the global $E_{8(8)}$ symmetry have been shown to be consistent gauge groups [27, 28]

$$\begin{aligned} G_0 &= E_8 ; \\ G_0 &= E_7 \times A_1 ; \\ G_0 &= E_6 \times A_2 ; \\ G_0 &= F_4 \times G_2 ; \\ G_0 &= D_4 \times D_4 ; \end{aligned} \tag{1}$$

and appear in all those real forms that ‘fit’ into $E_{8(8)}$ (including, in particular, the groups $SO(p, q) \times SO(p, q)$ for $p+q = 8$). The situation is similar for lower N supergravities [29, 6] where an equally rich variety of gauge groups has been found to exist.

However, it is clear that the list (1) cannot possibly exhaust the groups for which a gauged maximal supergravity can be constructed. First of all, it has been known for a long time that in higher dimensions there exist gaugings with non-semisimple groups [18, 19, 20, 1, 2, 21, 9], implying that similar non-semisimple gaugings should also exist in three dimensions. Secondly, the non-semisimple gaugings in three dimensions play a more prominent role than their higher dimensional cousins: as shown in [30, 6], any three-dimensional YM gauged supergravity with gauge group G_0 is on-shell equivalent to a CS gauged supergravity with non-semisimple gauge group $G_0 \times T$ with a certain translation group T . This class in particular includes all theories obtained by reduction of higher dimensional maximal gauged supergravities on a torus (i.e. a product of circles) or by Kaluza Klein compactification on some internal manifold (such as IIA/IIB supergravity on

the seven-sphere).

In this paper, we will identify these missing gauge groups, and in particular exhibit all those non-semisimple gaugings of the maximal $N = 16$ theory which are equivalent on shell to the torus reductions of the known gauged $N = 8$ theories of [18, 19, 20] after performing the elimination of translational gauge fields described in [30]. However, due to the large number of possibilities we will not aim for an exhaustive classification of non-semisimple gaugings but rather study and explain in detail some representative examples of such gaugings. Our results apply to lower $N < 16$ gauged supergravities as well, furnishing the $D = 3$ supergravities associated with various supersymmetric Kaluza Klein compactifications, and in particular the examples listed in the last section of [30].

The second, and perhaps more surprising main result of the present work is the admissibility of the *complex* gauge group $SO(8, \mathbb{C})$. The fact that the group $SO(8, \mathbb{C})$ can be embedded into the *real* Lie group $E_{8(8)}$ (in two inequivalent ways) seems to have escaped mathematicians' notice so far. Because it does not require an imaginary unit, this embedding exhibits some rather strange properties, which we will highlight in section 4. Like the semisimple gauge groups (1), the $SO(8, \mathbb{C})$ gauged supergravities cannot be derived from higher dimensions by any known mechanism. Furthermore, they feature a de Sitter stationary point at the origin breaking all supersymmetries, and with tachyonic instabilities. Similar complex gaugings are expected to exist for lower $N < 16$ supergravities in three dimensions, in particular an $SO(6, \mathbb{C})$ theory for $N = 12$ and an $SO(5, \mathbb{C})$ theory for $N = 10$. We note that CS gauge theories with complex gauge groups are of considerable interest ([32]; see also [17] and references therein for some very recent developments). The embedding of such theories into supergravity with non-trivial matter couplings may well provide interesting new perspectives.

We now list the main new gauge groups found in this paper for maximal $N = 16$ supergravity, all of which are contained in $E_{8(8)}$

$$\begin{aligned}
 G_0 &= SO(p, q) \times T_{28} \quad \text{for } p + q = 8 ; \\
 G_0 &= CSO(p, q; r) \times T_{p,q,r} \quad \text{for } p + q + r = 8 \text{ and } r > 0 ; \\
 G_0 &= SO(8, \mathbb{C}) .
 \end{aligned}
 \tag{2}$$

The first two of these are non-semisimple extensions of the groups $SO(p, q)$ and $CSO(p, q; r)$, respectively, and have not appeared in the supergravity literature before. Here, T_{28} is an abelian group of 28 translations transforming in the adjoint of $SO(p, q)$. Similarly, $T_{p,q,r}$ is a group of translations, but

of smaller dimension

$$\dim T_{p,q,r} = \dim CSO(p, q; r) = 28 - \frac{1}{2}r(r-1) . \quad (3)$$

According to [30], the elimination of the gauge fields associated with the translational subgroups produces a YM type gauged supergravity with YM gauge group $SO(p, q)$ or $CSO(p, q; r)$, respectively. The resulting YM gauged supergravities coincide with the ones that would be obtained by an S^1 reduction of the $SO(p, q)$ or $CSO(p, q; r)$ gauged $N = 8$ supergravities in four dimensions [18, 19, 20]. In addition to (2) we will exhibit some examples of purely nilpotent gaugings obtained by the boost method of section 2.

In (2) the groups $SO(8) \times T_{28}$ and $SO(8, \mathbb{C})$ are singled out because they admit *two* inequivalent gaugings corresponding to the two embeddings

$$\mathbf{248} = (\mathbf{28}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{28}) \oplus (\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_s, \mathbf{8}_s) \oplus (\mathbf{8}_c, \mathbf{8}_c) , \quad (4)$$

(type IIA) and

$$\mathbf{248} = (\mathbf{28}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{28}) \oplus (\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_s, \mathbf{8}_c) \oplus (\mathbf{8}_c, \mathbf{8}_s) . \quad (5)$$

(type IIB), which differ by a triality rotation. The crucial feature here is that only the compact real form $SO(8)$ admits 8-component *real* spinors and hence the phenomenon of triality. While the compact $SO(8) \times SO(8)$ gaugings based on (4) and (5) are still equivalent, these two embeddings define different diagonal $SO(8)$ subgroups, leading to inequivalent embeddings and gaugings of $SO(8) \times T_{28}$ and $SO(8, \mathbb{C})$. Let us mention that these groups can be written uniformly as $SO(8, \mathbb{C}^\times)$, where \mathbb{C}^\times is equal to the complex numbers \mathbb{C} , the ‘split complex numbers’ \mathbb{C}' or the ‘dual numbers’ \mathbb{C}^0 [31]

$$\begin{aligned} \mathbb{C}' &:= \mathbb{R} \oplus \mathbb{R}e_1 , \\ \mathbb{C}^0 &:= \mathbb{R} \oplus \mathbb{R}e_0 , \end{aligned} \quad (6)$$

with ‘imaginary units’ obeying $e_1^2 = +1$ and $e_0^2 = 0$, respectively, because we have the following group isomorphisms [31]

$$\begin{aligned} SO(8, \mathbb{C}') &\cong SO(8) \times SO(8) , \\ SO(8, \mathbb{C}^0) &\cong SO(8) \times T_{28} . \end{aligned} \quad (7)$$

By contrast, the other groups in (2) involving $SO(p, q)$ or $CSO(p, q; r)$ (with $p \neq 0, 8$) admit *only one* embedding. This is obvious from the decomposition of the $\mathbf{248}$ under the subgroup $SO(p, q) \times SO(p, q) \subset E_{8(8)}$

$$\mathbf{248} = (\mathbf{28}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{28}) \oplus (\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_v, \mathbf{8}_v) \oplus (\mathbf{8}_v, \mathbf{8}_v) . \quad (8)$$

Alternatively, it follows from the fact that in the vacuum the symmetry is always broken to some compact subgroup of the gauge group involving factors of $SO(p)$ with $p < 8$, for which there is no triality.

2 Generalities

As shown in our previous work, the essential information about a maximally supersymmetric gauged $D = 3$ supergravity is encoded into the so-called embedding tensor Θ . This tensor characterizes the embedding of the gauge group into the global symmetry of the ungauged supergravity under consideration (see [10] for a classification of locally supersymmetric σ -models in three space time dimensions), and allows us to immediately write down the Lagrangian and supersymmetry transformations from the formulas of [28] once Θ is explicitly given. For this reason and in order to keep this paper within reasonable proportions we will not present any explicit Lagrangians or supersymmetry transformations, but restrict attention to the embedding tensors and their properties. Readers are therefore advised to consult especially Refs. [27, 28, 30] before delving into the details of the present article.

Here, we briefly summarize some basic properties of the embedding tensor Θ and the scalar field potential, and then explain the boost method, which allows us to derive many non-semisimple gaugings from known (consistent) semisimple ones. This construction works for any number N of supersymmetries, and therefore the discussion in that subsection will be kept general, whereas the rest of this paper is mostly devoted to the maximal $N = 16$ theory.

2.1 The embedding tensor

Generally, gauging a subgroup $G_0 \subset G$ corresponds to promoting the group G_0 to a local symmetry by making the minimal substitution

$$\partial_\mu \longrightarrow \partial_\mu + g\Theta_{\mathcal{MN}}t^\mathcal{M}B_\mu^\mathcal{N}, \quad (9)$$

for any derivative acting on a field transforming under the global symmetry G (prior to gauging). Here, g is the gauge coupling constant, the fields $B_\mu^\mathcal{M}$ are the dual vector fields to the scalar fields, and before gauging transform in the adjoint of the global symmetry group G . By $\{t^\mathcal{M}\}$, we denote a basis of $\mathfrak{g} \equiv \text{Lie } G$ with

$$[t^\mathcal{M}, t^\mathcal{N}] = f^{\mathcal{MN}}{}_p t^p. \quad (10)$$

The so-called embedding tensor [27, 28]

$$\Theta \equiv \Theta_{\mathcal{MN}}t^\mathcal{M} \otimes t^\mathcal{N} \in \text{Sym}(\mathfrak{g} \otimes \mathfrak{g}), \quad (11)$$

characterizes the embedding of the gauge group G_0 into the global symmetry group G , or more succinctly, the embedding of the associated Lie algebras $\mathfrak{g}_0 \subset \mathfrak{g}$. A basis of the Lie algebra \mathfrak{g}_0 is then given by the generators $\Theta_{\mathcal{MN}} t^{\mathcal{N}}$. In particular, we have

$$\dim \mathfrak{g}_0 = \text{rank } \Theta . \quad (12)$$

Evidently, the components $\Theta_{\mathcal{MN}}$ of the embedding tensor depend on the chosen basis and are thus defined only up to the adjoint action of G . They can thus assume various equivalent forms for a given gauge group G_0 .

To facilitate the task of writing out the components of a given embedding tensor, we will use the notation

$$a \vee b := \frac{1}{2}(a \otimes b + b \otimes a) , \quad (13)$$

for the symmetric tensor product. We can also work with the dual embedding tensor $\Theta^{\mathcal{MN}} \equiv \eta^{\mathcal{MK}} \eta^{\mathcal{NL}} \Theta_{\mathcal{KL}}$, where indices are raised and lowered by means of the Cartan-Killing metric η on the Lie algebra \mathfrak{g} , which we always assume to be non-degenerate (this requirement is evidently satisfied for the homogeneous target space manifolds appearing for $N > 4$).

As shown in previous work [27, 28, 6], for a consistent gauged supergravity to exist, the embedding tensor Θ must satisfy two conditions. First, the generators $\Theta_{\mathcal{MN}} t^{\mathcal{N}}$ of the algebra \mathfrak{g}_0 must form a closed algebra, under which $\Theta_{\mathcal{MN}}$ is invariant, i.e.

$$\Theta_{\mathcal{KP}} \Theta_{\mathcal{L}(\mathcal{M}} f^{\mathcal{KL}}{}_{\mathcal{N})} = 0 , \quad (14)$$

with the structure constants $f^{\mathcal{KL}}{}_{\mathcal{N}}$ from (10). Second, the embedding tensor needs to satisfy the projector condition

$$\mathbb{P}_{\mathcal{MN}}{}^{\mathcal{PQ}} \Theta_{\mathcal{PQ}} = 0 , \quad (15)$$

where \mathbb{P} projects onto the subrepresentation in $\text{Sym}(\mathfrak{g} \otimes \mathfrak{g})$ which does *not* occur in the fermionic bilinears that can be built from the gravitinos and the propagating fermions, see [6] for a complete list (it is a non-trivial fact that the R -symmetry representations arising from the fermionic bilinears and compatible with local supersymmetry can always be assembled into representations of the global symmetry group G). We call a subgroup of G ‘admissible’ if its embedding tensor obeys (14) and (15), and hence gives rise to a consistent gauging.

Specializing to $N = 16$ supergravity, the embedding tensor transforms as an element of the tensor product

$$(248 \otimes 248)_{\text{sym}} = \mathbf{1} \oplus \mathbf{3875} \oplus \mathbf{27000} , \quad (16)$$

With the fermionic bilinears that can be built out of the gravitinos and the matter fermions of $N = 16$ supergravity equation (15) becomes

$$(\mathbb{P}_{\mathbf{27000}})_{MN}{}^{\mathcal{PQ}} \Theta_{\mathcal{PQ}} = 0. \quad (17)$$

Following [25, 23], we split the generators of $\mathfrak{g} = \mathfrak{e}_{8(8)}$ into 120 compact ones $X^{IJ} = -X^{JI}$ with $SO(16)$ vector indices $I, J = 1, \dots, 16$, and 128 noncompact ones $\{Y^A\}$ with $SO(16)$ spinor indices $A = 1, \dots, 128$. Then the condition (17) implies that only particular $SO(16)$ representations can appear in Θ ; we have

$$\Theta = \Theta_{IJ|KL} X^{IJ} \vee X^{KL} + 2\Theta_{IJ|A} X^{IJ} \vee Y^A + \Theta_{A|B} Y^A \vee Y^B, \quad (18)$$

with [28, 15]

$$\begin{aligned} \Theta_{IJ|KL} &= -2\theta\delta_{KL}^{IJ} + 2\delta_{\underline{I[K}\underline{\Xi}_{L]J}} + \Xi_{IJKL}, \\ \Theta_{IJ|A} &= -\frac{1}{7}\Gamma_{A\dot{A}}^{[I}\underline{\Xi}^{J]\dot{A}}, \\ \Theta_{A|B} &= \theta\delta_{AB} + \frac{1}{96}\Xi_{IJKL}\Gamma_{AB}^{IJKL}, \end{aligned} \quad (19)$$

and the $SO(16)$ Γ matrices $\Gamma_{A\dot{A}}^I$, where the indices $\dot{A} = 1, \dots, 128$ label the conjugate spinor representation. The tensors Ξ_{IJ} , Ξ_{IJKL} and $\Xi^{I\dot{A}}$ transform as the **135**, **1820** and **$\overline{1920}$** representations of $SO(16)$, respectively; hence $\Xi_{II} = 0 = \Gamma_{A\dot{A}}^I \Xi^{I\dot{A}}$, and Ξ_{IJKL} is completely antisymmetric in its four indices. Unlike for the semisimple gauge groups (1) the singlet contribution in (19) is absent for non-semisimple and complex gauge groups, and we will thus set $\theta = 0$ in the remainder.

For semisimple gaugings, the Lie algebra \mathfrak{g}_0 decomposes as a direct sum

$$\mathfrak{g}_0 = \bigoplus_i \mathfrak{g}_{0i}, \quad (20)$$

of simple Lie algebras \mathfrak{g}_{0i} . The embedding tensor can be written as a sum of projection operators

$$\eta^{\mathcal{M}\mathcal{P}} \Theta_{\mathcal{P}\mathcal{N}} = \sum_i \varepsilon_i (\Pi_i)^{\mathcal{M}}{}_{\mathcal{N}}, \quad (21)$$

where Π_i projects onto the i -th simple factor \mathfrak{g}_{0i} , and the constants ε_i characterize the relative strengths of the gauge couplings. There is only one overall gauge coupling constant g for the maximal theory ($N = 16$), but there may be several independent coupling constants for lower N . For the semisimple examples with maximal supersymmetry known up to now [27, 28], the

sum (21) contains at most two terms. Moreover, for all these gaugings we have

$$\Xi^{I\dot{A}} = 0 \quad (\text{for semisimple } \mathfrak{g}_0). \quad (22)$$

As shown in [28, 15], this implies that all these theories possess maximally supersymmetric (AdS or Minkowski) ground states.

For non-semisimple gaugings, (20) is replaced by

$$\mathfrak{g}_0 = \bigoplus_i \mathfrak{g}_{0i} \oplus \mathfrak{t}, \quad (23)$$

where \mathfrak{t} represents the solvable part of the gauge group. As we will see below, for the non-semisimple gauge groups which appear in our analysis, the latter subalgebra decomposes into

$$\mathfrak{t} = \mathfrak{t}_0 \oplus \mathfrak{t}', \quad (24)$$

where \mathfrak{t}_0 transforms in the adjoint of the semisimple part of the gauge group and pairs up with the semisimple subalgebra in the embedding tensor, which has non-vanishing components only in $\mathfrak{g}_{0i} \vee \mathfrak{t}_0$ and in $\mathfrak{t}' \vee \mathfrak{t}'$. We will also encounter examples of purely nilpotent gaugings, where the semi-simple part is absent.

For the non-semisimple gaugings, in general all components in (19) are non-vanishing, in particular

$$\Xi^{I\dot{A}} \neq 0 \quad (\text{for non-semisimple } \mathfrak{g}_0). \quad (25)$$

It is evident that (21) cannot be valid for non-semisimple gauge groups because the Cartan-Killing metric degenerates on the nilpotent part of the associated Lie algebra. Furthermore, the complex gauge group $SO(8, \mathbb{C})$ whose admissibility we shall demonstrate here, also fails to satisfy (20) when written in the real basis of $E_{8(8)}$; in fact,

$$\theta = \Xi_{IJ} = \Xi_{IJKL} = 0, \quad (\text{for } \mathfrak{g}_0 = \mathfrak{so}(8, \mathbb{C})), \quad (26)$$

so that $\Xi^{I\dot{A}}$ represents its *only* nonvanishing component in (19). As a consequence, the associated ground state is of de Sitter type and supersymmetry is completely broken.

We finally note that the consistency conditions (14), (15) remain covariant under the *complexified* global symmetry group $E_8(\mathbb{C})$. Indeed, non-semisimple gaugings in four dimensions were originally found in [18, 19, 20] by analytic continuation of $SO(8)$ in the complexified global symmetry group

$E_7(\mathbb{C})$. In three dimensions, a similar construction should exist relating the different non-compact real forms of the gauge groups (1), and explaining why the ratios of coupling constants between the two factor groups are the same independently of the chosen real form. Likewise, the gauge groups $SO(8) \times SO(8)$ and $SO(8, \mathbb{C})$ are presumably related by analytic continuation in $E_8(\mathbb{C})$. We will, however, present a more systematic and more direct construction based on an analysis of the consistency conditions (14), (15).

2.2 Some properties of the scalar potential

The embedding tensor $\Theta_{\mathcal{MN}}$ discussed in the previous section completely specifies the gauged supergravity, i.e. its Lagrangian and supersymmetry transformation rules. For the reader's convenience, and because we will refer to them later, we here briefly recall some pertinent formulas for the $N = 16$ theory from [27, 28]). Both the fermionic mass tensors and the scalar potential may be expressed in terms of the so-called T -tensor

$$T_{\mathcal{AB}} = \mathcal{V}_{\mathcal{A}}^{\mathcal{M}} \mathcal{V}_{\mathcal{B}}^{\mathcal{N}} \Theta_{\mathcal{MN}}, \quad (27)$$

where $\mathcal{V}_{\mathcal{A}}^{\mathcal{M}} \in E_{8(8)}$ is a group valued matrix (the 248-bein) that combines the scalar fields of the theory. The fermions in the theory are the 16 gravitini ψ_{μ}^I and the 128 spin-1/2 matter fermions $\chi^{\dot{A}}$. They arise in the Lagrangian in bilinear combinations contracted with the scalar (Yukawa) tensors

$$\begin{aligned} A_1^{IJ} &\equiv \frac{8}{7} \theta \delta_{IJ} + \frac{1}{7} T_{IK,JK}, \\ A_2^{I\dot{A}} &\equiv -\frac{1}{7} \Gamma_{A\dot{A}}^J T_{IJ,A}, \\ A_3^{\dot{A}\dot{B}} &\equiv 2\theta \delta_{\dot{A}\dot{B}} + \frac{1}{48} \Gamma_{\dot{A}\dot{B}}^{IJKL} T_{IJ,KL}. \end{aligned} \quad (28)$$

The scalar potential W of maximal $N = 16$ supergravity has a rather simple form in terms of the T -tensor (27), but becomes an extremely complicated function when expressed directly in terms of the 128 physical scalar fields [11, 12]. It reads

$$W \equiv \frac{1}{8} g^2 \left(A_1^{IJ} A_1^{IJ} - \frac{1}{2} A_2^{I\dot{A}} A_2^{I\dot{A}} \right). \quad (29)$$

In [28] it is shown that the extrema of the potential must obey the (necessary and sufficient) condition

$$3 A_1^{IM} A_2^{M\dot{A}} - A_3^{\dot{A}\dot{B}} A_2^{I\dot{B}} \stackrel{!}{=} 0, \quad (30)$$

This condition is met in particular if $A_2^{I\dot{A}} = 0$, as is the case for all semisimple gaugings in (1) at the trivial stationary points $\mathcal{V} = \mathbf{1}$. Moreover, the

vanishing of A_2^{IA} implies maximal supersymmetry of these groundstates, which are therefore stable by the general analysis of [16]. As we will see, the complex gauge group $SO(8, \mathbb{C})$ realizes another possibility to satisfy (30): there we have $A_1^{IJ} = 0$ and $A_3^{\dot{A}\dot{B}} = 0$ for $\mathcal{V} = \mathbf{1}$.

2.3 The boost method

To find gaugings with non-semisimple groups in three dimensions, one can either directly search for solutions of the above two conditions (14), (15), or try to generate new solutions to these equations from known semisimple ones. A convenient alternative method realizing this possibility is the ‘boost method’, which we will now explain.

The method can be applied to any admissible semisimple gauge group $G_0 \subset G$. Having chosen a suitable G_0 , one selects a non-compact (‘boost’) generator $N \in \mathfrak{g}$, such that $N \notin \mathfrak{g}_0$. This boost generator will preserve a (still semisimple) subgroup $\tilde{G}_0 \subset G_0$, i.e. $[N, \tilde{\mathfrak{g}}_0] = 0$ where $\tilde{\mathfrak{g}}_0$ is the associated stable subalgebra of \mathfrak{g}_0 . The deformation can be understood systematically by decomposing (‘grading’) the full Lie algebra \mathfrak{g} into eigenspaces of N under its adjoint action. That is,

$$\mathfrak{g} = \bigoplus_{j=-\ell}^{\ell} \mathfrak{g}^{(j)}, \quad (31)$$

where $[N, t] = jt$ for $t \in \mathfrak{g}^{(j)}$ and ℓ is the maximum eigenvalue under the adjoint action of N . Obviously $\tilde{\mathfrak{g}}_0 = \mathfrak{g}^{(0)} \cap \mathfrak{g}_0$.

Since the embedding tensor Θ has two indices in the adjoint representation of \mathfrak{g} , we have a similar decomposition for it, viz.

$$\Theta = \sum_{j=-2\ell}^{2\ell} \Theta^{(j)}. \quad (32)$$

Under the action of the boost $\exp(\lambda N)$ the embedding tensor scales as

$$\exp(\lambda N) : \Theta \longrightarrow \sum_{j=-2\ell}^{2\ell} e^{j\lambda} \Theta^{(j)}. \quad (33)$$

The graded pieces $\Theta^{(j)}$ themselves need not transform irreducibly under $\tilde{\mathfrak{g}}_0$. We now exploit two basic properties of Θ and the consistency conditions (14),(15), namely

- the covariance of Θ w.r.t. to the global symmetry group G , implying that a ‘rotated’ embedding tensor still satisfies the conditions (14) and (15), and
- the fact that these conditions remain valid under rescaling of Θ .

We thus consider the boosted embedding tensor (33) and simultaneously replace $g \rightarrow ge^{-\omega\lambda}$ for the gauge coupling constant multiplying Θ in (9), where ω is the maximum degree appearing in the decomposition (32) of Θ (and might be different from 2ℓ). While the resulting Θ is still equivalent to the original one for any finite λ , this needs no longer be true for the limit $\lambda \rightarrow \infty$. By continuity, the new embedding tensor

$$\bar{\Theta} := \lim_{\lambda \rightarrow \infty} \left(e^{-\omega\lambda} \exp(\lambda N)(\Theta) \right), \quad (34)$$

still satisfies the projector condition (15) and the quadratic condition (14). Given a particular grading as in (32), it is now easy to see that only the highest components survive in this limit, viz.

$$\bar{\Theta}^{(j)} := \begin{cases} \Theta^{(j)} & \text{for } j = \omega \\ 0 & \text{otherwise} \end{cases}. \quad (35)$$

We emphasize once more that the graded piece $\bar{\Theta}^{(j)}$ may have more than one irreducible component. Furthermore, it is easy to see that the limiting gauge group will be solvable unless the graded components $\bar{\Theta}^{(j)}$ contain a piece intersecting the compact subalgebra.

While the structure constants f of the global symmetry group are not affected by the boost because they are invariant w.r.t. to the global group G , the boosted structure constants of the gauge group will no longer be equivalent to the original ones.¹ If the embedding tensor allows more than one free gauge coupling constant, we have the freedom to also scale the independent gauge couplings independently in such a way that different limits $\lambda \rightarrow \infty$ give rise to inequivalent new solutions of (14), (15).

For each ‘seed’ gauge group G_0 the non-semisimple gauge groups that can be generated by this method can be systematically searched for by (i) identifying an appropriate boost generator N , and (ii) decomposing the embedding tensor Θ into graded pieces according to (32). The problem can therefore be reduced to the classification of all possible graded decompositions of the Lie algebra \mathfrak{g} , and to analyzing how Θ decomposes under

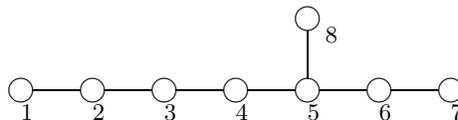
¹The limit (34) therefore realizes the well known Wigner-Inönü contraction.

them. The first problem, in turn, can be reformulated in terms of graded decompositions of the associated root systems.

An equally important consequence of the above derivation is that the projector condition (15) is in fact satisfied grade by grade in the decomposition (32). Given any embedding tensor satisfying (15) and (14) we can thus try not only to keep components with $j = \omega'$ for $\omega' < \omega$, but also to change the relative factors between the different components. In general, the quadratic constraint (14) will then fail to be satisfied, unless the generators appearing in (9) form again a closed algebra. However, this is relatively easy to ascertain by direct inspection. We will make use of this trick in order to establish the admissibility of the new gauge group $SO(8, \mathbb{C}) \subset E_{8(8)}$ (with two inequivalent embeddings) for the maximal $N = 16$ theory.

3 Non-semisimple gaugings

We first exemplify the boost method by deriving new non-semisimple gauge groups from the maximal $N = 16$ theory with compact gauge group $G_0 = SO(8) \times SO(8)$. The first two of our examples are especially important because they are directly related to the YM type maximal supergravities obtained by compactification of IIA and IIB supergravity on $AdS_3 \times S^7$.² The YM gauge group is $SO(8)$ in both cases, and by the general result of [30] the corresponding CS gauge group must be the non-semisimple extension of $SO(8)$ by a 28-dimensional group of translations T_{28} transforming in the adjoint of $SO(8)$; this is indeed one of the non-semisimple groups listed in (2). Different non-semisimple, and in particular purely nilpotent, gaugings can be obtained by boosting $SO(8) \times SO(8)$ with other boost generators $N \in E_{8(8)}$. The different boostings correspond to different gradings, and the associated semisimple gauge subgroups can be read off from the E_8 Dynkin diagram, which we give below with our numbering of the simple roots.



The most general grading is obtained by assigning real numbers s_i (usually taken to be non-negative integers) to the simple roots α_i and defining the

²For the type I theory, the reduction has been performed explicitly in [5]. The KK spectra of the IIA/IIB theories on S^7 have been given in [26].

degree \mathcal{D} of a given root $\alpha = \sum_i n_i \alpha_i$ as

$$\mathcal{D}(\alpha) = \sum_j n_j s_j . \quad (36)$$

The vector (s_1, s_2, \dots) will be referred to as the “grading vector”. The roots of the semisimple subalgebra $\tilde{\mathfrak{g}}_0 \subset \mathfrak{g}_0$ that is preserved by the boosting obviously satisfy $\mathcal{D}(\alpha) = 0$.

The extension of these considerations to lower N is immediate. In particular, the existence of gauged supergravities in four dimensions with $n < 8$ supersymmetries and gauge groups $SO(n)$ implies the existence of CS type gauged supergravities in three dimensions with $N = 2n$ supersymmetries and CS gauge groups $SO(n) \ltimes T$ with $\dim T = \frac{1}{2}n(n-1)$.³

3.1 $G_0 = SO(8) \ltimes T_{28}$ (type IIA)

Type IIA supergravity can be compactified on $\text{AdS}_3 \times S^7$, giving rise to a maximal YM gauged supergravity with gauge group $SO(8)$, which furthermore coincides with the S^1 reduction of maximal $SO(8)$ gauged supergravity in four dimensions. We now show how to obtain the required CS gauge group $G_0 = SO(8) \ltimes T_{28}$ from the compact gauge group $SO(8) \times SO(8)$ by an appropriate boost. In the next section, we will exhibit a second and inequivalent theory with the same gauge group based on the compactification of IIB supergravity on $\text{AdS}_3 \times S^7$.

The construction is based on the 5-graded decomposition (i.e. $\ell = 2$) of $E_{8(8)}$ under its subgroup $E_{7(7)} \times SL(2, \mathbb{R})$

$$\mathbf{248} = \mathbf{1} \oplus \mathbf{56} \oplus [\mathbf{1} \oplus \mathbf{133}] \oplus \mathbf{56} \oplus \mathbf{1} , \quad (37)$$

and associated with the grading vector

$$(s_1, \dots, s_8) = (1, 0, 0, 0, 0, 0, 0, 0) . \quad (38)$$

To give more details, we further decompose the generators w.r.t. the embedding $\mathfrak{so}(8) \subset \mathfrak{sl}(8, \mathbb{R}) \subset \mathfrak{e}_{7(7)}$. Using $SO(8)$ indices $a, \alpha, \dot{\alpha}$ for the representations $\mathbf{8}_v$, $\mathbf{8}_s$ and $\mathbf{8}_c$, respectively, the $\mathfrak{e}_{8(8)}$ generators X^{IJ} and Y^A decompose as

$$\begin{aligned} I & : \mathbf{8}_v + \mathbf{8}_v \implies [IJ] : \mathbf{28} + \mathbf{28} + \mathbf{1} + \mathbf{28} + \mathbf{35}_v , \\ A & : \mathbf{8}_s \times \mathbf{8}_s + \mathbf{8}_c \times \mathbf{8}_c = \mathbf{1} + \mathbf{28} + \mathbf{35}_s + \mathbf{1} + \mathbf{28} + \mathbf{35}_c . \end{aligned} \quad (39)$$

³The $n \leq 4$ and $n = 5$ gauged theories in four dimensions were found already long ago in [13, 14] and [7], respectively. The $SO(6)$ gauged supergravity was never explicitly constructed, but can be obtained by truncation of the maximal $SO(8)$ gauged theory.

Hence, $\mathfrak{e}_{8(8)}$ decomposes as (cf. (4))

$$\mathbf{248} = \mathbf{1} \oplus [\mathbf{28} \oplus \mathbf{28}] \oplus [\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_v \oplus \mathbf{35}_s \oplus \mathbf{35}_c] \oplus [\mathbf{28} \oplus \mathbf{28}] \oplus \mathbf{1} . \quad (40)$$

Now we see that the representation content in (37) matches indeed with that of $N = 8, D = 4$ gauged supergravity [8] reduced on a torus: after removal of the $\mathbf{1} + 3 \cdot \mathbf{28} + \mathbf{35}_v$ gauge degrees of freedom, we are left with the 70 scalars of the $D = 4$ theory, and 2×28 scalars coming from the 28 (YM) vector fields, together with two more scalar fields descending from the dilaton and the graviphoton, and residing in the coset $SL(2, \mathbb{R})/SO(2)$. The level zero generators consist of the grading generator N , the generators $E_{ab} = -E_{ba}$ and $S_{ab} = S_{ba}$ which obey

$$\begin{aligned} [E_{ab}, E_{cd}] &= 2\delta_{d[a}E_{b]c} - 2\delta_{c[a}E_{b]d} , \\ [E_{ab}, S_{cd}] &= 2\delta_{d[a}S_{b]c} + 2\delta_{c[a}S_{b]d} , \\ [S_{ab}, S_{cd}] &= 2\delta_{d(a}E_{b)c} + 2\delta_{c(a}E_{b)d} , \end{aligned} \quad (41)$$

and thus close into the Lie algebra of $SL(8, \mathbb{R}) \subset E_{7(7)}$, together with the 35 compact and the 35 non-compact (traceless) generators $S_{\alpha\beta} = S_{\beta\alpha}$ and $S_{\dot{\alpha}\dot{\beta}} = S_{\dot{\beta}\dot{\alpha}}$, respectively, which enlarge $SL(8, \mathbb{R})$ to the full $E_{7(7)}$. At level one, we have the $28 + 28$ generators (U_{ab}, V^{ab}) transforming in the fundamental $\mathbf{56}$ representation of $E_{7(7)}$, and at level -1 a conjugate set of $28 + 28$ generators (U^{ab}, V_{ab}) . These obey

$$[U_{ab}, U_{cd}] = [V^{ab}, V^{cd}] = 0 \quad , \quad [U_{ab}, V^{cd}] = \delta_{ab}^{cd} S^+ , \quad (42)$$

where S^+ is the level-2 singlet (the formulas for levels -1 and -2 are analogous). The compact group $SO(8) \times SO(8)$ is generated by the E_{ab} building the diagonal subgroup, and the linear combinations $U_{ab} - U^{ab}$ (or alternatively $V_{ab} - V^{ab}$). Its embedding tensor has been given in [27, 28] and is of the type (21) with the relative gauge coupling constant equal to -1 . In the above basis, it takes the form

$$\Theta = E_{ab} \vee U_{ab} - E_{ab} \vee U^{ab} , \quad (43)$$

(with the usual summation convention on the indices a, b) as one can easily check by writing out (9) explicitly. Therefore Θ has non-vanishing components only at grades ± 1 .

Boosting with N , we get

$$\exp(\lambda N)(\Theta) = e^\lambda E_{ab} \vee U_{ab} - e^{-\lambda} E_{ab} \vee U^{ab} . \quad (44)$$

Rescaling and taking the limit $\lambda \rightarrow +\infty$ as described above, we find the new embedding tensor

$$\bar{\Theta} = E_{ab} \vee U_{ab} . \quad (45)$$

Hence, the nonvanishing components of the new embedding tensor $\bar{\Theta}$ appear at grade +1. The associated Lie algebra is indeed the one corresponding to the group $SO(8) \times T_{28}$, and is spanned by the 28 $SO(8)$ generators E_{ab} and the 28 translation generators U_{ab} . To see this even more explicitly, we write out the minimal coupling (9)

$$\Theta_{\mathcal{MN}} B_{\mu}^{\mathcal{M}} t^{\mathcal{N}} = A_{\mu}^{ab} E_{ab} + C_{\mu}^{ab} U_{ab} . \quad (46)$$

Here A_{μ} and C_{μ} are those 28+28 vector fields out of the 248 vector fields $B_{\mu}^{\mathcal{M}}$ that are ‘excited’ by the gauging.

3.2 $G_0 = SO(8) \times T_{28}$ (type IIB)

Compactification of type IIB supergravity on $AdS_3 \times S^7$ gives rise to another maximal gauged supergravity of YM type in three dimensions with gauge group $SO(8)$. This theory is again equivalent on shell to a maximal gauged supergravity of CS type with non-semisimple gauge group $SO(8) \times T_{28}$. Although the gauge groups of the IIA and IIB compactifications are thus the same, their respective embeddings into $E_{8(8)}$ differ by a triality rotation, and there is no transformation that maps the two theories onto one another. Unlike the IIA theory given above which has its alternative origin in the maximal gauged D=4 theory of [8], the IIB theory may not be obtained by simple torus reduction from higher dimensions.

The embedding of $SO(8) \times SO(8)$ into $E_{8(8)}$ in the IIB basis and the identification of the requisite boost generator rely on the 7-graded decomposition (i.e. $\ell = 3$) of $E_{8(8)}$ w.r.t. its $SL(8, \mathbb{R})$ subgroup introduced in [4] (see also appendix B of [23])

$$248 = \mathbf{8} \oplus \overline{\mathbf{28}} \oplus \mathbf{56} \oplus [\mathbf{1} \oplus \mathbf{63}] \oplus \overline{\mathbf{56}} \oplus \mathbf{28} \oplus \overline{\mathbf{8}} . \quad (47)$$

The grade zero sector consists of the $\mathfrak{sl}(8, \mathbb{R})$ subalgebra and a singlet which will serve as the boost (and grading) generator. The grading vector is

$$(s_1, \dots, s_8) = (0, 0, 0, 0, 0, 0, 0, 1) . \quad (48)$$

Next we decompose the generators w.r.t. the subgroup $SO(8) \subset SL(8, \mathbb{R})$ which gives (cf. (5))

$$248 = \mathbf{8}_v \oplus \mathbf{28} \oplus \mathbf{56}_v \oplus [\mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35}_v] \oplus \mathbf{56}_v \oplus \mathbf{28} \oplus \mathbf{8}_v . \quad (49)$$

Indeed, this matches with the lowest floor of the KK tower of the IIB theory on S^7 [26]. Using the same notations as in the foregoing section, the $E_{8(8)}$ generators X^{IJ} and Y^A now decompose as (cf. (4))

$$\begin{aligned} I & : \mathbf{8}_s + \mathbf{8}_c \implies [IJ] : \mathbf{28} + \mathbf{28} + \mathbf{8}_v + \mathbf{56}_v , \\ A & : \mathbf{8}_v \times \mathbf{8}_v + \mathbf{8}_s \times \mathbf{8}_c = \mathbf{1} + \mathbf{28} + \mathbf{35}_v + \mathbf{8}_v + \mathbf{56}_v . \end{aligned} \quad (50)$$

The diagonal $SO(8)$ subgroup is generated by the elements

$$E_{ab} := \frac{1}{4} \left(\gamma_{\alpha\beta}^{ab} X^{\alpha\beta} + \gamma_{\dot{\alpha}\dot{\beta}}^{ab} X^{\dot{\alpha}\dot{\beta}} \right) , \quad (51)$$

with $SO(8)$ γ -matrices, and where $X^{\alpha\beta}$ and $X^{\dot{\alpha}\dot{\beta}}$ generate the compact $SO(8) \times SO(8)$ subgroup of $E_{8(8)}$. The commutation relations are the same as in (41).

The grading operator $N \equiv Y^{cc}$ commutes with $\mathfrak{sl}(8, \mathbb{R})$ (so we have in particular $[N, E_{ab}] = 0$), and therefore this subalgebra is unaffected by any boosting with N . We also need the 28+28 nilpotent abelian generators (cf. appendix B of [23])

$$\begin{aligned} Z_{ab} & = \frac{1}{8} \left(\gamma_{\alpha\beta}^{ab} X^{\alpha\beta} - \gamma_{\dot{\alpha}\dot{\beta}}^{ab} X^{\dot{\alpha}\dot{\beta}} \right) + Y^{[ab]} , \\ Z^{ab} & = -\frac{1}{8} \left(\gamma_{\alpha\beta}^{ab} X^{\alpha\beta} - \gamma_{\dot{\alpha}\dot{\beta}}^{ab} X^{\dot{\alpha}\dot{\beta}} \right) + Y^{[ab]} . \end{aligned} \quad (52)$$

The relevant commutation relations between these generators are

$$\begin{aligned} [E_{ab}, E_{cd}] & = 2\delta_{d[a} E_{b]c} - 2\delta_{c[a} E_{b]d} , \\ [E_{ab}, Z_{cd}] & = 2\delta_{d[a} Z_{b]c} - 2\delta_{c[a} Z_{b]d} , \\ [E_{ab}, Z^{cd}] & = 2\delta^{d[a} Z^{b]c} - 2\delta^{c[a} Z^{b]d} , \\ [Z_{ab}, Z_{cd}] & = [Z^{ab}, Z^{cd}] = 0 , \end{aligned} \quad (53)$$

together with

$$[N, Z_{ab}] = 2Z_{ab} , \quad [N, Z^{ab}] = -2Z^{ab} . \quad (54)$$

In this basis, the compact gauge group $SO(8) \times SO(8)$ is generated by the E_{ab} building the diagonal subgroup, and the linear combinations $Z_{ab} - Z^{ab}$. The embedding tensor (43) in this basis is

$$\Theta = E_{ab} \vee Z_{ab} - E_{ab} \vee Z^{ab} , \quad (55)$$

with all other components of Θ vanishing. Therefore, the embedding tensor has nonvanishing components only at levels ± 2 .

As before, we boost with N and make use of (54), to get the new embedding tensor

$$\bar{\Theta} = E_{ab} \vee Z_{ab} . \quad (56)$$

In accordance with our general arguments above $\bar{\Theta}$ thus has only the graded piece $\bar{\Theta}^{(2)}$. That (56) is indeed the embedding tensor for $SO(8) \times T_{28}$ inside $E_{8(8)}$ follows as in (46).

3.3 $G_0 = SO(p, 8 - p) \times T_{28}$

$N = 8$ supergravity in four dimensions admits gaugings for all gauge groups $CSO(p, q; r)$ with $p + q + r = 8$ [18, 19, 20]. These are semisimple for $r = 0$, and non-semisimple for $r > 0$ (and all contained in $E_{7(7)}$ regardless of the choice of p, q, r). By dimensional reduction on S^1 , each of these theories gives rise to a maximal gauged supergravity of YM type in three dimensions, whose CS type description requires the gauge groups $G_0 = CSO(p, q; r) \times T \subset E_{8(8)}$ such that the elimination of the translational gauge fields associated with T leads back to a YM type gauged supergravity with gauge group $CSO(p, q; r)$. We first discuss the case $r = 0$, which descends from the semisimple non-compact gaugings with gauge groups $SO(p, 8 - p)$ in four dimensions.

As explained in the introduction, there is no distinction between IIA and IIB for these non-compact embeddings. For definiteness, we will therefore use the IIB basis of section 3.2, where we already defined the $SO(8)$ generators E_{ab} and the nilpotent generators Z_{ab} and Z^{ab} . We also need the 35 non-compact generators

$$S_{ab} := 2Y^{(ab)} - \frac{1}{4}\delta^{ab}Y^{cc} , \quad (57)$$

which enlarge $\mathfrak{so}(8)$ to $\mathfrak{sl}(8, \mathbb{R})$. In addition to (53) we have the commutation relations

$$\begin{aligned} [S_{ab}, Z_{cd}] &= -2\delta_{d(a}Z_{b)c} + 2\delta_{c(a}Z_{b)d} - \frac{1}{2}\delta_{ab}Z_{cd} , \\ [S_{ab}, Z^{cd}] &= -2\delta^{d(a}Z^{b)c} + 2\delta^{c(a}Z^{b)d} - \frac{1}{2}\delta_{ab}Z^{cd} , \end{aligned} \quad (58)$$

and

$$[Z_{ab}, Z^{cd}] = \delta_{c[a}(E_{b]d} + S_{b]d}) - \delta_{d[a}(E_{b]c} + S_{b]c}) + \frac{1}{2}\delta_{ab}^{cd}N . \quad (59)$$

To identify the $SO(p, q) \times SO(p, q)$ subgroup inside $E_{8(8)}$, we split the $SO(8)$ indices a, b, \dots into indices $i, j, \dots \in \{1, \dots, p\}$ and $r, s, \dots \in \{p + 1, \dots, 8\}$.

Then the Lie algebras of the two factor groups $SO(p, q)$ are spanned by

$$\begin{aligned} E_{ij} - Z_{ij} + Z^{ij} \ , \quad E_{rs} + Z_{rs} - Z^{rs} \ , \quad S_{ir} + Z_{ir} + Z^{ir} \ , \\ E_{ij} + Z_{ij} - Z^{ij} \ , \quad E_{rs} - Z_{rs} + Z^{rs} \ , \quad S_{ir} - Z_{ir} - Z^{ir} \ , \end{aligned} \quad (60)$$

respectively, where the generators $E_{ij} \pm (Z_{ij} - Z^{ij})$ and $E_{rs} \pm (Z_{rs} - Z^{rs})$ are compact, whereas the pq generators $S_{ir} \pm (Z_{ir} + Z^{ir})$ are non-compact. In this basis, the embedding tensor of the maximal gauged supergravity with gauge group $SO(p, q) \times SO(p, q)$ [28] is given by

$$\Theta = E_{ij} \vee (Z_{ij} - Z^{ij}) - E_{rs} \vee (Z_{rs} - Z^{rs}) + S_{ir} \vee (Z_{ir} + Z^{ir}) \ . \quad (61)$$

Applying the boost method as before we get the new embedding tensor

$$\bar{\Theta} = E_{ij} \vee Z_{ij} - E_{rs} \vee Z_{rs} + S_{ir} \vee Z_{ir} \ , \quad (62)$$

corresponding to the non-semisimple group $G_0 = SO(p, q) \ltimes T_{28}$ whose $SO(p, q)$ subgroup is generated by $\{E_{ij}, E_{rs}, S_{ir}\}$, and whose nilpotent part is spanned by the 28 elements Z_{ab} .

In [15] we have given the embedding tensor (61) in the IIA basis, where it involves $SO(8)$ γ -matrices. The above IIB basis is triality rotated w.r.t. to the one used there, which explains the simpler form of the above embedding tensor. Just as for the two inequivalent theories constructed in sections 3.1, 3.2 above from the compact $SO(8) \times SO(8)$, we expect a second inequivalent theory with gauge group $SO(p, q) \ltimes T_{28}$ obtained from boosting the embedding tensor in the IIA basis.

3.4 $G_0 = CSO(p, q; r) \ltimes T_{p,q,r}$ for $r > 0$

For $r > 0$ the non-semisimple groups $CSO(p, q; r) \times CSO(p, q; r)$ cannot be embedded into $E_{8(8)}$ and thus are *not* admissible gauge groups for $N = 16$ supergravity. However, there are the non-semisimple groups containing only one $CSO(p, q; r)$ factor, namely the groups $CSO(p, q; r) \ltimes T_{p,q,r}$ from the list (2). As there seems to be no way to get these non-semisimple gauge groups by the boost method, we proceed directly to their description. For this purpose, we have to further refine the split of $SO(8)$ indices a, b, \dots into $i, j, \dots \in \{1, \dots, p\}$, $m, n, \dots \in \{p+1, \dots, p+q\}$ and $s, t, \dots \in \{p+q+1, \dots, 8\}$. The generators of the non-semisimple subgroup $CSO(p, q; r)$ are then E_{ij} , E_{mn} and S_{im} for the non-compact semisimple subgroup $SO(p, q)$, and the $(p+q)r$ translation generators

$$T_{is} := E_{is} + S_{is} \ , \quad T_{ms} := E_{ms} + S_{ms} \ , \quad (63)$$

both of which can be obtained by boosting E_{is} and E_{ms} within $E_{7(7)}$. To this algebra we adjoin the $\frac{1}{2}(p+q)(p+q-1) + (p+q)r$ commuting generators Z_{ij}, Z_{mn}, Z_{im} and Z_{is}, Z_{ms} : this defines the Lie algebra of the non-semisimple group $CSO(p, q; r) \times T_{p,q,r}$. The removal of the generators E_{st} from the definition of $CSO(p, q; r)$ is thus accompanied by the removal of the nilpotent elements Z_{st} from the translation group T_{28} yielding the subgroup $T_{p,q,r}$. Together with the $(p+q)r$ nilpotent generators of $CSO(p, q; r)$, we thus have altogether $\frac{1}{2}(p+q)(p+q-1) + 2(p+q)r$ nilpotent generators. Although this number may exceed 36 (for instance with the choice $(p, q, r) = (3, 2, 3)$), which is the maximal number of mutually commuting nilpotent generators in $E_{8(8)}$, there is no contradiction because

$$[T_{is}, Z_{jt}] = \delta_{st} Z_{ij}, \quad [T_{ms}, Z_{nt}] = \delta_{st} Z_{mn}, \quad (64)$$

do not commute. Observe again that Z_{st} does not appear in this commutation relation. The maximal number of mutually commuting generators in $CSO(p, q; r) \times T_{p,q,r}$ is thus equal to $\frac{1}{2}(p+q)(p+q-1) + (p+q)r < 28$ for any choice of (p, q, r) .

Analysis of the projector condition (17) reveals that the $CSO(p, q; r) \times T_{p,q,r}$ embedding tensor has the following non-vanishing components

$$\begin{aligned} \bar{\Theta} = & E_{ij} \vee Z_{ij} - E_{mn} \vee Z_{mn} + 2S_{im} \vee Z_{im} \\ & + T_{is} \vee Z_{is} - T_{ms} \vee Z_{ms}, \end{aligned} \quad (65)$$

with all other components vanishing.

3.5 Nilpotent gauge groups

There are many other gradings which one can use to boost $SO(8) \times SO(8)$ or the non-compact and non-semisimple gauge groups discussed in the foregoing sections. However, most of these will lead to nilpotent gauge groups because the compact part of the gauge group is boosted away completely. Moreover, different boosts do not necessarily lead to new and different gauge groups. For instance, due to the high symmetry of the $SO(8) \times SO(8)$ embedding tensor, the more difficult task is to obtain *different* gaugings from boosting from that group.

To give an example, still with maximal grade zero subalgebra, choose the grading vector $(s_1, \dots, s_8) = (0, 0, 0, 0, 0, 0, 1, 0)$, which yields the 5-graded decomposition

$$248 = 14 \oplus 64 \oplus [1 \oplus 91] \oplus 64 \oplus 14. \quad (66)$$

Inspection of the Dynkin diagram shows that the **91** at grade zero is the $\mathfrak{so}(7,7)$ algebra, under which the **14** and **64** transform as the vector and spinor representations, respectively. With regard to this subalgebra, the embedding tensor has non-vanishing graded pieces

$$\Theta \in \mathfrak{g}^{(2)} \vee \mathfrak{g}^{(2)} \oplus \mathfrak{g}^{(2)} \vee \mathfrak{g}^{(-2)} \oplus \mathfrak{g}^{(0)} \vee \mathfrak{g}^{(0)} \oplus \mathfrak{g}^{(-2)} \vee \mathfrak{g}^{(-2)} . \quad (67)$$

After boosting, we are thus left with a purely nilpotent embedding tensor

$$\bar{\Theta} \in \mathfrak{g}^{(2)} \vee \mathfrak{g}^{(2)} \quad (68)$$

yielding a 14-dimensional abelian nilpotent gauge group. We can identify this group with the gauge group

$$G_0 = CSO(1,0;7) \times T_{1,0,7} , \quad (69)$$

of the previous section.

Choosing instead the grading vector $(s_1, \dots, s_8) = (1, 0, 0, 0, 0, 0, 1, 0)$ gives rise to the 9-graded decomposition

$$\begin{aligned} \mathbf{248} = & \mathbf{1} \oplus \mathbf{12} \oplus [\mathbf{32} \oplus \mathbf{1}] \oplus [\mathbf{32} \oplus \mathbf{12}] \oplus [\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{66}] \\ & \oplus [\mathbf{32} \oplus \mathbf{12}] \oplus [\mathbf{32} \oplus \mathbf{1}] \oplus \mathbf{12} \oplus \mathbf{1} , \end{aligned} \quad (70)$$

with an $\mathfrak{so}(6,6)$ and *two* singlets in the middle, and the vector representation **12** and the spinor representation **32**. There are thus two possible boost generators, whose different linear combinations correspond to the different values of s_1 and s_7 (both of which have been chosen = 1 above). Now, Θ decomposes as

$$\begin{aligned} \Theta \in & \mathfrak{g}^{(4)} \vee \mathfrak{g}^{(2)} \oplus \mathfrak{g}^{(3)} \vee \mathfrak{g}^{(3)} \oplus \mathfrak{g}^{(4)} \vee \mathfrak{g}^{(-2)} \oplus \mathfrak{g}^{(1)} \vee \mathfrak{g}^{(1)} \\ & \oplus \mathfrak{g}^{(3)} \vee \mathfrak{g}^{(-3)} \oplus \mathfrak{g}^{(0)} \vee \mathfrak{g}^{(0)} \oplus \mathfrak{g}^{(-1)} \vee \mathfrak{g}^{(-1)} \\ & \oplus \mathfrak{g}^{(2)} \vee \mathfrak{g}^{(-4)} \oplus \mathfrak{g}^{(-3)} \vee \mathfrak{g}^{(-3)} \oplus \mathfrak{g}^{(-4)} \vee \mathfrak{g}^{(-2)} . \end{aligned} \quad (71)$$

Boosting with the above grading vector leaves us with

$$\bar{\Theta} \in \mathfrak{g}^{(4)} \vee \mathfrak{g}^{(2)} \oplus \mathfrak{g}^{(3)} \vee \mathfrak{g}^{(3)} . \quad (72)$$

Examining the representation content of the grade 6 contributions to $\bar{\Theta}$, one sees that the associated nilpotent gauge group contains only the $14 = 1+12+1$ nilpotent generators, and therefore coincides with the gauge group $CSO(1,0;7) \times T_{1,0,7}$ obtained above.

4 Complex gaugings: $G_0 = SO(8, \mathbb{C})$

We now come to our most surprising result, which is the admissibility of the *complex* group $G_0 = SO(8, \mathbb{C})$. This result is arrived at by exploiting the observation made at the end of section 2, according to which we can change the relative factors between the components of the embedding tensor at different grades, as long as the modified embedding tensor still defines a closed algebra. Here we simply need to switch the relative sign between the two terms in the $SO(8) \times SO(8)$ embedding tensor in (55) to get

$$\Theta = E_{ab} \vee Z_{ab} + E_{ab} \vee Z^{ab}, \quad (73)$$

again with all other components vanishing. Writing out (9) with (73) one immediately deduces that the associated Lie algebra is spanned by the $\mathfrak{e}_{8(8)}$ elements E_{ab} and the 28 elements

$$F_{ab} := Z_{ab} + Z^{ab} \equiv 2Y^{[ab]}, \quad (74)$$

which together again form a closed algebra:

$$\begin{aligned} [E_{ab}, E_{cd}] &= 2\delta_{d[a}E_{b]c} - 2\delta_{c[a}E_{b]d}, \\ [E_{ab}, F_{cd}] &= 2\delta_{d[a}F_{b]c} - 2\delta_{c[a}F_{b]d}, \\ [F_{ab}, F_{cd}] &= -2\delta_{d[a}E_{b]c} + 2\delta_{c[a}E_{b]d}. \end{aligned} \quad (75)$$

Note the relative minus sign between the first and the third line. It is easy to see that this Lie algebra is isomorphic to the complex Lie algebra $\mathfrak{so}(8, \mathbb{C})$, if we decree the generators E_{ab} to be ‘real’ and the generators F_{ab} to be ‘imaginary’ (so the latter can be thought of as ‘ iE_{ab} ’). A second $\mathfrak{so}(8, \mathbb{C})$ is obtained by replacing the IIB generators Z_{ab} and Z^{ab} by the corresponding IIA generators U_{ab} and U^{ab} .

Under the action of $\mathfrak{so}(8, \mathbb{C})$ in the IIB basis the adjoint **248** of $E_{8(8)}$ decomposes into three irreducible subspaces: the first of these is the $\mathfrak{so}(8, \mathbb{C})$ subalgebra itself, the second is the 64-dimensional subspace spanned by the generators $Z_{ab} - Z^{ab}$, S_{ab} (cf. (57)) and the grading operator N , and the third is the 128-dimensional subspace spanned by the level ± 3 and ± 1 generators in (47) (i.e. the generators $Z^a, Z_a, E^{abc}, E_{abc}$ in the notation of [4, 23]). For the IIA basis we find that the **248** decomposes instead into the subalgebra and *three* 64-dimensional irreducible subspaces. To understand these rather counterintuitive results, we recall that there exist simple real forms of Lie algebras whose complexification is no longer simple.⁴ In the

⁴A familiar example is the Lorentz group, where

$$\mathbb{C} \otimes \mathfrak{so}(1, 3) = \mathfrak{so}(4, \mathbb{C}) = \mathfrak{so}(3, \mathbb{C}) \oplus \mathfrak{so}(3, \mathbb{C}).$$

case at hand, $\mathfrak{so}(8, \mathbb{C})$ is embedded as a simple Lie algebra into $\mathfrak{e}_{8(8)}$, but its complexification in $\mathfrak{e}_8(\mathbb{C})$ is no longer simple:

$$\mathbb{C} \otimes \mathfrak{so}(8, \mathbb{C}) = \mathfrak{so}(8, \mathbb{C})' \oplus \mathfrak{so}(8, \mathbb{C})' , \quad (76)$$

where the prime on the r.h.s. is to indicate two copies of the standard complexified $\mathfrak{so}(8)$. The 64-dimensional irreducible subspace just identified may then be viewed as a real section of the complex $(\mathbf{8}, \mathbf{8})$ representation of $SO(8, \mathbb{C}) \times SO(8, \mathbb{C})$.

In terms of the $SO(16)$ decomposition (19) the $SO(8, \mathbb{C})$ embedding tensor is purely ‘off-diagonal’, viz.

$$\Theta_{IJ|A} = -\frac{1}{7}\Gamma_{AA}^I \Xi^{J\dot{A}} , \quad \Theta_{IJ|KL} = \Theta_{A|B} = 0 , \quad (77)$$

where, in the $SO(8)$ decomposition (50), $\Xi^{I\dot{A}}$ has the non-vanishing components

$$\Xi^{I\dot{A}} = \begin{cases} \gamma_{\alpha\dot{\alpha}}^a & \text{for } I = \alpha \text{ and } \dot{A} = (a\dot{\alpha}) \\ -\gamma_{\alpha\dot{\alpha}}^a & \text{for } I = \dot{\alpha} \text{ and } \dot{A} = (a\alpha) \end{cases} . \quad (78)$$

The relative sign is fixed by requiring $\Gamma_{AA}^I \Xi^{I\dot{A}} = 0$. Consequently the vacuum expectation values of both A_1^{IJ} and $A_3^{\dot{A}\dot{B}}$ vanish at the origin, see (28); from (29) we immediately obtain

$$\langle W \rangle = +\frac{1}{16}g^2 A_2^{I\dot{A}} A_2^{I\dot{A}} = \frac{1}{16}g^2 \Xi^{I\dot{A}} \Xi^{I\dot{A}} = 8g^2 > 0 , \quad (79)$$

which is a de Sitter vacuum with completely broken supersymmetry, where the $SO(8, \mathbb{C})$ symmetry is broken to its compact subgroup $SO(8) \equiv SO(8, \mathbb{R})$. The fermionic mass term is purely off-diagonal

$$\mathcal{L}_m^{(f)} = \frac{1}{7}ig\Xi^{I\dot{A}} \overline{\chi}^{\dot{A}} \gamma^\mu \psi_\mu^I . \quad (80)$$

An analysis of the scalar mass matrix [15]

$$\mathcal{M}_{AB}^2 = -\frac{3}{16}g^2 \left(\Gamma_{AA}^I \Xi^{J\dot{A}} \Xi^{J\dot{B}} \Gamma_{\dot{B}\dot{B}}^I - \Gamma_{AA}^I \Xi^{J\dot{A}} \Xi^{I\dot{B}} \Gamma_{\dot{B}\dot{B}}^J \right) , \quad (81)$$

yields the (mass)² eigenvalues

$$\underline{\underline{\begin{array}{c|c|c|c} m_S^2 & 16g^2 & 0 & -48g^2 \\ \hline SO(8) & \mathbf{35}_s + \mathbf{35}_c & \mathbf{28} + \mathbf{28} & \mathbf{1} + \mathbf{1} \end{array}}} , \quad (82)$$

and

$$\frac{m_S^2}{SO(8)} \parallel \begin{array}{c|c|c|c|c|c} 16g^2 & 12g^2 & 0 & -20g^2 & -48g^2 & \\ \hline \mathbf{35}_v & \mathbf{56}_v & \mathbf{28} & \mathbf{8}_v & \mathbf{1} & \end{array}, \quad (83)$$

for the IIA and the IIB embedding, respectively, in accordance with the spectrum of representations (39), (50). The fact that these spectra come out to be different confirms the inequivalence of the IIA and IIB embeddings of $SO(8, \mathbb{C})$ into $E_{8(8)}$. Because of the tachyonic directions present for both embeddings, the de Sitter vacua are unstable. Moreover, a preliminary analysis indicates that neither potential has any non-trivial stationary points: in fact, numerical checks suggest that the potential is a monotonic function along any geodesic starting from the origin $\mathcal{V} = \mathbf{1}$ in the scalar field space.

Starting from the $SO(p, q) \times SO(p, q)$ generators in either the IIA or the IIB basis, one finds the following alternative bases for $\mathfrak{so}(8, \mathbb{C})$ in $E_{8(8)}$:

$$\begin{array}{ll} E_{ij}, E_{rs}, S_{ir} & \text{(real generators) ,} \\ Z_{ij} + Z^{ij}, Z_{rs} + Z^{rs}, Z_{ir} - Z^{ir} & \text{(imaginary generators) ,} \end{array} \quad (84)$$

with the embedding tensor

$$\Theta = E_{ij} \vee (Z_{ij} + Z^{ij}) - E_{rs} \vee (Z_{rs} + Z^{rs}) + S_{ir} \vee (Z_{ir} - Z^{ir}) . \quad (85)$$

It might thus appear that there are more inequivalent ‘ $SO(p, q, \mathbb{C})$ gaugings’, but this is not the case – in agreement with the well known fact that there is only one complex group over $SO(8)$ (which might be embedded in inequivalent ways, though, as we have seen). However, an analysis of the scalar mass spectrum at the origin reveals that the putative $SO(p, q, \mathbb{C})$ theories have identical spectra: in the IIB basis adopted in (85), the scalar mass spectrum coincides with (83) for even p, q , and with (82) for odd p, q , and vice versa for the type IIA gauging of $SO(8, \mathbb{C})$. There should thus exist an explicit $E_{8(8)}$ transformation relating the different but equivalent bases.

Are there similar complex gaugings for other values and dimensions? Let us first note that complex embeddings analogous to the one discussed above (i.e. without an imaginary unit i) exist for all split real forms. More specifically, for $\mathbb{C}^\times = \mathbb{C}, \mathbb{C}'$ or \mathbb{C}^0 , and in analogy with the embedding $SO(8, \mathbb{C}^\times) \subset E_{8(8)}$, we have

$$\begin{array}{ll} SO(6, \mathbb{C}^\times) & \subset E_{7(7)} , \\ SO(5, \mathbb{C}^\times) & \subset E_{6(6)} , \\ SO(4, \mathbb{C}^\times) & \subset E_{5(5)} \equiv SO(5, 5) , \end{array} \quad (86)$$

as one can show by truncating the decomposition (47) to the relevant $SO(n)$ subgroups. These are the noncompact real forms appearing in $D \geq 4$ supergravities [3], but a quick counting argument shows that none of the groups on the l.h.s. are viable gauge groups for these theories (for instance, $SO(6, \mathbb{C}^\times)$ would require 30 vector fields in four dimensions). By contrast, in three dimensions we expect complex gaugings to exist for lower $N = 2n$, because the existence of non-semisimple gaugings with groups $SO(n) \ltimes T$ can be inferred from the existence of corresponding gauged theories in four dimensions. For instance, $SO(6) \ltimes T_{15}$ may be embedded in the isometry group of the coset space $E_{7(-5)}/(SO(12) \times SU(2))$ of the three-dimensional $N = 12$ theory; however, this embedding will necessarily break the compact $SU(2)$ factor, in accordance with the fact that the gauged theory has no global $SU(2)$ symmetry. Flipping signs as in (73) will then produce the desired gauged theories with $SO(n, \mathbb{C})$.

5 Potentials and supersymmetry breaking

It has been known for some time that dimensionally reduced gauged supergravities in general do not admit maximally supersymmetric ground states, even if the ancestor theories do possess such vacua. A prime example is the maximal $N = 8$ theory in four dimensions [8] which after torus reduction to three dimensions only admits a partially supersymmetric domain wall solution [24]. In this section, we would like to explain how this ‘loss of supersymmetric vacuum’ comes about by studying how the scalar potentials are affected when a semisimple gauge group is replaced by a non-semisimple one.

As explained in section 2.2, the scalar potential W (29) is expressed in terms of the T -tensor (27). Like Θ , the latter decomposes into a sum of graded pieces

$$T_{AB} = \sum_{j=-2\ell}^{j=2\ell} T_{AB}^{(j)} \ , \quad T_{AB}^{(j)} := \mathcal{V}_A^{\mathcal{M}} \mathcal{V}_B^{\mathcal{N}} \Theta_{\mathcal{MN}}^{(j)} \ . \quad (87)$$

The potential itself is a quadratic function of the T -tensor, and can therefore be decomposed into graded pieces with grades ranging between -4ℓ and $+4\ell$ (cf. (32))

$$W = \sum_{n=-4\ell}^{4\ell} W^{(n)} \ , \quad (88)$$

where each term $W^{(n)}$ receives contributions from the products $T^{(j)}T^{(k)}$ with $j + k = n$. Consequently, under a rescaling with the boost generator N , we obtain

$$W \longrightarrow \sum_{n=-4\ell}^{4\ell} e^{n\lambda} W^{(n)}. \quad (89)$$

For those non-semisimple gaugings which can be generated by the boost method, or by restricting the embedding tensor to a given grade, the corresponding potentials can be immediately deduced by replacing Θ by $\bar{\Theta}$

$$\bar{T}_{AB} = \mathcal{V}_A^{\mathcal{M}} \mathcal{V}_B^{\mathcal{N}} \bar{\Theta}_{\mathcal{MN}}. \quad (90)$$

The singular boost leading to the non-semisimple gauge group thus results in the removal of certain terms from the T -tensor, and therefore in the removal of certain terms from the potential itself: after rescaling and taking the limit, we are left with a truncated potential

$$\bar{W} = W^{(2\omega)}. \quad (91)$$

computed from (90) with the maximal grade ω from (35). It is this removal of lower grade contributions which may ‘destabilize’ a potential which originally did possess a stable groundstate. Roughly speaking, the removal of certain terms from the potential turns an initially ‘cosh-like’ potential into an exponential one, thus inducing a run-away behavior in special directions in the scalar field manifold.

In order to further analyze this decomposition of the potential and to elucidate the relation between the potentials obtained directly in three dimensions and those obtained by dimensional reduction from higher dimensional gauged supergravities, we define the ‘dilaton’ to be the scalar field ϕ associated with the grading generator N by extracting its dependence from the 248-bein

$$\mathcal{V}(\tilde{\phi}, \phi) = \tilde{\mathcal{V}}(\tilde{\phi}) \cdot \exp(\phi N). \quad (92)$$

This decomposition requires that we choose a basis $\{t^{\mathcal{M}}\}$ of \mathfrak{g} which is compatible with (i.e. diagonal w.r.t.) the grading (31) with grades $d_{\mathcal{M}} \equiv \mathcal{D}(t^{\mathcal{M}})$, such that

$$t^{\mathcal{M}} \in \mathfrak{g}^{(d_{\mathcal{M}})}. \quad (93)$$

The coset space G/H may then be parametrized in a triangular gauge by exponentiating the nilpotent positive-grade generators $\{t^{\mathcal{M}} \mid d_{\mathcal{M}} > 0\}$ together

with the non-compact generators at grade $d_{\mathcal{M}} = 0$. Corresponding to the grade of their generators, we may then assign a charge to the scalar fields. In the representation (92), the matrix $\tilde{\mathcal{V}}$ is an exponential containing only non-negative grade generators other than N with their associated fields $\tilde{\phi}$ (for which we will not need an explicit parametrization). We shall verify below that for those theories descending from higher dimensions, the field ϕ can indeed be identified with the usual dilaton which is defined as the ratio of metric determinants

$$\sqrt{g_D} = \sqrt{g_3} e^{\phi}, \quad (94)$$

where g_D and g_3 are the metric determinants in D and three dimensions, respectively. The parametrization (92) correspondingly yields a ‘free’ kinetic term $\propto \partial_{\mu}\phi\partial^{\mu}\phi$ for the dilaton ϕ , whereas the kinetic terms for the other fields $\tilde{\phi}$ come with a field dependent metric and certain powers of e^{ϕ} depending on the respective charges of the $\tilde{\phi}$.

Defining the dilaton independent part of the T -tensor

$$\tilde{T}_{AB}(\tilde{\phi}) = \tilde{\mathcal{V}}_{\mathcal{A}}^{\mathcal{M}} \tilde{\mathcal{V}}_{\mathcal{B}}^{\mathcal{N}} \Theta_{\mathcal{MN}}, \quad (95)$$

the dilaton dependence of the potential can be made completely explicit. To this aim, we first note that one must be careful in distinguishing for every expression between its ‘dilaton power’ (i.e. the integer n appearing in the factor $e^{n\phi}$ multiplying this expression) and its grade w.r.t. N : they are not the same, because, in (92) the grading operator N acts *from the left* on \mathcal{V} whereas the dilaton is factored out *on the right* of \mathcal{V} . Hence, the matrix $\mathcal{V}_{\mathcal{A}}^{\mathcal{M}}$ decomposes as

$$\mathcal{V}_{\mathcal{A}}^{\mathcal{M}} = \text{tr}[\mathcal{V}^{-1} t^{\mathcal{M}} \mathcal{V} t_{\mathcal{A}}] = e^{-d_{\mathcal{A}}\phi} \tilde{\mathcal{V}}_{\mathcal{A}}^{\mathcal{M}}(\tilde{\phi}). \quad (96)$$

Here $\tilde{\mathcal{V}}_{\mathcal{A}}^{\mathcal{M}}$ no longer depends on ϕ , its grade w.r.t. N is $d_{\mathcal{M}}$, and it has charge $(d_{\mathcal{A}} - d_{\mathcal{M}})$; in particular, $\tilde{\mathcal{V}}_{\mathcal{A}}^{\mathcal{M}} = 0$ for $d_{\mathcal{A}} < d_{\mathcal{M}}$ by triangularity. Similarly, the expansion (87) of the T -tensor takes the form

$$T_{AB}(\tilde{\phi}, \phi) = e^{-(d_{\mathcal{A}}+d_{\mathcal{B}})\phi} \sum_{j=-2\ell}^{2\ell} \tilde{T}_{AB}^{(j)}(\tilde{\phi}), \quad (97)$$

where $\tilde{T}_{AB}^{(j)}$ has charge $(d_{\mathcal{A}} + d_{\mathcal{B}} - j)$. At this point we can factor out the dilaton dependence by writing the potential (88) in the form

$$W(\tilde{\phi}, \phi) = \sum_{n=-4\ell}^{4\ell} \sum_{k=0}^{4\ell-n} e^{-(n+k)\phi} \tilde{W}^{(n,k)}(\tilde{\phi}), \quad (98)$$

where $\tilde{W}^{(n,k)}$ depends only on Θ -bilinears $\Theta^{(j_1)}\Theta^{(j_2)}$ with $j_1 + j_2 = n$, and has charge k . Moreover, since the potential (29) is obtained from contracting bilinears in the T -tensor $T_{AB}T_{CD}$ with a metric invariant under the compact subgroup of \mathfrak{g} , the components $\tilde{W}^{(n,k)}(\tilde{\phi})$ in (98) vanish for $n+k$ odd. After boosting, the potential becomes

$$\bar{W} = e^{-2\omega\phi} \sum_{k=0}^{2\ell-\omega} e^{-2k\phi} \tilde{W}^{(2\omega,2k)}. \quad (99)$$

From the form of this potential it is immediately evident that the boosted potential corresponding to a non-semisimple gauge group in general will not admit a fully supersymmetric groundstate at $\mathcal{V} = \mathbf{1}$, even if the original theory did have one, because of the unbalanced exponential terms.

As an illustration let us consider the theory discussed in section 3.1 which is obtained from the maximal four-dimensional gauged supergravity upon reduction on a circle S^1 . As discussed above, and in accordance with the grading (37), the scalar content of the three-dimensional theory comprises the 70 four-dimensional scalar fields (of charge 0), the $28 + 28$ contributions from the four-dimensional vector fields (of charge 1), and the two scalars coming from dilaton and graviphoton (of charge 0 and 2, respectively). With the dilaton defined in (94), the dimensional reduction is performed together with a Weyl rescaling of the three-dimensional metric $g_{\mu\nu} \rightarrow e^{-2\phi} g_{\mu\nu}$ in order to obtain a canonical Einstein-Hilbert term. It is straightforward to verify that the dilaton powers of the kinetic terms in three dimensions precisely correspond to the grading (37). Moreover, it is easy to see that the four-dimensional potential and the kinetic term of the vector fields in four dimensions give rise to the following scalar terms in three dimensions

$$\begin{aligned} \sqrt{g_4} W_4 &\rightarrow \sqrt{g_3} e^{-2\phi} \tilde{W}^{(2,0)}, \\ \sqrt{g_4} g^{A4} A_4^{ab} A_4^{cd} M_{ab,cd} &\rightarrow \sqrt{g_3} e^{-4\phi} \tilde{W}^{(2,2)}, \end{aligned} \quad (100)$$

(e.g. in the first line we have a factor e^ϕ from (94) and a factor $e^{-3\phi}$ from the Weyl rescaling, etc.). The above expressions thus precisely reproduce the first terms of the expansion (99). In this case the series (99) does not extend to all $2\ell - \omega + 1$ terms due to the fact that the highest level is a singlet under the gauge group. Although stationary points of the four-dimensional potential, i.e. of $\tilde{W}^{2,0}$ do not give rise to stationary points of the boosted potential (99), there are indications [11, 12] that they may all be lifted to stationary points of the full three-dimensional potential (98) of the compact gauged theory. The precise mechanism of the lift remains to be explored;

the series (99) provides a natural starting point, describing the embedding of the higher-dimensional potential into the three-dimensional one.

Let us finally mention that the expansion (99) may be extended to an expansion w.r.t. several scalar fields associated with an abelian subalgebra of \mathfrak{g} using the techniques developed in [4]. For particular choices, this corresponds to the theories coming from reduction of higher dimensional gauged supergravities, with the different terms in (99) corresponding to the terms of different higher dimensional origin.

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