UPR-1047-T
AEI-2003-062
hep-th/0308045

arXiv:hep-th/0308045 v2 6 Nov 2003

Supersymmetric Intersecting D6-branes and Fluxes in Massive Type IIA String Theory

Klaus Behrndt[a1] and Mirjam Cvetič[b2]

[a] *Max-Planck-Institut für Gravitationsphysik*
Am Mühlenberg 1, 14476 Golm, Germany

[b] *Institute for Advanced Study, Princeton, NJ 08540, USA*

ABSTRACT

We study N=1 supersymmetric four-dimensional solutions of massive Type IIA supergravity with intersecting D6-branes in the presence NS-NS three-form fluxes. We derive N=1 supersymmetry conditions for the D6-brane and flux configurations in an internal manifold X_6 and derive the intrinsic torsion (or SU(3)-structure) related to the fluxes. In the absence of fluxes, N=1 supersymmetry implies that D6-branes wrap supersymmetric three-cycles of X_6 that intersect at angles of SU(3) rotations and the geometry is deformed by SU(3)-structures. The presence of fluxes breaks the SU(3) structures to SU(2) and the D6-branes intersect at angles of SU(2) rotations; non-zero mass parameter corresponds to D8-branes which are orthogonal to the common cycle of all D6-branes. The anomaly inflow indicates that the gauge theory on intersecting (massive) D6-branes is not chiral.

[1]E-mail: `behrndt@aei.mpg.de`
[2]E-mail: `cvetic@cvetic.hep.upenn.edu`. On sabbatic leave from the University of Pennsylvania.

1 Introduction

Constructions of four-dimensional N=1 supersymmetric solutions of Type IIA string theory with intersecting D6-branes, wrapping three-cycles of orbifolds (with an orientifold projection) provide an explicit realization of supersymmetric ground sates of string theory with massless chiral super-multiplets [1, 2]. Explicit supersymmetric solutions, based on $Z_2 \times Z_2$ orbifolds, yielded the first examples with the Standard-like model [3, 4, 5] as well as Grand unified (GUT) $SU(5)$ Georgi-Glashow model [4, 6]. [Non-supersymmetric intersecting D6-brane constructions were initiated in [7, 8, 9, 10] and supersymmetric chiral constructions implicitly in [11].] Such constructions, when lifted on a circle to M-theory correspond to compactifications of M-theory on compact, singular G_2-holonomy metrics [4, 12, 13, 14].

N=1 supersymmetry in D=4 imposes conditions on angles of the three-cycles, wrapped by branes, relative to the orientifold plane. In particular, for orbifold (toroidal) compactifications, the six-torus T^6 can be written as a product of three two-tori T^2 and the three-cycles as a product of three one-cycles (one in each T^2). In this case the condition for supersymmetry [1, 4] becomes a condition that the sum of three angles, relative to the orientifold plane, which have to sum to zero, i.e. the rotation of the three-cycles relative to the orientifold three-plane is an SU(3) rotation. This condition is typically very constraining, since it has to be satisfied for all three-cycles wrapped by various D6-branes; if solved, it typically imposes conditions on the complex structure of toroidal moduli [4].

The consistency conditions [8, 4] are equivalent to the Gauss law for the D6-brane (positive charge) and O6-plane (negative charge) sources; they ensure the charge cancellation condition for D6-brane and O6-plane charges in the internal space and impose constrains on the number of D6-branes and the wrapping numbers of the supersymmetric three-cycles wrapped by D6-branes.

One can in principle generalize constructions on orbifolds (with orientifold projection) to general Calabi-Yau manifolds X_6 (with holomorphic Z_2 involution), see e.g. [15] and refs. therein. The supersymmetry conditions on the three-cycles become special Lagrangian conditions and the consistency conditions reduce to an analog of the Gauss law for D6-brane and O6-plane sources in the general Calabi-Yau background.

The supersymmetry conditions can be equivalently rephrased using SU(3)-structures generated by the non-trivial fluxes. Note however, that unlike the orbifold compactifications, where one has explicit conformal field theory techniques to calculate the spectrum and couplings of the resulting theory, for general Calabi-Yau compactifications techniques of algebraic and differential geometry may not suffice to solve explicitly consistency conditions and determine the full spectrum and correlation functions.

The purpose of this paper is to address modifications of such constructions due to the presence of fluxes associated with the Ramond-Ramond (R-R) and Neveu-Schwarz-Neveu-Schwarz (NS-NS) closed string sectors. Our primary focus would be on quantification of modified conditions that ensure N=1 supersymmetry in four-dimensions. The presence of fluxes typically modifies Gauss law for D6-brane and O6-brane sources. This modification, is due to the transgression (Chern-Simons) terms which act as a source on the right hand side (rhs) of the equation for the D6-brane field strengths (see, e.g., [16]). These transgression terms give a positive contribution to the "total" charge and thus fluxes typically modify the charge cancellation conditions by reducing the number D6-brane configurations.

A framework where we can study the effects of fluxes[3] for the intersecting D6-brane probes in a straightforward way turns out to be within masssive Type IIA supergravity [31]; it contains the Chern-Simons term that couples the D6-brane potential $C^{(7)}$ to the zero-form (mass) m and NS-NS 3-form field-strength $H^{(3)}$:

$$L_{CS} = \int_{M_4 \times X_6} m\, H^{(3)} \wedge C^{(7)} \,. \tag{1}$$

Note, that while massive Type IIA supergravity provides a straightforward framework that via the Chern-Simons term (1) couples D6-brane probes to supergravity NS-NS three-form fluxes, there are other possibilities. Within massless Type IIA supergravity with off-diagonal metric components on X_6 turned on, the kinetic energy term for the R-R sector fluxes may induce an effective transgression term that couples R-R sector and metric fluxes to $C^{(7)}$ [32]. Such examples could be related

[3]There is a growing literature on the subject of string compactifications with fluxes which was initiated in [17, 18], a partial list of subsequent works includes, e.g., [19, 20, 21, 22, 23], and for recent work, quantifying effects in terms of deformations of the original manifold (G-structures), see [24, 25, 26, 27, 28, 29, 30] and references therein.

by T-duality to Type IIB configurations with D3-brane probes and self-dual 3-form fluxes as studied in [33, 34, 35, 36] and for generalizations to magnetized D-branes, see [37, 32]. In this paper we would like to capture the explicit structure of the intersecting D6-branes in the presence of NS-NS fluxes within massive Type IIA superstring theory.

The Chern-Simons term (1) turns out to modify the equation of motion for $C^{(7)}$ as:

$$d\,F^{(2)} = m\,H^{(3)} \tag{2}$$

where the two-form $F^{(2)}$ is the magnetic field strength of $C^{(7)}$. Eq. (2) provides a modification of the original consistency conditions on the number of D6-branes, wrapping specific three-cycles, which are now modified by the rhs of (2). While for orbifold backgrounds the charge cancellation condition can be solved explicitly [4], for a general Calabi-Yau compactification (with Z_2 involution) these conditions are complicated and the explicit solutions may be hard to find.

Our main focus, however, will be on the modifications of the supersymmetry conditions and classification of the internal torsion of the resulting internal manifold which ensures N=1 supersymmetry in four-dimensions. We choose to turn on only the D6-brane sources ($C^{(7)}$), the mass parameter m and the NS-NS three-form $H^{(3)}$. Within this framework, our approach shall be general; we shall neither impose a priori conditions on the structure of the internal manifold nor shall we impose a priori conditions on the D6-brane and NS-NS three-form flux configurations. Such conditions will be derived as a consequence of supersymmetry conditions, i.e. from the Killing spinor equations of massive Type IIA supergravity. (This is analogous to the analysis in Type IIB string theory with D3-branes in the presence of three-form R-R and NS-NS fluxes [38].)

The upshot of the analysis yields strong constraints on the allowed D6-brane configurations: in the presence of mass parameter m and NS-NS fluxes D6-branes should intersect only at angles compatible with SU(2) (and not SU(3)) rotations, in order to preserve N=1 supersymmetry in four-dimensions, i.e. the G-strucuture

[4] Note however that the quantization conditions for fluxes on orbifolds can be subtle, c.f., [37, 32], one can over-saturate the charge cancellation leading to the introduction of anti-branes which explicitly break supersymmetry of the configuration.

of the internal manifold is that of SU(2). Without NS-NS fluxes (and m=0) the massless spectrum at the intersection of D6-brane probes (rotated by SU(2) angles) corresponds to those of N=2 hypermultiplets and is therefore *non-chiral*. However, in the presence of fluxes the supersymmetry is broken down to N=1 and we expect that the spectrum changes. By studying anomaly inflow we shall also address whether the gauge-theory on the world-volume of the D6-branes is chiral.

As a warm-up and to elucidate the derivation of the supersymmetry conditions (and the intrinsic torsion of underlying compact manifold) we shall also explicitly address supersymmetry conditions for intersecting D6-brane probes without fluxes, thus reducing the analysis to the framework of massless IIA superstring theory. In this case we obtain SU(3) torsion classes for the resulting internal space, which of course have a natural lift on a circle to M-theory on G_2 holonomy manifold. This derivation is closely related to the studies in [26].

The paper is organized in the following way: In Section II we define the supersymmetry transformations and the Ansatz for the metric and fluxes. In Section III we decompose the 10-d spinors into 4- and 6-d spinors and we use the standard technique to define a fundamental two-form and a three-form for the six-dimensional internal manifold X_6. We distinguish between two cases: in *case (i)* we assume that the 6-manifold has only a single (chiral) spinor and in *case (ii)* we consider two 6-d spinors. The existence of two spinors is equivalent to the existence of a nowhere vanishing vector field v on X_6 and this vector breaks the SU(3) invariance of the supersymmetry projectors in *case (i)* to SU(2) invariance in *case (ii)*. In Section IV we explicitly derive the supersymmetry conditions: *case (i)* is appropriate to the massless case, i.e. a D6-brane background without fluxes, whereas *case (ii)* corresponds to the massive case which, in the absence of 4-form fluxes, requires the existence of a vector field on X_6. In the limit $m = 0$, the NS-NS-three form flux is also turned off and both cases coincide. In Section V we discuss the back reaction on the geometry. The flux deformations correspond to specific SU(3) structures, i.e. the internal space is Calabi-Yau with torsion and we show that two of the five torsion components are zero. In fact, the vector field v implies that the SU(3) structures are broken to SU(2) structures. In Section VI we address the anomaly inflow in the presence of NS-NS 3-fluxes as well as D6-brane, NS5-brane and D8-brane sources. We conclude that there is no anomaly

inflow, thus indicating that the D6-brane world-volume gauge theory is not chiral. In Section VII conclude with a discussion of a number of open questions and possible generalizations of our approach. In particular, we comment on the structure of the four-dimensional superpotential and point out possible generalizations, by adding R-R 4-form fluxes in the massive Type IIA case.

2 Supersymmetry variations

In massive type IIA supergravity, one introduces a gauge invariant 2-form by

$$F^{(2)} = mB + dA^{(1)}$$

where the R-R 1-form $A^{(1)}$ can be gauged away to give a mass to the NS-B-field. In the field equation for the NS-NS 3-form[5] $H = dB$, the 2-form $m\,F^{(2)}$ appears as a source term and hence, whenever $m\,F^{(2)} \neq 0$, also the NS-NS 3-form has to be non-zero and has to be included into our consideration.

We will especially be interested in the modification of the supersymmetry constraints on intersecting D6-brane configurations due to a non-vanishing mass parameter. Hence, in the limit $m = 0$ we get back to the standard D6-brane configuration which couples to the one-form potential $A^{(1)}$ only. The massive B-field also enters the 4-form field strength and in order to ensure the absence of $F^{(4)}$:

$$F^{(4)} = dC^{(3)} + 6mB \wedge B = 0$$

we have to impose the constraint (besides $C^{(3)}$ =0)

$$m\,B \wedge B = 0\ . \tag{3}$$

For $m \neq 0$, one can gauge away the 1-form $A^{(1)}$ and this constraint is equivalent to

$$F \wedge F = 0$$

and means that we neglect effects due to 4-form fluxes and/or D4-branes, which might be interesting in it own (Such configurations may be related to the chiral brane-box

[5]In the rest of the paper we suppress the superscript (3) on $H^{(3)}$ and use simply the notation H and F for the NS-NS 3-form field strength and 2-form potential, respectively.

model of [39], supplemented with additional supergravity fluxes. We briefly comment on that in the Discussion section.)

Supersymmetry for purely bosonic background requires the vanishing of the gravitino and dilatino supersymmetry variation and is equivalent to the existence of a Killing spinor. In the canonical Einstein frame, these variations are given in [31], but we are going to use the string frame. Using the identity

$$\Gamma_M \Gamma^{N_1\cdots N_n} = \Gamma_M^{\ N_1\cdots N_n} + n\delta_M^{\ [N_1}\Gamma^{N_2\cdots N_n]} \ , \tag{4}$$

the variations can be written as

$$\begin{aligned} \delta\psi_M &= \Big\{ D_M - \tfrac{1}{4} H_M \Gamma_{11} - \tfrac{1}{8}\, e^{\phi}\Big[m\Gamma_M + (\Gamma_M F - 4F_M)\,\Gamma_{11}\Big]\Big\}\epsilon \ , \\ \delta\lambda &= \Big\{ -\tfrac{1}{2}\,\partial\phi - e^{\phi}\Big[\tfrac{5}{8}m - \tfrac{3}{8}F\,\Gamma_{11} - \tfrac{1}{12}\,H\,\Gamma_{11}\Big]\Big\}\epsilon \end{aligned} \tag{5}$$

where we used the abbreviations

$$\partial \equiv \Gamma^M \partial_M \ , \quad H = H_{PQR}\Gamma^{PQR} \ , \quad H_M = H_{MPQ}\Gamma^{PQ} \ , \quad \text{etc.} \tag{6}$$

Since we are interested in a compactification to a flat 4-d Minkowski space, i.e. up to warping $Y_{10} = R^{(1,3)} \times X_6$, we write the metric Ansatz as

$$ds^2 = e^{-2U(y)}\Big[-dt^2 + d\vec{x}^2 + h_{mn}(y)dy^m dy^n\Big] \ . \tag{7}$$

Consistent with this Ansatz is the assumption that the fluxes associated with the 2-form F (and 3-form H) have non-zero components only in the internal space X_6:

$$F = F_{mn}dy^m \wedge dy^n \quad , \qquad H = H_{mnp}dy^m \wedge dy^n \wedge dy^p \ . \tag{8}$$

The Γ-matrices are decomposed as usual

$$\begin{aligned} &\Gamma^\mu = \hat{\gamma}^\mu \otimes \mathbb{1} \quad , \qquad \Gamma^{m+3} = \hat{\gamma}^5 \otimes \gamma^m \quad , \qquad \Gamma^{11} = -\hat{\gamma}^5 \otimes \gamma^7 \ , \\ &\hat{\gamma}^5 = i\hat{\gamma}^0\hat{\gamma}^1\hat{\gamma}^2\hat{\gamma}^3 \quad , \qquad \gamma^7 = i\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5\gamma^6 \end{aligned} \tag{9}$$

and we use the Majorana representation so that Γ^{11}, $\hat{\gamma}^\mu$ are real and $\hat{\gamma}^5$, γ^7 and γ^m are imaginary and anti-symmetric.

With these expressions, it is now straightforward to decompose the supersymmetry variations into external an internal components. With our metric Ansatz the covariant

derivatives can be written as

$$D_\mu = -\tfrac{1}{2}\Gamma_\mu{}^m \partial_m U = -\tfrac{1}{2}\hat{\gamma}_\mu \hat{\gamma}^5 \otimes \partial U \ ,$$

$$D_m = \nabla_m - \tfrac{1}{2}\Gamma_m{}^n \partial_n U = \mathbb{1} \otimes [\nabla_m - \tfrac{1}{2}\gamma_m{}^n \partial_n U]$$

where $\partial \equiv \gamma^m \partial_m$ and ∇_m is the covariant derivative with respect to the metric h_{mn}. Thus, the external components of the gravitino variation become

$$\delta\psi_\mu = -\frac{1}{2}\hat{\gamma}_\mu \otimes \mathbb{1}\Big[e^U \hat{\gamma}^5 \otimes \partial U + \frac{1}{4}e^\phi \Big(m[\mathbb{1}\otimes\mathbb{1}] - e^{2U}\hat{\gamma}^5 \otimes F\gamma^7\Big)\Big]\epsilon = 0 \ ,$$

($F \equiv F_{mn}\gamma^{mn}$) and it is solved if

$$e^U (\hat{\gamma}^5 \otimes \partial U)\, \epsilon = -\frac{1}{4}e^\phi \Big(m[\mathbb{1}\otimes\mathbb{1}] - e^{2U}\hat{\gamma}^5 \otimes F\gamma^7\Big)\epsilon \ . \tag{10}$$

Using this expression, we can now bring the internal components of the gravitino variation into the form

$$\delta\psi_m = \Big[\mathbb{1}\otimes\Big(\nabla_m + \frac{1}{2}e^{\phi+\frac{U}{2}}F_m\gamma^7\Big) - \frac{1}{4}\,\hat{\gamma}^5 \otimes e^{\frac{3}{2}U}H_m\gamma^7\Big]\hat{\epsilon} \quad , \qquad \hat{\epsilon} \equiv e^{\frac{U}{2}}\epsilon \tag{11}$$

with $F_m \equiv F_{mn}\gamma^n$, $H_m \equiv H_{mpq}\gamma^{pq}$. In a similar way, we can also simplify the dilatino variation and find

$$\delta\lambda = -\frac{1}{2}\Big[\hat{\gamma}^5 \otimes e^U\,(\partial\phi + 3\partial U) + \mathbb{1}\otimes e^\phi\Big(m + \frac{1}{12}e^{3U}\,H\gamma^7\Big)\Big]\epsilon \ . \tag{12}$$

These three equations (10), (11) and (12) will finally fix the flux, dilaton and the warp factor e^{-2U} as well as the geometry of the internal space. But before we come to this, we have to discuss the decomposition of the Killing spinor and the supersymmetry projectors.

3 Killing spinors and supersymmetric projectors

Type IIA supergravity has two spinors of opposite chirality and hence, also the Killing spinor ϵ should decompose in two different Majorana-Weyl spinors $\epsilon_{L,R}$ as $\epsilon = \epsilon_L + \epsilon_R$. In some cases the Killing spinor equations can be solved with just one Majorana-Weyl spinor (ϵ_L or ϵ_R), which simplifies significantly the calculation. In general however, this is not the case (as we shall see for the massive case) and therefore in the decomposition of the 10-d spinor (ϵ) into 4-d spinors (θ's) and 6-d spinors (η's)

one has to sum over all independent spinors. We shall distinguish the following two cases:

$$\begin{aligned} &(i) \qquad \epsilon = \theta \otimes \eta + \theta^\star \otimes \eta^\star \ , \\ &(ii) \qquad \epsilon = \theta_1 \otimes \eta_1 + \theta_2 \otimes \eta_2 + \theta_1^\star \otimes \eta_1^\star + \theta_2^\star \otimes \eta_2^\star \end{aligned} \tag{13}$$

where $\eta_{\{1,2\}}$ are 6-d chiral spinors and the chirality properties of the 4-d spinors θ_i will be fixed later.

Comments on case (i).

This is the Ansatz suitable for massless Type IIA supergravity which can be lifted on a circle to 11-d supergravity where the internal space becomes 7-dimensional. In the simplest situation, there is only one 7-d Killing spinor η_0 which can always be written as a G_2 singlet. In the reduction back to 10-d dimensions, the internal space becomes 6-dimensional, and using γ^7 we can build two chiral spinors which can be combined into one complex spinor, representing a singlet under $SU(3) \subset G_2$ in the following way:

$$\eta = \frac{1}{\sqrt{2}}(\mathbb{1} - \gamma^7)\, e^{\alpha + i\beta}\, \eta_0 \tag{14}$$

with α and β as real functions. This is the complex 6-d spinor η appearing in the Ansatz for ϵ in case (i) and the function α and β have to be fixed by the Killing spinor equations. The G_2 singlet spinor η_0 has just one (real) component and is normalized to $\eta_0^T \eta_0 = 1$. The 6-d γ-matrices satisfy $(\gamma_m)^T = -\gamma_m = (\gamma_m)^\star$, which yields for the transposed spinor: $\eta^T = \frac{1}{\sqrt{2}} e^{\alpha + i\beta} \eta_0^T (\mathbb{1} + \gamma^7)$. Since the internal spinors commute, one obtains the identities

$$0 = \eta^T \eta = \eta^T \gamma_m \eta = \eta^T \gamma_m \eta^\star = \eta^T \gamma_{mn} \eta \quad , \qquad \eta^T \eta^\star = e^{2\alpha} \ . \tag{15}$$

The complex structure and holomorphic 3-form are introduced as usual

$$\eta\, \gamma_{mn} \eta^\star = i\, e^{2\alpha}\, J_{mn} \quad , \qquad \eta \gamma_{mnp} \eta = i\, e^{2(\alpha + i\beta)}\, \Omega_{mnp} \tag{16}$$

which are related to the G_2 invariant 3-form ($\varphi_{rst} = -i\, \eta_0 \gamma_{rst} \eta_0$) by

$$J_{mn} = \varphi_{mn7} \quad , \tag{17}$$

$$\Omega_{mnl} = \varphi_{mnl} + i\, J_m^{\ k} \varphi_{knl} = \chi^+_{mnl} + i\, \chi^-_{mnl} = i(\delta_m^{\ p} + i J_m^{\ p}) \chi^-_{pnl} \ . \tag{18}$$

The properties of the G_2-invariant 3-form φ yield the relation $\chi^- = J\chi^+$ which in turn implies that the 3-form Ω is holomorphic $[(\mathbb{1} - i\,J)_{mn}\Omega_{npq} = 0]$. Since the spinor η_0 is a G_2 singlet, it has to satisfying the constraint $(r, s, t, \ldots = 1, \ldots, 7)$

$$\mathbb{P}^{rs}_{+\ tu}\,\gamma_{rs}\,\eta_0 \equiv \frac{2}{3}\Big(\mathbb{1}^{rs}{}_{tu} + \frac{1}{4}\psi^{rs}{}_{tu}\Big)\gamma_{rs}\,\eta_0 = 0$$

which is the projector onto the **14** (adjoint of G_2) and ψ_{pqrs} is the G_2-invariant 4-index tensor. This projector is equivalent to the condition

$$(\gamma_{rs} - i\,\varphi_{rst}\gamma^t)\,\eta_0 = 0 \quad , \qquad r, s, t = 1, \ldots, 7\ . \tag{19}$$

We can derive the constraints satisfied by the spinors η by multiplying this equation with $(\mathbb{1} + \gamma^7)$. Using (17) we find

$$\begin{aligned} (\gamma_m - iJ_{mn}\gamma^n)\,\eta &= 0\ , \\ (\gamma_{mn} + i\,J_{mn})\,\eta &= \tfrac{i}{2}\,e^{2i\beta}\Omega_{mnp}\gamma^p\,\eta^\star\ , \\ (\gamma_{mnp} + 3iJ_{[mn}\gamma_{p]})\,\eta &= i\,e^{2i\beta}\Omega_{mnp}\eta^\star \end{aligned} \tag{20}$$

and employing the projector

$$P_\pm \equiv \frac{1}{2}(\mathbb{1} \pm iJ) \tag{21}$$

one can decompose the above constraints into holomorphic and anti-holomorphic components. Using $\{a, b, c\}$ for holomorphic and $\{\bar{a}, \bar{b}, \bar{c}\}$ for anti-holomorphic indices we find

$$\begin{aligned} \gamma_a\,\eta^\star = \gamma_{\bar{a}}\,\eta &= 0\ , \\ \gamma_{ab}\,\eta &= \tfrac{i}{2}\,e^{2i\beta}\,\Omega_{abc}\gamma^c\,\eta^\star\ , \\ (\gamma_{a\bar{b}} + i\,J_{a\bar{b}})\,\eta^\star &= 0\ . \end{aligned} \tag{22}$$

The complex conjugate of these equations gives analogous constraints for anti-holomorphic indices (note $\gamma^a = \delta^{a\bar{b}}\gamma_{\bar{b}}$). Moreover, one finds

$$\gamma_{abc}\,\eta = i\,e^{2i\beta}\,\Omega_{abc}\,\eta^\star\ . \tag{23}$$

If the spinor η is covariantly constant, the six-manifold has SU(3) holonomy. But non-trivial fluxes will introduce SU(3)-structures [(con-)torsion] and the space is in general neither complex, nor Kähler nor Ricci flat; we return to this point in Section V.

Comments on case (ii).

The existence of two (chiral) 6-d spinors $\eta_{\{1,2\}}$ implies the existence of a holomorphic vector ($\sim \eta_1\gamma_m\eta_2^\star$) and we can write the two spinors as

$$\eta_1 = \eta \quad , \qquad \eta_2 = v\,\eta \quad , \qquad v = v_m\gamma^m \quad (v_m v_m = 1) \tag{24}$$

and η is given as in eq. (14). Therefore, the two spinors have opposite chirality

$$\gamma^7\eta_1 = -\eta_1 \quad , \qquad \gamma^7\eta_2 = \eta_2 \ . \tag{25}$$

Note, the complex conjugate spinors $\eta_i^\star$ have opposite chirality of η_i. Since we have not yet specified the spinors θ_i, we do not make any restriction by the above choice of the chirality for $\eta_{\{1,2\}}$ (e.g. by exchanging $\eta_2 \leftrightarrow \eta_2^\star$ we would have two 6-d spinors of the same chirality).

The relations for the spinor η (15), imply now the following identities (up to the exchange $\eta_1 \leftrightarrow \eta_2$)

$$\begin{aligned} &\eta_1^T\eta_2 = \eta_1^T\eta_2^\star = \eta_1^T\gamma^m\eta_2 = \eta_1^T\gamma^m\eta_1^\star = 0 \ , \\ &\eta_1^T\eta_1^\star = \eta_2^T\eta_2^\star = \eta^T\eta^\star = e^{2\alpha} \ , \\ &\eta_1^T\gamma_m\eta_2^\star = e^{2\alpha}(\delta_{mn} + i\,J_{mn})\,v^n \ . \end{aligned} \tag{26}$$

The last equation implies: $(\delta^{mn} - i\,J^{mn})\,(\eta_1\gamma_n\eta_2^\star) = 0$ and therefore this vector is holomorphic. [Note, in the tangent space there is no distinction between upper and lower indices $(m, n, \ldots = 1, \ldots, 6)$, which is in contrast to the holomorphic notation where lowering and rising an index involves a complex conjugation.]

Using the γ-identity (4), we can derive analogous projector conditions as in case (i), cp. eq. (20). Defining

$$\hat{\Omega}_{mn} = \Omega_{mnp}v^p \quad , \qquad v_\pm^m = \frac{1}{2}(\delta^{mn} \pm iJ^{mn})v_n \tag{27}$$

we find

$$\begin{aligned} \gamma^m\,\eta_2 &= 2v_-^m\,\eta_1 - \tfrac{i}{2}e^{2i\beta}\hat{\Omega}^{mn}\gamma_n\,\eta_1^\star \ , \\ \gamma^{mn}\,\eta_2 &= -iJ^{mn}v^p\gamma_p\,\eta_1 + ie^{2i\beta}\hat{\Omega}^{mn}\,\eta_1^\star - 4v_-^{[m}\gamma^{n]}\,\eta_1 \ . \end{aligned} \tag{28}$$

If one takes into account the holomorphic structure of the quantities, i.e. that Ω is a (3,0)-form, $\hat{\Omega}$ a (2,0)-form, J a (1,1)-form and v_+ is a (1,0)-vector, it is straightforward to express these equations in holomorphic and anti-holomorphic indices

$$\begin{aligned} \gamma_a\,\eta_2 &= -\tfrac{i}{2}e^{2i\beta}\hat{\Omega}_{ab}\gamma^b\,\eta_1^\star \quad , \quad \gamma_{\bar{a}}\,\eta_2 = v_{\bar{a}}\,\eta_1 \ , \\ \gamma_{ab}\eta_2 &= ie^{2i\beta}\hat{\Omega}_{ab}\,\eta_1^\star \qquad , \quad \gamma_{a\bar{b}}\eta_2 = -iJ_{a\bar{b}}\,\eta_2 + 2v_{\bar{b}}\gamma_a\,\eta_1 \ . \end{aligned} \tag{29}$$

In the case that both Killing spinors are covariantly constant, also the vector v is covariantly constant and the holonomy of the internal space is further reduced to SU(2). In fact, in this case the space would factorize into $R^2 \times X_4$, where X_4 is a 4-d manifold with SU(2) holonomy and the covariantly constant holomorphic vector identifies the R^2 directions. Of course, this is only possible if the fluxes are trivial and as we will discuss in Section V the fluxes will deform the internal manifold by non-vanishing torsion components (SU(2) structures).

4 Conditions on Fluxes and intersecting D6-brane configurations

The relevant equations that fix the metric as well as the fluxes were given by [see eqs. (10 –(12)]

$$0 = e^U(\hat{\gamma}^5 \otimes \partial U)\,\epsilon + \frac{1}{4}e^\phi\left(m[\mathbb{1}\otimes\mathbb{1}] - e^{2U}\hat{\gamma}^5\otimes F\gamma^7\right)\epsilon \ , \tag{30}$$

$$0 = \left[\hat{\gamma}^5\otimes e^U\,(\partial\phi + 3\partial U) + \mathbb{1}\otimes e^\phi\left(m + \frac{1}{12}e^{3U}\,H\gamma^7\right)\right]\epsilon \ , \tag{31}$$

$$0 = \left[\mathbb{1}\otimes\left(\nabla_m + \frac{1}{2}e^{\phi+\frac{U}{2}}F_m\gamma^7\right) - \frac{1}{4}\,\hat{\gamma}^5\otimes e^{\frac{3}{2}U}H_m\gamma^7\right]\hat{\epsilon} \tag{32}$$

where $\hat{\epsilon} = e^{\frac{U}{2}}\epsilon$. In solving these equations, we shall again distinguish the two cases with a single and two chiral six-dimensional spinors. As we will see, the spinor Ansatz in case (i) can only be solved for trivial mass parameter and hence yields the massless case with only D6-branes turned on, whereas the mass deformation m requires two 6-d spinors as in case (ii).

4.1 Case (i): Massless case

Due to the relations (22) the different terms become

$$
\begin{aligned}
(\hat{\gamma}^5 \otimes \partial U)\,\epsilon &= \hat{\gamma}^5\theta \otimes \partial^a U \gamma_a \eta + cc \ , \\
m\,(\mathbb{1} \otimes \mathbb{1}\,)\epsilon &= m\,\theta \otimes \eta + cc \ , \\
(\hat{\gamma}^5 \otimes F\gamma^7)\,\epsilon &= -\hat{\gamma}^5\theta \otimes (\tfrac{i}{2} e^{2i\beta} F^{ab}\Omega_{abc}\gamma^c\eta^\star - iF_{a\bar{b}}J^{a\bar{b}}\eta) + cc \ .
\end{aligned}
$$

These expression have to be inserted in (30) and since η and $\gamma^a\eta$ are different spinors, we infer

$$
m = 0 \quad , \qquad F_{a\bar{b}}J^{a\bar{b}} = 0 \tag{33}
$$

and

$$
e^{-U}\partial_a U = \frac{i}{8}\, e^{\phi}\, \Omega_{abc}F^{bc} \ , \tag{34}
$$

if the 4-d spinor satisfies the relation

$$
e^{i\beta}\theta \equiv \hat{\theta} = \hat{\theta}^\star \ . \tag{35}
$$

As we will see below, in order to allow for massive deformations we need at least two internal spinors. But let us also mention, that a non-trivial 4-form flux might change the situation, because the 4-form contribution in the Killing spinor equations can naturally compensate the mass term (see also the example discussed already by Romans [31]). We plan to return to issue of non-vanishing 4-form flux in the future.

Next, consider the eq. (31) which for $m = H = 0$ is trivially solved by

$$
\phi = -3\,U \tag{36}
$$

(recall, in our setup H vanishes in massless case). Finally, we have to investigate eq. (32) which becomes for $H = 0$

$$
\begin{aligned}
0 &= \theta \otimes \hat{\nabla}_m\hat{\eta}^\star + \theta^\star \otimes \hat{\nabla}_m\hat{\eta}^\star \\
&= \theta \otimes [\nabla_m - \tfrac{1}{2}e^{-\frac{5}{2}U}F_{mn}\gamma^n]\hat{\eta} + \theta^\star \otimes [\nabla_m + \tfrac{1}{2}e^{-\frac{5}{2}U}F_{mn}\gamma^n]\hat{\eta}^\star
\end{aligned}
$$

with $\hat{\eta} = e^{\frac{U}{2}}\eta$. Note, η and $\gamma^n\eta$ are spinors of opposite chirality and using (35) and collecting the spinors of the same chirality, we find

$$
\nabla_m\hat{\eta} + \frac{1}{2}e^{2i\beta}e^{-\frac{5}{2}U}F_{mn}\gamma^n\hat{\eta}^\star = 0 \ . \tag{37}
$$

If we identify $\alpha = -\frac{U}{2}$, the spinor $\hat{\eta}$ is normalized by $\hat{\eta}^\star\hat{\eta} = 1$ and thus, multiplying this equations with $\hat{\eta}^\star$ and using the relations (15), one finds that $\partial_m\beta = 0$ and this phase can be dropped in this case. In the next Section, we will use this differential equation to determine the torsion components.

To summarize, for the spinor Ansatz (i) in (13) we found the following constraints on the fluxes $(m, n, \cdots = 1, \cdots, 6)$

$$\begin{aligned} & m = H \equiv dB = 0 \quad , \qquad J^{mn}F_{mn} = 0 \ , \\ & e^{2U}\partial_q U = -\tfrac{1}{8}\, h_{qp}\, \chi^{-\,pmn} F_{mn} \quad , \qquad \phi = -3U \ . \end{aligned} \tag{38}$$

In the special case, that the D6-brane lives in a flat non-compact 10-d space, these results reproduce the known D6-brane solution given in the string frame by

$$\begin{aligned} & ds^2 = \tfrac{1}{\sqrt{H}}\Big[-dt^2 + d\vec{x}^2\Big] + \tfrac{1}{\sqrt{H}}\Big[dy_1^2 + dy_2^2 + dy_3^2 + H\,(dy_4^2 + dy_5^2 + dy_6^2)\Big] \\ & e^{-4\phi} = H^3 \quad , \qquad F_{mn} = \epsilon_{mnp}\partial_p H \end{aligned} \tag{39}$$

where H is a harmonic function and $e^{2U} = H^{\frac{1}{2}} = e^{-\frac{2}{3}\phi}$ and moreover in this case $\chi^- = -dy^4 \wedge dy^5 \wedge dy^6$ [or $\chi^{-\,ijk} = -\frac{1}{\sqrt{g_3}}\epsilon^{ijk}$, where $g_3 = H^3$ is the determinant of the metric on the subspace spanned by the coordinates $\{y_4, y_5, y_6\}$].

4.2 Case (ii): Massive deformation

As next step we consider in (13) the spinor Ansatz (ii) and start again with the external gravitino variation as given in eq. (30). Using the relations (22) [recall $\eta_1 = \eta$] and (29), the different terms become

$$\begin{aligned} (\hat{\gamma}^5 \otimes \partial U)\,\epsilon &= \hat{\gamma}^5\theta_1 \otimes \partial^a U \gamma_a \eta_1 + \hat{\gamma}^5\theta_2 \otimes (\partial^{\bar{a}} U v_{\bar{a}}\eta_1 - \tfrac{i}{2}e^{2i\beta}\partial^a U \hat{\Omega}_{ab}\gamma^b\eta_1^\star) + cc \ , \\ m\,(\mathbb{1}\otimes\mathbb{1})\,\epsilon &= m\,(\theta_1 \otimes \eta_1 + \theta_2 \otimes v^a\gamma_a\eta_1 + cc) \ , \\ (\hat{\gamma}^5 \otimes F\gamma^7)\,\epsilon &= \hat{\gamma}^5\theta_1 \otimes (iF_{a\bar{b}}J^{a\bar{b}}\eta_1 - \tfrac{i}{2}e^{2i\beta}F^{ab}\Omega_{abc}\gamma^c\eta_1^\star) \\ & \quad -\hat{\gamma}^5\theta_2 \otimes (iF_{a\bar{b}}J^{a\bar{b}}v^c\gamma_c\eta_1 - ie^{2i\beta}F^{ab}\hat{\Omega}_{ab}\eta_1^\star + 2F^{a\bar{b}}v_{\bar{b}}\gamma_a\eta_1) + cc \ . \end{aligned} \tag{40}$$

Now, these expressions have to cancel when inserted into (30) and one obtains two complex equations; one proportional to the spinor η_1 and the other proportional to

$\gamma_a\eta_1$:

$$
\begin{aligned}
0 &= e^{-U} v_{\bar{a}} \partial^{\bar{a}} U \, \hat{\gamma}^5 \theta_2 + \frac{1}{4} e^{\phi} \Big(m \, e^{-2U} \theta_1 - i F_{a\bar{b}} J^{a\bar{b}} \, \hat{\gamma}^5 \theta_1 - i e^{-2i\beta} F_{ab} \hat{\Omega}^{ab} \, \hat{\gamma}^5 \theta_2^{\star} \Big) \ , \qquad (41) \\
0 &= e^{-U} \partial^a U \, \hat{\gamma}^5 \theta_1 + \frac{i}{2} e^{-U} e^{-2i\beta} \partial_b U \hat{\Omega}^{ba} \, \hat{\gamma}^5 \theta_2^{\star} \qquad (42) \\
&\quad + \frac{1}{4} e^{\phi} \Big(m v^a \, e^{-2U} \theta_2 - \frac{i}{2} e^{-2i\beta} F_{bc} \Omega^{bca} \, \hat{\gamma}^5 \theta_1^{\star} + i F_{b\bar{c}} J^{b\bar{c}} v^a \, \hat{\gamma}^5 \theta_2 + 2 \, F^{a\bar{b}} v_{\bar{b}} \, \hat{\gamma}^5 \theta_2 \Big) \ .
\end{aligned}
$$

In order to solve these equations, we relate the 4-d spinors θ_1 and θ_2 by

$$
\hat{\gamma}^5 \theta_2 = \theta_1 \qquad (43)
$$

and as in case (i) we take again $e^{i\beta}\theta_1 = \hat{\theta}$ as a real spinor. As consequence we get one equation proportional to $\hat{\theta}$ and another proportional to $\hat{\gamma}^5\hat{\theta}$. Since $\hat{\theta}$ is a Majorana spinor, these terms have to cancel separately which gives the eqs.

$$
\begin{aligned}
e^{-U} \partial_a U &= \tfrac{1}{4} e^{\phi} \Big(\tfrac{i}{2} \Omega_{abc} F^{bc} - m \, v_a \, e^{-2U} \Big) \ , \\
i \, e^{-U} \hat{\Omega}_{ab} \partial^b U &= - e^{\phi} F_{a\bar{b}} v^{\bar{b}} \ , \qquad (44) \\
F_{a\bar{b}} J^{a\bar{b}} &= 0
\end{aligned}
$$

where the (2,0)-form: $\hat{\Omega}_{ab} \equiv \Omega_{abc} v^c$ was introduced in (27). Inserting the first into the second equation, we find: $F_{ab} v^b + F_{a\bar{b}} v^{\bar{b}} = 0$ and therefore the 2-form F cannot have components along the vector v.

Next, using the same relation as in the massless case: $\phi = -3U$ the terms in the second equation (31) can be written as

$$
\begin{aligned}
& m \, (\mathbb{1} \otimes \mathbb{1}) \, \epsilon = m \, (\theta_1 \otimes \eta_1 + \theta_2 \otimes v^a \gamma_a \eta_1 + cc) \ , \\
& (\mathbb{1} \otimes H \gamma^7) \, \epsilon = - \theta_1 \otimes H \eta_1 + \theta_2 \otimes H \eta_2 + cc \ .
\end{aligned} \qquad (45)
$$

Because there is no $\hat{\gamma}^5$, each term proportional to θ_1 and θ_2 has to vanish separately. Using (4) we find

$$
H \eta_1 = -3 i H^{a\bar{b}c} J_{a\bar{b}} \gamma_c \, \eta_1 + i e^{2i\beta} H^{abc} \Omega_{abc} \, \eta_1^{\star}
$$

and the term $\mathcal{O}(\theta_1)$ gives

$$
\Omega_{abc} H^{abc} = 12 \, i \, m \quad , \qquad J_{a\bar{b}} H^{a\bar{b}c} = 0 \ .
$$

This relation simplifies also the calculation of $H\eta_2$, which we write as $H_{npq}\gamma^n\gamma^{pq}\eta_2$ and use (28). As a result we find

$$H\eta_2 = ie^{2i\beta}\Big(\frac{1}{2}H^{abc}\Omega_{bcd} + H_{dbc}\Omega^{bca}\Big)v_a\gamma^d\,\eta_1^\star$$

and get finally the constraint on the 3-form flux

$$H^{abc}\Omega_{bcd} = 4im\,\delta^a{}_d\ .$$

As the last equation we have to discuss eq. (32) for the spinor. Collecting again terms of the same chirality gives now the following equations for $\hat{\eta}_{\{1,2\}} \equiv e^{\frac{U}{2}}\eta_{\{1,2\}}$

$$\nabla_m\hat{\eta}_1 = \frac{1}{4}e^{\frac{3}{2}U}e^{2i\beta}H_m\hat{\eta}_2^\star - \frac{1}{2}e^{-\frac{5}{2}U}e^{2i\beta}F_m\hat{\eta}_1^\star\ , \tag{46}$$

$$\nabla_m\hat{\eta}_2 = -\frac{1}{4}e^{\frac{3}{2}U}e^{2i\beta}H_m\hat{\eta}_1^\star + \frac{1}{2}e^{-\frac{5}{2}U}e^{2i\beta}F_m\eta_2^\star \tag{47}$$

and since $\eta_2 = v_m\gamma^m\eta_1$, the second equation fixes the vector v_m. The first equation on the other hand determines again the torsion components (see next Section).

To summarize, by solving the Killing spinor equations of massive Type IIA supergravity (with trivial 4-form flux), we derived the following conditions for the bosonic background:

$$8\,e^{2U}\partial_a U = i\,\Omega_{abc}F^{bc} - 2m\,v_a\,e^{-2U} \tag{48}$$

and

$$F_{ab}v^b + F_{a\bar{b}}v^{\bar{b}} = 0 \quad , \quad F_{a\bar{b}}J^{a\bar{b}} = 0 \quad , \quad H_{a\bar{b}c}J^{a\bar{b}} = 0 \quad , \quad H^{abc}\Omega_{bcd} = 4im\,\delta^a{}_d\ . \tag{49}$$

Recall, the absence of 4-form fluxes implied the constraint: $m(B\wedge B) = 0$ and bcause $F = mB + dA^{(1)}$ the last condition means: $^\star(dF\wedge\Omega) \sim m^2$.

An obvious solution is to keep only the holomorphic components of the 2-form F, i.e. to set $F_{a\bar{b}} = 0$. For the special case that the D6-branes is embedded into flat space, we find $H \sim m\chi^-_{mnp}dy^m\wedge dy^n\wedge dy^p = m\,dy^4\wedge dy^5\wedge dy^6$ and our results agree with the solution found in [40] yielding the metric as in (39) where the harmonic function has to be replaced by

$$H \to e^{4U} = my^1 - \sum_p M_p y^p y^p + H(\vec{y}) \quad , \quad \sum M_p = \frac{m^2}{2} \tag{50}$$

where the vector field is given by $v_m dy^m = dy^1$. Now, if $\partial^2 H(\vec{y}) = -n_6 \delta^3(\vec{y})$ this solution describes n_6 (massive) D6-branes and replacing $y^1 \to -|y^1|$ corresponds to D8-branes at $y^1 = 0$ (O8-branes correspond $y^1 \to |y^1|$, see[41]).

The locations of the different branes can also be identified by investigating the supersymmetry projectors

$$\begin{aligned} \left(\mathbb{1} + \tfrac{1}{12m} H_{MNP}\Gamma^{MNP}\right)\epsilon &= 0 \ , \\ m\left(\mathbb{1} + v_M \Gamma^M\right)\epsilon &= 0 \ . \end{aligned}$$

By inserting the Γ-matrices as given in eq. (9), the first equation becomes equivalent to (31) [with $\phi = -3U$] and the second equation is identically fulfilled by our spinor Ansatz for case (ii) [with (24) and (43)]. In the massless case, the second projector is empty whereas the first projector gives the location of the 6-branes, i.e. the 3-form defines the 3-d transversal space of the 6-brane. In the massive case, the second projector identifies the location of the D8-branes.

5 Back reaction on the geometry and G-structures

It is obvious that, due to the fluxes, the 7-d spinors are not covariantly constant and hence also the complex structure J as well as the holomorphic 3-form Ω cannot be covariantly constant. The deviation is related to non-trivial torsion components (or G-structures) and in the following we shall summarize some aspect relevant for our setup. For details see e.g. [42, 43, 25, 27, 28, 44, 45, 29]. To include torsion, one replaces the covariant derivative of a spinor η by

$$\nabla_m \eta \to (\nabla_m - \frac{1}{4}\tau_m{}^{pq}\gamma_{pq})\eta$$

where the 3-index object τ is the intrinsic torsion[6]. Since the spinor η is an SU(3) singlet, $\gamma_{pq}\eta$ does not contain the adjoint of SU(3) and thus the intrinsic torsion is an element of $\Lambda_1 \otimes su(3)^\perp$, where Λ_1 denotes the space of 1-forms and $su(3)^\perp$ denotes the compliment to the SU(3) Lie algebra, i.e. $su(3) \oplus su(3)^\perp = so(6)$. Thus, although the 3-index object τ can have $\mathbf{6} \otimes \mathbf{15}$ components, only $\mathbf{6} \otimes \mathbf{7}$ components contribute

[6] In this spinorial context, it is also called (intrinsic) con-torsion.

to the intrinsic torsion and these components are decompose under SU(3) as follows

$$\mathbf{6}\otimes\mathbf{7}\rightarrow(\mathbf{1}+\mathbf{1})\oplus(\mathbf{8}+\mathbf{8})\oplus(\mathbf{6}+\bar{\mathbf{6}})\oplus(\mathbf{3}+\bar{\mathbf{3}})\oplus(\mathbf{3}+\bar{\mathbf{3}})=\mathcal{W}_{\mathbf{1}}\oplus\mathcal{W}_{\mathbf{2}}\oplus\mathcal{W}_{\mathbf{3}}\oplus\mathcal{W}_{\mathbf{4}}\oplus\mathcal{W}_{\mathbf{5}}$$

where $\mathcal{W}_1$ is a complex scalar, $\mathcal{W}_2$ a 2-form, $\mathcal{W}_3$ a 3-form and $\mathcal{W}_4$ as well as $\mathcal{W}_5$ are two vectors. These components can now be read-off from dJ and $d\Omega$ as

$$\begin{aligned} dJ &= \tfrac{3i}{4}\left(\mathcal{W}_1\bar{\Omega}-\bar{\mathcal{W}}_1\Omega\right)+\mathcal{W}_3+J\wedge\mathcal{W}_4\ , \\ d\Omega &= \mathcal{W}_1 J\wedge J+J\wedge\mathcal{W}_2+\Omega\wedge\mathcal{W}_5 \end{aligned} \tag{51}$$

with the constraints

$$J\wedge J\wedge\mathcal{W}_2=J\wedge\mathcal{W}_3=\Omega\wedge\mathcal{W}_3=0$$

and therefore $\mathcal{W}_2$ and $\mathcal{W}_3$ are a primitive two- and and three-form, respectively. By using the definition of J and Ω in terms of the spinor η (see eq. (16) and applying Fierz re-arrangements, one can also verify the usual relations

$$J\wedge J\wedge J=\frac{3i}{4}\,\Omega\wedge\bar{\Omega}\quad,\qquad J\wedge\Omega=0\ .$$

The components of $\mathcal{W}_{\{1,4,5\}}$ can also be written as

$$\mathcal{W}_1\sim\Omega^{pmn}\partial_p J_{mn}\ ,\quad (\mathcal{W}_4)_p\sim J^{mn}\partial_{[p}J_{mn]}\ ,\quad (\mathcal{W}_5)_p\sim(\Omega^\star)^{mnq}\partial_{[p}\Omega_{mnq]}\ . \tag{52}$$

Depending on the components which are non-trivial, one distinguishes between different complex and non-complex manifolds. For example, the manifold is non-complex if $\tau\in\mathcal{W}_1$ (nearly Kähler) and $\tau\in\mathcal{W}_2$ (almost Kähler) and examples of complex manifolds are $\tau\in\mathcal{W}_3$ (special-hermitian), $\tau\in\mathcal{W}_5$ (Kähler) and of course if $\tau=0$ we have a Calabi-Yau space (see [46, 42, 28] for more examples). Let us now determine the different components for our flux compactification.

Massless case (i)

For this case $dF=0$ and the spinor has to satisfy the equation

$$\nabla_m\hat{\eta}+\frac{1}{2}e^{-\frac{5}{2}U}F_{mn}\gamma^n\hat{\eta}^\star=0$$

where we set $\beta=0$ and $\alpha=-\frac{U}{2}$ so that $\hat{\eta}=\frac{1}{\sqrt{2}}(\mathbb{1}-\gamma^7)\eta_0$ which is normalized as $\hat{\eta}^T\hat{\eta}=1$ [see (48) and (14)]. Using the fact, that $\hat{\eta}$ is a SU(3) singlet so that

the conditions (20) are satisfied, we can solve this equation by writing the covariant derivative as

$$\nabla_m \hat{\eta} = \partial_m \hat{\eta} + \frac{1}{4}\omega_m^{pq}\gamma_{pq}\hat{\eta} = \frac{1}{4}\,\omega_m^{pq}\Big[-iJ_{pq} + \frac{i}{2}\Omega_{pqr}\gamma^r\Big]\hat{\eta}$$

and since $\hat{\eta} = const.$ we get first order differential equations for the Vielbein $e_r{}^s$: $0 = \omega_m^{pq}\,J_{pq}$ and $\omega_m^{pq}\Omega_{pqs}e_r{}^s \sim F_{mr}$.

In order to get the torsion components, we will consider the complex structure J as well as the holomorphic 3-form Ω written in terms of the spinor $\hat{\eta}$ and find for the covariant derivative

$$D_p J_{mn} = -e^{-\frac{5}{2}U}F_p{}^r\,\chi^-_{rnl}\,J_{lm} = \frac{1}{2}e^{-\frac{5}{2}U}F_p{}^r\,(\Omega_{rmn} + \Omega^\star_{rmn})\ , \tag{53}$$

$$D_p\Omega_{mnq} = \frac{i}{2}\,e^{-\frac{5}{2}U}\,F_p{}^r\,J_{r[m}J_{np]} \tag{54}$$

where $m, n = 1, \ldots, 6$. Using the formulae (52) we find

$$\mathcal{W}_1 = 0 \quad , \qquad (\mathcal{W}_4)_m \sim (\mathcal{W}_5)_m \sim \chi^-_{mpq}F^{pq} = -8e^{2U}\partial_m U$$

where we used in the last equation the monopole equation (38). So, the non-zero values of $\mathcal{W}_{\{4,5\}}$ are related to the non-trivial warping of the metric. In order to fix the remaining components, we should use holomorphic coordinates and we can write $d\Omega \sim F \wedge J$. Since the (3,0) part in $d\Omega$ vanishes ($\mathcal{W}_1 = 0$) we infer that

$$\mathcal{W}_2 \sim F^{(1,1)} \tag{55}$$

On the other hand, since $\mathcal{W}_1$ vanishes dJ has only a (2,1) and (1,2) part and therefore only the $F^{(2,0)}$ part and its complex conjugate contributes to dJ. But since this holomorphic part of F is equivalent to $\Omega_{abc}F^{bc} \sim (\mathcal{W}_{\{4,5\}})_a$ we conclude

$$\mathcal{W}_3 = 0\ . \tag{56}$$

These results are in agreement with those derived in [45].

Massive case (ii)

It is now straightforward to repeat the analysis for the massive case, where $dF = mH$ and the differential equation for the spinor η_1, which defines J and Ω, was given in

(46) and can be written as

$$\begin{aligned} \nabla_m \hat{\eta} &= M_m \hat{\eta} + e^{2i\beta}\, N_{mn}\gamma^n \hat{\eta}^\star \ , \\ \text{with :} \quad M_m &\equiv -\tfrac{i}{4} e^{\frac{3}{2}U} H_{mpq} (\hat{\Omega}^\star)^{pq} \ , \\ N_{mn} &\equiv -\tfrac{1}{2}\, e^{-\frac{5}{2}U} \Big(F_{mn} - 2e^{4U} H_{mnp} v_+^p \Big) \ . \end{aligned} \tag{57}$$

Again, $\hat{\eta} = e^{\frac{U}{2}}\eta_1$ and we identified again $\alpha = -\frac{U}{2}$ and introduced M_m and N_{mn} to simplify the notation (note β is non-trivial in this case). One gets again a set of first order differential equations for the spin connection, if one uses the fact that $\hat{\eta}$ is an SU(3) singlet, i.e. obeys the relations (20). If one further takes into account the non-trivial phase β, this calculations is analogous to the massless case.

The covariant derivatives of J and Ω now become

$$\begin{aligned} D_p J_{mn} &= 2M_p J_{mn} - N_p{}^q (\, \Omega^\star_{qmn} + \Omega_{qmn}\,) \ , \\ D_q \Omega_{mnp} &= 2(M_q - i\, \partial_q \beta)\, \Omega_{mnp} + 2i\, N_q{}^r\, J_{r[p} J_{mn]} \end{aligned}$$

where $m, n, \cdots = 1, \cdots, 6$. Using holomorphic coordinates, we find again

$$\begin{aligned} \mathcal{W}_1 &= 0 \quad , \\ (\mathcal{W}_4)_a &\sim 4M_a + i\, \Omega_{abc} N^{bc} \ , \\ (\mathcal{W}_5)_a &\sim 3(M_a - i\partial_a \beta) + i\, \Omega_{abc} N^{bc} \end{aligned}$$

where we used now the massive monopole equation (44) or (48), combined with (49). To find $\mathcal{W}_2$, it is enough to look on the last term in $d\Omega$, which is proportional to $J \wedge N$ and hence

$$(\mathcal{W}_2)_{a\bar{b}} \ \sim \ N_{a\bar{b}} \ \sim \ F_{a\bar{b}} - 2e^{4U}\, H_{a\bar{b}c} v^c \ .$$

which is primitive because H and F are primitive. Finally, to get $\mathcal{W}_3$, we have to consider the (1,2)-piece of dJ which is not part of $\mathcal{W}_4$. Therefore, only the term $N_{ab}\Omega^b{}_{\bar{c}\bar{d}}$ and its complex conjugate can contribute to $\mathcal{W}_3$. However, since $N_{ab} \sim \Omega_{abc}\partial^c U$ (i.e. $\partial_c U \sim \Omega_{cab} N^{ab}$ which follows from (48) and (49)) and using the identity $\Omega_{abc}\Omega^{cde} \sim \delta_a{}^{[d}\delta_b{}^{e]}$ we find that all terms of dJ are part of $\mathcal{W}_4$ and conclude that also for the massive case

$$\mathcal{W}_3 = 0 \ .$$

The appearance of the vector v implies a breaking of the SU(3) to SU(2) structures. In order to decompose our expressions in SU(2) representations we have to separate the components of $\mathcal{W}_2$, $\mathcal{W}_4$ and $\mathcal{W}_5$ parallel and transverse to v. The (1,1)-form $\mathcal{W}_2$ decomposes into: $\mathbf{1}+(\mathbf{2}+\bar{\mathbf{2}})+\mathbf{3}$ given by $v^a N_{a\bar{b}} v^{\bar{b}}$, $N_{a\bar{b}} v^{\bar{b}}$, $N_{\bar{a}b} v^b$ and the remaining components comprise the $\mathbf{3}$. Similarly, by contracting the vectors $\mathcal{W}_{\{4,5\}}$ with v, we get a $\mathbf{1}$ and the remaining components become $\mathbf{2}+\bar{\mathbf{2}}$.

6 Interesecting branes and chirality

In the limit of vanishing mass parameter, our results are invariant under SU(3) rotations and therefore intersecting brane solutions can be build by SU(3) rotations as proposed in [1]. A non-zero mass parameter implies a massive NS-NS B-field yielding a 3-form flux ($H = dB$) and as we discussed this mass parameter can only be non-zero, if the 6-manifold allows for a (no-where vanishing) vector v. This puts already constraints on the (compact) manifold, as e.g., a vanishing Euler number (Hopf theorem), and corresponds to the existence of two 6-d spinors with opposite chirality. The massless case on the other hand, is described by a single 6-d spinor, which is a SU(3) singlet.

From the supergravity point of view, a mass parameter is related to the appearance of D8-branes which are perpendicular to the vector v. At the same time, this vector breaks the SU(3) rotations known from the massless case to SU(2) rotations and therefore the D6-branes can be localized only in four of the six internal directions and are aligned along one internal direction. An example is given by the following picture

	y_1	y_2	y_3	y_4	y_5	y_6
$D6$	$\times$	o	$\times$	o	$\times$	o
$D6'$	$\times$	o	o	$\times$	o	$\times$
$D8$		$\times$	$\times$	$\times$	$\times$	$\times$

where $y_1, \ldots, y_6$ comprises the internal directions and "$\times$" indicates the world-volume directions of D6-branes and "o" the constant H-flux, e.g., in the simplest case: $H = (h\, dy^4 \wedge dy^6 + h'\, dy^3 \wedge dy^5) \wedge dy^2$ for some constants $h, h' \sim m$. Due to the constraint

$B \wedge B = 0$ (coming from $F_4 = 0$), F becomes

$$B = (h\, y^4 dy^6 + h'\, y^3 dy^5) \wedge dy^2 \ . \tag{58}$$

From the supergravity point of view one should distinguish between localized branes and branes that are dissolved into fluxes and so far we discussed only the latter ones. However, it is straightforward to add also localized branes. E.g., we can add n_6 localized D6-branes by changing the Bianchi identity

$$dF = mH \to mH - n_6\, \delta^{(3)} \tag{59}$$

where $\delta^{(3)}$ is a 3-form δ-function which projects onto the world-volume of the D6-brane. In order to add sources for D8-branes we replace $m \to -m\, \theta(y_1 - y^0)$ and NS5-brane source correspond to $dH = n_5\, \delta^{(4)}$, where the 4-d δ-function projects onto the NS5-brane world-volume and n_5 is the number of NS5-branes. If we ignore for the moment D8-brane sources and consider only NS5- and D6-branes, only, then from $ddF = 0$ one infers that the D6-branes *must* end on the NS5-branes and the number of D6-branes is given by the number of NS5-branes and the mass parameter [39]:

$$ddF = 0 = (mn_5 - n_6)\, \delta^{(4)} \ .$$

Therefore, if one adds localized NS5-branes, one has necessarily to include open D6-branes that end on these NS5-branes.

With the intersecting D6-brane configuration discussed above we would now like to address whether the D6-brane world-volume 4-d gauge theory is chiral. This can be addressed by studying a possible anomaly inflow [47] from the bulk to the 4-d subspace. [Without invoking the constraints imposed by supersymmetry, this anomaly inflow in the presence of a NS-NS 3-form flux has also been discussed in [48].] Anomaly inflow [47] takes place when the Wess-Zumino action associated with the given Dp-brane is not invariant under the gauge transformation. In this case the anomaly of the world-volume field theory has to cancel the anomaly inflow contribution, thus rendering the gauge theory chiral. The Wess-Zumino action associated with the specific Dp-brane world-volume has the form:

$$S^p_{WZ} = \int_{Dp} \mathcal{C} \wedge Y \tag{60}$$

where $\mathcal{C}$ is a sum over all R-R potentials, $Y \equiv ch(F)\sqrt{\hat{A}(R)}$ with $ch(F)$ denoting the Chern class of the world-volume gauge bundle and $\hat{A}$ is the A-roof genus, which depends on the curvature form (see [47] and references therein). In our specific consideration X_6 is flat and $\hat{A}$ plays no role.

Since Y is exact, it can be written as $Y = dY^{(0)}$ and we can integrate (60) *per partes* to obtain an integral $\int_{Dp} d\mathcal{C} \wedge Y^{(0)}$. Now, $Y^{(0)}$ transforms under a gauge transformation ($\delta Y^{(0)} = dY^{(1)}$) and if we denote the field strengths of the R-R gauge potentials by $\mathcal{G} \equiv d\mathcal{C}$, one finds for the variation of the Wess-Zumino action (60):

$$\delta S^p_{WZ} = \int_{Dp} d\mathcal{G}\, Y^{(1)} \ . \tag{61}$$

Whenever $d\mathcal{G}$, projected on the Dp-brane world-volume, is non-zero, (61) is non-zero and thus Wess-Zumino action (60) is not gauge invariant. Its contribution should then be cancelled by the gauge-anomaly contribution of the D-brane world-volume gauge theory.

We shall now apply this inflow mechanism for the world-volume theory of a D6-brane, i.e. $d\mathcal{G} = dF$. As seen from eq. (59) both the NS-NS 3-flux as well as D6-brane sources can potentially contribute to the anomaly inflow. The integral (61) is non-zero only if the dF projected onto the world-volume of one D6-brane is non-zero. However, the constraints on the configuration are such that this is not the case. As it is obvious from the example in the table above, the H-flux always extends in the y^2 direction, which is not part of any of the D6-brane world-volume directions. The same is also true for the D6'-source term when we consider the Wess-Zumino term for D6-brane (and vice versa): $\delta^{(3)}$ includes always the δ-function in y^2 direction. Hence neither the NS-NS 3-form fluxes (or NS5-brane sources) nor the D6-brane sources can give rise to a non-zero anomaly inflow term.

Note however, that in our analysis we have to include also effects from D8-branes, which appear as domain walls in the common world-volume direction of D6-branes. If we put them at $y_1 = 0$ we have to replace the two-form field strength F with $-m\,\theta(y_1)\,B + dA_1$ and the effect of D8-brane sources modifies the right hand side of the Bianchi identity for F in the following way:

$$dF = -\delta(y_1)\, v \wedge B - m\,\theta(y_1)\, H \ . \tag{62}$$

Recall, the vector field v is orthogonal the D8-branes. Again, the H-term cannot give a non-zero contribution to the inflow integral. In addition also the first term, which describes the coupling of the D8-brane background to the world-volume of a D6-brane, can only be nonzero if the B-field, projected onto the D6-brane world-volume, is nonzero. This however, is only possible if $B \wedge B \neq 0$ (because the B-field has always components in the transverse space of the D6-brane). However, for our configuration, without R-R 4-form fluxes, $B \wedge B = 0$, and thus there is no anomaly inflow from D8-brane sources.

The same analysis could be repeated for the Wess-Zumino coupling for the world-volume of the D8-brane, again giving no anomaly inflow. Hence, we conclude that for our specific supersymmetric configuration (without R-R 4-form turned on), there is no anomaly inflow from the bulk to the world-volume field theory and thus the gauge theory is not chiral.

7 Discusssion

We have discussed the constraints imposed by supersymmetry of D6-brane configurations in massive type IIA superstring theory. In the simplest case of parallel 6-branes wrapping a 3-cycle of a torus, the massive deformation of the warp factor or dilaton is given in eq. (50) with the metric (39), which agrees with the result found in [40]. The supergravity solution exhibits a naked singularity at a finite distance in the transversal space, which is given by a zero of e^{4U}, which is reminiscent to the deformation of the M2-brane due to a self-dual 4-form potential [49]. A similar singularity occurs in brane world scenarios with positive tension branes where the warp factor has a zero and in the AdS/CFT language, this singularity was resolved by non-trivial IR effects. [Note, from the world-volume point of view the IR regime corresponds to a small warp factor.] In the case at hand however, the 10-d warp factor in both, the Einstein- and string frame, is infinite at the singularity indicating that the theory is UV "incomplete". Better understanding of this singularity deserves further investigations. Note, however, that in the case of non-flat internal space X_6 one may allow for the non-singular configurations with the NS-NS 3-form flux corresponding to a regular L^2 integrable harmonic 3-form on X_6, which would in turn, due to transgression

Chern-Simons terms, render the R-R 2-form field strength and the metric regular, thus making the configuration regular (c.f., [16]).

The superpotential and fixing of moduli

We would also like to comment on moduli dependence of the 4-d superpotential W generated by fluxes. A non-zero superpotential can be determined from the Killing spinor equation for a 4-d spinor θ which is not covariantly constant, but satisfied the relation, written schematically as $D_\mu\theta \sim W\gamma_\mu\theta$. Implementing this relation into the calculation of the Killing spinor equations we obtain the following two contributions to the superpotential W:

$$\sim \int F \wedge J \wedge J \quad , \qquad \sim 12m\, i + \int H \wedge \Omega$$

For our vacuum the superpotential W and its Kähler covariant derivatives vanish. A contribution to W from the 2-form F flux yields a dependence of the Kähler class moduli whereas that from H yields a dependence on the complex structure moduli. Note, however, that the 2-form F flux has to satisfy the constraint $F \wedge F = 0$ and it has to be transverse to the vector v ($F_{mn}v^n = 0$) and therefore it cannot fix the 2-cycle which is related to the holomorphic vector v. In addition, since $dF = mH$, the constraints on F imply analogous constraints on the 3-form H and hence the contribution to W from H yields a dependence on only some complex structure moduli.

A way to understand the moduli dependence of the superpotential and the fixing of moduli in the vacuum is to consider the supergravity theory obtained after dimensional reduction (see [18, 23, 50]). One obtains moduli from Kähler class deformations which are in Type IIA string theory related to scalar fields in vector multiplets and complex structure moduli related to scalar fields in hypermultiplets. General R-R 2- and 4-form fluxes yield a (complex) superpotential that fixes in the generic case all scalars in the vector multiplets and the vacuum is described by a BPS domain wall solution of N=2 supergravtiy [51] which becomes flat space time in the limit of vanishing superpotential. An additional NS-NS 3-form flux will result in an additional dependence of W on the complex structure moduli. Our case at hand, however, is not generic because we have only special R-R 2-form and NS-NS 3-form fluxes which

are subject to the constraints mentioned above. The absence of a R-R 4-form flux would also result in the absence of certain Kähler class moduli in the superpotiential. (Due to the presence of the NS-NS 3-form flux we cannot "turn on" the 4-flux by a symplectic rotation; see also [50].) In conclusion, due to the constrained structure of the turned on fluxes only a limited class of moduli can be fixed in the vacuum.

Further open questions

There are a number of directions for further exploration. The construction described in this paper was severely constrained due to the existence of the vector field v, which, e.g., has forced us to align all D6-brane along this vector in the internal space. There is however a possibility of a more more general setting that does not require the existence of such a vector. This seems to be possible if one allows for additional R-R 4-form fluxes. In this case the additional terms in the Killing spinor equations can in fact naturally compensate for the mass terms without the necessity of two (opposite chirality) 6-d internal spinors. There is a strong indication that the inclusion of a R-R 4-form fluxes (and D4-brane sources) would yield a chiral gauge theory on the D4-brane world-volume (as considered in [39] in the absence of supergavity fluxes). One also expects that in this case most of the moduli (except the dilaton) would be fixed in the N=1 supersymmetric vacuum.

An alternative approach to derive 4-dimensional N=1 supersymmetric solutions of (massless) Type IIA superstring theory with intersecting D6-branes and supergravity fluxes would be to address its strongly coupled limit as M-theory compactified on 7-dimensional manifold with 4-form field strength G fluxes turned on, resulting in manifolds with G_2 structures with torsion. Reduction of this seven-dimensional space on a circle would in turn yield a (massless) Type IIA superstring theory with intersecting D6-branes (and O6-planes) and a six-dimensional manifold with SU(3) torsion classes. This approach may shed a complementary light on the possible intersecting D6-brane configurations and the spectrum of the resulting $N = 1$ supersymemtric D6-brane world-volume gauge theory in the presence of fluxes and is a subject of further study. Some work in this direction has been done already, e.g., in [52, 53, 54]. However, a more general study of the possible flux configurations and the implications for the D6-brane world-volume gauge theory is a subject of further research.

Acknowledgments

We would like to thank Ralph Blumenhagen, Amihay Hanany, Dario Martelli and Angel Uranga for useful discussions. M.C. would like to thank Institute for Advanced Study, Princeton, and we would like to thank CERN, Theory Division, for hospitality and support during the course of this work. Research is supported in part by DOE grant DOE-EY-76-02-3071 (M.C.), NSF grant INT02-03585 (M.C.) and Fay R. and Eugene L. Langberg endowed Chair (M.C.).

References

[1] M. Berkooz, M. R. Douglas, and R. G. Leigh, "Branes intersecting at angles," *Nucl. Phys.* **B480** (1996) 265–278, `hep-th/9606139`.

[2] C. Bachas, "A way to break supersymmetry," `hep-th/9503030`.

[3] M. Cvetic, G. Shiu, and A. M. Uranga, "Three-family supersymmetric standard like models from intersecting brane worlds," *Phys. Rev. Lett.* **87** (2001) 201801, `hep-th/0107143`.

[4] M. Cvetic, G. Shiu, and A. M. Uranga, "Chiral four-dimensional N = 1 supersymmetric type IIA orientifolds from intersecting D6-branes," *Nucl. Phys.* **B615** (2001) 3–32, `hep-th/0107166`.

[5] M. Cvetic and I. Papadimitriou, "More supersymmetric standard-like models from intersecting D6-branes on type IIA orientifolds," `hep-th/0303197`.

[6] M. Cvetic, I. Papadimitriou, and G. Shiu, "Supersymmetric three family SU(5) grand unified models from type IIA orientifolds with intersecting D6-branes," `hep-th/0212177`.

[7] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, "Noncommutative compactifications of type I strings on tori with magnetic background flux," JHEP **0010** (2000) 006 [arXiv:hep-th/0007024].

[8] R. Blumenhagen, B. Körs, and D. Lüst, "Type I strings with F- and B-flux," *JHEP* **02** (2001) 030, `hep-th/0012156`.

[9] G. Aldazabal, S. Franco, L. E. Ibáñez, R. Rabadan, and A. M. Uranga, "Intersecting brane worlds," *JHEP* **02** (2001) 047, hep-ph/0011132.

[10] R. Blumenhagen, B. Körs, D. Lüst, and T. Ott, "The standard model from stable intersecting brane world orbifolds," *Nucl. Phys.* **B616** (2001) 3–33, hep-th/0107138.

[11] C. Angelantonj, I. Antoniadis, E. Dudas, and A. Sagnotti, "Type-I strings on magnetised orbifolds and brane transmutation," *Phys. Lett.* **B489** (2000) 223–232, hep-th/0007090.

[12] E. Witten, "Anomaly cancellation on G(2) manifolds," hep-th/0108165.

[13] B. Acharya and E. Witten, "Chiral fermions from manifolds of G(2) holonomy," hep-th/0109152.

[14] M. Cvetic, G. Shiu, and A. M. Uranga, "Chiral type II orientifold constructions as M theory on G(2) holonomy spaces," hep-th/0111179.

[15] I. Brunner and K. Hori, "Orientifolds and mirror symmetry," hep-th/0303135.

[16] M. Cvetic, H. Lu, and C. N. Pope, "Brane resolution through transgression," *Nucl. Phys.* **B600** (2001) 103–132, hep-th/0011023.

[17] A. Strominger, "Superstrings with torsion," *Nucl. Phys.* **B274** (1986) 253.

[18] J. Polchinski and A. Strominger, "New vacua for type II string theory," *Phys. Lett.* **B388** (1996) 736–742, hep-th/9510227.

[19] S. Gukov, C. Vafa, and E. Witten, "CFT's from Calabi-Yau four-folds," *Nucl. Phys.* **B584** (2000) 69–108, hep-th/9906070.

[20] K. Dasgupta, G. Rajesh, and S. Sethi, "M theory, orientifolds and G-flux," *JHEP* **08** (1999) 023, hep-th/9908088.

[21] T. R. Taylor and C. Vafa, "RR flux on Calabi-Yau and partial supersymmetry breaking," *Phys. Lett.* **B474** (2000) 130–137, hep-th/9912152.

[22] K. Behrndt and S. Gukov, "Domain walls and superpotentials from M theory on Calabi-Yau three-folds," *Nucl. Phys.* **B580** (2000) 225–242, hep-th/0001082.

[23] G. Curio, A. Klemm, D. Lüst, and S. Theisen, "On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes," *Nucl. Phys.* **B609** (2001) 3–45, hep-th/0012213.

[24] M. Cvetic, G. W. Gibbons, H. Lu, and C. N. Pope, "Almost special holonomy in type IIA and M theory," *Nucl. Phys.* **B638** (2002) 186–206, hep-th/0203060.

[25] J. P. Gauntlett, D. Martelli, S. Pakis, and D. Waldram, "G-structures and wrapped NS5-branes," hep-th/0205050.

[26] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, "Kaluza-Klein bundles and manifolds of exceptional holonomy," *JHEP* **09** (2002) 033, hep-th/0206213.

[27] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, "Mirror symmetry in generalized Calabi-Yau compactifications," *Nucl. Phys.* **B654** (2003) 61–113, hep-th/0211102.

[28] G. L. Cardoso *et al.*, "Non-Kaehler string backgrounds and their five torsion classes," *Nucl. Phys.* **B652** (2003) 5–34, hep-th/0211118.

[29] J. P. Gauntlett, D. Martelli, and D. Waldram, "Superstrings with intrinsic torsion," hep-th/0302158.

[30] D. Martelli and J. Sparks, "G-structures, fluxes and calibrations in M-theory," hep-th/0306225.

[31] L. J. Romans, "Massive N=2a supergravity in ten-dimensions," *Phys. Lett.* **B169** (1986) 374.

[32] J. F. G. Cascales and A. M. Uranga, "Chiral 4d N = 1 string vacua with D-branes and NSNS and RR fluxes," *JHEP* **05** (2003) 011, hep-th/0303024.

[33] S. B. Giddings, S. Kachru, and J. Polchinski, "Hierarchies from fluxes in string compactifications," *Phys. Rev.* **D66** (2002) 106006, `hep-th/0105097`.

[34] S. Kachru, M. B. Schulz, and S. Trivedi, "Moduli stabilization from fluxes in a simple IIB orientifold," `hep-th/0201028`.

[35] A. R. Frey and J. Polchinski, "N = 3 warped compactifications," Phys. Rev. D **65** (2002) 126009 [arXiv:hep-th/0201029].

[36] S. Kachru, M. B. Schulz, P. K. Tripathy, and S. P. Trivedi, "New supersymmetric string compactifications," *JHEP* **03** (2003) 061, `hep-th/0211182`.

[37] R. Blumenhagen, D. Lüst, and T. R. Taylor, "Moduli stabilization in chiral type IIB orientifold models with fluxes," `hep-th/0303016`.

[38] M. Grana and J. Polchinski, "Supersymmetric three-form flux perturbations on AdS(5)," *Phys. Rev.* **D63** (2001) 026001, `hep-th/0009211`.

[39] A. Hanany and A. Zaffaroni, "Chiral symmetry from type IIA branes," *Nucl. Phys.* **B509** (1998) 145–168, `hep-th/9706047`.

[40] B. Janssen, P. Meessen, and T. Ortin, "The D8-brane tied up: String and brane solutions in massive type IIA supergravity," *Phys. Lett.* **B453** (1999) 229–236, `hep-th/9901078`.

[41] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, and A. Van Proeyen, "New formulations of D = 10 supersymmetry and D8 - O8 domain walls," *Class. Quant. Grav.* **18** (2001) 3359–3382, `hep-th/0103233`.

[42] S. Chiossi and S. Salamon, "The intrinsic torsion of $SU(3)$ and G_2 structures," `math.dg/0202282`.

[43] T. Friedrich and S. Ivanov, "Parallel spinors and connections with skew-symmetric torsion in string theory," `math.dg/0102142`.

[44] G. L. Cardoso, G. Curio, G. Dall'Agata, and D. Lüst, "BPS action and superpotential for heterotic string compactifications with fluxes," `hep-th/0306088`.

[45] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, "Nontrivial RR two-form field strength and SU(3)-structure," hep-th/0301063.

[46] A. Grey and L. Hervella, "The sixteen classes of almost hermitian manifolds and their linear invariants," *Ann. Math. Pura Appl.* **123** (1980) 35–58.

[47] M. B. Green, J. A. Harvey, and G. W. Moore, "I-brane inflow and anomalous couplings on D-branes," *Class. Quant. Grav.* **14** (1997) 47–52, hep-th/9605033.

[48] A. M. Uranga, "D-brane, fluxes and chirality," *JHEP* **04** (2002) 016, hep-th/0201221.

[49] M. Cvetic, H. Lu, and C. N. Pope, "Penrose limits, pp-waves and deformed M2-branes," hep-th/0203082.

[50] J. Louis and A. Micu, "Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes," *Nucl. Phys.* **B635** (2002) 395–431, hep-th/0202168.

[51] K. Behrndt, G. Lopes Cardoso, and D. Lüst, "Curved BPS domain wall solutions in four-dimensional N = 2 supergravity," *Nucl. Phys.* **B607** (2001) 391–405, hep-th/0102128.

[52] M. Cvetič, H. Lü, and C. N. Pope, "Massless 3-branes in M-theory," *Nucl. Phys.* **B613** (2001) 167–188, hep-th/0105096.

[53] K. Behrndt and C. Jeschek, "Fluxes in M-theory on 7-manifolds and G structures," *JHEP* **04** (2003) 002, hep-th/0302047.

[54] P. Kaste, R. Minasian, and A. Tomasiello, "Supersymmetric M-theory compactifications with fluxes on seven-manifolds and G-structures," *JHEP* **07** (2003) 004, hep-th/0303127.