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Stochastic background from extra dimensions

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Abstract

The existence of extra dimensions is a common feature in almost all quantum theories of gravity. These extra dimensions, whose size and number vary from theory to theory, have a signature on the gravitational stochastic background. Here we review the predictions of the individual theories and the hope of revealing such signals with earthbound detectors.

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1. The stochastic background

The gravitational stochastic background is, by definition, the radiation produced by sources which are independent, uncorrelated and unresolved [1]. Cosmologically speaking, we are looking for relic gravitons produced during the phase transitions that took place in the early universe. We have two bounds from the theory for the sources predicted by the cosmological standard model [2]:

$$h_{70}^2 \Omega_{\mathrm{GW}} \leqslant 2.3 \times 10^{-9} \tag{1}$$

for topological defects and

$$h_{70}^2 \Omega_{\mathrm{GW}} \leqslant 8 \times 10^{-14} \tag{2}$$

for inflation. These bounds also apply in the frequency range of interest for earthbound interferometers. We also have astrophysical sources, such as unresolved white dwarf binary systems or young, rapidly rotating neutron stars. The typical frequency of emission of an object of mass M is

$$f \leqslant 10^4 \frac{M_\odot}{M}\ \mathrm{Hz}. \tag{3}$$

These frequencies seem to fall outside the detection possibility of earthbound detectors, but could be the main source for a space-based detector like LISA [2]. An experimental bound comes from the nucleosynthesis theory, the formation of light elements when the universe was $t = 180$ s old. According to this theory we have [2]

$$\int_{f=0}^{f=\infty} \mathrm{d}(\ln f) h_{70}^2 \Omega_{\mathrm{GW}}(f) \leqslant 5.6 \times 10^{-6}(N_\nu - 3) \tag{4}$$

where N_ν is the number of massless neutrino species at the time of nucleosynthesis. Various estimates exist, but it is reasonable to assume $4 \leqslant N_\nu < 5$. Some more conservative cosmologists even restrict this bound, $N_\nu \leqslant 4$. This rather stringent bound applies of course only to the cosmological background, already present at nucleosynthesis time. Another bound comes from pulsar timing, but it is irrelevant so long as $f < 10^{-7}$ Hz.

2. Extra dimensions

Extra dimensions are an essential ingredient in almost all quantum theories of gravity, canonical quantum gravity being the only remarkable exception [3]. Quantum effects not yet fully understood let them decouple from the ordinary 4D universe forcing them to remain small. Their size might range from the Planck length to a few microns. Some theories predict the existence of new particles, radions, associated with the extra dimensions [4]. The number, like the size, of extra dimensions varies from theory to theory. String theory, and M-theory, require $D = 11$ [5, 6], Randall–Sundrum theories do not fix the number *a priori* [7]; inflation per se is formulated in 4D, but recently an attempt to incorporate it into a larger quantum frame has given birth to a 5D inflationary theory [8]. The ekpyrotic universe, and other cyclical models of universe, require $D = 5$ as well [9]. In theories with n extra dimensions, the Planck mass is no longer a fundamental quantity, but rather it is derived from the fundamental mass M_* [10]:

$$M_{\mathrm{Pl}}^2 = M_*^{n+2} R^n \tag{5}$$

where R is the typical size of extra dimensions which also satisfies the hierarchy relation $R \gg M_*^{-1}$ [4].

2.1. 5D inflation

In this theory, equation (5) becomes

$$M_{\mathrm{Pl}}^2 = (8\pi G)^{-1} = 2\pi R M_*^3. \tag{6}$$

The extra dimension does not alter the cosmological framework described by general relativity (GR) if the radion stabilization mechanism remains constant,

$$\pi R H \leqslant 1 \tag{7}$$

where H is the Hubble parameter at decoupling time. The power spectrum of the tensor modes is enhanced by a factor [8]

$$\left(1 - \frac{2\pi^2 R^2 H^2}{3}\right)^{-1}. \tag{8}$$

We can thus achieve at most an improvement of an order of magnitude, reaching a gravitational wave (GW) density of $\Omega_{\mathrm{GW}} \sim 10^{-13}$. This is still too little for earthbound detectors: Advanced LIGO has a designed sensitivity of $\Omega_{\mathrm{GW}} \sim 10^{-10}$, with correlation of the two detectors and one year integration time, but such an energy density is well within LISA's possibilities, since for LISA $\Omega_{\mathrm{GW}} \sim 10^{-16}$. This result has been obtained by assuming that the inflaton lives on the (3 + 1)-dimensional surface (3-brane) that corresponds to our visible universe, as do all other fields except the graviton, which lives on the 5D bulk. We can release this assumption and find a qualitatively similar result. The enhancement (8) is due to the fact that the graviton zero mode feels a 'smaller' Planck mass than in the 4D case: gravity was stronger during inflation. Extra dimensions, on the other hand, do not enhance the power spectrum of the scalar modes. The consistency relations remain unaltered at the lowest order in the slow roll parameter,

$$\frac{\mathcal{P}_T(k)}{\mathcal{P}_S(k)} = -8n_T \tag{9}$$

where the tensor spectral index is defined as usual [11]

$$n_T \equiv \frac{\mathrm{d}\ln \mathcal{P}_T(k)}{\mathrm{d}\ln k}. \tag{10}$$

These relations also hold for an arbitrary number of extra dimensions $n > 1$ if all extra dimensions have the same size; but they do not hold if extra dimensions have different sizes.

2.2. Pre-big bang cosmology

This theory assumes the fundamental action of string theory with a dilaton field. This field is responsible for a pre-big bang phase, or dilaton-driven phase [12]. This phase ends with a transition to the usual Friedmann–Robertson–Walker (FRW) cosmology [13]. The most interesting feature of this theory is that the spectrum of GW grows with frequency [2]; this prediction seems independent of the number of extra dimensions,

$$h_{70}^2\Omega_{\mathrm{GW}} \leqslant 4.5\times10^{-7} \qquad f = 100\,\mathrm{Hz}. \tag{11}$$

This model is thus very promising for earthbound detectors, in particular for Advanced LIGO. Advanced LIGO and spherical resonant antennas could also eventually reveal a background of scalar waves [14], whose theoretical predictions though are very uncertain.

2.3. Braneworld cosmologies

These cosmological models are derived essentially from M-theory. The typical emission frequency for gravitational waves depends on the size of the largest extra dimension b [4]:

$$f_{\mathrm{peak}} \sim 10^{-4}\ \mathrm{Hz}\left(\frac{1\,\mathrm{mm}}{b}\right)^{1/2}. \tag{12}$$

From this formula it follows that earthbound detectors, whose sensitivity is typically around 100 Hz–1 kHz, can probe dimensions of the order $b \sim 10^{-15}$ m, whilst LISA can probe larger, Randall–Sundrum dimensions, $b \sim 10^{-5}$ m. This corresponds to the door to the yet partially unexplored realm of submillimetric ($d < 0.1$ mm) gravity [15, 16], which cannot be ruled out by present data and which has recently been proposed as a solution of the cosmological constant problem (CCP) [17]. A very interesting case is

$$b > H^{-1}(T = M_*). \tag{13}$$

The Nambu–Goldstone modes are excited by the Kibble mechanism and produce a GW background, whose spectrum is peaked at [4]

$$f_{H_0}(H^{-1} = b). \tag{14}$$

The energy density can be as high as $\Omega_{\mathrm{GW}} \sim 10^{-10}$, giving us a concrete hope of detecting such a background with earthbound interferometers. This energy density falls off as a power law for higher and lower frequencies; in particular, for lower frequencies it falls off as $\Omega_{\mathrm{GW}}(\Delta f = f) \propto f^7$.

The opposite case is $b < H^{-1}(T = M_*)$. In this case, the main mechanism of GW production is radion stabilization. The amplitude of the spectrum seems much smaller than in the previous case [4]. If all extra dimensions are of the Planck scale

$$b = M_{\mathrm{Pl}}^{-1} = M_*^{-1} = 10^{-35}\,\mathrm{m} \tag{15}$$

then the GW spectrum has a peak for

$$f = 10^{12}\ \mathrm{Hz}, \qquad \lambda = 3 \times 10^{-4}\,\mathrm{m}. \tag{16}$$

Although there have been some proposals aimed at detecting GWs in the MHz range [18] and beyond [19], frequencies as high as equation (16) seem at the moment beyond our experimental reach. That is rather unfortunate, since there is no astrophysical object which can emit at such high frequencies [2] and thus the detection of a stochastic background in this high-frequency range would be certainly of cosmological origin.

2.4. The ekpyrotic universe

This term describes some 'cyclic' models, where the universe is eternal and oscillates between big bangs and big crunches [20]. Such theories are particularly interesting because they provide a strong link between quantum theories of gravity and cosmology, without any further assumption. They also address the shortcomings of the cosmological standard model, i.e. the horizon and flatness problem. These models heavily rely on heterotic M-theory, whose consistent and ultimate formulation is far from completed, and thus present some unclear features: in particular there is no convincing mechanism for galaxy formation starting from Bogomoln'yi–Prasad–Sommerfield (BPS) states for the branes. Fluctuations are generated as the bulk 3-brane, whose collision with the visible brane that corresponds to our universe marks the beginning of cosmic time, moves through the 5D bulk [9]. The Hubble radius for an observer on the visible brane is decreasing and thus the spectrum is blue-shifted. This is a key difference from the inflationary models: in inflationary theories wavelengths are stretched superluminally, whilst the horizon remains nearly constant; in the ekpyrotic universe, wavelengths remain almost constant whilst the horizon shrinks. The excitations that produce scalar gravitational waves are ripples on the moving 3-brane. On the other hand, tensor fluctuations are excitations of the gravitational field, the field which lives on the bulk; as a result, tensor modes and scalar ones have different scale factors. The power spectrum of tensor fluctuations is [9]

$$|\delta h_{\vec{k}}|^2 \equiv \frac{k^3 h_{\vec{k}}^2}{(2\pi)^3} = \left(\frac{H_{\mathrm{c}}}{M_{\mathrm{Pl}}}\right)^2 k^2. \tag{17}$$

The spectrum is strongly blue-shifted. This can be seen also by comparing the tensor spectral index, $n_T = 2$, with the inflationary case $n_T \leqslant 0$. So, the spectrum predicted by inflation is almost scale- (and frequency-) invariant, whilst the spectrum predicted by the ekpyrotic model is blue-shifted. The gravitational wave energy density in the frequencies of interest for earthbound detectors $\nu \sim 1$ kHz is [21]

$$\Omega_{\mathrm{GW}} \leqslant 10^{-32} \tag{18}$$

and even smaller for LISA frequencies. There is a peak in energy density for $\nu = 10^{10}$ Hz, but again, of no practical interest for present day technologies and sensitivities. A detection of a cosmological stochastic background in the LISA frequencies would therefore be consistent with inflation and would violate and falsify the ekpyrotic models.

3. Conclusions

The existence of extra dimensions seems to be a predominant feature of almost all quantum theories of gravity. These extra dimensions could create a detectable signature on the stochastic background, which could help us to discriminate amongst various theories and to reach a far better understanding of the early universe. Unfortunately, no quantum theory of gravity has at present a self-consistent formulation, and thus our prediction power is limited. Nevertheless, as we saw in the previous subsection, even a negative result, i.e. no detection as LISA starts, would be of major significance for cosmology and astronomy.

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