

Remarks on type IIB pp waves with Ramond-Ramond fluxes and massive two dimensional nonlinear sigma models

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Abstract

We continue the study of supersymmetric type IIB pp-wave solutions by Maldacena and Maoz (hep-th/0207284), who showed Ramond-Ramond five-forms can induce potential terms in the light cone string actions which are nonlinear sigma models with special holonomy target spaces. We show that nonvanishing Ramond-Ramond three-forms provide extra potential terms involving Killing vectors in the string action and identify the supersymmetry requirements. In particular, in solutions with $(1,1)$ worldsheet supersymmetry, the Killing vectors are required to be self-dual in $\text{Spin}(7)$.

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I. INTRODUCTION

Recently Maldacena and Maoz [1] have constructed an interesting class of supersymmetric pp-wave solutions in ten dimensional type IIB supergravity, which includes the maximally supersymmetric plane-wave solution [2] as a special case. The nontrivial curvature is supported by null Ramond-Ramond (RR) five-forms which are non-constant, and it is argued that the light cone string actions in Green-Schwarz formalism are nonlinear sigma models with potential terms. When the target space is curved the spacetime supersymmetry requires it to have special holonomy. For solutions with $(2, 2)$ worldsheet supersymmetry the target spaces are Calabi-Yau four-folds in general, and RR five-forms give holomorphic superpotential and Killing vector terms, whereas when one demands only $(1, 1)$ supersymmetry the target geometry has reduced Spin(7) holonomy most generally and non-constant five-forms are translated into real harmonic superpotentials on the worldsheet. These solutions are shown to be exact string backgrounds using the U(4) formalism [3] and also by considering possible higher order correction terms to the string effective actions [4]. Thus a large class of interacting two dimensional field theories, including integrable models, are added to the list of quantizable RR backgrounds. For subsequent works on these pp-wave string theories see [5, 6, 7, 8, 9, 10, 11].

Although the solutions in [1] comprise quite general class of massive nonlinear sigma models, certainly they do not have the *most general* potential terms. The supersymmetry requirements of potential terms in two dimensional nonlinear sigma models are summarised in [12]. With $\mathcal{N} = (2, 2)$ the target spaces are Kähler and *two* commuting holomorphic Killing vector terms and one holomorphic superpotential term can be present. And for $\mathcal{N} = (1, 1)$ the target space can be any real manifold and the potential terms include one real superpotential and one Killing vector term. Supersymmetry also requires that the Lie derivative of the superpotential should be constant along the Killing vectors. Compared to this, first of all the field theories from pp-waves are special in the sense that the target space is always eight dimensional with special holonomy, and secondly we see that one Killing vector contribution is missing both in $\mathcal{N} = (2, 2)$ and $\mathcal{N} = (1, 1)$ solutions. It is conceivable that RR three-forms can provide the missing potential terms on the worldsheet. It is the aim of this paper to show it is indeed the case.

In sec. II we closely follow and repeat the analysis of [1] with nontrivial RR three-forms

as well as five-forms. We identify the spacetime supersymmetry requirements on the Killing vectors from RR three-forms and find they are consistent with the results in [12] as quoted above. For the analysis of $\mathcal{N} = (1, 1)$ solutions we find it more illuminating to assume that the transverse space has Spin(7) holonomy, and find supersymmetry requires the Killing vector to preserve the Spin(7) structure. In sec. III we conclude with a few comments.

II. IIB PP-WAVE SOLUTIONS WITH RAMOND-RAMOND BACKGROUNDS

For IIB supergravity in ten dimensions we follow the conventions of [13], and take the ansatz

$$\begin{aligned} ds^2 &= -2dx^+ dx^- + H(x^i)(dx^+)^2 + ds_8^2, \\ F^{(5)} &= dx^+ \wedge \xi(x^i), \\ F^{(3)} &= dx^+ \wedge \theta(x^i), \end{aligned} \tag{1}$$

where $i = 1, 2 \dots 8$ denote the transverse eight dimensional space with Euclidean signature, $F^{(5)}, F^{(3)}$ are the RR fields and all other fields are set to zero. The Einstein's equation has only one nontrivial component

$$\nabla^2 H = -\frac{2}{3}\xi_{ijkl}\xi^{ijkl} - \frac{1}{2}\theta_{ij}\theta^{ij}, \tag{2}$$

and from other field equations and the Bianchi identities ξ is anti-self-dual, closed and co-closed, and θ is a closed two-form in eight dimensions.

The supersymmetry transformations for the dilatino λ and the gravitino ψ_a are given in terms of a Weyl spinor ϵ ,

$$\delta\lambda = \frac{1}{24}G_{abc}\Gamma^{abc}\epsilon, \tag{3}$$

$$\delta\psi_a = D_a\epsilon - \Omega_a\epsilon - \Lambda_a\epsilon^*, \tag{4}$$

with

$$\begin{aligned} \Omega_a &= -\frac{i}{480}F_{bcdef}^{(5)}\Gamma^{bcdef}\Gamma_a, \\ \Lambda_a &= -\frac{1}{96}(\Gamma_a G_{bcd}\Gamma^{bcd} + 2G_{bcd}\Gamma^{bcd}\Gamma_a). \end{aligned}$$

$a, b, c \dots$ are used to denote the ten dimensional frame indices and the gamma matrices are constants subsequently. G_{abc} is the complexified three-form, and since we have set Neveu-Schwarz (NS) fields to zero G is pure imaginary, i.e. $G = iF^{(3)}$.

Given the ansatz Eq.(1), it is natural to employ the following decomposition of ϵ ,

$$\epsilon = -\frac{1}{2}\Gamma_+\Gamma_-\epsilon - \frac{1}{2}\Gamma_-\Gamma_+\epsilon \equiv \epsilon_+ + \epsilon_-.$$

It is important to note that ϵ_+, ϵ_- have opposite chiralities in $SO(8)$. Now we write down the Killing spinor equations in terms of ϵ_{\pm} . First by setting the variation of the dilatino to zero,

$$\not{\theta}\epsilon_+ = 0, \tag{5}$$

and from $\delta\psi_a = 0$ we get

$$\partial_-\epsilon_+ = \partial_-\epsilon_- = 0, \tag{6}$$

$$\partial_+\epsilon_+ + \frac{i}{8}\not{\theta}\epsilon_+^* = 0, \tag{7}$$

$$\nabla_i\epsilon_+ = 0, \tag{8}$$

$$\partial_+\epsilon_- + \frac{i}{2}\not{\xi}\epsilon_- + \frac{i}{4}\not{\theta}\epsilon_-^* - \frac{1}{4}\Gamma_-\not{\nabla}H\epsilon_+ = 0, \tag{9}$$

$$\nabla_i\epsilon_- - \frac{i}{4}\Gamma_-\not{\xi}\Gamma_\mu\epsilon_+ + \frac{i}{16}\Gamma_-(\Gamma_\mu\not{\theta} - 2\not{\theta}\Gamma_\mu)\epsilon_+^* = 0, \tag{10}$$

where $\not{\xi} = \frac{1}{4!}\xi_{ijkl}\Gamma^{ijkl}$, $\not{\theta} = \frac{1}{2}\theta_{ij}\Gamma^{ij}$. First from Eq.(6) we see that the Killing spinors are independent of x^- .

We find it useful to recall here the interpretation of different Killing spinor solutions given in [1]. The Killing spinor solutions with nonvanishing ϵ_+ are related to dynamical supersymmetries in the light cone worldsheet action when it is rearranged to give a nonlinear sigma model. On the other hand for the other half of Killing spinor components ϵ_+ can be set to zero and we can try to solve the remaining equations of ϵ_- . These Killing spinors are related to the kinematic part of the supersymmetries in the light cone gauge, and in the nonlinear sigma models they are related to the number of free fields. Note that these Killing spinors in general depend on x^+ , and the x^+ dependence of ϵ_+ -nonvanishing Killing spinors can be ignored by exploiting that we are free to superpose two different types of Killing spinor solutions. Since our aim here is to show that RR three-forms give potentials involving Killing vector terms to the string worldsheet action, and in flat spaces the isometries are trivial, we concern ourselves particularly with nontrivial target geometry. We thus look for Killing spinors with nonzero ϵ_+ only.

A. Solutions with four supercharges

In order to have two linearly independent complex spinors with nonvanishing ϵ_+ , we see first from Eq.(8) that the transverse eight-dimensional space should most generally be a Calabi-Yau four-fold. It is thus useful to consider how ξ, θ , originally in **35, 28** of SO(8) are decomposed into SU(4).

$$\mathbf{35} \rightarrow \mathbf{15} + \mathbf{10} + \overline{\mathbf{10}}, \quad (11)$$

$$\mathbf{28} \rightarrow \mathbf{15} + \mathbf{6} + \overline{\mathbf{6}} + \mathbf{1}. \quad (12)$$

When we choose the basis where the Killing spinors satisfy

$$\Gamma_{12}\epsilon_+ = \Gamma_{34}\epsilon_+ = \Gamma_{56}\epsilon_+ = \Gamma_{78}\epsilon_+, \quad (13)$$

it becomes natural to choose the Fock space notations with $\gamma_\mu = \Gamma_{2\mu-1} + i\Gamma_{2\mu}$, $\mu = 1, 2, 3, 4$ and states $|0\rangle, |\tilde{0}\rangle$ satisfying $\gamma^\mu|0\rangle = \gamma^{\bar{\mu}}|\tilde{0}\rangle = 0$.

Now ϵ_+ can be written as

$$\epsilon_+ = \alpha|0\rangle + \zeta|\tilde{0}\rangle, \quad (14)$$

where α, ζ are complex constants, and we used the fact that in Calabi-Yau spaces a gauge choice for the spin connections can be made such that the Killing spinors are constants. Using this basis, it is easy to see from Eq.(5) that only **15**, a traceless (1,1)-form, can be nonzero since they annihilate the Killing spinor ϵ_+ in Eq.(14) for arbitrary α, ζ . When **6** and $\overline{\mathbf{6}}$, holomorphic and anti-holomorphic two-forms of SU(4) respectively, are dual to each other with respect to the holomorphic four-form $\epsilon_{\mu\nu\lambda\rho}$, Eq.(14) can be satisfied with a relation between α, ζ , making the number of worldsheet supercharges reduced to two, i.e. $\mathcal{N} = (1, 1)$. Together with **15**, there are 21 components of θ which now satisfy Eq.(14) and it is better described as **21** of Spin(7) in SO(8). This will be studied in more detail in the next subsection and here we consider the solutions with arbitrary α, ζ .

Using the fact that ϵ_+, ϵ_- have opposite SO(8) chiralities we can write

$$\epsilon_- = \Gamma_-(\beta_{\bar{\mu}}\gamma^{\bar{\mu}}|0\rangle + \delta_{\mu}\gamma^{\mu}|\tilde{0}\rangle). \quad (15)$$

Now it is straightforward to find equations for $\alpha, \zeta, \beta_{\bar{\mu}}, \delta_{\mu}$ from the Killing spinor equations. Following [1] we introduce a holomorphic tensor $\xi_{\mu\nu} \equiv \frac{1}{3!}\xi_{\mu\lambda\rho\sigma}\epsilon^{\lambda\rho\sigma\nu}g_{\bar{\nu}}$ and a hermitian tensor, $\xi_{\mu\bar{\nu}} \equiv \frac{1}{2}g^{\lambda\bar{\lambda}}\xi_{\mu\bar{\nu}\lambda\bar{\lambda}}$.

From Eq.(9), we have

$$2i(-\beta_{\bar{\mu}}\xi^{\bar{\mu}}_{\bar{\nu}} + \delta_{\mu}\xi^{\mu}_{\nu}) - \frac{i}{2}\theta^{\bar{\mu}}_{\bar{\nu}}\delta_{\bar{\mu}} = \frac{\alpha}{4}\partial_{\bar{\nu}}H, \quad (16)$$

$$2i(+\beta_{\bar{\mu}}\xi^{\bar{\mu}}_{\nu} - \delta_{\mu}\xi^{\mu}_{\nu}) + \frac{i}{2}\theta^{\mu}_{\nu}\beta_{\mu} = \frac{\zeta}{4}\partial_{\nu}H, \quad (17)$$

with $\beta_{\mu} \equiv \beta_{\bar{\mu}}^*$, $\delta_{\bar{\mu}} \equiv \delta_{\mu}^*$. Eq.(10) gives

$$\nabla_{\bar{\mu}}\beta_{\bar{\nu}} - i\zeta\xi_{\bar{\mu}\bar{\nu}} = 0, \quad (18)$$

$$\nabla_{\mu}\delta_{\nu} - i\alpha\xi_{\mu\nu} = 0, \quad (19)$$

$$\nabla_{\mu}\beta_{\bar{\nu}} + i\alpha\xi_{\mu\bar{\nu}} + \frac{i}{4}\zeta^*\theta_{\mu\bar{\nu}} = 0, \quad (20)$$

$$\nabla_{\bar{\mu}}\delta_{\nu} + i\zeta\xi_{\bar{\mu}\nu} - \frac{i}{4}\alpha^*\theta_{\mu\bar{\nu}} = 0. \quad (21)$$

Of course apart from the terms involving θ 's the above equations are the same as the ones found in [1], and we can proceed in the same spirit to find the solutions for $\beta_{\bar{\mu}}, \delta_{\mu}$ in terms of arbitrary α, ζ and identify the requirements on ξ, θ from consistency.

By exploiting the properties like closure and anti-self-duality of ξ, θ and considering the integrability conditions we find we can write

$$\xi_{\mu\nu} = 2\nabla_{\mu}\nabla_{\nu}W, \quad (22)$$

$$\xi_{\mu\bar{\nu}} = 2\nabla_{\mu}\nabla_{\bar{\nu}}G_1, \quad (23)$$

$$\theta_{\mu\bar{\nu}} = 8\nabla_{\mu}\nabla_{\bar{\nu}}G_2, \quad (24)$$

with the coefficients chosen for later convenience. W is a holomorphic function and G_1, G_2 are real harmonic functions which serve as the potentials for holomorphic Killing vectors $V_{\mu}^1 = i\nabla_{\mu}G_1$ and $V_{\mu}^2 = i\nabla_{\mu}G_2$. The fact that G_1, G_2 are harmonic is obvious from the group theory consideration that they are **15**, i.e. (1,1)-form which is *traceless*.

The solution for ϵ_{-} is given by

$$\beta_{\bar{\mu}} = 2i(-\alpha\nabla_{\bar{\mu}}G_1 + \zeta\nabla_{\bar{\mu}}\bar{W} + \zeta^*\nabla_{\bar{\mu}}G_2), \quad (25)$$

$$\delta_{\mu} = 2i(-\zeta\nabla_{\mu}G_1 + \alpha\nabla_{\mu}W + \alpha^*\nabla_{\mu}G_2). \quad (26)$$

When they are substituted into Eqs.(16) and (17) we find from consistency

$$\nabla_{\nu}G_2\nabla^{\nu}\nabla_{\mu}G_1 - \nabla_{\nu}G_1\nabla^{\nu}\nabla_{\mu}G_2 = 0,$$

$$\nabla_{\nu}(\nabla^{\nu}G_1\nabla_{\mu}W) = 0,$$

$$\nabla_{\nu}(\nabla^{\nu}G_2\nabla_{\mu}W) = 0,$$

and the complex conjugates. It is obvious what they mean. As promised, the spacetime supersymmetry requires that the two Killing vectors should commute with each other and the Lie derivative of the holomorphic superpotential along the Killing vectors should vanish. This precisely matches the field theory supersymmetry conditions in [12].

Finally the metric is given in terms of

$$H = -32g^{\mu\bar{\nu}}(\nabla_{\mu}G_1\nabla_{\bar{\nu}}G_1 + \nabla_{\mu}G_2\nabla_{\bar{\nu}}G_2 + \nabla_{\mu}W\nabla_{\bar{\nu}}\bar{W}).$$

B. Solutions with two supercharges

The existence of one Killing spinor solution to $\nabla_i\epsilon_+ = 0$ implies that the transverse eight-dimensional space has a reduced holonomy $\text{Spin}(7)$ in general. The Killing spinor which is left invariant under $\text{Spin}(7)$ subgroup of $\text{SO}(8)$ will be denoted as η and is chosen to be real. As it is well-known $\text{Spin}(7)$ holonomy is characterised by the Cayley four-form Ψ which can be constructed from the Killing spinor

$$\Psi_{ijkl} = \eta^T \Gamma_{ijkl} \eta, \quad (27)$$

which is covariantly constant by construction and self-dual in $\text{SO}(8)$ when we take the convention that ξ is anti-self-dual. We assume that η is normalised appropriately and Ψ can take values $0, \pm 1$. In the standard basis the non-zero components of Ψ are given as

$$\begin{aligned} 1 &= \Psi_{1234} = \Psi_{5678} = \Psi_{3478} = \Psi_{2468} = \Psi_{2367} \\ &= \Psi_{1368} = \Psi_{1256} = \Psi_{1357} = \Psi_{1458} = \Psi_{2457} \\ &= \Psi_{1287} = \Psi_{1476} = \Psi_{3465} = \Psi_{2385}. \end{aligned}$$

The basic identity involving Ψ is

$$\Psi_{ijkp} \Psi^{lmnp} = \frac{1}{6} \delta_{[i}^{[l} \delta_j^m \delta_k^{n]} - \frac{1}{4} \Psi_{[ij}^{[lm} \delta_k^{n]}, \quad (28)$$

which will prove useful in verifying the statements in the following discussions.

Now in order to solve the Killing spinor equations involving ξ, θ , we first recall that

$$\mathbf{35}_{asd} \rightarrow \mathbf{35}, \quad (29)$$

$$\mathbf{28} \rightarrow \mathbf{21} + \mathbf{7}. \quad (30)$$

It turns out that **35** of Spin(7) can be alternatively described as a traceless symmetric rank-two tensor when we make use of Ψ . It is straightforward to show that

$$(\Psi \cdot \xi)_{ij} \equiv \Psi_{iklm} \xi_j{}^{klm} \quad (31)$$

is symmetric and traceless.

The decomposition of the adjoint representation Eq.(30) and its implication on the solution of Eq.(5) is rather famous. Projection operators for two-forms in eight dimensions can be explicitly constructed using Ψ , so that

$$\theta_{ij}^\pm = \lambda_\pm \Psi_{ij}{}^{kl} \theta_{kl}^\pm, \quad (32)$$

where θ^+ is **21** with $\lambda^+ = \frac{1}{2}$, while θ^- is **7** with $\lambda^- = -\frac{1}{6}$. This was first considered in [15] as a generalization of the four-dimensional self-dual gauge fields, and extended to nontrivial special holonomy manifolds in [16]. It is customary to call **21** self-dual and **7** anti-self-dual. Since ϵ_+ is proportional to η , the invariance of the dilatino means that we should keep **21** while **7** should be set to zero. This is consistent with what we observed in the previous subsection with transverse Calabi-Yaus. When we demand two complex ϵ_+ spinors we keep only **15** of SU(4), but when **6** and $\bar{\mathbf{6}}$ are nonzero and dual to each other we still have one spinor solution to Eq.(5) $\eta = |0\rangle + |\tilde{0}\rangle$.

Now we take the following ansatz for Killing spinors,

$$\epsilon_+ = \alpha \eta, \quad \epsilon_- = -i \Gamma_- v_i \Gamma^i \eta, \quad (33)$$

where α is a constant and v_i is an unknown nonconstant vector to be determined. From Eq.(10) we have

$$\nabla_i v_j = \frac{\alpha}{4!} (\Psi \cdot \xi)_{ij} + \frac{1}{8} \theta_{ij} \alpha^*. \quad (34)$$

In deriving this and other equations we have chosen the gauge for spin connections which makes η constant, like the discussions about Calabi-Yau four-folds in the last subsection. A proof that it is possible also with exceptional holonomy manifolds can be found for instance in [14].

Since Ψ, ξ are closed, $(\Psi \cdot \xi)_{ij}$ is curl-free. Therefore we can write locally

$$\frac{1}{4!} (\Psi \cdot \xi)_{ij} = \nabla_i \nabla_j U, \quad (35)$$

where U is a real harmonic function since $(\Psi \cdot \xi)_{ij}$ is traceless, and U becomes the real superpotential of $\mathcal{N} = (1, 1)$ nonlinear sigma models in the light cone lagrangian. Then Eq.(34) implies that the gauge potential for θ can be chosen to be a Killing vector, i.e.

$$\frac{1}{8}\theta_{ij} = D_i G_j, \quad D_{(i} G_{j)} = 0. \quad (36)$$

When we substitute

$$v_i = \alpha \nabla_i U + \alpha^* G_i, \quad (37)$$

into Eq.(9) we get a consistency condition on the Killing vector G ,

$$D_i(G^j \nabla_j U) = 0 \quad \text{or} \quad \mathcal{L}_G U = \text{const} \quad (38)$$

which matches with the condition for potential terms of $\mathcal{N} = (1, 1)$ supersymmetric nonlinear sigma models. When we integrate what remains we obtain

$$H = -4(G_i G^i + \nabla_i U \nabla^i U), \quad (39)$$

which serves as the scalar potential of the light cone worldsheet lagrangian.

Before we finish let us point out that an alternative interpretation can be given to the requirement that the Killing vector should be in the adjoint representation **21** of Spin(7). We note that for any Killing vector K ,

$$\begin{aligned} \mathcal{L}_K \Psi &\equiv (di_K + i_K d)\Psi \\ &= \nabla_j K^n \Psi_{nkml} dx^j \wedge dx^k \wedge dx^l \wedge dx^m \\ &= (\nabla_j K^n)^- \Psi_{nkml} dx^j \wedge dx^k \wedge dx^l \wedge dx^m. \end{aligned}$$

In order to get the third line Eq.(28) and Eq.(32) are used. We thus see that the Killing vector G 's being **21** means it preserves the Spin(7) structure, in the sense that the Lie derivative of Ψ vanishes along G .

III. DISCUSSIONS

In this paper we have presented a general class of IIB pp-wave solutions with Ramond-Ramond backgrounds. In the light cone gauge the bosonic string action can be simply read off from the metric and it is obvious we have nonlinear sigma models with eight dimensional

special holonomy manifold target spaces followed by potential terms given by H . Then the worldsheet supersymmetries inherited by the Killing spinors found above dictates how the terms with fermions should be written. In particular the non-vanishing RR fields give fermionic mass terms or Yukawa couplings more generally.

The potential terms of supersymmetric nonlinear sigma models are studied in [12]. The analysis does not make use of the superfield formalism; it started with the most general lagrangian and supersymmetry transformation rules allowed by Lorentz invariance and found the consistency conditions. The potential is given in terms of one holomorphic superpotential and two commuting holomorphic Killing vectors for $\mathcal{N} = (2, 2)$ models and one real superpotential and one Killing vector for $\mathcal{N} = (1, 1)$ solutions. The Lie derivatives of the superpotentials along the Killing vectors are required to be constants. The supergravity analysis presented in [1] and here is found to be consistent with the field theory results, but in general the spacetime supersymmetry is more restrictive. It is particularly distinctive with $\mathcal{N} = (1, 1)$ solutions; the superpotential is a harmonic function and the Killing vector should be self-dual with respect to Spin(7). For $\mathcal{N} = (2, 2)$ solutions the Killing potentials are required to be harmonic.

Perhaps it is useful to consider an example of the pp-wave with nonzero three-form. In flat transverse space mass terms can be given to the worldsheet fields by a holomorphic Killing vector, for instance one can consider a pure RR three-form background such as

$$F_{+12}^{(3)} = F_{+34}^{(3)} = F_{+56}^{(3)} = F_{+78}^{(3)} = m. \quad (40)$$

This solution is already considered in [17] where generic supersymmetric plane-wave solutions with nonvanishing RR fields are studied. This solution in fact preserves 28 spacetime supersymmetries. What is special with this solution is that the worldsheet fields all have the same mass. With bosons it is obvious from the metric and the worldsheet supersymmetry guarantees that fermion masses are the same. In other words, the light cone string spectrum of this background is the same as that of the maximally supersymmetric solution with nontrivial five-form [18]. This of course is a natural consequence of our claim that for $\mathcal{N} = (2, 2)$ solutions the same Killing vector terms on the worldsheet can come from either RR five-forms or three-forms. Our result is also consistent with the observation made in [17] that the plane-wave solutions can be superposed. In this paper we have extended it to nontrivial target geometries and non-constant form fields, i.e. general pp-waves. Although

we expect the degeneracy will be lifted once string interactions are taken into account, it will be interesting if we can find a field theory dual of this solution using the Penrose limit as in [19, 20]. It is not immediately obvious how or whether the solution Eq.(40) can be obtained from an AdS solution as the Penrose limit [17]. It might be also worth mentioning that our results can be used to regularize the orbifolds of solutions found in [17] by replacing the orbifolded part of the target spaces with, e.g., Eguchi-Hanson space.

In [4] pp-wave solutions with nonvanishing NS and RR three-forms are considered and the authors concluded that three-forms cannot induce worldsheet interactions without breaking supersymmetry. Our result does not contradict theirs, since in flat target spaces the Killing vectors can give mass terms at most. It is essentially the target space curvature and the superpotential which make the worldsheet action interacting, but the message of our analysis is that the RR three-forms can be succinctly incorporated into interacting models.

Finally we reckon it is an important enterprise to study the field theoretical properties of the massive nonlinear sigma models found in this paper. They are special in the sense that although manifestly non-conformal they can be embedded into exact superconformal theories. By working out the quantum corrections one might be able to identify the hidden conformal covariance of these massive two dimensional field theories. It should also help rectifying the definition of supersymmetric D-branes in the class of nonlinear sigma models considered in this paper.

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