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# Chern-Simons vs. Yang-Mills gaugings in three dimensions

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**Abstract**

Recently, gauged supergravities in three dimensions with Yang-Mills and Chern-Simons type interactions have been constructed. In this article, we demonstrate that any gauging of Yang-Mills type with semisimple gauge group $G_0$, possibly including extra couplings to massive Chern-Simons vectors, is equivalent on-shell to a pure Chern-Simons type gauging with non-semisimple gauge group $G_0 \ltimes T \subset G$, where T is a certain translation group, and where G is the maximal global symmetry group of the ungauged theory. We discuss several examples.

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# 1 Introduction

As borne out by recent work, gauged supergravities in three spacetime dimensions come in more guises than the corresponding models in dimensions $D \geq 4$. This variety of theories is not least due to the fact that in three dimensions vector and scalar fields are related by duality (see e.g. [1] for a general discussion of such dualities). The on-shell equivalence of scalars and vectors in three dimensions is not only reflected in a much larger choice of gauge groups but also in the (co-)existence of both Chern-Simons (CS) type gaugings and Yang-Mills (YM) type gaugings, depending on which fields carry the propagating bosonic degrees of freedom.

YM type gaugings can be obtained alternatively by direct construction, by torus reduction of gauged supergravities in higher dimensions to three dimensions, or by Kaluza Klein reduction on non-flat internal manifolds to three space-time dimensions (see [2,3,4] for examples of such constructions). Examples of CS type gaugings were first obtained for $N=2$ supergravity with an abelian gauge group [5]. Maximal ($N=16$) and half maximal ($N=8$) gauged supergravities of CS type with various compact and non-compact semisimple gauge groups were constructed in [6] and [7], respectively (the relevant gauge groups are subgroups of $\mathrm{E}_{8(8)}$ for $N=16$ and $\mathrm{SO}(8,n)$ for $N=8$). The CS type supergravities are based on those versions of the ungauged theories, in which all propagating bosonic degrees of freedom reside in the scalar fields, and which therefore exhibit the largest global symmetry group G; the scalar fields parametrizing the coset space manifold G/H. By contrast, YM type gaugings are deformations of a dualized version of the ungauged theory where some of the propagating bosonic degrees of freedom are carried by abelian vector fields. The actual construction of the CS type gauged Lagrangians is greatly facilitated by exploiting the group structure; a simple and universal group-theoretical consistency condition determines all the admissible gauge groups for an arbitrary number of local supersymmetries, as first shown for the $N=16$ and $N=8$ theories [6,7], and subsequently for all other theories with $N<16$ [8]. The direct construction of the YM type theories, on the other hand, seems more involved, because the global symmetry group G is broken to a smaller group $\mathrm{G}' \subset \mathrm{G}$, and furthermore the remaining scalars cannot be assigned to a single coset space any more in general. The relative simplicity of the CS type formulation in comparison with the YM type formulation is evident from inspection of the resulting on-shell equivalent Lagrangians (2.1) and (3.8) below.

In this paper we establish, independently of the number $N$ of local supersymmetries, that YM gaugings in three dimensions are in fact equivalent on-shell to CS type gaugings, in the following sense. The equations of motion of any gauged supergravity of YM type with (semisimple) gauge group $\mathrm{G}_0$ coincide with the equations of motion obtained from a corresponding CS type gauged supergravity with non-semisimple gauge group $\mathrm{G}_0 \ltimes \mathrm{T}_\nu$, where $\mathrm{T}_\nu$ is a group of $\nu = \dim \mathrm{G}_0$ (abelian) translations transforming non-trivially

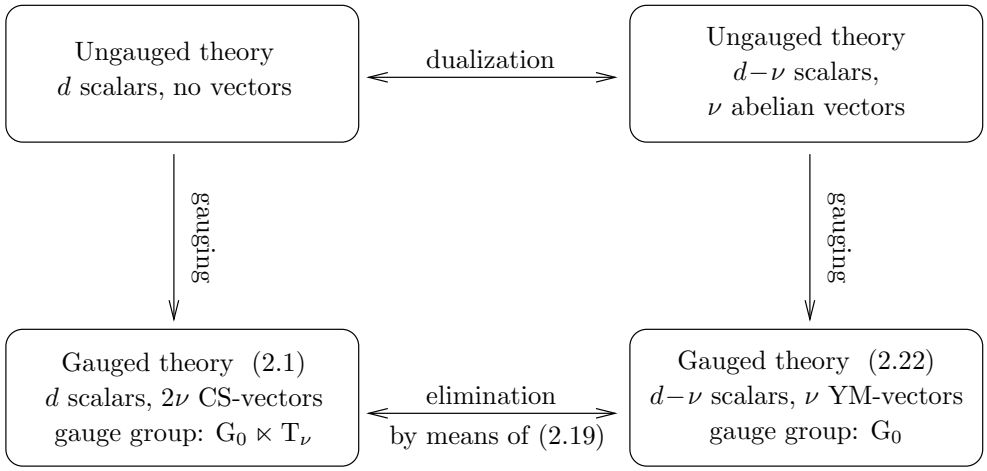


Figure 1: CS and YM gauged supergravity in three dimensions

under $G_0$. Our second main result is the extension of this construction to include couplings of massive CS vector fields to the YM type Lagrangian. On the CS side these correspond to additional nilpotent directions in the gauge group with a particular algebra structure, see (3.1) below.

Generally, the scalar fields of the CS gauged theory (whose number we denote by $d$) parametrize a coset space G/H with H the maximal compact subgroup of G. Here G is the maximal global symmetry of the ungauged theory that can be achieved by dualizing all propagating bosonic degrees of freedom into scalar fields. Obviously, the group we wish to gauge must then satisfy $G_0 \ltimes T_\nu \subset G$. Given a gauge group of this kind, we show that $\nu$ scalar and $\nu$ vector fields may be eliminated together, whereupon the theory turns into a YM gauged theory with $(d-\nu)$ scalars and $\nu$ propagating vector fields gauging the group $G_0$. It is important that the scalar potential is independent of the scalar fields in question and is therefore not affected by this elimination procedure. After the elimination, the $(d-\nu)$ scalars of the YM gauged theory in general can no longer be uniformly described as a coset space; only part of them can be assigned to a (smaller) coset space $G'/H'$ with $G' \subset G$, $H' \subset H$ (of course, the YM gauge group $G_0$ must be contained in $G'$). Allowing also for some bosonic degrees of freedom to be realized as massive CS vectors (see section 3), the following general matching condition for the bosonic degrees of freedom must evidently be satisfied

$$d \;=\; \dim G/H \;=\; \#(\text{scalars}) + \#(\text{YM vectors}) + \#(\text{massive CS vectors})\,. \qquad (1.1)$$

The procedure relating the two types of theories is schematically represented in figure 1.

The necessity of flat directions in the CS gauge group, and hence of non-semisimple gauge groups, for the transmutation of a CS type theory into a YM type theory may be

understood by noting that only those scalar fields on which the scalar potential of the gauged theory does not depend can be dualized away and replaced by YM vector fields. Hence the associated translations along these directions on the target space manifold must be among the local symmetries of the CS type Lagrangian. There exist numerous results on non-semisimple gaugings in $D \geq 4$ [9,10,11,12,13], but non-semisimple gauge groups have not been considered so far in the context of CS gauged supergravities in three dimensions. These will be treated in detail in a forthcoming publication [14], so we here only note that there are two methods to search for them. The first is to directly solve the group-theoretical consistency conditions of [6,7,8]. For the second, one performs an infinite "boost" on a known admissible semisimple gauge group with a suitable non-compact element of the global symmetry group G; this boost must be accompanied by a singular rescaling of the coupling constants, which is adjusted in such a way that the embedding tensor $\Theta$ has a finite limit.

In summary, the set of CS gauged supergravities in three dimensions with non-semisimple gauge groups contains all the known types of YM gauged supergravities. Because there exist numerous admissible semisimple gauge groups, that cannot be related to YM type gaugings, it follows that the CS gauged supergravities encompass a much larger class of models than those of YM type. Combining the classification of ungauged three-dimensional theories [15] with the group-theoretical results on the existence of CS gaugings [6,7,8] thus provides a straightforward route to constructing gauged supergravity theories in three dimensions for a given field content, gauge group and number of supersymmetries.

The paper is organized as follows. In section 2 we provide details of the correspondence outlined in figure 1, exhibiting the YM type gaugings as a particular subclass of CS gauged theories. Section 3 describes the generalization to coupling additional massive vector fields. We close by briefly discussing several examples, including the compactifications of type I, IIA, IIB supergravity in $D = 10$ on $\mathrm{AdS}_3 \times S^7$ and the six-dimensional supergravity on $\mathrm{AdS}_3 \times S^3$.

# 2 Chern-Simons vs. Yang-Mills gauging

The bosonic Lagrangian of a CS gauged supergravity theory in three dimensions is always of the form (see [6,7] for our conventions and notations; we use the metric $(+--)$)

$$e^{-1}\mathcal{L} \;=\; -\tfrac{1}{4}R + e^{-1}g\mathcal{L}_{\mathrm{CS}} + \tfrac{1}{4}g^{\mu\nu}\,\mathcal{P}^A_\mu\mathcal{P}^A_\nu - W + \ldots\ . \tag{2.1}$$

The dots stand for fermionic terms which we will here ignore because they are not relevant for the argument we are going to present; see however [6,7,8] for further details concerning

the fermionic Lagrangian and the supersymmetry variations. The first term in (2.1) is just the usual Einstein term, while the second is the CS Lagrangian

$$\mathcal{L}_{\rm CS} \;=\; \tfrac{1}{4}\varepsilon^{\mu\nu\rho} B_\mu^{\mathcal{M}}\, \Theta_{\mathcal{M}\mathcal{N}}\, \big(\partial_\nu B_\rho^{\mathcal{N}} + \tfrac{1}{3} g\, f^{\mathcal{N}\mathcal{P}}{}_{\mathcal{L}}\, \Theta_{\mathcal{P}\mathcal{K}}\, B_\nu^{\mathcal{K}} B_\rho^{\mathcal{L}}\big) \;. \tag{2.2}$$

with the constant symmetric embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$, which characterizes the CS gauge group, see (2.3) below. Note that $\mathcal{L}_{\rm CS}$ comes with a factor $g$, the gauge coupling constant. $g \to 0$ describes the (smooth) limit to the ungauged theory.

To explain the remaining two terms in (2.1) we recall that the $d$ scalar fields parametrize a coset space G/H where H is the maximal compact subgroup of G. Of course, the choice of possible coset spaces depends on the number $N$ of local supersymmetries and becomes more and more restricted with increasing $N$ [15]. Explicitly, the scalar fields are described by a group valued matrix $\mathcal{S} \in$ G such that the current

$$\mathcal{Q}_\mu + \mathcal{P}_\mu \;\equiv\; \mathcal{S}^{-1}\big(\partial_\mu + g\, \Theta_{\mathcal{M}\mathcal{N}}\, B_\mu^{\mathcal{M}}\, t^{\mathcal{N}}\big)\, \mathcal{S} \;, \tag{2.3}$$

takes values in the associated Lie algebra $\mathfrak{g}$, with a suitably normalized basis $\{t^{\mathcal{M}}\}$, where $\mathcal{M}, \mathcal{N} = 1, \ldots, \dim$ G. The quantity $\mathcal{P}_\mu = \mathcal{P}_\mu^A t^A$ appearing in the kinetic term of the scalar fields in the Lagrangian (2.1) is the projection of this current onto the noncompact part of $\mathfrak{g}$, which is spanned by the generators $\{t^A\}$ with labels $A, B, \ldots$. The compact part $\mathcal{Q}_\mu$, on the other hand, serves as a (composite) connection for the maximal compact subgroup H and governs the scalar-fermion couplings of the ungauged theory. The embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ describes the coupling of the vector fields to the generators of the action of the symmetry group, hence the embedding of the CS gauge group into G.

The potential $W$ is likewise a function of the scalar fields $\mathcal{S}$. More specifically, it is a quadratic polynomial in the entries of the $T$-tensor $T_{\mathcal{A}\mathcal{B}}$ which in turn is given in terms of the matrix $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ representing the group element $\mathcal{S}$ in the adjoint representation:

$$T_{\mathcal{A}\mathcal{B}} \;\equiv\; \Theta_{\mathcal{M}\mathcal{N}}\, \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}} \mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \;, \qquad \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\, t^{\mathcal{A}} \;\equiv\; \mathcal{S}^{-1} t^{\mathcal{M}} \mathcal{S} \;. \tag{2.4}$$

The exact dependence of $W$ on $T_{\mathcal{A}\mathcal{B}}$ as well as the possible gauge groups and their embedding matrices $\Theta_{\mathcal{M}\mathcal{N}}$ can be found in [6,7,8] and is completely determined by supersymmetry.

The above Lagrangian is invariant under the (infinitesimal) gauge transformations

$$\delta\mathcal{S} \;=\; -g\, \Theta_{\mathcal{M}\mathcal{N}}\, \Lambda^{\mathcal{M}}\, t^{\mathcal{N}}\, \mathcal{S} \;, \qquad \delta B_\mu^{\mathcal{M}} \;=\; \partial_\mu \Lambda^{\mathcal{M}} + g\, f^{\mathcal{M}\mathcal{P}}{}_{\mathcal{L}}\, \Theta_{\mathcal{P}\mathcal{K}}\, B_\mu^{\mathcal{K}}\, \Lambda^{\mathcal{L}} \;. \tag{2.5}$$

Variation of the Lagrangian (2.1) with respect to the vector fields gives rise to the first order duality equations (again omitting fermionic contributions):

$$\Theta_{\mathcal{M}\mathcal{N}}\, \mathcal{B}_{\mu\nu}^{\mathcal{N}} \;\equiv\; \Theta_{\mathcal{M}\mathcal{N}}\, \big(2\partial_{[\mu} B_{\nu]}^{\mathcal{N}} + g\, \Theta_{\mathcal{K}\mathcal{P}}\, f^{\mathcal{N}\mathcal{K}}{}_{\mathcal{Q}}\, B_\mu^{\mathcal{P}} B_\nu^{\mathcal{Q}}\big) \;=\; e\varepsilon_{\mu\nu\rho}\, \Theta_{\mathcal{M}\mathcal{N}}\, \mathcal{V}^{\mathcal{N}}{}_A\, \mathcal{P}^{A\rho} \;. \tag{2.6}$$

To proceed we now assume a very particular type of gauge group, namely a non-semisimple group of the form $\mathrm{G}_0 \ltimes \mathrm{T}_\nu \subset \mathrm{G}$, where $\mathrm{T}_\nu$ is a set of $\nu = \dim \mathrm{G}_0$ translations transforming in the adjoint representation of $\mathrm{G}_0$. A systematic discussion and representative examples of such gaugings will be given in [14]. Denoting the generators of $\mathfrak{g}_0 \equiv \mathrm{Lie}\,\mathrm{G}_0$ by $\{\mathcal{J}^m \equiv t^m\}$ and those of $\mathfrak{t}_\nu \equiv \mathrm{Lie}\,\mathrm{T}_\nu$ by $\{\mathcal{T}^{\underline{m}} \equiv t^{\underline{m}}\}$, respectively, where both $m$ and $\underline{m}$ range over $1, \ldots, \nu \equiv \dim \mathrm{G}_0$, we have the commutation relations

$$[\mathcal{J}^m, \mathcal{J}^n] \;=\; f^{mn}{}_k\, \mathcal{J}^k \;, \qquad [\mathcal{J}^m, \mathcal{T}^{\underline{n}}] \;=\; f^{mn}{}_k\, \mathcal{T}^{\underline{k}} \;, \qquad [\mathcal{T}^{\underline{m}}, \mathcal{T}^{\underline{n}}] \;=\; 0 \;, \tag{2.7}$$

where $f^{mn}{}_k$ denote the structure constants of $\mathrm{G}_0$, so the translation generators transform in the adjoint of $\mathrm{G}_0$. It can now be shown that for this particular choice of gauge group, a consistent gauging is possible only if the embedding tensor $\Theta_{\mathcal{MN}}$ is of the form

$$g\,\Theta_{m\underline{n}} \;=\; g\,\Theta_{\underline{m}n} \;=\; g_1\,\eta_{mn} \;, \qquad g\,\Theta_{\underline{mn}} \;=\; g_2\,\eta_{mn} \;, \tag{2.8}$$

with all remaining components equal to zero. Here $\eta_{mn}$ is the Cartan-Killing form on $\mathfrak{g}_0$. The ansatz (2.8) is uniquely fixed by demanding invariance of $\Theta_{\mathcal{MN}}$ under the gauge group (2.7). In particular, this invariance requires $\Theta_{mn} = 0$; happily, this is also the condition needed for our elimination procedure to work. The real constants $g_1$, $g_2$ in general cannot be freely chosen, but are determined as a function of the one free gauge coupling constant $g$ by how $\mathrm{G}_0 \ltimes \mathrm{T}_\nu$ is embedded in G, and by the fact that the embedding tensor (2.8) must satisfy the group-theoretical identities of [6,7,8] to ensure compatibility with supersymmetry. After the elimination procedure we are about to describe, the coupling $g_1$ will play the role of the YM gauge coupling constant while $g_2$ corresponds to an inequivalent deformation of the theory by an additional CS term.

We denote the vector fields associated with $\mathrm{G}_0$ and $\mathrm{T}_\nu$ by $C^m_\mu \equiv B^m_\mu$ and $A^m_\mu \equiv B^{\underline{m}}_\mu$, respectively. Their transformation properties follow from (2.5):

$$\begin{aligned} \delta A^m_\mu \;&=\; \partial_\mu \Lambda^{\underline{m}} + g_1\, f^m{}_{kl}\, A^k_\mu\, \Lambda^{\underline{l}} \;, \\ \delta C^m_\mu \;&=\; \partial_\mu \Lambda^m + g_1 f^m{}_{kl}\, C^k_\mu \Lambda^{\underline{l}} + f^m{}_{kl}\, A^k_\mu (g_1 \Lambda^l + g_2 \Lambda^{\underline{l}}) \;, \end{aligned} \tag{2.9}$$

where $f^m{}_{kl} \equiv \eta_{nk} f^{mn}{}_l$. The associated field strengths can be read off from (2.6); they are

$$\begin{aligned} \mathcal{A}^m_{\mu\nu} \;\equiv\; \mathcal{B}^{\underline{m}}_{\mu\nu} \;&=\; \partial_\mu A^m_\nu - \partial_\nu A^m_\mu + g_1\, f^m{}_{kl}\, A^k_\mu A^l_\nu \;, \\ \mathcal{C}^m_{\mu\nu} \;\equiv\; \mathcal{B}^m_{\mu\nu} \;&=\; \partial_\mu C^m_\nu - \partial_\nu C^m_\mu + 2\, g_1\, f^m{}_{kl}\, C^k_{[\mu} A^l_{\nu]} + g_2\, f^m{}_{kl}\, A^k_\mu A^l_\nu \;. \end{aligned} \tag{2.10}$$

Next we split off the translation part from the scalar field matrix $\mathcal{S}$

$$\mathcal{S}(\phi, \tilde{\phi}) \;\equiv\; e^{\phi_m\, \mathcal{T}^{\underline{m}}}\, \tilde{\mathcal{S}}(\tilde{\phi}) \;, \tag{2.11}$$

in terms of $\nu$ scalars $\phi_m$ associated with the translation generators $\{\mathcal{T}^{\underline{m}}\}$, and the remaining scalar fields $\tilde{\phi}$ some of which coordinatize the YM coset manifold G′/H′. What is important is that the matrix $\tilde{\mathcal{S}}$ no longer depends on the translational degrees of freedom $\phi_m$. Defining the modified field strength

$$\tilde{\mathcal{C}}^m_{\mu\nu} \;\equiv\; \mathcal{C}^m_{\mu\nu} - f^{mn}{}_k\,\phi_n \mathcal{A}^k_{\mu\nu}\;, \tag{2.12}$$

we have the transformation properties

$$\begin{aligned} \delta\mathcal{A}^m_{\mu\nu} &= g_1\, f^m{}_{kl}\,\mathcal{A}^k_{\mu\nu}\,\Lambda^{\underline{l}}\,, \qquad \delta\tilde{\mathcal{S}} \;=\; -\,g_1\eta_{mn}\Lambda^{\underline{m}}\,\mathcal{J}^n\tilde{\mathcal{S}}\;, \\ \delta\tilde{\mathcal{C}}^m_{\mu\nu} &= g_1\, f^m{}_{kl}\,\tilde{\mathcal{C}}^k_{\mu\nu}\,\Lambda^{\underline{l}}\,, \qquad \delta\phi_m \;=\; -\,g_1\eta_{mn}\Lambda^n - \left(g_2\eta_{mn} + g_1 f^l{}_{mn}\phi_l\right)\Lambda^{\underline{n}}\;, \end{aligned} \tag{2.13}$$

from (2.5). Note that the fields $\phi_m$ and $C^m_\mu$ are the only ones to transform with the translation parameters $\Lambda^m$, and that $\phi_m$ is shifted under such transformation and hence could be gauged away altogether. Accordingly, we define the quantities

$$\begin{aligned} \tilde{\mathcal{V}}^{\mathcal{M}}{}_{\mathcal{A}}\,t^{\mathcal{A}} &\equiv \tilde{\mathcal{S}}^{-1}t^{\mathcal{M}}\tilde{\mathcal{S}}\;, \\ \tilde{\mathcal{Q}}_\mu + \tilde{\mathcal{P}}_\mu &\equiv \tilde{\mathcal{S}}^{-1}\left(\partial_\mu + g_1\eta_{mn}\,A^m_\mu\,\mathcal{J}^n\right)\tilde{\mathcal{S}}\;, \end{aligned} \tag{2.14}$$

which do not depend on the $\phi_m$ either. It is then easy to check that, for $\mathcal{M} = m, \underline{m}$

$$\tilde{\mathcal{V}}^{\underline{m}}{}_{\mathcal{A}} = \mathcal{V}^{\underline{m}}{}_{\mathcal{A}}\;, \qquad \tilde{\mathcal{V}}^m{}_{\mathcal{A}} = \mathcal{V}^m{}_{\mathcal{A}} - f^{mn}{}_k\,\phi_n\,\mathcal{V}^{\underline{k}}{}_{\mathcal{A}}\;. \tag{2.15}$$

As expected, the $T$-tensor (2.4) does not depend on $\phi_m$. This follows from the fact that by construction it is gauge invariant, and more specifically invariant under the (local) $\Lambda^m$ translations on $\phi_m$, see (2.13), but it is also easy to verify directly that

$$T_{\mathcal{AB}} \;=\; \Theta_{\mathcal{MN}}\,\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}} \;=\; \Theta_{\mathcal{MN}}\,\tilde{\mathcal{V}}^{\mathcal{M}}{}_{\mathcal{A}}\tilde{\mathcal{V}}^{\mathcal{N}}{}_{\mathcal{B}}\;, \tag{2.16}$$

with $\Theta_{\mathcal{MN}}$ from (2.8). Consequently, the scalar potential $W$ in (2.1) does not depend on $\phi_m$. After a little algebra, the current $\mathcal{Q}_\mu + \mathcal{P}_\mu$ can be rewritten as

$$\begin{aligned} \mathcal{Q}_\mu + \mathcal{P}_\mu &= \tilde{\mathcal{Q}}_\mu + \tilde{\mathcal{P}}_\mu + \left(\partial_\mu\phi_m + \eta_{mn}\left(g_1 C^n_\mu + g_2 A^n_\mu\right) + g_1 f^k{}_{mn}\,A^n_\mu\,\phi_k\right)\tilde{\mathcal{V}}^{\underline{m}}{}_{\mathcal{A}}t^{\mathcal{A}} \\ &\equiv \tilde{\mathcal{Q}}_\mu + \tilde{\mathcal{P}}_\mu + D_\mu\phi_m\,\tilde{\mathcal{V}}^{\underline{m}}{}_{\mathcal{A}}t^{\mathcal{A}}\;. \end{aligned} \tag{2.17}$$

with the definition of the covariant derivative in accordance with (2.13). The first order duality equations (2.6) for the gauge group (2.7), (2.8), take the form

$$\begin{aligned} \mathcal{A}^m_{\mu\nu} &= e\varepsilon_{\mu\nu\rho}\,\tilde{\mathcal{V}}^{\underline{m}}{}_A\left(\tilde{\mathcal{P}}^{A\rho} + \tilde{\mathcal{V}}^{\underline{n}}{}_A\,D^\rho\phi_n\right)\;, \\ \tilde{\mathcal{C}}^m_{\mu\nu} &= e\varepsilon_{\mu\nu\rho}\,\tilde{\mathcal{V}}^m{}_A\left(\tilde{\mathcal{P}}^{A\rho} + \tilde{\mathcal{V}}^{\underline{n}}{}_A\,D^\rho\phi_n\right)\;, \end{aligned} \tag{2.18}$$

with the modified field strength $\tilde{\mathcal{C}}^m_{\mu\nu}$ from (2.12). Hence they may be formulated exclusively in terms of objects that are invariant under $\Lambda^m$ and transform covariantly under $\Lambda^{\underline{m}}$.

Our aim is now to eliminate all $\phi_m$ dependence from the equations of motion. To this end, assume the matrix $M^{mn} \equiv \tilde{\mathcal{V}}^{\underline{m}}{}_A \tilde{\mathcal{V}}^{\underline{n}}{}_A$ to be invertible with inverse $M_{mn}$. This allows to solve equations (2.18) for $D_\mu \phi_m$ and $\tilde{\mathcal{C}}^m_{\mu\nu}$

$$\begin{aligned} e\varepsilon_{\mu\nu\rho}\, D^\rho \phi_m &= M_{mn}\, \mathcal{A}^n_{\mu\nu} - e\varepsilon_{\mu\nu\rho}\, M_{mn}\, \tilde{\mathcal{V}}^{\underline{n}}{}_A\, \tilde{\mathcal{P}}^{A\rho}\ , \\ \tilde{\mathcal{C}}^m_{\mu\nu} &= e\varepsilon_{\mu\nu\rho} \left( \tilde{\mathcal{V}}^m{}_A - \tilde{\mathcal{V}}^m{}_B \tilde{\mathcal{V}}^{\underline{k}}{}_B\, M_{kl}\, \tilde{\mathcal{V}}^{\underline{l}}{}_A \right) \tilde{\mathcal{P}}^{A\rho} + \tilde{\mathcal{V}}^m{}_A \tilde{\mathcal{V}}^{\underline{k}}{}_A\, M_{kn}\, \mathcal{A}^n_{\mu\nu}\ . \end{aligned} \tag{2.19}$$

These equations can now be used to eliminate both $\phi_m$ and $C^m_\mu$ from the theory. Solubility of the first equation in (2.19) implies an integrability condition on the r.h.s. which is straightforwardly computed using

$$[D_\mu, D_\nu]\, \phi_m = \eta_{mn} \left( g_1 \tilde{\mathcal{C}}^n_{\mu\nu} + g_2 \mathcal{A}^n_{\mu\nu} \right)\ , \tag{2.20}$$

and leads to the following second order field equation for the vector fields $A^m_\mu$

$$\begin{aligned} D^\nu \left( M_{mn} \mathcal{A}^n_{\mu\nu} \right) &= e\varepsilon_{\mu\nu\rho}\, D^\nu \left( M_{mn} \tilde{\mathcal{V}}^{\underline{n}}{}_A \tilde{\mathcal{P}}^{A\rho} \right) + g_1\, \eta_{mn} \tilde{\mathcal{V}}^n{}_A \left( \delta_{AB} - \tilde{\mathcal{V}}^{\underline{k}}{}_A M_{kl} \tilde{\mathcal{V}}^{\underline{l}}{}_B \right) \tilde{\mathcal{P}}^B_\mu \\ &\quad + \tfrac{1}{2} e\varepsilon_{\mu\nu\rho} \left( g_2 \eta_{mn} + g_1 \eta_{mk} \tilde{\mathcal{V}}^k{}_A \tilde{\mathcal{V}}^{\underline{l}}{}_A M_{ln} \right) \mathcal{A}^{n\nu\rho}\ . \end{aligned} \tag{2.21}$$

Hence this field equation is equivalent to the set of first order equations (2.18) while the fields $\phi_m$ and $C^m_\mu$ are completely decoupled and may be restored from (2.19). (Integrability of the second equation in (2.19) in addition requires also part of the scalar field equations.) Equation (2.21) may be derived from the Lagrangian

$$\begin{aligned} e^{-1}\tilde{\mathcal{L}} &= -\tfrac{1}{4} R - e^{-1} g_2 \tilde{\mathcal{L}}_{\rm CS}(A) - \tfrac{1}{8}\, M_{mn}\, \mathcal{A}^{m\mu\nu} \mathcal{A}^n_{\mu\nu} + \tfrac{1}{4} g^{\mu\nu}\, G_{AB}\, \tilde{\mathcal{P}}^A_\mu \tilde{\mathcal{P}}^B_\nu - W \\ &\quad + \tfrac{1}{4} e^{-1}\, \varepsilon^{\mu\nu\rho}\, M_{mn}\, \tilde{\mathcal{V}}^{\underline{n}}{}_A\, \mathcal{A}^m_{\mu\nu}\, \tilde{\mathcal{P}}^A_\rho\ , \end{aligned} \tag{2.22}$$

with

$$\begin{aligned} G_{AB} &\equiv \delta_{AB} - \tilde{\mathcal{V}}^{\underline{m}}{}_A M_{mn} \tilde{\mathcal{V}}^{\underline{n}}{}_B\ , \qquad M_{mn} \equiv (\tilde{\mathcal{V}}^{\underline{m}}{}_A \tilde{\mathcal{V}}^{\underline{n}}{}_A)^{-1}\ , \\ \tilde{\mathcal{L}}_{\rm CS}(A) &= \tfrac{1}{4}\, \varepsilon^{\mu\nu\rho} A^m_\mu\, \eta_{mn} \left( \partial_\nu A^n_\rho + \tfrac{1}{3} g_1 f^n{}_{kl}\, A^k_\nu A^l_\rho \right)\ . \end{aligned} \tag{2.23}$$

It requires a little more work to show that the scalar field equations derived from (2.22) reproduce those descending from (2.1) upon eliminating $D_\mu \phi_m$ by means of (2.19). Note that the metric $G_{AB}$ on the scalar target space is degenerate along the $\nu$ directions $\tilde{\mathcal{V}}^{\underline{m}}{}_A$. This just means that the elimination of the scalar fields $\phi_m$ has effectively reduced the dimension of the scalar manifold in (2.22) by $\nu$. The Chern-Simons term in (2.22) collects

only part of the terms from the corresponding term in (2.1). The scalar potentials $W$ in (2.1) and (2.22) coincide. The resulting YM type theory thus has $d-\nu$ scalar fields and $\nu$ propagating Yang-Mills vectors. The residual gauge group $\mathrm{G}_0$ acts canonically as

$$\delta\tilde{\mathcal{S}} \;=\; -\,g_1\eta_{mn}\Lambda^{\underline{m}}\,\mathcal{J}^n\tilde{\mathcal{S}}\;, \qquad \delta A^m_\mu \;=\; \partial_\mu\Lambda^{\underline{m}} + g_1\, f^m{}_{kl}\, A^k_\mu\,\Lambda^{\underline{l}}\,. \tag{2.24}$$

The fermionic part of the Lagrangian (2.22) as well as the supersymmetry transformation rules may be directly obtained from those of (2.1) upon eliminating $D_\mu\phi_m$ and $\mathcal{C}^m_{\mu\nu}$ by means of (2.19).

In the (smooth) limit $g_1 \to 0$, $g_2 \to 0$, the Lagrangian (2.22) reduces to the ungauged theory with $d-\nu$ scalar fields and $\nu$ abelian vectors. The metrics $M_{mn}$ and $G_{AB}$ in the kinetic terms remain unchanged in this limit. As anticipated above, the two constants $g_1$ and $g_2$ from (2.8) in (2.22) correspond to deformations of the ungauged theory of two different types: The constant $g_1$ arises as gauge coupling constant of the gauge group $\mathrm{G}_0$ while $g_2$ appears as proportionality factor of the Chern-Simons term which may be viewed as another deformation of the ungauged YM theory. In addition, both constants appear in the $T$-tensor and thereby in the scalar potential $W$ which is a quadratic polynomial in $g_1$, $g_2$. Let us however stress once more that in general $g_1$ and $g_2$ are not free parameters in (2.8) but related by some consistency relation implied by supersymmetry. In the degenerate case $g_1 = 0$, the dualized theory (2.22) appears with an abelian gauge group $U(1)^\nu$ which does not act on the scalar fields, and is deformed only by the presence of the Chern-Simons term and a scalar potential. This has been worked out in [16] in the context of the $N = 2$ theories describing the Calabi-Yau fourfold compactifications of M-theory with flux [17].

# 3 Coupling massive vector fields

The above elimination procedure can be extended in a straightforward fashion to include couplings to massive vector fields in the framework of pure CS gaugings (2.1). As we will now explain these massive vector fields correspond to additional nilpotent directions in the CS gauge group.

To this aim we consider an extension of the CS gauge group $\mathrm{G}_0 \ltimes \mathrm{T}_\nu$ by a set $\hat{\mathrm{T}}_p$ of $p$ nilpotent generators transforming in some representation of $\mathrm{G}_0$ and closing into $\mathrm{T}_\nu$. Accordingly, the Lie algebra relations (2.7) are extended by

$$\Big[\mathcal{J}^m, \hat{\mathcal{T}}^\alpha\Big] \;=\; t^{m\alpha}{}_\beta\,\hat{\mathcal{T}}^\beta\;, \qquad \Big[\hat{\mathcal{T}}^\alpha, \hat{\mathcal{T}}^\beta\Big] \;=\; t^{\alpha\beta}{}_m\,\mathcal{T}^{\underline{m}}\;, \qquad \Big[\mathcal{T}^{\underline{m}}, \hat{\mathcal{T}}^\alpha\Big] \;=\; 0\;, \tag{3.1}$$

while the embedding tensor $\Theta_{\mathcal{M}\mathcal{N}}$ has the additional components

$$g\,\Theta_{\alpha\beta} \;=\; g_1\,\kappa_{\alpha\beta}\;, \tag{3.2}$$

with the structure constants in (3.1) and the symmetric tensor $\kappa_{\alpha\beta}$ being related by $\eta_{mn}t^{n\alpha}{}_{\beta} = \kappa_{\beta\gamma}t^{\alpha\gamma}{}_{m}$ in order to have $\Theta_{\mathcal{MN}}$ invariant under the gauge group. We denote the group corresponding to (2.7), (3.1) by $\mathrm{G}_0 \ltimes (\hat{\mathrm{T}}_p, \mathrm{T}_\nu)$. In addition to the vector fields (2.9) there are now also vector fields $B^\alpha_\mu$ corresponding to the nilpotent generators $\hat{\mathcal{T}}^\alpha$. Similar to (2.11), we may also split off the scalars associated with the generators $\hat{\mathcal{T}}^\alpha$ from the scalar field matrix $\mathcal{S}$

$$\mathcal{S}(\phi, \tilde{\phi}) \;\equiv\; e^{\phi_m \mathcal{T}^{\underline{m}}} e^{\phi_\alpha \hat{\mathcal{T}}^\alpha}\, \tilde{\mathcal{S}}(\tilde{\phi})\ . \tag{3.3}$$

Explicitly, the individual parts of (3.3) transform under gauge transformations as

$$\begin{aligned} \tilde{\mathcal{S}} &= -g_1\eta_{mn}\Lambda^{\underline{m}}\mathcal{J}^n\tilde{\mathcal{S}}\ , \qquad \delta\phi_\alpha \;=\; -\,g_1\kappa_{\alpha\beta}\Lambda^\beta + g_1 t_{m\alpha}{}^{\beta}\phi_\beta\,\Lambda^{\underline{m}}\ , \\ \delta\phi_m &= -\eta_{mn}g_1\Lambda^n - \tfrac{1}{2}g_1 t_{m\alpha}{}^{\beta}\phi_\beta\Lambda^\alpha - \big(g_2\eta_{mn} + g_1 f^l{}_{mn}\phi_l - \tfrac{1}{2}g_1 t^{\alpha\gamma}{}_m t^{\beta}{}_{\gamma n}\,\phi_\alpha\phi_\beta\big)\,\Lambda^{\underline{n}}\ , \end{aligned} \tag{3.4}$$

and correspondingly this defines their covariant derivatives. The elimination procedure described in the last section may now straightforwardly be generalized to this setting. We refrain from giving details of the computation and just note that (2.17) generalizes to

$$\mathcal{P}^A_\mu \;=\; \tilde{\mathcal{P}}^A_\mu + \tilde{\mathcal{V}}^\alpha{}_A\, D_\mu\phi_\alpha + \tilde{\mathcal{V}}^{\underline{m}}{}_A\,(D_\mu\phi_m - \tfrac{1}{2}t^{\alpha\beta}{}_m\phi_\alpha D_\mu\phi_\beta)\ , \tag{3.5}$$

with $\tilde{\mathcal{V}}$, $\tilde{\mathcal{P}}$ defined as in (2.14). The first order duality equations (2.6) then allow to express $D_\mu\phi_m$ and the field strength $\mathcal{C}^m_{\mu\nu}$ in terms of the remaining fields, thereby eliminating them from the theory. As above, integrability of these equations implies a second order field equation for the vector fields $A^m_\mu$ that generalizes (2.21). In addition, we remain with a first order equation for the vector fields $B^\alpha_\mu$ associated with the nilpotent generators $\hat{\mathcal{T}}^\alpha$

$$\begin{aligned} \tilde{\mathcal{B}}^\alpha_{\mu\nu} &\equiv D_\mu B^\alpha_\nu - D_\nu B^\alpha_\mu - t^{\alpha\beta}{}_m\,\phi_\beta\,A^m_{\mu\nu} \\ &= \tilde{\mathcal{V}}^\alpha{}_A\tilde{\mathcal{V}}^{\underline{m}}{}_A\,M_{mn}\,A^n_{\mu\nu} + e\epsilon_{\mu\nu\rho}\tilde{\mathcal{V}}^\alpha{}_A\,G_{AB}\,(\tilde{\mathcal{P}}^{B\rho} + \tilde{\mathcal{V}}^\beta{}_B\,D^\rho\phi_\beta)\ , \end{aligned} \tag{3.6}$$

with $G_{AB}$ from (2.23), and where left and right hand side are separately invariant under gauge transformations with parameters $\Lambda^\alpha$, $\Lambda^m$. Finally, the entire set of field equations may be derived from the Lagrangian

$$\begin{aligned} e^{-1}\tilde{\mathcal{L}} \;=\;& -\tfrac{1}{4}R + \tfrac{1}{4}G_{AB}\,(\tilde{\mathcal{P}}^A_\mu + \tilde{\mathcal{V}}^\alpha{}_A\,D_\mu\phi_\alpha)(\tilde{\mathcal{P}}^{B\mu} + \tilde{\mathcal{V}}^\beta{}_B\,D^\mu\phi_\beta) - \tfrac{1}{8}\,M_{mn}\,\mathcal{A}^m_{\mu\nu}\mathcal{A}^{n\mu\nu} \\ &+ \tfrac{1}{4}e^{-1}\,\varepsilon^{\mu\nu\rho}\,M_{mn}\tilde{\mathcal{V}}^{\underline{n}}{}_A\,\mathcal{A}^m_{\mu\nu}\,(\tilde{\mathcal{P}}^A_\rho + \tilde{\mathcal{V}}^\alpha{}_A\,D_\rho\phi_\alpha) - \tfrac{1}{8}e^{-1}\,\varepsilon^{\mu\nu\rho}\,\tilde{\mathcal{B}}^\alpha_{\mu\nu}\,D_\rho\phi_\alpha \\ &+ e^{-1}g_2\tilde{\mathcal{L}}_{\mathrm{CS}}(A) - W\ , \end{aligned} \tag{3.7}$$

which generalizes (2.22) by including a coupling to the additional vector fields $B^\alpha_\mu$ which arise with the first order field equation (3.6). The Lagrangian (3.7) has the additional

gauge symmetry $\Lambda^\alpha$ exclusively acting on $B^\alpha_\mu$ and shifting $\phi_\alpha$. In particular, this symmetry may be gauge fixed by imposing $\phi_\alpha = 0$ which leads to the Lagrangian

$$\begin{aligned} e^{-1}\tilde{\mathcal{L}} &= -\tfrac{1}{4}R + \tfrac{1}{4}G_{AB}\,\tilde{\mathcal{P}}^A_\mu\tilde{\mathcal{P}}^{B\mu} - \tfrac{1}{8}\,M_{mn}\,\mathcal{A}^m_{\mu\nu}\mathcal{A}^{n\mu\nu} + \tfrac{1}{4}e^{-1}\varepsilon^{\mu\nu\rho}\,M_{mn}\tilde{\mathcal{V}}^{\underline{n}}{}_A\,\mathcal{A}^m_{\mu\nu}\,\tilde{\mathcal{P}}^A_\rho \\ &\quad + \tfrac{1}{4}\,g_1^2\,\tilde{\mathcal{V}}_{\alpha A}\,G_{AB}\,\tilde{\mathcal{V}}_{\beta B}\,B^\alpha_\mu B^{\beta\mu} + \tfrac{1}{4}g_1\,e^{-1}\varepsilon^{\mu\nu\rho}\,M_{mn}\tilde{\mathcal{V}}^{\underline{n}}{}_A\tilde{\mathcal{V}}_{\alpha A}\;B^\alpha_\rho\,\mathcal{A}^m_{\mu\nu} \\ &\quad + \tfrac{1}{2}g_1\,G_{AB}\tilde{\mathcal{V}}_{\alpha B}\,\tilde{\mathcal{P}}^A_\mu\,B^{\alpha\mu} - \tfrac{1}{8}g_1\,e^{-1}\varepsilon^{\mu\nu\rho}\,B^\alpha_\rho\,\kappa_{\alpha\beta}\,\mathcal{B}^\beta_{\mu\nu} + e^{-1}g_2\tilde{\mathcal{L}}_{\rm CS}(A) - W\ . \end{aligned} \tag{3.8}$$

This is a theory describing $(d-\nu-p)$ scalar fields combined in the matrix $\tilde{\mathcal{S}}$, together with $p$ massive CS vector fields $B^\alpha_\mu$, and $\nu$ YM vector fields $A^m_\mu$ gauging the group $G_0$ with the gauge symmetry acting as

$$\begin{aligned} \delta A^m_\mu &= \partial_\mu\Lambda^{\underline{m}} + g_1\,f^m{}_{kl}\,A^k_\mu\,\Lambda^{\underline{l}}\,, \\ \delta\tilde{\mathcal{S}} &= -g_1\eta_{mn}\Lambda^{\underline{m}}\mathcal{J}^n\tilde{\mathcal{S}}\,, \qquad \delta B^\alpha_\mu = -\,g_1\,t^\alpha{}_{m\beta}\,B^\beta_\mu\,\Lambda^{\underline{m}}\,. \end{aligned} \tag{3.9}$$

Lagrangians of the type (3.8) typically arise in dimensional reduction on nontrivial internal manifolds [3,4], see the following section. Note that in contrast to (2.1), (3.7), this Lagrangian no longer has a smooth limit $g_1 \to 0$ because some propagating degrees of freedom decouple with the $B^\alpha_\mu$. It is worthwhile to emphasize the simplicity of the original Lagrangian (2.1) in comparison with (3.8), which is due to the fact that all the different scalar tensors which describe the couplings of the various fields in (3.8) are encoded in the original G/H coset structure. Moreover, the gauge deformation of (2.1) is uniformly described in terms of the embedding tensor $\Theta$ which transforms covariantly under the maximal global symmetry G underlying the CS description of the gauged theory.

# 4 Examples

Having shown that the CS gauged supergravities (2.1) contain the YM type theories (2.22) and (3.8) as special cases, we now have the means to construct any three-dimensional supergravity given the number of supersymmetries, gauge group and field content. To do so, one first identifies that version of the underlying ungauged supergravity for which all propagating bosonic degrees of freedom appear as scalar fields and are uniformly described by a maximal coset space G/H. By contrast, the YM type theory is based on a description where only part of the bosonic degrees of freedom correspond to scalar fields, such that (1.1) holds.

Together with the precise representation content under the gauge group and for sufficiently large number $N$ of supersymmetries, this is already sufficient to identify the corresponding theory in the list of [15]. Next, the gauge group must be chosen as a subgroup of G such as to reproduce the correct representation content while its non-semisimple

part determines the nature of the vector couplings as explained in sections 2, 3. In addition, the embedding tensor $\Theta_{\mathcal{MN}}$ of this group is constrained by the group-theoretical consistency condition of [6,7,8] where the complete form of the CS gauged theory is then found. Finally, one may apply the constructions presented in this paper to cast the vector couplings into the desired form (2.1), (2.22), or (3.8).

We conclude with some examples that reproduce the mass spectra and symmetries of known $\mathrm{AdS}_3$ compactifications. Most of these theories have not been constructed before. Recall that in dimensional reduction one typically encounters YM gauged theories, i.e. the Lagrangian obtained directly by compactification will take the form (2.22), (3.8) rather than the equivalent simpler form (2.1). As pointed out in [6] the gauged CS type theories with semisimple gauge groups have no obvious higher dimensional ancestor. It is therefore remarkable that the CS type models which can be linked to higher dimensional supergravities, all have non-semisimple gauge groups. Could there be a higher-dimensional theory that gives rise to these models in a singular limit akin to the boost limit producing non-semisimple gaugings from semisimple ones? We should also stress that at this stage we restrict attention to the (unique) lower-dimensional theories with the correct field content, and are not concerned with the consistency of the truncations from the higher-dimensional point of view.

- One of the main examples of the AdS/CFT correspondence is the duality between type IIB string theory on $\mathrm{AdS}_3 \times S^3 \times M^4$ and certain two-dimensional conformal field theories [18]. The spectrum of $N = (2,0)$ supergravity on $\mathrm{AdS}_3 \times S^3$ has been computed in [19,20,21], in three dimensions this is a half-maximal, i.e. $N = 8$ theory. In [7] we have shown that the lowest multiplets of this spectrum together with the expected global and local symmetries are reproduced by a three-dimensional theory (2.1) with coset space $\mathrm{SO}(8,n)/(\mathrm{SO}(8)\times\mathrm{SO}(n))$, and CS gauge group SO(4), where $n$ denotes the number of tensor multiplets in six dimensions. An outstanding question has been the coupling of this theory to the YM vector multiplet containing additional 26 scalars. In view of the above results, one may now verify that this larger theory may be described by a coset space $\mathrm{SO}(8,4{+}n)/(\mathrm{SO}(8)\times\mathrm{SO}(4{+}n))$ and CS gauge group $\mathrm{SO}(4) \ltimes \mathrm{T}_6$. This SO(4) is embedded as a certain diagonal of two factors in the SO(8) and the SO(4+$n$), respectively, such that the scalar spectrum decomposes as $(8,(n{+}4)) \to n\cdot 4 + 4n\cdot 1 + 1 + 9 + 4\cdot 4 + 3_+ + 3_-$. Eliminating the six abelian translations as described in section 2 leads to a YM SO(4) gauged theory (2.22) coupling 26+8$n$ scalar fields. Details of this construction will be presented in [22]. Surprisingly, using the results of section 3 and a particular coset space, it is even possible to describe the coupling of multiplets from arbitrary (!) levels of the massive spin-1 KK towers.

- The near horizon limit of the so-called "double D1-D5 system" describes an $\mathrm{AdS}_3 \times S^3 \times S^3$ geometry. The supergravity spectrum on this background has been computed in [23]. The three-dimensional theory describing the lowest mass multiplets again is organized by a coset space $\mathrm{SO}(8,n)/(\mathrm{SO}(8)\times\mathrm{SO}(n))$, now with gauge group $\mathrm{SO}(4)\times\mathrm{SO}(4)$ [7]. Similar to the construction given above, one may further couple the two YM multiplets by a proper enlargement of the coset space to $\mathrm{SO}(8,8+n)/(\mathrm{SO}(8)\times\mathrm{SO}(8+n))$ and embedding a gauge group $\mathrm{SO}(4)_{\mathrm{diag}} \times \mathrm{SO}(4)_{\mathrm{diag}}$ together with the corresponding nilpotent directions.

- Recently, the reduction of of six-dimensional $N = (1,0)$ supergravity on $\mathrm{AdS}_3 \times \mathrm{SU}(2)$, has been performed in [4]. The field content of the three-dimensional $N = 4$ theory comprises three YM gauge fields together with three massive vector fields and six scalars parametrizing the coset space $\mathrm{GL}(3)/\mathrm{SO}(3)$. This spectrum suggests that the CS version (2.1) of this theory is governed by the larger coset space $\mathrm{SO}(4,3)/(\mathrm{SO}(4)\times\mathrm{SO}(3))$ with gauge group $\mathrm{SO}(3)_{\mathrm{diag}} \ltimes (\hat{\mathrm{T}}_3, \mathrm{T}_3)$. The group $\mathrm{SO}(4,3)$ indeed has a unique subgroup of this type, whose algebra generators satisfy relations of the type (2.7), (3.1). Its semi-simple part is embedded as the diagonal of the three $\mathrm{SO}(3)$ factors in the compact $\mathrm{SO}(4) \times \mathrm{SO}(3)$, such that the 12 scalars decompose as $12 \to 1+3+3+5$. Eliminating the three abelian translations and gauge-fixing the other three nilpotent directions as described in section 3 leads to a theory (3.8) with the desired spectrum. Indeed, the couplings appearing in (3.8) for this case are of the form found in [4].

- The reduction of six-dimensional $N = (1,0)$ supergravity on $\mathrm{AdS}_3 \times S^3$ leads to a three-dimensional $N = 4$ theory, whose spectrum has been computed in [19,21]. The resulting theory will be described by a coset space $\mathrm{SO}(4,4)/(\mathrm{SO}(4)\times\mathrm{SO}(4))$ and CS gauge group $\mathrm{SO}(4)_{\mathrm{diag}} \ltimes \mathrm{T}_6$ embedded such that the scalar spectrum correctly decomposes as $16 \to 1 + 3_+ + 3_- + 9$, of which the $3_+ + 3_-$ transforming in the adjoint representation of $\mathrm{SO}(4)_{\mathrm{diag}}$ are eliminated according to section 2.

- The compactification of simple five-dimensional supergravity on $S^2$ whose KK spectrum has been analyzed in [21,24,25] should be related to proper gaugings of the $N=4$ theory with coset space $\mathrm{G}_{2(2)}/\mathrm{SO}(4)$. The CS gauge group is a $\mathrm{SO}(3)_{\mathrm{diag}} \ltimes \mathrm{T}_3$ under whose semisimple part the scalar spectrum decomposes as $8 \to 3+5$.

- The most interesting example is the (warped) compactification of ten-dimensional supergravity on $S^7$. For the type I theory, the reduction has been performed explicitly in [3]. We may recover that theory by starting from an $N = 8$ theory (2.1) with coset space $\mathrm{SO}(8,8)/(\mathrm{SO}(8) \times \mathrm{SO}(8))$, and CS gauge group $\mathrm{SO}(8)_{\mathrm{diag}} \ltimes \mathrm{T}_{28}$ upon eliminating the 28 translations, leading to an $\mathrm{SO}(8)$ gauged YM theory (2.22) with 36 scalar fields. Using the above results, it is then straightforward to extend this construction to the maximally supersymmetric theories which are supposed to

describe the compactifications of type IIA/IIB theory on $S^7$, and whose spectra have been given in [26]. These two different compactifications correspond to two inequivalent embeddings of the gauge group $\mathrm{SO}(8)_{\mathrm{diag}} \ltimes \mathrm{T}_{28}$ into $\mathrm{E}_{8(8)}$. Details will appear in [14].

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