Time-dependent backgrounds from supergravity with gauged non-compact R-symmetry

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ABSTRACT

We obtain a general class of time-dependent, asymptotically de Sitter backgrounds which solve the first order bosonic equations that extremize the action for supergravity with gauged non-compact R-symmetry. These backgrounds correspond only to neutral fields with the correct sign of kinetic energy. Within N=2 five-dimensional supergravity with vector-superfields we provide examples of multi-centered charged black holes in asymptotic de Sitter space, whose spatial part is given by a time-dependent hyper-Kähler space. Reducing these backgrounds to four dimensions yields asymptotically de Sitter multi-centered charged black hole backgrounds and we show that they are related to an instanton configuration by a massive T-duality over time. Within N=2 gauged supergravity in four (and five)-dimensions with hyper-multiplets there could also be neutral cosmological backgrounds that are regular and correspond to the different de Sitter spaces at early and late times.

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1 Introduction

Time-dependent backgrounds in fundamental theory are much less understood than stationary solutions. In addition these solutions are typically singular, leading cosmological singularities of the big bang or big crunch type.

The purpose of this paper is to provide a large class of cosmological solutions that have an interpretation of solutions of gauged supergravity with non-compact R-symmetry gauged. In this case the positive cosmological constant leads to asymptotically de Sitter space. The obtained class of solutions satisfies first order bosonic equations that extremize the action, and can be in a broader sense referred to as supersymmetric (see [1, 2, 3] and references therein).

Gauging of non-compact R-symmetry introduces an effective imaginary gauge coupling via Wick rotation $g = i\lambda$, and thus the covariant derivatives for charged fields are typically complex. In order to have a real Lagrangian, one would have to impose further Wick rotation for the charged matter fields or the gauge fields. In either case, this will change the sign of the kinetic energy terms for their respective fields, which in fact is the case for the supergravity models based on supergroups that include the de Sitter group [4].

In this paper we shall consider only the neutral backgrounds that have the correct signs of the kinetic energy terms. Nevertheless at the quantum level one has to face the problem with the ghost-like contribution from charged sectors. We do not have much to say about this problem, and focus only on the neutral, bosonic part, where the classical backgrounds are turned on.

The time-dependent asymptotically de Sitter backgrounds of supergravity theory with non-compact R-symmetry gauged inherit a number of properties of the static, asymptotically flat BPS solutions of ungauged supergravity: there exist multi-centered solutions, and the flat transverse space can be replaced by a general hyper-Kähler space.

In order to motive the basic set-up, we start with the static asymptotically flat BPS solution of five- (or four-) dimensional ungauged supergravity and then generate the corresponding cosmological solution, that solves the same first order integrability conditions, but now for (non-compact) gauged supergravity with the *positive cosmological constant.*

As an illustration let us demonstrate the generating technique for five-dimensional $N=2$ ungauged supergravity without couplings to matter supermultiplets. The static BPS black hole solution is given by

$$ds^2 = -\frac{1}{H^2}dt^2 + Hdx^m dx^m \quad , \qquad A = \frac{dt}{H} \tag{1}$$

where H is a harmonic function in the transverse four-dimensional space and A is the $U(1)$ gauge field one-form. For $H = 1 + \frac{q}{|x|^2}$ this is the extreme Reissner-Nordström black hole in an asymptotically flat Minkowski space.

If one shifts H by a general linear function in the time, i.e.

$$H(x) \rightarrow \lambda t + H(x) \tag{2}$$

the asymptotic space becomes de Sitter and the solution (1) solves the equations coming for the five-dimensional bosonic action

$$S_5 = \int \left[\frac{R}{2} - \frac{1}{4}F^2 + \frac{3}{2}\lambda^2 \right] . \tag{3}$$

To verify this, one can calculate the Ricci tensor for the general function $H = H(r,t)$ and if we separate terms proportional to time derivatives, the Einstein equations become

$$R_m{}^n \rightarrow R_m{}^n + [(\partial_0 H)^2 + \tfrac{1}{2}H\partial_0^2 H]\, g_m{}^n = (F^2)_m{}^n - \tfrac{1}{6}\,g_m{}^n F^2 + \lambda^2 g_m{}^n \ ,$$

$$R_0{}^0 \rightarrow R_0{}^0 + (\partial_0 H)^2 + 2H\partial_0^2 H = (F^2)_0{}^0 - \tfrac{1}{6}\,F^2 + \lambda^2 \ , \tag{4}$$

$$R_0{}^m \sim \partial_m \partial_0 H = 0 \ .$$

If $\partial_0^2 H = \partial_m \partial_0 H = 0$ the positive cosmological constant is compensated by the linear time-dependent part in H. Note, the gauge field equations are not effected by the cosmological constant and with $F_{0m} = \frac{\partial_m H}{H^2}$ one finds

$$\partial_0(\sqrt{g}F^{0m}) = \partial_0 \partial_m H = 0 \quad , \quad \partial_m(\sqrt{g}F^{m0}) = \partial_m \partial_m H = 0 \ . \tag{5}$$

Thus the equations of motion are in fact solved with the Ansatz (2) and the solution becomes after time reparameterization equivalent to the ones found in [5].

If one turns off the $U(1)$ gauge field charge, $H(x) = 1$, and the space-time metric becomes

$$ds^2 = -\frac{dt^2}{(\lambda t + 1)^2} + (\lambda t + 1)dx^m dx^m \tag{6}$$

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which corresponds to the five-dimensional de Sitter space.

We shall see that the harmonic function Ansatz of the type (2) solves the equations of motion even for more general Lagrangians, that involve more $U(1)$ gauge fields and the scalar fields. In a more general set-up, we focus on the prototype example of five-dimensional $N=2$ gauged supergravity with vector-superfields. We will also allow for the initial configuration to be stationary. In Section 2 we discuss multi-centered, multiple charged Abelian black holes with asymptotic de Sitter space, which have been found in [6] as solutions of first order equations that extremize the action.

In Subsection 3.1 we further reduce these solutions to four-dimensions, by first replacing the spatial part of the five dimensional solution with a specific time-dependent hyper-Kähler space with $U(1)$ isometry. Reduction of these backgrounds to four dimensions yields four-dimensional, asymptotically de Sitter multi-centered charged black hole backgrounds.

By turning off the gauge fields we further discuss the backgrounds with only scalar fields coupled to gravity in Subsection 3.2. These backgrounds correspond to the cosmological flows, that are complementary to the BPS domain walls for supergravity theories with gauged compact symmetry [3], i.e. renormalization group (RG) flows [10] in the context of AdS/CFT correspondence [11].

In Subsection 3.3 we also address the possible resolutions of the early time (cosmological) singularity that are generic in the set up with vector-supermultiplets. Within $N=2$ gauged supergravity coupled to hyper-multiplets, one can also obtain regular cosmological backgrounds with a different value of the de Sitter cosmological constant at early and late times and we comment on this possibility.

Examples of four-dimensional charged de Sitter black hole backgrounds can be obtained by performing (massive) T-duality over the time if one starts with the BPS instanton solution of ungauged supergravity. We discuss the procedure in Section 4. We also note that the obtained five- and four-dimensional gauged supergravity backgrounds should be related to a specific reduction of D=10 type II* theories [1] on non-compact spaces.

Conclusions and open questions are addressed in Section 5.

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[3]BPS domain wall solutions of supergravity theory we first found in the context of $N=1$ D=4 ungauged supergravity in [7]. (See also [8] for generalizations and [9] for a review.)

2 Cosmological background in five dimensions

In the Introduction we demonstrated that in the case of static BPS solutions of $N=2$ ungauged supergravity without couplings to matter multiplets, time-dependent backgrounds of gauged supergravity with a positive cosmological constant can be obtained by replacing the integration constants in the harmonic functions by a function linear in time (2). Via this procedure static, asymptotically flat BPS solutions of ungauged supergravity become time-dependent, asymptotically de Sitter solutions of gauged supergravity (with non-compact R-symmetry gauged).

In this section we will show that such time-dependent backgrounds, in a general set-up, solve first order differential equations which extremize the action. As a prototype example we shall focus on the five-dimensional $N=2$ gauged supergravity which couples to vector-supermultiplets, only.

The gravity supermultiplet has, besides the graviton and gravitino, one Abelian vector field and each vector-supermultiplet has one Abelian vector field and a (real) scalar field ϕ^A ($A = 1, ..., n_v$) that parameterize a manifold $\mathcal{M}$ defined by the constraint

$$\mathcal{V} = \frac{1}{6} C_{IJK} X^I X^J X^K = 1 \tag{7}$$

with $I = 0, 1, ..., n_v$. The constants C_{IJK} enter the Chern-Simons term of the action

$$S_5 = \int \left[\frac{R}{2} - g^2 V - \frac{1}{2} g_{AB} \partial \phi^A \partial \phi^B - \frac{1}{4} G_{IJ} F^I \cdot F^J \right] + \frac{1}{12} \int C_{IJK} F^I \wedge F^J \wedge A^K \tag{8}$$

and F^I are the field strength for the $U(1)$ gauge fields and the potential reads

$$V = 6 \left(\frac{3}{4} g^{AB} \partial_A W \partial_B W - W^2 \right) . \tag{9}$$

The couplings in the Lagrangians are now defined by

$$G_{IJ} = \frac{1}{2} \partial_I \partial_J \mathcal{V} \Big|_{\mathcal{V}=1} \quad , \qquad g_{AB} = \partial_A X^I \partial_B X^J G_{IJ} \tag{10}$$

where $\partial_A X^I \equiv \frac{d}{d\phi^A} X^I(\phi^A)$. With this definitions one finds the relations

$$X_I = \tfrac{2}{3} G_{IJ} X^J \quad , \qquad dX_I = -\tfrac{2}{3} G_{IJ} dX^J ,$$

$$\mathcal{V} = X^I X_I = 1 \quad , \qquad X_I dX^I = X^I dX_I = 0 . \tag{11}$$

Unbroken supersymmetry implies the existence of at least one Killing spinor ϵ which gives a zero for the gravitino and gaugino supersymmetry variation. If one includes

an Abelian gauging of the $SU(2)$-R-symmetry with the gauge field $A = \alpha_I A^I$ (with $\alpha_I = const.$), these variations contain a superpotential of the form [12]

$$W = \alpha_I X^I \ . \tag{12}$$

With the definition of the scalar field metric, we can write $g_{AB} d\phi^B = -\frac{3}{2}\partial_A X^I dX_I$ and $\partial_A W = \partial_A X^I \alpha_I$ and the supersymmetry variations become

$$
\begin{aligned}
\delta\Psi_\mu &= \left[D_\mu + \frac{i}{8}\left(\Gamma_\mu{}^{\alpha\beta} - 4\delta_\mu{}^\alpha\Gamma^\beta\right)X_I F^I_{\alpha\beta} + \frac{1}{2}g\,\Gamma_\mu\,W\right]\epsilon = 0 \ , \\
\delta\lambda_A &= \tfrac{3i}{2}\partial_A X^I\left[\Gamma^\mu\partial_\mu X_I + \tfrac{2i}{3}G_{IJ}F^J_{\mu\nu}\Gamma^{\mu\nu} + \tfrac{1}{2}g\alpha_I\right]\epsilon = 0 \ , \\
D_\mu &= \partial_\mu + \tfrac{1}{4}\omega_\mu^{ab}\Gamma_{ab} - \tfrac{3i}{2}g\alpha_I A^I_\mu \ .
\end{aligned}
\tag{13}
$$

To keep the notation as simple as possible we have not used symplectic Majorana spinors but used instead the conventions of [12]. Solutions of these first order differential equation solve the equations of motion if in addition the gauge field equations are satisfied, i.e.

$$dF^I = 0 \quad , \qquad d\left(G_{IJ}{}^\star F^J + \frac{1}{2}C_{IJK}A^J \wedge F^K\right) = 0 \ . \tag{14}$$

Let us recall first the supersymmetric solution in the ungauged case $(g = 0)$. It has a time-like isometry and can be written in a proper coordinate system as [13, 14]

$$
\begin{aligned}
ds^2 &= -e^{-4U}(dt + \omega)^2 + e^{2U}dx^m dx^m \ , \\
A^I &= e^{-2U}X^I(dt + \omega) \ , \\
e^{2U}X_I &= \tfrac{1}{3}H_I \quad , \qquad d\omega + {}^\star d\omega = 0 \ .
\end{aligned}
\tag{15}
$$

The one-form $\omega = \omega_m dx^m$ corresponds to $U(1)$ fibration of the transversal space and without specifying the functions H_I, these fields solve the equations (13). However, in order to fulfill the equations of motion we have to consider the gauge field equations (14) that are solved only if

$$\partial_m \partial^m H_I \ . \tag{16}$$

Recall, the fields X^I are subject to the constraint (7) so the $n_v + 1$ harmonic functions determine the scalar fields ϕ^A as well as the metric function e^{2U}. Note, $g = 0$ corresponds to the case of ungauged supergravity.

If we consider a gauging of non-compact R-symmetry, i.e. $g^2 < 0$, the equations of motion arising from the action (8) are solved if

$$(-\partial_0^2 + \partial_m\partial_m)H_I = 0 \quad , \qquad \partial_0 H_I = \alpha_I \lambda \quad , \qquad g = i\lambda \qquad (17)$$

are i.e. H_I is now harmonic in all five coordinates and the vacuum, given by stationary point of the superpotential ($dW = 0$), yield now a de Sitter space (since $g^2 < 0$). Note that we are tacitly assuming that the first order equations arising from the fermionic supersymmetry variations (13) are formally still valid, in spite of the fact that $g = i\lambda$ is an imaginary coupling.

The proof that the equations of motion are solved with the above Ansatz for the modified harmonic functions has already been given in [6] (partly based on [5]). In our context we use a different time parameterization where H_I depend linearly on time. We have summarized the proof that the first order equations (13) are in fact solved in the Appendix.

In the following we shall discuss only the gauge field equations (14), which yield harmonic functions as a solution. Using the form for the gauge field strength (57), the Chern-Simon term can be written as

$$\frac{1}{2}C_{IJK}F^J \wedge F^K = d\left[e^{-6U}H_I(dt + \omega) \wedge d\omega\right] \qquad (18)$$

and moreover, using the explicit form (60) (and $\omega_m\partial_m H_I = \omega_m\partial_m U = 0$) we find

$$\partial_n\left(G_{IJ}\sqrt{g}F^{I\,n0}\right) = \partial_n\partial_n H_I - \partial_n\left(e^{-6U}H_I\omega_m\partial_{[n}\omega_{m]}\right) , \qquad (19)$$

$$\partial_\mu\left(G_{IJ}\sqrt{g}F^{I\,\mu n}\right) = \partial_0(\omega_n\partial_0 H_I) - \partial_0\partial_n H_I - \partial_\mu\left(e^{-6U}H_I\omega_m\partial_{[n}\omega_{m]}\right) \qquad (20)$$

where $\mu = 0, 1, 2, 3, 4$. For trivial ω, these equations agree with (5) and are obviously solved, but if ω is non-trivial we have to use the fact that $d\omega$ is anti-selfdual in order to verify that the equations are solved if (17) holds.

It is obvious from eq. (17) that the equations of motion are also satisfied for the multi-centered solution, and that the flat transverse space can be replaced by a hyper-Kähler space (see the next Section). Therefore, these asymptotically de Sitter solutions inherit features of the asymptotically flat BPS solutions of ungauged supergravity.

3 Cosmological background in four dimensions

The procedure, described in the previous Section to obtain time-dependent backgrounds from BPS solutions can of course also be applied to other dimensions. Again, one has to shift the harmonic functions by functions linear in time, combined with the replacement $g = i\lambda$. Again, we consider a truncation to the neutral sector only where all kinetic terms have the correct sign.

We shall however not give a general proof of the procedure, but will focus instead on deriving the four-dimensional cosmological background by dimensional reduction of the five-dimensional one described in the previous Section.

3.1 Transversal hyper-Kähler space and the reduction to four dimensions

Following the calculations in the Appendix, it is straightforward to realize that one can replace the flat transversal space of the five-dimensional metric in (15) by any four-dimensional hyper-Kähler space. To see this, let us write the metric as

$$ds^2 = -e^{-4U}(dt + \omega)^2 + e^{2U}h_{mn}dx^m dx^n \ . \tag{21}$$

This yield instead of (63) the equation

$$[\nabla^{(h)}_\mu + (\partial_\mu U)]\epsilon = 0 \ . \tag{22}$$

with the covariant derivative corresponding to the metric

$$ds^2 = -dt^2 + h_{mn}dx^m dx^n \ . \tag{23}$$

As a solution one finds that h_{mn} can be any Ricci-flat metric. On any constant time slice, the integrability condition for this equation means that the Riemann tensor has to be self-dual and hence the space is hyper-Kähler (and therefore Ricci flat).

For concreteness we consider the four-dimensional Gibbons-Hawking metric [15]

$$h_{mn}dx^m dx^n = \frac{1}{V}(d\chi + \sigma)^2 + V dx^i dx^i \quad , \qquad \partial_i V = \epsilon_{ijk}\partial_j \sigma_k \tag{24}$$

with the isometry ∂_χ. The equation for V is solved with any harmonic function in the three-dimensional space spanned by the coordinates x^i. We are however interested in

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a time-dependent functions, i.e. $V = V(t, x)$. Inserting in (23) one can verify by an explicit calculation that the five-dimensional space remains Ricci-flat as long as

$$\partial_0 \partial_0 V = \partial_0 \partial_i V = 0 \ . \tag{25}$$

Thus, this harmonic function is on equal footing to the harmonic functions introduced in (17).

Assuming that ∂_χ is also an isometry of the complete five-dimensional metric, it is straightforward to reduce the model to four dimensions. In order to obtain a diagonal metric in four dimensions, which may be most interesting from the cosmological point of view, we identify the $U(1)$ fibre $(d\chi + \sigma)$ with ω in the five-dimensional metric (see also [16]), which yield the four-dimensional metric

$$ds^2 = -e^{-2\hat{U}} dt^2 + e^{2\hat{U}} dx^i dx^i \ . \tag{26}$$

In order to obtain the Einstein-frame metric (26) after the reduction, there is a conformal rescaling of the metric, and the function $\hat{U}$ is given by

$$e^{4\hat{U}} = V e^{6U} \ . \tag{27}$$

We expect, that the general four-dimensional solution can also be obtained directly as a generalization of the general BPS solution [17], where the complex scalar fields $z^A = \frac{Y^A}{Y^0}$ and the metric function $e^{2\hat{U}}$ are now a solution of the equations

$$e^{2\hat{U}} = i(\bar{Y}^I F_I - Y^I \bar{F}_I) \quad ,$$
$$i(Y^I - \bar{Y}^I) = H^I(t, x) \quad , \qquad i(F_I - \bar{F}_I) = H_I(t, x) \tag{28}$$

with: $(H^I(t, x) = \lambda \beta^I t + h^I(x)$ and $H_I(t, x)) = \lambda \alpha_I t + h_I(x))$. As in five dimensions, these equations should solve the BPS equations of four-dimensional supergravity with a gauged R-symmetry (and with $g = i\lambda$). Note, in this solution the symplectic section (Y^I, F_I) is rescaled so that it becomes invariant under Kähler transformations (see [18, 17] for details). A number of explicit solutions of these algebraic equations have been discussed in the literature [19]. [Note, $(h^I(x), h_I(x))$ is again a set of any harmonic functions and includes especially also multi-center black holes.] We will show in the next Section that the shift by a linear function in time is the result of a massive

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T-duality over time applied on an instanton solution of ungauged supergravity. The massive T-duality generates in the dual theory the correct potential.

We conclude this Subsection by presenting a specific example where the Chern-Simons cubic form (7) reads

$$\mathcal{V} = X^1 X^2 X^3 . \tag{29}$$

This is the so-called STU model ($S = X^1, T = X^2, U = X^3$) with three Abelian gauge fields and two real scalars, e.g., by $\phi^1 = X^2/X^1$ and $\phi^2 = X^3/X^1$. For the single center case, one obtains for the five-dimensional solution reads

$$e^{6U} = H_1 H_2 H_3 = \left(\lambda t + \frac{q^1}{r^2}\right)\left(\lambda t + \frac{q^2}{r^2}\right)\left(\lambda t + \frac{q^3}{r^2}\right) , \qquad \omega = 0 , \tag{30}$$

$$X^I = \frac{e^{2U}}{H_I} , \qquad A^I = \frac{dt}{H_I} . \tag{31}$$

In order to reduce this model to four dimensions, we will replace the transversal space by a specific hyper-Kähler space given by the Gibbons-Hawking metric. This requires, that the harmonic function in e^{6U} do not depend on χ, which has to be a isometric direction. Hence, we have to replace the harmonic functions: $\left(\lambda t + \frac{q^i}{r^2}\right) \rightarrow \left(\lambda t + \frac{q^i}{r}\right)$, where r is now the three-dimensional radial direction ($dx^i dx^i \equiv dr^2 + r^2 d\Omega_2$). Denoting $V = \lambda t + \frac{q^0}{r}$ the four-dimensional metric function becomes

$$e^{4\hat{U}} = \left(\lambda t + \frac{q^0}{r}\right)\left(\lambda t + \frac{q^1}{r}\right)\left(\lambda t + \frac{q^2}{r}\right)\left(\lambda t + \frac{q^3}{r}\right) . \tag{32}$$

Of course we can replace $\frac{q^m}{r}$ by any harmonic function giving a general multi-center solution living in an asymptotic de Sitter space. This is the de Sitter analog of the four-charge black hole of ungauged supergravity [20]. In fact, for large t the metric (26) becomes

$$ds^2 = -\frac{dt^2}{(\lambda t)^2} + (\lambda t)^2 dx^i dx^i . \tag{33}$$

On the other hand, if we go back in time, we will necessarily reach the point where $e^{4\hat{U}}$ becomes zero giving a curvature singularity, i.e. a big bang singularity.

3.2   Cosmological renormalization group flow

If we turn off the $U(1)$ gauge fields, the background becomes only time-dependent (any dependence on the spatial coordinates disappears) and we can interpret the time evolution as a cosmological renormalization group (RG) flows within in the proposed

dS/CFT correspondence [21]. Let us formulate the flow equations in D dimension and write the (Robertson-Walker) metric as

$$ds^2 = -d\tau^2 + e^{2A(\tau)} \, dx^i dx^i \ . \tag{34}$$

If we write the Lagrangian as

$$S_D = \int \left[\frac{R}{2} - g^2 V - \frac{1}{2} g_{AB} \partial \phi^A \partial \phi^B \right] \tag{35}$$

where the potential is given in terms of a real superpotential $\tilde{W}$

$$V = \frac{(D-2)^2}{2} \partial_A \tilde{W} \partial^A \tilde{W} - \frac{(D-1)(D-2)}{2} \tilde{W}^2 \ . \tag{36}$$

In five dimensions (35) corresponds to Lagrangian (8) without $U(1)$ gauge fields. In four dimensions the $\tilde{W}$ is related to the complex superpotential by $\tilde{W}^2 = e^K W \bar{W}$. Because $g^2 = -\lambda^2$ the potential has the opposite sign and we can square the action for the time-dependent metric (34) in the same way as for BPS domain walls (see also [22, 7]) and we find

$$S = S_{bulk} + S_{boundary}$$

$$S_{bulk} = \int e^{(D-1)A} \left[-(D-1)(D-2)(\dot{A} - \lambda \tilde{W})^2 + \tfrac{1}{2} |\partial \phi^A + (D-2) \lambda \, g^{AB} \partial_B \tilde{W}|^2 \right] \ . \tag{37}$$

From here we obtain the first order equations

$$\dot{A} = \tilde{W} \quad , \qquad \dot{\phi}^A = -(D-2) \, g^{AB} \partial_B \tilde{W} \ . \tag{38}$$

Extrema of $\tilde{W}$ correspond obviously to dS vacua and the solution fulfilling these first order flow equations extremizes the action and de Sitter vacua are approached with vanishing velocity with no oscillations.

These first order equations corresponds to the cosmological RG-flow equations. Regular flow requires necessarily a superpotential $\tilde{W}$ with at least two (connected) extrema. For the examples at hand (i.e. vector-multiplets, only), one can however show [23] that on any given component of the scalar manifold, there is at most one critical point and therefore a given solution will always run towards a singularity related to a pole in $\tilde{W}$. In the cosmological setting this means that the big bang singularity is un-avoidable in these models. However one should of course ask, whether

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one can trust the model all the way to the singularity or whether one should replace the model by a different one – as it has been discussed for the cusp singularity for ordinary AdS domain walls.

3.3 Resolution of the singularities

There are different ways in dealing with a singularity, e.g.:

(i) it may be an artifact of the approximation or truncation of the theory,

(ii) it appears in a non-physical region, which should be replaced or

(iii) the singularity could be smoothed out by a second de Sitter space.

Let us comment on either case.

Case (i): It is well known that the appearance of singularities in supergravity reflects often the failure of the approximation. However, in most interesting examples one is forced to consider certain approximations as e.g., the restriction to the lowest order in the expansions in α' and/or g_s. It is likely that higher derivative corrections coming from the α' expansion can smooth out singularities, e.g., as it is known in Born-Infeld instead of Maxwell theory. Another source of singularities is the truncation to Abelian gauge fields instead of non-Abelian, which yields a regular background, such as an example discussed in [24].

Another interesting way to regulate supergravity solutions is via the so-called transgression mechanism [25], where the incorporation of additional differential forms yield a regular background due to corrections coming from the Chern-Simons term. For five-dimensional supergravity, this mechanism was in fact used in [16] to obtain regular BPS solutions in ungauged supergravity. In our derivation it was important that $e^{-2U} d\omega$ as well as the spatial components of the gauge fields are anti-selfdual, which yielded the harmonic equations for H^I. In the case with no time-dependence ($\lambda = 0$), it was shown in [16] that one can add a self-dual 2-form $G^{(+)}$ to $e^{-2U} d\omega$ as well as to the spatial components of the gauge field strength. As a result, the Chern-Simons transgression term gives an additional contribution from $G^{(+)} \wedge G^{(+)} = |G^{(+)}|^2$ and the gauge field equations do not yield the harmonic equation for H_I, but: $\partial_m \partial_m H \sim |G^{(+)}|^2$. This modification via the transgression term resolves singularities [25] related to poles in H. However, for $\lambda \neq 0$ the cosmological singularity appears if one goes back in time, i.e. H decreases and may eventually vanish. At this point however,

the time-derivatives of H are still smooth and hence, the transgression mechanism cannot resolve these singularities. Note, that even for the pole singularities, this mechanism seems to fail if one includes the time-dependence ($\lambda \neq 0$), this however deserves further investigations.

Case (ii). The cosmological singularity of the discussed solutions is reminiscent of the repulson singularity discussed for black holes [26]. This singularity appears in the supergravity solution as a zero of the metric function e^{2U} (or $e^{2\hat{U}}$ in four dimensions) due to a vanishing or negative harmonic function. Since the harmonic functions parameterize the radii of the internal cycles, this singularity indicates interesting physics related to gauge symmetry enhancement, flop or conifold transitions. By using a probe analysis for time independent black holes one can show, that even before one reaches the repulson singularity the tension of the probe brane becomes tensionless and at the enhancement point the supergravity solution cannot be trusted anymore and additional massless degrees of freedom have to be taken into account [27]. An analogous mechanism does also apply for (static) domain walls, where the enhancon locus appears before one reaches the space-time singularity [28] and integrating-in the additional modes, the singularity can be avoided (see [29] and references therein).

We would like to argue that also for the cosmological solutions at hand the repulson singularity is only an artifact of the approximation and can be smoothed out. For the four-dimensional case discussed in Subsection 3.1 there is a powerful tool to regulate supergravity solutions, by including the term proportional to the Euler number in the prepotential. This is an α'-correction which is added to the prepotential as

$$F = -i\frac{\chi\zeta(3)}{2(2\pi)^3}(Y^0)^2 + \tilde{F}(Y) = -i(Y^0)^2\Big[c + \mathcal{F}(z)\Big] \tag{39}$$

with $c = \frac{\chi\zeta(3)}{2(2\pi)^3}$ and χ is the Euler number of the internal space. Recall, we use the rescaled section $(F_I(Y), Y^I)$, which ensures that the supergravity solution is invariant under Kähler transformations (by rescaling of the section with the supersymmetry central charge [18]). In this rescaled section we should not set $Y^0 = 1$ because this would eliminate one harmonic function in (28). Now, if the scalars flow to smaller values and even if one reaches the point where $\mathcal{F}(z)$ vanishes, the metric will still be regular as long as $c \sim \chi \neq 0$ (see also [19]). In fact, if we solve the equations (28) for

this prepotential the metric function becomes

$$e^{2\hat{U}} \sim H_0^2 \qquad \text{for} \quad \mathcal{F}(z) \simeq 0 \ . \tag{40}$$

and we obtain a de Sitter space if we choose $H_0 = \lambda(t - t_0)$. [In the context of transgression mechanism for the static BPS configurations the α' correction due to the gravitational Chern-Simons term, proportional to the Euler number of the internal space, was studied in [30].]

Another form of singularity is of course given by scalars that run to infinite values, which do not correspond to divergencies of the internal space but correspond to a large volume limit. In this case, the singularity reflects the appearance of an extra dimension and the corresponding higher dimensional solution should be regular. We have not much to say about this singularity (see [31] for a recent discussion), but this remark also brings us to a discussion of the situation in five dimensions. There, the supergravity solution describes also a flow with a de Sitter vacuum at late time and may run towards a phase transition point. In five dimensions however, we do not have the Euler number term in the pre-potential and hence the mechanism as described in four dimensions is not applicable. Nevertheless, the solution is regular as long as at least some cycles stay finite as, e.g., at the flop transition point [32, 33], but also at boundaries of the vector moduli space. Note, that although scalars disappear at this point, other scalars may continue to flow along this boundary to infinity, indicating the appearance of additional large extra dimensions.

Case (iii). The resolution that we discussed so far, rely on the picture that scalar fields parameterize cycles of an internal space and the model may change if the scalars vanish. From the supergravity point of view however, all these flows are singular. To regularize the flow, one has to consider a superpotential with a second extremum. Complementary to BPS domain walls, this flow would interpolate between two anti de Sitter vacua; in this case we would have a cosmological flow connecting an early de Sitter phase with a late time de Sitter space. In fact, if one includes hyper-supermultiplets it is possible to construct smooth flow [34], which represents a truncation of the flow found in [10] on N=2 supergravity. However, we have to truncate the model onto the neutral scalar sector and one should investigate if this can be done consistently. Similarly, it would be interesting to see whether the superpotential yielding a realization of the Randall-Sundrum model in supergravity

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[35] can be employed to obtain a regular time-dependent solution. Since in this case the warp factor is exponentially small at both sides of the wall, it would describe a bubble that first exponentially expands followed by an collapse.

4 Time-like "massive" T-duality in D=4

We will now show that the cosmological solution described in Section 3 can be obtained from a BPS configuration of ungauged supergravity by a T-duality along the time. It is however not the usual T-duality transformation, but the "massive" one, where one takes into account a linear dependence with respect to the isometry direction [36]. For concreteness we will use a "massive" generalization of the c-map, which maps type IIA and IIB superstring compactification on the same internal space onto each other and which is nothing but a time-like T-duality [37, 38].

To explain the procedure we start with the simple case, where we have only the gravity supermultiplet, but no vector-supermultiplets and no potential. In this case, the BPS solution is just the Reissner-Nordström black-hole (with or without rotation). As a result from the c-map, the two on-shell degrees of freedom of the metric as well as of the two degrees of freedom of the gauge field combine to four scalars of the universal hyper-supermultiplet. Therefore, in type II compactifications two scalars are in the NS-NS sector and two coming from the RR sector and the BPS configuration becomes an instanton solution with a flat Einstein frame metric (see [39]). The scalars of the classical universal hyper-supermultiplet spans a manifold given by the coset $\frac{SU(2,1)}{U(2)}$ and can be decomposed into two complex scalars. For a BPS configuration, one complex scalar is trivial (constant) and the other is fixed by the BPS equations and parameterizes the coset $\frac{SU(1,1)}{U(1)}$ as given by the sigma model

$$\frac{\partial S \partial \bar{S}}{(\mathrm{Im} S)^2} \, . \tag{41}$$

There are now two different possibilities to embed this coset into $\frac{SU(2,1)}{U(2)}$. In one case one combines the two NS-NS-scalars into the complex field

$$S = a + i e^{-2\varphi} \tag{42}$$

which is the well-known combination in heterotic string models, where the axion a is dual to the anti-symmetric tensor field and φ is the dilaton. The other possibility is

to combine one NS-NS and one RR-scalar yielding

$$S = a + ie^{-\varphi} \tag{43}$$

which is the combination appearing in type IIB compactifications. This is also the representation that we obtain after the c-map of the type IIA model.

As the next step, we show that a solution of the instanton equation can be mapped onto cosmological solutions as given in eqs. (26) with $e^{2\hat{U}}$ as a harmonic function. Following [40] we are looking for an instanton solution in Euclidean time with a vanishing energy-momentum tensor. Due to the Wick rotation the sign of the axionic part in the Lagrangian is changed

$$\frac{\partial S \partial \bar{S}}{(\text{Im}S)^2} = (\partial\varphi)^2 + e^{2\varphi}(\partial a)^2 \quad \longrightarrow \quad \frac{\partial S_+ \partial S_-}{\frac{1}{2}(S_+ - S_-)^2} = (\partial\varphi)^2 - e^{2\varphi}(\partial a)^2 \tag{44}$$

with $S_\pm = e^{-\varphi} \pm a$. For the instanton configuration the energy momentum tensor has to vanish, implying $\partial a = \pm \partial e^{-\varphi}$ and the equations of motion for the axion becomes

$$\partial_\mu(e^{2\varphi}\partial^\mu a) = \pm\partial_\mu\partial^\mu e^{\varphi} = 0 \tag{45}$$

and therefore the solution is expressed in terms of a harmonic function H

$$e^{\varphi} = H \quad , \qquad a = \pm\frac{1}{H} + const. \tag{46}$$

In general the harmonic function can depend on all four Euclidean coordinates, but for the case at hand we do not consider a dependence on the Euclidean time.

If one is doing the standard T-duality along the Euclidean time followed by a Wick rotation to the Minkowskean time one obtains the static Reissner-Nordström black hole [39]. In order to generate a time-dependence we will employ the massive T-duality, which, as the usual T-duality, defines a map of two models dimensionally reduced over inverse radii, where one model is massless (i.e. no potential) and the other is massive (i.e. with a potential). To make the map explicit one has to use the Scherk-Schwarz reduction [41]. This is a generalized dimensional reduction where one allows for a linear dependence on the coordinate along which one makes the reduction, but in a way that the reduced model is still independent of this coordinate. As a consequence one obtains the correct potential of massive supergravity, at least as long as one is doing the T-duality along the spatial direction [42, 36, 43]. For the

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above instanton solution this implies, that we have to perform the usual dimensional reduction along the Euclidean time followed by an inverse Scherk-Schwarz reduction over the inverse radius. The correct time dependence can be fixed using the global symmetries. Namely, the time independence of the reduced model is ensured if it can be absorbed into a $SL(2, R)$ symmetry transformation of the scalar matrix of the form [43]

$$\mathcal{M} = e^{\varphi} \begin{pmatrix} |S|^2 & a \\ a & 1 \end{pmatrix} \longrightarrow \Omega^{-1}(t)\, \mathcal{M}\, \Omega(t) \quad , \qquad \Omega^{-1}(t)\, \partial_0 \Omega(t) = C \tag{47}$$

where $\Omega \in SL(2, R)$ and the mass matrix C has to be time independent, yielding the potential [43]

$$V \sim \text{tr}(C^2 + C^T \mathcal{M} C \mathcal{M}^{-1}) \ . \tag{48}$$

Next, the harmonic function that defines the solution in (46) can be shifted by a constant $H \to H + h$ by the following $SL(2, R)$ transformation

$$\Omega = \Omega_1^{-1} \Omega_2\, \Omega_1 = \begin{pmatrix} 1 & 0 \\ \frac{h}{2} & 1 \end{pmatrix} \tag{49}$$

with the two generators $\Omega_1 : \ S_{\pm} \to S_{\pm}^{-1}$ and $\Omega_2 : \ S_{\pm} \to S_{\pm} + \frac{h}{2}$ (see [44] where this transformation was used in type IIB string theory). In order to fix the time-dependence of Ω we have to ensure that the mass matrix C does not depend on time, which is the case for $h = g\, t$. So, as result of the "massive" T-duality we added to H a linear function in time and the potential (48) reads

$$V \sim g^2 e^{2\varphi} \ . \tag{50}$$

Finally, to obtain a Minkowski signature one has to perform a Wick rotation in the time $(t \to it)$ and in $g = i\lambda$, so that H remains real, which in turn changes the sign of the potential. Applying standard rules of T-duality, one finds that the new dilaton is constant and $e^{2\hat{U}} = e^{2\varphi}$ and hence the factor $e^{2\varphi}$ in the potential is nothing but $\sqrt{\det(g_{mn})}$ for the Reissner-Nordström black hole (see eq. (26)). Therefore there is only a positive cosmological constant in the dual Lagrangian.

The procedure that we described here for the supergravity supermultiplet can be generalized to include any number of vector-supermultiplets, which for ungauged

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supergravity was done in [39, 45]. In this case one obtains $n_V + 1$ hyper-supermultiplet with the scalar fields: $S, z^A, \xi^I, \tilde{\xi}_I$ (recall: $I = 0, 1, ..., n_V$, $A = 1, ..., n_V$). The explicit transformation Ω is now much more involved, but it is again basically given by shifts of the axionic scalars $(\xi^I, \tilde{\xi}_I)$, which represent isometries of the scalar manifold and which are related to harmonic functions (see also [45]). However, we do not need the explicit form of Ω; the correct form of the potential can also be derived if one replaces the axionic scalars as follows

$$(\tilde{\xi}_I, \xi^I) \;\rightarrow\; (\tilde{\xi}_I + g\alpha_I t \,,\; \xi^I + g\beta^I t) \tag{51}$$

which for the single axion a in (44) yields exactly the potential (50). Note, that up to a U-duality transformation (inversion of the dilaton field) the transformation Ω corresponds precisely to a linear time shift in the axion. Using the notation from [46] the corresponding part of the hyper-supermultiplet Lagrangian reads

$$e^{2\varphi}(\partial\tilde{\xi}_I + \mathcal{N}_{IL}\partial\xi^L)(\mathrm{Im}\mathcal{N}^{-1})^{IJ}(\partial\tilde{\xi}_J + \bar{\mathcal{N}}_{JK}\partial\xi^K) \tag{52}$$

where the complex matrix $\mathcal{N}$ defines the gauge field couplings in the Lagrangian. This yields, after the time shifts (51), the potential

$$V \sim g^2 e^{2\varphi}(\alpha_I + \mathcal{N}_{IL}\beta^L)(\mathrm{Im}\mathcal{N}^{-1})^{IJ}(\alpha_J + \bar{\mathcal{N}}_{JK}\beta^K) \tag{53}$$

which can be written in the standard potential form appearing in gauged supergravity [46]. As before, after the time-like T-duality one has to do a Wick rotation in time, combined with the replacement $g = i\lambda$, which changes the sign of the potential resulting in a de Sitter instead of anti de Sitter vacuum.

In addition to the shifts in the axionic scalars $(\tilde{\xi}^I, \xi_I)$ one could also allow for shifts in the axion $S + \bar{S}$, but as result, the dilaton now appears in the potential and hence, as a result of the c-map one obtains a model that contains also a universal hyper-supermultiplet. It is of course interesting to explore this possibility further.

In summary, we have seen that a time-dependent background can be obtained by a time-like (massive) T-duality, which includes a Scherk-Schwarz reduction over time. In comparison to massive T-duality over a space-like direction, the overall sign of the potential has changed, but otherwise it is the same potential. Hence, it is similar to the models introduced by Hull [1].

5 Discussion

In this paper we presented a procedure to generate time-dependent backgrounds starting from stationary BPS solutions of ungauged supergravity. We focused on examples of $N=2$ supergravity coupled to vector-supermultiplets. These solutions are given by a set of harmonic functions that can be shifted by linear functions in time and solve the first order differential equations which coincides with the BPS equations of gauged supergravity, but with an imaginary gauge coupling $g = i\lambda$. Since only the R-symmetry is gauged, an imaginary g reflects the non-compactness of the R-symmetry. Since the bosonic fields are not charged under the R-symmetry, the bosonic model is well-defined. All kinetic terms in the bosonic Lagrangian have the correct sign, but the potential has the opposite sign yielding stable de Sitter vacua, instead of anti de Sitter vacua known from supergravity with compact R-symmetry.

We also showed in Section 4 that for the four-dimensional case, these solutions can be generated by a "massive" T-duality over the time, which employs Scherk-Schwarz reductions to map the two models. The model is similar to the type II* models introduced by Hull [1]. In our case all kinetic terms have the correct sign and this was possible due to a truncation of the model on the bosonic sector.

Prototype examples of solutions of five- and four-dimensional are of the form

$$
\begin{aligned}
ds_5 &= -e^{-4U}dt^2 + e^{2U}dx_4^2 \quad, \qquad e^{6U} = \prod_{i=1}^{3}[\alpha_i\, t + h_i(x)] \\
ds_4 &= -e^{-2\hat{U}}dt^2 + e^{2\hat{U}}dx_3^2 \quad, \qquad e^{4\hat{U}} = \prod_{i=1}^{4}[\alpha_i\, t + h_i(x)]
\end{aligned}
\tag{54}
$$

where $h_i(x)$ are arbitrary harmonic functions of the four- or three-dimensional flat transversal spatial space. If all constants α_i vanish, we get back the well-known BPS solutions of ungauged supergravity as, e.g., the multi-center extremal black holes. On the other hand, if these constants are non-zero these multi-center black holes live in an space-time that asymptots at late time to a de Sitter space with the cosmological constant given by $\prod_i \alpha_i$.

These solutions are not supersymmetric, at least not in the usual sense. However, they inherit properties well known from supersymmetric solutions, e.g., the existence of multi-center solutions and the appearance of hyper-Kähler spaces. Hence, we expect that this de Sitter solution is stable, especially because this the solution satisfies first order equations that coincide with the BPS equations for a non-compact R-

symmetry. The existence of multi-center solutions implies a balance of forces, namely the de Sitter expansion compensates the gravitational attraction of the black holes.

Moreover, in the asymptotic de Sitter space the solution is in a minimum of the potential with no tachyonic directions. The potential coincides, up to the overall sign, with the potential appearing in gauged supergravity with compact R-symmetry and this potential has a number of interesting properties [23]. First of all, there is only one extremum of the superpotential on a given component of the scalar manifold and it corresponds to a maximum of the potential. For the model at hand the potential thus has a stable minimum. Also, there are no flat directions, due to the fact, that in the vacuum all scalars of the vector-supermultiplets are fixed. However, the situation changes if one takes into account also scalar fields of hyper-supermultiplets, which cannot be fixed completely in gauged supergravity.

In general, there are no static coordinates to describe these backgrounds. One can introduce static coordinates only around a given center or for the single-center solution [6]. Otherwise, this solution is intrinsically time-dependent and if we naively continue to early times, it exhibits a big bang singularity where the spatial part of the metric collapses to a point. Depending on the choice for the harmonic function, the qualitative behavior near the singularity differs. If e.g. in eq. (54) $e^{4\hat{U}}$ or e^{6U} vanishes quadratically (setting $h_i(x) = const.$) the solution exhibits a singularity as in the Milne universe, otherwise the cosmological expansion near the singularity is decelerating for a single zero or accelerating[4]. Note also, there is no reason to stick to a time $t \geq 0$ in the solutions above, and the singularity is a repulson-type. If this solution can be understood as coming from a compactification, the scalar fields cannot naively be extended to negative values. In fact, the solution may run towards a phase transition point which has to be treated with care. In Subsection 3.3 we argued that, due to α' corrections, the solution can run towards a regular de Sitter space at early time.

A different way of dealing with the singularity is to cut-off the region by introducing a space-like brane at $t = 0$, which is done by replacing: $t \rightarrow |t|$. This however is a subtle issue, because such a replacement requires a brane source. However, one should allow for variations of the location of the brane source and is seems that this

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[4]We would like to thank Miguel S. Costa for a discussion on this point.

will necessarily move the brane towards the singularity, or at least to the point where it becomes tensionless. Although the solution remains basically the same, these s-branes may provide a physical picture of the endpoint of the flow. [Note, the s-branes appearing in our context are closely related to the ones discussed in [47, 1], however, they are of different nature as the ones discussed in [48, 49] and references therein. In addition, the de Sitter vacua appearing in our context are of different nature as the ones discussed for non-compact gaugings as, e.g., in [50].]

We mentioned also the possibility that the flow becomes completely regular if one considers more general models with hyper-multiplets where the superpotential has two continuously connected extrema [10, 35]. These potentials would describe a cosmological scenario with an early and late time de Sitter vacuum, where, depending on the nature of the fixed point, the space-time is exponentially large or small. However, one should investigate whether for this case the truncation to the neutral sector can consistently been done.

Moreover, the examples that we presented here are closely related to BPS solutions and it would be interesting whether the non-extreme solutions as, e.g., discussed in [51], have also a cosmological analog. Moreover, for the four-dimensional model we considered only the models with diagonal metrics, which may be of most interest. However, it remains to be shown that the general stationary BPS solutions can also be generalized to cosmological backgrounds.

Appendix

In this appendix, we will show that the fields given in eqs. (15) and (17) solve in fact the first order differential equations (13). Basically, we will repeat the calculations done in [6], but we use a different notation and in order to compare both results one has to rescale the harmonic functions combined with a reparameterization of the time $(e^{-gt} \rightarrow gt)$.

Since we know the BPS solution as given in eq. (15) and (16) [14], it is sufficient to look at the terms, which are proportional to time-like derivatives – all other terms will cancel, because our starting point was a BPS configuration. Using the relation $X_I dX^I$, see (11), and the harmonic functions as given in eq. (17) one finds

$$\partial_0 e^{2U} = X^I \partial_0(e^{2U} X_I) = \lambda \alpha_I X^I = \lambda W \ . \tag{55}$$

This yields for the term containing the superpotential

$$
\begin{aligned}
g\Gamma_0 W &= g\,\Gamma_{\underline{0}}\,e^{-2U} W = \tfrac{g}{\lambda}\,\dot{U}\,\Gamma_{\underline{0}} \ , \\
g\Gamma_m W &= (e^{-2U}\omega_m \Gamma_{\underline{0}} + e^U \Gamma_{\underline{m}})\,W = (\omega_m \Gamma_{\underline{0}} + e^{3U}\Gamma_{\underline{m}})\tfrac{1}{\lambda}\dot{U}\,W
\end{aligned}
\tag{56}
$$

where we have underlined the tangent space indices. Moreover, the gauge field component gives

$$G_{IJ}F^I = -\frac{1}{2}e^{-4U}[dH_I \wedge (dt + \omega_m dx^m) - H_I d\omega] \tag{57}$$

and because $X^I dH_I = 3de^{2U}$ we find for the time-like component (note factors of 2/3 in (11)!)

$$X_I F^I_{0m} = (\partial_0 e^{-2U})e^{2U} X_I A^I_m - X_I \partial_m A^I_0 = -2\,\dot{U}e^{-2U}\omega_m - \partial_m e^{-2U} \ . \tag{58}$$

Using the inverse metric

$$g^{00} = -e^{4U} + e^{-2U}\omega_m \omega_m \quad , \qquad g^{0n} = -e^{-2U}\omega_n \quad , \qquad g^{mn} = e^{-2U}\delta_{mn} \tag{59}$$

we can also calculate the field strength with upper indices. The fact that $\omega_m \partial_m H_I = \omega_m \partial_m U = 0$ means that $F^{I\,mn}$ does not contain any time derivatives and that

$$G_{IJ}F^{J\,0n} = e^{-2U}\left[\partial_n H_I - \omega_n \partial_0 H_I - e^{-6U}H_I \omega_m \partial_{[n}\omega_{m]}\right] \ . \tag{60}$$

With these expressions it is now straightforward to calculate the expression appearing in (13). We get

$$
\begin{aligned}
X_I(\Gamma_{0mn}F^{I\,mn} - 4F^I_{0n}\Gamma^n) &= 8\,e^{-3U}\dot{U}\omega_n \Gamma^{\underline{n}} + ... \\
X_I(2\,\Gamma_{mn0}F^{I\,n0} - 4F^I_{m0}\Gamma^0) &= -2\,\dot{U}\Gamma_{\underline{mn0}}\,\omega^n - 8\,\dot{U}\omega_m(\Gamma^{\underline{0}} - e^{-3U}\omega_n \Gamma^{\underline{n}}) + ...
\end{aligned}
\tag{61}
$$

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(we dropped all terms that have no time derivatives). Next, for the covariant derivative: $\nabla_\mu \epsilon = (\partial_\mu + \frac{1}{4}\omega_\mu^{mn}\Gamma_{\underline{mn}})\,\epsilon$ we find the modifications due to the time-dependence

$$
\begin{aligned}
\nabla_0\,\epsilon &= (\hat{\nabla}_0 + e^{-3U}\dot{U}\omega_m\Gamma_{\underline{0}}\Gamma^{\underline{m}})\,\epsilon\ , \\
\nabla_m\,\epsilon &= (\hat{\nabla}_m + \dot{U}[e^{-3U}\omega_m\omega_n - \tfrac{1}{2}e^{3U}\delta_{mn}]\Gamma_{\underline{0}}\Gamma^{\underline{n}} - \tfrac{1}{4}\dot{U}\omega_n\Gamma_{\underline{m}}^{\,\underline{n}})\,\epsilon\ .
\end{aligned}
\tag{62}
$$

Inserting all terms into (13) one gets finally the differential equation for ϵ

$$
[\partial_\mu + (\partial_\mu U)]\,\epsilon = 0
\tag{63}
$$

which gives $\epsilon = e^{-U}\epsilon_0$ with the constant spinor ϵ_0 fulfilling the projector equation

$$
\Gamma_{\underline{0}}\epsilon_0 = i\epsilon_0\ .
\tag{64}
$$

As next step, we have to verify the second equation in (13). Inserting the field X_I from eq. (15) we find

$$
\partial_A X^I\Big[\Gamma^\mu \partial_\mu X_I\Big] = \partial_A X^I\Big[(\Gamma^{\underline{0}} - e^{-3U}\omega_m\Gamma^{\underline{m}})\frac{1}{3}\alpha_I\lambda + ...\Big]
\tag{65}
$$

$$
\partial_A X^I\Big[G_{IJ}F^{J\,\mu m}\Gamma_{\mu m}\Big] = \partial_A X^I\Big[-e^{-3U}\omega_m\Gamma_{\underline{0m}}\frac{1}{3}\alpha_I\lambda + ...\Big]
\tag{66}
$$

and with the projector (64) we have verified also this equation.

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