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On the Uniqueness of Plane-wave String Field Theory

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Abstract

We prove that the two interaction Hamiltonians of light-cone closed superstring field theory in the plane-wave background present in the literature are identical.

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1 Introduction

Following the discovery of the plane-wave solution of Type IIB supergravity [1] [1], the spectrum and superalgebra of the free superstring theory in this background were found in the light-cone gauge [4, 5]. The theory possesses a unique groundstate and a tower of states with energies proportional to

$$\omega_n = \sqrt{n^2 + (\mu\alpha' p^+)^2}\,, \tag{1.1}$$

where $n \in \mathbb{Z}$ and μ and p^+ are the R-R field-strength and light-cone momentum respectively. The plane-wave background has also become important because of its interpretation as a Penrose limit [6] of the $AdS_5 \times S^5$ space-time. In this setting, the AdS/CFT correspondence has been identified as a relation between string theory in the large μ limit and the $\mathcal{N} = 4$ $U(N)$ SYM gauge theory in a non-'t Hooft limit where not only N, but also J, a chosen U(1) R-charge, is taken to be large, with the ratio J^2/N fixed [7]. A subset of so-called BMN operators has been identified in the gauge theory which corresponds to string states. These operators have an expansion in terms of an effective coupling constant $\lambda' = g^2_{\rm YM}N/J^2 = (\mu\alpha' p^+)^{-2}$ and effective genus counting parameter $g_2^2 = (J^2/N)^2 = g_s^2(\mu\alpha' p^+)^4$ [8, 9], and the gauge/gravity correspondence in this background is given by [10]

$$\frac{1}{\mu} H_s = \Delta - J\,, \tag{1.2}$$

viewed as an operator identity between the Hilbert spaces of string theory and the BMN sector of gauge theory. This correspondence has been placed on a firm footing at the level of planar graphs, or equivalently at the level of free string theory [7, 8, 9, 10, 11]. At the non-planar/string interaction level there is also good evidence that, at least for so-called impurity preserving amplitudes, the operator identity above is valid [12, 13, 14, 15, 16, 17, 18, 19], see also [20, 21] for recent reviews.

An essential ingredient in the understanding of string theory in the plane-wave background is the knowledge of string interactions. Unfortunately, the background has only been quantized in light-cone gauge and so conformal field theory tools such as vertex operators cannot be used here. [2] The only known way of studying string interactions in the plane-wave comes from light-cone string field theory [23, 24, 25, 26, 27, 28, 29]. In this formalism the generators of the supersymmetry algebra are divided into two sets of operators: the kinematical and the

[1]For previous work on supergravity plane-wave solutions see [2, 3].

[2]In flat space it is possible to develop vertex operator techniques even in light-cone gauge [22]. This is aided by the presence of a classical conformal invariance of the equations of motion in light-cone gauge, as well as by the existence of angular momentum generators J^{-I}.

dynamical. The former, such as the transverse momenta P^I, do not receive corrections in the string coupling g_s, while the latter, which include the Hamiltonian, are modified order by order in the string coupling. For example

$$H_s = H_2 + g_s H_3 + \dots \,, \tag{1.3}$$

where H_2 is the free-string Hamiltonian and H_3 represents the process of one string splitting into two (as well as the time-reversal of this interaction). When computing string interactions it is most convenient to write H_3 as an operator in the three-string Hilbert space [28, 29].

The interaction Hamiltonian H_3 is constructed by requiring two conditions. Firstly, the process is to be smooth on the world-sheet; this is equivalent to demanding the supercommutation relations between the interaction Hamiltonian and the kinematical generators be satisfied. In the operator formalism this is enforced by a coherent state of the three-string Hilbert space often denoted by $|V\rangle$. Secondly, H_3 is required to satisfy the supersymmetry algebra relations involving the Hamiltonian and the dynamical supercharges at next-to-leading order in the string coupling. These conditions require that

$$|H_3\rangle = \mathcal{P}|V\rangle \,, \tag{1.4}$$

where $\mathcal{P}$ is the so-called prefactor which, in the oscillator basis, is polynomial in the creation operators.

Originally [30, 31, 32, 33], when H_3 was constructed in the plane-wave background, the oscillator basis expression was built on the state $|0\rangle$ which has energy 4μ and (hence) is not the groundstate of the theory.[3] Rather, it is smoothly connected to the $SO(8)$ invariant flat space state $|0\rangle_{\mu=0}$ on which the flat spacetime interaction vertex was built [27]. We will refer to H_3 constructed on this state as the $SO(8)$ solution throughout this paper

$$|H_3\rangle_{SO(8)} = \mathcal{P}_{SO(8)}|V\rangle_{SO(8)} \,. \tag{1.5}$$

The presence of the R-R flux in the plane-wave background breaks the transverse $SO(8)$ symmetry of the metric to $SO(4)\times SO(4)\times \mathbb{Z}_2$, where the discrete $\mathbb{Z}_2$ is an $SO(8)$ transformation that exchanges the two transverse $\mathbb{R}^4$ subspaces of the plane-wave. Based on this $\mathbb{Z}_2$ symmetry it was argued [34] that one should in fact construct H_3 on the true groundstate of the theory: $|v\rangle$. A solution of the kinematical constraints based on this state was given in [35], while the dynamical constraints were solved in [36]; this solution will be called the $SO(4)^2$ solution here

$$|H_3\rangle_{SO(4)^2} = \mathcal{P}_{SO(4)^2}|V\rangle_{SO(4)^2} \,. \tag{1.6}$$

[3] For the precise definitions of $|0\rangle$ and $|v\rangle$ see section 2.

The two interaction Hamiltonians appeared to be quite different, and it was not, *a priori* clear, if they should give the same physics.[4]

In this paper we prove that the two interaction Hamiltonians are identical when viewed as operators acting on the three-string Hilbert space. The proof is presented in section 2 for the supergravity modes only, and generalized in section 3 to the full three-string Hamiltonian. Two appendices are provided in which our conventions are summarized and some of the computational details are presented.

2 The equivalence of the $SO(8)$ and $SO(4)^2$ formalisms in supergravity

In this section we prove that the supergravity three-string interaction vertices constructed in the $SO(8)$ formalism in [30] and in the $SO(4)^2$ formalism in [36] are identical to each other. Recall the fermionic part of the light-cone action on the plane wave [4]

$$S_{\mathrm{ferm.}(r)} = \frac{1}{8\pi}\int d\tau \int_0^{2\pi|\alpha_r|} d\sigma_r [i(\bar\vartheta_r\dot\vartheta_r + \vartheta_r\dot{\bar\vartheta}_r) - \vartheta_r\vartheta'_r + \bar\vartheta_r\bar\vartheta'_r - 2\mu\bar\vartheta_r\Pi\vartheta_r]\,, \tag{2.1}$$

where $r = 1,\, 2,\, 3$ denotes the rth string, $\alpha_r \equiv \alpha' p_r^+$ and $e(\alpha_r) \equiv \alpha_r/|\alpha_r|$. ϑ^a_r is a complex, positive chirality SO(8) spinor, $\dot\vartheta_r \equiv \partial_\tau\vartheta_r$, $\vartheta'_r \equiv \partial_{\sigma_r}\vartheta_r$ and $\Pi_{ab} \equiv (\gamma^1\gamma^2\gamma^3\gamma^4)_{ab}$ is symmetric, traceless and squares to one. The mode expansions of ϑ^a_r and its conjugate momentum $\lambda^a_r \equiv \bar\theta^a_r/4\pi$ at $\tau = 0$ are

$$\begin{aligned}
\vartheta^a_r(\sigma_r) &= \vartheta^a_{0(r)} + \sqrt{2}\sum_{n=1}^\infty\big(\vartheta^a_{n(r)}\cos\frac{n\sigma_r}{|\alpha_r|} + \vartheta^a_{-n(r)}\sin\frac{n\sigma_r}{|\alpha_r|}\big)\,,\\
\lambda^a_r(\sigma_r) &= \frac{1}{2\pi|\alpha_r|}\big[\lambda^a_{0(r)} + \sqrt{2}\sum_{n=1}^\infty\big(\lambda^a_{n(r)}\cos\frac{n\sigma_r}{|\alpha_r|} + \lambda^a_{-n(r)}\sin\frac{n\sigma_r}{|\alpha_r|}\big)\big]\,.
\end{aligned} \tag{2.2}$$

The Fourier modes satisfy $2\lambda^a_{n(r)} = |\alpha_r|\bar\vartheta^a_{n(r)}$ and the canonical anti-commutation relations for the fermionic coordinates yield the anti-commutation rules

$$\{\vartheta^a_r(\sigma_r), \lambda^b_s(\sigma_s)\} = \delta^{ab}\delta_{rs}\delta(\sigma_r-\sigma_s) \qquad \Leftrightarrow \qquad \{\vartheta^a_{n(r)}, \lambda^b_{m(s)}\} = \delta^{ab}\delta_{nm}\delta_{rs}\,. \tag{2.3}$$

The fermionic normal modes are defined via ($e(0)\equiv 1$)

$$\vartheta_{n(r)} = \frac{c_{n(r)}}{\sqrt{|\alpha_r|}}\Big[(1+\rho_{n(r)}\Pi)b_{n(r)} + e(\alpha_r)e(n)(1-\rho_{n(r)}\Pi)b^\dagger_{-n(r)}\Big]\,, \qquad n\in\mathbb{Z}\,, \tag{2.4}$$

[4]Some evidence that they were in fact identical was already presented in [36].

and break the $SO(8)$ symmetry to $SO(4)\times SO(4)$. Here

$$\rho_{n(r)}=\rho_{-n(r)}=\frac{\omega_{n(r)}-|n|}{\mu\alpha_r}\,,\qquad c_{n(r)}=c_{-n(r)}=\frac{1}{\sqrt{1+\rho_{n(r)}^2}}\,. \tag{2.5}$$

These modes satisfy $\{b^a_{n(r)},b^{b\,\dagger}_{m(s)}\}=\delta^{ab}\delta_{nm}\delta_{rs}$. The two states $|v\rangle$ and $|0\rangle$, on which the interaction Hamiltonians are constructed, are then annihilated by all $b_n(r)$ for $n\neq 0$ with

$$\theta^a_0|0\rangle=0\,,\qquad b^a_0|v\rangle=0\,. \tag{2.6}$$

We use a γ-matrix representation in which

$$\Pi=\begin{pmatrix}\delta_{\alpha_1}^{\beta_1}\delta_{\alpha_2}^{\beta_2} & 0\\ 0 & -\delta^{\dot\alpha_1}_{\dot\beta_1}\delta^{\dot\alpha_2}_{\dot\beta_2}\end{pmatrix}\,, \tag{2.7}$$

where α_k, $\dot\alpha_k$ (β_k, $\dot\beta_k$) are two-component Weyl indices of $SO(4)_k$.[5] Hence, $(1\pm\Pi)/2$ projects onto the $(\mathbf{2},\mathbf{2})$ and $(\mathbf{2}',\mathbf{2}')$ of $SO(4)\times SO(4)$, respectively, and

$$\{b_{n(r)\,\alpha_1\alpha_2},b^{\beta_1\beta_2\,\dagger}_{m(s)}\}=\delta_{\alpha_1}^{\beta_1}\delta_{\alpha_2}^{\beta_2}\delta_{nm}\delta_{rs}\,,\qquad \{b_{n(r)\,\dot\alpha_1\dot\alpha_2},b^{\dot\beta_1\dot\beta_2\,\dagger}_{m(s)}\}=\delta_{\dot\alpha_1}^{\dot\beta_1}\delta_{\dot\alpha_2}^{\dot\beta_2}\delta_{nm}\delta_{rs}\,. \tag{2.8}$$

The fermionic contribution to the free string light-cone Hamiltonian is

$$H_{2(r)}=\frac{1}{\alpha_r}\sum_{n\in\mathbb{Z}}\omega_{n(r)}\big(b^{\alpha_1\alpha_2\,\dagger}_{n(r)}b_{n(r)\,\alpha_1\alpha_2}+b^{\dot\alpha_1\dot\alpha_2\,\dagger}_{n(r)}b_{n(r)\,\dot\alpha_1\dot\alpha_2}\big)\,, \tag{2.9}$$

and we have neglected the zero-point energy that is canceled by the bosonic contribution.

2.1 The kinematical part of the vertex

The fermionic contributions to $|V\rangle$ - the kinematical part of the supergravity vertices - in the $SO(8)$ and $SO(4)^2$ formalisms are respectively ($\beta_r\equiv-\frac{\alpha_r}{\alpha_3}$ and $\alpha_1+\alpha_2+\alpha_3=0$)

$$|E^0_b\rangle_{SO(8)}=\prod_{a=1}^{8}\left[\sum_{r=1}^{3}\lambda^a_{0(r)}\right]|0\rangle_{123}\,, \tag{2.10}$$

$$|E^0_b\rangle_{SO(4)^2}=\exp\left(\sum_{r=1}^{2}\sqrt{\beta_r}\big(b^{\alpha_1\alpha_2\,\dagger}_{0(3)}b^\dagger_{0(r)\,\alpha_1\alpha_2}+b^{\dot\alpha_1\dot\alpha_2\,\dagger}_{0(3)}b^\dagger_{0(r)\,\dot\alpha_1\dot\alpha_2}\big)\right)|v\rangle_{123}\,. \tag{2.11}$$

[5]See appendix A for our conventions.

To relate these two expressions recall that (*cf.* equation (2.4))

$$\lambda_{0(3)}^{\alpha_1\alpha_2} = -\sqrt{-\frac{\alpha_3}{2}}\, b_{0(3)}^{\alpha_1\alpha_2}\,, \qquad \lambda_{0(3)}^{\dot\alpha_1\dot\alpha_2} = \sqrt{-\frac{\alpha_3}{2}}\, b_{0(3)}^{\dot\alpha_1\dot\alpha_2\,\dagger}\,, \tag{2.12}$$

$$\lambda_{0(r)}^{\alpha_1\alpha_2} = \sqrt{\frac{\alpha_r}{2}}\, b_{0(r)}^{\alpha_1\alpha_2\,\dagger}\,, \qquad \lambda_{0(r)}^{\dot\alpha_1\dot\alpha_2} = \sqrt{\frac{\alpha_r}{2}}\, b_{0(r)}^{\dot\alpha_1\dot\alpha_2}\,, \tag{2.13}$$

and

$$|0\rangle_3 = -\prod_{\alpha_1,\,\alpha_2} b^\dagger_{0(3)\,\alpha_1\alpha_2}|v\rangle_3\,, \quad |0\rangle_r = \prod_{\dot\alpha_1,\,\dot\alpha_2} b^\dagger_{0(r)\,\dot\alpha_1\dot\alpha_2}|v\rangle_r\,. \tag{2.14}$$

The relative sign in (2.14) is not fixed and has been chosen for convenience. Then it is easy to show that

$$|E_b^0\rangle_{SO(8)} = -\left(\frac{\alpha_3}{2}\right)^4 \prod_{\dot\alpha_1,\,\dot\alpha_2}\big(\sqrt{\beta_1}b^\dagger_{0(2)} - \sqrt{\beta_2}b^\dagger_{0(1)}\big)_{\dot\alpha_1\dot\alpha_2}|E_b^0\rangle_{SO(4)^2}\,. \tag{2.15}$$

By construction, both $|E_b^0\rangle_{SO(8)}$ and $|E_b^0\rangle_{SO(4)^2}$ satisfy the world-sheet continuity conditions. Hence, the combination $\prod_{\dot\alpha_1,\,\dot\alpha_2}\big(\sqrt{\beta_1}b^\dagger_{0(2)} - \sqrt{\beta_2}b^\dagger_{0(1)}\big)_{\dot\alpha_1\dot\alpha_2}$ has to commute with the kinematical constraints, and so can be re-written in terms of the (zero-mode of the) fermionic prefactor constituent $Z_{\dot\alpha_1\dot\alpha_2}$ (in the notation of [32]). In fact

$$\left(\frac{2}{\alpha_3}\right)^4(1-4\mu\alpha K)^2|E_b^0\rangle_{SO(8)} = -\prod_{\dot\alpha_1,\,\dot\alpha_2} Z_{0\,\dot\alpha_1\dot\alpha_2}|E_b^0\rangle_{SO(4)^2} \equiv \frac{1}{12}Z_0^4|E_b^0\rangle_{SO(4)^2}\,. \tag{2.16}$$

The factor of $\left(\frac{2}{\alpha_3}\right)^4(1-4\mu\alpha K)^2$ was introduced in the $SO(8)$ formalism as the overall normalization of the cubic vertex.

2.2 Prefactor

In order to proceed further, we have to re-write the prefactor of the $SO(8)$ formulation in a manifestly $SO(4)\times SO(4)$ invariant form using the γ-matrix representation given in appendix A. The prefactor is [33, 30][6]

$$\mathcal{P}_{SO(8)} = \big(K^I\widetilde{K}^J - \frac{\mu\alpha}{\alpha'}\delta^{IJ}\big)v_{IJ}(Y)\,. \tag{2.17}$$

Here K^I and $\widetilde{K}^I$ are the bosonic constituents commuting with the world-sheet continuity conditions (for their explicit expressions see e.g. [33]) and $v_{IJ} = w_{IJ} + y_{IJ}$ with[7]

$$w^{IJ} = \delta^{IJ} + \frac{1}{4!}t^{IJ}_{abcd}Y^aY^bY^cY^d + \frac{1}{8!}\delta^{IJ}\varepsilon_{abcdefgh}Y^a\cdots Y^h\,, \tag{2.18}$$

$$y^{IJ} = -\frac{i}{2!}\gamma^{IJ}_{ab}Y^aY^b - \frac{i}{2\cdot 6!}\gamma^{IJ}_{ab}\varepsilon^{ab}{}_{cdefgh}Y^c\cdots Y^h\,, \tag{2.19}$$

[6]When no confusion arises we will suppress the subscript '0' in what follows.

[7]Compared to [33] we have redefined $\sqrt{-\frac{\alpha'}{\alpha}}Y_{\rm there} = Y_{\rm here}$.

and $t^{IJ}_{abcd} = \gamma^{IK}_{[ab}\gamma^{JK}_{cd]}$. The positive and negative chirality parts of Y^a are[8]

$$Y^{\alpha_1\alpha_2} = \sum_{r=1}^{3}\sum_{n\geq 0}\bar{G}_{n(r)}b^{\dagger\,\alpha_1\alpha_2}_{n(r)}\,, \tag{2.20}$$

$$Y^{\dot\alpha_1\dot\alpha_2} = -(1-4\mu\alpha K)^{-1/2}\sum_{r,s=1}^{2}\varepsilon^{rs}\sqrt{\beta_s}b^{\dot\alpha_1\dot\alpha_2}_{0(r)} + \sum_{r=1}^{3}\sum_{n>0}U_{n(r)}\bar{G}_{n(r)}b^{\dagger\,\dot\alpha_1\dot\alpha_2}_{n(r)}\,, \tag{2.21}$$

where $\bar{G}$ is defined in [36]. Note in particular that the zero-mode of $Y^{\dot\alpha_1\dot\alpha_2}$ is an annihilation operator. If we want to suppress the spinor indices of $Y^{\dot\alpha_1\dot\alpha_2}$, we will denote these components by $\bar{Y}$. We have the useful relations

$$\{Y_{0\,\dot\alpha_1\dot\alpha_2}, Z_0^{\dot\beta_1\dot\beta_2}\} = \delta^{\dot\beta_1}_{\dot\alpha_1}\delta^{\dot\beta_2}_{\dot\alpha_2}\,, \qquad Y_{0\,\dot\alpha_1\dot\alpha_2}|E_b^0\rangle_{SO(4)^2} = 0\,. \tag{2.22}$$

Using identities (A.8)–(A.16) of appendix A, the $SO(8)$ prefactor decomposes into the following $SO(4)\times SO(4)$ expressions[9]

$$\begin{aligned}
K_I\widetilde{K}_J w^{IJ} &= K_I\widetilde{K}_J\delta^{IJ}\Big(1+\frac{1}{144}Y^4\bar{Y}^4\Big) \\
&+ \frac{1}{12}K_i\widetilde{K}_j\Big(\delta^{ij}\big(Y^4+\bar{Y}^4\big) - 3\big(Y^2\bar{Y}^2\big)^{ij}\Big) \\
&- \frac{1}{12}K_{i'}\widetilde{K}_{j'}\Big(\delta^{i'j'}\big(Y^4+\bar{Y}^4\big) + 3\big(Y^2\bar{Y}^2\big)^{i'j'}\Big) \\
&+ \frac{1}{3}\big(K^{\dot\alpha_1\alpha_1}\widetilde{K}^{\dot\alpha_2\alpha_2} + \widetilde{K}^{\dot\alpha_1\alpha_1}K^{\dot\alpha_2\alpha_2}\big)\big(Y^3_{\alpha_1\alpha_2}Y_{\dot\alpha_1\dot\alpha_2} + Y_{\alpha_1\alpha_2}Y^3_{\dot\alpha_1\dot\alpha_2}\big)\,,
\end{aligned} \tag{2.23}$$

and

$$\begin{aligned}
2iK_I\widetilde{K}_J y^{IJ} &= K_i\widetilde{K}_j\Big(Y^{2\,ij}\big(1+\frac{1}{12}\bar{Y}^4\big) + \bar{Y}^{2\,ij}\big(1+\frac{1}{12}Y^4\big)\Big) \\
&+ K_{i'}\widetilde{K}_{j'}\Big(Y^{2\,i'j'}\big(1-\frac{1}{12}\bar{Y}^4\big) + \bar{Y}^{2\,i'j'}\big(1-\frac{1}{12}Y^4\big)\Big) \\
&+ 2\big(K^{\dot\alpha_1\alpha_1}\widetilde{K}^{\dot\alpha_2\alpha_2} - \widetilde{K}^{\dot\alpha_1\alpha_1}K^{\dot\alpha_2\alpha_2}\big)\big(Y_{\alpha_1\alpha_2}Y_{\dot\alpha_1\dot\alpha_2} - \frac{1}{9}Y^3_{\alpha_1\alpha_2}Y^3_{\dot\alpha_1\dot\alpha_2}\big)\,,
\end{aligned} \tag{2.24}$$

where we use the notation of [36], for example

$$K^{\dot\alpha_1\alpha_1} = K^i\sigma^{i\,\dot\alpha_1\alpha_1}\,, \qquad Y^{2ij} = Y^{2\,\alpha_1\beta_1}\sigma^{ij}_{\alpha_1\beta_1}\,, \qquad \big(Y^2\bar{Y}^2\big)^{ij} = Y^{2\,k(i}\bar{Y}^{2\,j)k}\,, \tag{2.25}$$

and $Y^2_{\alpha_1\beta_1}$ etc. are defined in appendix B. Commuting the terms involving $\bar{Y}$ through the Z^4 term in equation (2.16) using equations (2.22) and (B.9)–(B.16), one can show the equivalence of the two interaction Hamiltonians at the supergravity level

$$\big(\mathcal{P}|V\rangle\big)_{SO(8)\,,\,\mathrm{Sugra}} = \big(\mathcal{P}|V\rangle\big)_{SO(4)^2\,,\,\mathrm{Sugra}}\,. \tag{2.26}$$

[8]Here the chirality refers to either of the two $SO(4)$'s.

[9]For the derivation of the decomposition of the $\mathcal{O}(Y^6)$ term see equations (B.17)–(B.21).

Here [36]

$$
\begin{aligned}
\mathcal{P}_{SO(4)^2} = &\big(\frac{1}{2}K^{\dot\alpha_1\alpha_1}\widetilde{K}^{\dot\beta_1\beta_1} - \frac{\mu\alpha}{\alpha'}\varepsilon^{\alpha_1\beta_1}\varepsilon^{\dot\alpha_1\dot\beta_1}\big)t_{\alpha_1\beta_1}(Y)t^*_{\dot\alpha_1\dot\beta_1}(Z) \\
&- \big(\frac{1}{2}K^{\dot\alpha_2\alpha_2}\widetilde{K}^{\dot\beta_2\beta_2} - \frac{\mu\alpha}{\alpha'}\varepsilon^{\alpha_2\beta_2}\varepsilon^{\dot\alpha_2\dot\beta_2}\big)t_{\alpha_2\beta_2}(Y)t^*_{\dot\alpha_2\dot\beta_2}(Z) \\
&- K^{\dot\alpha_1\alpha_1}\widetilde{K}^{\dot\alpha_2\alpha_2}s_{\alpha_1\alpha_2}(Y)s^*_{\dot\alpha_1\dot\alpha_2}(Z) - \widetilde{K}^{\dot\alpha_1\alpha_1}K^{\dot\alpha_2\alpha_2}s^*_{\alpha_1\alpha_2}(Y)s_{\dot\alpha_1\dot\alpha_2}(Z)\,,
\end{aligned} \tag{2.27}
$$

and the spinorial quantities are

$$
s(Y) \equiv Y + \frac{i}{3}Y^3\,, \qquad t(Y) \equiv \varepsilon + iY^2 - \frac{1}{6}Y^4\,. \tag{2.28}
$$

3 Extension to non-zero-modes

In this section, we prove that the string theory three-string interaction vertex constructed in the $SO(8)$ formalism in [30, 31, 32, 33] and in the $SO(4)^2$ formalism in [34, 35, 36] are identical. In the $SO(8)$ formulation, the complete fermionic contribution to the kinematical part of the vertex is [32, 30]

$$
|E_b\rangle_{SO(8)} = \exp\Big[\sum_{r,s=1}^{3}\sum_{m,n=1}^{\infty} b^\dagger_{-m(r)}Q^{rs}_{mn}b^\dagger_{n(s)} - \sqrt{2}\Lambda\sum_{r=1}^{3}\sum_{m=1}^{\infty}Q^r_m b^\dagger_{-m(r)}\Big]|E^0_b\rangle_{SO(8)}\,. \tag{3.1}
$$

In the $SO(4)^2$ formalism the fermionic contribution to the kinematical part of the vertex is [35]

$$
\begin{aligned}
|E_b\rangle_{SO(4)^2} = \exp\Big[&\sum_{r,s=1}^{3}\sum_{m,n=1}^{\infty}\big(b^{\alpha_1\alpha_2\,\dagger}_{-m(r)}b^\dagger_{n(s)\,\alpha_1\alpha_2}\bar{Q}^{rs}_{mn} - b^{\dot\alpha_1\dot\alpha_2\,\dagger}_{-m(r)}b^\dagger_{n(s)\,\dot\alpha_1\dot\alpha_2}\bar{Q}^{sr}_{nm}\big) \\
&- \sqrt{2}\Lambda^{\alpha_1\alpha_2}\sum_{r=1}^{3}\sum_{m=1}^{\infty}\bar{Q}^r_m b^\dagger_{-m(r)\,\alpha_1\alpha_2} + \frac{\alpha}{\sqrt{2}}\Theta^{\dot\alpha_1\dot\alpha_2}\sum_{m=1}^{\infty}\bar{Q}^r_m b^\dagger_{m(r)\,\dot\alpha_1\dot\alpha_2}\Big]|E^0_b\rangle_{SO(4)^2}\,,
\end{aligned} \tag{3.2}
$$

and we have the following relations between the fermionic Neumann matrices of the two vertices

$$
Q^{rs}_{mn} = \left(\frac{1+\Pi}{2} + \frac{1-\Pi}{2}U_{m(r)}U_{n(s)}\right)\bar{Q}^{rs}_{mn}\,, \tag{3.3}
$$

$$
Q^r_m = \left(\frac{1+\Pi}{2} + \frac{1-\Pi}{2}(1-4\mu\alpha K)^{-1}U^{-1}_{m(r)}\right)\bar{Q}^r_m\,. \tag{3.4}
$$

The positive chirality parts of the vertices agree in both formulations. In what follows we concentrate on the contribution with negative chirality. Recall that $\Theta|E^0_b\rangle_{SO(8)} = 0$, ($\alpha_3\Theta \equiv \vartheta_{0(1)} - \vartheta_{0(2)}$) and

$$
\bar{Q}^{sr}_{nm} = \frac{\alpha_r n}{\alpha_s m}\bar{Q}^{rs}_{mn}\,, \tag{3.5}
$$

$$
\bar{Q}^{sr}_{nm} - \big(U_{(r)}\bar{Q}^{rs}U_{(s)}\big)_{mn} = \bar{G}_{m(r)}\big(U_{(s)}\bar{G}_{(s)}\big)_n\,. \tag{3.6}
$$

Equation (3.6) can be derived using the factorization theorem for the bosonic Neumann matrices [37, 32]. Using these identities, one can show that the generalization of (2.16) to include the stringy modes is

$$\left(\frac{2}{\alpha_3}\right)^4 (1-4\mu\alpha K)^2 |E_b\rangle_{SO(8)} = \frac{1}{12} Z^4 |E_b\rangle_{SO(4)^2} \,. \tag{3.7}$$

Finally, note that

$$\{Y_{\dot\alpha_1\dot\alpha_2}, Z_{\dot\beta_1\dot\beta_2}\} = \delta_{\dot\alpha_1}^{\dot\beta_1}\delta_{\dot\alpha_2}^{\dot\beta_2}\,, \qquad Y_{\dot\alpha_1\dot\alpha_2}|E_b\rangle_{SO(4)^2} = 0\,. \tag{3.8}$$

Since equations (3.7) and (3.8) are algebraically the same as (2.16) and (2.22), the results of section 2 imply that

$$\big(\mathcal{P}|V\rangle\big)_{SO(8)} = \big(\mathcal{P}|V\rangle\big)_{SO(4)^2}\,, \tag{3.9}$$

as conjectured in [36].

4 Conclusions

In this paper, we have proved that the plane-wave light-cone superstring field theory Hamiltonians constructed on the states $|0\rangle_{123}$ and $|v\rangle_{123}$ are identical. This analysis could be easily extended to show the equivalence of the dynamical supercharges as well. We have thereby resolved one of the puzzling features of the $SO(4)^2$ formalism, namely that it appeared not to have a smooth $\mu \to 0$ flat space limit to the vertex of [27]. In fact $Z^4|E_b\rangle_{SO(4)^2} \sim |E_b\rangle_{SO(8)}$ and $\mathcal{P}_{SO(4)^2}\bar{Y}^4 \sim \mathcal{P}_{SO(8)}$ have well-defined limits as $\mu \to 0$ rather than $|E_b\rangle_{SO(4)^2}$ and $\mathcal{P}_{SO(4)^2}$. Moreover, since it is known that $|E_b\rangle_{SO(8)}$ and $|E_b\rangle_{SO(4)^2} \sim \bar{Y}^4|E_b\rangle_{SO(8)}$ have opposite $\mathbb{Z}_2$ parity [36, 34], it follows that $\mathcal{P}_{SO(4)^2}$ and $\mathcal{P}_{SO(8)}$ also have opposite parity and, therefore, $\mathcal{P}_{SO(4)^2}$ is odd under the $\mathbb{Z}_2$.

The existence of a smooth flat space limit, together with $\mathbb{Z}_2 \subset SO(8)$ invariance, suggests that the assignment of negative $\mathbb{Z}_2$ parity to $|v\rangle$ (equivalently positive $\mathbb{Z}_2$ parity to $|0\rangle$) is correct: only then the plane-wave interaction Hamiltonian is invariant under $SO(4)\times SO(4)\times\mathbb{Z}_2$ and the latter is continuously connected to the $SO(8)$ symmetry of the flat space vertex. This suggests the uniqueness[10] of the interaction Hamiltonian at this order in the string coupling as a solution of the world-sheet continuity and supersymmetry algebra constraints.[11]

[10]Up to the overall normalization, which due to the absence of the J^{-I} generator can be any suitable function of the light-cone momenta.

[11]Recently, a different solution of these conditions has been presented [38]. However, it does not have a smooth flat space limit and is not $\mathbb{Z}_2$ invariant with the above parity assignment.

The presence of apparently different interaction Hamiltonians has already been encountered in flat space, where two such objects were constructed. These had an explicit $SO(8)$ or $SU(4)$ symmetry, respectively [39], and at first sight appear to be quite different. It is clear that our proof can be easily applied to show that the two expressions are, in fact, equivalent. Similarly for the open string interaction Hamiltonian in the plane-wave background, two apparently different expressions exist [40, 41]. Again our proof can be easily adapted to this case to show that the two are identical as operators in the three-string Hilbert space.

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A Conventions and Notation

The R-R flux in the plane wave geometry breaks the $SO(8)$ symmetry of the metric into $SO(4)\times SO(4)\times \mathbb{Z}_2$. Then

$$\mathbf{8}_v \longrightarrow (\mathbf{4},\mathbf{1})\oplus(\mathbf{1},\mathbf{4})\,,\qquad \mathbf{8}_s \longrightarrow (\mathbf{2},\mathbf{2})\oplus(\mathbf{2}',\mathbf{2}')\,,\qquad \mathbf{8}_c \longrightarrow (\mathbf{2},\mathbf{2}')\oplus(\mathbf{2}',\mathbf{2})\,, \tag{A.1}$$

where $\mathbf{2}$ and $\mathbf{2}'$ are the inequivalent Weyl representations of $SO(4)$. We decompose $\gamma^I_{a\dot a}$ and $\gamma^I_{\dot a a}$ into $SO(4)\times SO(4)$ as follows

$$\gamma^i_{a\dot a} = \begin{pmatrix} 0 & \sigma^i{}_{\alpha_1\dot\beta_1}\delta^{\beta_2}_{\alpha_2} \\ \sigma^{i\,\dot\alpha_1\beta_1}\delta^{\dot\alpha_2}_{\dot\beta_2} & 0 \end{pmatrix}\,,\qquad \gamma^i_{\dot a a} = \begin{pmatrix} 0 & \sigma^i{}_{\alpha_1\dot\beta_1}\delta^{\dot\alpha_2}_{\dot\beta_2} \\ \sigma^{i\,\dot\alpha_1\beta_1}\delta^{\beta_2}_{\alpha_2} & 0 \end{pmatrix}\,, \tag{A.2}$$

$$\gamma^{i'}_{a\dot a} = \begin{pmatrix} -\delta^{\beta_1}_{\alpha_1}\sigma^{i'}{}_{\alpha_2\dot\beta_2} & 0 \\ 0 & \delta^{\dot\alpha_1}_{\dot\beta_1}\sigma^{i'\,\dot\alpha_2\beta_2} \end{pmatrix}\,,\qquad \gamma^{i'}_{\dot a a} = \begin{pmatrix} -\delta^{\beta_1}_{\alpha_1}\sigma^{i'\,\dot\alpha_2\beta_2} & 0 \\ 0 & \delta^{\dot\alpha_1}_{\dot\beta_1}\sigma^{i'}{}_{\alpha_2\dot\beta_2} \end{pmatrix}\,. \tag{A.3}$$

Here, the σ-matrices consist of the usual Pauli-matrices, together with the 2d unit matrix

$$\sigma^i_{\alpha\dot\alpha} = \left(i\tau^1, i\tau^2, i\tau^3, -1\right)_{\alpha\dot\alpha} \tag{A.4}$$

and we raise and lower spinor indices with the two-dimensional Levi-Civita symbols, e.g.

$$\sigma^i_{\alpha\dot{\alpha}} = \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\sigma^{i\,\dot{\beta}\beta} \equiv \varepsilon_{\alpha\beta}\sigma^{i\,\beta}{}_{\dot{\alpha}} \equiv \varepsilon_{\dot{\alpha}\dot{\beta}}\sigma^{i\,\dot{\beta}}{}_{\alpha}\,. \tag{A.5}$$

The σ-matrices obey the relations

$$\sigma^i_{\alpha\dot{\alpha}}\sigma^{j\,\dot{\alpha}\beta} + \sigma^j_{\alpha\dot{\alpha}}\sigma^{i\,\dot{\alpha}\beta} = 2\delta^{ij}\delta^\beta_\alpha\,, \qquad \sigma^{i\,\dot{\alpha}\alpha}\sigma^j_{\alpha\dot{\beta}} + \sigma^{j\,\dot{\alpha}\alpha}\sigma^i_{\alpha\dot{\beta}} = 2\delta^{ij}\delta^{\dot{\alpha}}_{\dot{\beta}}\,. \tag{A.6}$$

In particular, in this basis

$$\Pi_{ab} = \begin{pmatrix} \left(\sigma^1\sigma^2\sigma^3\sigma^4\right)^{\beta_1}_{\alpha_1}\delta^{\beta_2}_{\alpha_2} & 0 \\ 0 & \left(\sigma^1\sigma^2\sigma^3\sigma^4\right)^{\dot{\alpha}_1}_{\dot{\beta}_1}\delta^{\dot{\alpha}_2}_{\dot{\beta}_2} \end{pmatrix} = \begin{pmatrix} \delta^{\beta_1}_{\alpha_1}\delta^{\beta_2}_{\alpha_2} & 0 \\ 0 & -\delta^{\dot{\alpha}_1}_{\dot{\beta}_1}\delta^{\dot{\alpha}_2}_{\dot{\beta}_2} \end{pmatrix}, \tag{A.7}$$

and $(1 \pm \Pi)/2$ projects onto $(\mathbf{2},\mathbf{2})$ and $(\mathbf{2}',\mathbf{2}')$, respectively. The following identities are used throught the paper

$$\varepsilon_{\alpha\beta}\varepsilon^{\gamma\delta} = \delta^\delta_\alpha\delta^\gamma_\beta - \delta^\gamma_\alpha\delta^\delta_\beta\,, \tag{A.8}$$

$$\sigma^i_{\alpha\dot{\beta}}\sigma^{j\,\dot{\beta}}{}_{\beta} = -\delta^{ij}\varepsilon_{\alpha\beta} + \sigma^{ij}_{\alpha\beta}\,, \qquad (\sigma^{ij}_{\alpha\beta} \equiv \sigma^{[i}_{\alpha\dot{\alpha}}\sigma^{j]\,\dot{\alpha}}{}_{\beta} = \sigma^{ij}_{\beta\alpha}) \tag{A.9}$$

$$\sigma^i_{\alpha\dot{\alpha}}\sigma^{j\,\alpha}{}_{\dot{\beta}} = -\delta^{ij}\varepsilon_{\dot{\alpha}\dot{\beta}} + \sigma^{ij}_{\dot{\alpha}\dot{\beta}}\,, \qquad (\sigma^{ij}_{\dot{\alpha}\dot{\beta}} \equiv \sigma^{[i}_{\alpha\dot{\alpha}}\sigma^{j]\,\alpha}{}_{\dot{\beta}} = \sigma^{ij}_{\dot{\beta}\dot{\alpha}}) \tag{A.10}$$

$$\sigma^k_{\alpha\dot{\alpha}}\sigma^k_{\beta\dot{\beta}} = 2\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\,, \tag{A.11}$$

$$\sigma^{ik}_{\alpha\beta}\sigma^k_{\gamma\dot{\delta}} = \varepsilon_{\alpha\gamma}\sigma^i_{\beta\dot{\delta}} + \varepsilon_{\beta\gamma}\sigma^i_{\gamma\dot{\delta}}\,, \tag{A.12}$$

$$\sigma^{ik}_{\alpha\beta}\sigma^{jk}_{\gamma\delta} = \delta^{ij}(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma}) - \frac{1}{2}\left(\sigma^{ij}_{\alpha\gamma}\varepsilon_{\beta\delta} + \sigma^{ij}_{\beta\delta}\varepsilon_{\alpha\gamma} + \sigma^{ij}_{\alpha\delta}\varepsilon_{\beta\gamma} + \sigma^{ij}_{\beta\gamma}\varepsilon_{\alpha\delta}\right), \tag{A.13}$$

$$\sigma^{kl}_{\alpha\beta}\sigma^{kl}_{\gamma\delta} = 4(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma})\,, \tag{A.14}$$

$$\sigma^{kl}_{\alpha\beta}\sigma^{kl}_{\dot{\gamma}\dot{\delta}} = 0\,, \tag{A.15}$$

$$2\sigma^i_{\alpha\dot{\alpha}}\sigma^j_{\beta\dot{\beta}} = \delta^{ij}\varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}} + \sigma^{k(i}_{\alpha_1\beta_1}\sigma^{j)k}_{\dot{\alpha}_1\dot{\beta}_1} - \varepsilon_{\alpha\beta}\sigma^{ij}_{\dot{\alpha}\dot{\beta}} - \sigma^{ij}_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\,. \tag{A.16}$$

B Useful relations

We define the following quantities, which are quadratic in Y and symmetric in spinor indices

$$Y^2_{\alpha_1\beta_1} \equiv Y_{\alpha_1\alpha_2}Y^{\alpha_2}_{\beta_1}\,, \qquad Y^2_{\alpha_2\beta_2} \equiv Y_{\alpha_1\alpha_2}Y^{\alpha_1}_{\beta_2}\,, \tag{B.1}$$

cubic in Y

$$Y^3_{\alpha_1\beta_2} \equiv Y^2_{\alpha_1\beta_1}Y^{\beta_1}_{\beta_2} = -Y^2_{\beta_2\alpha_2}Y^{\alpha_2}_{\alpha_1}\,, \tag{B.2}$$

and, finally, quartic in Y and antisymmetric in spinor indices

$$Y^4_{\alpha_1\beta_1} \equiv Y^2_{\alpha_1\gamma_1}Y^{2\,\gamma_1}{}_{\beta_1} = -\frac{1}{2}\varepsilon_{\alpha_1\beta_1}Y^4\,, \qquad Y^4_{\alpha_2\beta_2} \equiv Y^2_{\alpha_2\gamma_2}Y^{2\,\gamma_2}{}_{\beta_2} = \frac{1}{2}\varepsilon_{\alpha_2\beta_2}Y^4\,, \tag{B.3}$$

where

$$Y^4 \equiv Y^2_{\alpha_1\beta_1} Y^{2\,\alpha_1\beta_1} = -Y^2_{\alpha_2\beta_2} Y^{2\,\alpha_2\beta_2}\,. \tag{B.4}$$

These multi-linears in Y satisfy

$$Y_{\alpha_1\alpha_2} Y_{\beta_1\beta_2} = -\frac{1}{2}\big(\varepsilon_{\alpha_1\beta_1} Y^2_{\alpha_2\beta_2} + \varepsilon_{\alpha_2\beta_2} Y^2_{\alpha_1\beta_1}\big)\,, \tag{B.5}$$

$$Y_{\alpha_1\alpha_2} Y^2_{\beta_2\gamma_2} = -\frac{1}{3}\big(\varepsilon_{\alpha_2\gamma_2} Y^3_{\alpha_1\beta_2} + \varepsilon_{\alpha_2\beta_2} Y^3_{\alpha_1\gamma_2}\big)\,, \tag{B.6}$$

$$Y_{\alpha_1\alpha_2} Y^2_{\beta_1\gamma_1} = \frac{1}{3}\big(\varepsilon_{\alpha_1\beta_1} Y^3_{\gamma_1\alpha_2} + \varepsilon_{\alpha_1\gamma_1} Y^3_{\alpha_1\alpha_2}\big)\,, \tag{B.7}$$

$$Y^3_{\beta_1\gamma_2} Y_{\alpha_1\delta_2} = \frac{1}{4}\varepsilon_{\beta_1\alpha_1}\varepsilon_{\gamma_2\delta_2} Y^4\,. \tag{B.8}$$

Analogous relations hold for Z.

To derive equations (2.16) and (3.7) we need the following (anti)commutators

$$[Y_{\dot\alpha_1\dot\alpha_2}, Z^2_{\dot\beta_1\dot\gamma_1}] = \varepsilon_{\dot\alpha_1\dot\beta_1} Z_{\dot\gamma_1\dot\alpha_2} + \varepsilon_{\dot\alpha_1\dot\gamma_1} Z_{\dot\beta_1\dot\alpha_2}\,, \tag{B.9}$$

$$[Y_{\dot\alpha_1\dot\alpha_2}, Z^2_{\dot\beta_2\dot\gamma_2}] = \varepsilon_{\dot\alpha_2\dot\beta_2} Z_{\dot\alpha_1\dot\gamma_2} + \varepsilon_{\dot\alpha_2\dot\gamma_2} Z_{\dot\alpha_1\dot\beta_2}\,, \tag{B.10}$$

$$\{Y_{\dot\alpha_1\dot\alpha_2}, Z^3_{\dot\beta_1\dot\beta_2}\} = -3 Z_{\dot\alpha_1\dot\beta_2} Z_{\dot\beta_1\dot\alpha_2}\,, \tag{B.11}$$

$$[Y_{\dot\alpha_1\dot\alpha_2}, Z^4] = -4 Z^3_{\dot\alpha_1\dot\alpha_2}\,, \tag{B.12}$$

$$[Y^2_{\dot\alpha_1\dot\beta_1}, Z^4]|E_b\rangle_{SO(4)^2} = -12 Z^2_{\dot\alpha_1\dot\beta_1}|E_b\rangle_{SO(4)^2}\,, \tag{B.13}$$

$$[Y^2_{\dot\alpha_2\dot\beta_2}, Z^4]|E_b\rangle_{SO(4)^2} = 12 Z^2_{\dot\alpha_2\dot\beta_2}|E_b\rangle_{SO(4)^2}\,, \tag{B.14}$$

$$[Y^3_{\dot\alpha_1\dot\alpha_2}, Z^4]|E_b\rangle_{SO(4)^2} = -36 Z_{\dot\alpha_1\dot\alpha_2}|E_b\rangle_{SO(4)^2}\,, \tag{B.15}$$

$$[\bar Y^4, Z^4]|E_b\rangle_{SO(4)^2} = 144|E_b\rangle_{SO(4)^2}\,, \tag{B.16}$$

Finally, to rewrite the $\mathcal{O}(Y^6)$ term in the $SO(8)$ prefactor in a manifestly $SO(4)\times SO(4)$ invariant form, it is useful to employ the identity [27]

$$-\frac{1}{6!}\gamma^{IJ}_{ab}\,\varepsilon^{ab}{}_{cdefgh} Y^c Y^d Y^e Y^f Y^g Y^h = \int d^8\Lambda\,\gamma^{IJ}_{ab}\Lambda^a\Lambda^b e^{-Y\cdot\Lambda}\,, \tag{B.17}$$

and

$$\int \prod_{\alpha_1\alpha_2} d\Lambda_{\alpha_1\alpha_2}\, e^{-Y_{\gamma_1\gamma_2}\Lambda^{\gamma_1\gamma_2}} = -\frac{1}{12} Y^4\,, \tag{B.18}$$

$$\int \prod_{\alpha_1\alpha_2} d\Lambda_{\alpha_1\alpha_2}\,\Lambda_{\beta_1\beta_2} e^{-Y_{\gamma_1\gamma_2}\Lambda^{\gamma_1\gamma_2}} = -\frac{1}{3} Y^3_{\beta_1\beta_2}\,, \tag{B.19}$$

$$\int \prod_{\alpha_1\alpha_2} d\Lambda_{\alpha_1\alpha_2}\,\Lambda^2_{\beta_1\delta_1} e^{-Y_{\gamma_1\gamma_2}\Lambda^{\gamma_1\gamma_2}} = Y^2_{\beta_1\delta_1}\,, \tag{B.20}$$

$$\int \prod_{\alpha_1\alpha_2} d\Lambda_{\alpha_1\alpha_2}\,\Lambda^2_{\beta_2\delta_2} e^{-Y_{\gamma_1\gamma_2}\Lambda^{\gamma_1\gamma_2}} = -Y^2_{\beta_2\delta_2}\,. \tag{B.21}$$

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