Singular 7-manifolds with G_2 holonomy and intersecting 6-branes

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ABSTRACT

A 7-manifold with G_2 holonomy can be constructed as a $\mathbb{R}^3$ bundle over a quaternionic space. We consider a quaternionic base space which is singular and its metric depends on three parameters, where one of them corresponds to an interpolation between $\mathbb{S}^4$ and $\mathbb{CP}^2$ or its non-compact analogs. This 4-d Einstein space has four isometries and the fixed point set of a generic Killing vector is discussed. When embedded into M-theory the compactification over a given Killing vector gives intersecting 6-branes as IIA configuration and we argue that membrane instantons may resolve the curvature singularity.

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1 Introduction

Minimal supersymmetric field theories in four dimensions have many phenomenologically interesting features and can be obtained by Calabi-Yau compactification of heterotic string theory. Over the years this has been the standard way for their construction, because one obtains naturally chiral fermions and non-Abelian gauge groups. On the other hand, it is much more difficult to obtain phenomenological interesting $N = 1$ models directly from M-theory, see [1, 2]. One reason is that the 7-manifold has to have G_2-holonomy and these spaces are not yet well understood. Only few non-compact examples, that rely on the construction done in [3, 4], are explicitly known. Another reason are the difficulties to obtain a model with chiral fermions in four dimensions [5]. In fact, chiral fermions, but also non-Abelian gauge groups, require that the 7-manifold is singular [6, 7, 8, 9] and a supergravity approximation may become questionable.

There are mainly two classes of known metrics: one is topologically a $\mathbb{R}^3$ bundle over a quaternionic space and the other a $\mathbb{R}^4$ bundle over $\mathbb{S}^3$. They have been introduced in [3, 4] and many generalizations, with more parameters or functions, have been discussed in the past year. It is impossible to give a complete list of refs., but relevant aspects for our consideration can be found in [10, 11, 12, 9, 13, 14, 15, 16, 17, 18]. In the first class one can further distinguish between different quaternionic spaces as e.g. the 4-sphere $\mathbb{S}^4 = \frac{SO(5)}{SO(4)}$ but also the complex projective space $\mathbb{CP}^2 = \frac{SU(3)}{U(2)}$, which are the only compact homogeneous quaternionic 4-dimensional spaces [19]. Apart from their non-compact analogs, there are also non-homogeneous quaternionic spaces, which we want to use for the construction of explicit metrics.

Quaternionic spaces have been discussed in the physical literature mainly as moduli spaces of $N = 2$ supergravity in four and five dimensions [20, 21], for a more recent discussion see also [22, 23]. The quantum moduli space is expected to be non-homogeneous, but unfortunately not many examples are explicitly known. In a long quest to incorporate brane world scenarios into gauged supergravity and to overcome numerous problems, see e.g. [24, 25], a non-homogeneous quaternionic space has been explored recently in [26]. This space interpolates between the two homogeneous spaces and it is obvious to consider this space also in the construction of metrics with G_2 holonomy and therewith "to unify" the two spaces representing $\mathbb{R}^3$ bundles over $\mathbb{S}^4$ and $\mathbb{CP}^2$. More important is however, that the Killing vectors of this non-homogeneous space have additional fixed points which become additional D6-branes after compactification along this Killing vector [27]. To be more clear, a co-dimension four fixed point set of a given isometry extends in 6+1 dimensions representing a NUT (=point) on the quaternionic space can be identified as a 6-brane. There are also co-dimension two fixed points, but the interpretation of these bolts on the quaternionic space is unclear.

As it has been shown in [28] on any component of a non-compact *homogeneous* quaternionic spaces can be at most one fixed point and for compact we expect at most two. Hence, in order to find a configuration with three or more 6-branes one necessarily needs in this setup non-homogeneous quaternionic spaces. As we will see,

depending on the choice of parameters, our space can have up to five NUT fixed points. The additional fixed points reflect the topological non-trivial nature of this manifold, because, due to the Lefschetz fixed point theorem, every NUT fixed point adds one unit to the Euler characteristic of the manifold, see also [9, 14].

The paper is organized as follows. In the next section we basically follow the literature and solve the Killing spinor equations followed by a discussion of the closed and co-closed 3-form. This consideration is very general without using any specific quaternionic space. In section 3 we will investigate examples corresponding to different quaternionic spaces. In section 3.1 we start with the homogenous cases and in section 3.2 we discuss the following basic features of the non-homogenous space: it has four Killing vectors; it exhibits a curvature singularity, which separates two asymptotic regions and finally it interpolates between the two homogeneous spaces. In section 4 we give a detailed analysis of the isometries and the fixed point set; many details for the non-compact case were already derived in [26]. Due to their interpretation as D6-branes, we especially identify NUT fixed points. For compact quaternionic spaces there can be three or five and for non-compact we found three or four non-degenerate fixed points, depending on the isometry groups $U(1) \times SU(2)$ or $U(1) \times SL(2, R)$. Unfortunately, in the IIA description the string coupling constant is not bounded – every model contains at least one non-compact direction in which the dilaton diverges. We conclude with a discussion of two aspects that could be interesting for future investigations: (i) resolution of the curvature singularity e.g. by of membrane instantons (or general G-fluxes) and (ii) the construction of new Spin(7) manifold in complete analogy to the procedure of this paper.

2 Constructing metrics with G_2 holonomy

Before we can discuss explicit examples let us summarize some aspects of the procedure described in [3, 29, 30] which will also fix our notations. After discussing the ansatz for the metric, we will derive the Killing spinor and the closed and co-closed 3-form. Both conditions are sufficient to ensure supersymmetry for this background.

2.1 The metric ansatz

The construction of the 7-manifold relies on a 4-d quaternionic base space and before we discuss the metric ansatz we need some basic properties of these spaces; for a recent resume about quaternionic geometry we refer to [22] and the appendix of [23]. Quaternionic spaces are generalizations of complex spaces that allow for three complex structures J^i $(i = 1, 2, 3)$ defined by the algebra

$$J^i \cdot J^j = -\mathbb{I}\, \delta^{ij} + \epsilon^{ijk} J^k \tag{1}$$

and denoting the quaternionic vielbein by e^m, one obtains three 2-forms Ω^i by

$$\Omega^i = e^m \wedge J^i_{mn} e^n \ . \tag{2}$$

The holonomy of a 4n-dimensional quaternionic spaces is contained in $Sp(n) \times SU(2)$. This statement is trivial for $n = 1$ and is replaced by the requirement that the Weyl-tensor of 4-dimensional quaternionic space has to be anti-selfdual [20]

$$W + {}^\star W = 0 \ .$$

For a quaternionic space in any dimensions the triplet of 2-forms Ω^i is expressed in terms of the $SU(2)$-part of the quaternionic connection A^i as

$$dA^i + \frac{1}{2} \epsilon^{ijk} A^j \wedge A^k = \kappa \, \Omega^i \tag{3}$$

which ensures that the triplet of 2-forms is covariantly constant. Moreover, any quaternionic space is an Einstein space with the curvature κ implying that its metric g_{mn} solves the equation

$$R_{mn} = 3 \, \kappa \, g_{mn} \ . \tag{4}$$

The complex structures can be selfdual or anti-selfdual and in our notation we will take the latter one $(J^i_{mn} = -\frac{1}{2}\epsilon_{mnpq} J^i_{pq})$ so that the triplet of 2-forms can be written as

$$\Omega^1 = e^4 \wedge e^7 - e^5 \wedge e^6 \ ,$$
$$\Omega^2 = e^4 \wedge e^6 + e^5 \wedge e^7 \ , \tag{5}$$
$$\Omega^3 = -e^4 \wedge e^5 + e^6 \wedge e^7 \ .$$

Moreover, the $SU(2)$ connection is given as the anti-selfdual part of the spin connection ω^{mn} of the quaternionic space

$$A^i = \frac{1}{2}\omega^{mn} J^i_{mn} \ . \tag{6}$$

In the same way, the selfdual part gives the $Sp(n)$ connection.

Having the basic relations for the quaternionic base space, the metric of the 7-manifold is introduced by the ansatz [3, 4]

$$\hat{ds}^2 = e^{2f}\alpha^i\alpha^i + e^{2g}e^m e^m \tag{7}$$

where e^m $(m = 4...7)$ is the vielbein of the quaternionic space and α^i $(i = 1..3)$ is defined by

$$\alpha^i \equiv \nabla u^i = du^i + \epsilon^{ijk}A^j u^k \tag{8}$$

with u^i as local coordinates (in addition to the quaternionic once). Using the relation (3), it is straightforward to verify that

$$\nabla\alpha^i \equiv d\alpha^i + \epsilon^{ijk}A^j\alpha^k = \kappa \, \epsilon^{ijk} \, \Omega^j \, u^k \ . \tag{9}$$

The two unknown functions f and g in the metric ansatz are now fixed by the requirement of G_2 holonomy implying the existence of a Killing spinor or equivalently the existence of a closed and co-closed 3-form. We will check both conditions.
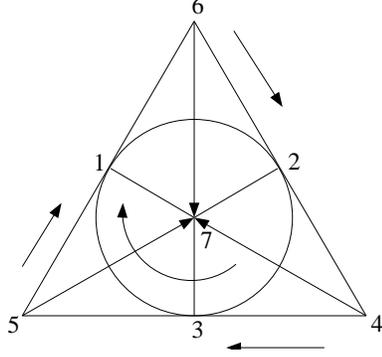
Figure 1: The multiplication table of imaginary components of octonians: $i_1 i_2 = i_3$, $i_6 i_2 = i_4$, $i_4 i_7 = i_1$, ... can be obtained from this figure by following the lines in the direction of the arrows.

2.2 Solving the Killing spinor equations

Consider M-theory on the manifold $M_4 \times X_7$ and assume that M_4 is the flat 4-d Minkowski space. If we moreover assume the absence of G-fluxes, a Killing spinor ϵ is a solution of the equation $(a, b = 1...7)$

$$(d + \frac{1}{4}\hat{\omega}^{ab}\Gamma_{ab})\, \epsilon = 0 \tag{10}$$

where $\hat{\omega}^{ab}$ is the spin connection 1-form and $\Gamma_{ab} = \Gamma_{[a}\Gamma_{b]}$ with Γ_a as the 7-d gamma matrices. If this equation has exactly one solution, the resulting 4-dimensional field theory has $N = 1$ supersymmetry and the manifold X_7 has G_2 holonomy. Recall, the unrestricted holonomy of a 7-manifold is $SO(7)$, but the existence of a (covariantly constant) Killing spinor implies that the holonomy of the manifold is restricted. A generic spinor transforms as representation $\mathbf{8}$ of $SO(7)$, while a Killing spinor implies the decomposition $\mathbf{8} \to \mathbf{7} + \mathbf{1}$, which is exactly the decomposition under $G_2 \subset SO(7)$. The exceptional group G_2 appears as automorphism group of octonions: $o = x^0\mathbb{I} + x^a i_a$, where i_a satisfy the algebra

$$i_a i_b = -\delta_{ab} + \psi_{abc}\, i_c \ ,$$

for more details see e.g. in [31, 29, 32, 33]. The G_2-invariant 3-index tensor ψ_{abc} can be obtained from figure 1 and is given in the standard basis by

$$
\begin{aligned}
\tfrac{1}{3!}\psi_{abc}\, e^a \wedge e^b \wedge e^c \ =\ & e^1 \wedge e^2 \wedge e^3 + e^4 \wedge e^3 \wedge e^5 + e^5 \wedge e^1 \wedge e^6 + e^6 \wedge e^2 \wedge e^4 + \\
& e^4 \wedge e^7 \wedge e^1 + e^5 \wedge e^7 \wedge e^2 + e^6 \wedge e^7 \wedge e^3 \ , \\
=\ & e^1 \wedge e^2 \wedge e^3 + e^i \wedge \Omega^i
\end{aligned}
\tag{11}
$$

where we used the complex structures as introduced in (2) and (5).

If the Killing spinor equation (10) has more than one solution, the holonomy is smaller than G_2 resulting in a 4-d field theory with $N > 1$ supersymmetry. The

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holonomy is equal to G_2 if the Killing spinor equation has exactly one solution, which is equivalent to the absence of covariantly constant 1-forms [3]. The existence of such a 1-form would imply the existence of a covariantly constant Killing vector and hence to a factorization of the space. As we will discuss in the next section this is not the case for the examples that we consider and hence our models yield $N = 1$ supersymmetry in 4 dimensions.

Following the arguments from [29, 30] (see also [34, 35]), one can define two orthogonal projectors $\mathbb{P}_\pm$ that decompose the spin connection into two parts: $\hat{\omega}^{ab} = \hat{\omega}_+^{ab} + \hat{\omega}_-^{ab}$ with

$$
\begin{aligned}
\hat{\omega}_+^{ab} &= \mathbb{P}_+^{ab}{}_{cd}\,\hat{\omega}^{cd} \equiv \tfrac{2}{3}(\hat{\omega}^{ab} + \tfrac{1}{4}\psi^{abcd}\hat{\omega}_{cd}) \;, \\
\hat{\omega}_-^{ab} &= \mathbb{P}_-^{ab}{}_{cd}\,\hat{\omega}^{cd} \equiv \tfrac{1}{3}(\hat{\omega}^{ab} - \tfrac{1}{2}\psi^{abcd}\hat{\omega}_{cd})
\end{aligned}
\tag{12}
$$

where ψ_{abcd} is the G_2 invariant 4-index tensor which is dual to the 3-index tensor $(\psi_{abcd} = \tfrac{1}{3!}\epsilon_{abcdefg}\,\psi_{efg})$. In order to solve (10) one imposes on the (constant) Killing spinor ϵ and the spin connection $\hat{\omega}^{cd}$ the projector equations

$$
\mathbb{P}_+^{ab}{}_{cd}\,\Gamma^{cd}\epsilon = 0 \quad, \qquad \mathbb{P}_-^{ab}{}_{cd}\,\hat{\omega}^{cd} = 0
\tag{13}
$$

and finds as solution for the Killing spinor [29]

$$
\epsilon^\alpha = c\,\delta^{\alpha 8}
\tag{14}
$$

where $\alpha = 1\dots 8$ is the $SO(7)$ spinor index and c is constant. The projector condition on the spin connection can be simplified by contracting with the 3-index tensor ψ_{abc} and using the relation: $\psi_{abc}\psi_{bcde} = -4\psi_{abc}$. One infers [30]

$$
\psi_{abc}\,\hat{\omega}^{bc} = 0
\tag{15}
$$

yielding first order differential equations for the unknown functions. We will assume in the following that in the metric ansatz (7) $f = f(|u|)$ and $g = g(|u|)$ and obtain for the spin connection

$$
\begin{aligned}
\hat{\omega}^{ij} &= \tfrac{f'}{|u|}\,\alpha^{[i}u^{j]} - \epsilon^{ijk}A^k \;, \\
\hat{\omega}^{mn} &= \omega^{mn} - \kappa e^{2(f-g)}\epsilon^{ijk}u^k J_{mn}^j\alpha^i \;, \\
\hat{\omega}^{mi} &= \tfrac{g'}{|u|}e^{g-f}\,e^m u^i - \kappa e^{f-g}\,\epsilon^{ijk}\,u^k J_{mn}^j e^n
\end{aligned}
\tag{16}
$$

where $(...)'$ denotes the derivative with respect to $|u|$. In the contraction with the 3-index tensor (11) we use the relation (6) and find

$$
\begin{aligned}
0 &= \psi_{iab}\,\hat{\omega}^{ab} = \psi_{ijk}\hat{\omega}^{jk} + \psi_{imn}\hat{\omega}^{mn} = \Big[\tfrac{f'}{|u|} + \kappa\,e^{2(f-g)}\Big]\epsilon^{ijk}\,\alpha^j u^k \;, \\
0 &= \psi_{mab}\,\hat{\omega}^{ab} = \psi_{mni}\,\hat{\omega}^{ni} = J_{mn}^i\,\hat{\omega}^{ni} = \Big[\tfrac{g'}{|u|} - \kappa\,e^{2(f-g)}\Big]e^{g-f}\,J_{mn}^i e^n\,u^i \;.
\end{aligned}
\tag{17}
$$

These equations have the symmetry: $f \to f+\lambda$, $g \to g+\lambda$, giving a constant conformal rescaling of the metric (7) and a second symmetry: $f \to f - \lambda$, $u \to e^\lambda u$ leaves the

metric invariant. Using these symmetries, only one (discrete) integration constant $c = \pm 1, 0$ appears in the solution that can be written as [3]

$$e^{-4f} = 2\kappa |u|^2 + c \qquad , \qquad e^{4g} = 2\kappa |u|^2 + c \tag{18}$$

with $|u|^2 = (u^1)^2 + (u^2)^2 + (u^3)^2$. Notice if $\kappa < 0$ (i.e. a non-compact quaternionic space), the parameter range of u is bounded by $2\kappa |u|^2 + c > 0$, while for $\kappa \geq 0$ u is unbounded.

2.3 The closed and co-closed 3-form

The holonomy reduction from $SO(7)$ to G_2 is equivalent to the existence a 3-form Φ which is closed and co-closed. Again following the procedure done in [3] we write this 3-form as

$$\Phi = \frac{1}{3!} \psi_{abc} \, \hat{e}^a \wedge \hat{e}^b \wedge \hat{e}^c = e^{3f} \, \alpha^1 \wedge \alpha^2 \wedge \alpha^3 + e^{f+2g} \, \alpha^i \wedge \Omega^i \tag{19}$$

and we will show now that the functions (18) ensure that: $d\Phi = d^\star \Phi = 0$, i.e. it is closed and co-closed. Since $d(u^i u^i) = d|u|^2 = 2|u|d|u| = 2u^i \alpha^i$ and $\nabla \Omega^i = 0$ as well as using the relation (9) it follows

$$\begin{aligned}
\tfrac{1}{3!} d(\epsilon_{ijk}\alpha^i \wedge \alpha^j \wedge \alpha^k) &= \tfrac{1}{2}\epsilon_{ijk}(\nabla\alpha^i) \wedge \alpha^j \wedge \alpha^k = -\kappa \, |u|d|u| \wedge \alpha^i \wedge \Omega^i \ , \\
d(\alpha^i \wedge \Omega^i) &= (\nabla\alpha^i) \wedge \Omega^i - \alpha^i(\nabla\Omega^i) = 0 \ , \\
(de^{3f}) \wedge \alpha^1 \wedge \alpha^2 \wedge \alpha^3 &= (e^{3f})' \, d|u| \wedge \alpha^1 \wedge \alpha^2 \wedge \alpha^3 = 0 \ .
\end{aligned} \tag{20}$$

Therefore, from $d\Phi = 0$ we derive the equation

$$\kappa \, |u| \, e^{3f} = (e^{f+2g})' \ . \tag{21}$$

Next, for the co-closure we have to check whether the dual 4-form

$$\Psi = e^{2(f+g)} \frac{1}{2}\epsilon_{ijk} \, \alpha^i \wedge \alpha^j \wedge \Omega^k + e^{4g} \, e^4 \wedge e^5 \wedge e^6 \wedge e^7 \tag{22}$$

is also closed. Employing the relation $\Omega^i \wedge \Omega^j = -2\,\delta^{ij} \, e^4 \wedge e^5 \wedge e^6 \wedge e^7$ and again (9) one obtains

$$\frac{1}{2}d(\epsilon_{ijk}\alpha^i \wedge \alpha^j \wedge \Omega^k) = \epsilon_{ijk}(\nabla\alpha^i) \wedge \alpha^j \wedge \Omega^k = -4\kappa \, u^i\alpha^i \wedge e^4 \wedge e^5 \wedge e^6 \wedge e^7 \tag{23}$$

and thus (recall $u^i \alpha^i = |u|d|u|$)

$$\begin{aligned}
0 = d\Psi &= (e^{2(f+g)})' d|u| \wedge \tfrac{1}{2}\epsilon_{ijk}\alpha^i \wedge \alpha^j \wedge \Omega^k + \\
&\quad \left[-4\kappa|u| \, e^{2(f+g)} + (e^{4g})' \right] d|u| \wedge e^4 \wedge e^5 \wedge e^6 \wedge e^7 \ .
\end{aligned} \tag{24}$$

This gives the two differential equations

$$\left(e^{2(f+g)} \right)' = 0 \qquad , \qquad (e^{4g})' = 4\kappa|u| \, e^{2(f+g)} \tag{25}$$

and it is straightforward to verify that these two equations together with (21) are equivalent to the first order equations derived from (17) with the solution (18).

3 Explicit metric of the 7-manifold

In the previous section we have verified that the metric of the 7-manifold with G_2 holonomy is given by [3, 4]

$$ds^2 = \frac{1}{\sqrt{2\kappa|u|^2 + c}} \left(du^i + \epsilon^{ijk}A^j u^k\right)^2 + \sqrt{2\kappa|u|^2 + c}\, e^m e^m \;, \tag{26}$$

where e^m is the vielbein of the quaternionic space with the $SU(2)$ connection A^j as introduced in (6). By choosing different quaternionic spaces we can now discuss explicit models.

3.1 Homogeneous quaternionic spaces

We will start with examples that played an important role in the recent literature. They base on homogeneous quaternionic spaces, which appear in two classes [19, 36], namely

(i) the maximal symmetric spaces $\frac{SO(5)}{SO(4)}$ and $\frac{SO(4,1)}{SO(4)}$ with 10 isometries and

(ii) the complex projective spaces $\frac{SU(3)}{SU(2)\times U(1)}$ and $\frac{SU(2,1)}{SU(2)\times U(1)}$ each having 8 isometries.

The metric of the quaternionic space and the corresponding $SU(2)$ connection for the maximal symmetric case (i) can be written as

$$e^m e^m = \frac{d\rho^2}{1-\kappa\rho^2} + (1 - \kappa\rho^2)\, d\psi^2 + \rho^2(d\theta^2 + \sin^2\theta d\varphi^2) \;,$$

$$A^1 = -\sqrt{1 - \kappa\rho^2}\, \sin\theta\, d\varphi \;, \quad A^2 = -\sqrt{1 - \kappa\rho^2}\, d\theta \;, \tag{27}$$

$$A^3 = -\kappa\rho\, d\psi - \cos\theta\, d\varphi \;.$$

For $\kappa = +1$ this metric describes the coset $\frac{SO(5)}{SO(4)} = \mathbb{S}^4$, and for $\kappa = -1$ the corresponding non-compact analog, $\frac{SO(4,1)}{SO(4)}$. These spaces with the maximal number of Killing vectors become trivial for $\kappa = 0$ and have a vanishing Weyl tensor. The complex projective spaces (ii) have less isometries and the Weyl tensor is non-trivial. The corresponding expressions read (cp. [37])

$$e^m e^m = \frac{2\, d\rho^2}{(1+\kappa\rho^2)^2} + \frac{\rho^2}{2(1+\kappa\rho^2)^2}\left(d\psi - \cos\theta\, d\varphi\right)^2 + \frac{\rho^2}{2(1+\kappa\rho^2)}(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$A^1 = -\frac{\sin\theta}{\sqrt{1+\kappa\rho^2}}\, d\varphi \;, \quad A^2 = -\frac{1}{\sqrt{1+\kappa\rho^2}}\, d\theta \;, \tag{28}$$

$$A^3 = -\frac{\kappa\rho^2}{2(1+\kappa\rho^2)}\, d\psi - \frac{(2+\kappa\rho^2)\cos\theta}{2(1+\kappa\rho^2)}\, d\varphi \;.$$

As before, the space is compact for $\kappa > 0$ and otherwise non-compact.

Both spaces are spherical symmetric related to the $\mathbb{S}^2$ parameterized by (θ, φ). But there is also a generalization to any 2-space with constant curvature ϵ (as it will appear in the next example). If we take into account this additional parameter, the resulting G_2 metric (26) depends in total on three parameters: ϵ, c and κ.

3.2 Quaternionic spaces with four isometries

More general, non-homogeneous quaternionic spaces may be classified by the number of isometries, but not many concrete examples are known; see however [38, 39]. One example with four isometries has been discussed in [26] and when regarded as a hyper multiplet moduli space yields upon gauging a supersymmetric Randall-Sundrum scenario. The metric and connection for this space is given by

$$
e^m e^m = \frac{d\rho^2}{V(\rho)} + V(\rho)\left[d\tau - n\,\frac{ydx-xdy}{1+\frac{\epsilon}{4}(x^2+y^2)}\right]^2 + (\rho^2 - n^2)\frac{dx^2+dy^2}{\left[1+\frac{\epsilon}{4}(x^2+y^2)\right]^2}
$$
$$
A^1 = -\sqrt{\frac{\rho+n}{\rho-n}}\,V\,\frac{dy}{1+\frac{\epsilon}{4}(x^2+y^2)}\ , \qquad A^2 = -\sqrt{\frac{\rho+n}{\rho-n}}\,V\,\frac{dx}{1+\frac{\epsilon}{4}(x^2+y^2)}\ , \tag{29}
$$
$$
A^3 = -\kappa(\rho-n)\,d\tau - \left[\epsilon + 2n\kappa(n-\rho)\right]\frac{ydx-xdy}{1+\frac{\epsilon}{4}(x^2+y^2)}\ ,
$$

with

$$
V = -\kappa\,\frac{(\rho-n)}{\rho+n}\,(\rho-\rho_+)(\rho-\rho_-)\ , \quad \rho_\pm = -n \pm \sqrt{4n^2 + \frac{\epsilon}{\kappa}}\ . \tag{30}
$$

As we mentioned in the last paragraph the parameter $\epsilon = 0, \pm 1$ determines the symmetry group: for $\epsilon = +1$ it is a spherical symmetric solution with the isometry group $U(1) \times SU(2)$; for $\epsilon = -1$ the isometry group becomes $U(1) \times SL(2, R)$ and for $\epsilon = 0$ it is the solvable sub-algebra of $SO(1, 4)$.

To understand this space better consider $\epsilon = 1$, where it becomes the metric of the Taub-NUT-(A)dS space [40], with n as NUT parameter and the mass parameter had been fixed to ensure the quaternionic property (3) (or equivalently the anti-selfduality of the Weyl tensor). For vanishing cosmological constant ($\kappa = 0$) one obtains the well known Taub-NUT metric, and hence, this space represents topologically an orbifold. In fact, this orbifold is related to the non-trivial periodicity of τ, which ensures the absence of conical singularities:

$$
\tau \simeq \tau + 4\pi n\ . \tag{31}
$$

By complete analogy to the Ricci-flat Taub–NUT case we can make the orbifold action explicit. For $\kappa = 1$ (the other case goes in complete analogy, see also [26]) the coset $\mathbb{S}^4 = SO(5)/SO(4)$ is defined by

$$
(X_0)^2 + (X_1)^2 + (X_2)^2 + (X_3)^2 + (X_4)^2 = 1 \tag{32}
$$

with the metric

$$
ds^2 = (dX_0)^2 + (dX_1)^2 + (dX_2)^2 + (dX_3)^2 + (dX_4)^2 \tag{33}
$$

subject to the constraint (32). Before imposing the constraint, the $SO(5)$ symmetry group is manifest, but afterwards only a subclass of these isometries are realized linearly and the other symmetries are not manifest. Since we are interested here in the spherical symmetric case ($\epsilon = 1$) we introduce polar coordinates in $X_{1,2,3,4}$: $(dX_1)^2 + (dX_2)^2 +$
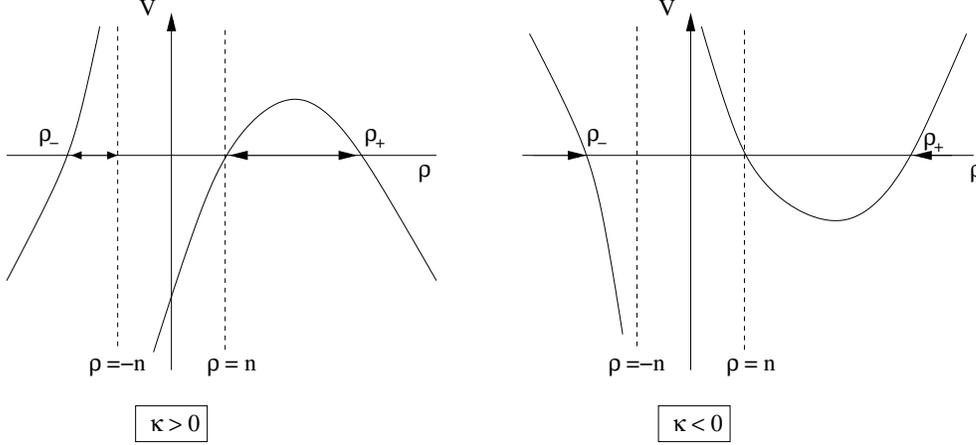
Figure 2: A positive definite metric requires: $V > 0$ as well as $\rho^2 > n^2$. Hence, there are two allowed coordinate regions, as we have indicated by the arrows.

$(dX_3)^2 + (dX_4)^2 = d\rho^2 + \rho^2 d\Omega_3$ and the constraint becomes $X_0^2 + \rho^2 = 1$. Since $X_0 \, dX_0 = -\rho \, d\rho$ we can eliminate X_0 and find for the metric

$$ds^2 = \frac{d\rho^2}{1-\rho^2} + \rho^2 d\Omega_3 = \frac{d\bar{r}^2 + \bar{r}^2 d\Omega_3}{(1+\frac{\bar{r}^2}{4})^2} = \frac{dz_1 d\bar{z}_1 + dz_2 d\bar{z}_2}{\left(1 + \frac{|z_1|^2 + |z_2|^2}{4}\right)^2} \tag{34}$$

with $\rho = \frac{\bar{r}}{1 + \bar{r}^2/4}$. As for the Taub-NUT space the $\mathbb{Z}_n$ orbifold acts on the two complex coordinates as

$$z_1 \simeq e^{\frac{2\pi i}{n}} z_1 \qquad , \qquad z_2 \simeq e^{-\frac{2\pi i}{n}} z_2 \tag{35}$$

which, after the change of coordinates

$$z_1 = r \, \cos(\theta/2) \, e^{i\frac{\varphi+\psi}{2}} \,, \quad z_2 = r \, \sin(\theta/2) \, e^{i\frac{\varphi-\psi}{2}} \,, \tag{36}$$

is equivalent to (31). Hence, this $\mathbb{Z}_n$ orbifold acts on the $\mathbb{S}^3$ sub-space as: $\mathbb{S}^3 \to \mathbb{S}^3/\mathbb{Z}_n$ in the usual way; for a discussion of orbifolds see also [10, 15, 16].

In addition to the reduced number of isometries, given by the subgroup that commutes with the orbifold action, there are more significant differences to the homogeneous cases that we discussed before. First, in the limit $\kappa = 0$ it becomes the Taub-NUT space, which is hyper-Kaehler and since it is still non-trivial it may be of interest for G_2 manifolds as well. Second, the quaternionic space (29) has a curvature singularity at $\rho = -n$ as indicated by the curvature invariant

$$R_{mnst} R^{mnst} = 24 \Big[\kappa^2 + \frac{4n^2(\epsilon + 4\kappa n^2)^2}{(\rho + n)^6} \Big] \,. \tag{37}$$

This singularity exist for any value of u^i and hence also the G_2 manifold (26) becomes singular, but it is shielded by the Killing horizon where $V = 0$ representing a fixed

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point set of $k = \partial_\tau$, see below. Note, the zeros of V are regular points as it is obvious from the curvature invariant (37). But at these zeros V changes its sign and therefore the metric is well-defined only on the coordinate patch where

$$V \geq 0 \quad , \quad \rho^2 - n^2 \geq 0$$

which allows for two physical regions that are disconnected by the curvature singularity, see figure 2:

$$\rho \leq \rho_- \quad , \quad \rho \geq \max\{n, \rho_+\} \ .$$

Since the metric is invariant under $\rho \to -\rho$ and $n \to -n$ we can assume that $n > 0$.

Before we discuss in the next section the Killing vectors and their fixed point set, let us note that the solution (29) interpolates between the two homogeneous quaternionic spaces that we introduced in eqs. (27) and (28). To see this, let us set $\epsilon = +1$. Obviously, if $n = 0$ we get $V = 1 - \kappa\rho^2$ and obtain the maximal symmetric spaces $\mathbb{S}^4$ or $EAdS_4$ in eq. (27). On the other hand, if we transform

$$\rho = \frac{\hat{\rho}}{n} + n \quad , \quad \tau = 2n\psi \tag{38}$$

and take the limit

$$n \to \infty \quad \text{keeping} \quad \hat{\rho} = fix$$

one finds the metric

$$ds^2 = \frac{2d\hat{\rho}^2}{\hat{\rho}(1 - 4\kappa\hat{\rho})} + 2\hat{\rho}(1 - 4\kappa\hat{\rho})\left[d\psi - \frac{1}{2}\frac{ydx - xdy}{1 + \frac{x^2+y^2}{4}} \right]^2 + 2\hat{\rho}\frac{dx^2 + dy^2}{(1 + \frac{x^2+y^2}{4})^2} \ . \tag{39}$$

Finally, the transformation

$$\hat{\rho} = \frac{r^2}{4(1 + \kappa\, r^2)} \quad , \quad x + \mathrm{i}\, y = 2\tanh\frac{\theta}{2}\, e^{\mathrm{i}\varphi} \ .$$

brings us to the complex projective space in eq. (28).

Since negative n are equivalent to positive (after $\rho \to -\rho$), the two homogenous spaces appear at the endpoints of the parameter space of n and in both limits the curvature singularity disappears and the spaces become smooth. Let us also note, that in the limit of vanishing cosmological constant the space (29) is the standard Taub-NUT space, whereas the space (39) which appears in the large n limit becomes for $\kappa = 0$ the Eguchi-Hanson space.

4 Isometries and IIA description

Having the metric of the manifold X_7 we can consider M-theory on $M_4 \times X_7$ and if X_7 has at least one isometry the dimensional reduction will give a IIA description. Especially interesting are Killing vector fields with a fixed point set L of co-dimension four implying that L extends in 6+1 dimensions, and hence, become D6-branes upon compactification [27]. But before we come to the fixed point set let us discuss the Killing vectors.

4.1 Isometries

An isometry is related to the existence of a Killing vector field k satisfying the equation $D_{(m}k_{n)} = 0$. In a given coordinate patch one can introduce proper coordinates so that the Killing vector field becomes $k = \partial_\chi$ and the (χ-independent) metric reads

$$ds^2 = e^{2\phi(x)}\Big[d\chi + \omega_m(x)dx^m\Big]^2 + \hat{g}_{mn}(x)dx^m dx^n \ . \tag{40}$$

In these (local) coordinates the Killing vector field becomes $k_M = e^{2\phi}\{1, \omega_m\}$ and due to the Killing property one find $D_M k_N = D_{[M}k_{N]} = \partial_{[M}k_{N]}$. Obviously any covariantly constant vector field is also a Killing vector with the consequence that ϕ and ω_m are trivial (i.e. $d\omega = 0$). This in turn implies that the space factorizes and the holonomy is reduced. Therefore, the holonomy in our case is equal to G_2 iff there are no covariantly constant (Killing) vector fields. The absence of such Killing vectors for 7-manifolds given as $\mathbb{R}^3$ bundles over $\mathbb{S}^4$ and $\mathbb{CP}^2$ was shown shown in [3, 4]. Since our case represents an interpolation between these two spaces, where only a subclass of isometries survive, we are still dealing with a manifold where the holonomy is equal to G_2.

To be more concrete, the isometries of X_7 with the metric (26) are given by the isometries of the quaternionic space (29) and in addition $SU(2)$ transformations of the coordinates u^i. The quaternionic space has four Killing vectors [40] which can be written as [26]

$$k_1 = ny\,\partial_\tau - (1 + \epsilon\tfrac{x^2-y^2}{4})\,\partial_x - \epsilon\tfrac{xy}{2}\,\partial_y \ ,$$

$$k_2 = -nx\,\partial_\tau - \epsilon\tfrac{xy}{2}\,\partial_x - (1 - \epsilon\tfrac{x^2-y^2}{4})\,\partial_y \ , \tag{41}$$

$$k_3 = 2n\,\partial_\tau + \epsilon\,y\,\partial_x - \epsilon\,x\,\partial_y \quad , \qquad k_4 = \partial_\tau$$

and fulfill the algebra

$$[\,k_i, k_4\,] = 0 \quad , \qquad [\,k_i, k_j\,] = f_{ijl}\,k_l \tag{42}$$

with $f_{123} = 1$, $f_{231} = f_{312} = \epsilon$. For the spherical symmetric case $\epsilon = 1$ we get thus the symmetry $SU(2) \times U(1)$, where the $U(1)$ corresponds to the orbifold action (35) and the $SU(2)$ is the subgroup of $SU(3)$ or $SU(2,1)$ that commutes with the orbifold. For the hyperbolic case ($\epsilon = -1$) one obtains the non-compact analog, namely the algebra $SL(2,R) \times U(1)$. For $\epsilon = 0$ two Killing vectors become equivalent ($k_3 \sim k_4$) and we should take a different parameterization, namely

$$k_1 = ny\,\partial_\tau - \partial_x \ , \quad k_2 = -nx\,\partial_\tau - \partial_y \ , \quad k_3 = y\,\partial_x - x\,\partial_y \ , \quad k_4 = \partial_\tau \ . \tag{43}$$

They satisfy the algebra

$$[\,k_i, k_4\,] = 0 \ , \quad [\,k_1, k_2\,] = 2nk_4 \ , \quad [\,k_2, k_3\,] = k_1 \ , \quad [\,k_3, k_1\,] = k_2 \tag{44}$$

which is not not anymore a direct product of the two Lie algebras.

From our discussion above it is clear that a covariantly constant Killing vector would commute with the other Killing vectors. But the only commuting Killing vector $k_4 = \partial_\tau$ is not covariantly constant, due to the non-trivial $U(1)$ fibration in (29). Therefore the space cannot be factorized and the holonomy is equal to G_2, which we infered already from the relation to the other known G_2 manifolds.

4.2   Fixed point set

For a given Killing vector k, the hypersurface L defined by $|k|^2 = 0$ is the fixed point set and L degenerates if the surface gravity vanishes, i.e. if $|Dk|^2 = 0$ at $|k|^2 = 0$; see e.g. [41, 42]. For $\epsilon \neq 0$ the fixed points of all Killing vectors are non-degenerate implying a periodic identification along the Killing direction to ensure the absence of conical singularities. This fact is well known from *non-extreme* black holes where the event horizon is a non-degenerate fixed point set for a timelike Killing vector. In the case here, the compactness of the Killing direction ensures that the Kaluza-Klein gauge group becomes $U(1)$ and that the D6-branes, which are identified as the fixed point set L [27], are at finite geodesic distance.

In order to identify the D6-branes we need a Killing vector that has a co-dimension four fixed point set L, or in other words, L extends over three coordinates in X_7 which, in addition to M_4, become the world volume of the D6-branes. We should therefore not consider the Killing vectors related rotations of the u^i coordinates. They have a fixed point set at $|u| = 0$ which is a point in the 3-d u^i-space and hence a co-dimension three fixed point set. On the other hand, the fixed points of the quaternionic Killing vectors can be NUTs or bolts, depending on the rank of the 2-from dk; see [41]. The NUTs are points on the quaternionic space and since the $SU(2)$ connection A^i becomes trivial, these NUTs correspond to isotropic D6-branes. On the other hand, if L is a bolt on the quaternionic space it has co-dimension two and since at least one $SU(2)$ connection remains non-trivial, there is no isotropic brane interpretation. After dimensional reduction we obtain a supergravity solution which is singular at the fixed point set L and if L is a NUT the singularity can be identified with the location of the D6-brane, but if L is a bolt there is no clear interpretation. It may be related to some deformation of a given D6-brane, since, as we will see, the bolts are connected with the NUTs for specific Killing vectors.

A generic Killing vector of the quaternionic space, can be introduced as a linear combination of the four Killing vectors in (41)

$$
\begin{aligned}
k &= \alpha_1 k_1 + \alpha_2 k_2 + \alpha_3 k_3 + \alpha_4 k_4 \\
&= \left[ny\,\alpha_1 - nx\,\alpha_2 + 2n\,\alpha_3 + \alpha_4 \right] \partial_\tau \\
&\quad - \left[(1 + \epsilon\tfrac{x^2-y^2}{4})\,\alpha_1 + \epsilon\tfrac{xy}{2}\,\alpha_2 + \epsilon\,y\,\alpha_3 \right] \partial_x \\
&\quad - \left[\epsilon\tfrac{xy}{2}\,\alpha_1 + (1 - \epsilon\tfrac{x^2-y^2}{4})\,\alpha_2 + \epsilon\,x\,\alpha_3 \right] \partial_y \,.
\end{aligned}
\tag{45}
$$

We will now investigate the fixed point set of this Killing vector following the discussion in [26]. Neglecting zeros of $V(\rho)$, zeros $|k|^2$ are given by zeros of all components k^m. Since the Killing vector depends only on x and y the equations $k^m = 0$ are not solvable for a generic choice of α_m. We can solve, however, $k^x = k^y = 0$ and find as solution [26]

$$x = x_\pm = \frac{2\alpha_2}{\alpha_1^2 + \alpha_2^2}\Big(\alpha_3 \pm |\alpha|\Big) \quad , \qquad y = y_\pm = -\frac{2\alpha_1}{\alpha_1^2 + \alpha_2^2}\Big(\alpha_3 \pm |\alpha|\Big) \qquad (46)$$

where $|\alpha|^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2$. If one inserts these values into $k^\tau = 0$ one obtains a constraint on α_4, but one would obtain bolts since ρ and τ are arbitrary. In order to find a NUT we keep α_4 arbitrary and set instead

$$V(\rho) = 0 \qquad \text{or}: \quad \rho = (n, \rho_\pm) \qquad (47)$$

which ensures that $g_{\tau\tau}k^\tau k^\tau = 0$ and represent points in the (ρ, τ) space[2]. The point $\rho = n$ is special, because without fixing x, y we find $|k|^2 \equiv 0$.

Therefore, we have on the quaternionic space the following NUT fixed points of the Killing vector k

$$
\begin{aligned}
\kappa > 0: \ (\rho, x, y) &= \big\{\, (n, x, y)\,, \, (\rho_-, x_\pm, y_\pm)\,(\rho_+, x_\pm, y_\pm)_{\text{if } \epsilon > 0} \,\big\} \\
\kappa < 0: \ (\rho, x, y) &= \big\{\, (\max[n, \rho_+], x_\pm, y_\pm)\,, \, (\rho_-, x_\pm, y_\pm)\,\big\}
\end{aligned}
\qquad (48)
$$

with $\rho_\pm = -n \pm \sqrt{4n^2 + \frac{\epsilon}{\kappa}}$.

The dimensional reduction along the Killing vector (45) will yield a bound state of D6-branes, located on the quaternionic space at these fixed points. The number depends on the choice of parameters, but can be at most five (for $\epsilon, \kappa > 0$) where two of them (at $\rho = \rho_-$) are disconnected from the other three (at $\rho = n$ and $\rho = \rho_+$) by the curvature singularity at $\rho = -n$. Recall, there are different (physical) regions for ρ (see figure 2) and hence some fixed points may not have a physical sensible interpretation. The correct identification of the D6-branes is, however, a subtle point. E.g. because $x_+ x_- + y_+ y_- = -4$ the two points (x_+, y_+) and (x_-, y_-) are antipodal points in the (x, y)-space (i.e. for $\epsilon = 1$ on the $\mathbb{S}^2$) and one may be tempted to identify the two branes. But note, since the Killing vector is zero at both points and non-singular in between, the eigenvalue of dk have to be different at both fixed points, which implies that the 6-brane charge or tension will be different. It would be interesting to see whether our model corresponds to one of the supersymmetric intersecting brane worlds or the relation to the non-supersymmetric ones; see e.g. [7, 43] and refs. therein.

A subtle question concerns also the dilaton. In adapted coordinates one can write the 7-metric as in eq. (40), where x_{11} correspond to χ and ϕ is proportional to the dilaton. Unfortunately, $e^{2\phi} \sim |k|^2$, which becomes the string coupling constant, is not bounded for our solution. While for $\kappa > 0$ the range of ρ is bounded, see figure 2, but $|u|$ is unbounded, see eq. (18). On the other hand for a non-compact quaternionic

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[2]Notice, $V = 0$ is a conical singularity which is resolved by the proper periodicity in τ.

space, with $\kappa \leq 0$, is situation is opposite. Therefore, in any case there are asymptotic regions where $|k|^2 \sim e^{2\phi}$ blows up and one should look for similar deformation as the ones discussed in [12] to obtain a finite string coupling constant.

5 Outlook: Membrane instantons and new spin(7) metric

We mentioned already a few aspects that deserve further investigations, but let us add another two.

A. Resolution of the curvature singularity by Membrane instantons

One genuine feature of our G_2 manifold is the appearance of a curvature singularity; otherwise the continues deformation of two topological different spaces (like $\mathbb{S}^4$ and $\mathbb{CP}^2$) would be difficult to understand. Although for $\kappa < 0$ the singularity is inside an unphysical coordinate range (with timelike coordinates), the validity of the supergravity solution is a subtle question. One may therefore look for possibilities to resolve the singularity. An interesting possibility could be to turn on appropriate fluxes along the lines discussed in [44, 45]. But non-trivial fluxes play an important role also in a somewhat different aspect. Namely the curvature singularity in eq. (37) disappears if $\epsilon + 4\kappa n^2 = 0$ (or $\rho_- = \rho_+$) and since κ is the constant curvature of the 4-d quaternionic space, this value can effectively be changed by turning on 4-form fluxes. In fact, this possibility has been discussed as "Neutralization of the cosmological constant" and corresponds to take into account membrane instanton effects [46, 47]. What will happen with the quaternionic space in the limit $\rho_+ = \rho_-$? Setting $\epsilon = 1$ and $\kappa = -1$ so that $n = 1/2$ we transform $\rho = \frac{1}{2}\cosh R$ and the metric becomes

$$ds^2 = dR^2 + \sinh^2 R \, d\Omega_3 \tag{49}$$

which is one parameterization of Euclidean AdS_4 space. This metric covers only the region $\rho > n$, but for $\rho < -n$ one obtains the same metric so that both physical regions are decoupled and the region with the curvature singularity disappeared and moreover all fixed points are joined at $\rho = \pm n$ which corresponds to the point $R = 0$ where the metric becomes flat. But note, the 4-form flux will not only change the effective value of the curvature of the quaternionic space, it will also cause a back reaction on the 7-metric which requires a detailed analysis. Needless to say, that it would be also interesting to calculate the superpotential caused by these 4-form fluxes [48, 49, 14].

There is also another effect that may weaken or resolve the curvature singularity, namely to change effectively n by a multi-center solution (as for Taub-NUT or the Eguchi-Hanson space). It is unclear whether these multi-center solution for quaternionic spaces exist, but one could try to construct them similar to the multi-center supergravity solutions of [50, 51].

B. New Spin(7) metrics

We have considered here only generalizations of known 7-manifolds with G_2 holonomy, but analogous calculations will also yield more general 8-manifolds with Spin(7) holonomy. If one follows again the work by Bryant and Salamon, the corresponding metric of the 8-manifold becomes

$$ds^2 = f^2 \alpha \bar{\alpha} + g^2 e^m e^m \tag{50}$$

where $e^m e^m$ is again the metric of a quaternionic space and $\alpha = du - uA$, where we used the quaternionic notation $u = u^0 + iu^1 + ju^2 + ku^3$ and $A = iA^1 + jA^2 + kA^3$ is the $SU(2)$ connection of the quaternionic space. The function $f = f(|u|)$ and $g = g(|u|)$ are again the same as in [3] and new fixed points are again encoded by the quaternionic space. Also, one may turn on fluxes and calculate the superpotential or replace the $\mathbb{R}^4$, spanned by the coordinates u^m, by a hyper Kaehler space and finds a IIA description of 6-branes wrapping the quaternionic space. Some related work in these directions has been done e.g. in [52, 53, 54, 55, 56].

Acknowledgments

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