

# Gravitational Waves from Neutron Stars with Large Toroidal B-Fields

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We show that NS's with large toroidal B-fields tend naturally to evolve into potent gravitational-wave (gw) emitters. The toroidal field  $B_t$  tends to distort the NS into a prolate shape, and this magnetic distortion dominates over the oblateness “frozen into” the NS crust for  $B_t \gtrsim 3.4 \times 10^{12} \text{G} (\nu_s/300 \text{Hz})^2$ . An *elastic* NS with frozen-in B-field of this magnitude is clearly secularly unstable: the wobble angle between the NS's angular momentum  $J^i$  and the star's magnetic axis  $n_B^i$  grow on a dissipation timescale until  $J^i$  and  $n_B^i$  are orthogonal. This final orientation is clearly the optimal one for gw emission. The basic cause of the instability is quite general, so we conjecture that the same final state is reached for a realistic NS, with superfluid core. Assuming this, we show that for LMXB's with  $B_t \sim 2 \times 10^{12} - 2 \times 10^{14} \text{G}$ , the spindown from gw's is sufficient to balance the accretion torque—supporting a suggestion by Bildsten. The spindown rates of most millisecond pulsars can also be attributed to gw emission sourced by toroidal B-fields, and both these sources could be observed by LIGO II. While the first-year spindown of a newborn NS is most likely dominated by electromagnetic processes, reasonable values of  $B_t$  and the (external) dipolar field  $B_d$  can lead to detectable levels of gw emission, for a newborn NS in our own galaxy.

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## I. INTRODUCTION

Nearly periodic gravitational waves (gw’s) from distorted or wobbling neutron stars (NS’s) are among the most promising sources for the large-scale gravitational-wave detectors now coming online. In this paper we show that NS’s with large toroidal B-fields tend naturally to evolve into potent gw emitters.

While most known, young pulsars have dipolar magnetic field strengths  $B_d \sim 10^{12}$  G, studies of soft-gamma repeaters and anomalous X-ray pulsars suggest that a substantial fraction of NS’s are born with larger magnetic fields,  $B \sim 10^{14} - 10^{15}$  G [1–4]. In cases where the B-field is initially “wound up” by differential rotation (as opposed to convective motions), one would expect the field’s toroidal piece  $B_t$  to dominate the poloidal part  $B_p$ .

The toroidal field  $B_t$  tends to distort a NS into a prolate shape, and we show that this magnetic distortion dominates over the oblateness “frozen into” the NS crust for  $B_t \gtrsim 3.4 \times 10^{12} (\nu_s/300\text{Hz})^2$  G, where  $\nu_s$  is the NS spin frequency. An *elastic* NS with frozen-in B-field of this magnitude is clearly secularly unstable: the wobble angle between the NS’s angular momentum  $J^i$  and the star’s magnetic axis  $n_B^i$  grow on a viscous timescale until  $J^i$  and  $n_B^i$  are orthogonal. This final orientation is the optimal one for gw emission. The basic cause of the instability seems so general that we conjecture that the same final state is reached for a realistic NS (with crust and superfluid core). In this paper we consider NS’s where  $B_t > B_p$ , we assume that most of the results for elastic NS’s can be carried over to the realistic NS case, and examine the consequences. We show that for LMXB’s with  $B_t \sim 2 \times 10^{12} - 2 \times 10^{14}$  G, the spindown from gw’s is sufficient to balance the accretion torque—supporting a suggestion by Bildsten [5]. The spindown rates of most millisecond pulsars can also be attributed to gw emission sourced by toroidal B-fields of comparable size,  $B_t \sim 10^{12} - 10^{13}$  G. For several known millisecond pulsars, the gravitational waves would be detectable by LIGO II. Finally, while the first-year spindown of a newborn NS is most likely dominated by electromagnetic (em) processes, reasonable values of  $B_t$  and the (external) dipolar field  $B_d$  can still lead to detectable levels of gw emission, for a newborn NS in our own galaxy.

The idea that a large, toroidal magnetic field should drive the magnetic axis orthogonal to  $J^i$  is not new; a literature search revealed that P. B. Jones had this idea 28 years ago [6]. Jones [7–10] went on to develop a scenario in which the dipole axes of young pulsars (which he assumed have  $B_t > B_p$ ) are *first* driven orthogonal to  $J^i$  by dissipative forces, on a timescale of  $\sim 10^3$  yr, but then are slowly driven back parallel to  $J^i$  by em radiation reaction, for times  $\gtrsim 10^4$  yr. That is, Jones estimated that at around age  $10^3 - 10^4$  yr, dissipative forces would diminish to the point that em backreaction would drive the dynamics. Clearly, Jones was strongly

guided by the observation that most young pulsars do *not* appear to have dipole axes orthogonal to  $J^i$  (nor can they be perfectly aligned with  $J^i$ , as one would expect for  $B_p > B_t$ , since they pulse). Given his strong argument that dissipation should drive the magnetic axis orthogonal to  $J^i$ , and the apparent contradiction by observation, there are three likely ways out: 1) assume dissipative effects become negligible on a timescale of order the ages of known, young pulsars, 2) assume the pulsar emission region is not aligned along a principal axis of the *internal* B-field, or 3) assume that the internal B-field is either not born with, or quickly loses, any strong directionality. Jones developed a model around the first option, but found little support for it in the data.

Here we essentially resurrect the Jones argument, but concentrate on a different class of objects possessing much smaller external dipole fields  $B_d$ : the LMXB’s and millisecond pulsars. For these objects, it seems clear that dissipation must dominate over the em torque in determining the alignment of the NS’s axes. We are agnostic on the question of why the *normal* (i.e., young, slow) pulsars are generally neither aligned nor orthogonal rotators. It is irrelevant to our argument whether option 1 or 2 above is correct; option 3 seems the least likely to us, but if option 3 is always correct it would render this paper irrelevant.

Of course, it has been noted before that magnetically distorted NS’s can generate gw’s; see, e.g., Bonazzola & Gourgouhlon [11] and D. I. Jones [12] and references therein. Also, Melatos & Phinney [13] recently showed how the poloidal B-field on accreting NS’s might build up “mountains” of accreted matter on the magnetic poles, leading to gw emission. What is new here is our observation that NS’s with large toroidal B-fields should naturally evolve to configurations with large gw emission, and our identification of LMXB’s and millisecond pulsars as likely “sites” for this phenomenon.

In §II we derive the basic energetics and timescales. These are applied to LMXB’s and millisecond pulsars in §III, and to newborn NS’s in §IV. For our estimates, we shall always use the following fiducial NS parameters:  $M = 1.4M_\odot$ ,  $R = 10$  km, and  $I = 10^{45}$  g-cm<sup>2</sup>.

## II. ENERGETICS AND TIMESCALES

Let  $n_B^i$  be the NS’s magnetic axis (so that if we call  $n_B^i$  the  $z$ -axis, then the toroidal field  $B_t$  points in the  $\phi$ -direction). Following Pines & Shaham [14], we can write the NS’s inertia tensor  $I^{ij}$  as the sum of four pieces—a spherical piece and three quadrupolar distortions—as follows:

$$I^{ij} = I_0 \left[ e^{ij} + \epsilon_\Omega n_\Omega^i n_\Omega^j + \epsilon_d n_d^i n_d^j + \epsilon_B n_B^i n_B^j \right] \quad (2.1)$$

Here  $e^{ij}$  is the flat, spatial 3-metric,  $I_0 e^{ij}$  is the spherically symmetric part of the inertia tensor, and the

terms proportional to  $\epsilon_\Omega$ ,  $\epsilon_d$ , and  $\epsilon_B$  are the quadrupolar distortions due to the star's spin, crustal shear stresses, and the magnetic field, respectively.

The centrifugal piece “follows” the instantaneous (unit-)spin vector  $n_\Omega^i$ , but the unit-vectors  $n_d^i$  and  $n_B^i$  are assumed fixed in the body frame. (However on long timescales,  $n_d^i$  can be “re-set” by crustal relaxation.)

The centrifugal piece  $\epsilon_\Omega$  in Eq. (2.1) is approximately given by  $\epsilon_\Omega \approx 0.3(\nu_s/\text{kHz})^2$ , where  $\nu_s$  is the spin frequency. The term  $\epsilon_d$  is due to the elastic crust's “memory” of some preferred shape. Absent the magnetic field, it is the residual oblateness the NS *would* have if it were spun down to zero frequency, without the crust breaking or otherwise relaxing. (To understand this, consider a rotating NS with relaxed crust. Removing the centrifugal force would decrease the NS's oblateness, but then shear stresses would also build up that tended to push the crust back to its relaxed, oblate shape.) For a NS with relaxed crust,  $\epsilon_d$  is proportional to the centrifugal oblateness:  $\epsilon_d = b\epsilon_\Omega$ . The coefficient  $b$  was recently calculated by Cutler et al. [15], who solved the coupled hydroelastic equations describing the deformed crust and found  $b \approx 2 \times 10^{-7}$ . (An older, back-of-the-envelope estimate of  $b \sim 10^{-5}$  used by many authors turned out to be too high by a factor  $\sim 40$ .) Therefore we adopt the estimate

$$\epsilon_d \approx 6 \times 10^{-8} \left( \frac{\nu_s}{\text{kHz}} \right)^2. \quad (2.2)$$

Again, this estimate assumes the crust is nearly relaxed. While this might not be a good assumption for young, slow NS's, it seems safe for the fast, old NS's of interest to us here, for two reasons. First, during the LMXB phase, the entire crust is replaced by accreted matter as the NS spins up, and it seems unlikely the new crust would “remember” the preferred shape from an earlier era. Second, the alternative requires the crust to withstand average strains of order  $3 \times 10^{-3}(\nu_s/300 \text{ Hz})^2$  for  $\sim 10^7 - 10^{10}$  yr without relaxing [15], which also seems unlikely.

Next we need  $\epsilon_B$ . Of course, the actual distortion produced by some toroidal field  $B_t$  depends on the precise distribution of B-field (and hence currents) within the star, which is highly uncertain. So we must be satisfied with a rough estimate of  $\epsilon_B$ , which we derive as follows. Let  $n_B^i$  be along the  $z$ -axis. For an incompressible, constant- $\rho$  star that is not superconducting,  $\epsilon_B \equiv (I_{zz} - I_{xx})/I_{zz}$  depends only on the total energy in the toroidal field [17,18]:

$$\epsilon_B = -\frac{15}{4}E_G^{-1} \int \frac{1}{8\pi} B_t^2 dV, \quad (2.3)$$

where  $E_G = (3/5)GM^2/R$  is the star's gravitational binding energy. The sign of  $\epsilon_B$  is negative because the toroidal magnetic field lines are like a rubber belt, pulling in the star's waist at the magnetic equator.

Now, it is generally believed that for interior temperatures  $T < 10^9 - 10^{10} \text{ }^\circ\text{K}$ , the protons in the NS interior are a type II superconductor; hence the magnetic field is confined to flux tubes with field strength  $B_{c1} \approx 10^{15} \text{ G}$ . (For  $B > B_{c1}$ , the magnetic fluxoids overlap.) Then for  $B < B_{c1}$ , the anisotropic part of the mean magnetic stress tensor is increased over its non-superconducting value by a factor  $B_{c1}/B$ , and  $\epsilon_B$  increases by the same factor [6,19]. (The virial-theorem-based derivation of  $\epsilon_B$  in Ostriker & Gunn [18] makes this completely clear.) Putting this factor together with Eq. (2.3), but using  $E_G = (3/4)GM^2/R$  (i.e., the  $n = 1$  polytrope result, which quite accurately gives the binding energy of realistic NS's), we adopt the estimate

$$\epsilon_B = \begin{cases} -1.6 \times 10^{-6} \langle B_t \rangle / 10^{15} \text{ G} & B_t < B_{c1}, \\ -1.6 \times 10^{-6} \langle B_t^2 \rangle / 10^{30} \text{ G} & B_t > B_{c1}, \end{cases} \quad (2.4)$$

where  $\langle \dots \rangle$  means “volume-averaged over the NS interior.” Of course, the first line of Eq. (2.4) only applies to NS's older than  $\sim 0.1 - 1$  yr (i.e., old and cold enough to be superconducting).

For simplicity we are assuming, here and below, that  $B_t$  is sufficiently greater than  $B_p$  that we can neglect the latter. When is this a good approximation? For an incompressible, constant- $\rho$ , non-superconducting NS, with  $B^i$  of the form  $B_p \hat{z}^i + B_t \hat{\phi}^i$  in the interior and matched to a dipolar field in the exterior, the deformation is [17,18]<sup>1</sup>

$$\epsilon_B = \frac{25R^4}{24GM^2} \left( \frac{21}{10} \langle B_p^2 \rangle - \langle B_t^2 \rangle \right). \quad (2.5)$$

For a superconducting NS, with  $B_t, B_p \ll B_{c1}$ , we claim this becomes

$$\epsilon_B = \frac{25R^4}{24GM^2} \left( 2 \langle B_p B_{c1} \rangle - \langle B_t B_{c1} \rangle \right). \quad (2.6)$$

(The small change  $(21/10) \rightarrow 2$  reflects the fact that the *external* poloidal magnetic field energy becomes a negligible fraction of the total field energy in the superconducting case.) Clearly then, as long as  $\langle B_t \rangle \gtrsim 4 < B_p \rangle$ , our estimates should be fairly reliable.

We next consider the kinetic energy KE of the spinning NS as the orientation of the body changes, for fixed angular momentum  $J^i$ . While in the inertial frame the NS's angular velocity  $\Omega^i$  clearly must remain approximately fixed (near the star's angular momentum  $J^i$ ),  $\Omega^i$  will migrate with respect to the body axes as the star precesses

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<sup>1</sup>Ostriker & Gunn [18] actually give 3, not 2.1, as the coefficient of  $\langle B_p^2 \rangle$  in Eq. (2.6), but we claim they made an algebra error; the coefficient on the rhs of their Eq. (A19) should be  $-7/40$ , not their  $-1/4$ . With this correction, their coefficient 3 becomes 2.1, in agreement with Wentzel [17].

(and as the precession angle secularly changes). Using  $J^i = I^{ij}\Omega_j$ ,  $KE = \frac{1}{2}J^i\Omega_i$ , and expanding everything to lowest order in  $\epsilon_\Omega$ ,  $\epsilon_d$  and  $\epsilon_B$ , we find:

$$KE = \frac{J^2}{2I_0} \left[ 1 - \epsilon_\Omega - \epsilon_d(n_d^i n_{Ji})^2 - \epsilon_B(n_B^i n_{Ji})^2 \right], \quad (2.7)$$

where  $n_j^i$  is the unit vector along  $J^i$ .

Now consider the case where  $\epsilon_B < 0$  and  $|\epsilon_B| \gg \epsilon_d$ , and let  $\theta$  be the ‘‘wobble angle’’ between  $J^i$  and  $n_B^i$ . Then the  $\theta$ -dependent piece of KE is  $(2I_0)^{-1}J^2(-\epsilon_B)\cos^2\theta$ , which is clearly minimized for  $\theta = \pi/2$ . (More precisely—including the effect of non-zero  $\epsilon_d$ —KE is minimized for  $|n_B^i n_{Ji}| = \frac{1}{2}|(\epsilon_d/\epsilon_B)\sin 2\beta|$ , where  $\beta$  is the angle between  $n_B^i$  and  $n_d^i$ .) In addition to the kinetic energy, the wobbling NS stores potential energy (PE) in the stressed crust and the distorted fluid. One easily convinces oneself that the  $\theta$ -dependent pieces of PE and KE have the same sign and same functional form ( $\propto \cos^2\theta$ ), and that they have comparable magnitudes for NS’s with relaxed crusts. Therefore, considering the total energy  $E = KE + PE$  leads to the same conclusion about energy-minimization/stability as considering KE alone.

It is worth emphasizing that the NS need not actually be prolate to be unstable; indeed, for the cases of interest here ( $\nu_s > 100$  Hz), the centrifugal oblateness will be orders of magnitude larger than  $|\epsilon_B|$ . But what matters, for both precessional dynamics and stability, is the net prolateness/oblateness that is ‘‘frozen into’’ the star’s body frame, not the centrifugal piece that ‘‘follows’’ the star’s angular velocity. Therefore the condition for instability is  $-\epsilon_B > \epsilon_d$ , or

$$B_t \gtrsim 3.4 \times 10^{12} \text{G} \left( \frac{\nu_s}{300 \text{Hz}} \right)^2. \quad (2.8)$$

Assuming inequality (2.8) is satisfied, how does the instability grow? For an elastic star with frozen-in magnetic field, this is completely clear. In the body frame,  $\Omega^i$  precesses around  $n_B^i$  with precession frequency  $\nu_s \epsilon_B$  (neglecting a small correction due to nonzero  $\epsilon_d$ ). Due to dissipation, the angle between  $\Omega^i$  and  $n_B^i$  steadily increases until they are nearly orthogonal. (For a review of NS precessional dynamics, see Cutler & Jones [16] and references therein.)

For a realistic NS, with superfluid core, the precessional dynamics could be more complicated. But as long as those dynamics do not reduce  $-\epsilon_B$  below  $\epsilon_d$  (i.e., by destroying the large-scale coherence of the internal field or building up an equally large internal poloidal piece), it seems inevitable, by simple energetics, that dissipation will tend to drive  $n_B^i$  orthogonal to  $J^i$ . In what follows, we shall assume this is true. Some support for this assumption comes from the work of Goldreich & Reisenegger [20], who show that the composition of a NS interior is stably stratified, impeding convective motions that might otherwise distort the interior B-field.

We next turn to timescales. There are four relevant ones:  $\tau_{GW}$ ,  $\tau_{EM}$ ,  $\tau_{ACC}$  and  $\tau_{DIS}$ . Once  $n_B^i$  has been

re-aligned perpendicular to  $J^i$ , the NS spins down due to gw emission on a timescale  $\tau_{GW} \equiv J/(2\dot{J}_{GW})$ , given by

$$1/\tau_{GW} = 5.50 \times 10^{-13} \text{s}^{-1} \left( \frac{\epsilon_B}{10^{-7}} \right)^2 \left( \frac{\nu_s}{\text{kHz}} \right)^4. \quad (2.9)$$

We must compare this to the timescale  $\tau_{EM} \equiv J/(2\dot{J}_{EM})$  on which the NS spins down due to electromagnetic processes. Define  $B_d \equiv 2\mathcal{M}/R^3$ , where  $\mathcal{M}$  is the NS’s magnetic dipole moment. (With this definition,  $B_d$  is the value of the field at the magnetic pole, on the NS surface, for an external field that is perfectly dipolar.) Then  $\tau_{EM}$  is given by

$$1/\tau_{EM} = 4.88 \times 10^{-16} \text{s}^{-1} \left( \frac{B_d}{10^9 \text{G}} \right)^2 \left( \frac{\nu_s}{\text{kHz}} \right)^2. \quad (2.10)$$

For a magnetic dipole rotating in vacuum, the rhs of Eq. (2.10) would be multiplied by a factor  $\sin^2\alpha$ , where  $\alpha$  is the angle between the spin vector and magnetic dipole direction. But Goldreich & Julian [21] showed that a realistic NS is surrounded by plasma, and as a result Eq. (2.10) is a reasonable estimate of the spindown rate even for an aligned (i.e.,  $\alpha = 0$ ) rotator.

From Eqs. (2.9)-(2.10), spindown due to gw emission dominates em spindown for

$$B_t/B_d > 6.2 \times 10^3 \left( \frac{300 \text{Hz}}{\nu_s} \right), \quad B_t < B_{c1} \quad (2.11a)$$

$$B_t/B_d > 2.5 \times 10^2 \left( \frac{300 \text{Hz}}{\nu_s} \right)^{1/2} \left( \frac{10^{14} \text{G}}{B_d} \right)^{1/2}, \quad B_t > B_{c1} \quad (2.11b)$$

Such a large ratio  $B_t/B_d$  would not be expected in a normal pulsar, but could occur in a LMXB/recycled pulsar. For these sources, accretion apparently ‘‘buries’’ or otherwise reduces the external dipolar field, but the toroidal field in the deep interior could be left largely intact.

The em torque from the external dipole field causes the wobble angle  $\theta$  between  $J^i$  and  $n_B^i$  to damp (or grow, for  $\sin^2\xi > 2/3$ ) on roughly the em spindown timescale  $\tau_{EM}$  [22]:

$$\frac{1}{\sin\theta} \frac{d\sin\theta}{dt} = -\frac{1}{2}\cos^2\theta \left( 1 - \frac{3}{2}\sin^2\xi \right) / \tau_{EM}, \quad (2.12)$$

where  $\xi$  is the angle between the external dipole direction and the magnetic axis  $n_B^i$  of the internal B-field. Similarly, gravitational radiation reaction tends to align the distortion axis  $n_B^i$  with the spin direction (independent of the sign of  $\epsilon_B$ ) on the gw spindown timescale  $\tau_{GW}$  [16]:  $\dot{\theta} = -\frac{1}{32}\theta/\tau_{GW}$  for small wobble angle  $\theta$ , where here  $\tau_{GW}$  is defined as the rhs of Eq. (2.9).

Next there is the timescale  $\tau_{ACC}$  on which accretion can significantly change the NS’s angular momentum (either magnitude or direction). We approximate  $\tau_{ACC} \equiv J/2\dot{J}_{ACC}$  by [23]

$$1/\tau_{ACC} = 9.3 \times 10^{-16} \text{s}^{-1} \left( \frac{\dot{M}}{10^{-9} M_\odot/\text{yr}} \right) \left( \frac{300 \text{Hz}}{\nu_s} \right). \quad (2.13)$$

Lastly, we consider the dissipation timescale  $\tau_{DIS}$  on which the instability acts. Define  $n$  to be the dissipation timescale  $\tau_{DIS}$  divided by the wobble period, so  $\tau_{DIS} \equiv nP/\epsilon_B$ , where  $\tau_{DIS} \sim |\epsilon_B|[J^2\theta^2/(2I)]/\dot{E}_{DIS}$ . We can rewrite this as

$$1/\tau_{DIS} = 3.0 \times 10^{-8} \text{s} \left( \frac{10^4}{n} \right) \left( \frac{\nu_s}{300 \text{ Hz}} \right) \left( \frac{\epsilon_B}{10^{-7}} \right). \quad (2.14)$$

The  $n$  factor is hard to estimate, but fortunately most of our conclusions are essentially independent of  $n$  over a huge range. Previous authors who have considered precessional damping in NS's have generally concluded that precession damps quickly. E.g., Chau & Henriksen [24] considered dissipation in the elastic crust as  $\Omega^i$  (and hence the centrifugal bulge) precess through the body frame. The crust is periodically distorted by the wobbling motion, and the elastic energy stored in the crust is some fraction  $F$  of the total wobble energy, where we estimate  $0.05 < F < 0.5$ . (For the Earth,  $F$  is 0.11 [25].) If some fraction  $1/Q$  of the elastic energy is dissipated in each wobble period, then  $n \sim Q/F$ . Typically  $Q \sim 10^4$  for terrestrial metals, so one might estimate  $n \lesssim 10^5$ . Other mechanisms seem likely to lead to faster damping: the core and crust will not wobble together rigidly, and relative internal motions will lead to frictional damping, e.g., at the crust-core interface, or through interactions of neutron superfluid vortices with electrons and nuclei in the inner crust. (E.g., Alpar & Sauls [26] estimated  $n \approx 10^2 - 10^4$  for slowly rotating NS's, with the dissipation due to electron scattering off pinned superfluid vortices. However superfluid vortices certainly will *not* remain pinned in the wobble regimes of interest to us—large wobble angle and  $\nu_s \gtrsim 300$  Hz—so this particular estimate does not seem immediately relevant to our case; see Link & Cutler 2002 [27].) We therefore leave  $n$  as unknown parameter to be determined by observation or improved theory. The point is that, for LMXB's and millisecond pulsars, any  $n$  less than  $\sim 10^9$  means the dissipation timescale is much shorter than all the others, and so should successfully drive  $n_B^i$  orthogonal to  $J^i$ .

We have collected all the necessary formulae. We now apply them to interesting cases: LMXB's, millisecond pulsars, and newborn NS's.

### III. LMXB'S AND MILLISECOND PULSARS

#### A. Evolution Scenario

The arguments and timescales above lead to the following evolution scenario for (some large fraction of) LMXB's and millisecond pulsars. The NS is born with  $B_t \gtrsim 4B_p$  and  $B_d \sim 10^{12} - 10^{14}$  G. At birth,  $n_B^i$  is likely nearly aligned with the NS's spin (and so is nearly aligned with  $n_d^i$ ). The NS spins down electromagnetically, but much later it is recycled by accretion from a

companion. Accretion reduces the exterior field  $B_d$  below  $\sim 10^9$  G; however in the interior,  $B_t$  remains in the range  $\sim 10^{12} - 10^{15}$  G. As the NS spins back up and its external B-field decays, it reaches a state where  $\tau_{DIS}$  is shorter than the other timescales (including  $\tau_{EM}$ , thanks to the decay of  $B_d$ ), while  $|\epsilon_B| > \epsilon_d$ . Then dissipation rapidly “flips”  $n_B^i$  perpendicular to  $J^i$ . Initially  $n_d^i$  flips along with  $n_B^i$ , but we expect the crust of a rapidly spinning NS will crack/relax such that its new preferred axis  $n_d^i$  is again aligned with  $\Omega^i$ , but now is perpendicular to  $n_B^i$ . This final configuration is clearly a minimum of both KE and PE (for fixed  $J$ ), independent of the relative sizes of  $\epsilon_d$  and  $|\epsilon_B|$ . Thus while the NS continues to spin up, dissipation will ensure that  $n_d^i$  remains aligned with  $J^i$ , with  $n_B^i$  perpendicular to both. This is important because  $\epsilon_d$  increases quadratically with spin frequency, and so (as we shall see) eventually surpasses  $|\epsilon_B|$  for the fastest LMXB's and millisecond pulsars. Our point is that as long as  $n_d^i$  has already been “re-set” along  $J^i$  before that happens, then  $n_B^i$  will remain orthogonal to  $J^i$ . After accretion stops, the NS continues to spin down by gw emission. And because  $n_B^i$  is time-varying (rotating around  $J^i$  with frequency  $\nu_s$ ) while  $n_d^i$  is not, it is the magnetic distortion that sources the gravitational waves.

A potential objection to this scenario arises if one assumes that the pulsar beam is aligned with the axis  $n_B^i$  of the B-field in deep interior, since then we would predict that (a large fraction of) millisecond pulsars should be orthogonal rotators, which is not observed. So for our picture to be sensible, we should assume that the B-field near the NS surface evolves (e.g., via accretion and crust-cracking) in such a way that that the pulsar beam is *not* aligned with  $n_B^i$ .

#### B. LMXB's

The distribution of LMXB spin periods suggest that many have “hit a wall” at  $\nu_s \sim 260 - 590$  Hz—far below the likely maximum rotation rate of  $1 - 1.5$  kHz [5,28]. Bildsten [5] proposed that they had reached an equilibrium where accretion torque was balanced by spin-down from gravitational radiation (resuscitating an idea of Wagoner's [29]).

Equilibrium between accretion torque and gw spin-down implies an ellipticity

$$\epsilon = 4.5 \times 10^{-8} \left( \frac{\dot{M}}{10^{-9} M_\odot/\text{yr}} \right)^{1/2} \left( \frac{300 \text{ Hz}}{\nu_s} \right)^{5/2}. \quad (3.1)$$

For the LMXB's with known spin rates, the range of  $\dot{M}$ 's is  $\sim 10^{-11} - 2 \times 10^{-8} M_\odot/\text{yr}$ , with implied  $\epsilon$ 's in the range  $\sim 3 \times 10^{-9} - 3 \times 10^{-7}$  [28].

On this assumption, gw's from the brightest LMXB's (especially Sco X-1) should be detectable by LIGO II. Ushomirsky et al. [23] showed that crustal “mountains” can provide the required ellipticity, but only if the crustal breaking strain  $\sigma_{max}$  is larger than  $\sim 10^{-3} - 10^{-2}$ .

Toroidal B-fields place no such requirement on the crust’s breaking strain:  $B_t \sim 2 \times 10^{12} - 2 \times 10^{14} \text{G}$  provides the required ellipticity. Our picture requires that gw torque dominates em torque, or  $B_d \lesssim 5 \times 10^8 - 5 \times 10^{10} \text{G}$ , by Eq. (2.11a). There are no direct measurements of  $B_d$  in LMXB’s to compare this to, but  $B_d \lesssim 10^9 \text{G}$  is indicated by the lack of X-ray pulses.

While for simplicity we have assumed  $B_t \gg B_p$ , it should be clear that the basic picture remains the same if they are comparable, so long as the total  $\epsilon_B$  is negative and  $|\epsilon_B| \sim 10^{-8} - 10^{-7}$ .

### C. Millisecond Pulsars

We turn now to millisecond radio pulsars (which we define as those with spin periods  $P < 10 \text{ms}$ ), which are commonly assumed to be the descendants of LMXB’s. Our picture suggests that many should now be spinning down by gw emission. The NS’s characteristic age  $\tau_c \equiv P/(2\dot{P})$  is then just  $\tau_{GW}$ . We can re-write Eq. (2.9) as

$$|\epsilon_B| = 6.0 \times 10^{-9} \left( \frac{10^{10} \text{yr}}{\tau_c} \right) \left( \frac{P}{5 \text{msec}} \right)^2, \quad (3.2)$$

or  $B_t \sim 4 \times 10^{12} \text{G}$  for our fiducial millisecond pulsar values ( $\tau_c = 10^{10} \text{yr}$  and  $P = 5 \text{msec}$ )—consistent with the lower end of the range we inferred for LMXB’s. (A comparison of derived  $\epsilon_B$ ’s in the LMXB and millisecond pulsar populations would be interesting, but is clearly complicated by selection effects; e.g., low- $|\epsilon_B|$  millisecond pulsars live longer, and so will dominate the observed sample.) Note that  $|\epsilon_B| > \epsilon_d$  only for  $P \gtrsim 4.0 \text{msec} (\tau_c / 10^{10} \text{yr})^{1/2}$  (from Eqs. 2.2 and 3.2), so the oldest/fastest millisecond pulsars would not be unstable in the way we described in §II. However, as explained in §III.A, that probably does not matter: their magnetic axes were flipped orthogonal to  $J^i$  while they were being spun up, and will not flip back if the crust has relaxed.

Given a millisecond pulsar at distance  $D$ , spinning down by gw emission, it is straightforward to compute the detectability of the gw signal  $h(t)$ . The rms  $S/N$  (averaged over source direction and polarization) is

$$S/N = 1.1 \left( \frac{1 \text{kpc}}{D} \right) \left( \frac{T_o}{1 \text{yr}} \right)^{1/2} \left( \frac{10^{10} \text{yr}}{\tau_c} \right)^{1/2} \left( \frac{2 \times 10^{-24}}{S_h^{1/2}(f)} \right), \quad (3.3)$$

where  $f = 2\nu_s$  is the gravity-wave frequency,  $S_h(f)$  is the (single-sided) noise spectral density, and  $T_o$  is the observation time. The broad-band LIGO-II noise curve has a broad minimum at  $f_{gw} \sim 400 \text{Hz}$ , with  $S_h^{1/2}(f) \approx 2 \times 10^{-24}$  there [31]. Using this fiducial value for  $S_h^{1/2}(f)$ , one finds there are at least 4 known millisecond pulsars with  $S/N > 2.7$  in a 1-yr observation. PSR J0437-4715 ( $f_{gw} = 347 \text{Hz}$ ,  $\tau_c = 0.6 \times 10^{10} \text{yr}$ , and  $D = 0.18 \text{kpc}$  [32]) gives the highest  $S/N$  ratio:  $S/N = 7.9$ . Also, PSR

J1744-1134 ( $f_{gw} = 491 \text{Hz}$ ,  $\tau_c = 0.72 \times 10^{10} \text{yr}$ ,  $D = 0.36 \text{kpc}$  [34]) yields  $S/N = 3.6$ ; PSR J1024-0719 ( $f_{gw} = 387 \text{Hz}$ ,  $\tau_c = 2.7 \times 10^{10} \text{yr}$ ,  $D = 0.20 \text{kpc}$  [32]) yields  $S/N = 3.3$ ; and PSR J1012+5307 ( $f_{gw} = 381 \text{Hz}$ ,  $\tau_c = 0.6 \times 10^{10} \text{yr}$ ,  $D = 0.52 \text{kpc}$  [33]) gives  $S/N = 2.7$ . Factor  $\sim 2.5$  improvements in these SNR’s could be obtained by operating the detector in narrow-band, signal-recycling mode.

Note that since the gw signal is almost completely known from radio observations (i.e., up to polarization, amplitude and overall phase),  $S/N \gtrsim 2.7$  is enough for confident detection.

### IV. NEWBORN PULSARS

As another application of our basic idea, consider the spindown gw’s from a supernova in our own galaxy ( $D \sim 10 \text{kpc}$ ). (The supernova rate in our galaxy is  $\sim 1/40 \text{yr}$ , so this author expects to be around for the next one.) It does not seem outlandish to posit field strengths  $B_d \sim 10^{14} \text{G}$  and  $B_t \sim 10^{15} \text{G}$ . Now em radiation dominates the spindown torque, but there is significant gw emission as well.

The matched-filtering  $(S/N)^2$  for a NS spinning down *solely* due  $l = m = 2$  gw’s is [30]

$$\left( S/N \right)^2 = \frac{2G}{5\pi D^2 c^3} \int_{f_{min}}^{f_{max}} \frac{dJ/df}{f S_h(f)} df. \quad (4.1)$$

For our case, we just need to multiply the numerator in Eq. (4.1) by  $\dot{J}_{GW}/\dot{J}_{EM} = \tau_{EM}/\tau_{GW} \propto f^2$ . Approximating the high-frequency part of the broad-band, LIGO-II noise spectrum by  $S_h(f) = 2.1 \times 10^{-47} \text{s} (f/10^3 \text{Hz})^2$  [31], we find

$$S/N = 11.7 \left( \frac{10 \text{kpc}}{D} \right) \left( \frac{\epsilon_B}{10^{-6}} \right) \left( \frac{10^{14} \text{G}}{B_d} \right) \left[ \ln \left( \frac{f_{max}}{f_{min}} \right) \right]^{1/2}. \quad (4.2)$$

Here we can take  $f_{min}$  to be the frequency ( $\sim 500 \text{Hz}$ ) where the actual LIGO-II broad-band noise curve “flattens out.”

E.g., consider a NS with  $B_t = 2 \times 10^{15} \text{G}$  and  $B_d = 10^{14} \text{G}$ , which spins down from  $\nu_s \sim 1 \text{kHz}$  to  $\nu_s = 250 \text{Hz}$  in  $\sim 10$  days (and to  $\nu_s \approx 80 \text{Hz}$  after one year). In order for  $n_B^i$  to “flip” before the NS spins down substantially, we need  $\tau_{DIS} < \tau_{EM}$ , or  $n \lesssim 1.3 \times 10^3$ , by Eqs. (2.10) and (2.14). Assuming  $n$  is this small, and taking  $[\ln(f_{max}/f_{min})]^{1/2} \approx 1$ , we find  $S/N = 75$  for  $D = 10 \text{kpc}$ , which would likely be detectable even in a semi-blind search—i.e., one where neutrino detectors and other channels give the collapse time and a rough position, but where there is no radio signal to give us the rotational phase as function of time.

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[34] M. Toscano et al., ApJ **523**, L171 (1999); astro-ph/99906372.

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- [1] M. Lyutikov, C. Thompson, and S. R. Kulkarni, in *Neutron Stars in Supernova Remnants* (ASP, 2002); astro-ph/0111319.
  - [2] C. Thompson, astro-ph/0110679.
  - [3] S. Mereghetti, L. Chiarlone, G. L. Israel, and L. Stella; astro-ph/0205122.
  - [4] R.A. Chevalier, in *Neutron Stars in Supernova Remnants*, (ASP, 2002); astro-ph/0201295.
  - [5] L. Bildsten, ApJ **501**, L89 (1998).
  - [6] P. B. Jones, Ap. and Sp. Sci. **33**, 215 (1975).
  - [7] P. B. Jones, Nature **262**, 120 (1976).
  - [8] P. B. Jones, ApJ **209**, 602 (1976).
  - [9] P. B. Jones, Ap. and Sp. Sci. **45**, 669 (1976).
  - [10] P. B. Jones, MNRAS **178**, 87 (1977).
  - [11] S. Bonazzola and E.ourgoulhon, A&A **312**, 675 (1996).
  - [12] D.I. Jones, Ph.D. thesis, University of Wales, Cardiff, 2000.
  - [13] A. Melatos and E. S. Phinney, PASA **18**, 421 (2001).
  - [14] D. Pines and J. Shaham, Nature Physical Science **235**, 43 (1972); Phys. Earth Planet. Interiors **6**, 103 (1972)
  - [15] C. Cutler, G. Ushomirsky, and B. Link, in preparation.
  - [16] C. Cutler and D. I. Jones, Phys. Rev. D **63**, 24002 (2001).
  - [17] D. G. Wentzel, ApJ Supp. **47**, 187 (1960).
  - [18] J. P. Ostriker and J. E. Gunn, ApJ **157**, 1395 (1969).
  - [19] I. Easson and C. J. Pethick, Phys. Rev. D **16**, 275 (1977).
  - [20] P. Goldreich and A. Reisenegger, ApJ **395**, 250 (1992).
  - [21] P. Goldreich and W.H. Julian, ApJ **157**, 869 (1969).
  - [22] P. Goldreich, ApJ **160**, L11 (1970).
  - [23] G. Ushomirsky, C. Cutler, and L. Bildsten, MNRAS **319**, 902 (2000).
  - [24] W.Y. Chau and R.N. Henriksen, Astrophys. Letters **8**, 49 (1971)
  - [25] F.D. Stacey, *Physics of the Earth, 3rd Ed.* (Brookfield Press, 1992).
  - [26] A. Alpar and J.A. Sauls, ApJ **327**, 723 (1988)
  - [27] B. Link and C. Cutler, to appear in MNRAS; astro-ph/0108281.
  - [28] G. Ushomirsky, L. Bildsten, and C. Cutler, in *Proceedings of the Third Edoardo Amaldi Conference* (AIP Press, 2000); astro-ph/0001129.
  - [29] R. V. Wagoner, ApJ **278**, 345 (1984).
  - [30] B. J. Owen and L. Lindblom, Class. Quant. Grav. **19**, 1247 (2002).
  - [31] K. S. Thorne, LIGO Internal Document G000025-00-M.
  - [32] M. Toscano et al., MNRAS **307**, 925 (1999); astro-ph/9811398.
  - [33] D. Lorimer, Living Reviews in Relativity (2001); astro-ph/0104388.