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Vacua of maximal gauged $D = 3$ supergravities

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Abstract

We analyse the scalar potentials of maximal gauged three-dimensional supergravities which reveal a surprisingly rich structure. In contrast to maximal supergravities in dimensions $D \geqslant 4$, all these theories possess a maximally supersymmetric ($N = 16$) ground state with negative cosmological constant $\Lambda < 0$, except for the $SO(4,4)^2$ gauged theory, whose maximally supersymmetric groundstate has $\Lambda = 0$. We compute the mass spectra of bosonic and fermionic fluctuations around these vacua and identify the unitary irreducible representations of the relevant background (super)isometry groups to which they belong.

In addition, we find several stationary points which are not maximally supersymmetric, and determine their complete mass spectra as well. In particular, we show that there are analogues of all stationary points found in higher dimensions, among them are de Sitter (dS) vacua in the theories with noncompact gauge groups $SO(5,3)^2$ and $SO(4,4)^2$, as well as anti-de Sitter (AdS) vacua in the compact gauged theory preserving $\frac{1}{4}$ and $\frac{1}{8}$ of the supersymmetries. All the dS vacua have tachyonic instabilities, whereas there do exist nonsupersymmetric AdS vacua which are stable, again in contrast to the $D \geqslant 4$ theories.

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1. Introduction

Maximal ($N = 16$) gauged supergravities [1, 2] are the most symmetric of all known field theories in three spacetime dimensions. Their unique position is not least a consequence of the presence of the 'maximally extended' Lie algebra $E_{8(8)}$ which plays a very special role in their construction. In contrast to gauged supergravities in higher dimensions, the vector fields appear via a non-Abelian Chern–Simons term rather than the usual Yang–Mills term, implying

a non-Abelian duality between scalars and vectors which has no analogue in dimensions $D > 3$ because suitable non-Abelian extensions of higher-rank tensor gauge theories do not appear to exist. As required by supersymmetry and the matching of bosonic and fermionic degrees of freedom on-shell, these vectors do not introduce new propagating degrees of freedom over and above the scalar fields already present in these theories (in the ungauged version of the theory obtained by torus reduction from eleven dimensions, eight Kaluza–Klein vectors and 28 vectors coming from the rank-3 antisymmetric tensor are dualized to scalar fields [3]). The fact that the number of gauge fields is not *a priori* fixed entails a much greater variety of gaugings with both compact and noncompact gauge groups $G_0 \subset E_{8(8)}$ than in higher dimensions.

In this paper, we will focus on the semisimple gaugings obtained in [1, 2] and investigate the associated scalar field potentials, which arise through the gauging. The existence of maximal gauged supergravities with non-semisimple gauge groups will be demonstrated in a separate publication; again, there are more possibilities than in higher dimensions as well as new phenomena without higher-dimensional analogues. The potentials of gauged $N = 16$ supergravity are substantially more complicated than the potentials of maximal gauged supergravities in dimensions $D \geqslant 4$, and arguably the most intricate potentials ever encountered in the context of supergravity (and perhaps beyond). A glimpse of their structural wealth is already offered by their maximally supersymmetric stationary points (at the origin $\mathcal{V} = I$) which exist for all the semisimple gauge groups G_0, and which we study in some detail here. Our analysis nicely exemplifies the representation theory of supergroups $\mathfrak{G}$ containing the $D = 3$ AdS group $SO(2, 2)$ [4]. In fact, the model contains representative examples of almost all such supergroups, including the exceptional ones $G(3)$ and $F(4)$.

Although a general study of the extremal properties of the potentials appears to be beyond reach with present techniques, considerable progress can be made by adapting a technique first introduced by Warner [5], which consists in studying the potential on a restricted subspace of scalar fields which are singlets under some fixed subgroup of the gauge group. In a previous paper by one of the authors [6], this technique was already employed to identify a number of nontrivial stationary points for the $SO(8) \times SO(8)$ gauged theory. Here, we continue this analysis by working out the potentials for various other gauge groups and singlet sectors, and exhibit several new nontrivial stationary points. In addition, we give general mass formulae which allow us to compute the full mass spectra at each of these stationary points. A good part of our analysis relies on the computational methods developed in [6], which are described in greater detail in [7].

Let us briefly summarize the most interesting facets of our findings. The compact gauge group $G_0 = SO(8)^2$ admits AdS vacua preserving $\frac{1}{4}$ and $\frac{1}{8}$, respectively, of the supersymmetries. In addition, for gauge groups $G_0 = SO(8)^2$ and $SO(7, 1)^2$ we identify nonsupersymmetric AdS vacua which unlike their known higher-dimensional analogues are stable in the sense that all scalar fields satisfy the Breitenlohner–Freedman bound [8]. For the noncompact gauge group $G_0 = SO(5, 3)^2$ we find the first example of a maximal supersymmetric model with both AdS and dS stationary points. The potential corresponding to the gauge group $G_0 = SO(4, 4)^2$ even interpolates between a dS stationary point and a maximally supersymmetric vacuum with vanishing cosmological constant. As a more exotic example, we investigate the potential of the theory with exceptional gauge group $G_2 \times F_{4(-20)}$, and find a nontrivial supersymmetric AdS stationary point, which breaks the maximal $N = (7, 9)$ supersymmetry in an asymmetric way to a residual $N = (0, 1)$ supersymmetry and an unbroken $SU(3) \times SO(7)^-$ symmetry.

Further motivation for our studies comes from the appearance of gauged supergravities in the AdS/CFT correspondence [9]. In particular, their scalar potentials turn out to carry

the information about holographic renormalization group flows in the boundary quantum field theories, see e.g. [10–12] for work in higher dimensions. Flows in three-dimensional gauged supergravities have recently been studied in the $N = 8$ theories related to the D1–D5 system [13]. The maximal ($N = 16$) theories remain to be fully exploited in this context; in particular, they may have a role to play in the supergravity description of matrix string theories [14, 15]. We should like to emphasize that not much is known about the boundary (super)conformal field theories related to the maximal AdS supergravities in three dimensions. What is known is that, in the absence of propagating degrees of freedom in the bulk, the pure CS theories reduce to Liouville and WZNW theories on the boundary (or their supersymmetric extensions) [16–18]. An important question concerns the extendibility of the background superisometries with more than $N = 4$ supersymmetry on the boundary worldsheet to infinite-dimensional superalgebras containing the Witt–Virasoro algebra.

Finally, pure CS theories in three dimensions are known to occupy a central place in the classification of knot invariants *à la* Jones Witten. While the significance of the gauge groups found here, in particular the noncompact ones, in that context is not clear (most of the previous work [19–21] is based on the compact gauge groups $SU(2)$), one could hope that gauged supergravity might also provide a much wider framework for investigations in $D = 3$ differential geometry and topology.

This paper is organized as follows. In section 2, we give a brief review of the three-dimensional maximal gauged supergravities, in particular their scalar potentials, stationarity conditions, and the computation of the mass matrices around a given stationary point. In section 3, we analyse in some detail the maximally $N = (8, 8)$ supersymmetric vacua of the theories with gauge group $SO(p, 8-p) \times SO(p, 8-p)$. The spectra of physical fields are organized by the corresponding superextensions of the AdS_3 group $SO(2, 2)$, except for $p = 4$ for which the ground state is Minkowskian. In section 4, we extend this analysis to the exceptional gauge groups which all admit a maximally supersymmetric AdS vacua. Section 5 finally is devoted to further extremal points in the scalar potentials which do not preserve the full supersymmetry.

2. Potential and mass matrices

Let us briefly recall the pertinent facts about gauged maximal ($N = 16$) supergravity in three dimensions, which we will need here, especially those concerning the scalar potential. For further information, we refer readers to [1, 2] where the construction of the gauged theories has been explained in great detail. In addition, we here present some new formulae which will enable us to calculate the various mass matrices at the stationary points under consideration.

The gauging of $N = 16$ supergravity was achieved in [1, 2] by minimally coupling the scalar fields to their dual vector fields. This induces a non-Abelian duality between vectors and scalars, which has no analogue in higher dimensions. Due to the fact that the number of gauge fields is not determined *a priori* (unlike in dimensions $D \geqslant 4$), there is a richer variety of possible gauge groups G_0, all of which are subgroups of the rigid $E_{8(8)}$ symmetry of ungauged $N = 16$ supergravity. As in the ungauged theory [3, 22] the 128 propagating scalar fields of the theory are conveniently described as elements of the coset space $E_{8(8)}/SO(16)$, i.e. in terms of a matrix $\mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}$ in the adjoint (fundamental) representation of $E_{8(8)}$ subject to the transformations

$$\mathcal{V}(x) \longrightarrow g(x)\mathcal{V}(x)h^{-1}(x), \qquad g(x) \in G_0 \subset E_{8(8)}, \quad h(x) \in SO(16). \tag{2.1}$$

Here, we use indices $\mathcal{A}, \mathcal{B}, \ldots$ and $\mathcal{M}, \mathcal{N}, \ldots = 1, \ldots, 248$ as collective labels for the $E_{8(8)}$ generators, such that the former transform under local $SO(16) \subset E_{8(8)}$ while the latter

transform under the full $E_{8(8)}$ in the ungauged theory, and under the gauge group G_0 in the gauged theory. Under the $SO(16)$ subgroup, the $E_{8(8)}$ generators $\{t^{\mathcal{M}}\}$ split into 120 compact ones $X^{IJ} \equiv -X^{JI}$ and 128 noncompact ones Y^A, with $SO(16)$ vector indices $I, J, \ldots \in \underline{16}$, and spinor indices $A, B, \ldots \in \underline{128}$, with a corresponding split in the collective labels $\mathcal{M}, \mathcal{N}, \ldots = ([IJ], A), \ldots$. We will also need the conjugate $SO(16)$ spinors labelled by dotted indices $\dot{A}, \dot{B}, \ldots$.

The scalar potential of $N = 16$ gauged supergravity is thus a function on the coset space $E_{8(8)}/SO(16)$. It is explicitly given by

$$V = -\tfrac{1}{8}g^2\left(A_1^{IJ}A_1^{IJ} - \tfrac{1}{2}A_2^{I\dot{A}}A_2^{I\dot{A}}\right), \tag{2.2}$$

where the tensors A_1 and A_2, respectively, transform in the $\mathbf{1}+\mathbf{135}$ and $\overline{\mathbf{1920}}$ representations of $SO(16)$. A third tensor $A_3^{\dot{A}\dot{B}}$, transforming in the $\mathbf{1}+\mathbf{1820}$, governs the Yukawa couplings of the matter fermions to the scalars; unlike its analogues in dimensions $D \geqslant 4$, it is algebraically independent of A_1 and A_2. These tensors are defined as

$$\begin{aligned} A_1^{IJ} &= \tfrac{8}{7}\theta\delta_{IJ} + \tfrac{1}{7}T_{IK|JK}, \\ A_2^{I\dot{A}} &= -\tfrac{1}{7}\Gamma^J_{A\dot{A}}T_{IJ|A}, \\ A_3^{\dot{A}\dot{B}} &= 2\theta\delta_{\dot{A}\dot{B}} + \tfrac{1}{48}\Gamma^{IJKL}_{\dot{A}\dot{B}}T_{IJ|KL}, \end{aligned} \tag{2.3}$$

in terms of the so-called T-tensor

$$T_{\mathcal{A}|\mathcal{B}} = \mathcal{V}^{\mathcal{M}}{}_{\mathcal{A}}\mathcal{V}^{\mathcal{N}}{}_{\mathcal{B}}\Theta_{\mathcal{M}\mathcal{N}}, \qquad \theta = \tfrac{1}{248}\eta^{\mathcal{M}\mathcal{N}}\Theta_{\mathcal{M}\mathcal{N}} \tag{2.4}$$

where $\Gamma^I_{A\dot{A}}$ are the standard $SO(16)$ Γ-matrices. The numerical tensor $\Theta_{\mathcal{M}\mathcal{N}}$ is the embedding tensor of the gauge group $G_0 \subset E_{8(8)}$, i.e. it may be thought of as the restriction of the $E_{8(8)}$ Cartan–Killing form $\eta_{\mathcal{M}\mathcal{N}}$ to G_0. As shown in [1, 2] all consistency conditions, and in particular the maximal supersymmetry of the gauged theory, are satisfied as a consequence of a single algebraic condition on this embedding tensor, namely

$$\mathbb{P}_{\mathbf{27\,000}}\Theta = 0 \tag{2.5}$$

where $\mathbb{P}_{\mathbf{27\,000}}$ is the projector onto the $\mathbf{27\,000}$ representation in the decomposition

$$(\mathbf{248}\times\mathbf{248})_{\text{sym}} = \mathbf{1}\oplus\mathbf{3875}\oplus\mathbf{27\,000}. \tag{2.6}$$

The condition (2.5) entails that only the $SO(16)$ representations $\mathbf{1}$, $\mathbf{135}$, $\mathbf{1820}$ and $\overline{\mathbf{1920}}$ can appear in Θ. More specifically, we have

$$\begin{aligned} \Theta_{IJ|KL} &= -2\theta\delta^{IJ}_{KL} + 2\delta_{\underline{I[K}}\Xi_{\underline{L]J}} + \Xi_{IJKL}, \\ \Theta_{IJ|A} &= -\tfrac{1}{7}\Gamma^{[I}_{A\dot{A}}\Xi^{J]\dot{A}}, \\ \Theta_{A|B} &= \theta\delta_{AB} + \tfrac{1}{96}\Xi_{IJKL}\Gamma^{IJKL}_{AB}, \end{aligned} \tag{2.7}$$

where Ξ_{IJ}, Ξ_{IJKL} and $\Xi^{I\dot{A}}$ denote the $\mathbf{135}$, $\mathbf{1820}$ and $\overline{\mathbf{1920}}$ representations of $SO(16)$, respectively (hence $\Xi_{II} = 0$ and $\Gamma^I_{A\dot{A}}\Xi^{I\dot{A}} = 0$, and Ξ_{IJKL} is completely antisymmetric in its four indices). For the semisimple gauge groups identified in [2] the embedding tensor Θ has no component transforming as the $\overline{\mathbf{1920}}$ representation, and we will therefore set

$$\Xi^{I\dot{A}} = 0, \tag{2.8}$$

in the remainder of this paper. As we will explain elsewhere, however, this component is needed for the non-semisimple gaugings.

Stationary points of the scalar potential (2.2) are characterized by

$$\frac{\delta V}{\delta\Sigma^A} = 0 \quad\Longleftrightarrow\quad 3A_1^{IM}A_2^{M\dot{A}} = A_3^{\dot{A}\dot{B}}A_2^{I\dot{B}}, \tag{2.9}$$

where the derivative is taken with respect to a left invariant vector field Σ^A along the coset manifold $E_{8(8)}/SO(16)$. By an adaptation of the arguments of [23], it has been shown in [2] that the number of unbroken supersymmetries at a stationary point is determined by the number of eigenvalues α_i of A_1^{IJ} satisfying

$$16\alpha_i^2 = A_1^{IJ}A_1^{IJ} - \tfrac{1}{2}A_2^{I\dot{A}}A_2^{I\dot{A}} = \frac{4}{g^2L^2}. \tag{2.10}$$

Here L denotes the AdS scale, which is set by the value V_0 of the potential at the stationary point, namely

$$\frac{4}{g^2L^2} \equiv -\frac{8}{g^2}V_0, \tag{2.11}$$

(as is well known, unbroken supersymmetry requires the value of V_0 to be non-positive). Maximal supersymmetry is then equivalent to $A_2^{I\dot{A}} = 0$.

By use of the formulae given in section 4.3 of [2] it is straightforward to compute the scalar mass matrix at any given stationary point, which is given by the matrix of second derivatives,

$$\begin{aligned}-4g^{-2}\mathcal{M}_{AB} &\equiv -8g^{-2}\frac{\delta^2}{\delta\Sigma^A\delta\Sigma^B}V \\ &= \tfrac{3}{4}\left(\Gamma^I_{A\dot{A}}A_2^{J\dot{A}}A_2^{J\dot{B}}\Gamma^I_{\dot{B}B} + \Gamma^I_{A\dot{A}}A_2^{J\dot{A}}A_2^{I\dot{B}}\Gamma^J_{\dot{B}B}\right) + \tfrac{3}{4}A_1^{IJ}A_1^{IJ}\delta_{AB} - \tfrac{3}{4}A_1^{II}T_{A|B} \\ &\quad + \tfrac{1}{2}\Gamma^I_{A\dot{A}}A_1^{IJ}A_3^{\dot{A}\dot{B}}\Gamma^J_{\dot{B}B} - \tfrac{1}{4}\Gamma^I_{A\dot{A}}A_3^{\dot{A}\dot{C}}A_3^{\dot{C}\dot{B}}\Gamma^I_{\dot{B}B} + \tfrac{1}{4}\Gamma^I_{A\dot{A}}A_3^{\dot{A}\dot{C}}\Gamma^I_{\dot{C}C}T_{C|B}.\end{aligned} \tag{2.12}$$

Because the derivatives have been taken with respect to a left invariant vector field, the scalar kinetic term is uniformly normalized,

$$\mathcal{L}_{\rm kin} = \tfrac{1}{4}e\partial^\mu\Sigma^A\partial_\mu\Sigma^A + \cdots, \tag{2.13}$$

independently of which stationary point of the potential one is expanding around.

A substantial part of this paper will be devoted to studying the mass matrices at the origin $\mathcal{V} = I$. By (2.4) the T-tensor then coincides with the embedding tensor, i.e. $T = \Theta$. Since $\Theta_{IJ|A} = 0$ for all the semisimple gaugings considered in this paper, we have

$$A_2^{I\dot{A}}\big|_{\mathcal{V}=I} = 0, \tag{2.14}$$

and the stationarity condition (2.9) is trivially satisfied. By the same token, all these stationary points preserve maximal supersymmetry. Observe that this is not true in dimensions $D \geqslant 4$ where the origin $\mathcal{V} = I$ is not a stationary point of the potential, unless the gauge group is compact. Not unexpectedly, the scalar mass matrix (2.12) simplifies considerably when $\mathcal{V} = I$: with

$$A_1^{IJ} = -\theta\delta^{IJ} + \Xi_{IJ} \qquad A_3^{\dot{A}\dot{B}} = 2\theta\delta^{\dot{A}\dot{B}} + \tfrac{1}{48}\Xi_{IJKL}\Gamma^{IJKL}_{\dot{A}\dot{B}}, \tag{2.15}$$

we obtain

$$\begin{aligned}-4g^{-2}\mathcal{M}_{AB}|_{\mathcal{V}=I} &= \left(\tfrac{3}{4}\Xi^{IJ}\Xi^{IJ} - \tfrac{1}{32}\Xi_{KLMN}\Xi_{KLMN}\right)\delta_{AB} \\ &\quad + \left(-\tfrac{1}{12}\Xi_{IM}\Xi_{MJKL} + \tfrac{1}{32}\Xi_{IJMN}\Xi_{MNKL}\right)\Gamma^{IJKL}_{AB} \\ &\quad + \tfrac{1}{2304}\Xi_{IJKL}\Xi_{MNPQ}\Gamma^{IJKLMNPQ}_{AB},\end{aligned} \tag{2.16}$$

for the semisimple gaugings with $\Theta_{IJ|A} = 0$. This expression is independent of θ (that this should be so is obvious for $G_0 = E_{8(8)}$, where $\Theta_{\mathcal{MN}} = \theta\eta_{\mathcal{MN}}$ and the potential is constant). Note, however, that θ is not a free parameter, but fixed by group theory in relation to the other components of the embedding tensor. The only tunable free parameter is the overall gauge coupling constant g.

Let us next turn to the vector bosons. They carry indices $\mathcal{M}, \mathcal{N}, \ldots$, transforming in the adjoint representation of G_0. In the absence of mass terms, the vectors do not represent propagating degrees of freedom. However, at the stationary points the G_0 symmetry is spontaneously broken to its maximal compact subgroup H_0. Consequently, the vector fields associated with the noncompact generators of G_0 will absorb the corresponding Goldstone bosons and thereby acquire a mass in a $D = 3$ (topological) variant of the Brout–Englert–Higgs effect. This can be directly seen from the vector field equation of motion, namely equation (3.32) of [2], which we quote here for the reader's convenience:

$$\varepsilon^{\mu\nu\rho}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{B}_{\nu\rho}{}^{\mathcal{N}} = 2e\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}}{}_{A}\mathcal{P}^{\mu A} + \cdots, \tag{2.17}$$

(the dots stand for fermionic terms not relevant for our argument). At a given stationary point, this equation reduces to

$$\varepsilon^{\mu\nu\rho}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{B}_{\nu\rho}{}^{\mathcal{N}} = 2eg\Theta_{\mathcal{M}\mathcal{K}}\mathcal{V}^{\mathcal{K}}{}_{A}\mathcal{V}^{\mathcal{L}}{}_{A}\Theta_{\mathcal{N}\mathcal{L}}\mathcal{B}_{\mu}{}^{\mathcal{N}}, \tag{2.18}$$

forming a set of massive self-duality equations [24]. The vector mass matrix at this vacuum is hence obtained by restricting $g\mathcal{V}^{\mathcal{M}}{}_{A}\mathcal{V}^{\mathcal{L}}{}_{A}\Theta_{\mathcal{N}\mathcal{L}}$ to the subgroup G_0. Since the eigenvalues of a product of matrices are invariant under cyclic permutation of the factors, the vector masses may equivalently be extracted from the matrix

$$g^{-1}\mathcal{M}^{\text{vec}}_{AB} = \mathcal{V}^{\mathcal{M}}{}_{A}\Theta_{\mathcal{M}\mathcal{N}}\mathcal{V}^{\mathcal{N}}{}_{B} = T_{A|B}. \tag{2.19}$$

This shows that in accordance with the counting of degrees of freedom, never more than 128 vector fields can simultaneously become massive—although there are gauge groups with higher dimension, such as the noncompact $E_{7(7)} \times SL(2)$. At the symmetric vacuum $\mathcal{V} = I$, the matrix (2.19) simplifies to the projection of the embedding tensor onto the noncompact part of the gauge group, cf (2.7),

$$g^{-1}\mathcal{M}^{\text{vec}}_{AB}\big|_{\mathcal{V}=I} = \Theta_{A|B} = \theta\delta_{AB} + \tfrac{1}{96}\Xi_{IJKL}\Gamma^{IJKL}_{AB}. \tag{2.20}$$

It is then evident that only those vector fields corresponding to noncompact generators in the gauge group acquire a mass at $\mathcal{V} = I$. In this way, some propagating bosonic degrees of freedom are shifted from the scalar sector to the vector fields. As we will see very explicitly, this effect is beautifully realized for all gaugings.

For the maximally supersymmetric stationary points, for which $A_2^{I\dot{A}} = 0$, the fermion masses are simply given by the eigenvalues of the matrix $A_3^{\dot{A}\dot{B}}$. All gravitinos remain massless (with a formal mass term dictated by the ambient AdS geometry) and will pair up with the dreibein and the massless vector fields transforming under the unbroken compact subgroup $H_0 \subset G_0$ according to the sign of the associated eigenvalues α_i in (2.10) into (nonpropagating) supermultiplets so as to reproduce the purely topological Lagrangians of [25]. For all other stationary points supersymmetry is partially broken, and we have $A_2^{I\dot{A}} \neq 0$. In that case, some of the fermions become Goldstinos, and the gravitinos acquire a mass by the super Brout–Englert–Higgs effect. To find out which supersymmetries are preserved one must look for Killing spinors satisfying $\delta\psi^I = \delta\chi^{\dot{A}} = 0$. Designating the variations along the direction of broken supersymmetry by φ^I, we split the matter fermions as

$$\chi^{\dot{A}} = \eta^{\dot{A}} + A_2^{I\dot{A}}\varphi^I, \qquad \psi^I_\mu = \tilde{\psi}^I_\mu + \widehat{D}_\mu\varphi^I, \tag{2.21}$$

where $A_2^{I\dot{A}}\eta^{\dot{A}} = 0$, thereby diagonalizing the fermionic mass terms, such that the masses can be read off directly from the eigenvalues of A_1^{IJ} and $A_3^{\dot{A}\dot{B}}$ at the stationary point in question. In summary, there is thus a fermionic analogue of the transferral of physical degrees of freedom from the matter fields to some of the previously nonpropagating gauge fields, in precise agreement with the supermultiplet structure required by the background superisometries.

3. Maximally supersymmetric vacua for gauge groups $G_0 = SO(p, 8-p) \times SO(p, 8-p)$

In this section and the following one, we concentrate on the maximally supersymmetric stationary points and determine the mass matrices for all gauge groups identified in [2]. In particular, we will demonstrate that the mass spectra for the various gauge groups are indeed consistent with the representation theory of the corresponding supergroups as far as it has been developed [4, 26]. In addition, we determine the representations and spectra for the exceptional supergroups $G(3)$ and $F(4)$, which apparently do not admit an oscillator construction of the type considered in [4, 26].

3.1. Embedding of the gauge groups

The gauge groups $SO(p, 8-p) \times SO(p, 8-p)$ for $p = 0, 1, \ldots, 4$ are the only known solutions of (2.5) with vanishing singlet contribution, i.e. $\theta = 0$. They are embedded into $E_{8(8)}$ via its $SO(8, 8)$ subgroup (with $p + q = 8$),

$$SO(p, q) \times SO(p, q) \subset SO(8, 8) \subset E_{8(8)}. \tag{3.1}$$

At the supersymmetric extremum for $\mathcal{V} = I$, the symmetry is broken down to the maximally compact subgroup

$$H_0 = SO(p) \times SO(q) \times SO(p) \times SO(q) \subset SO(8) \times SO(8). \tag{3.2}$$

It is useful to note here that, apart from the case $p = 1$, the verification that $\mathcal{V} = I$ is indeed a (maximally supersymmetric) stationary point does not even require explicit knowledge of the embedding tensor (2.5) and the vanishing of $\Theta_{IJ|A}$ for the gauge groups considered here, but is a direct consequence of the fact that there is no H_0-invariant tensor in the decomposition of the $\overline{\mathbf{1920}}$. Thus A_2 must vanish at the H_0-invariant point, implying stationarity and maximal supersymmetry. We emphasize this point because in dimensions $D \geqslant 4$ the A_2-tensor does contain singlets with respect to the unbroken compact gauge group when $\mathcal{V} = I$, violating supersymmetry and the stationarity condition, see section 5.1.

Let us now study the embedding in somewhat more detail. Under the $SO(8) \times SO(8)$ subgroup of $E_{8(8)}$ the relevant $SO(16)$ representations decompose as follows[3]:

$$\begin{aligned}
\mathbf{16_v} &\longrightarrow (\mathbf{8_v}, \mathbf{1}) + (\mathbf{1}, \mathbf{8_v}) \\
\mathbf{120} &\longrightarrow (\mathbf{28}, \mathbf{1}) + (\mathbf{1}, \mathbf{28}) + (\mathbf{8_v}, \mathbf{8_v}) \\
\mathbf{128_s} &\longrightarrow (\mathbf{8_s}, \mathbf{8_s}) + (\mathbf{8_c}, \mathbf{8_c}) \\
\mathbf{128_c} &\longrightarrow (\mathbf{8_s}, \mathbf{8_c}) + (\mathbf{8_c}, \mathbf{8_s}).
\end{aligned} \tag{3.3}$$

Accordingly, we split the vector indices I as $a \equiv I$ for $I \in \{1, \ldots, 8\}$, and $\bar{a} \equiv I - 8$ for $I \in \{9, \ldots, 16\}$. The compact part of $SO(8, 8)$ is then composed of the two **28** representations occurring in the decomposition of **120**, while its noncompact part is identified with the $(\mathbf{8_s}, \mathbf{8_s})$. In terms of $SO(8)$ γ-matrices, the embedding tensor reads

$$\begin{aligned}
\Theta_{ab|cd} &= \tfrac{1}{4}(P_{\alpha\gamma} P_{\beta\delta} - Q_{\alpha\gamma} Q_{\beta\delta})\gamma^{ab}_{\alpha\beta}\gamma^{cd}_{\gamma\delta} \\
\Theta_{\bar{a}\bar{b}|\bar{c}\bar{d}} &= \tfrac{1}{4}(Q_{\alpha\gamma} Q_{\beta\delta} - P_{\alpha\gamma} P_{\beta\delta})\gamma^{\bar{a}\bar{b}}_{\alpha\beta}\gamma^{\bar{c}\bar{d}}_{\gamma\delta} \\
\Theta_{\alpha\beta|\gamma\delta} &= 2(P_{\alpha\gamma} Q_{\beta\delta} - Q_{\alpha\gamma} P_{\beta\delta}),
\end{aligned} \tag{3.4}$$

[3] Contact with the results for $D = 4$, $N = 8$ is established by noting that with respect to the diagonal $SO(8)$ the $E_{7(7)}$ subgroup consists of the representations $\mathbf{28} + \mathbf{35}_v + \mathbf{35}_s + \mathbf{35}_c$, while the $SL(2)$, which commutes with it, is made out of the three singlets arising in the decomposition of the **120** and the $\mathbf{128}_s$.

with all other components vanishing (in particular, all components with dotted indices: $\Theta_{\dot\alpha\dot\beta|\dot\gamma\dot\delta} = \cdots = 0$). The symbols P and Q are defined by

$$\begin{aligned} P_{\alpha\beta} &= \begin{cases} \delta_{\alpha\beta} & \text{for} \quad \alpha, \beta \in \{1, \ldots, p\} \\ 0 & \text{otherwise,} \end{cases} \\ Q_{\alpha\beta} &= \begin{cases} \delta_{\alpha\beta} & \text{for} \quad \alpha, \beta \in \{p+1, \ldots, 8\} \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{3.5}$$

such that $P_{\alpha\beta} + Q_{\alpha\beta} = \delta_{\alpha\beta}$, which shows explicitly the embedding (3.2), cf [2]. From the form of Θ it is evident that the ratio of the two coupling constants is -1. Furthermore, it is straightforward to verify that this tensor indeed satisfies the projection condition (2.5). To this end, we rewrite the compact part $\Theta_{IJ|KL}$ as

$$\Theta_{ab|cd} = -\tfrac{1}{2}\gamma^{abcd}_{\alpha\beta} P_{\alpha\beta} + \tfrac{3}{8}(p-q)\delta^{cd}_{ab}, \qquad \Theta_{\bar a\bar b|\bar c\bar d} = \tfrac{1}{2}\gamma^{\bar a\bar b\bar c\bar d}_{\alpha\beta} P_{\alpha\beta} - \tfrac{3}{8}(p-q)\delta^{\bar c\bar d}_{\bar a\bar b},$$

which shows that they are indeed of the form (2.7) with $\theta = 0$. Using a triality rotated version of the decomposition of $SO(16)$ Γ-matrices given in [27] (see appendix A), one likewise verifies the last equation in (2.7) for $\Theta_{A|B}$.

For later use, we also record the results for the tensors A_1 and A_3 for $\mathcal{V} = I$, which are

$$A_1^{ab} = \tfrac{1}{4}(p-q)\delta^{ab}, \qquad A_1^{\bar a\bar b} = -\tfrac{1}{4}(p-q)\delta^{\bar a\bar b}, \tag{3.6}$$

and (using the decomposition of the $\mathbf{128}_c$ in (3.3))

$$A_3^{\alpha\dot\gamma\beta\dot\delta} = \tfrac{1}{2}\delta_{\dot\gamma\dot\delta}(q P_{\alpha\beta} + p Q_{\alpha\beta}), \qquad A_3^{\dot\alpha\gamma\dot\beta\delta} = -\tfrac{1}{2}\delta_{\dot\alpha\dot\beta}(q P_{\gamma\delta} + p Q_{\gamma\delta}). \tag{3.7}$$

The value of the cosmological constant at the maximally supersymmetric vacuum is hence given by

$$\Lambda = -\frac{2}{L^2} = -2g^2(p-q)^2. \tag{3.8}$$

In particular, Λ vanishes for $p = q$, i.e. for gauge group $G_0 = SO(4,4) \times SO(4,4)$.

3.2. $SO(8) \times SO(8)$

For the compact gauging, the gauge group remains unbroken at the origin. The background isometry group around this point is $\mathfrak{G} = OSp(8|2,\mathbb{R}) \times OSp(8|2,\mathbb{R})$ which has the bosonic part $(SL(2,\mathbb{R}) \times SO(8))^2$. The physical spectrum is given by the tensor product of two (left and right) singleton supermultiplets according to

$$(8_v + 8_s, 8_v + 8_s), \tag{3.9}$$

under $SO(8)_L \times SO(8)_R$. Representation theory of $OSp(8|2,\mathbb{R})$ gives the conformal weights of the states in this multiplet [4]:

$SO(8)$	8_v	8_s
ℓ_0	$\frac{1}{4}$	$\frac{3}{4}$

(3.10)

From this, one reads that the physical spectrum around the origin consists of 128 scalars and 128 spin-$\frac{1}{2}$ fields with

Fields	$SO(8)_L \times SO(8)_R$	$(\ell_0, \bar\ell_0)$	m^2L^2
Scalars	$(8_v, 8_v)$	$\left(\frac{1}{4}, \frac{1}{4}\right)$	$-\frac{3}{4}$
	$(8_s, 8_s)$	$\left(\frac{3}{4}, \frac{3}{4}\right)$	$-\frac{3}{4}$
Fermions	$(8_v, 8_s)$	$\left(\frac{1}{4}, \frac{3}{4}\right)$	0
	$(8_s, 8_v)$	$\left(\frac{3}{4}, \frac{1}{4}\right)$	0

(3.11)

The relation between mass and conformal dimension $\Delta = \ell_0 + \bar{\ell}_0$ in three dimensions is given by [28, 29]

$$\begin{aligned} &\Delta(\Delta-2) = m^2L^2 && \text{for scalars} \\ &(\Delta-1)^2 = m^2L^2 && \text{for fermions} \\ &(\Delta-1)^2 = m^2L^2 && \text{for massive self-dual vectors,} \end{aligned} \tag{3.12}$$

which gives the mass values in the last column of (3.11) in units of the inverse AdS length (2.11). They indeed agree with the spectrum computed from (2.15), (2.16). All mass eigenvalues satisfy the Breitenlohner–Freedman bound [8, 30]

$$m^2L^2 \geqslant -1. \tag{3.13}$$

The stationary point hence is stable as is implied by supersymmetry. The metric and the massless gravitino fields form a separate (unphysical) 'multiplet' together with the massless self-dual vector fields. They transform in the adjoint representation of $\mathfrak{G}$.

3.3. $SO(7,1) \times SO(7,1)$

The background isometry group at the origin is $\mathfrak{G} = F(4) \times F(4)$, whose bosonic part is $(SL(2,\mathbb{R}) \times SO(7))^2$. The physical spectrum around this point is given by the tensor product of two (left and right) massless unitary supermultiplets according to

$$(1+8+7, 1+8+7), \tag{3.14}$$

under $SO(7)_L \times SO(7)_R$. To the best of our knowledge, the representation theory of $F(4)$ has not been worked out so far. We can however invert the reasoning which for the compact gauge group led to (3.11), and derive the conformal weights from the masses of the supergravity fields. Computing the masses from (2.15), (2.16) and (2.20) gives rise to

Fields	H_0	$(\ell_0, \bar{\ell}_0)$	m^2L^2
Scalars	$(1, 1)$	$\left(\frac{4}{3}, \frac{4}{3}\right)$	$\frac{16}{9}$
	$(8, 8)$	$\left(\frac{5}{6}, \frac{5}{6}\right)$	$-\frac{5}{9}$
	$(7, 7)$	$\left(\frac{1}{3}, \frac{1}{3}\right)$	$-\frac{8}{9}$
Fermions	$(1, 8)$	$\left(\frac{4}{3}, \frac{5}{6}\right)$	$\frac{49}{36}$
	$(7, 8)$	$\left(\frac{1}{3}, \frac{5}{6}\right)$	$\frac{1}{36}$
Vectors	$(1, 7)$	$\left(\frac{4}{3}, \frac{1}{3}\right)$	$\frac{4}{9}$
	$(7, 1)$	$\left(\frac{1}{3}, \frac{4}{3}\right)$	$\frac{4}{9}$

(3.15)

For simplicity, we have omitted half of the fermionic fields which arise with opposite chirality. Note that fourteen vector fields have become massive due to the Brout–Englert–Higgs-like effect, corresponding to the noncompact directions in the gauge group. The corresponding massless scalar (Goldstone) fields have not been included in the table. The conformal dimensions in (3.15) have been computed via (3.12). This confirms the structure of the spectrum as a tensor product of $F(4)$ supermultiplets (3.14) whose conformal dimensions are given by

$SO(7)$	7	8	1
ℓ_0	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{4}{3}$

(3.16)

Note, that this poses a highly nontrivial consistency check on the masses obtained in our supergravity computation. Furthermore, it is obvious from these values that the oscillator

construction developed in [4] does not apply here because it can only produce values ℓ_0 which are multiples of $\frac{1}{4}$.

3.4. $SO(6,2) \times SO(6,2)$

The background isometry group at the origin is $\mathfrak{G} = SU(4|1,1) \times SU(4|1,1)$. The physical spectrum around this point is given by the tensor product of two (left and right) massless unitary supermultiplets according to

$$(1^{+2} + 4^{+} + 6^{0} + \overline{4}^{-} + 1^{-2},\ 1^{+2} + \overline{4}^{+} + 6^{0} + 4^{-} + 1^{-2}). \tag{3.17}$$

Representation theory of $SU(4|1,1)$ gives the conformal weights of the states in this multiplet (cf [4], table 3, $n = 4$):

$SO(6) \times U(1)$	1^{+2}	4^{+}	6^{0}	$\overline{4}^{-}$	1^{-2}
ℓ_0	$\frac{3}{2}$	1	$\frac{1}{2}$	1	$\frac{3}{2}$

(3.18)

Note that this is the unique supermultiplet of $SU(4|1,1)$ which upon tensoring a left with a right copy reproduces the correct spins for the supergravity fields, including the 24 massive (self-dual) vector fields which correspond to the noncompact directions of the gauge group. In particular, this rules out the similar multiplet of [4] (table 2), whose states combine the same $SU(4)$ quantum numbers (3.17) with different values of ℓ_0, giving rise to massive spin-2 states which do not occur in the supergravity.

From (3.18) one may read the physical spectrum around the origin,

Fields	H_0	$(\ell_0, \bar{\ell}_0)$	m^2L^2
Scalars	$(1^{+2}, 1^{+2})$	$\left(\frac{3}{2}, \frac{3}{2}\right)$	3
	$(4^{+}, \overline{4}^{+})$	$(1, 1)$	0
	$(6^{0}, 6^{0})$	$\left(\frac{1}{2}, \frac{1}{2}\right)$	-1
	$(\overline{4}^{-}, 4^{-})$	$(1, 1)$	0
	$(1^{-2}, 1^{-2})$	$\left(\frac{3}{2}, \frac{3}{2}\right)$	3
	$(1^{+2}, 1^{-2})$	$\left(\frac{3}{2}, \frac{3}{2}\right)$	3
	$(1^{-2}, 1^{+2})$	$\left(\frac{3}{2}, \frac{3}{2}\right)$	3
	$(\overline{4}^{-}, \overline{4}^{+})$	$(1, 1)$	0
	$4^{+}, 4^{-})$	$(1, 1)$	0
Fermions	$(1^{+2}, \overline{4}^{+})$	$(\frac{3}{2}, 1)$	$\frac{9}{4}$
	$(4^{+}, 6^{0})$	$\left(1, \frac{1}{2}\right)$	$\frac{1}{4}$
	$(6^{0}, 4^{-})$	$\left(\frac{1}{2}, 1\right)$	$\frac{1}{4}$
	$(\overline{4}^{-}, 1^{2-})$	$\left(1, \frac{3}{2}\right)$	$\frac{9}{4}$
	$(1^{2+}, 4^{-})$	$\left(\frac{3}{2}, 1\right)$	$\frac{9}{4}$
	$(4^{+}, 1^{-2})$	$\left(1, \frac{3}{2}\right)$	$\frac{9}{4}$
Vectors	$(1^{+2}, 6^{0})$	$\left(\frac{3}{2}, \frac{1}{2}\right)$	1
	$(1^{-2}, 6^{0})$	$\left(\frac{1}{2}, \frac{3}{2}\right)$	1

(3.19)

which again gives complete agreement with the masses computed in supergravity from (2.15), (2.16) and (2.20). As above, we have omitted half of the vector and half of the fermion fields which arise with opposite chirality.

3.5. $SO(5,3) \times SO(5,3)$

The background isometry group at the origin is $\mathfrak{G} = OSp(4^*|4) \times OSp(4^*|4)$. The physical spectrum around this point is given by the tensor product of two (left and right) massless unitary supermultiplets according to

$$((1,3) + (4,2) + (5,1), (1,3) + (4,2) + (5,1)) . \tag{3.20}$$

Representation theory of $OSp(4^*|4)$ gives the conformal weights of the states in this multiplet [26],

$SO(5) \times SO(3)$	(5, 1)	(4, 2)	(1, 3)
ℓ_0	1	$\frac{3}{2}$	2

(3.21)

From this one may read the physical spectrum around the origin,

Fields	H_0	$(\ell_0, \bar{\ell}_0)$	m^2L^2
Scalars	(5, 1, 5, 1)	(1, 1)	0
	(4, 2, 4, 2)	$\left(\frac{3}{2}, \frac{3}{2}\right)$	3
	(1, 3, 1, 3)	(2, 2)	8
Fermions	(5, 1, 4, 2)	$\left(1, \frac{3}{2}\right)$	$\frac{9}{4}$
	(4, 2, 1, 3)	$\left(\frac{3}{2}, 2\right)$	$\frac{25}{4}$
Vectors	(5, 1, 1, 3)	(1, 2)	4
	(1, 3, 5, 1)	(2, 1)	4

(3.22)

This again agrees with the spectrum computed from (2.15), (2.16) and (2.20).

3.6. $SO(4,4) \times SO(4,4)$

From (3.6) it follows that for this gauge group both tensors A_1 and A_2 vanish at the origin. The theory hence possesses a maximally supersymmetric Minkowski vacuum[4]. From (2.15), (2.16) and (2.20), we find the spectrum

Fields	#	m^2
Scalars	96	$4g^2$
Fermions	128	$4g^2$
Vectors	32	$4g^2$

(3.23)

4. Maximally supersymmetric vacua for exceptional gaugings

One notable peculiarity of three-dimensional maximal gauged supergravity is the possibility of having exceptional gauge groups. Thus, the question naturally arises whether these gaugings

[4] In [2], this vacuum was misidentified as an AdS stationary point.

which do not have any higher-dimensional counterparts also possess nontrivial extremal structures. Again, all of these theories admit a maximally supersymmetric AdS vacuum at the scalar origin $\mathcal{V} = I$. If we gauge the full $E_{8(+8)}$, the scalar potential reduces to a cosmological constant. The smaller we choose the gauge group the richer we expect the structure of the potential to become, and therefore, it is particularly interesting to study the second-simplest case, $E_{7(+7)} \times SL(2)$, as well as the most compact form of the smallest exceptional gauge group, $G_2 \times F_{4(-20)}$. We will discuss the embedding of these two gauge groups and their maximally supersymmetric vacua in some detail. For all the other possible exceptional gauge groups [2], we simply list the supermultiplet structures of the physical field content around the maximally supersymmetric AdS vacuum.

4.1. $G_2 \times F_{4(-20)}$

For this gauge group, the embedding tensor assumes a rather simple form. The relevant $SO(16)$ representations decompose as follows under the subgroup $G_2 \times SO(9)$:

$$\begin{aligned} \mathbf{16}_v &\longrightarrow (\mathbf{7},\mathbf{1}) + (\mathbf{1},\mathbf{9}) \\ \mathbf{120} &\longrightarrow (\mathbf{14},\mathbf{1}) + (\mathbf{1},\mathbf{36}) + (\mathbf{7},\mathbf{1}) + (\mathbf{7},\mathbf{9}) \\ \mathbf{128}_s &\longrightarrow (\mathbf{1},\mathbf{16}) + (\mathbf{7},\mathbf{16}). \end{aligned} \tag{4.1}$$

Accordingly, we split the $SO(16)$ vector indices as $I = (i, \hat{j})$. The embedding tensor then reads

$$\Theta_{ij|kl} = 12 P_{ij}{}^{kl} \equiv 8\delta_{ij}^{kl} + 2C_{ijkl}, \qquad \Theta_{\hat{i}\hat{j}|\hat{k}\hat{l}} = -8\delta_{\hat{i}\hat{j}}^{\hat{k}\hat{l}}, \qquad \Theta_{\alpha\beta} = -\delta_{\alpha\beta}, \tag{4.2}$$

with all other components zero. Here P is the projector onto the G_2 subgroup of $SO(7)$, with C_{ijkl} the G_2-invariant tensor made out of the octonionic structure constants [31], obeying

$$C_{ijmn}C_{mnkl} = 8\delta_{ij}^{kl} - 2C_{ijkl}. \tag{4.3}$$

We see that the ratio of coupling constants is indeed $\left(-\frac{3}{2}\right)$. Furthermore, Θ can be brought into the form (2.7) with

$$\theta = -1, \qquad \Xi_{IJ} = \mathrm{diag}(9\delta_{ij}, -7\delta_{\hat{i}\hat{j}}), \qquad \Theta_{AB} = -\delta_{AB} + C_{AB}, \tag{4.4}$$

with the G_2-invariant tensor

$$C_{AB} = \delta_{\alpha\beta}\,\mathrm{diag}(-7, \delta_{ij}), \tag{4.5}$$

showing that this indeed gives a solution of the projection condition (2.5). At the origin $\mathcal{V} = I$, the gauge group is broken down to its maximally compact subgroup $G_2 \times SO(9)$. Together with the 16 supercharges and the AdS_3 group $SO(2,2)$, this combines into the background isometry group $\mathfrak{G} = G(3)_L \times OSp(9|2,\mathbb{R})_R$, i.e. the supersymmetries split as $N = (7,9)$. The spectrum around this point is given by the tensor product of two (left and right) supermultiplets of $G(3)$ and $OSp(9|2,\mathbb{R})$, respectively. Comparing to the masses computed from (2.15), (2.16) and (2.20), we identify their conformal weights as

$(G_2)_L$		7	1
ℓ_0		$\frac{3}{4}$	$\frac{5}{4}$
$\times$			
$SO(9)_R$	16	16	
$\bar{\ell}_0$	$\frac{1}{4}$	$\frac{3}{4}$	

(4.6)

With (3.12), this gives the correct supergravity masses, which we do not write explicitly here. Note that (4.6) consistently describes 16 massive self-dual vector fields, corresponding to the noncompact directions in the gauge group.

4.2. $E_{7(7)} \times SL(2)$

In terms of the $SO(8) \times SO(8)$ decomposition of section 2.1, the embedding tensor of this gauge group is given by

$$\begin{aligned}
&\Theta_{ab|cd} = (P_1)_{ab}{}^{cd} \equiv \delta_{ab}^{cd},\\
&\Theta_{ab|\bar{c}\bar{d}} = (P_1)_{ab}{}^{\bar{c}\bar{d}} \equiv \delta_{ab}^{\bar{c}\bar{d}},\\
&\Theta_{\bar{a}\bar{b}|cd} = (P_1)_{\bar{a}\bar{b}}{}^{cd} \equiv \delta_{\bar{a}\bar{b}}^{cd},\\
&\Theta_{\bar{a}\bar{b}|\bar{c}\bar{d}} = (P_1)_{\bar{a}\bar{b}}{}^{\bar{c}\bar{d}} \equiv \delta_{\bar{a}\bar{b}}^{\bar{c}\bar{d}},\\
&\Theta_{a\bar{b}|c\bar{d}} = -\Theta_{\bar{b}a|c\bar{d}} = -\Theta_{a\bar{b}|\bar{d}c} = \Theta_{\bar{b}a|\bar{d}c} = (P_1)_{a\bar{b}}{}^{c\bar{d}} - 3(P_2)_{a\bar{b}}{}^{c\bar{d}},\\
&\Theta_{\alpha\beta|\gamma\delta} = \delta_{(\alpha}^{\gamma}\delta_{\beta)}^{\delta} - \tfrac{1}{2}\delta_{\alpha\beta}\delta^{\gamma\delta},\\
&\Theta_{\dot{\alpha}\dot{\beta}|\dot{\gamma}\dot{\delta}} = \delta_{(\dot{\alpha}}^{\dot{\gamma}}\delta_{\dot{\beta})}^{\dot{\delta}} - \tfrac{1}{2}\delta_{\dot{\alpha}\dot{\beta}}\delta^{\dot{\gamma}\dot{\delta}},
\end{aligned} \tag{4.7}$$

where P_1 and P_2 are projectors onto the $SU(8)$ and $U(1)$ subgroups of $SO(16)$, respectively, with

$$(P_1)_{a\bar{b}}{}^{c\bar{d}} := \delta_{(a}^{c}\delta_{b)}^{d} - \tfrac{1}{8}\delta_{ab}\delta^{cd} \qquad (P_2)_{a\bar{b}}{}^{c\bar{d}} := \tfrac{1}{8}\delta_{ab}\delta^{cd}. \tag{4.8}$$

We see that the relative coupling strength is indeed (-3). Furthermore, the embedding tensor is invariant under triality rotations interchanging $\mathbf{35}_v \to \mathbf{35}_s \to \mathbf{35}_c \to \mathbf{35}_v$. The background isometry group at $\mathcal{V} = I$ is given by the $N = (16, 0)$ supergroup $\mathfrak{G} = SU(8|1, 1)_L \times SU(1, 1)_R$. The physical spectrum is described by tensoring a (left) supermultiplet of $SU(8|1, 1)$ [4], with a singlet on the right:

$SU(8)_L \times U(1)_L$	70^0	$56^+ + \overline{56}^-$	$28^{+2} + \overline{28}^{-2}$	$8^{+3} + \overline{8}^{-3}$	$1^{+4} + 1^{-4}$
ℓ_0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$
$\times$					
I_R			1		
$\bar{\ell}_0$			$\frac{3}{2}$		

(4.9)

The supergravity masses are again obtained from (3.12). The tensor product (4.9) consistently includes 70 massive self-dual vector fields $\left(\Delta = \left(\frac{1}{2}, \frac{3}{2}\right)\right)$ corresponding to the noncompact directions of $E_{7(7)}$ and two massive self-dual vector fields of opposite spin $\left(\Delta = \left(\frac{5}{2}, \frac{3}{2}\right)\right)$ associated with the noncompact directions of $SL(2)$.

4.3. Spectra of the other exceptional gaugings

Here, we list the physical mass spectra around the maximally supersymmetric vacuum for the remaining exceptional gaugings. They again factor into tensor products under the two factors of the background isometry group $\mathfrak{G} = \mathfrak{G}_L \times \mathfrak{G}_R$. For simplicity, we restrict ourselves to giving the conformal dimensions $\ell_0, \bar{\ell}_0$ for the states in these factors, from which three-dimensional spins and masses may be extracted via $s = |\ell_0 - \bar{\ell}_0|$, $\Delta = \ell_0 + \bar{\ell}_0$, and (3.12).

- $G_{2(2)} \times F_{4(4)}$: $\qquad \mathfrak{G} = D^1\left(2, 1; -\frac{2}{3}\right)_L \times OSp(4^*|6)_R$

$SU(2)_L \times SU(2)_L$		$(1, 2)$	$(2, 1)$	
ℓ_0		$\frac{3}{2}$	2	
$\times$				
$SU(2)_R \times USp(6)_R$	$(1, 14)$	$(2, 14')$	$(3, 6)$	$(4, 1)$
$\bar{\ell}_0$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

(4.10)

- $E_{6(6)} \times SL(3)$: $\mathfrak{G} = OSp(4^*|8)_L \times SU(1,1)_R$

$SU(2)_L \times USp(8)_L$	$(1,42)$	$(2,48)$	$(3,27)$	$(4,8)$	$(5,1)$
ℓ_0	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
$\times$					
I_R			1		
$\bar{\ell}_0$			2		

(4.11)

- $E_{6(2)} \times SU(2,1)$: $\mathfrak{G} = SU(6|1,1)_L \times D^1\big(2,1;-\frac{1}{2}\big)_R$

$SU(6)_L \times U(1)_L$	20^0	$15^+ + \overline{15}^-$	$6^{+2} + \overline{6}^{-2}$	$1^{+3} + 1^{-3}$
ℓ_0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$\times$				
$SU(2)_R \times SU(2)_R$		$(2,1)$	$(1,2)$	
$\bar{\ell}_0$		1	$\frac{3}{2}$	

(4.12)

- $E_{6(-14)} \times SU(3)$: $\mathfrak{G} = OSp(10|2,\mathbb{R})_L \times SU(3|1,1)_R$

$SO(10)_L$	16	$\overline{16}$	
ℓ_0	$\frac{1}{4}$	$\frac{3}{4}$	
$\times$			
$SU(3)_R \times U(1)_R$		$3^+ + \overline{3}^-$	$1^{+2} + 1^{-2}$
$\bar{\ell}_0$		$\frac{3}{4}$	$\frac{5}{4}$

(4.13)

- $E_{7(-5)} \times SU(2)$: $\mathfrak{G} = OSp(12|2,\mathbb{R})_L \times D^1\big(2,1;-\frac{1}{3}\big)_R$

$SO(12)_L$	32	$\overline{32}$	
ℓ_0	$\frac{1}{4}$	$\frac{3}{4}$	
$\times$			
$SU(2)_R \times SU(2)_R$		$(2,1)$	$(1,2)$
$\bar{\ell}_0$		$\frac{3}{4}$	$\frac{5}{4}$

(4.14)

- $E_{8(8)}$: $\mathfrak{G} = OSp(16|2,\mathbb{R})_L \times SU(1,1)_R$
  This gauging is special in that the scalar potential becomes trivial (a negative cosmological constant), and all scalar fields are absorbed into vector fields. The theory can thus be considered as a maximally supersymmetric $SO(16)$ CS theory coupled to 128 massive self-dual vectors and 128 spin-$\frac{1}{2}$ fields. The spectrum is simply obtained from tensoring the singleton multiplet of $OSp(16,2|\mathbb{R})$ with a singlet on the right:

$SO(16)_L$	128	$\overline{128}$	
ℓ_0	$\frac{1}{4}$	$\frac{3}{4}$	
$\times$			
I_R			1
$\bar{\ell}_0$			$\frac{5}{4}$

(4.15)

5. Vacua without maximal supersymmetry

We here do not aim at an exhaustive classification of nontrivial stationary points, but rather would like to discuss and to illustrate some salient features, in particular those which have no analogues in higher-dimensional gauged supergravities. Highly efficient symbolic algebra tools together with worked out examples that illustrate how to use them for the calculation of potentials like (5.5) can be found in [7].

5.1. Vacua in higher-dimensional noncompact gaugings

Let us first briefly recall the known structure of extrema in higher-dimensional maximal noncompact gaugings. In $D = 4$, the noncompact $SO(p, q)$ gaugings ($p+q = 8$) were originally obtained in [32, 33] by analytic continuation of the compact gauged theory [34]. The structure of the scalar potential is similar to (2.2),

$$V = -g^2 \left(\tfrac{3}{4} A_1^{ij} A_1^{ij} - \tfrac{1}{24} A_{2\,jkl}^i A_{2\,jkl}^i \right), \tag{5.1}$$

where the indices $i, j, \ldots$ denote $SU(8)$ indices. The tensors A_1 and A_2 are functions on the coset manifold $E_{7(7)}/SU(8)$ and transform in the **36** and **420** of $SU(8)$, respectively. Together, they form the **912** of $E_{7(7)}$.

At the origin $\mathcal{V} = I$, the gauge group is broken down to its maximally compact subgroup $H_0 = SO(p) \times SO(q)$. But unlike in three dimensions, this point is *not* a stationary point in the noncompact gaugings. This is because, except for the compact gauged theory, the tensor $A_{2\,jkl}^i$ does not vanish at $\mathcal{V} = I$, as may be anticipated from the fact that the **420** contains singlets under $H_0 = SO(p) \times SO(q)$ unless $p = 0$. Vanishing $A_{2\,jkl}^i$ would imply stationarity and maximal supersymmetry, but this would be incompatible with the non-existence of proper superextensions of the four-dimensional AdS group, i.e. simple supergroups containing $SO(3, 2) \times (SO(p) \times SO(8 - p))$ as maximal bosonic subgroup for $p \neq 0$ [35, 36]. Recall that, by contrast, in three dimensions no singlets appear in the decomposition of A_2 under H_0, and this was sufficient to imply the existence of maximally supersymmetric stationary points.

The search for stationary points of the noncompact potentials has been pursued in [37] by restricting the potential to singlets under certain subgroups of the gauge group. Summarizing their results, there is no stationary point of the $SO(7, 1)$ gauged theory which leaves the $G_2 \subset SO(7)$ invariant, and no stationary point with at least $SU(3)$ invariance in the $SO(6, 2)$ gauged theory. The potential of the $SO(5, 3)$ gauged theory on the other hand does exhibit a stationary point away from the origin with $SO(5) \times SO(3)$ residual symmetry [38]. It is found by computing the potential in the truncation to the only singlet under $SO(5) \times SO(3)$ and has a positive cosmological constant. Similarly, it has been found that the $SO(4, 4)$ gauged theory admits a dS vacuum with remaining $SO(4) \times SO(4)$ symmetry. Both these dS points have been shown to be unstable in the sense that they admit tachyonic scalar fluctuations with $V'' = -2V$ [39, 40].

In five dimensions the potential for the $SO(p, q)$ ($p + q = 6$) gauged theory is given by [23]

$$V = -g^2 \left(\tfrac{6}{45^2} A_1^{ab} A_1^{ab} - \tfrac{1}{96} A_2^{abcd} A_2^{abcd} \right), \tag{5.2}$$

where the indices $a, b, \ldots$ now denote $USp(8)$ indices. The tensors A_1 and A_2 transform in the **36** and **315** of $USp(8)$, respectively, and together combine into the **351** of $E_{6(6)}$.

Again, the scalar origin $\mathcal{V} = I$ is not a stationary point in the noncompact gaugings—the **315** under $H_0 = SO(p) \times SO(q)$ contains singlets unless $p = 0$. The existence of critical points in the $SO(5, 1)$ and $SO(4, 2)$ gauged theories which preserve at least an $SO(5)$ and

$SO(4) \times SO(2)$ subgroup, respectively, has been excluded in [23]. A stationary point with positive cosmological constant, again for $\mathcal{V} \neq I$, has been identified in the $SO(3,3)$ gauged theory. Presumably it, too, is unstable.

5.2. $SO(8) \times SO(8)$

Several stationary points breaking the diagonal of this group down to a group containing $SU(3)$ have been presented in [6]. All of these correspond to known stationary points of $D = 4$, $N = 8$ supergravity. Here, we want to complete this list by also giving analogues of $D = 4$ stationary points breaking $SO(8)$ down to $SO(7)^-$ and G_2.

In what follows, we will designate by $SO(7)^+$ the subgroup of $SO(8)$ stabilizing the spinor $\psi_\alpha = \delta_{\alpha 8}$, and by $SO(7)^-$ the subgroup stabilizing the co-spinor $\phi_{\dot\alpha} = \delta_{\dot\alpha \dot 8}$. With the conventions of appendix A, their intersection G_2 will also stabilize the vector $v^i = \delta^{i8}$. Those generators of E_8 which are invariant under $(G_2)_{\text{diag}}$ form an $SL(2) \times SL(2)$ subalgebra (where one of these $SL(2)$ is just the $SL(2)$ from $E_{7(+7)} \times SL(2) \subset E_{8(+8)}$ which will show up whenever we form a diagonal $SO(p, 8-p)$). Hence we parametrize the four-dimensional manifold of $(G_2)_{\text{diag}}$ singlets in the coset $E_8/SO(16)$ by

$$\mathcal{V} = \exp(vV)\exp(sS)\exp(-vV)\exp(wW)\exp(zZ)\exp(-wW), \tag{5.3}$$

where the generators V, S, respectively W, Z, correspond to one compact and one noncompact generator of each $SL(2)$. Using the same decomposition as in (3.3), these generators read explicitly

$$\begin{aligned} V^{\mathcal{C}}{}_{\mathcal{B}} &= \left(2\delta^{a8}\delta^{\bar b 8} - \tfrac{1}{4}\delta^{a\bar b}\right) f_{\underline{[a\bar b]}\mathcal{B}}{}^{\mathcal{C}} & S^{\mathcal{C}}{}_{\mathcal{B}} &= \left(2\delta^{\alpha 8}\delta^{\beta 8} - \tfrac{1}{4}\delta^{\alpha\beta}\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}} \\ W^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{4}\delta^{a\bar b} f_{\underline{[a\bar b]}\mathcal{B}}{}^{\mathcal{C}} & Z^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{4}\delta^{\alpha\beta} f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}. \end{aligned} \tag{5.4}$$

Using the $SO(8) \times SO(8)$ embedding tensor (3.4), the corresponding potential reads

$$\begin{aligned} -8g^{-2}V = {} & \tfrac{243}{8} + \tfrac{7}{2}\cosh(2s) + \tfrac{49}{8}\cosh(4s) + \tfrac{1141}{64}\cosh(s)\cosh(z) \\ & + \tfrac{427}{64}\cosh(3s)\cosh(z) - \tfrac{7}{64}\cosh(5s)\cosh(z) - \tfrac{25}{64}\cosh(7s)\cosh(z) \\ & + \tfrac{21}{8}\cos(4v) - \tfrac{7}{2}\cos(4v)\cosh(2s) + \tfrac{7}{8}\cos(4v)\cosh(4s) \\ & - \tfrac{21}{64}\cos(4v)\cosh(s)\cosh(z) + \tfrac{21}{64}\cos(4v)\cosh(3s)\cosh(z) \\ & + \tfrac{7}{64}\cos(4v)\cosh(5s)\cosh(z) - \tfrac{7}{64}\cos(4v)\cosh(7s)\cosh(z) \\ & - \tfrac{1645}{128}\cos(v-w)\sinh(z)\sinh(s) + \tfrac{651}{128}\cos(v-w)\sinh(z)\sinh(3s) \\ & + \tfrac{7}{128}\cos(v-w)\sinh(z)\sinh(5s) - \tfrac{49}{128}\cos(v-w)\sinh(z)\sinh(7s) \\ & - \tfrac{315}{64}\cos(3v+w)\sinh(z)\sinh(s) + \tfrac{133}{64}\cos(3v+w)\sinh(z)\sinh(3s) \\ & - \tfrac{7}{64}\cos(3v+w)\sinh(z)\sinh(5s) - \tfrac{7}{64}\cos(3v+w)\sinh(z)\sinh(7s) \\ & + \tfrac{35}{128}\cos(7v+w)\sinh(z)\sinh(s) - \tfrac{21}{128}\cos(7v+w)\sinh(z)\sinh(3s) \\ & + \tfrac{7}{128}\cos(7v+w)\sinh(z)\sinh(5s) - \tfrac{1}{128}\cos(7v+w)\sinh(z)\sinh(7s). \end{aligned} \tag{5.5}$$

Setting $v = 0$, this reduces to the potential on the three-dimensional manifold of $SO(7)^+_{\text{diag}}$ singlets,

$$\begin{aligned} -8g^{-2}V = {} & 33 + 7\cosh(4s) + \tfrac{35}{2}\cosh(s)\cosh(z) + 7\cosh(3s)\cosh(z) \\ & - \tfrac{1}{2}\cosh(7s)\cosh(z) - \tfrac{35}{2}\cos(w)\sinh(z)\sinh(s) \\ & + 7\cos(w)\sinh(z)\sinh(3s) - \tfrac{1}{2}\cos(w)\sinh(z)\sinh(7s) \end{aligned} \tag{5.6}$$

whose only nontrivial stationary point is located at $w = \pi, s = -z = \frac{1}{2}\operatorname{arccosh} 2$, with remaining $SO(7)^+ \times SO(7)^+$ invariance and completely broken supersymmetry [6][5]. The value of the cosmological constant at this vacuum is $\Lambda = -50g^2$. Recalling that the central charge of the associated conformal algebra on the boundary goes proportional in $\sqrt{1/\Lambda}$, [41, 42], we find from (3.8) that

$$\frac{c_{SO(7)}}{c_{SO(8)}} = \sqrt{\frac{\Lambda_{SO(8)}}{\Lambda_{SO(7)}}} = \frac{4}{5}, \tag{5.7}$$

i.e. a *rational* value for the ratio of central charges of the boundary theories associated with the different vacua. The scalar masses at this extremum are computed with (2.12) and give

$SO(7)^+ \times SO(7)^+$	$(1, 1)$	$(8, 8)$	$(7, 7)$
m^2L^2	$\frac{96}{25}$	$-\frac{9}{25}$	$-\frac{24}{25}$

(5.8)

in units of the inverse AdS length L, together with 14 Goldstone scalars. The full mass spectrum is collected in (B.1). In particular, this vacuum despite being nonsupersymmetric and in contrast to its higher-dimensional analogue is *stable* in the sense that all scalar fields satisfy the Breitenlohner–Freedman bound (3.13)[6]. Moreover, their associated conformal dimensions computed from (3.12), (5.8) are all rational.

The corresponding potential on the manifold of $SO(7)^-_{\text{diag}}$ singlets is obtained by replacing the generator S by $\tilde{S}^{\mathcal{C}}{}_{\mathcal{B}} = \left(2\delta^{\dot\alpha\dot\delta}\delta^{\dot\beta\dot\delta} - \frac{1}{4}\delta^{\dot\alpha\dot\beta}\right) f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}}$ and reads

$$\begin{aligned} -8g^{-2}V = {} & 33 + 7\cosh(4s) + \tfrac{35}{2}\cosh(s)\cosh(z) + 7\cosh(3s)\cosh(z) \\ & - \tfrac{1}{2}\cosh(7s)\cosh(z) + \tfrac{35}{2}\sinh(z)\sin(w)\sinh(s) \\ & - 7\sinh(z)\sin(w)\sinh(3s) + \tfrac{1}{2}\sinh(z)\sin(w)\sinh(7s) \end{aligned} \tag{5.9}$$

which differs from the previous one only by a rotation in w. This means that in contrast to $N = 8$, $D = 4$, the other $SO(7)$ stationary point, here at $w = -\pi/2, s = z = \frac{1}{2}\operatorname{arccosh} 2$, with $SO(7)^- \times SO(7)^-$ symmetry, has the same value of the cosmological constant, and the same mass spectrum.

Although (5.5) is too complicated for a detailed analytic treatment, it is nevertheless possible to extract information about the location of further extrema either numerically[7] or by educated inspection. This allows us to identify a further stationary point at $v = w = \frac{1}{4}\pi, s = z = \frac{1}{2}\operatorname{arccosh}\frac{7}{3}$ which breaks $SO(8) \times SO(8)$ down to $G_2 \times G_2$, preserving $N = (1, 1)$ supersymmetry. Again, the ratio of central charges associated with this vacuum and the origin comes out to be rational,

$$\frac{c_{G_2}}{c_{SO(8)}} = \sqrt{\frac{\Lambda_{SO(8)}}{\Lambda_{G_2}}} = \frac{3}{4}. \tag{5.10}$$

The scalar mass spectrum is given by

$G_2 \times G_2$	$(1, 1)$	$(1, 1)$	$(7, 7)$	$(7, 7)$
m^2L^2	$\frac{65}{16}$	$\frac{9}{16}$	$-\frac{7}{16}$	$-\frac{15}{16}$

(5.11)

[5] Whenever we give coordinates for stationary points, we list only one representative and do not consider trivial sign flips.

[6] Several nonsupersymmetric stable AdS vacua in the three-dimensional half-maximal ($N = 8$) gauged supergravities have been found in [13].

[7] Note that $\partial_v V = 0$ can readily be solved for v in terms of w, z, s, thereby considerably reducing the complexity of this problem.

together with 28 Goldstone scalars. By further computation, one may verify that the physical spectrum around this vacuum is organized in terms of $N = (1, 1)$ supermultiplets and the external $G_2 \times G_2$. More precisely, the original chiral multiplet (3.10) breaks into $N = 1$ multiplets according to

G_2	1	7
ℓ_0	$\frac{13}{8}$, $\frac{9}{8}$	$\frac{5}{8}$, $\frac{1}{8}$

(5.12)

Tensoring a left and a right copy of (5.12) reproduces the full physical spectrum at this vacuum given in (B.2), in particular the 14 massive spin-$\frac{3}{2}$ fields and 28 massive vector fields corresponding to broken super and gauge symmetries.

Although numerical evidence suggests that there is no further stationary point of (5.5) which is not equivalent to one of those given here, a proof is still lacking. For completeness, we include here the physical spectrum around the $N = (2, 2)$ supersymmetric vacuum with remaining $SU(3) \times SU(3) \times U(1) \times U(1)$ symmetry, found in [6]. This vacuum is located at

$$\mathcal{V}^{\mathcal{C}}{}_{\mathcal{B}} = \exp\Big(\tfrac{1}{8}\operatorname{arccosh}(3)\Big(f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}\Big(\Gamma^{1234}_{\alpha\beta} + \Gamma^{1256}_{\alpha\beta} + \Gamma^{1278}_{\alpha\beta} - \delta_{\alpha\beta}\Big) - f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}}\Big(\Gamma^{1357}_{\dot\alpha\dot\beta} - \Gamma^{1467}_{\dot\alpha\dot\beta} + \Gamma^{1458}_{\dot\alpha\dot\beta} + \Gamma^{1368}_{\dot\alpha\dot\beta}\Big)\Big)\Big). \tag{5.13}$$

The spectrum is organized in terms of $N = (2, 2)$ supermultiplets and the external $SU(3) \times SU(3)$ (note that $U(1)$ is the R-symmetry of the associated $N = 2$ superconformal algebra). The original chiral multiplet (3.10) breaks into $N = 2$ multiplets according to

$SU(3)$	1	6
ℓ_0	$\frac{10}{6}$, $\frac{7}{6}$, $\frac{7}{6}$, $\frac{4}{6}$	$\frac{4}{6}$, $\frac{1}{6}$

(5.14)

from which one again reproduces the physical mass spectrum (B.3) via (3.12).

5.3. $SO(7, 1) \times SO(7, 1)$

In four dimensions, there is no stationary point with G_2 symmetry in the theory with noncompact gauge group $SO(7, 1)$. For comparison, we will again compute the potential restricted to the four-dimensional manifold of $(G_2)_{\text{diag}}$ singlets. Since our conventions are just such that G_2 leaves the last vector, spinor and co-spinor index invariant, this calculation parallels the $SO(8) \times SO(8)$ case, only with a different embedding tensor Θ.

Here, the potential on the manifold of $G_{2,\text{diag}}$ singlets is

$$\begin{aligned}
-8g^{-2}V = {} & \tfrac{909}{32} - \tfrac{7}{8}\cosh(2s) - \tfrac{49}{32}\cosh(4s) + \tfrac{6461}{512}\cosh(s)\cosh(z) \\
& - \tfrac{1001}{512}\cosh(3s)\cosh(z) - \tfrac{203}{512}\cosh(5s)\cosh(z) - \tfrac{137}{512}\cosh(7s)\cosh(z) \\
& + \tfrac{21}{8}\cos(2v) + \tfrac{63}{32}\cos(4v) + \tfrac{7}{2}\cos(2v)\cosh(2s) \\
& - \tfrac{49}{8}\cos(2v)\cosh(4s) - \tfrac{21}{8}\cos(4v)\cosh(2s) + \tfrac{21}{32}\cos(4v)\cosh(4s) \\
& + \tfrac{5145}{1024}\cos(2v)\cosh(s)\cosh(z) - \tfrac{5229}{1024}\cos(2v)\cosh(3s)\cosh(z) \\
& + \tfrac{217}{1024}\cos(2v)\cosh(5s)\cosh(z) - \tfrac{133}{1024}\cos(2v)\cosh(7s)\cosh(z) \\
& - \tfrac{21}{512}\cos(4v)\cosh(s)\cosh(z) - \tfrac{63}{512}\cos(4v)\cosh(3s)\cosh(z) \\
& + \tfrac{147}{512}\cos(4v)\cosh(5s)\cosh(z) - \tfrac{63}{512}\cos(4v)\cosh(7s)\cosh(z) \\
& - \tfrac{105}{1024}\cos(6v)\cosh(s)\cosh(z) + \tfrac{189}{1024}\cos(6v)\cosh(3s)\cosh(z) \\
& - \tfrac{105}{1024}\cos(6v)\cosh(5s)\cosh(z) + \tfrac{21}{1024}\cos(6v)\cosh(7s)\cosh(z) \\
& - \tfrac{28\,987}{2048}\cos(v - w)\sinh(z)\sinh(s) - \tfrac{651}{512}\cos(v + w)\sinh(z)\sinh(s)
\end{aligned}$$

$$
\begin{aligned}
&- \tfrac{2835}{2048}\cos(v-w)\sinh(z)\sinh(3s) - \tfrac{3255}{512}\cos(v+w)\sinh(z)\sinh(3s)\\
&- \tfrac{623}{2048}\cos(v-w)\sinh(z)\sinh(5s) + \tfrac{21}{512}\cos(v+w)\sinh(z)\sinh(5s)\\
&- \tfrac{343}{2048}\cos(v-w)\sinh(z)\sinh(7s) - \tfrac{63}{512}\cos(v+w)\sinh(z)\sinh(7s)\\
&+ \tfrac{1323}{1024}\cos(3v-w)\sinh(z)\sinh(s) - \tfrac{7875}{2048}\cos(3v+w)\sinh(z)\sinh(s)\\
&- \tfrac{385}{1024}\cos(3v-w)\sinh(z)\sinh(3s) + \tfrac{2541}{2048}\cos(3v+w)\sinh(z)\sinh(3s)\\
&+ \tfrac{35}{1024}\cos(3v-w)\sinh(z)\sinh(5s) + \tfrac{609}{2048}\cos(3v+w)\sinh(z)\sinh(5s)\\
&- \tfrac{49}{1024}\cos(3v-w)\sinh(z)\sinh(7s) - \tfrac{399}{2048}\cos(3v+w)\sinh(z)\sinh(7s)\\
&+ \tfrac{35}{2048}\cos(5v-w)\sinh(z)\sinh(s) + \tfrac{315}{1024}\cos(5v+w)\sinh(z)\sinh(s)\\
&- \tfrac{189}{2048}\cos(5v-w)\sinh(z)\sinh(3s) + \tfrac{63}{1024}\cos(5v+w)\sinh(z)\sinh(3s)\\
&+ \tfrac{175}{2048}\cos(5v-w)\sinh(z)\sinh(5s) - \tfrac{189}{1024}\cos(5v+w)\sinh(z)\sinh(5s)\\
&- \tfrac{49}{2048}\cos(5v-w)\sinh(z)\sinh(7s) + \tfrac{63}{1024}\cos(5v+w)\sinh(z)\sinh(7s)\\
&+ \tfrac{315}{2048}\cos(7v+w)\sinh(z)\sinh(s) - \tfrac{189}{2048}\cos(7v+w)\sinh(z)\sinh(3s)\\
&+ \tfrac{63}{2048}\cos(7v+w)\sinh(z)\sinh(5s) - \tfrac{9}{2048}\cos(7v+w)\sinh(z)\sinh(7s)
\end{aligned}
\tag{5.15}
$$

which unfortunately is again too complicated for a detailed analytic treatment. Again, by using numerical guidance we find a nontrivial AdS extremum located at $v = w = -\frac{\pi}{2}$, $s = z = \frac{1}{2}\operatorname{arccosh} 2$, with a remaining symmetry of $G_2 \times G_2$ and mass spectrum collected in (B.4). The cosmological constant takes the value $\Lambda = -211g^2/8$. Note that also this vacuum is *stable* although it does not preserve any supersymmetry. Further stationary points of this potential might exist. Upon restriction to $SO(7)_{\rm diag}$ singlets, we obtain

$$
\begin{aligned}
-8g^{-2}V = {}& 33 - 7\cosh(4s) + \tfrac{35}{2}\cosh(s)\cosh(z) - 7\cosh(3s)\cosh(z)\\
&- \tfrac{1}{2}\cosh(7s)\cosh(z) - \tfrac{35}{2}\cos(w)\sinh(z)\sinh(s)\\
&- 7\cos(w)\sinh(z)\sinh(3s) - \tfrac{1}{2}\cos(w)\sinh(z)\sinh(7s)
\end{aligned}
\tag{5.16}
$$

which does not possess a nontrivial stationary point.

5.4. $SO(6,2) \times SO(6,2)$

The four-dimensional theory with gauge group $SO(6,2)$ has no stationary point with remaining $SU(3)$ symmetry. In three dimensions, there are twelve $SU(3)_{\rm diag}$ singlets among the 128 scalars, seven $SO(6)_{\rm diag}$ singlets and five singlets under $(SO(6) \times SO(2))_{\rm diag}$, so we consider breaking the gauge group down to $SO(6)_{\rm diag}$ here. The seven singlets come from the noncompact directions of $SL(3) \times SL(2)$ commuting with $SO(6)$ whose generators $p_{1\ldots8}$, $q_{1\ldots3}$ are given by

$$
\begin{aligned}
p_1{}^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{2}\left(\delta^i_1\delta^j_7 + \delta^i_2\delta^j_8 - \delta^i_3\delta^j_6 + \delta^i_4\delta^j_5\right)\left(\delta^I_i\delta^{J-8}_j - \delta^{J-8}_i\delta^I_j + \delta^{I-8}_i\delta^J_j - \delta^J_i\delta^{I-8}_j\right) f_{\underline{[IJ]}\mathcal{B}}{}^{\mathcal{C}}\\
p_2{}^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{2}\delta^{ij}\left(\delta^{I-8}_i\delta^J_j - \delta^I_i\delta^{J-8}_j\right) f_{\underline{[IJ]}\mathcal{B}}{}^{\mathcal{C}}\\
p_3{}^{\mathcal{C}}{}_{\mathcal{B}} &= -\tfrac{1}{2}\left(\delta^i_1\delta^j_7 + \delta^i_2\delta^j_8 - \delta^i_3\delta^j_6 + \delta^i_4\delta^j_5\right)\left(\delta^I_i\delta^J_j - \delta^J_i\delta^I_j - \delta^{I-8}_i\delta^{J-8}_j + \delta^{J-8}_i\delta^{I-8}_j\right) f_{\underline{[IJ]}\mathcal{B}}{}^{\mathcal{C}}\\
p_4{}^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{2}\left(\delta^{\dot\gamma}_1\delta^{\dot\delta}_2 - \delta^{\dot\gamma}_3\delta^{\dot\delta}_4 + \delta^{\dot\gamma}_5\delta^{\dot\delta}_6 + \delta^{\dot\gamma}_7\delta^{\dot\delta}_8\right)\left(\delta^{\dot\alpha}_{\dot\gamma}\delta^{\dot\beta}_{\dot\delta} - \delta^{\dot\beta}_{\dot\gamma}\delta^{\dot\alpha}_{\dot\delta}\right) f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}}\\
p_5{}^{\mathcal{C}}{}_{\mathcal{B}} &= \tfrac{1}{2}\delta^{\dot\alpha\dot\beta} f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}}
\end{aligned}
$$

$$p_6{}^{\mathcal{C}}{}_{\mathcal{B}} = \left(\delta_7^\alpha \delta_8^\beta - \delta_8^\alpha \delta_7^\beta\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}$$
$$p_7{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{2}\delta^{\alpha\beta} f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}$$
$$p_8{}^{\mathcal{C}}{}_{\mathcal{B}} = p_7{}^{\mathcal{C}}{}_{\mathcal{B}} - 2\left(\delta_7^\alpha \delta_7^\beta + \delta_8^\alpha \delta_8^\beta\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}$$
$$q_1{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{4}\left(\delta_1^i \delta_7^j + \delta_2^i \delta_8^j - \delta_3^i \delta_6^j + \delta_4^i \delta_5^j\right)\left(\delta_i^I \delta_j^J - \delta_i^J \delta_j^I + \delta_i^{I-8} \delta_j^{J-8} - \delta_i^{J-8} \delta_j^{I-8}\right) f_{\underline{[IJ]}\mathcal{B}}{}^{\mathcal{C}}$$
$$q_2{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{2}\left(\delta_7^\alpha \delta_7^\beta - \delta_8^\alpha \delta_8^\beta\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}$$
$$q_3{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{2}\left(\delta_7^\alpha \delta_8^\beta + \delta_8^\alpha \delta_7^\beta\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}.$$

The p, respectively q, generators stand in one-to-one correspondence to the following matrices that satisfy the same commutation relations:

$$\tilde{p}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \qquad \tilde{p}_2 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \tilde{p}_3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$
$$\tilde{p}_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \tilde{p}_5 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \tilde{p}_6 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$
$$\tilde{p}_7 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \tilde{p}_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$
$$\tilde{q}_1 = \begin{pmatrix} 0 & 1/2 \\ -1/2 & 0 \end{pmatrix} \qquad \tilde{q}_2 = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \qquad \tilde{q}_3 = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}.$$

Since acting with $SO(3)$ on traceless diagonal matrices gives all traceless symmetric matrices, we parametrize $SL(3) \times SL(2)$ by

$$\mathcal{V} = \exp(r_1 p_1)\exp(r_2 p_3)\exp(r_3 p_2)\exp(z p_8 - s p_7)\exp(-r_3 p_2)\exp(-r_2 p_3)\exp(-r_1 p_1) \times \exp(r_5 q_1)\exp(v q_2)\exp(-r_5 q_1), \tag{5.17}$$

and obtain the potential given in (C.3), which is independent of r_5. Since the intersection of the $SL(3)$ algebra and the gauge group algebra is one dimensional, this potential does possess one trivial flat direction, apart from which we were not able to find any further stationary points numerically.

It is tempting to reuse the parametrization (5.17) to compute the potential of the $SO(8) \times SO(8)$ and $SO(7,1) \times SO(7,1)$ gauged theories on the manifold of $SO(6)_{\text{diag}}$ singlets, since one only has to repeat the calculation with a different embedding tensor. The corresponding results are collected in the appendix. For $SO(8) \times SO(8)$, we obtain (C.1), while for $SO(7,1) \times SO(7,1)$ the potential is given in (C.2).

5.5. $SO(5,3) \times SO(5,3)$

We have seen that in four and five dimensions there is a de Sitter vacuum in the theories with gauge groups $SO(5,3)$ and $SO(3,3)$, respectively, which completely breaks supersymmetry, but preserves the maximally compact $SO(5) \times SO(3)$ and $SO(3) \times SO(3)$ subgroup of the gauge group, respectively. Extrapolating these results, one may expect an analogue of this point in the $SO(5,3) \times SO(5,3)$ gauged theory in three dimensions. We shall show now that this is indeed the case.

From (3.22), it follows that the spectrum contains no singlet under $H_0 = SO(5) \times SO(3) \times SO(5) \times SO(3)$, i.e. the only point preserving the full H_0 is the AdS ground state described above. Considering the diagonal $SO(5, 3)$, the spectrum contains three singlets: the obvious two from $SL(2)$ as well as a further one

$$M^{\mathcal{C}}{}_{\mathcal{B}} = \left(\tfrac{3}{4}P^{(5)}_{\alpha\beta} - \tfrac{5}{4}Q^{(5)}_{\alpha\beta}\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}. \tag{5.18}$$

Parametrizing this three-dimensional manifold via

$$\mathcal{V} = \exp(sM)\exp(wW)\exp(zZ)\exp(-wW), \tag{5.19}$$

where W, Z are given in (5.4), we get the potential

$$\begin{aligned} -8g^{-2}V = {} & 25 - 15\cosh(4s) - 15\cosh(s)\cosh(z) + \tfrac{15}{2}\cosh(3s)\cosh(z) \\ & + \tfrac{3}{2}\cosh(5s)\cosh(z) + 15\cos(w)\sinh(z)\sinh(s) \\ & + \tfrac{15}{2}\cos(w)\sinh(z)\sinh(3s) - \tfrac{3}{2}\cos(w)\sinh(z)\sinh(5s) \end{aligned} \tag{5.20}$$

which has a nontrivial stationary point at $w = \pi, s = \frac{1}{4}\operatorname{arccosh} 5, z = 3s$. Since the corresponding generator $\tilde{M} = M - 3Z$ according to (3.22) is precisely the only $H_0 = SO(5) \times SO(5) \times SO(3)_{\text{diag}}$ singlet, the symmetry is broken down to this group. The cosmological constant takes the value $\Lambda = 22g^2 > 0$. The mass spectrum at this point is collected in (B.5). In particular, the scalar mass squares are given by

H_0	(5, 5, 1)	(4, 4, 3)	(4, 4, 1)	(1, 1, 5)	(1, 1, 3)	(1, 1, 1)
$m^2 L^2_{\text{dS}}$	$\frac{24}{11}$	$\frac{45}{11}$	$-\frac{3}{11}$	$\frac{96}{11}$	0	$-\frac{48}{11}$

(5.21)

in units of the inverse dS length L_{dS}, together with 33 Goldstone bosons. Hence, this de Sitter vacuum is unstable like its counterparts in higher dimensions. In our case, this instability is already implied by the fact that it is smoothly connected with the maximally supersymmetric AdS vacuum at the origin (3.22).

5.6. $SO(4, 4) \times SO(4, 4)$

For the gauge group $SO(4, 4) \times SO(4, 4)$, we may find stationary points breaking its compact subgroup down to a diagonal $SO(4) \times SO(4)$. If we split $SO(8)_{L,R}$ via $\alpha \to (\alpha_1, \alpha_2)$ and accordingly label the $SO(4)$ factors, the compact subgroup of our gauge group is $(SO(4)_{L1} \times SO(4)_{R2}) \times (SO(4)_{R1} \times SO(4)_{L2})$. In this particular case, there is more than one obvious way to form a diagonal subgroup of the compact part of the gauge group, but we will only consider the case corresponding to the constructions employed above. We hence again have the two singlets W, Z from $SL(2)$ as in (5.4) as well as two additional singlets from another $SL(2)$,

$$S_1{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{4}\left(P^{(4)}_{\alpha\beta} - Q^{(4)}_{\alpha\beta}\right) f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}}, \qquad S_2{}^{\mathcal{C}}{}_{\mathcal{B}} = \tfrac{1}{4}\left(P^{(4)}_{\dot\alpha\dot\beta} - Q^{(4)}_{\dot\alpha\dot\beta}\right) f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}}. \tag{5.22}$$

We parametrize this four-dimensional manifold as

$$\mathcal{V} = \exp(wW)\exp(zZ)\exp(-wW)\exp(v[S_1, S_2])\exp(sS_1)\exp(-v[S_1, S_2]), \tag{5.23}$$

and get the potential

$$-8g^{-2}V = 24 - 16\cosh(z) - 16\cosh(s) + 8\cosh(s)\cosh(z), \tag{5.24}$$

which does not depend on w and v. Besides the origin, there is a second stationary point at $|s| = |z| = z_0 := \operatorname{arccosh} 2$, at which the gauge group is broken down to $H_0 = SO(4)_{L2} \times SO(4)_{R2} \times SO(4)_{L1,R1}$. Evaluating the potential, one verifies that this

vacuum is de Sitter with $\Lambda = 4g^2$. The mass spectrum is collected in (B.7). The scalar mass squares are given by

#	49	32	8	1
$m^2 L_{\mathrm{dS}}^2$	12	9	0	−12

(5.25)

together with 38 Goldstone bosons. Note that there is just one unstable direction which is required in order to run into the maximally supersymmetric Minkowski vacuum (3.23) at the origin. Comparing this mass with (5.21), we find that there seems to be no universal value for the highest tachyonic mass square, contrary to the situation in higher dimensions [40].

5.7. Exceptional gauge groups

Finally, we will compute some of the potentials of the exceptional gaugings discussed in section 4. For the gauge group $E_{7(+7)} \times SL(2)$ and using the tools we developed in previous sections, it is natural to consider the scalar potential on the manifold of $SO(6) \subset SO(8)$ singlets by using the parametrization (5.17). Since the $SL(2)$ parametrized by v, r_5 is part of the gauge group, and hence corresponds to flat directions in the potential, these two parameters drop out. Furthermore, three of the five noncompact directions of the $SO(6)$-invariant $SL(3)$ singlets lie in the gauge group, and the smallest group containing the remaining two orthogonal directions is $SL(2)$, which we can parametrize by

$$S = 2\delta^{\alpha}_{[6}\delta^{\beta}_{7]} f_{\alpha\beta\mathcal{B}}{}^{\mathcal{C}} \qquad V = \tfrac{1}{2}\delta^{i\bar{j}} f_{i\underline{\bar{j}}\mathcal{B}}{}^{\mathcal{C}} \qquad \mathcal{V} = \exp(vV)\exp(sS)\exp(-vV), \tag{5.26}$$

and obtain the potential

$$-8g^{-2}V = 22 - 6\cosh(4s), \tag{5.27}$$

which obviously does not have any nontrivial stationary points.

A richer structure is found for the exceptional gauge group $G_2 \times F_{4(-20)}$. The main problem in this case is to find an appropriate invariance subgroup of the gauge group small enough to show nontrivial structure, yet big enough to produce not too many singlets. Since none of the parametrizations given so far work well here, we choose that particular subgroup $SU(3) \times SU(3)$ of the group $SO(8)_L \times SO(8)_R$ which stabilizes the vectors $v_1^i = \delta^{i7}$, $v_2^i = \delta^{i8}$, $v_3^{\bar{i}} = \delta^{\bar{i}7}$, $v_4^{\bar{i}} = \delta^{\bar{i}8}$ as well as the spinors $\psi^{\alpha_L} = \delta^{\alpha_L 8}$, $\psi^{\alpha_R} = \delta^{\alpha_R 8}$ (and which is also a subgroup of $G_2 \times F_{4(-20)}$).

This group is stabilized by a subgroup $SU(2,1) \times SU(2,1)$ of $E_{8(8)}$, hence we have to deal with an eight-dimensional submanifold of the supergravity scalars here. The intersection of this eight-dimensional manifold with the gauge group is four dimensional, but unfortunately, unlike the parametrization considered in the $E_{7(7)} \times SL(2)$ case, the smallest group containing the four directions orthogonal to the gauge group is the full $SU(2,1) \times SU(2,1)$, hence we parametrize the full eight-dimensional manifold[8]. Using the generators $X_{(A,B)}$ of both $SO(3)$ subalgebras as well as those of two noncompact directions $Y_{(A,B)}$,

$$\begin{aligned}
Y^{\mathcal{C}}_{(A)\mathcal{B}} &= -\tfrac{1}{2}\left(\delta^{\dot{\alpha}}_2\delta^{\dot{\beta}}_8 - \delta^{\dot{\alpha}}_8\delta^{\dot{\beta}}_2\right) f_{\dot{\alpha}\dot{\beta}\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(A)1\mathcal{B}} &= 2\left(\delta^i_7\delta^{\bar{j}}_8 - \delta^i_8\delta^{\bar{j}}_7\right) f_{\underline{[i\bar{j}]}\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(A)2\mathcal{B}} &= 2\left(\delta^i_7\delta^{\bar{j}}_7 + \delta^i_8\delta^{\bar{j}}_8\right) f_{\underline{[i\bar{j}]}\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(A)3\mathcal{B}} &= 2\left(\delta^i_7\delta^j_8 f_{\underline{[ij]}\mathcal{B}}{}^{\mathcal{C}} - \delta^{\bar{i}}_7\delta^{\bar{j}}_8 f_{\underline{[\bar{i}\bar{j}]}\mathcal{B}}{}^{\mathcal{C}}\right)
\end{aligned}$$

[8] Admittedly, motivation to do so comes in part from the urge to test the limits of our now improved symbolic algebra tools. Calculation of this potential takes less than four hours on a decent modern x86-based Linux workstation.

$$
\begin{aligned}
Y^{\mathcal{C}}_{B\mathcal{B}} &= -\tfrac{1}{2}\left(\delta^{\dot\alpha}_2\delta^{\dot\beta}_8 + \delta^{\dot\alpha}_8\delta^{\dot\beta}_2\right) f_{\dot\alpha\dot\beta\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(B)1\mathcal{B}} &= -2\left(\delta^i_7\delta^{\bar j}_8 + \delta^i_8\delta^{\bar j}_7\right) f_{\underline{[i\,\bar j]}\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(B)2\mathcal{B}} &= -2\left(\delta^i_7\delta^{\bar j}_7 - \delta^i_8\delta^{\bar j}_8\right) f_{\underline{[i\,\bar j]}\mathcal{B}}{}^{\mathcal{C}} \\
X^{\mathcal{C}}_{(B)3\mathcal{B}} &= -2\left(\delta^i_7\delta^j_8 f_{\underline{[ij]}\mathcal{B}}{}^{\mathcal{C}} + \delta^{\bar i}_7\delta^{\bar j}_8 f_{\underline{[\bar i\,\bar j]}\mathcal{B}}{}^{\mathcal{C}}\right),
\end{aligned}
\tag{5.28}
$$

we parametrize the eight-dimensional singlet manifold as

$$
\begin{aligned}
\mathcal{V} = {} & \exp(r_1 X_{(A)1})\exp(r_2 X_{(A)2})\exp(r_3 X_{(A)3})\exp(r_4 X_{(B)1})\exp(r_5 X_{(B)2})\exp(r_6 X_{(B)3}) \\
& \times \exp(sY_{(A)})\exp(zY_{(B)})\exp(-r_6 X_{(B)3})\exp(-r_5 X_{(B)2})\exp(-r_4 X_{(B)1}) \\
& \times \exp(-r_3 X_{(A)3})\exp(-r_2 X_{(A)2})\exp(-r_1 X_{(A)1}),
\end{aligned}
\tag{5.29}
$$

and obtain for the potential the somewhat lengthy expression given in (C.4). A nontrivial stationary point is located at $r_i = 0, z = -s = \frac{1}{2}\operatorname{arccosh} 7$, with remaining symmetry $SU(3) \times SO(7)^-$. The value of the cosmological constant is $\Lambda = -25g^2/2$, i.e. again the ratio of associated central charges of this vacuum and the origin (4.6) comes out to be rational:

$$
\frac{c_{SU(3)\times SO(7)}}{c_{G_2\times SO(9)}} = \sqrt{\frac{\Lambda_{G_2\times SO(9)}}{\Lambda_{SU(3)\times SO(7)}}} = \frac{4}{5}. \tag{5.30}
$$

The full mass spectrum is collected in (B.9). In particular, inspection of the gravitino masses shows that this vacuum preserves $N = (0, 1)$ supersymmetries.

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Appendix A. $E_{8(+8)}$ conventions

Since some of the results in the main text depend on our particular choice of conventions for $E_{8(8)}$ structure constants (e.g. the fact that G_2 can be embedded in such a way into $SO(8)$ that the stabilized vector, spinor and co-spinor all carry the index 8), we state them here for reference.

Using the conventions of [43], we define

$$
\begin{aligned}
\sigma_1 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad & \sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
\sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad & \sigma_e &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}
\end{aligned}
\tag{A.1}
$$

from which we obtain $SO(8)$ γ-matrices using the tensor $G_{i\lambda\mu\rho}$ implementing the $2 \times 2 \times 2 \to 8$ mapping[9]

$$
\begin{aligned}
&G_{1111} = 1 \qquad G_{2112} = 1 \qquad G_{3121} = 1 \qquad G_{4122} = 1 \\
&G_{5211} = 1 \qquad G_{6212} = 1 \qquad G_{7221} = 1 \qquad G_{8222} = 1
\end{aligned}
\tag{A.2}
$$

[9] Note that this convention, which seems to be more widespread, accidentally is just the opposite of that implicitly used in [6].

as well as the abbreviation

$$Z(\sigma_{(A)};\sigma_{(B)};\sigma_{(C)}) = \sigma_{(A)\alpha_1\dot\beta_1}\sigma_{(B)\alpha_2\dot\beta_2}\sigma_{(C)\alpha_3\dot\beta_3}G^{\alpha\alpha_1\alpha_2\alpha_3}G^{\beta\beta_1\beta_2\beta_3} \tag{A.3}$$

via

$$\begin{aligned}
\gamma^1 &= Z(\sigma_e;\sigma_e;\sigma_e) & \gamma^2 &= Z(\sigma_1;\sigma_z;\sigma_e)\\
\gamma^3 &= Z(\sigma_e;\sigma_1;\sigma_z) & \gamma^4 &= Z(\sigma_z;\sigma_e;\sigma_1)\\
\gamma^5 &= Z(\sigma_1;\sigma_x;\sigma_e) & \gamma^6 &= Z(\sigma_e;\sigma_1;\sigma_x)\\
\gamma^7 &= Z(\sigma_x;\sigma_e;\sigma_1) & \gamma^8 &= Z(\sigma_1;\sigma_1;\sigma_1)
\end{aligned} \tag{A.4}$$

from which we form $SO(16)$ Γ-matrices using the splitting $J \to (j, \bar k)$ of $SO(16)$ vector and $A \to (\alpha\beta, \dot\gamma\dot\delta)$, $\dot A \to (\alpha\dot\beta, \dot\gamma\delta)$ of MW spinor and co-spinor indices by

$$\begin{aligned}
\Gamma^i_{\alpha\beta\dot\gamma\dot\delta} &= \delta_{\alpha\dot\gamma}\gamma^i_{\beta\dot\delta} & \Gamma^i_{\dot\alpha\dot\beta\dot\gamma\delta} &= \delta_{\dot\alpha\dot\gamma}\gamma^i_{\delta\dot\beta}\\
\Gamma^{\bar\imath}_{\alpha\beta\dot\gamma\delta} &= \delta_{\beta\delta}\gamma^{\bar\imath}_{\alpha\dot\gamma} & \Gamma^{\bar\imath}_{\dot\alpha\dot\beta\gamma\delta} &= -\delta_{\dot\beta\delta}\gamma^{\bar\imath}_{\gamma\dot\alpha}.
\end{aligned} \tag{A.5}$$

If we denote $SO(16)$ adjoint indices by $\underline{[IJ]}$, which naturally decompose into $SO(16)$ vector indices[10] I, J and split $E_{8(8)}$ adjoint indices $\mathcal{A} \to (A, \underline{[IJ]})$, then $E_{8(8)}$ structure constants are given by

$$\begin{aligned}
f_{\underline{[IJ][KL]}}{}^{\underline{[MN]}} &= -8\delta_{[I[K}}\delta^{MN}_{L]J]} & f_{\underline{[IJ]}A}{}^{B} &= \tfrac{1}{2}\Gamma^{IJ}_{AB}\\
f_{B\underline{[IJ]}}{}^{A} &= \tfrac{1}{2}\Gamma^{IJ}_{AB} & f_{AB}{}^{\underline{[IJ]}} &= -\tfrac{1}{2}\Gamma^{IJ}_{AB}.
\end{aligned} \tag{A.6}$$

Appendix B. Vacua and mass spectra

In this appendix, we collect the mass spectra computed around all the stationary points identified in this paper. The tables give the eigenvalues of $\mathcal{M}$, $\mathcal{M}^{\rm vec}$, A_1 and A_3, where the multiplicity of each eigenvalue is given by the subscript in parentheses. For the AdS vacua, the associated conformal dimensions may be obtained from (3.12). The Goldstone modes are contained in the $m^2 = 0$ eigenvalues of $\mathcal{M}$. The Goldstino modes among the eigenvalues of A_3 are more difficult to disentangle as their identification requires projection with the A_2 tensor, cf (2.21) and the subsequent discussion. They are marked with an asterisk and do not appear in the effective physical spectrum.

- $G_0 = SO(8) \times SO(8)$, remaining symmetry $SO(7)^{\pm} \times SO(7)^{\pm}$:

$\Lambda/2g^2$	-25
$\mathcal{M}/g^2$	$96_{(\times 1)}, 0_{(\times 14)}, -9_{(\times 64)}, -24_{(\times 49)}$
$\mathcal{M}^{\rm vec}/g$	$6_{(\times 7)}, 0_{(\times 114)}, -6_{(\times 7)}$
A_1	$7/2_{(\times 8)}, -7/2_{(\times 8)}$
A_3	$21/2_{(\times 8)^*}, 3/2_{(\times 56)}, -3/2_{(\times 56)}, -21/2_{(\times 8)^*}$

(B.1)

- $G_0 = SO(8) \times SO(8)$, remaining symmetry $G_2 \times G_2$, $N = (1, 1)$:

$\Lambda/2g^2$	$-256/9$
$\mathcal{M}/g^2$	$1040/9_{(\times 1)}, 16_{(\times 1)}, 0_{(\times 28)}, -112/9_{(\times 49)}, -80/3_{(\times 49)}$
$\mathcal{M}^{\rm vec}/g$	$20/3_{(\times 7)}, 4/3_{(\times 7)}, 0_{(\times 100)}, -4/3_{(\times 7)}, -20/3_{(\times 7)}$
A_1	$4_{(\times 7)}, 8/3_{(\times 1)}, -8/3_{(\times 1)}, -4_{(\times 7)}$
A_3	$12_{(\times 7)^*}, 28/3_{(\times 1)}, 4_{(\times 7)}, 4/3_{(\times 49)},$ $-4/3_{(\times 49)}, -4_{(\times 7)}, -28/3_{(\times 1)}, -12_{(\times 7)^*}$

(B.2)

[10] Note that a sum over all adjoint indices has to include a double-counting correction factor $\frac{1}{2}$ if it is performed as a sum over antisymmetric vector indices. *Whenever we implicitly sum over an adjoint index* $\underline{[IJ]}$*, we include every pair of indices* I, J *only once.*

- $G_0 = SO(8) \times SO(8)$, remaining symmetry $SU(3) \times SU(3) \times U(1) \times U(1)$, $N = (2, 2)$:

$\Lambda/2g^2$	-36
$\mathcal{M}/g^2$	$160_{(\times 1)}, 28_{(\times 4)}, 0_{(\times 38)}, -20_{(\times 36)}, -32_{(\times 49)}$
$\mathcal{M}^{\text{vec}}/g$	$8_{(\times 7)}, 2_{(\times 12)}, 0_{(\times 90)}, -2_{(\times 12)}, -8_{(\times 7)}$
A_1	$5_{(\times 6)}, 3_{(\times 2)}, -3_{(\times 2)}, -5_{(\times 6)}$
A_3	$15_{(\times 6)^*}, 11_{(\times 2)}, 5_{(\times 14)}, 1_{(\times 42)}, -1_{(\times 42)}, -5_{(\times 14)}, -11_{(\times 2)}, -15_{(\times 6)^*}$

(B.3)

- $G_0 = SO(7, 1) \times SO(7, 1)$, remaining symmetry $G_2 \times G_2$:

$\Lambda/2g^2$	$-211/16$
$\mathcal{M}/g^2$	$195/4_{(\times 1)}, 45/2_{(\times 1)}, 0_{(\times 28)}, -9/2_{(\times 49)}, -33/4_{(\times 49)}$
$\mathcal{M}^{\text{vec}}/g$	$9/2_{(\times 7)}, 3_{(\times 7)}, 0_{(\times 100)}, -3_{(\times 7)}, -9/2_{(\times 7)}$
A_1	$35/8_{(\times 1)}, 19/8_{(\times 7)}, -19/8_{(\times 7)}, -35/8_{(\times 1)}$
A_3	$105/8_{(\times 1)^*}, 57/8_{(\times 7)^*}, 33/8_{(\times 7)}, 15/8_{(\times 49)}$, $-15/8_{(\times 49)}, -33/8_{(\times 7)}, -57/8_{(\times 7)^*}, -105/8_{(\times 1)^*}$

(B.4)

- $G_0 = SO(5, 3) \times SO(5, 3)$, remaining symmetry $SO(5) \times SO(5) \times SO(3)_{\text{diag}}$:

$\Lambda/2g^2$	11
$\mathcal{M}/g^2$	$96_{(\times 5)}, 45_{(\times 48)}, 24_{(\times 25)}, 0_{(\times 33)}, -3_{(\times 16)}, -48_{(\times 1)}$
$\mathcal{M}^{\text{vec}}/g$	$6_{(\times 15)}, 0_{(\times 98)}, -6_{(\times 15)}$
A_1	$5/2_{(\times 8)}, -5/2_{(\times 8)}$
A_3	$15/2_{(\times 8^*, \times 16)}, 9/2_{(\times 40)}, -9/2_{(\times 40)}, -15/2_{(\times 8^*, \times 16)}$

(B.5)

- $G_0 = SO(4, 4) \times SO(4, 4)$, remaining symmetry $SO(4)^4$, $N = 16$:

$\Lambda/2g^2$	0
$\mathcal{M}/g^2$	$4_{(\times 96)}, 0_{(\times 32)}$
$\mathcal{M}^{\text{vec}}/g$	$2_{(\times 16)}, 0_{(\times 96)}, -2_{(\times 16)}$
A_1	$0_{(\times 16)}$
A_3	$2_{(\times 64)}, -2_{(\times 64)}$

(B.6)

- $G_0 = SO(4, 4) \times SO(4, 4)$, remaining symmetry $SO(4) \times SO(4) \times SO(4)_{\text{diag}}$:

$\Lambda/2g^2$	2
$\mathcal{M}/g^2$	$12_{(\times 49)}, 9_{(\times 32)}, 0_{(\times 46)}, -12_{(\times 1)}$
$\mathcal{M}^{\text{vec}}/g$	$3_{(\times 16)}, 0_{(\times 96)}, -3_{(\times 16)}$
A_1	$1_{(\times 8)}, -1_{(\times 8)}$
A_3	$3_{(\times 8^*, \times 56)}, -3_{(\times 8^*, \times 56)}$

(B.7)

- $G_0 = G_2 \times F_{4(-20)}$, remaining symmetry $G_2 \times SO(9)$, $N = (7, 9)$:

$\Lambda/2g^2$	-4
$\mathcal{M}/g^2$	$0_{(\times 16)}, -3_{(\times 112)}$
$\mathcal{M}^{\text{vec}}/g$	$1_{(\times 16)}, 0_{(\times 112)}$
A_1	$1_{(\times 7)}, -1_{(\times 9)}$
A_3	$2_{(\times 16)}, 0_{(\times 112)}$

(B.8)

- $G_0 = G_2 \times F_{4(-20)}$, remaining symmetry $SU(3) \times SO(7)^-$, $N = (0, 1)$:

$\Lambda/2g^2$	$-25/4$
$\mathcal{M}/g^2$	$24_{(\times 1)}, 0_{(\times 37)}, -9/4_{(\times 48)}, -6_{(\times 42)}$
$\mathcal{M}^{\text{vec}}/g$	$4_{(\times 1)}, 3_{(\times 6)}, 3/2_{(\times 8)}, 1_{(\times 7)}, 0_{(\times 91)}, -1/2_{(\times 8)}, -3_{(\times 7)}$
A_1	$11/4_{(\times 1)}, 7/4_{(\times 6)}, -5/4_{(\times 1)}, -7/4_{(\times 8)}$
A_3	$33/4_{(\times 1)^*}, 21/4_{(\times 6)^*}, 17/4_{(\times 1)}, 11/4_{(\times 8)}, 9/4_{(\times 7)}, 3/4_{(\times 48)}, -3/4_{(\times 42)}, -7/4_{(\times 7)}, -21/4_{(\times 8)^*}$

(B.9)

Appendix C. Explicit scalar potentials

In this appendix, we collect some of the scalar potentials which are too lengthy to be given in the main text. These are the potentials for gauge groups $SO(8) \times SO(8)$, $SO(7, 1) \times SO(7, 1)$ and $SO(6, 2) \times SO(6, 2)$, restricted to the seven-dimensional manifold of $SO(6)_{\text{diag}}$ singlets, as well as the potential for the exceptional gauge group $G_2 \times F_{4(-20)}$ restricted to the manifold of $SU(3) \times SU(3)$ singlets.

- $G_0 = SO(8) \times SO(8)$, potential restricted to singlets under $SO(6)_{\text{diag}}$:

$$
\begin{aligned}
-8g^{-2}V = {} & 27 + 3\cosh(4z) + 3\cosh(4z)\cos(2r_2) - 3\cosh(4z)\cos(2r_1) \\
& - 3\cosh(4z)\cos(2r_1)\cos(2r_2) + \tfrac{1}{4}\cosh(4s) + \tfrac{1}{4}\cosh(4s)\cos(2r_2) \\
& - \tfrac{1}{4}\cosh(4s)\cos(2r_1) - \tfrac{1}{4}\cosh(4s)\cos(2r_1)\cos(2r_2) \\
& + 9\cosh(2s)\cosh(2z) + \tfrac{3}{4}\cosh(2s)\cosh(6z) \\
& - 3\cos(2r_3)\sinh(2z)\sinh(2s) + \tfrac{1}{4}\cos(2r_3)\sinh(6z)\sinh(2s) \\
& - 3\cosh(2s)\cosh(2z)\cos(2r_2) - \tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_2) \\
& - 3\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s) \\
& + \tfrac{1}{4}\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s) \\
& + 3\cosh(2s)\cosh(2z)\cos(2r_1) + \tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_1) \\
& - 9\cos(2r_1)\cos(2r_3)\sinh(2z)\sinh(2s) \\
& + \tfrac{3}{4}\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s) \\
& + 3\cosh(2s)\cosh(2z)\cos(2r_1)\cos(2r_2) \\
& + \tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2) \\
& - 12\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2z)\sinh(2s) \\
& + \sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s) \\
& + 3\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s) \\
& - \tfrac{1}{4}\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s) + \cosh(2v) \\
& + 9\cosh(v)\cosh(4z) - 3\cosh(v)\cosh(4z)\cos(2r_2) \\
& + 3\cosh(v)\cosh(4z)\cos(2r_1) + 3\cosh(v)\cosh(4z)\cos(2r_1)\cos(2r_2) \\
& - \tfrac{1}{4}\cosh(2v)\cosh(4s) - \tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_2) \\
& + \tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_1) + \tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2) \\
& + 15\cosh(v)\cosh(2s)\cosh(2z) \\
& - \tfrac{3}{4}\cosh(2v)\cosh(2s)\cosh(6z) + 3\cosh(v)\cos(2r_3)\sinh(2z)\sinh(2s) \\
& - \tfrac{1}{4}\cosh(2v)\cos(2r_3)\sinh(6z)\sinh(2s)
\end{aligned}
$$

$$
\begin{aligned}
&+3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_2)\\
&+\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_2)\\
&+3\cosh(v)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_1)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\\
&+9\cosh(v)\cos(2r_1)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{3}{4}\cosh(2v)\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_1)\cos(2r_2)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\\
&+12\cosh(v)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2z)\sinh(2s)\\
&-\cosh(2v)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&-3\cosh(v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+\tfrac{1}{4}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s). \qquad \text{(C.1)}
\end{aligned}
$$

- $G_0 = SO(7,1) \times SO(7,1)$, potential restricted to singlets under $SO(6)_{\text{diag}}$:

$$
\begin{aligned}
-8g^{-2}V = {}&\tfrac{1663}{64} + \tfrac{43}{64}\cos(2r_5) + \tfrac{9}{64}\cos(4r_3)\\
&-\tfrac{3}{64}\cos(4r_3)\cos(2r_5) - \tfrac{3}{16}\cos(2r_2) + \tfrac{21}{64}\cos(4r_2)\\
&+\tfrac{1}{16}\cos(2r_2)\cos(2r_5) - \tfrac{7}{64}\cos(4r_2)\cos(2r_5)\\
&+\tfrac{3}{16}\cos(2r_2)\cos(4r_3) + \tfrac{3}{64}\cos(4r_2)\cos(4r_3)\\
&-\tfrac{1}{16}\cos(2r_2)\cos(4r_3)\cos(2r_5) - \tfrac{1}{64}\cos(4r_2)\cos(4r_3)\cos(2r_5)\\
&-\tfrac{9}{64}\cos(2r_1) + \tfrac{3}{64}\cos(2r_1)\cos(2r_5) - \tfrac{15}{64}\cos(2r_1)\cos(4r_3)\\
&+\tfrac{5}{64}\cos(2r_1)\cos(4r_3)\cos(2r_5) + \tfrac{3}{16}\cos(2r_1)\cos(2r_2)\\
&+\tfrac{21}{64}\cos(2r_1)\cos(4r_2) - \tfrac{1}{16}\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&-\tfrac{7}{64}\cos(2r_1)\cos(4r_2)\cos(2r_5) - \tfrac{3}{16}\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&-\tfrac{3}{16}\sin(4r_3)\sin(3r_2)\sin(2r_1) - \tfrac{3}{16}\cos(2r_1)\cos(2r_2)\cos(4r_3)\\
&+\tfrac{3}{64}\cos(2r_1)\cos(4r_2)\cos(4r_3) + \tfrac{1}{16}\cos(2r_5)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cos(2r_5)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cos(2r_1)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&-\tfrac{1}{64}\cos(2r_1)\cos(4r_2)\cos(4r_3)\cos(2r_5) + 3\cosh(4z)\\
&+3\cosh(4z)\cos(2r_2) - 3\cosh(4z)\cos(2r_1) - 3\cosh(4z)\cos(2r_1)\cos(2r_2)\\
&-\tfrac{15}{64}\cosh(4s) + \tfrac{5}{64}\cosh(4s)\cos(2r_5) - \tfrac{9}{64}\cosh(4s)\cos(4r_3)\\
&+\tfrac{3}{64}\cosh(4s)\cos(4r_3)\cos(2r_5) - \tfrac{3}{16}\cosh(4s)\cos(2r_2)\\
&+\tfrac{3}{64}\cosh(4s)\cos(4r_2) + \tfrac{1}{16}\cosh(4s)\cos(2r_2)\cos(2r_5)\\
&-\tfrac{1}{64}\cosh(4s)\cos(4r_2)\cos(2r_5) - \tfrac{3}{16}\cosh(4s)\cos(2r_2)\cos(4r_3)\\
&-\tfrac{3}{64}\cosh(4s)\cos(4r_2)\cos(4r_3) + \tfrac{1}{16}\cosh(4s)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&+\tfrac{1}{64}\cosh(4s)\cos(4r_2)\cos(4r_3)\cos(2r_5) + \tfrac{9}{64}\cosh(4s)\cos(2r_1)\\
&-\tfrac{3}{64}\cosh(4s)\cos(2r_1)\cos(2r_5) + \tfrac{15}{64}\cosh(4s)\cos(2r_1)\cos(4r_3)
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{5}{64}\cosh(4s)\cos(2r_1)\cos(4r_3)\cos(2r_5) + \tfrac{3}{16}\cosh(4s)\cos(2r_1)\cos(2r_2)\\
&+ \tfrac{3}{64}\cosh(4s)\cos(2r_1)\cos(4r_2) - \tfrac{1}{16}\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&- \tfrac{1}{64}\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(2r_5)\\
&+ \tfrac{3}{16}\cosh(4s)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&+ \tfrac{3}{16}\cosh(4s)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&+ \tfrac{3}{16}\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(4r_3)\\
&- \tfrac{3}{64}\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(4r_3)\\
&- \tfrac{1}{16}\cosh(4s)\cos(2r_5)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(4s)\cos(2r_5)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&+ \tfrac{1}{64}\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(4r_3)\cos(2r_5)\\
&+ 9\cosh(2s)\cosh(2z) - \tfrac{3}{4}\cosh(2s)\cosh(6z)\\
&+ \tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_5)\\
&- 3\cos(2r_3)\sinh(2z)\sinh(2s) - 3\cosh(2s)\cosh(2z)\cos(2r_2)\\
&+ \tfrac{3}{8}\cosh(2s)\cosh(6z)\cos(2r_2) - \tfrac{3}{8}\cosh(2s)\cosh(6z)\cos(4r_2)\\
&- \tfrac{1}{8}\cosh(2s)\cosh(6z)\cos(2r_2)\cos(2r_5)\\
&+ \tfrac{1}{8}\cosh(2s)\cosh(6z)\cos(4r_2)\cos(2r_5)\\
&-3\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&- \tfrac{3}{8}\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&- \tfrac{3}{8}\cos(4r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+ \tfrac{1}{8}\cos(2r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&+ \tfrac{1}{8}\cos(4r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&+ 3\cosh(2s)\cosh(2z)\cos(2r_1) - 9\cos(2r_1)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&- \tfrac{3}{4}\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+ \tfrac{1}{4}\cos(2r_1)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&+ 3\cosh(2s)\cosh(2z)\cos(2r_1)\cos(2r_2)\\
&- \tfrac{3}{8}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\\
&- \tfrac{3}{8}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(4r_2)\\
&+ \tfrac{1}{8}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&+ \tfrac{1}{8}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(4r_2)\cos(2r_5)\\
&- 12\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2z)\sinh(2s)\\
&- \tfrac{3}{4}\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+ \tfrac{3}{4}\sin(2r_3)\sin(3r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+ 3\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+ \tfrac{3}{8}\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&- \tfrac{3}{8}\cos(2r_1)\cos(4r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+ \tfrac{1}{4}\cos(2r_5)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{4}\cos(2r_5)\sin(2r_3)\sin(3r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&- \tfrac{1}{8}\cos(2r_1)\cos(2r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&+ \tfrac{1}{8}\cos(2r_1)\cos(4r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&- \tfrac{43}{64}\cosh(2v) - \tfrac{43}{64}\cosh(2v)\cos(2r_5) + \tfrac{3}{64}\cosh(2v)\cos(4r_3)\\
&+ \tfrac{3}{64}\cosh(2v)\cos(4r_3)\cos(2r_5) - \tfrac{1}{16}\cosh(2v)\cos(2r_2)\\
&+ \tfrac{7}{64}\cosh(2v)\cos(4r_2) - \tfrac{1}{16}\cosh(2v)\cos(2r_2)\cos(2r_5)\\
&+ \tfrac{7}{64}\cosh(2v)\cos(4r_2)\cos(2r_5) + \tfrac{1}{16}\cosh(2v)\cos(2r_2)\cos(4r_3)\\
&+ \tfrac{1}{64}\cosh(2v)\cos(4r_2)\cos(4r_3) + \tfrac{1}{16}\cosh(2v)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&+ \tfrac{1}{64}\cosh(2v)\cos(4r_2)\cos(4r_3)\cos(2r_5) - \tfrac{3}{64}\cosh(2v)\cos(2r_1)\\
&- \tfrac{3}{64}\cosh(2v)\cos(2r_1)\cos(2r_5) - \tfrac{5}{64}\cosh(2v)\cos(2r_1)\cos(4r_3)\\
&- \tfrac{5}{64}\cosh(2v)\cos(2r_1)\cos(4r_3)\cos(2r_5) + \tfrac{1}{16}\cosh(2v)\cos(2r_1)\cos(2r_2)\\
&+ \tfrac{7}{64}\cosh(2v)\cos(2r_1)\cos(4r_2) + \tfrac{1}{16}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&+ \tfrac{7}{64}\cosh(2v)\cos(2r_1)\cos(4r_2)\cos(2r_5)\\
&- \tfrac{1}{16}\cosh(2v)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(2v)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(4r_3)\\
&+ \tfrac{1}{64}\cosh(2v)\cos(2r_1)\cos(4r_2)\cos(4r_3)\\
&- \tfrac{1}{16}\cosh(2v)\cos(2r_5)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(2v)\cos(2r_5)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&- \tfrac{1}{16}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&+ \tfrac{1}{64}\cosh(2v)\cos(2r_1)\cos(4r_2)\cos(4r_3)\cos(2r_5)\\
&+ 3\cos(r_5)\sinh(4z)\sinh(v) - 9\cos(2r_2)\cos(r_5)\sinh(4z)\sinh(v)\\
&- 3\cos(2r_1)\cos(r_5)\sinh(4z)\sinh(v)\\
&- 3\cos(2r_1)\cos(2r_2)\cos(r_5)\sinh(4z)\sinh(v)\\
&- \tfrac{5}{64}\cosh(2v)\cosh(4s) - \tfrac{5}{64}\cosh(2v)\cosh(4s)\cos(2r_5)\\
&- \tfrac{3}{64}\cosh(2v)\cosh(4s)\cos(4r_3) - \tfrac{3}{64}\cosh(2v)\cosh(4s)\cos(4r_3)\cos(2r_5)\\
&- \tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_2) + \tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(4r_2)\\
&- \tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_2)\cos(2r_5)\\
&+ \tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(4r_2)\cos(2r_5)\\
&- \tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_2)\cos(4r_3)\\
&- \tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(4r_2)\cos(4r_3)\\
&- \tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&- \tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(4r_2)\cos(4r_3)\cos(2r_5)\\
&+ \tfrac{3}{64}\cosh(2v)\cosh(4s)\cos(2r_1) + \tfrac{3}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_5)\\
&+ \tfrac{5}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_3)
\end{aligned}
$$

$$
\begin{aligned}
&+\tfrac{5}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_3)\cos(2r_5)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2)\\
&+\tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_2)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&+\tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(2r_5)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(4r_3)\\
&-\tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(4r_3)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_5)\sin(4r_3)\sin(r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_5)\sin(4r_3)\sin(3r_2)\sin(2r_1)\\
&+\tfrac{1}{16}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2)\cos(4r_3)\cos(2r_5)\\
&-\tfrac{1}{64}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(4r_2)\cos(4r_3)\cos(2r_5)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)+3\cosh(2s)\cos(r_5)\sinh(2z)\sinh(v)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_5)\\
&-9\cosh(2z)\cos(2r_3)\cos(r_5)\sinh(2s)\sinh(v)\\
&+\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_2)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(4r_2)\\
&-9\cosh(2s)\cos(2r_2)\cos(r_5)\sinh(2z)\sinh(v)\\
&+\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_2)\cos(2r_5)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(4r_2)\cos(2r_5)\\
&-\tfrac{1}{8}\cosh(2v)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-\tfrac{1}{8}\cosh(2v)\cos(4r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-9\cosh(2z)\cos(2r_2)\cos(2r_3)\cos(r_5)\sinh(2s)\sinh(v)\\
&-\tfrac{1}{8}\cosh(2v)\cos(2r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&-\tfrac{1}{8}\cosh(2v)\cos(4r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&-3\cosh(2s)\cos(2r_1)\cos(r_5)\sinh(2z)\sinh(v)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+9\cosh(2z)\cos(2r_1)\cos(2r_3)\cos(r_5)\sinh(2s)\sinh(v)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_1)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(4r_2)\\
&-3\cosh(2s)\cos(2r_1)\cos(2r_2)\cos(r_5)\sinh(2z)\sinh(v)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\cos(2r_5)\\
&-\tfrac{1}{8}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(4r_2)\cos(2r_5)\\
&-\tfrac{1}{4}\cosh(2v)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+\tfrac{1}{4}\cosh(2v)\sin(2r_3)\sin(3r_2)\sin(2r_1)\sinh(6z)\sinh(2s)
\end{aligned}
$$

$$
\begin{aligned}
&+\tfrac{1}{8}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-\tfrac{1}{8}\cosh(2v)\cos(2r_1)\cos(4r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+12\cosh(2z)\cos(r_5)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2s)\sinh(v)\\
&-3\cosh(2z)\cos(2r_1)\cos(2r_2)\cos(2r_3)\cos(r_5)\sinh(2s)\sinh(v)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_5)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+\tfrac{1}{4}\cosh(2v)\cos(2r_5)\sin(2r_3)\sin(3r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+\tfrac{1}{8}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s)\\
&-\tfrac{1}{8}\cosh(2v)\cos(2r_1)\cos(4r_2)\cos(2r_3)\cos(2r_5)\sinh(6z)\sinh(2s).
\end{aligned}
\tag{C.2}
$$

- $G_0 = SO(6,2) \times SO(6,2)$, potential restricted to singlets under $SO(6)_{\text{diag}}$:

$$
\begin{aligned}
-8g^{-2}V = {}& 27+3\cosh(4z)+3\cosh(4z)\cos(2r_2)-3\cosh(4z)\cos(2r_1)\\
&-3\cosh(4z)\cos(2r_1)\cos(2r_2)+\tfrac{1}{4}\cosh(4s)\\
&+\tfrac{1}{4}\cosh(4s)\cos(2r_2)-\tfrac{1}{4}\cosh(4s)\cos(2r_1)\\
&-\tfrac{1}{4}\cosh(4s)\cos(2r_1)\cos(2r_2)+9\cosh(2s)\cosh(2z)\\
&+\tfrac{3}{4}\cosh(2s)\cosh(6z)-3\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+\tfrac{1}{4}\cos(2r_3)\sinh(6z)\sinh(2s)-3\cosh(2s)\cosh(2z)\cos(2r_2)\\
&-\tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_2)-3\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+\tfrac{1}{4}\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)+3\cosh(2s)\cosh(2z)\cos(2r_1)\\
&+\tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_1)-9\cos(2r_1)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+\tfrac{3}{4}\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+3\cosh(2s)\cosh(2z)\cos(2r_1)\cos(2r_2)\\
&+\tfrac{1}{4}\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\\
&-12\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2z)\sinh(2s)\\
&+\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+3\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{1}{4}\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+\cosh(2v)-9\cosh(v)\cosh(4z)+3\cosh(v)\cosh(4z)\cos(2r_2)\\
&-3\cosh(v)\cosh(4z)\cos(2r_1)-3\cosh(v)\cosh(4z)\cos(2r_1)\cos(2r_2)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(4s)-\tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_2)\\
&+\tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_1)\\
&+\tfrac{1}{4}\cosh(2v)\cosh(4s)\cos(2r_1)\cos(2r_2)-15\cosh(v)\cosh(2s)\cosh(2z)\\
&-\tfrac{3}{4}\cosh(2v)\cosh(2s)\cosh(6z)-3\cosh(v)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&-3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_2)\\
&+\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_2)\\
&-3\cosh(v)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{1}{4}\cosh(2v)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s)
\end{aligned}
$$

$$
\begin{aligned}
&+3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_1)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\\
&-9\cosh(v)\cos(2r_1)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&-\tfrac{3}{4}\cosh(2v)\cos(2r_1)\cos(2r_3)\sinh(6z)\sinh(2s)\\
&+3\cosh(v)\cosh(2s)\cosh(2z)\cos(2r_1)\cos(2r_2)\\
&-\tfrac{1}{4}\cosh(2v)\cosh(2s)\cosh(6z)\cos(2r_1)\cos(2r_2)\\
&-12\cosh(v)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(2z)\sinh(2s)\\
&-\cosh(2v)\sin(2r_3)\sin(r_2)\sin(2r_1)\sinh(6z)\sinh(2s)\\
&+3\cosh(v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(2z)\sinh(2s)\\
&+\tfrac{1}{4}\cosh(2v)\cos(2r_1)\cos(2r_2)\cos(2r_3)\sinh(6z)\sinh(2s). \qquad \text{(C.3)}
\end{aligned}
$$

- $G_0 = G_2 \times F_{4(-20)}$, potential restricted to singlets under $SU(3)\times SU(3)$:

$$
\begin{aligned}
-8g^{-2}V = {}&\tfrac{24\,125}{2048}+\tfrac{9}{2048}\cos(8r_5)+\tfrac{9}{2048}\cos(8r_2)\\
&+\tfrac{115}{128}\sin(4r_5)\sin(4r_2)-\tfrac{27}{2048}\cos(8r_2)\cos(8r_5)\\
&-\tfrac{1}{64}\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5)-\tfrac{9}{2048}\cos(8r_1-8r_4)\\
&-\tfrac{9}{2048}\cos(8r_1-8r_4)\cos(8r_5)-\tfrac{1}{64}\cos(4r_1-4r_4)\cos(4r_3-4r_6)\\
&+\tfrac{1}{64}\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)-\tfrac{9}{2048}\cos(8r_1-8r_4)\cos(8r_2)\\
&-\tfrac{9}{512}\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&+\tfrac{115}{128}\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&-\tfrac{9}{2048}\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{64}\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)\\
&-\tfrac{1}{64}\cos(4r_1-4r_4)\cos(4r_3-4r_6)\sin(4r_5)\sin(4r_2)+\tfrac{449}{512}\cosh(z)\\
&-\tfrac{65}{2048}\cosh(2z)-\tfrac{3}{512}\cosh(z)\cos(8r_5)+\tfrac{3}{2048}\cosh(2z)\cos(8r_5)\\
&-\tfrac{3}{512}\cosh(z)\cos(8r_2)+\tfrac{3}{2048}\cosh(2z)\cos(8r_2)\\
&-\tfrac{29}{32}\cosh(z)\sin(4r_5)\sin(4r_2)+\tfrac{1}{128}\cosh(2z)\sin(4r_5)\sin(4r_2)\\
&+\tfrac{9}{512}\cosh(z)\cos(8r_2)\cos(8r_5)-\tfrac{9}{2048}\cosh(2z)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{64}\cosh(2z)\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5)+\tfrac{3}{512}\cosh(z)\cos(8r_1-8r_4)\\
&-\tfrac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)+\tfrac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)\\
&-\tfrac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)\\
&+\tfrac{1}{64}\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)\\
&-\tfrac{1}{64}\cosh(2z)\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)\\
&+\tfrac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\\
&-\tfrac{3}{2048}\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\\
&+\tfrac{3}{128}\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&-\tfrac{3}{512}\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&-\tfrac{29}{32}\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&+\tfrac{3}{512}\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{128}\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{3}{2048}\cosh(2z)\cos(8r_1 - 8r_4)\cos(8r_2)\cos(8r_5)\\
&- \tfrac{1}{64}\cosh(2z)\sin(4r_3 - 4r_6)\sin(4r_2)\sin(4r_1 - 4r_4)\\
&+ \tfrac{1}{64}\cosh(2z)\cos(4r_1 - 4r_4)\cos(4r_3 - 4r_6)\sin(4r_5)\sin(4r_2)\\
&+ \tfrac{449}{512}\cosh(s) - \tfrac{65}{2048}\cosh(2s) - \tfrac{3}{512}\cosh(s)\cos(8r_5)\\
&+ \tfrac{3}{2048}\cosh(2s)\cos(8r_5) - \tfrac{3}{512}\cosh(s)\cos(8r_2)\\
&+ \tfrac{3}{2048}\cosh(2s)\cos(8r_2) - \tfrac{29}{32}\cosh(s)\sin(4r_5)\sin(4r_2)\\
&+ \tfrac{1}{128}\cosh(2s)\sin(4r_5)\sin(4r_2) + \tfrac{9}{512}\cosh(s)\cos(8r_2)\cos(8r_5)\\
&- \tfrac{9}{2048}\cosh(2s)\cos(8r_2)\cos(8r_5)\\
&+ \tfrac{1}{64}\cosh(2s)\cos(4r_2)\cos(4r_3 - 4r_6)\cos(4r_5)\\
&+ \tfrac{3}{512}\cosh(s)\cos(8r_1 - 8r_4) - \tfrac{3}{2048}\cosh(2s)\cos(8r_1 - 8r_4)\\
&+ \tfrac{3}{512}\cosh(s)\cos(8r_1 - 8r_4)\cos(8r_5)\\
&- \tfrac{3}{2048}\cosh(2s)\cos(8r_1 - 8r_4)\cos(8r_5)\\
&+ \tfrac{1}{64}\cosh(2s)\cos(4r_1 - 4r_4)\cos(4r_3 - 4r_6)\\
&- \tfrac{1}{64}\cosh(2s)\sin(4r_5)\sin(4r_3 - 4r_6)\sin(4r_1 - 4r_4)\\
&+ \tfrac{3}{512}\cosh(s)\cos(8r_1 - 8r_4)\cos(8r_2)\\
&- \tfrac{3}{2048}\cosh(2s)\cos(8r_1 - 8r_4)\cos(8r_2)\\
&+ \tfrac{3}{128}\cosh(s)\cos(4r_1 - 4r_4)\sin(8r_5)\sin(8r_2)\\
&- \tfrac{3}{512}\cosh(2s)\cos(4r_1 - 4r_4)\sin(8r_5)\sin(8r_2)\\
&- \tfrac{29}{32}\cosh(s)\cos(4r_1 - 4r_4)\cos(4r_2)\cos(4r_5)\\
&+ \tfrac{3}{512}\cosh(s)\cos(8r_1 - 8r_4)\cos(8r_2)\cos(8r_5)\\
&+ \tfrac{1}{128}\cosh(2s)\cos(4r_1 - 4r_4)\cos(4r_2)\cos(4r_5)\\
&- \tfrac{3}{2048}\cosh(2s)\cos(8r_1 - 8r_4)\cos(8r_2)\cos(8r_5)\\
&- \tfrac{1}{64}\cosh(2s)\sin(4r_3 - 4r_6)\sin(4r_2)\sin(4r_1 - 4r_4)\\
&+ \tfrac{1}{64}\cosh(2s)\cos(4r_1 - 4r_4)\cos(4r_3 - 4r_6)\sin(4r_5)\sin(4r_2)\\
&+ \tfrac{341}{128}\cosh(s)\cosh(z) - \tfrac{21}{512}\cosh(s)\cosh(2z)\\
&- \tfrac{21}{512}\cosh(2s)\cosh(z) - \tfrac{107}{2048}\cosh(2s)\cosh(2z)\\
&+ \tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_5) - \tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_5)\\
&- \tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_5) + \tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_5)\\
&+ \tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_2) - \tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_2)\\
&- \tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_2) + \tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_2)\\
&+ \tfrac{7}{8}\cosh(s)\cosh(z)\sin(4r_5)\sin(4r_2)\\
&+ \tfrac{1}{32}\cosh(s)\cosh(2z)\sin(4r_5)\sin(4r_2)\\
&+ \tfrac{1}{32}\cosh(2s)\cosh(z)\sin(4r_5)\sin(4r_2)\\
&- \tfrac{5}{128}\cosh(2s)\cosh(2z)\sin(4r_5)\sin(4r_2)\\
&- \tfrac{3}{128}\cosh(s)\cosh(z)\cos(8r_2)\cos(8r_5)\\
&+ \tfrac{3}{512}\cosh(s)\cosh(2z)\cos(8r_2)\cos(8r_5)
\end{aligned}
$$

$$
\begin{aligned}
&+\tfrac{3}{512}\cosh(2s)\cosh(z)\cos(8r_2)\cos(8r_5)\\
&-\tfrac{3}{2048}\cosh(2s)\cosh(2z)\cos(8r_2)\cos(8r_5)\\
&-\tfrac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_2)\cos(4r_3-4r_6)\cos(4r_5)\\
&-\tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\\
&+\tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\\
&+\tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\\
&-\tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\\
&-\tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)\\
&+\tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)\\
&+\tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_5)\\
&-\tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_5)\\
&-\tfrac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_3-4r_6)\\
&+\tfrac{1}{64}\cosh(2s)\cosh(2z)\sin(4r_5)\sin(4r_3-4r_6)\sin(4r_1-4r_4)\\
&-\tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\\
&+\tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\\
&+\tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\\
&-\tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\\
&-\tfrac{1}{32}\cosh(s)\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&+\tfrac{1}{128}\cosh(s)\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&+\tfrac{1}{128}\cosh(2s)\cosh(z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&-\tfrac{1}{512}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\sin(8r_5)\sin(8r_2)\\
&+\tfrac{7}{8}\cosh(s)\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&-\tfrac{1}{128}\cosh(s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{32}\cosh(s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&+\tfrac{1}{512}\cosh(s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{32}\cosh(2s)\cosh(z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&+\tfrac{1}{512}\cosh(2s)\cosh(z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&-\tfrac{5}{128}\cosh(2s)\cosh(2z)\cos(4r_1-4r_4)\cos(4r_2)\cos(4r_5)\\
&-\tfrac{1}{2048}\cosh(2s)\cosh(2z)\cos(8r_1-8r_4)\cos(8r_2)\cos(8r_5)\\
&+\tfrac{1}{64}\cosh(2s)\cosh(2z)\sin(4r_3-4r_6)\sin(4r_2)\sin(4r_1-4r_4)\\
&+\tfrac{1}{64}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)\\
&+\tfrac{229}{64}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)\\
&-\tfrac{1}{64}\sin(2r_3-2r_6)\sin(6r_2-2r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)\\
&+\tfrac{3}{64}\sin(2r_3-2r_6)\sin(6r_2+6r_5)\sin(2r_1-2r_4)\sinh(z)\sinh(s)\\
&+\tfrac{1}{64}\sin(2r_3-2r_6)\sin(2r_2-6r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)\\
&+\tfrac{1}{64}\sin(2r_3-2r_6)\sin(2r_2+2r_5)\sin(6r_1-6r_4)\sinh(z)\sinh(s)
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{1}{64}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(s)\\
&- \tfrac{1}{64}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(s)\\
&- \tfrac{5}{128}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(s)\\
&- \tfrac{3}{128}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(2r_1 - 2r_4)\sinh(z)\sinh(2s)\\
&- \tfrac{5}{128}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(2r_1 - 2r_4)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(2r_1 - 2r_4)\sinh(z)\sinh(2s)\\
&- \tfrac{3}{128}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(2r_1 - 2r_4)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{128}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(6r_1 - 6r_4)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(2s)\\
&- \tfrac{27}{256}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(2s)\\
&+ \tfrac{3}{256}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(2r_1 - 2r_4)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(2r_2 - 6r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(2r_2 + 2r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(6r_2 - 2r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{256}\sin(2r_3 - 2r_6)\sin(6r_2 + 6r_5)\sin(6r_1 - 6r_4)\sinh(2z)\sinh(2s)\\
&- \tfrac{229}{64}\cos(2r_1 - 2r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&- \tfrac{1}{64}\cos(2r_1 - 2r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{3}{64}\cos(2r_1 - 2r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&- \tfrac{1}{64}\cos(2r_1 - 2r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{1}{64}\cos(6r_1 - 6r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{1}{64}\cos(6r_1 - 6r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{1}{64}\cos(6r_1 - 6r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{1}{64}\cos(6r_1 - 6r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(s)\\
&+ \tfrac{5}{128}\cos(2r_1 - 2r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&+ \tfrac{1}{128}\cos(2r_1 - 2r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)
\end{aligned}
$$

$$
\begin{aligned}
&- \tfrac{3}{128}\cos(2r_1 - 2r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&+ \tfrac{1}{128}\cos(2r_1 - 2r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(s)\\
&+ \tfrac{5}{128}\cos(2r_1 - 2r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{128}\cos(2r_1 - 2r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&- \tfrac{3}{128}\cos(2r_1 - 2r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&+ \tfrac{1}{128}\cos(2r_1 - 2r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&- \tfrac{1}{128}\cos(6r_1 - 6r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(z)\sinh(2s)\\
&+ \tfrac{27}{256}\cos(2r_1 - 2r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{256}\cos(2r_1 - 2r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&+ \tfrac{3}{256}\cos(2r_1 - 2r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{256}\cos(2r_1 - 2r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\cos(6r_1 - 6r_4)\cos(2r_2 - 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\cos(6r_1 - 6r_4)\cos(2r_2 + 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\cos(6r_1 - 6r_4)\cos(6r_2 - 6r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&+ \tfrac{1}{256}\cos(6r_1 - 6r_4)\cos(6r_2 + 2r_5)\cos(2r_3 - 2r_6)\sinh(2z)\sinh(2s)\\
&- \tfrac{1}{64}\cosh(2s)\cosh(2z)\cos(4r_1 - 4r_4)\cos(4r_3 - 4r_6)\sin(4r_5)\sin(4r_2).
\end{aligned}
\tag{C.4}
$$

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