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Boundary Superstring Field Theory Annulus Partition Function in the Presence of Tachyons

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Abstract

We compute the Boundary Superstring Field Theory partition function on the annulus in the presence of independent linear tachyon profiles on the two boundaries. The R-R sector is found to contribute non-trivially to the derivative terms of the space-time effective action. In the process we construct a boundary state description of D-branes in the presence of a linear tachyon. We quantize the open string in a tachyonic background and address the question of open/closed string duality.

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1 Introduction

In the last few years significant progress has been made in understanding the open string tachyon dynamics. It has become clear that open string tachyon condensation describes the decay of unstable D-branes into stable ones or into the closed string vacuum. Initially the discussion was based on the first quantized string theory [1, 2, 3]. Subsequently tachyon condensation has been investigated with a remarkable degree of accuracy in Cubic Open String Field Theory [4] by using the level truncation approximation [5]. From the world-sheet point of view the tachyon condensation process is then viewed as the RG flow relating conformal field theories with Neumann and Dirichlet boundary conditions [6, 7]. More recently it has been argued [8, 9, 10, 11, 12, 13, 14] that the Boundary String Field Theory (BSFT) [15, 16] also provides a suitable description of the tachyon condensation. In particular the exact tree level tachyon potential, the ratios of the brane tensions [8, 9] as well as the low-energy effective action for massless fluctuations around a tachyonic soliton [17] are obtained quite naturally in this setting.

A basic object of the BSFT is an effective space-time action S_{eff} considered as a functional of the open string background fields. In the supersymmetric case, with which we will be mainly concerned here, the BSFT action is known [11, 18, 19, 20] to coincide with the partition function Z of the open string boundary sigma model [21]. In the sigma model approach the Weyl and diffeomorphism invariant string action on a world-sheet Σ with boundaries is modified by including boundary perturbations which correspond to turning on space-time background fields. For example one may turn on a background gauge field A_μ by including a boundary perturbation[1]

$$S_{\text{gauge}} = -i \int_{\partial\Sigma} \mathrm{d}s\, A_\mu(X) \frac{\partial}{\partial s} X^\mu \,, \tag{1.1}$$

This perturbation is marginal as it preserves both Weyl and diffeomorphism invariance. In the study of the open superstring background tachyon one includes a boundary perturbation

$$S_{\text{tachyon}} = \frac{1}{2} \int_{\partial\Sigma} \mathrm{d}s\, T^2(X)\,, \tag{1.2}$$

where $T(X)$ is a tachyon profile. A particularly simple case is the linear profile $T(X) = u_\mu X^\mu$, for some non-negative constants u_μ and $\mathrm{d}s$ is the diffeomorphism invariant length element on the boundary $\partial\Sigma$ of the world-sheet. In this case the boundary sigma model is exactly solvable. The higher open string background fields can also be incorporated in this approach [22] though the corresponding action defines a non-renormalizable theory.

One important part of the investigation of the supersymmetric boundary sigma model is a study of contributions of higher genus Riemann surfaces to Z. This should give new insight into the effective action S_{eff}, provided that the correspondence $S_{\text{eff}} = Z$ holds for higher Riemann

[1] In the introduction we restrict ourselves to the bosonic part of the boundary action.

surfaces as well. In this paper we consider the case of the annulus, a subject already being under discussion in the current literature [23]-[27].

In general one defines (as in the case of Polyakov's string [28]) the path integral to include an integration over world-sheet metrics, modulo the symmetries of the theory. In particular the sigma model action is still invariant with respect to a subgroup of the diffeomorphism group compatible with the boundary conditions. The boundary action (1.2) is not Weyl-invariant, while the bulk action is. This too should be accounted for in a path integral formulation. Since such a path integral formalism, properly accounting for the remaining gauge symmetry, is not presently available, the first problem to study is to consider some fixed metric $g_{\alpha\beta}$ on the annulus and to compute $Z[g_{\alpha\beta}]$, the path integral over the X^μ's.

In this paper we consider the simplest case of the flat metric on the annulus for which the bosonic part of the action is (we set $\alpha' = 2$)

$$\begin{aligned} S &= \frac{1}{8\pi}\int_a^1 \int_0^{2\pi} r\,\mathrm{d}r\mathrm{d}\phi \left(\frac{1}{r^2}(\partial_\phi X(r,\phi))^2 + (\partial_r X(r,\phi))^2\right) \\ &\quad + \frac{1}{8\pi}u_\mu u_\nu \int_0^{2\pi} \mathrm{d}\phi X^\mu X^\nu(1,\phi) + \frac{1}{8\pi} a v_\mu v_\nu \int_0^{2\pi} \mathrm{d}\phi X^\mu X^\nu(a,\phi)\,, \end{aligned} \tag{1.3}$$

where $u\,,v$ are tachyons on the $r=1$ and $r=a\leq 1$ boundaries of the annulus, respectively; the presence of a in the last term of the action is due to the diffeomorphism invariant measure ds. A more detailed discussion of the action involving fermions will be given in Section 2.

The computation of the annulus partition function can be done in several ways and below we briefly summarise those used in this paper. Perhaps the most direct approach is via the Green's function method, discussed in Section 3. The classical field configurations minimizing the bosonic action (1.3) are subjected to the following boundary conditions[2]

$$\partial_r X^\mu(1,\phi) + u_\mu u_\nu X^\nu(1,\phi) = 0\,, \qquad -\partial_r X^\mu(a,\phi) + v_\mu v_\nu X^\nu(a,\phi) = 0 \tag{1.4}$$

and computing the path integral by the Green's function approach one may conveniently choose the Green's functions to obey the same type of boundary conditions. The two limiting cases $u = 0\,,\infty$ correspond to Neumann and Dirichlet boundary conditions, respectively.

In Section 3 we derive the boundary conditions for fermions, taking into account the different spin structures on the annulus. In particular we show that the choice of spin structure is equivalent to identifying the boundary fermion, treated as a Lagrangian multiplier, in terms of the bulk fermions. In Section 3 we construct the Green's functions both in the NS-NS and R-R sectors for different spin structures and use them to derive the corresponding contributions to the partition function.[3]

[2]Our boundary conditions differ from the ones used in [23, 25, 27]. They do agree with [24] and [26] who also use the diffeomorphism invariant measure.

[3]The Green's functions in the NS-NS sector of the boundary sigma model were considered in [25] and [26] but only for one particular choice of the spin structure.

In Section 4 we discuss another way to compute the partition function. We map the annulus to the cylinder via

$$\tau_{\rm c} = \ln r\,, \qquad \sigma_{\rm c} = \phi\,, \tag{1.5}$$

and construct boundary states [29, 30] $|B,u\rangle$, $\langle B,v|$ corresponding to the absorption and emission of closed strings from the D-branes with background tachyons turned on.[4] Specialising to a tachyon profile with u_μ and v_μ in the same direction (say $\mu = 9$) the bosonic part of the boundary states (in that direction) satisfy

$$\Big(\partial_{\tau_{\rm c}} X^9(1,\sigma_{\rm c}) + u^2 X^9(1,\sigma_{\rm c})\Big)\,|B,u,0\rangle = 0\,, \qquad \langle B,v,-l|\,\Big(-\partial_{\tau_{\rm c}} X^9(l,\sigma_{\rm c}) + a v^2 X^9(l,\sigma_{\rm c})\Big) = 0\,, \tag{1.6}$$

with $l = -\ln a$. The partition function on the cylinder is

$$Z_{\rm cylinder}(u,v) = \int_0^\infty {\rm d}l \langle B,v| e^{-lH_c} |B,u\rangle\,, \tag{1.7}$$

where H_c is the (conformal) closed string Hamiltonian. In this formalism the disc partition function is

$$Z_{\rm disc}(u) = \langle {\rm vacuum}\, |B,u\rangle\,, \tag{1.8}$$

which can be shown to coincide with the one computed in [8, 9, 11, 15]. The boundary state approach emphasizes the conformal nature of the bulk theory - in the bulk the usual Virasoro generators $L_n, \tilde{L}_n$ are well defined and satisfy the standard algebra. For a conformal boundary perturbation the boundary states satisfy [32]

$$(L_n - \tilde{L}_{-n})|B\rangle = 0\,, \tag{1.9}$$

indicating that Weyl invariance is not broken on the boundary. The $|B,u\rangle$ satisfy no such simple relation.

The construction of the boundary states involves finding suitably normalized coherent states which satisfy (1.6) and fermionic boundary conditions in the NS-NS and R-R sectors. These states have to be invariant under the closed string GSO projection. Due to the presence of fermionic zero-modes this places a restriction on the allowed boundary states in the R-R sector [3]. In particular it is well known that with no background tachyon the Dp-brane R-R boundary state is GSO invariant for p even/odd in Type II A/B, respectively. So for example there is no GSO invariant R-R boundary state corresponding to the non-BPS D9-brane of Type IIA. We show that in the presence of a non-zero background tachyon in one direction (say $u^9 \neq 0$) a GSO invariant non-BPS D9-brane R-R boundary state does exist. The normalization of this state depends linearly on u^9 and so becomes zero in the non-BPS D9-brane limit. In the limit $u^9 \to \infty$ it reduces to the boundary state of the BPS D8-brane.

[4]Boundary states for bosonic D-branes and the NS-NS sector have been independently constructed in [31, 27].

Finally, we change coordinates on the annulus again, taking the world-sheet time to be periodic

$$\sigma_{\mathrm{o}} = -\tau_{\mathrm{c}}\frac{\pi}{l}\,, \qquad \tau_{\mathrm{o}} = \sigma_{\mathrm{c}}\frac{\pi}{l}\,, \tag{1.10}$$

and compute the functional integral on the annulus as an open string partition function. In this case the boundary conditions are

$$\partial_{\sigma_{\mathrm{o}}} X^9(\tau_{\mathrm{o}},0) - \frac{u^2 l}{\pi}X^9(\tau_{\mathrm{o}},0) = 0\,, \qquad \partial_{\sigma_{\mathrm{o}}} X^9(\tau_{\mathrm{o}},\pi) + \frac{a v^2 l}{\pi}X^9(\tau_{\mathrm{o}},\pi) = 0\,. \tag{1.11}$$

In the first part of Section 5 the open string on a strip with boundary conditions relevant to tachyonic perturbations is analysed. The system is canonically quantised and found to have a countably infinite spectrum. Our partition function is found to factorise on closed string poles (with residues depending on the tachyons) giving it the interpretation of a transition amplitude for a closed string propagating between two non-BPS D9-branes in the presence of background tachyons. We compute a renormalised, tachyon dependent normal-ordering constant of the open string Hamiltonian which we expect to be compatible with the open/closed string duality. The path integral on the annulus is then computed as an open string partition function with boundary conditions (1.11) in the second part of the section. Some details about Green's functions and the derivation of the partition function are relegated to the Appendix.

To summarise, we compute the superstring partition function on the annulus in the presence of background open string tachyonic fields and show in particular that the R-R sector contributes non-trivially. In the process we construct boundary states in the presence of linear tachyons; as a corrolary we compute the WZ couplings of non-BPS D-branes. Furthermore we discuss the quantisation of an open string in the presence of a background tachyon, comment on the fate of the open string GSO projection and open/closed string duality.

2 The superstring action in a tachyon and gauge field background

The world sheet action for the superstring in the background of a tachyon and abelian gauge field is

$$S = S_{\mathrm{bulk}} + S_{\mathrm{bndy}}, \tag{2.1}$$

with the standard NSR action in the bulk (we set $\alpha' = 2$)

$$S_{\mathrm{bulk}} = \frac{1}{4\pi}\int_{\Sigma} \mathrm{d}^2 z \left(\partial_z X^\mu \partial_{\bar z} X_\mu + \psi^\mu \partial_{\bar z}\psi_\mu + \tilde\psi^\mu \partial_z \tilde\psi_\mu\right), \tag{2.2}$$

and the boundary action in superspace [11], [14]

$$S_{\mathrm{bndy}} = -\frac{1}{2\pi}\int_{\partial\Sigma} \mathrm{d}s\mathrm{d}\Theta \left(\Gamma D\Gamma + T(\mathbf{X})\Gamma + \frac{i}{2}A_\mu(\mathbf{X})D\mathbf{X}^\mu\right). \tag{2.3}$$

The boundary superspace coordinates are (s, Θ), where Θ is the boundary Grassmann coordinate and $s = r\phi$, ϕ being the angular coordinate on the boundary. Here $\mathbf{X} = X + \Theta\theta$, where θ is a boundary fermion, whose precise relation to the bulk fermions ψ and $\tilde{\psi}$ will be determined below and $D = \partial_\Theta + \Theta\partial_s$. $\Gamma = \rho + \Theta K$ is an auxiliary boundary superfield [2, 6].

The world-sheet Σ is the annulus with inner radius $a < 1$ and outer radius equal to one. In this case there is generically an independent set of background and auxiliary fields on each component of the boundary, though to avoid cluttering of notation this is not explicitly indicated in the above. Performing the integral over Θ and integrating out the auxiliary field K one obtains

$$S_{\mathrm{bndy}} = -\frac{1}{2\pi}\int_{\partial\Sigma} \mathrm{d}s \left(\dot{\rho}\rho + \partial_\mu T(X)\theta^\mu\rho - \frac{1}{4}T(X)^2 + \frac{i}{2}A_\mu(X)\dot{X}^\mu + \frac{i}{4}F_{\mu\nu}(X)\theta^\mu\theta^\nu\right), \tag{2.4}$$

where $\dot{\rho} = \partial_s\rho$ *etc.* The world-sheet theory is exactly solvable in the presence of a linear tachyon profile and a constant abelian field strength

$$T(X) = u_\mu X^\mu, \qquad A_\mu(X) = -\frac{1}{2}F_{\mu\nu}X^\nu. \tag{2.5}$$

In this case the boundary action is

$$S_{\mathrm{bndy}} = \frac{1}{8\pi}\int_{\partial\Sigma} \mathrm{d}s\, (u_{\mu\nu}X^\mu X^\nu + iF_{\mu\nu}\partial_s X^\mu X^\nu - iF_{\mu\nu}\theta^\mu\theta^\nu - 4\partial_s\rho\rho - 4u_\mu\theta^\mu\rho)\,, \tag{2.6}$$

where we defined $u_{\mu\nu} \equiv u_\mu u_\nu$.

The relationship between the boundary and bulk fermions can be determined as follows. On-shell the variation of the fermionic bulk term reduces to the boundary contribution

$$\delta S_{\mathrm{bulk}} = -\frac{i}{4\pi}\int \mathrm{d}s \left(\psi^\mu(s)\delta\psi_\mu(s) + \tilde{\psi}^\mu(s)\delta\tilde{\psi}_\mu(s)\right), \tag{2.7}$$

where we took into account the transformation of the bulk fermions from $z = re^{i\phi}$ to $s = r\phi$ variables. As above we introduce the boundary action with the boundary fermion θ, which is a *new* field and relate it to the bulk fermions by treating it as the Lagrange multiplier in the following modified boundary action

$$S'_{\mathrm{bndy}} = S_{\mathrm{bndy}} - \frac{i}{8\pi}\int_{\partial\Sigma} \mathrm{d}s\, \theta^\mu(\psi_\mu - i\eta\tilde{\psi}_\mu)\,, \tag{2.8}$$

with $\eta = \pm 1$. As we will see in a moment this (and a similar choice $\tilde{\eta} = \pm 1$ on the other component of the boundary) corresponds to the spin structure, since it leads to different ways of identifying the boundary fermion in terms of the bulk fermions.[5] Introducing $\psi_\pm = \psi \pm i\eta\tilde{\psi}$ the variation coming from the fermionic parts of the action now reads (on the $r = 1$ boundary)

$$\delta S = -\frac{i}{8\pi}\int \mathrm{d}\phi \Big[\psi_-\delta\psi_+ + (\psi_+ + \theta)\delta\psi_- - \psi_-\delta\theta\Big] + \delta S_{\mathrm{bndy}}\,. \tag{2.9}$$

[5] Note that the equation of motion for θ reduces to the standard relation $\psi = i\eta\tilde{\psi}$ in the case of vanishing background fields.

We define the boundary conditions for fermions by requiring the variation of the total action to vanish on-shell. Since $\delta\psi_-$ and $\delta\theta$ are independent variables on the boundary the vanishing of the coefficient of $\delta\psi_-$ yields

$$-\theta = \psi_+ = \psi + i\eta\tilde{\psi}, \tag{2.10}$$

i.e. it now relates the bulk and boundary fermions. This relation implies $\delta\psi_+ = -\delta\theta$ and the remaining part of δS gives the boundary conditions for θ (*cf.* Section 3).

The choice of boundary fermion $-\theta = \psi + i\eta\tilde{\psi}$ in terms of bulk fermions is precisely the choice of spin structure. Since we have two boundaries for the annulus we have in sum four different possibilities to identify the boundary fermions with the bulk fermions (*cf.* [30]). These cases should be combined with the conditions for bulk fermions to be antiperiodic or periodic around the circle, so together we would get eight different sectors. However, as we will show in Section 3, there are only four different sectors since the spin structure enters in the final answers only through the combination $\eta\tilde{\eta}$.

3 The annulus partition function via Green's functions

In this section we determine the complete partition function in the closed channel by the method of Green's functions [33]. For the sake of clarity we will analyse the different sectors separately and summarise the results here. The computational details are presented in the Appendix.

3.1 The bosonic sector

Besides the Laplace equation in the bulk the X^μ's satisfy[6]

$$\begin{aligned}(z\partial_z + \bar{z}\partial_{\bar{z}})X_\mu + u_{\mu\nu}X^\nu + F_{\mu\nu}(z\partial_z - \bar{z}\partial_{\bar{z}})X^\nu &= 0, \qquad |z| = 1\,, \\ -(z\partial_z + \bar{z}\partial_{\bar{z}})X_\mu + av_{\mu\nu}X^\nu + L_{\mu\nu}(z\partial_z - \bar{z}\partial_{\bar{z}})X^\nu &= 0, \qquad |z| = a\,,\end{aligned} \tag{3.1}$$

where $z = re^{i\phi}$, $L_{\mu\nu}$ and $v_{\mu\nu} = v_\mu v_\nu$ are respectively the gauge field and tachyon on the $r = a$ boundary. The Green's function corresponding to these boundary conditions is

$$\begin{aligned}G(z,w) &= -\ln|z-w|^2 + A - \frac{1}{2}Cu\ln|z|^2\ln|w|^2 + C\ln|z|^2 + C^T\ln|w|^2 \\ &+ \sum_{k>0}(\alpha_k z^k + \alpha_{-k}z^{-k} + \tilde{\alpha}_k\bar{z}^k + \tilde{\alpha}_{-k}\bar{z}^{-k})\,,\end{aligned} \tag{3.2}$$

where

$$A = 2(1 - av\ln a)(u + av - auv\ln a)^{-1}, \tag{3.3}$$

$$C = av(u + av - auv\ln a)^{-1}. \tag{3.4}$$

[6]The normal and tangential derivatives are $\partial_r = \frac{1}{|z|}(z\partial_z + \bar{z}\partial_{\bar{z}})$, $\partial_s = \frac{i}{|z|}(z\partial_z - \bar{z}\partial_{\bar{z}})$, respectively.

The explicit expressions for the oscillators are given in the Appendix . $G(z,w)$ satisfies $G_{\mu\nu}(z,w) = G_{\nu\mu}(w,z)$ and is in fact the propagator $\langle X_\mu(z) X_\nu(w)\rangle$.

One finds the bosonic contribution to the partition function (up to normalization) by differentiating the boundary action (2.6) with respect to the world-sheet couplings [15] and using this Green's function (for more details see the Appendix). The resulting expression is

$$\begin{aligned} Z_{\mathrm{bos}}(a) &= \det(u + av - auv\ln a)^{-1/2} \prod_{k=1}^{\infty} \det(1 + u/k + F)^{-1} \det(1 + av/k + L)^{-1} \\ &\quad \times \det\Big(1 - a^{2k} S_k(u,F) S_k(av,L)\Big)^{-1} , \end{aligned} \tag{3.5}$$

where

$$S_k(u,F) = \frac{k - u - kF}{k + u + kF} . \tag{3.6}$$

This result is purely formal and has to be regularized. The infinite product above diverges and is treated in Section 4.2 (*cf.* equations (4.30) and (4.31) below). As in the disc case the bosonic and fermionic divergences combine to produce a finite result [11, 12, 14]. Furthermore the matrix $(u + av - auv\ln a)$ has rank one or two depending on whether the tachyons u_μ , v_μ are switched on in one or more directions.[7] The determinant above should be understood as a product of the determinant of the maximal rank sub-matrix and the regularised volume of the remaining space-time directions.

In principle there may also be an overall dependence on the modulus a in the partition function that could not be fixed by the previous considerations. By looking at the change of the action under variations of the modulus [33] one can also derive an equation for $\partial_a \ln Z$. It turns out that this equation is consistent with the above expression (3.5) for the partition function, *i.e.* no extra dependence on a appears (for details see the Appendix). Note also that in the limit $a \to 0$ one recovers the (bosonic) partition function on the disc (setting $L = 0$) [15].

The partition function obtained above agrees with the one computed in section 4 using the boundary state formalism. When comparing the two results one should note that the correct integration measure on the annulus is $\frac{da}{a^2}$.

3.2 The NS-NS sector

In the NS-NS sector neither ρ nor θ have zero-modes and the auxiliary boundary fermion ρ can be integrated out. The fermionic part of the boundary action becomes

$$S'^{F}_{\mathrm{bndy}} = \frac{1}{8\pi} \int_{\partial\Sigma} \Big(u_{\mu\nu} \theta^\mu \partial_s^{-1} \theta^\nu - i F_{\mu\nu} \theta^\mu \theta^\nu - i\theta^\mu (\psi_\mu - i\eta\tilde{\psi}_\mu) \Big) , \tag{3.7}$$

[7]One can always rotate to reduce u and v to two-dimensional vectors $(u_1, 0)$ and (v_1, v_2).

and the fermionic boundary conditions following from the (on-shell) vanishing of the total variation of $S_{\text{bulk}} + S'_{\text{bndy}}$ (*cf.* the discussion in Section 2) are

$$\begin{aligned} \left(\delta_{\mu\nu} + iu_{\mu\nu}\partial_s^{-1} + F_{\mu\nu}\right)\psi^\nu &= i\eta\left(\delta_{\mu\nu} - iu_{\mu\nu}\partial_s^{-1} - F_{\mu\nu}\right)\tilde{\psi}^\nu, \qquad r = 1, \\ \left(\delta_{\mu\nu} - iav_{\mu\nu}\partial_s^{-1} - L_{\mu\nu}\right)\psi^\nu &= i\tilde{\eta}\left(\delta_{\mu\nu} + iav_{\mu\nu}\partial_s^{-1} + L_{\mu\nu}\right)\tilde{\psi}^\nu, \qquad r = a\,. \end{aligned} \tag{3.8}$$

Introducing the four kinds of Green's functions on the boundaries

$$\begin{aligned} G_{++}(z,w) &\equiv \langle\psi(z)\psi(w)\rangle = -i\frac{\sqrt{zw}}{z-w} + \sum_{r=1/2}^{\infty}\left(\psi_r(w)z^r + \psi_{-r}(w)z^{-r}\right), \\ G_{--}(\bar{z},\bar{w}) &\equiv \langle\tilde{\psi}(\bar{z})\tilde{\psi}(\bar{w})\rangle = -i\frac{\sqrt{\bar{z}\bar{w}}}{\bar{z}-\bar{w}} + \sum_{r=1/2}^{\infty}\left(\tilde{\psi}_r(\bar{w})\bar{z}^r + \tilde{\psi}_{-r}(\bar{w})z^{-r}\right), \\ G_{+-}(z,\bar{w}) &\equiv \langle\psi(z)\tilde{\psi}(\bar{w})\rangle = \sum_{r=1/2}^{\infty}\left(a_r(\bar{w})z^r + a_{-r}(\bar{w})z^{-r}\right), \\ G_{-+}(\bar{z},w) &\equiv \langle\tilde{\psi}(\bar{z})\psi(w)\rangle = \sum_{r=1/2}^{\infty}\left(b_r(w)\bar{z}^r + b_{-r}(w)\bar{z}^{-r}\right), \end{aligned} \tag{3.9}$$

the boundary conditions on the Green's functions can be written as

$$\begin{aligned} (1+F)z\partial_z + u)\,G_{+\pm} + i\eta\,((1-F)\bar{z}\partial_{\bar{z}} + u)\,G_{-\pm} &= 0, \qquad |z| = 1, \\ (-(1-L)z\partial_z + av)\,G_{+\pm} + i\tilde{\eta}\,(-(1+L)\bar{z}\partial_{\bar{z}} + av)\,G_{-\pm} &= 0, \qquad |z| = a. \end{aligned} \tag{3.10}$$

The boundary fermion at $r = 1$ is related to the bulk fermions by $\theta = -(\psi + i\eta\tilde{\psi})$ (*cf.* equation (2.10)) and therefore the propagator is

$$\langle\theta\theta\rangle = G_{++} - G_{--} + i\eta(G_{+-} + G_{-+})\,. \tag{3.11}$$

On the boundary $r = a$ the second boundary fermion $\tilde{\theta}$ is

$$\tilde{\theta} = -(\psi + i\tilde{\eta}\tilde{\psi}) \tag{3.12}$$

and consequently has propagator

$$\langle\tilde{\theta}\tilde{\theta}\rangle = G_{++} - G_{--} + i\tilde{\eta}(G_{+-} + G_{-+})\,. \tag{3.13}$$

A straightforward, though tedious calculation determines the oscillators of the Green's functions (whose explicit expressions are again collected in the Appendix) and, using the expressions for the boundary fermion propagators, one finds that the resulting contribution to the partition function from the NS-NS sector spin structures is formally

$$Z_{\text{NS-NS}}\,(a,\eta\tilde{\eta}) = \prod_{r=1/2}^{\infty}\det(1+u/r+F)\det(1+av/r+L)\det\left(1 - \eta\tilde{\eta}a^{2r}S_r(u,F)S_r(av,L)\right)\,. \tag{3.14}$$

Due to the closed string GSO projection the contributions from different spin structures ($\eta\tilde{\eta} = \pm 1$) should be added with the opposite sign. This removes the closed string tachyon (*cf.* Section 4).

3.3 The R-R sector

The R-R sector is more subtle, due to the appearance of the ρ and θ zero-modes. Since the zero-mode drops out of the kinetic term of the auxiliary boundary fermion ρ one cannot integrate out ρ completely as in the NS-NS sector. Instead we will integrate out the non-zero modes and treat the zero-modes separately. Then the boundary condition on the non-zero modes of ψ, $\tilde{\psi}$ are exactly as in (3.8). The Green's functions now read

$$
\begin{aligned}
G_{++}(z,w) &\equiv \langle \psi(z)\psi(w)\rangle \\
&= -i\frac{1}{z-w}\left(w\Theta(|z|-|w|)+z\Theta(|w|-|z|)\right)+\sum_{r=1}^{\infty}(\psi_r(w)z^r+\psi_{-r}(w)z^{-r})\,, \\
G_{--}(\bar{z},\bar{w}) &\equiv \langle \tilde{\psi}(\bar{z})\tilde{\psi}(\bar{w})\rangle \\
&= -i\frac{1}{\bar{z}-\bar{w}}\left(\bar{w}\Theta(|z|-|w|)+\bar{z}\Theta(|w|-|z|)\right)+\sum_{r=1}^{\infty}(\tilde{\psi}_r(\bar{w})\bar{z}^r+\tilde{\psi}_{-r}(\bar{w})z^{-r})\,, \\
G_{+-}(z,\bar{w}) &\equiv \langle \psi(z)\tilde{\psi}(\bar{w})\rangle = \sum_{r=1}^{\infty}(a_r(\bar{w})z^r+a_{-r}(\bar{w})z^{-r})\,, \\
G_{-+}(\bar{z},w) &\equiv \langle \tilde{\psi}(\bar{z})\psi(w)\rangle = \sum_{r=1}^{\infty}(b_r(w)\bar{z}^r+b_{-r}(w)\bar{z}^{-r})\,,
\end{aligned}
\tag{3.15}
$$

where $\Theta(|z|-|w|)$ is the step function. A completely analogous calculation to the one for the NS-NS sector shows that the contribution of the R-R non-zero modes to the partition function is

$$
Z_{\text{R-R}}\,(a,\eta\tilde{\eta}) = \prod_{r=1}^{\infty}\det(1+u/r+F)\det(1+av/r+L)\det\Big(1-\eta\tilde{\eta}a^{2r}S_r(u,F)S_r(av,L)\Big)\,. \tag{3.16}
$$

For the zero-modes the kinetic terms of the auxiliary boundary fermions ρ, $\tilde{\rho}$ are absent and the relevant part of the boundary action reads[8]

$$
S^{(0)}_{\text{bndy}} = -\frac{1}{8\pi}\int_{\partial\Sigma} ds\,(4u_\mu\theta_0^\mu\rho_0 + iF_{\mu\nu}\theta_0^\mu\theta_0^\nu)\,. \tag{3.17}
$$

Integrating out ρ_0 and $\tilde{\rho}_0$ we have

$$
Z^{(0)}_{\text{R-R}} = \Big\langle \Big(u_\mu\theta_0^\mu\exp(\frac{i}{4}F_{\rho\lambda}\theta_0^\rho\theta_0^\lambda)\Big)_{r=1}\Big(av_\mu\tilde{\theta}_0^\mu\exp(\frac{i}{4}L_{\rho\lambda}\tilde{\theta}_0^\rho\tilde{\theta}_0^\lambda)\Big)_{r=a}\Big\rangle\,. \tag{3.18}
$$

[8]For the R-R zero-modes we impose the free boundary conditions, thereby relating the left and right movers.

Translating this into Hilbert space language, we see that the zero-mode part of the partition function in the R-R sector is given by the amplitude of the "in-states" at $r = a$ and the "out-states" at $r = 1$. The explicit expression for the in- and out-states will be derived in the following. The zero-modes of the bulk fermions satisfy

$$\{\psi_0^\mu, \psi_0^\nu\} = \delta^{\mu\nu} = \{\tilde{\psi}_0^\mu, \tilde{\psi}_0^\nu\}, \qquad \{\psi_0^\mu, \tilde{\psi}_0^\nu\} = 0. \tag{3.19}$$

The action of ψ_0^μ and $\tilde{\psi}_0^\mu$ on the R-R vacuum (in a non-chiral basis)

$$|A, \tilde{B}\rangle \equiv \lim_{z,\bar{z}\to 0} S^A(z)\tilde{S}^B(\bar{z})|0\rangle, \quad A, B = 1, \ldots, 32 \tag{3.20}$$

can be realized as[9]

$$\begin{aligned} \psi_0^\mu|A, \tilde{B}\rangle &= \frac{1}{\sqrt{2}}(\Gamma^\mu)^A{}_C(1)^B{}_D|C, \tilde{D}\rangle\,, \\ \tilde{\psi}_0^\mu|A, \tilde{B}\rangle &= \frac{1}{\sqrt{2}}(\Gamma_{11})^A{}_C(\Gamma^\mu)^B{}_D|C, \tilde{D}\rangle\,. \end{aligned} \tag{3.21}$$

The vacuum 'in-state' is defined by the free boundary condition

$$(\psi_0^\mu - i\tilde{\eta}\tilde{\psi}_0^\mu)| - \tilde{\eta}\rangle = 0. \tag{3.22}$$

Explicitly it is

$$| - \tilde{\eta}\rangle = \mathcal{M}_{AB}^{(\tilde{\eta})}|A, \tilde{B}\rangle, \qquad \mathcal{M}_{AB}^{(\tilde{\eta})} = \left[C\Gamma_{11}\frac{1 + i\tilde{\eta}\Gamma_{11}}{1 + i\tilde{\eta}}\right]_{AB}, \tag{3.23}$$

where C is the charge conjugation matrix. Similarly, the vacuum "out-state" is

$$\langle\eta| = \langle A, \tilde{B}|\mathcal{N}_{AB}^{(\eta)}, \qquad \mathcal{N}_{AB}^{(\eta)} = -\left[C\Gamma_{11}\frac{1 - i\eta\Gamma_{11}}{1 + i\eta}\right]_{AB}, \tag{3.24}$$

and satisfies

$$\langle\eta|(\psi_0^\mu + i\eta\tilde{\psi}_0^\mu) = 0\,. \tag{3.25}$$

We have

$$\langle\eta| - \tilde{\eta}\rangle = -32\delta_{\eta,-\tilde{\eta}}\,, \tag{3.26}$$

so that, as usual, only one of the two spin structure contributions of the R-R sector is non-zero. Since the boundary fermions anti-commute the expansion of the exponentials will in general terminate at fourth order in the gauge field strengths and we have

$$Z_{\text{R-R}}^{(0)} = au_\mu v_\nu\langle\theta_0^\mu|\tilde{\theta}_0^\nu\rangle - \frac{1}{16}au_\mu v_\nu F_{\rho\lambda}L_{\sigma\tau}\langle\theta_0^\mu\theta_0^\rho\theta_0^\lambda|\tilde{\theta}_0^\nu\tilde{\theta}_0^\sigma\tilde{\theta}_0^\tau\rangle + \cdots \tag{3.27}$$

[9]We follow here the approach of [34].

Since $\tilde{\theta}_0^\mu$ and θ_0^μ act as creation and annihilation operators on the in and out vacua respectively, only terms with the same number of θ_0^μ and $\tilde{\theta}_0^\mu$ give a non-zero contribution. We explicitly compute this in two particular cases. First consider turning on tachyons $u\,,v$ in directions transversal to a gauge field $L = F$. In this case the zero-modes contribute as

$$Z^{(0)}_{\text{R-R}} \sim a u_\mu v^\mu \det(1+F)\,. \tag{3.28}$$

Next consider again $L = F$ restricted to, say, four directions $\mu = 1, 2, 3, 4$ with the tachyons non-zero in the same directions. As long as the tachyons u, v are general we may in fact rotate F to bring it to a block-diagonal form consisting of two antisymmetric 2×2 matrices with independent entries f_1, f_2. After some algebra the result becomes

$$Z^{(0)}_{\text{R-R}} \sim a u_i v^i + a f_1^2(u^3v^3 + u^4v^4) + a f_2^2(u^1v^1 + u^2v^2)\,. \tag{3.29}$$

Covariantly this is written as

$$Z^{(0)}_{\text{R-R}} \sim a u_\mu v^\mu + a u^\mu F^2_{\mu\nu} v^\nu - \frac{a}{2} u_\mu v^\mu F_{\rho\lambda}F^{\rho\lambda} \tag{3.30}$$

and indicates a mixing of the tachyons with the gauge field in the space-time effective action.

Summarising, the contribution of the R-R sector to the full partition function in the closed string channel is

$$Z_{\text{R-R}}\,(a) = Z^{(0)}_{\text{R-R}} \prod_{r=1}^{\infty} \det(1+u/r+F)\det(1+av/r+L)\det\Big(1-\eta\tilde{\eta}a^{2r}S_r(u,F)S_r(av,L)\Big)\,, \tag{3.31}$$

where due to the zero-modes only the spin structure $\eta\tilde{\eta} = -1$ gives a non-vanishing contribution. The above expression should contribute with an overall minus sign to the total partition function so as to respect open/closed string duality. This is discussed in more detail in Section 4.

4 Boundary states in the presence of a tachyon

In this section we construct coherent states which represent D-branes in the presence of a boundary tachyon perturbation. These states are first obtained as solutions of the boundary conditions viewed as an eigenvector equation; the functional approach [32] is then used to normalise them. Finally the cylinder diagram is computed for a closed string propagating between two parallel D-branes with tachyon perturbations turned on.

4.1 Boundary states as eigenvector solutions of boundary conditions

In this sub-section we construct the boundary state for an unstable D9-brane in Type IIA in the presence of a linear tachyon. Our solution will be complete apart from normalisation, since the boundary state shall be obtained, in the usual way, by interpreting the boundary conditions as eigenvector equations. We construct a boundary state as an eigenvector satisfying at $\tau_c = 0$

$$\begin{aligned} \partial_{\tau_c} X^9 + u_c X^9 &= 0\,, \\ \partial_{\tau_c} \psi^9 + u_c \psi^9 &= i\eta(\partial_\tau \tilde{\psi}^9 - u_c \tilde{\psi}^9)\,, \end{aligned} \tag{4.1}$$

where η corresponds to the two spin structures each in the NS-NS and R-R sectors and u_c is some constant. Comparing with boundary conditions (1.6) we obtain the boundary states relevant to us by taking $u_c = u^2, -e^l v^2$ respectively. This identification is consistent with the boundary conditions (3.8) on the annulus with no gauge fields and tachyon only in one direction. The boundary conditions (4.1) in modes read

$$2ip^9 + u_c x^9 = 0\,, \qquad (n + u_c)\alpha_n^9 = (n - u_c)\tilde{\alpha}_{-n}^9\,, \qquad (r + u_c)\psi_r^9 = i\eta(r - u_c)\tilde{\psi}_{-r}^9\,, \tag{4.2}$$

for $n = \pm 1\,, \pm 2\,, \ldots$, $r = \pm\frac{1}{2}\,, \pm\frac{3}{2}\,, \ldots$ in the NS-NS sector and $r = \pm 1\,, \pm 2\,, \ldots$ in the R-R sector. We shall discuss the bosonic and R-R sector zero-modes below. The coherent state which solves these equations is

$$\begin{aligned} |B, u_c, \eta\rangle_{\text{NS-NS ,R-R}} &= \mathcal{N}_{\text{NS-NS ,R-R}}\,(u_c) \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \frac{n - u_c}{n + u_c} \alpha_{-n}^9 \tilde{\alpha}_{-n}^9 + i\eta \sum_{r>0}^{\infty} \frac{r - u_c}{r + u_c} \psi_{-r}^9 \tilde{\psi}_{-r}^9 \right) \\ &\quad \times |B, u_c, \eta\rangle^{(0)}_{\text{NS-NS ,R-R}}\, |B \text{ other}\rangle_{\text{NS-NS ,R-R}}\ . \end{aligned} \tag{4.3}$$

Here $\mathcal{N}_{\text{NS-NS ,R-R}}\,(u_c)$ is the u_c dependent normalisation, $|B, u_c, \eta\rangle^{(0)}$ contains the zero-mode dependence (see below) and $|B \text{ other}\rangle$ is the contribution of the other (Neumann) directions. With the present boundary conditions the images of fields outside the disc are rather complicated; for example for the world-sheet fermion

$$\psi^9(z) = i\eta \frac{\partial_\tau + u_c}{\partial_\tau - u_c} \tilde{\psi}^9(1/\bar{z})\,. \tag{4.4}$$

The bosonic zero-mode conditions

$$(2ip + u_c x)|B, u_c\rangle^{(0)}_{\text{bosonic}} = 0\,, \tag{4.5}$$

are solved by

$$|B, u_c\rangle^{(0)}_{\text{bosonic}} = e^{-\frac{1}{4}u_c x^2}|0\rangle_p\,, \tag{4.6}$$

in the momentum basis. To treat the R-R sector fermionic zero modes we define in the usual fashion (see for example [35])

$$\psi^9_\eta = \frac{1}{\sqrt{2}}(\psi^9_0 + i\eta\tilde{\psi}^9_0)\,, \tag{4.7}$$

which satisfy for non-zero $u_{\rm c}$ the (Dirichlet) boundary condition (see equation (4.4))

$$\psi^9_\eta |B, u_{\rm c}, -\eta\rangle_{\text{R-R}} = 0\,, \qquad u_{\rm c} \neq 0\,. \tag{4.8}$$

In the NS-NS sector there are no fermionic zero-modes and requiring closed string GSO invariance produces a unique boundary state

$$|B, u_{\rm c}\rangle_{\text{NS-NS}} = \frac{1}{2}(|B, u_{\rm c}, +\rangle_{\text{NS-NS}} - |B, u_{\rm c}, -\rangle_{\text{NS-NS}}\,)\,. \tag{4.9}$$

Similarly, in the R-R sector the GSO invariant boundary state is

$$|B, u_{\rm c}\rangle_{\text{R-R}} = 2i(|B, u_{\rm c}, +\rangle_{\text{R-R}} + |B, u_{\rm c}, -\rangle_{\text{R-R}}\,)\,, \qquad u_{\rm c} \neq 0\,. \tag{4.10}$$

4.2 Normalisation of the boundary states

In principle the normalisation of the above constructed boundary states can be fixed by computing the cylinder amplitude and performing a modular transformation to compare with the one loop open string partition function. As we will see it is quite difficult to determine the modular properties of the functions obtained in the cylinder channel directly. A second way [32] involves integrating out the boundary degrees of freedom. This approach has been used in [31] to normalise the NS-NS boundary state in the presence of a tachyon. We review briefly the considerations of [32] and apply them to the problem at hand. Firstly one must set up a complete orthonormal set of bosonic and fermionic coordinates. Define

$$\bar{x}^\mu_m = a^{\mu\dagger}_m + \tilde{a}^\mu_m\,, \qquad x^\mu_m = a^\mu_m + \tilde{a}^{\mu\dagger}_m\,, \tag{4.11}$$

with $m > 0$. Together with q^μ, the centre of mass position, this gives a complete commuting set of bosonic coordinates. The state

$$|x, \bar{x}\rangle = \exp\left\{-\frac{1}{2}(\bar{x}|x) - (a^\dagger|\tilde{a}^\dagger) + (a^\dagger|x) + (\bar{x}|\tilde{a}^\dagger)\right\}|0\rangle\,, \tag{4.12}$$

satisfies the eigenvector equation

$$\begin{aligned} \left[a^{\mu\dagger}_m + \tilde{a}^\mu_m - \bar{x}^\mu_m\right]|x, \bar{x}\rangle &= 0\,, & (4.13)\\ \left[a^\mu_m + \tilde{a}^{\mu\dagger}_m - x^\mu_m\right]|x, \bar{x}\rangle &= 0\,. & (4.14) \end{aligned}$$

In the above

$$(\bar{x}|x) = \sum_{\mu=1}^{10} \sum_{m=1}^{\infty} \bar{x}_m^\mu x_{m,\mu} \,. \tag{4.15}$$

The states $|x, \bar{x}\rangle$ are complete as can be seen from

$$\int \mathcal{D}x \mathcal{D}\bar{x} |x, \bar{x}\rangle\langle x, \bar{x}| = 1 \,. \tag{4.16}$$

For fermions one defines

$$\psi^\mu + i\eta\tilde{\psi}^\mu \equiv \theta^\mu \equiv \sum_n \theta_n^\mu e^{-in\sigma} \tag{4.17}$$

and

$$\bar{\theta}_n^\mu \equiv \theta_{-n}^{\mu\dagger} \,. \tag{4.18}$$

These anti-commute in the usual fashion

$$\{\theta_m^\mu, \theta_n^\nu\} = 0 \,. \tag{4.19}$$

Ignoring for the time being the R-R zero-modes we look for eigenvectors satisfying

$$(\bar{\theta}_m^\mu - \psi_m^{\mu\dagger} - i\eta\tilde{\psi}_m^\mu)|\theta, \bar{\theta}; \eta\rangle = 0 \,, \tag{4.20}$$

$$(\theta_m^\mu - \psi_m^\mu + i\eta\tilde{\psi}_m^{\mu\dagger})|\theta, \bar{\theta}; \eta\rangle = 0 \,. \tag{4.21}$$

They are

$$|\theta, \bar{\theta}; \eta\rangle = \exp\left\{ -\frac{1}{2}(\bar{\theta}|\theta) + i\eta(\psi^\dagger|\tilde{\psi}^\dagger) + (\tilde{\psi}^\dagger|\theta) - i\eta(\bar{\theta}|\tilde{\psi}^\dagger) \right\} |0; \eta\rangle \,, \tag{4.22}$$

and satisfy the completeness relations

$$\int \mathcal{D}\bar{\theta}\mathcal{D}\theta |\theta, \bar{\theta}; \eta\rangle\langle\theta, \bar{\theta}; \eta| = 1 \,. \tag{4.23}$$

The inclusion of bosonic and fermionic zero-modes are discussed in detail in [32] which should be consulted by the interested reader. Here we simply point out that these act directly on the zero field vacuum and are not integrated over. The boundary state can be written as

$$|B, u_c, \eta\rangle = \int \mathcal{D}\bar{x}\mathcal{D}x\mathcal{D}\bar{\theta}\mathcal{D}\theta e^{-S(x,\bar{x},q;\theta,\bar{\theta},\theta_0)} |x, \bar{x}\rangle|\theta, \bar{\theta}, \eta\rangle \,. \tag{4.24}$$

where S in our case is the boundary action for the linear tachyon. Evaluating the functional integrals explicitly yields

$$\begin{aligned} |B, u_c, \eta\rangle_{\text{NS-NS , R-R}} &= \mathcal{N}_{\text{NS-NS ,R-R}} \frac{\prod_{r>0}^\infty (1 + \frac{u_c}{r})}{\prod_{n=1}^\infty (1 + \frac{u_c}{n})} \exp\left(\sum_{n=1}^\infty \frac{1}{n} \alpha_{-n}^\mu S_{\mu\nu}^n \tilde{\alpha}_{-n}^\mu \right) \\ &\quad \times \exp\left(i\eta \sum_{r>0}^\infty \psi_{-r}^\mu S_{\mu\nu}^r \tilde{\psi}_{-r}^\mu \right) |B, u_c, \eta\rangle_{\text{NS-NS , R-R}}^{(0)} |B \text{ other}\rangle_{\text{NS-NS ,R-R}} \,, \end{aligned} \tag{4.25}$$

where r is half-integral in the NS-NS sector and integral in the R-R sector, and the zero-mode part of the boundary state is as discussed above. $\mathcal{N}$ is the normalisation of the zero-mode part of the boundary state (and the other directions). The matrix S is

$$S^n_{\mu\nu} = \mathrm{diag}(-1, \ldots, -1, 1, \ldots, 1, \frac{n-u_\mathrm{c}}{n+u_\mathrm{c}}) \,, \tag{4.26}$$

with entries $-1, 1$ in the Neumann, Dirichlet directions, respectively. In the R-R sector the above infinite products cancel between the bosons and fermions, while in the NS-NS sector they need to be regularised. This gives the u_c dependence of the normalisation as [11]

$$\mathcal{N}_{\text{NS-NS}}\,(u_\mathrm{c}) = \frac{1}{2} u_\mathrm{c} 4^{u_\mathrm{c}} B(u_\mathrm{c}, u_\mathrm{c}) \mathcal{N}_{\text{NS-NS}} \tag{4.27}$$

with B the Euler Beta function and $\mathcal{N}_{\text{NS-NS}}$ is a u_c independent constant.[10]

Viewed as an eigenvector the boundary state thus obtained agrees with the one constructed in sub-section 4.1. Further, we have determined the normalisation of the non-zero mode part by integrating out the boundary degrees of freedom. The bosonic and R-R fermionic zero-modes are not integrated; instead they act directly on the closed string vacuum [32]. In particular the bosonic zero-mode's action is

$$\exp\left(-\frac{1}{4} u_\mathrm{c} x^2\right) |0\rangle_p \,, \tag{4.28}$$

fixing the normalisation of equation (4.6). The action of the fermionic zero-mode is discussed in the paragraph around equation (3.17) giving the zero-mode part of the D9-boundary state in the presence of a tachyon as

$$\sqrt{u_\mathrm{c}} \psi^9_\eta |B9, -\eta\rangle^{(0)}_{\text{R-R}} \,, \tag{4.29}$$

where $|B9, \eta\rangle^{(0)}_{\text{R-R}}$ is the zero-mode part of the usual D9-brane R-R boundary state (*cf.* [35]). This fixes the normalisation of equation (4.8). The normalisation constant $\mathcal{N}_{\text{NS-NS}}$ is[11]

$$\mathcal{N}_{\text{NS-NS}} = T_{\text{non-BPS D9}} \,. \tag{4.30}$$

Similarly the normalization of the R-R boundary state is

$$\mathcal{N}_{\text{R-R}}\,(u_\mathrm{c}) = \sqrt{u_\mathrm{c}} \mathcal{N}_{\text{R-R}} = \sqrt{u_\mathrm{c}} \frac{\mu_8}{\sqrt{2\pi}} \,, \tag{4.31}$$

μ_8 being the charge density of the BPS D8-brane of Type IIA. Here, we have absorbed the factor of $\sqrt{u_\mathrm{c}}$ from equation (4.29) into the normalisation of the R-R sector boundary state for

[10] A similar infinite product was also encountered in Section 3 and should be treated in an analogous fashion.

[11] With this normalisation the NS-NS boundary state reduces in the two limits $u_\mathrm{c} \to 0$, $u_\mathrm{c} \to \infty$ to the usual NS-NS boundary states for a non-BPS D9-brane and a BPS D8-brane of Type IIA with no background tachyon, respectively.

convenience. For $u_{\rm c} = 0$ the R-R sector boundary state is zero while in the $u_{\rm c} \to \infty$ limit we reproduce the usual BPS D8-brane R-R boundary state. As a check we note that

$$\langle 0|\ B, u_{\rm c}, \eta\rangle_{\text{NS-NS}} \ , \tag{4.32}$$

reproduce the disc partition function computed in [8, 9, 11, 15].

As a corollary to the above construction of the R-R sector boundary state for a non-BPS D-brane in the presence of a linear tachyon it is straightforward to generalise the scattering amplitudes of [36] (see also [37]) to obtain the non-BPS D9-brane WZ couplings, including the gravitational piece

$$S_{\rm WZ} = \frac{\mu_8}{2\sqrt{\pi}} \int C_\wedge dT_\wedge {\rm Tr} e^F{}_\wedge \sqrt{\hat{A}(R)} e^{-1/4T^2} \, . \tag{4.33}$$

These are in agreement with the results of [38, 11].

4.3 The cylinder channel

Having constructed normalised boundary states representing D-branes with a background tachyon perturbation, we compute the cylinder diagram corresponding to the exchange of a closed string between parallel D-branes. Specifically we are interested in

$$Z_c(u_{\rm c}, v_{\rm c}, l) = \int_0^\infty {\rm d}l \langle B, v_{\rm c}| e^{-lH_{\rm c}} |B, u_{\rm c}, 0\rangle \, , \tag{4.34}$$

where the bra is computed at $\tau_{\rm c} = -l$, the ket at $\tau_{\rm c} = 0$. To match equation (1.6) the values of the tachyons are

$$u_{\rm c} = u^2 \, , \qquad v_{\rm c} = -v^2 e^{-l} \, . \tag{4.35}$$

In the NS-NS sector the partition function is

$$\begin{aligned} Z_{c,\,\text{NS-NS}}\,(u_{\rm c}, v_{\rm c}) \;\; &= \;\; \frac{V_9}{128(2\pi)^{10}} \int_0^\infty {\rm d}l u_{\rm c}(-v_{\rm c}) 4^{u_{\rm c}-v_{\rm c}} B(u_{\rm c}, u_{\rm c}) B(-v_{\rm c}, -v_{\rm c}) (u_{\rm c} - v_{\rm c} - l u_{\rm c} v_{\rm c})^{-1/2} \\ &\qquad \times \frac{f_3^7(q) f_3^{(u_{\rm c}, v_{\rm c})}(q) - f_4^7(q) f_4^{(u_{\rm c}, v_{\rm c})}(q)}{f_1^7(q) f_1^{(u_{\rm c}, v_{\rm c})}(q)} \, , \end{aligned} \tag{4.36}$$

and in the R-R sector

$$Z_{c,\,\text{R-R}}\,(u_{\rm c}, v_{\rm c}) \;\; = \;\; -\frac{V_9}{64(2\pi)^9} \int_0^\infty {\rm d}l \sqrt{-u_{\rm c} v_{\rm c} q} (u_{\rm c} - v_{\rm c} - l u_{\rm c} v_{\rm c})^{-1/2} \frac{f_2^7(q) f_2^{(u_{\rm c}, v_{\rm c})}(q)}{f_1^7(q) f_1^{(u_{\rm c}, v_{\rm c})}(q)} \, . \tag{4.37}$$

where $q = e^{-l}$ and V_9 is the (infinite) volume of the directions along which the D-brane extends apart from x^9. The $f_i^{(u_c,v_c)}$ are defined as

$$
\begin{aligned}
f_1^{(u_c,v_c)}(q) &= q^{1/12}\prod_{n=1}^{\infty}(1-\frac{n-u_c}{n+u_c}\frac{n+v_c}{n-v_c}q^{2n})\,, \\
f_2^{(u_c,v_c)}(q) &= \sqrt{2}\,q^{1/12}\prod_{n=1}^{\infty}(1+\frac{n-u_c}{n+u_c}\frac{n+v_c}{n-v_c}q^{2n})\,, \\
f_3^{(u_c,v_c)}(q) &= q^{-1/24}\prod_{r=1/2}^{\infty}(1+\frac{r-u_c}{r+u_c}\frac{r+v_c}{r-v_c}q^{2r})\,, \\
f_4^{(u_c,v_c)}(q) &= q^{-1/24}\prod_{r=1/2}^{\infty}(1-\frac{r-u_c}{r+v_c}\frac{r+v_c}{r-v_c}q^{2r})\,,
\end{aligned}
\tag{4.38}
$$

and $f_i(q) = f_i^{(0,0)}(q)$. Z_c reproduces the cylinder diagrams in the four conformal limits $u, v \to 0, \infty$, which correspond to NN, ND, DN and DD boundary conditions in the x^9 direction[12]. The above partition functions are in agreement with the ones computed using the Green's function method in Section 3 for the case of vanishing gauge fields and one-dimensional tachyons. Due to the closed string GSO projection these integrals do not have divergences corresponding to the closed string tachyon. They do however, have an open string tachyon divergence signaling an instability of the D9-brane vacuum [39, 27]. Further there is a divergence due to the massless exchange; it would be interesting to see if this can be treated using the Fischler-Susskind mechanism [40].

Finally, we have found that the above amplitudes factorise on poles at the on-shell closed string mass levels. The residues of these poles are tachyon dependent. This suggests that the above partition functions may be interpreted as transition amplitudes for on-shell closed string states between D-branes with turned on tachyons.[13]

5 Open string in the presence of a tachyon

In this section we first quantise the open superstring on a strip in the presence of a tachyon; we use these results to compute the one-loop partition function for such a string and identify it with the BSFT functional integral on the annulus in the presence of tachyon perturbations. The analysis in this section follows the same lines as [33]. Consider the action

$$
S = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\int_0^{\pi}d\sigma\,(\partial_\tau X\partial_\tau X - \partial_\sigma X\partial_\sigma X) - \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\left[u_o X^2(\sigma=0) + v_o X^2(\sigma=\pi)\right]\,. \tag{5.1}
$$

[12]The $u_c, v_c \to 0$ limit is a little more subtle as the Gaussian integral in the direction of the tachyon field now becomes part of the volume integral. When evaluating the momentum part of the cylinder amplitude we obtained $(u_c - v_c - l u_c v_c)^{-1/2}$, which is only valid away from the zero tachyon.

[13]We would like to acknowledge a discussion with C. Schweigert on this subject.

The constants $u_{\rm o}\,, v_{\rm o}$ are related to the tachyons on the annulus via (see equation (1.11))

$$u_{\rm o} = \frac{u_9^2 l}{\pi}\,, \qquad v_{\rm o} = \frac{e^{-l} v_9^2 l}{\pi}\,. \tag{5.2}$$

Varying the action one obtains the usual wave equation

$$(\partial_\tau^2 - \partial_\sigma^2) X = 0\,, \tag{5.3}$$

with boundary conditions

$$\begin{aligned} \partial_\sigma X &= u_{\rm o} X \qquad \text{at } \sigma = 0\,, \\ \partial_\sigma X &= -v_{\rm o} X \qquad \text{at } \sigma = \pi\,. \end{aligned} \tag{5.4}$$

The solution is

$$X = i \sum_{n \neq 0} \alpha_{\epsilon_n} \chi_{\epsilon_n}(\tau, \sigma)\,, \tag{5.5}$$

where

$$\chi_{\epsilon_n} = \frac{|c_n|}{\epsilon_n} \cos\left[\epsilon_n \sigma - \tan^{-1}(u_{\rm o}/\epsilon_n)\right] e^{-i\epsilon_n \tau}\,, \tag{5.6}$$

and ϵ_n is the n-th root of the equation

$$e^{2i(\tan^{-1}(u_{\rm o}/\epsilon_n) + \tan^{-1}(v_{\rm o}/\epsilon_n))} = e^{2\pi i \epsilon_n}\,. \tag{5.7}$$

There is a countably infinite number of such solutions satisfying $\epsilon_{-n} = -\epsilon_n$. In the above the normalisation constant $c_n = c_{-n}$ is

$$\frac{1}{c_n^2} = \frac{u_{\rm o} + v_{\rm o}}{\pi} \frac{\epsilon_n^2 + u_{\rm o} v_{\rm o}}{(u_{\rm o}^2 + \epsilon_n^2)(v_{\rm o}^2 + \epsilon_n^2)} + 1\,. \tag{5.8}$$

The mode functions then satisfy the orthogonality relation

$$\int_0^\pi \frac{d\sigma}{\pi} \bar{\chi}_{\epsilon_m}(\tau, \sigma)(i \overleftrightarrow{\partial_\tau}) \chi_{\epsilon_n}(\tau, \sigma) = \frac{1}{|\epsilon_n|} \delta_{mn}\,, \tag{5.9}$$

where $\phi \overleftrightarrow{\partial_\tau} \psi \equiv \phi \partial_\tau \psi - \psi \partial_\tau \phi$. The canonical momentum $P(\tau, \sigma)$ is defined in the usual way

$$P(\tau, \sigma) = \frac{\partial \mathcal{L}}{\partial(\partial_\tau X)} = \frac{1}{\pi} \partial_\tau X(\tau, \sigma)\,. \tag{5.10}$$

Inverting the expression for X we have

$$\alpha_{\epsilon_n} = \epsilon_n \int_0^\pi \frac{d\sigma}{\pi} \bar{\chi}_{\epsilon_n} (P + \frac{i}{\pi} \epsilon_n X)\,, \tag{5.11}$$

and given the canonical commutation relations

$$[X(\tau,\sigma), X(\tau,\sigma')] = 0\,, \qquad [P(\tau,\sigma), P(\tau,\sigma')] = 0\,, \qquad [X(\tau,\sigma), P(\tau,\sigma')] = i\delta(\sigma-\sigma')\,, \tag{5.12}$$

we find that the Fourier modes satisfy the commutation relations

$$[\alpha_{\epsilon_n}, \alpha_{\epsilon_m}] = \epsilon_m \delta_{m,-n}\,. \tag{5.13}$$

The Hamiltonian is

$$\begin{aligned} H_o^{\rm bos} &= \frac{1}{2\pi}\int_0^\pi d\sigma(\partial_\tau^2+\partial_\sigma^2)X(\tau,\sigma) + u_{\rm o}X^2(\tau,\sigma)\delta(\sigma) + v_{\rm o}X^2(\tau,\sigma)\delta(\sigma-\pi) \\ &= \frac{1}{2}\sum_{n=1}^{\infty}\alpha_{\epsilon_{-n}}\alpha_{\epsilon_n} + c(u_{\rm o},v_{\rm o})\,. \end{aligned} \tag{5.14}$$

In the above $c(u_{\rm o},v_{\rm o})$ is the normal ordering constant written formally as

$$c(u_{\rm o},v_{\rm o}) = \frac{1}{2}\sum_{n=1}^{\infty}\epsilon_n\,, \tag{5.15}$$

which needs to be regularised. A way to compute the regularised c was recently suggested in [27] for the case when the tachyons on the two boundaries are the same. Below we generalise slightly this computation for the case of distinct tachyons. Consider

$$\phi(z) = e^{i\pi z}\frac{z-iu_{\rm o}}{z+iu_{\rm o}} - e^{-i\pi z}\frac{z+iv_{\rm o}}{z-iv_{\rm o}}\,. \tag{5.16}$$

This function has zeros at $z=\epsilon_n$ and is well defined for all values of the tachyons except at the poles $z=-iu_{\rm o}\,, iv_{\rm o}$.[14] Define

$$I = \frac{1}{4\pi i}\oint z e^{-\delta z}{\rm d}\ln\phi\,, \tag{5.17}$$

where the contour encloses the positive real line and therefore the integral is equal to $c(u_{\rm o},v_{\rm o})$ when the regularisation parameter (chosen to have an imaginary part) $\delta\to 0$. Now we open up the contour making it run along the imaginary axis avoiding the two poles at $z=-iu_{\rm o}\,, iv_{\rm o}$ as in Figure 1. The integral reduces to

$$\begin{aligned} I &= \frac{1}{2\pi}\int_0^\infty \ln\left(1-e^{-2\pi x}\frac{x-u_{\rm o}}{x+u_{\rm o}}\frac{x-v_{\rm o}}{x+v_{\rm o}}\right){\rm d}(x\cos(\delta x)) - \frac{1}{2}\int_0^\infty x\cos(\delta x){\rm d}x \\ &\quad - \frac{1}{4\pi}\int_0^\infty x e^{-i\delta x}\left(\frac{1}{x+v_{\rm o}}+\frac{1}{x+u_{\rm o}}\right){\rm d}x + J(u_{\rm o}) + J(v_{\rm o})\,, \end{aligned} \tag{5.18}$$

[14] ϕ is different from the one used in [27]. However as we will see, this does not change the answer for the integral I.

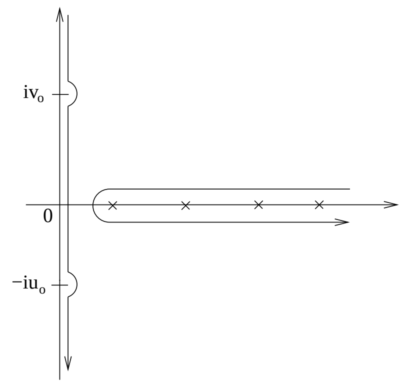

Figure 1: Change of the integration contour for I. The ϵ_i are denoted by crosses.

where we have separated out the terms $J(u_o)\,,J(v_o)$ of ϕ which have poles on the imaginary axis and are defined as

$$J(v_o) = -\frac{i}{4\pi}\int_C \frac{ze^{-\delta z}}{z - iv_o}\mathrm{d}z\,. \tag{5.19}$$

Here C is a contour consisting of three parts: $0 \leq z \leq i(v_o - \epsilon)$ for C_1, C_2 is a small semi-circle of radius ϵ around iv_o and for C_3, $i(v+\epsilon) \leq z \leq \infty$. In the limit $\epsilon \to 0$ we obtain

$$J(v_o) = -\frac{i}{4\pi\delta} - \frac{v_o}{4\pi}e^{-i\delta v_o}\mathrm{Ei}(i\delta v_o)\,, \tag{5.20}$$

where $\mathrm{Ei}(z)$ is the exponential integral function. The original integral now becomes

$$\begin{aligned} I \;\; = \;\; & \frac{1}{2\pi}\int_0^\infty \ln\left(1 - e^{-2\pi x}\frac{x-u_o}{x+u_o}\frac{x-v_o}{x+v_o}\right)\mathrm{d}(x\cos(\delta x)) \\ & + \frac{1}{2\delta^2} + \frac{1}{4\pi}\left(\frac{i}{\delta} + v_o e^{i\delta v_o}\Gamma(0, i\delta v_o)\right) - \frac{1}{4\pi}\left(\frac{i}{\delta} + v_o e^{-i\delta v_o}\mathrm{Ei}(i\delta v_o)\right) \\ & + \frac{1}{4\pi}\left(\frac{i}{\delta} + u_o e^{i\delta u_o}\Gamma(0, i\delta u_o)\right) - \frac{1}{4\pi}\left(\frac{i}{\delta} + u_o e^{-i\delta u_o}\mathrm{Ei}(i\delta u_o)\right)\,, \end{aligned} \tag{5.21}$$

where $\Gamma(x,y)$ is the incomplete Gamma function. In the limit $\delta \to 0$ we find

$$\begin{aligned} I \;\; \to \;\; & \frac{1}{2\delta^2} - \frac{u_o + v_o}{2\pi}\ln(i\delta) - \frac{1}{2\pi}\left(\gamma(u_o + v_o) + u_o \ln u_o + v_o \ln v_o\right) \\ & + \frac{1}{2\pi}\int_0^\infty \mathrm{d}x \ln\left(1 - e^{-2\pi x}\frac{x-u_o}{x+u_o}\frac{x-v_o}{x+v_o}\right)\,. \end{aligned} \tag{5.22}$$

For $v_o = u_o$ this expression agrees with the one obtained in [27]. The regularised normal ordering constant is

$$c(u_o, v_o) \;\; = \;\; \frac{1}{2\pi}\int_0^\infty \mathrm{d}x \ln\left(1 - e^{-2\pi x}\frac{x-u_o}{x+u_o}\frac{x-v_o}{x+v_o}\right)$$

$$- \frac{1}{2\pi}\left(\gamma(u_{\rm o}+v_{\rm o}) + u_{\rm o}\ln u_{\rm o} + v_{\rm o}\ln v_{\rm o}\right) . \tag{5.23}$$

The integral above reproduces the NN and DD normal ordering constants ($-\frac{1}{24}$) corresponding to $u_{\rm o} = v_{\rm o} = 0\,,\infty$, respectively. Further, it also reproduces the normal ordering constant of an ND string ($\frac{1}{48}$) obtained by taking $u_{\rm o} = 0\,, v_{\rm o} = \infty$. The terms divergent when the tachyons go to infinity will cancel with terms coming from the fermion normal ordering constant.[15]

Returning to the computation of the annulus diagram in the open string channel it is not difficult to compute the partition function for a single boson with boundary conditions (5.4)

$$Z_{\rm bosonic}(u_{\rm o},v_{\rm o}) = {\rm Tr}(e^{-2\pi t H_o^{\rm bos}}) = \tilde{q}^{2c(u_{\rm o},v_{\rm o})}\prod_{n=1}^{\infty}(1-\tilde{q}^{2\epsilon_n})^{-1} , \tag{5.24}$$

with $\tilde{q} = e^{-\pi t}$ and $t = \pi/l$.

A similar analysis has been carried out for the fermions. In the R sector these have the same moding as the bosons; canonically quantised they satisfy the anti-commutation relations

$$\{\psi_{\epsilon_n},\psi_{\epsilon_n}\} = \delta_{m,-n} , \tag{5.25}$$

and have the Hamiltonian

$$H_{\rm o}^{\rm R} = \frac{1}{2}\sum_{n=1}^{\infty}\psi_{\epsilon-n}\psi_{\epsilon n} - c(u_{\rm o},v_{\rm o}) . \tag{5.26}$$

In the NS sector the moding is different with the ϵ_r now satisfying

$$e^{2i(\tan^{-1}(u_{\rm o}/\epsilon_r)+\tan^{-1}(v_{\rm o}/\epsilon_r))} = -e^{2\pi i\epsilon_r} . \tag{5.27}$$

The anti-commutation relations and Hamiltonian are

$$\{\psi_{\epsilon_r},\psi_{\epsilon_s}\} = \delta_{r,-s} , \qquad H_{\rm o}^{\rm NS} = \frac{1}{2}\sum_{r=1}^{\infty}\psi_{\epsilon-r}\psi_{\epsilon_r} - c_{\rm NS}(u_{\rm o},v_{\rm o}) , \tag{5.28}$$

where in the NS sector the regularised normal ordering constant is

$$\begin{aligned} c_{\rm NS}(u_{\rm o},v_{\rm o}) &= \frac{1}{2\pi}\int_0^{\infty}dx\ln\left(1+e^{-2\pi x}\frac{x-u_{\rm o}}{x+u_{\rm o}}\frac{x-v_{\rm o}}{x+v_{\rm o}}\right) \\ &\quad - \frac{1}{2\pi}\left(\gamma(u_{\rm o}+v_{\rm o}) + u_{\rm o}\ln u_{\rm o} + v_{\rm o}\ln v_{\rm o}\right) . \end{aligned} \tag{5.29}$$

The partition function for a NS fermion is

$$Z_{\rm NS}(u_{\rm o},v_{\rm o}) = {\rm Tr}_{\rm NS}(e^{-2\pi t H_o^{\rm NS}}) = \tilde{q}^{-2c_{\rm NS}(u_{\rm o},v_{\rm o})}\prod_{r>0}^{\infty}(1+\tilde{q}^{2\epsilon_r}) , \tag{5.30}$$

[15] In the bosonic case these terms diverge and should match the corresponding divergences of the partition function in the closed string channel. We thank A. Konechny for a discussion on this point.

while that of an R fermion is

$$Z_{\rm R}(u_{\rm o},v_{\rm o}) = {\rm Tr}_{\rm R}(e^{-2\pi t H_o^{\rm R}}) = \tilde{q}^{-2c(u_{\rm o},v_{\rm o})}\prod_{n=1}^{\infty}(1+\tilde{q}^{2\epsilon_n})\,. \tag{5.31}$$

It is clear by construction and inspection that the coefficients of $\tilde{q}$ are integral. This is expected of an open string partition function.

Combining equations (5.24), (5.30) and (5.31) the partition function for open strings on a non-BPS D-brane in the presence of background tachyons is

$$Z_{\rm open} = \int_0^\infty \frac{{\rm d}t}{2t}{\rm Tr}_{\text{NS-R}}(e^{-2\pi t H_{\rm o}}) = \frac{V_9}{(2\pi)^9}\int_0^\infty \frac{{\rm d}t}{2t}(2t)^{-9/2}\frac{f_3^7(\tilde{q})g_3^{(u_{\rm o},v_{\rm o})}(\tilde{q}) - f_2^7(\tilde{q})g_2^{(u_{\rm o},v_{\rm o})}(\tilde{q})}{f_1^7(\tilde{q})g_1^{(u_{\rm o},v_{\rm o})}(\tilde{q})}\,, \tag{5.32}$$

where V_9 is the volume of space-time with no background tachyon, and the g functions are defined as

$$g_1^{(u_{\rm o},v_{\rm o})}(\tilde{q}) = \tilde{q}^{-2c(u_{\rm o},v_{\rm o})}\prod_{r>0}^{\infty}(1-\tilde{q}^{2\epsilon_n})\,, \qquad g_2^{(u_{\rm o},v_{\rm o})}(\tilde{q}) = \tilde{q}^{-2c(u_{\rm o},v_{\rm o})}\prod_{r>0}^{\infty}(1+\tilde{q}^{2\epsilon_n})\,, \tag{5.33}$$

$$g_3^{(u_{\rm o},v_{\rm o})}(\tilde{q}) = \tilde{q}^{-2c_{\rm NS}(u_{\rm o},v_{\rm o})}\prod_{r>0}^{\infty}(1+\tilde{q}^{2\epsilon_r})\,, \qquad g_4^{(u_{\rm o},v_{\rm o})}(\tilde{q}) = \tilde{q}^{-2c_{\rm NS}(u_{\rm o},v_{\rm o})}\prod_{r>0}^{\infty}(1-\tilde{q}^{2\epsilon_r})\,. \tag{5.34}$$

It is easy to see that in equation (5.32) the terms divergent for $u_{\rm o}\,,v_{\rm o}\to\infty$ in the normal ordering constants of bosons and fermions cancel.

In this section we have computed the partition function on the annulus by an operator method, slicing time in the periodic direction. In the previous section we used a different operator formalism with time running from one boundary of the annulus to the other. Since both approaches compute the same quantity we expect that equations (4.36) and (5.32) should give the same result. For conformal theories this is easily checked using the $t\to l$ transformation properties of the f_i functions. Unfortunately we do not know the corresponding transformations for the $g_i^{(u_{\rm o},v_{\rm o})}$ and $f_i^{(u_{\rm c},v_{\rm c})}$ functions and are, as a result, unable to verify this claim directly.

In the closed string channel discussed in Section 4 the R-R partition function gave a non-zero answer (for $u_{\rm c}\,,v_{\rm c}\neq 0$). This can be re-interpreted as the statement that there is a GSO-like projection acting on open strings in the presence of non-zero tachyons on the boundary. This projection should presumably be defined as a mod 2 number operator for world-sheet fermions just as in the case without background tachyon. This suggests a further contribution to the partition function of interest of the form

$$Z_{\rm open} = \int_0^\infty \frac{{\rm d}t}{2t}{\rm Tr}_{\text{NS-R}}(e^{-2\pi t H_{\rm o}}(-1)^F) = -\frac{V_9}{(2\pi)^9}\int_0^\infty \frac{{\rm d}t}{2t}(2t)^{-9/2}\frac{f_4^7(\tilde{q})g_4^{(u_{\rm o},v_{\rm o})}(\tilde{q})}{f_1^7(\tilde{q})g_1^{(u_{\rm o},v_{\rm o})}(\tilde{q})}\,, \tag{5.35}$$

where due to zero-modes the R sector trace is zero.

6 Conclusion

We have computed the partition function in the supersymmetric boundary sigma model on the annulus for the case of the exactly solvable linear tachyon profile. We showed how the one and the same answer can be achieved by means of different techniques: the Green's function method and the boundary state formalism, justifying thereby the latter for the case of non-conformal boundary deformations. An interesting feature of our results is that the R-R sector provides a non-trivial contribution to the partition function. If the interpretation of the annulus partition function as a one-loop correction to the space-time effective action S_{eff} is correct one may determine the corresponding change in S_{eff}. Taking the tachyon profile to be the same on the two boundaries one may expand the partition function around small u and interpret it as coming from an effective space-time action for the tachyon field T. In the R-R sector only zero-modes contribute to the leading order in u-expansion and one finds

$$S^{1-loop}_{\text{eff, R-R}} \sim \int \mathrm{d}^{10}x \int_0^1 \frac{\mathrm{d}a}{a} K(a) e^{-\frac{1}{4}(1+a)T^2} [\partial_\mu T \partial^\mu T + F^2_{\mu\nu} \partial^\mu T \partial^\nu T - \frac{1}{2} \partial_\mu T \partial^\mu T F_{\nu\rho} F^{\nu\rho} + \ldots] \,, \quad (6.36)$$

where we have indicated by dots the higher derivative terms and the mixing between the tachyon and the gauge field comes from the zero-modes as discussed in Section 3.3. In the above

$$K(a) = \frac{f_2^8(a)}{f_1^8(a)} \,. \quad (6.37)$$

Thus, the R-R sector contributes only to the derivative terms and not to the tree level potential. Similarly expanding the NS-NS partition function in u one finds its contribution to the effective action as

$$S^{1-loop}_{\text{eff, NS-NS}} \sim \int d^{10}x \int_0^1 \frac{da}{a} K(a) e^{-\frac{1}{4}(1+a)T^2} [1 + b(a) \partial_\mu T \partial^\mu T + \ldots] \,. \quad (6.38)$$

Here $b(a)$ is the next-to-leading term in the u-expansion of the NS-NS partition function.

The integral over a diverges due to the $a \to 1$ behaviour of the integrand. This arises from the open string tachyon and should be subtracted in a manner compatible with open/closed string duality. It is desirable to find such a subtraction scheme. This divergence indicates the instability of the $T = 0$ vacuum [39, 27]. In obtaining the partition function in the closed string channel we have summed over the spin structures, thus removing the closed string tachyon. This manifests itself in the fact that the a integral above has no linear divergences for $a \to 0$. However, there is a logarithmic divergence in the $a \to 0$ limit corresponding to a massless exchange. It is an interesting question whether this divergence (in the case of the superstring) can be treated using a Fischler-Susskind type mechanism [40].

We have also discussed the canonical quantization of the open string in the presence of a linear tachyon background. An important issue here is a generalization of the open/closed duality for this background. To compute the partition function in the open string channel one should know the normal ordering constant of the open string Hamiltonian that depends on the tachyon profile in a non-trivial way. Clearly the normal ordering constant is divergent and usually one picks up a subtraction scheme to define a finite quantity c. We discussed such a scheme (generalising the one in [27]) accounting for two different boundary tachyons. The part of c that is finite when u or v goes to infinity reproduces the normal ordering constants for the NN,DD and ND cases.

Note added. When our work was completed an interesting paper [41] appeared where the some issues related to the construction of the loop corrected tachyon potential were discussed.

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A Derivation of Green's functions and the partition function

In the Appendix we present the explicit expressions for the various Green's functions. We discuss in some detail the bosonic boundary conditions, their relation to Gauss's theorem and the transposition properties of the Green's function. Finally, we briefly outline how to obtain the partition function from the Green's function.

A.1 Green's functions

A.1.1 The bosonic sector

Consider the more general boundary conditions for the bosonic Green's function

$$\begin{aligned} (z\partial_z+\bar z\partial_{\bar z})G(z,w)+uG(z,w)+F(z\partial_z-\bar z\partial_{\bar z})G(z,w) &= D, \qquad |z|=1\,,\\ -(z\partial_z+\bar z\partial_{\bar z})G(z,w)+avG(z,w)+L(z\partial_z-\bar z\partial_{\bar z})G(z,w) &= E, \qquad |z|=a\,, \end{aligned} \tag{A.1}$$

where D and E are yet unknown matrices that may depend on the fields. Since $G_{\mu\nu}(z,w) = \langle X_\mu(z) X_\nu(w)\rangle$ the Green's function must satisfy

$$G_{\mu\nu}(z,w) = G_{\nu\mu}(w,z). \tag{A.2}$$

To find the Green's function corresponding to the boundary conditions (A.1) we make the ansatz

$$G(z,w) = G_f(z,w) + A + B\ln|z|^2\ln|w|^2 + C\ln|z|^2 + C^T\ln|w|^2 + \sum_{k\in\mathbf{Z}\setminus\{0\}}(\alpha_k(w)z^k + \bar{\alpha}_k(w)\bar{z}^k), \tag{A.3}$$

where $G_f(z,w) = -\ln|z-w|^2$ is the fiducial Green's function, obeying ($\alpha' = 2$)

$$-\frac{1}{2\pi}\partial_z\partial_{\bar{z}}G_f(z,w) = \delta^{(2)}(z,w). \tag{A.4}$$

Moreover we require that A, B and C are real matrices, satisfying

$$A = A^T, \qquad B = B^T. \tag{A.5}$$

The derivation of the oscillators is straightforward and one finds

$$\begin{aligned}
\alpha_k &= \frac{1}{k}\left(1 - a^{2k}S_k(u,F)S_k(av,L)\right)^{-1} S_k(u,F)\left(\bar{w}^k + a^{2k}w^{-k}S_k(av,L)\right),\\
\alpha_{-k} &= \frac{a^{2k}}{k}\left(1 - a^{2k}S_k^T(av,L)S_k^T(u,F)\right)^{-1} S_k^T(av,L)\left(\bar{w}^{-k} + w^k S_k^T(u,F)\right),\\
\tilde{\alpha}_k &= \frac{1}{k}\left(1 - a^{2k}S_k^T(u,F)S_k^T(av,L)\right)^{-1} S_k^T(u,F)\left(w^k + a^{2k}\bar{w}^{-k}S_k^T(av,L)\right),\\
\tilde{\alpha}_{-k} &= \frac{a^{2k}}{k}\left(1 - a^{2k}S_k(av,L)S_k(u,F)\right)^{-1} S_k(av,L)\left(w^{-k} + \bar{w}^k S_k(u,F)\right).
\end{aligned} \tag{A.6}$$

It is easy to see that the oscillator dependent parts of the Green's function indeed satisfy (A.2). For the zero-modes we find the conditions

$$-2 + 2C + uA - D + (2B + uC^T)\ln|w|^2 = 0\,, \quad |z| = 1\,, \tag{A.7}$$

$$-2C + av(A + 2C\ln a) - E + (-2B - av + 2avB\ln a + avC^T)\ln|w|^2 = 0\,, \quad |z| = a\,. \tag{A.8}$$

Taking into account (A.5), the cancellation of the $\ln|w|^2$ dependent terms in (A.7) and (A.8) requires that

$$B = -\frac{1}{2}Cu \quad \text{and} \quad uC^T = Cu \tag{A.9}$$

and C is explicitly found to be

$$C = av(u + av - auv\ln a)^{-1}. \tag{A.10}$$

From this expression one may check that C^T indeed satisfies the constraint $uC^T = Cu$. Solving (A.7) one finds

$$uA = (2(1-C)+D)\,, \qquad uD^T = Du \tag{A.11}$$

and (A.8) implies

$$Eu = avD^T\,. \tag{A.12}$$

Thus, the matrices D and E are related by (A.12) but not completely fixed by the boundary conditions. Since a non-vanishing D only results in a change of the overall normalisation of the (bosonic) partition function it is convenient to choose $D = 0 = E$. Then A is explicitly given by

$$A = 2(1 - av\ln a)(u + av - auv\ln a)^{-1}, \tag{A.13}$$

modulo elements in the kernel of u which we suppress for the same reasons as a non-vanishing D.

As a final check, we prove that Gauss's theorem is satisfied for any D. Since the Green's function satisfies

$$\Box G(\sigma_1,\sigma_2) = -4\pi\delta^{(2)}(\sigma_1,\sigma_2) \tag{A.14}$$

together with the boundary conditions (A.1), Gauss's theorem requires

$$-4\pi = \int_{\partial\Sigma}\partial_r G ds = -u\int_{|z|=1} d\phi\, G(z,w) - av\int_{|z|=a} d\phi\, G(z,w) + 2\pi(D+E)\,. \tag{A.15}$$

Only the integrals over the zero-modes will be non-vanishing and, therefore we confirm that

$$0 = 4\pi - 2\pi(uA + avA + 2avC\ln a - D - E + \ln|w|^2(uC^T - av + 2avB\ln a + avC^T))\,. \tag{A.16}$$

A.1.2 The NS-NS sector

Here we present the explicit expressions for the oscillators of the various Green's functions in the NS-NS sector. For example, the G_{++} oscillators are

$$\begin{aligned}
\psi_r(w) &= i\eta\tilde\eta a^{2r}\left(1-\eta\tilde\eta a^{2r}S_r(u,F)S_r(av,L)\right)^{-1}S_r(u,F)S_r(av,L)w^{-r}\,,\\
\psi_{-r}(w) &= -i\eta\tilde\eta a^{2r}\left(1-\eta\tilde\eta a^{2r}S_r^T(av,L)S_r^T(u,F)\right)^{-1}S_r^T(av,L)S_r^T(u,F)w^{r}\,.
\end{aligned} \tag{A.17}$$

Similarly

$$\begin{aligned}
\tilde\psi_r(\bar w) &= i\eta\tilde\eta a^{2r}\left(1-\eta\tilde\eta a^{2r}S_r^T(u,F)S_r^T(av,L)\right)^{-1}S_r^T(u,F)S_r^T(av,L)\bar w^{-r}\,,\\
\tilde\psi_{-r}(\bar w) &= -i\eta\tilde\eta a^{2r}\left(1-\eta\tilde\eta a^{2r}S_r(av,L)S_r(u,F)\right)^{-1}S_r(av,L)S_r(u,F)\bar w^{r}\,,
\end{aligned} \tag{A.18}$$

$$\begin{aligned}
a_r(\bar w) &= \eta\left(1-\eta\tilde\eta a^{2r}S_r(u,F)S_r(av,L)\right)^{-1}S_r(u,F)\bar w^{r}\,,\\
a_{-r}(\bar w) &= -\tilde\eta a^{2r}\left(1-\eta\tilde\eta a^{2r}S_r^T(av,L)S_r^T(u,F)\right)^{-1}S_r^T(av,L)\bar w^{-r}\,,
\end{aligned} \tag{A.19}$$

and

$$\begin{aligned} b_r(w) &= -\eta \left(1 - \eta\tilde{\eta} a^{2r} S_r^T(u,F) S_r^T(av,L)\right)^{-1} S_r^T(u,F) w^r\,, \\ b_{-r}(w) &= \tilde{\eta} a^{2r} \left(1 - \eta\tilde{\eta} a^{2r} S_r(av,L) S_r(u,F)\right)^{-1} S_r(av,L) w^{-r}\,. \end{aligned} \tag{A.20}$$

A.1.3 The R-R sector

For the non-zero modes the oscillators are exactly the same as in the NS-NS sector, the only difference being that now r is an integer.

A.2 The partition function

In this sub-section we outline the derivation of the partition function in the closed channel following a technique used in [15]. For simplicity we restrict ourselves to the bosonic contribution, the derivation of the fermionic parts proceeds exactly along the same lines.

Differentiation of the (bosonic part of the) boundary action with respect to the couplings u and v results in the differential equations

$$\begin{aligned} \frac{\partial \ln Z}{\partial u^{\mu\nu}} &= -\frac{1}{8\pi}\int_0^{2\pi} d\phi \langle X_\mu(\phi) X_\nu(\phi)\rangle = -\frac{1}{8\pi}\int_0^{2\pi} d\phi\, G_{\mu\nu}(e^{i\phi}, e^{i\phi})\,, \\ \frac{\partial \ln Z}{\partial v^{\mu\nu}} &= -\frac{1}{8\pi}\int_0^{2\pi} d\phi\, a G_{\mu\nu}(ae^{i\phi}, ae^{i\phi}) \end{aligned} \tag{A.21}$$

and similar equations when differentiating with respect to F and L. Using the result for the bosonic Green's function it is not difficult to verify that the solution to these equations (up to the normalization) is

$$\begin{aligned} Z_{\rm bos} &= \det(u + av - auv \ln a)^{-1/2} \prod_{k=1}^{\infty} \det(1 + u/k + F)^{-1} \det(1 + av/k + L)^{-1} \\ &\quad \times \det\left(1 - a^{2k} S_k(u,F) S_k(v,L)\right)^{-1}\,. \end{aligned} \tag{A.22}$$

Z also satisfies the equations obtained by differentiating with respect to $F_{\mu\nu}$ and $L_{\mu\nu}$. In principle there may be an overall dependence on the modulus a in the partition function that could not be fixed by the previous considerations. However, one can also derive an equation for $\frac{\partial \ln Z}{\partial a}$ by looking at the change of the action under variations of the modulus [33]. Following [33] this equation reads

$$\begin{aligned} \partial_a \ln Z &= -\langle \partial_a S_{\rm bulk}\rangle - \langle \partial_a S_{\rm bndy}\rangle \\ &= \frac{1}{2\pi}\frac{a}{1-a^2}\int_\Sigma d^2 z \left(\frac{1}{\bar{z}^2}\langle T_{zz}\rangle + \frac{1}{z^2}\langle T_{\bar{z}\bar{z}}\rangle\right) - \frac{1}{8\pi}\int_0^{2\pi} d\phi\, v_{\mu\nu} G^{\mu\nu}(ae^{i\phi}, ae^{i\phi})\,. \end{aligned} \tag{A.23}$$

Using the explicit expression for the bosonic Green's function and

$$\langle T_{zz}(z)\rangle = -\frac{1}{2}\lim_{w\to z}\left[\partial_z\partial_w G^{\mu}_{\mu}(z,w) + \frac{10}{(z-w)^2}\right] \tag{A.24}$$

and similarly for $\langle T_{\bar{z}\bar{z}}\rangle$ we find that $\partial_a \ln Z$ integrates to (3.5), so that there is no further dependence on the modulus.

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