

LETTER TO THE EDITOR

Gravitational waves from black hole collisions via an eclectic approach

John Baker[†], Bernd Brügmann[†], Manuela Campanelli[†]
and Carlos O Lousto^{†‡}

[†] Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1,
D-14476 Golm, Germany

[‡] Instituto de Astronomía y Física del Espacio-CONICET, Buenos Aires, Argentina

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Abstract. We present the first results in a new program intended to make the best use of all available technologies to provide an effective understanding of waves from inspiralling black hole binaries in time to assist imminent observations. In particular, we address the problem of combining the close-limit approximation describing ringing black holes and full numerical relativity, required for essentially nonlinear interactions. We demonstrate the effectiveness of our approach using general methods for a model problem, the head-on collision of black holes. Our method allows a more direct physical understanding of these collisions indicating clearly when non-linear methods are important. The success of this method supports our expectation that this unified approach will be able to provide relevant results for black hole binaries in time to assist gravitational wave observations.

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(Some figures in this article are in colour only in the electronic version; see www.iop.org)

Binary black hole systems pose one of the most exciting and challenging problems of general relativity, constituting not only a fundamental gravitational two-body problem, but also an important astrophysical problem of direct and immediate observational relevance. Gravitational waves from binary black hole mergers are considered one of the most promising candidates for experimental detection by the first wave of large interferometric gravitational wave observatories coming on line over the next few years. These imminent observations present an urgent call to the theoretical relativity community to immediately provide any information possible about the radiation that might be expected from these collisions.

The problem divides physically into three phases. Initially, a slow adiabatic inspiral lasting until the black holes are so close that the orbital motion destabilizes, a brief period of strong, essentially non-linear two-body interaction, and the linear ring-down of the newly formed remnant black hole to stationarity. Correspondingly, theorists have approached the problem along three primary avenues: the post-Newtonian (PN) slow-motion approximation applicable in the inspiral phase, the ‘close-limit’ (CL) single perturbed black hole approximation handling the ring-down, and the full numerical simulation (FN) of Einstein’s equations, which could ideally handle the entire problem on a large computer, but is so far limited to brief evolutions on small 3D domains. Nevertheless, the full numerical approach should be vital to treating the intermediate, essentially non-linear phase.

In order to form the best theoretical model possible for the radiation from these systems we feel it is vital to combine these three approaches focusing the numerical simulations squarely

on the intermediate phase of the interaction where no perturbative approach is applicable. The state-of-the-art in these three fields has advanced to the point where we can expect such an eclectic approach to provide a reasonable model for binary black hole radiation. In the cases where it has been applied the close-limit model has proved to be a reliable model for radiation after the system has formed a common event horizon, and work in this field has advanced sufficiently so that arbitrary perturbations can now be calculated routinely [1]. In full numerical relativity, parts of the plunge of rather general black hole systems, such as the grazing collision of two black holes with linear momenta and spins, can be simulated [2]. And the post-Newtonian method has advanced to the point where it might be trusted even for black holes approaching the last stable orbit (LSO). Recent estimates [3] suggest that in the absence of spins there are 0.6 orbits left for full-numerical treatment, and numerical relativity should today be able to handle this part of the plunge (roughly $50M$ evolution time). The primary obstacle to proceeding with the combined model is the construction of appropriate interfaces between the three existing models. Recent interest within the post-Newtonian and gravitational wave research community in providing Cauchy data for simulations may soon lead to the development of a practical PN–FN interface [3]. In this letter we introduce a general approach to providing the FN–CL interface.

The nominal result of a numerical simulation of Einstein’s equations is a time succession of values on a 3D grid for the spatial metric and extrinsic curvature holding the geometric spacetime information. For binary black hole simulations we expect the late time behaviour of the system to be best characterized as a ‘ringing’ black hole with outgoing radiation, with perturbation theory providing a good model for the dynamics. The perturbative model not only allows an inexpensive continuation of the evolution, but also supplies a clear interpretation of the dynamics not manifest in the generic numerical simulation. The dynamics reduces to the evolution of a single complex field, the Newman–Penrose Weyl scalar $\psi_4 = C_{\alpha\beta\gamma\delta} n^\alpha \bar{m}^\beta n^\gamma \bar{m}^\delta$, obeying a linear hyperbolic equation and being directly related to the outgoing gravitational radiation. Because of the axisymmetry of the background Kerr black hole the problem can be further simplified by Fourier (in φ coordinate) decomposition of ψ_4 , reducing to a series of 2D evolution problems for the axial m -mode components of $\psi_4^m(t, r, \theta)$ evolving according to the Teukolsky equation [4].

Several important steps are required to concretely implement the FN–CL interface:

- (1) Specify the background black hole by its mass M and angular momentum $a = J/M^2$.
- (2) Construct a space-like slice from the late-time region of the numerical spacetime which will be mapped to a constant time slice in the perturbative calculation. In general this slice may not be identified with the numerical foliation.
- (3) Specify an embedding explicitly mapping the numerical slice to the corresponding slice in the background spacetime.
- (4) Specify a (null and complex) tetrad, $(l^\mu, n^\mu, m^\mu, \bar{m}^\mu)$, on the numerical slice which will map, on the background slice, to an approximation of the standard tetrad used in the perturbative formalism where l^μ and n^μ are conveniently chosen to lie along the two-degenerated principal null directions of the background spacetime.
- (5) Using the specified tetrad and the numerical data for the metric g_{ij} and the extrinsic curvature K_{ij} on the slice calculate ψ_4 and $\partial_t \psi_4$. These will provide the Cauchy data for the perturbative evolution.
- (6) Evolve with the time-domain Teukolsky equation to determine the subsequent perturbative dynamics.

The heart of the problem is making the specifications required in (2)–(4). The general idea is to numerically compute physical quantities or geometrical invariants and relate them

to their analytic expressions in the perturbatively preferred coordinate system. There is generally no unique way to make these specifications but the first order gauge and tetrad invariance of the perturbative formalism implies that the results will not depend strongly on small variations in these choices. Step (5) was explicitly worked out in [5]. Since a concrete implementation requires us to make choices for which there is no clear mathematical preference, we will proceed by trying first the simplest possible specifications and adding sophistication only when it seems to be necessary. We begin with a model binary black hole problem which has already been solved by 2D numerical relativity and close-limit perturbation theory, head-on collisions of initially resting equal-mass black holes (Misner initial data). At the same time, we will try not to tune our techniques too closely to this particular example so that our method can be readily generalized. For this reason we will perform our numerical evolutions generically in 3D, using well-tested, numerical techniques and codes (Cactus [6]) ‘off the shelf’ with no fine-tuning for this problem. We also apply perturbation theory as described by the Teukolsky equation, allowing for a rotating black hole background, without multipole decomposition. Specifically, for the numerical evolutions we have used the ADM system of Einstein’s equations with maximal slicing for the lapse and vanishing shift, finite differenced on a 128^3 (octant mode) numerical grid, initially mapped non-uniformly to the standard Misner coordinates to allow a distant outer boundary without sacrificing resolution in the inner region. We express the Teukolsky equation in Boyer–Lindquist coordinates, although it may be convenient in the future to evolve the perturbations in another gauge, such as a Kerr–Schild representation of the Kerr metric [7].

We implement the steps listed above as follows:

- (1) In this case there is no angular momentum so the background reduces to Schwarzschild, $a = 0$. Since only about 0.1% of the system’s mass will be lost as radiation we specify the background mass as equal to the initial ADM mass.
- (2) We make the simplest choice of background slice by identifying the numerical slice with a Schwarzschild time slice. Numerical experience with Schwarzschild black hole evolutions in maximal slicing suggest a strong correspondence.
- (3) For the embedding, it is clear that the trivial choice, identification of numerical and background coordinates is inadequate because the black hole horizon must invariably expand in this numerical gauge (we use vanishing shift). On the other hand the same expansion has the tendency to drive the exterior region toward manifest spherical symmetry. A reasonable estimate for the map into the background Schwarzschild slice is a trivial identification of the numerical and background θ and ϕ coordinates, with some relabelling of the constant- r spheres. We account for the radial rescaling by choosing the background radius r' so that the value of Weyl-curvature invariant $\mathcal{I} = \tilde{C}_{abcd}\tilde{C}^{abcd}$ averaged over θ in the numerical slice coincides with its background value $\mathcal{I} = 3M^2/r'^6$ in the background slice.
- (4) We define an appropriate, manifestly orthonormal, tetrad primarily by identifying timelike normal, radial, and azimuthal directions. The unit normal and radial direction vectors providing the spatial components of l^μ and n^μ . The complex vectors m^μ and \bar{m}^μ point within the spherical 2-surface. At each step, a Gram–Schmidt procedure is first used to ensure that the triad remains orthonormal. Then a type III (boost) null rotation fixes the relative normalization of the two real-valued vectors to make it consistent with the tetrad assumed in the perturbative calculation.
- (5) Within the full numerical simulation we compute the values ψ_4 and $\partial_t\psi_4$ consistent with our tetrad specification using the formulas in [5].

- (6) Interpolate these Cauchy data (using splines) to generate data directly usable by the Teukolsky code developed in [8]. For the perturbative evolutions we use the tortoise coordinate $r^* = r + 2M \ln(r/2M - 1)$ in the range $-20 < r^*/M < 50$ with $n_\theta \times n_{r^*} = 48 \times 700$.

We evolved Einstein's equations numerically from Misner initial data for several different initial separations labelled by the proper distance L as shown in figure 1. A typical duration of the total evolution was $t = 10M$ and we extracted Cauchy data every $t = 1M$. A transition time t_T was determined by methods detailed below. After each Teukolsky code evolution we extract the full relevant signal of the waveforms, which typically lasted for $t < 100M$. The resulting radiation energies are shown in figure 1 where we compare our 3D results with the results of [9] where explicit use of the symmetries of the problem have been implemented in a 2D simulation. The other case for comparison is the Price–Pullin [10] curve providing the pure close-limit result. While all three predictions agree very well for small initial proper separations $L/M < 3$, it is clear that for larger separations the close limit and full numerical curves deviate considerably. Our results follow quite precisely the 2D computations. A minimal full numerical evolution time (given by our linearization time below) is essential in obtaining the above agreement. Evolution of exact initial data only perturbatively does not reproduce the full numerical results for large separations, but follows the Price–Pullin curve [11].

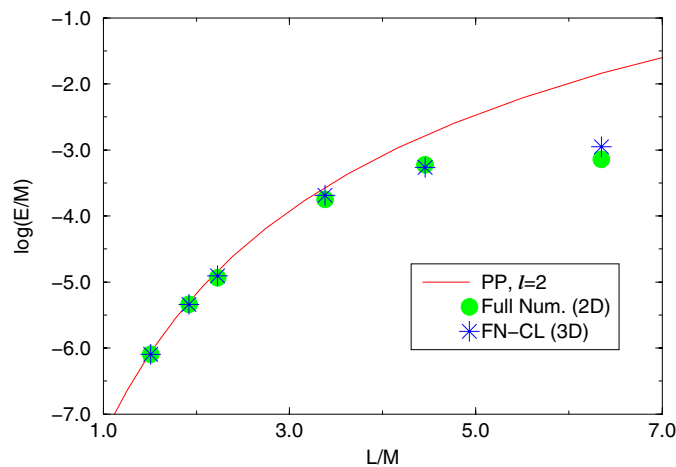


Figure 1. The total radiated energy from two black holes initially at rest (Misner initial data). The solid line represents Price–Pullin results [10] (labelled as PP, $\ell = 2$) for the close limit approximation. 2D full numerical results [9] are here given by full circles and the new FN–CL results are labelled with stars. The minimal amount of evolution needed for ($L \approx 3.4, 4.5, 6.4$) to reach the perturbative regime are approximately $1M, 3M, 6M$ respectively.

Extracting waveforms every $1M$ of non-linear numerical evolution allows us to study the transition to linear dynamics, and to perform important consistency tests on our results. If we have made a good definition of the perturbative background in steps (1)–(4) above then we can expect our radiation waveform results to be independent of the transition time, t_T , once the linear regime is reached and for as long as the numerical simulation continues to be accurate. We apply two independent criteria for estimating the onset of linear dynamics, the speciality invariant prediction based only on the Cauchy data and another estimate based on the stability of the radiation waveform phase. The speciality invariant introduced in [12]

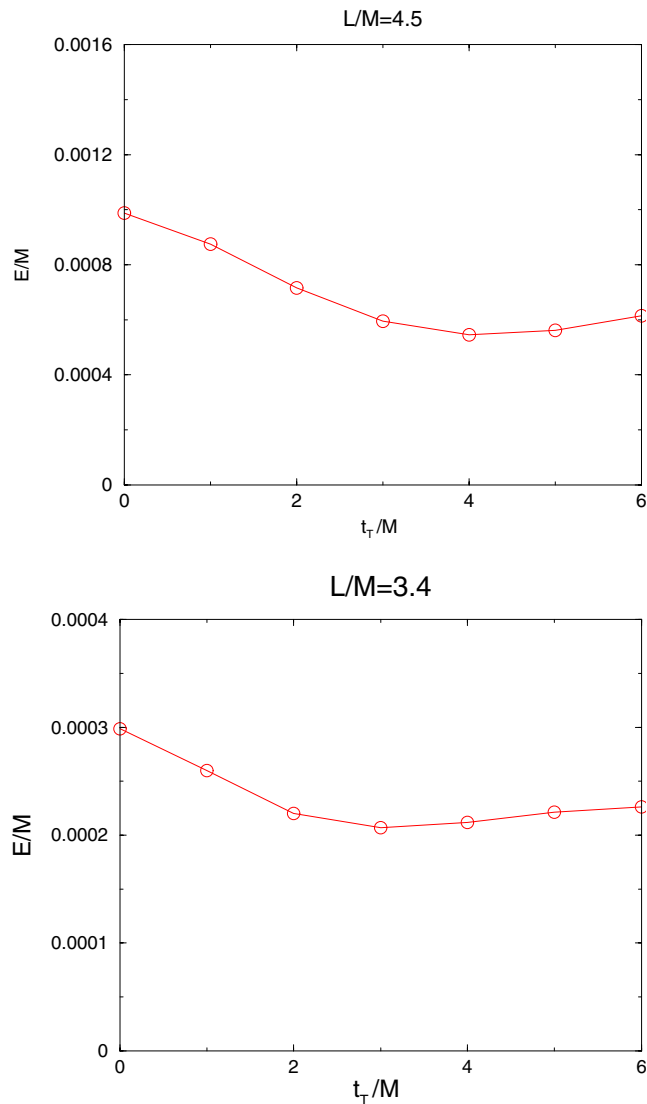


Figure 2. Radiated energy versus transition time. These figures show a clear plateau after linearization until numerical error begins to cause problems after $6M$.

predicts linear dynamics when $S = 27\mathcal{J}^2/\mathcal{I}^3$ differs from its background value of unity by less than a factor of two outside the (background) horizon, implying that not significant distortion is left outside the potential barrier. Such a deviation from algebraic speciality implies significant ‘second order’ perturbations. The phase of the waveform also provides an indicator of linear dynamics. Starting with detached black holes, we expect an initial period of weak bremsstrahlung radiation followed by the appearance of quasinormal ringing. On the other hand, switching to perturbative evolution immediately leads to prematurely ringing. Hence we first observe a series of phase delays for the beginning of the ringing until the actual ringing takes place, thereafter no phase shift should be seen. The value of t_T when the phase freezes gives a precise estimate of time for linearization of the system. We find that both estimates

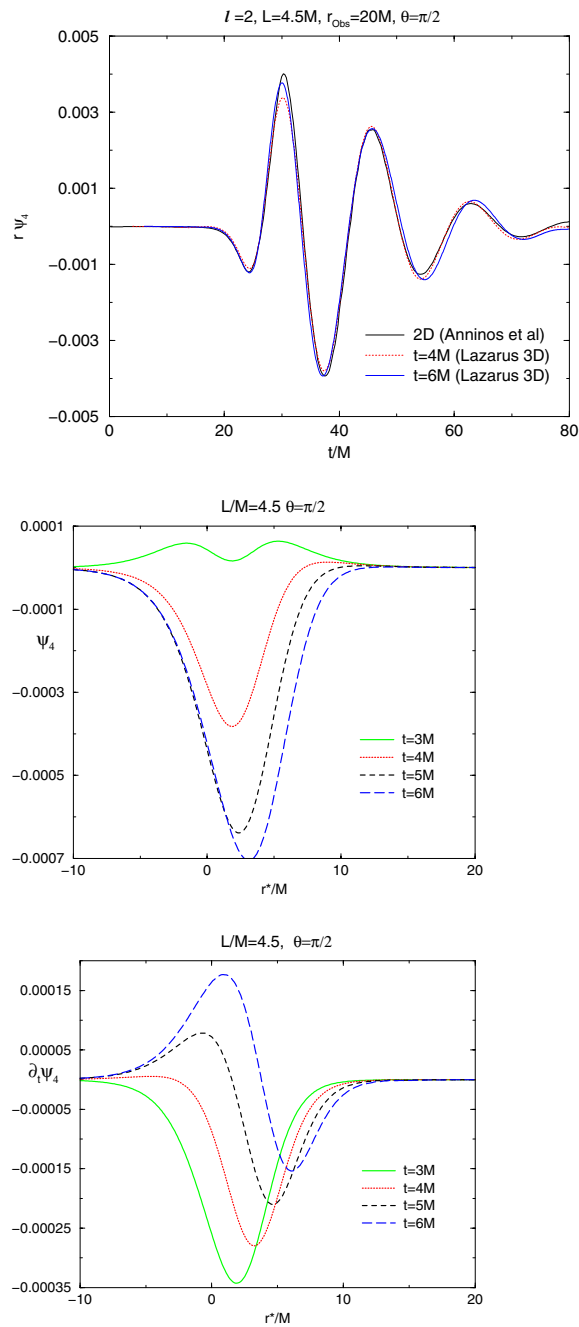


Figure 3. Waveforms (above) and Cauchy data (below) for initial separation $L/M = 4.5$. The observer location is $r^* = 20M$ and $\theta = \pi/2$. The data have been extracted after $t = 4M$ and $5M$ of full 3D nonlinear evolution respectively. There are evident differences in the extracted Cauchy data at the times considered because of the intervening evolution. Nevertheless the resulting waveforms agree, demonstrating the equivalence of the linear and non-linear evolution through this period, as well as the robustness of our methods.

for linearization time are in good agreement, yielding that $t < 1M$ for the $L/M = 2.2$ case, $t \approx 1M$ for $L/M = 3.4$, $3M$ for $L/M = 4.5$, and $6M$ for $L/M = 6.4$. The linearization time is somewhat longer than the ‘ringing times’ reported in figure 7 of [9] indicating that linearization occurs slightly after the onset of ‘ringing’ for the stronger collisions. Our linearization times are still much shorter than those for the appearance of a common apparent horizon and should be closer to the formation of a common *event* horizon.

Two example curves of energy versus t_T are shown in figure 2. Before the linearization time the premature application of perturbation theory tends to result in an overestimate of the energy. After linearization there is a plateau region when the energy is insensitive to t_T as is required if we have defined a useful FN–CL interface. Eventually, after $6M$ in these cases, numerical errors caused by the ‘grid-stretching’ inherent in the use of maximal slicing tend to result again in an overestimate of the energy. A stronger indication of the robustness of our method is evident in the waveforms themselves. After linearization, the waveforms should also be independent of t_T . Figure 3 shows an example of this comparing $t_T = 4M$ and $5M$ for the $L/M = 4.5$ case. Despite the fact that the Cauchy data at transition time is very different, the waveform is almost identical. The waveform quite agrees (apart from the reversed sign) with the ψ_4 published in [9], figure 13. It is worth noting here that our waveforms for the Misner data seem to be the first complete ones computed using 3D full numerical relativity.

Perturbation theory is very useful to gain information about waveforms from numerical spacetimes. Customarily this interface has been implemented only on a time-like surface to determine radiation content ‘far-away’ from the black hole. A much more natural boundary between the linear and non-linear regimes occurs on a spacelike interface defined by the time beyond which non-linear black hole perturbations no longer contribute significantly to the radiation. We have taken a general approach to the problem of providing such a FN–CL interface which we believe is essential to providing timely estimates of binary black hole waveforms. We are aware of only one previous attempt to make a combined use of numerical and close-limit evolution implemented in the case of two black holes formed by collapsing matter [13], using a 2D code and $l = 2$ metric perturbations (*à la* Zerilli) of the Schwarzschild background. Our method aims toward complete generality using full 3D numerical simulations and applying perturbation theory as described by the Teukolsky equation, applicable to arbitrary remnant black hole backgrounds. This approach is directly applicable to a unified eclectic model of colliding black holes joining the close-limit, full numerical relativity, and post-Newtonian methods. To our knowledge this is the first time such an approach has been proposed and turned into a concrete and generic scheme. The success in this test case encourages our hopes for providing theoretical results on black hole merger waveforms in time to assist the first gravitational wave observations. We will direct our future work toward a fully combined PN–FN–CL model for estimating astrophysically relevant binary black hole collision waveforms.

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