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On the M-theory approach to (compactified) 5D field theories [1]

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Abstract

We construct M-theory curves associated with brane configurations of $SU(N)$, $SO(N)$ and $Sp(2N)$ 5D supersymmetric gauge theories compactified on a circle. From the curves we can account for all the existing different $SU(N)$ field theories with $N_f \leq 2N$. This is the correct bound for $N \geq 3$. We remark on the exceptional case $SU(2)$. The bounds obtained for $SO(N)$ and $Sp(2N)$ are $N_f \leq N - 4$ and $N_f \leq 2N + 4$, respectively. © 1997 Elsevier Science B.V.

1. Introduction

Recently many interesting result in field theory and string theories were obtained using branes in superstring theories. In particular, brane configurations based on the construction of Hanany and Witten [1] led to realizations of various aspects of SUSY gauge theories. In this note we follow [2] (see also [3]) and focus on the brane configurations which are relevant to five dimensional $N = 1$ supersymmetric gauge theories and five dimensional theories compactified on a circle.

We briefly review the brane construction of 5D gauge theories and then discuss 5D theories compactified on a circle. As in the 4D case [4] it is possible to describe these theories using a smooth brane configuration in M theory given by a holomorphic curve [5]. The curves for $SU(N)$ theories can be determined using symmetries, periodicity and requiring their 4D limits $R_B \to 0$ to be the Seiberg-Witten curves. For $N > 2$ we find agreement with field theory results [6]. In particular, we find the same bound on N_f for consistent theories and are able to identify for fixed N and N_f the different theories which are classified in the field theory picture by the coefficient c_{cl} of the Chern-Simons term in the prepotential [6]. Also in the M-theory approach one does not find the field theories with $N_f = 5,6,7,8$ for $N = 2$. The curves we find can (for some values of c_{cl}) be identified with the spectral curves of integral models which were conjectured in Ref. [7] to be relevant for the non-perturbative solution of 5D theo-

ries compactified on a circle. We also discuss the generalizations to $SO(N)$ and $Sp(2N)$ gauge groups.

2. 5D field theories and type IIB description

We begin this section with a brief review of some of the results for five dimensional gauge theories with eight supercharges and simple gauge group. This analysis was initiated by Seiberg [8] and generalized in [9,6,10]. The two possible multiplets are: the vector multiplet with a real scalar in the adjoint representation of the gauge group G, denoted by ϕ^a, and a set of hypermultiplets. We will only discuss matter in the fundamental representation. This might be a limitation of the brane set-up, but not of the field theory. The Coulomb branch is parameterized by the scalars ϕ^i in the Cartan subalgebra of G, $i = 1 \ldots r = \mathrm{rank}(G)$. The moduli space is $\mathbf{R}^r / \mathscr{W}(G)$, where $\mathscr{W}(G)$ is the Weyl group of G. An important quantity is the quantum prepotential which is of the general form

$$\mathscr{F} = \frac{1}{2g_0^2} \phi^i \phi^i + \frac{c_{\mathrm{cl}}}{6} d_{ijk} \phi^i \phi^j \phi^k$$
$$+ \frac{1}{12} \left(\sum_\alpha |\alpha_i \phi^i|^3 - \sum_{l=1}^{N_f} \sum_w |w_i \phi^i + m_l|^3 \right). \tag{1}$$

The sums are over the roots and fundamental weights, respectively. The necessary condition for the existence of a non-trivial UV fixed point (in the limit $g_0 \to \infty$) is that the prepotential be a convex function over the Coulomb branch. Note that the third rank symmetric tensor d_{ijk} only exist for $SU(N)$ with $N > 2$. In all other cases $c_{\mathrm{cl}} = 0$. We now summarize the relevant results of [6]: For $SU(N)$ there is a quantization condition $c_{\mathrm{cl}} + N_f/2 \in \mathbf{Z}$ and only $N_f + 2|c_{\mathrm{cl}}| \leq 2N$ are allowed. By integrating out massive matter an effective $c_{\mathrm{eff}} = c_{\mathrm{cl}} - (n_+ - n_-)/2$ is generated where $n_+(n_-)$ denotes the number of hypermultiplets whose mass m_l was sent to $+\infty(-\infty)$. For $SO(N)$ ($Sp(2N)$) we have $c_{\mathrm{cl}} = 0$ and the condition is $N_f \leq N - 4$ ($N_f \leq 2N + 4$).

In [7] supersymmetric 5D gauge theories compactified on a circle of radius R were studied. The contributions to the perturbative prepotential from

Kaluza-Klein states was found, which exhibits the correct limits at small and large radius of the fifth dimension. The non-perturbative corrections were conjectured to be encoded in the spectral curves of relativistic Toda systems.

The brane description of $N = 1$ 5D gauge theories [2] is related to the configurations in [1] by T-duality. The world-volume of the NS 5-branes spans the x^0, x^1, x^2, x^3, x^4 and x^5 directions and the D5 branes are along the x^0, x^1, x^2, x^3, x^4 and x^6 directions. The coordinates x^0, x^1, x^2, x^3, x^4 which are common to the NS 5-branes and D5 branes are the coordinates of the 5D field theory. Actually, as pointed out in Ref. [1], the naive configurations obtained by T-duality should be reconsidered taking charge conservation into account. In addition to NS and D five-branes, there are, more generally, (n,m) five-branes with tension

$$T_5^{(n,m)} = \frac{1}{(2\pi)^5 \alpha'^3} \sqrt{\frac{n^2}{\lambda^4} + \frac{m^2}{\lambda^2}}, \tag{2}$$

n is the NS charge and m the R charge; λ is the IIB string coupling constant. [2] Charge conservation implies that when a $(0,m)$ brane ends on a $(n,0)$ brane, a (n,m) brane is formed. The zero force condition implies that it satisfies [2,5] $\left| \frac{\Delta x^6}{\Delta x^5} \right| = \left| \frac{m\lambda}{n} \right|$. This (n,m) brane with this orientation in the (x^5, x^6) plane does not break the supersymmetry any further.

The brane configuration is expected to be related to the 5D gauge theories when $\lambda \ll 1$. Then (n,m) branes with $n \neq 0$ are much heavier then the D5 fivebranes, hence they can be treated as a classical background.

3. M theory description

The M-theory origin of the type IIB 5D theories was discussed in [5]. In the usual type IIA picture we start with NS branes (012345) corresponding to unwrapped M-theory fivebranes (M5) and D4-branes (01236) corresponding to M5-branes wrapped on

x^{10}. To obtain the description of the type IIB 5D theories we further compactify the x^4 direction on a circle of radius R_A. Thus we consider M-theory on a torus. Type IIB string theory is obtained in the limit of a zero area torus $R_A R_{10} \to 0$ while keeping the string coupling $\lambda \sim \frac{R_{10}}{R_A}$ fixed. In view of the relation [11] (l_{11} is the 11-dimensional Planck length) $R_B \sim \frac{l_{11}^3}{R_{10} R_A}$, $R_B \to \infty$ in this limit; it becomes the new tenth type IIB dimension. Note that in effect we are performing here the usual T-duality transformation with $R_B \sim \alpha'/R_A$ and $\alpha' \sim l_{11}^3/R_{10}$.

In the M-theory description there is a $SL(2,\mathbf{Z})$ acting on the complex structure of the two-torus on which we compactify and thus also on the doublet of winding numbers, (p,q), of the M5 brane which wraps around the torus. This involves the coordinates x^4 and x^{10}. The fact that (p,q) specifies the winding of the $M5$ brane on T^2 as well as the orientation of the resulting (p,q) branes in the (x^5, x^6) plane, means that $SL(2,\mathbf{Z})$ acts on the complex coordinates $v = (x^4 + ix^5)/R_A$ and $s = (x^6 + ix^{10})/R_{10}$, which transform as a doublet.

We are, therefore, considering the type IIA theory for which an M-theory description exists with compactified x^4 and recover the 5D theories in the limit $R_A \to 0$ or, equivalently, $R_B \to \infty$.

4. SU(N) gauge theories

In [4] a description of brane configurations which are relevant to $N = 2$ gauge theories in four dimensional space-time was given via a smooth curve in M theory

$$F(v,t) = 0, \tag{3}$$

where $t = \exp(-s/R_{10})$. The brane configurations which we consider in the present paper are related to the brane configuration of [1] by T duality along the x^3, x^4 directions, taking into account charge conservation as mentioned in the previous section. It is possible therefore to describe them via a curve in M theory [5]. Before we continue we should note that the curve which was used to describe the four dimensional theories is smooth in the M theory limit. To be more precise the maximal curvature (at the

ends of the D4-branes on the NS fivebrane) is of the order of $1/R_{10}^2$ [12]. The M theory description of the type IIB theory is singular. Therefore one might expect that for the brane configuration which is relevant to describe five dimensional gauge theories the curve is singular in the type IIB limit $R_B \to \infty$.

Indeed, using the curve (which is defined below) one can find that the maximal curvature (at the ends of the D5 branes on the NS fivebrane) is proportional to $R_A^2 R_{10}^2/(R_A^2 + R_{10}^2)^3$, so by going to the type IIB limit a curvature singularity appears. Nevertheless the M theory description will provide some insight into the relation between the brane configurations and 5D field theories compactified on a circle of radius R_B. In the four-dimensional case by going from the type IIA brane configuration to the smooth brane configuration in M theory we obtained all non-perturbative corrections. The holomorphic curve turned out to be the Seiberg-Witten curve and the condition that the BPS states come from supersymmetry preserving M2 branes which end on the holomorphic curve lead to the Seiberg-Witten differential [13] For recent discussions within the context of M theory, see also [14–16]. Together they contain the information about the full IR dynamics of the $N = 2$ gauge theory including all instanton corrections [17]. The situation we encountered in the present work is quite similar. If we consider the 5D case the M theory description becomes singular and the type IIB brane configuration receives no corrections. There are no instanton corrections in five dimensions [8]. However, as we compactify this five dimensional theory on a circle the situation changes. To the usual 4D one-loop prepotential, we have to add corrections coming from an infinite tower of Kaluza-Klein states and instanton corrections [7]. Both types of corrections are again found by going to the M theory description where they are encoded in the explicit form of a complex curve which is uniquely fixed by the asymptotic behavior of the branes and the condition of supersymmetry, i.e. holomorphic embedding in the (v,s) space. Some of the curves we find agree with the spectral curves of relativistic Toda systems, which were used in Ref. [7] to study the non-perturbative solution of 5D theories compactified on a circle. The meromorphic differential, which one also needs to specify the theory, is the same the on in [13].

After compactifying x^4 on a circle, v is no longer single-valued. Following [5] we introduce

$$w = \exp\left(-i\frac{v}{R_A}\right) \qquad (4)$$

and describe the curve by

$$F(w,t) = 0. \qquad (5)$$

We are interested in finding all inequivalent theories. The S generator of $SL(2,\mathbf{Z})$ interchanges the NS5 and D5 branes. It, thus, rotates the corresponding brane configuration by 90 degrees taking $\lambda \rightarrow -1/\lambda$. Therefore, in the new configuration the (new) NS5 branes will be the lighter ones and we shall consider the effective field theory on them. This is precisely the same theory as the original one. We can, therefore, limit ourselves to $SL(2,\mathbf{Z})$ transformations which keep the D5 branes lighter than the NS branes. Since we look at theories with only one gauge factor we restrict ourselves to configurations with two NS5 branes i.e. the polynomial should be quadratic in t. This leaves $SL(2,\mathbf{Z})$ transformations of the form T^l. Matter is introduced via semi-infinite D5 branes.

$$P_1(w)t^2 + P_2(w)t + P_3(w) = 0. \qquad (6)$$

Moreover, we always have the freedom to move the semi-infinite D5 branes to one side keeping the coefficient of t^2 normalized to one. This can be achieved by the transformation $t \rightarrow P_3(w)/t$ (such a transformation moves all semi-infinite four branes to the right hand side of the NS branes) which yields

$$t^2 + P_2(w)t + P_4(w) = 0, \qquad (7)$$

where $P_4 = P_1 P_3$. If we express the curves as vanishing conditions in w and t, as opposed to v and t, we have to take into account that $w = \infty$ and $w = 0$ correspond to the asymptotic region (whereas $v = 0$ does not). That means that the multiplicity of the zero roots of the polynomials P_2 and P_4 will be relevant. We therefore write them in the following general form:

$$P_2(w) = c_2 \, w^n \prod_{i=1}^{N} (w - \tilde{a}_i),$$

$$P_4(w) = c_4 \, w^m \prod_{j=1}^{N_f} (w - \tilde{m}_j). \qquad (8)$$

The integers n, m characterize the underlying field theory, as will become explicit below. To specify the field theory which is described by the brane configuration associated with a given curve we need to study the asymptotic behavior of the branes.

Let us first consider $w \rightarrow \infty$. To leading order we get (after a rescaling)

$$t^2 + cw^{N+n}t + w^{N_f+m} = 0. \qquad (9)$$

The asymptotic behavior is therefore

$$t_1 \sim w^{N+n}, \quad t_2 \sim w^{N_f+m-N-n}$$

when $2(N+n) > N_f + m$,

$$t_{1,2} \sim w^{N+n} \quad \text{when} \quad 2(N+n) = N_f + m,$$

$$t_{1,2} \sim w^{(N_f+m)/2} \quad \text{when} \quad 2(N+n) < N_f + m. \qquad (10)$$

For $2(N+n) < N_f + m$ the asymptotic behavior of the branes depends only on N_f. The type IIB description of such theories leads to crossing of the NS fivebranes and hence new massless excitations will appear. The analog case in [4] leads to gauge theories with positive beta function. In the case of finite R_B no singularity associated with brane crossing appears in the curve. The branes do not really cross but they approach each other asymptotically. It would be interesting to further investigate this range since it may lead to new theories as was conjectured by [2]. One does not expect these theories to be $SU(N)$ field theories with N_f flavors since those do not lead to non-trivial fixed points for $N_f > 2N$ ($N \geq 3$) [6]. For ordinary $SU(N)$ theories we should focus on $2(N+n) \geq N_f + m$. In four dimensions they correspond to asymptotically free theories, whereas in five dimensions they will lead to non-trivial IR fixed points. The case $2(N+n) = N_f + m$ corresponds to a situation where in the IIB picture there are two parallel five branes which go off to $x^5 = +\infty$ at a finite distance apart. In the 4D limit this corresponds to the superconformal theories where the distance between the parallel fivebranes amounts to a choice of the gauge coupling τ.

Consider now $w \rightarrow 0$ ($x^5 \rightarrow -\infty$) the leading order polynomial is

$$t^2 + c'w^n t + w^m = 0 \qquad (11)$$

and the asymptotic behavior is

$$t_{1,2} \sim w^{m/2} \quad \text{when} \quad 2n > m,$$

$$t_{1,2} \sim w^n \quad \text{when} \quad 2n = m,$$

$$t_1 \sim w^n, \quad t_2 \sim w^{m-n} \quad \text{when} \quad 2n < m. \tag{12}$$

CCurves with $2n > m$ correspond to type IIB configurations with crossing semi-infinite fivebranes (or asymptotically approaching fivebranes for finite R_B). Thus we concentrate on curves with $2n \leq m$. As mentioned above, the only $SL(2,Z)$ transformations which are not yet fixed by the Ansatz (7) are T^l. Such a transformation acts as $t \to tw^l$, $w \to w$ which takes $m \to m + 2l$ and $n \to n + l$. Since $2n \leq m$ one can set $n = 0, m \geq 0$. We are thus left with the single constraint

$$2N \geq N_f + m, \tag{13}$$

which means that $m = 0,1,\ldots,2N - N_f$. The parity operator $w \to 1/w$ ($v \to -v$) takes

$$m \to 2N - N_f - m. \tag{14}$$

Therefore the number of allowed values for m which yields different curves is $[(2N - N_f + 1)/2]$. For $N > 2$ this result is in agreement with the field theory result [6] where it was found that the number of allowed values for $|c_{cl}|$ is $[(2N - N_f + 1)/2]$. Recall that c_{cl} is the numerical coefficient of the bare cubic term in the prepotential which characterizes the theory. We have thus found that the brane configurations we have obtained from M theory simply reproduce the known superconformal field theories.

To find the relation between $|c_{cl}|$ and m we note that the parity transformation $w \to 1/w$ acts as charge conjugation as it reverses the orientation of the elementary strings. Since charge conjugation takes $c_{cl} \to -c_{cl}$ [6] we identify, using Eq. (14)

$$c_{cl} = N - m - N_f/2. \tag{15}$$

Below we show, using the M theory approach, that the brane description agrees with the field theory result [6] (this was already shown using the type IIB description [3]). The one exceptional case is $SU(2)$ were it was argued that theories with $N_f = 5,6,7$ are also consistent [8] while no consistent brane configuration could be constructed [2].

The $SU(2)$ theories with $N_f = 5,6,7$ correspond to interacting fixed points with global symmetry groups

E_6, E_7, E_8 [8]. $N_f = 8$ is also consistent but does not lead to an interesting fixed point. But these theories are outside the range $N_f \leq 2N$ of allowed brane configuration. In Ref. [2] it was explained that by going beyond this bound two semi-infinite branes necessarily cross and this crossing would induce additional massless states. In our M theory configurations (7) this singularity does not occur but the two branes approach each other asymptotically. Nevertheless also in this setup we expect additional light states which have no conventional field theory interpretation.

After having given the general form for the curves in Eq. (8), we will now rewrite them in a form that allows us to go to the $d = 4$ limit by taking $R_B \to 0$ or, equivalently, $R_A \to \infty$. By an appropriate choice of the constants c_2 and c_4 one obtains

$$t^2 + 2tw^{N/2} \prod_{i=1}^{N} R_A \sin\left(\frac{v - a_i}{2R_A}\right)$$

$$+ w^{N_f/2+m} \prod_{j=1}^{N_f} R_A \sin\left(\frac{v - m_j}{2R_A}\right) = 0. \tag{16}$$

a_i are the positions of the finite D branes in v space and m_i, the bare masses of the hypermultiplets, are the positions of the semi-infinte D branes. Note that whereas in the five-dimensional theories the masses are real, they are complex in the compactified theory. The parameters are related to those in Eq. (8) via $\tilde{a}_i = e^{-a_i/R_A}$, $\tilde{m}_i = e^{-im_i/R_A}$. For $R_A \to \infty$ or $R_B \to 0$ this becomes

$$t^2 + 2t \prod_{i=1}^{N} (v - a_i) + \prod_{j=1}^{N_f} (v - m_j) = 0. \tag{17}$$

which agrees with the curves for $SU(N)$ $N = 2$ SQCD with N_f flavors of [18,19]

On the other hand we can study the large R_B region of these curves and integrate out hypermultiplets by sending their masses to infinity. There are two possibilities $m \to \pm\infty$ which leads to two different curves which correspond to two theories with equal matter content but different values of c_{cl}. Starting from the unique curve for $N_f = 2N$ the flow pattern of the curves reproduces all possible theories labeled by c_{cl}. Also the relation of c_{cl} to the number of quarks with positive and negative masses has an explanation in the M theory picture.

In the 4D case one can obtain the effective gauge coupling by considering the asymptotic bending of the NS branes which can be read directly from the associated M-theory curve. This does not hold in the 5D case. It is reflected by the fact that calculating $\log(t_1/t_2)$ where $t_{1,2}$ are the two roots associated with the curve (16) does not give the effective gauge coupling in the limit $R_B \to \infty$. This bending calculation reflects, however, the $N_f \le 2N$ bound. The correct way to define and obtain the 5D effective gauge coupling is by calculating some BPS mass as is explicitly demonstrated in Ref. [2]. It would be important to rederive it geometrically via the M-theory approach.

5. SO(N) and Sp(2N) gauge theories

The M theory description of $SO(N)$ and $Sp(2N)$ gauge theories in four dimensions was given in [14,21]. A subtlety in $d = 4$ was how the orientifold planes, which are present in the type IIA formulation would appear in the M theory description. Here we will determine curves which have the correct behavior in the four dimensional limit and which respect the symmetries, i.e. they must be symmetric under [3] $w \to 1/w$, corresponding to $v \to -v$, and be periodic in v. We first note that the first of these conditions does not allow for the introduction of the parameter m which distinguished different theories in the $SU(N)$ case. For $SO(2N)$ we thus arrive at the curve

$$t^2 \left(\sin\frac{v}{R_A} \right)^2 + 2t \prod_{i=1}^{N} \sin\left(\frac{v - a_i}{2R_A} \right) \sin\left(\frac{v + a_i}{2R_A} \right)$$
$$+ c \left(\sin\frac{v}{R_A} \right)^2 \prod_{j=1}^{N_f} \sin\left(\frac{v - m_j}{2R_A} \right) \sin\left(\frac{v + m_j}{R_A} \right) = 0.$$
$$(18)$$

By appropriate choice of c and rescaling of t one can take the limit $R_A \to \infty$ and arrives at the well

known Seiberg-Witten curves for $SO(2N)$ with N_f fundamental hypermultiplets.

Note that m_i and $-m_i$ enter in the curve symmetrically due to the reflection symmetry $v \to -v$. In particular, this implies that $c_{cl} = 0$ in this case since the flows $m \to +\infty$ and $m \to -\infty$ are equivalent (in contrast to the $SU(N)$ case). This is also in agreement with field theory results [6]. This will also carry over to the $SO(2N + 1)$ and $Sp(2N)$ theories to be discussed below. This curve has the expected behavior as we go to $R_A \to \infty$ i.e. it turns into the four-dimensional curve (after a rescaling of t and v). On the other hand if we investigate the curve for large R_B and study the behavior for $|v| \gg |a_i|,|m_j|$ one should reproduce the bending (tilting) of the five dimensional brane configurations. Indeed we find

$$t^2 \exp(2|v|/R_A) + t \exp(N|v|/R_A)$$
$$+ \exp\left((2 + N_f)|v|/R_A\right) = 0.$$
$$(19)$$

For

$$2N - 4 \ge N_f$$
$$(20)$$

one finds

$$\log(t_1/t_2) \sim R_B(4N - 8 - 2N_f)|v|,$$
$$(21)$$

i.e. in the asymptotic region there are two branes which diverge from each other or move off to infinity at a finite distance. For $4N - 8 < 2N_f$, on the other hand, they asymptotically approach one another. The condition (20) agrees with Ref. [3]. Note that it differs from the condition for asymptotically free $SO(2N)$ theories in four dimensions ($2N - 2 > N_f$).

The discussion for $SO(2N + 1)$ and $Sp(2N)$ is analogous, so we will be very brief.

For $Sp(2N)$ gauge groups the curve is

$$t^2 + 2t \left(\sin\frac{v}{R_A} \right)^2 \prod_{i=1}^{N} \sin\left(\frac{v - a_i}{2R_A} \right) \sin\left(\frac{v + a_i}{2R_A} \right)$$
$$+ c \prod_{j=1}^{N_f} \sin\left(\frac{v - m_j}{2R_A} \right) \sin\left(\frac{v + m_j}{2R_A} \right) = 0.$$
$$(22)$$

The allowed range of theories is now $2N + 4 \ge N_f$, in agreement with field theory results [6].

Finally, the $SO(2N + 1)$ curve follows from the $SO(2N)$ curve by realizing that one of the a_i should

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[3] As in the four-dimensional case, for $SO(2n + 1)$ we must at the same time transform $t \to -t$; cf. [21]. Also, a shift $v \to v + 2\pi R_A$ must be accompanied by a shift $s \to s + \pi R_{10}$.

vanish and that this zero should be simple. This gives

$$
t^2 \left(\sin \frac{v}{R_A} \right)^2
$$

$$
+ 2t \sin \left(\frac{v}{2R_A} \right) \prod_{i=1}^{N} \sin \left(\frac{v - a_i}{2R_A} \right) \sin \left(\frac{v + a_i}{2R_A} \right)
$$

$$
+ c \left(\sin \frac{v}{R_A} \right)^2 \prod_{j=1}^{N_f} \sin \left(\frac{v - m_j}{2R_A} \right) \sin \left(\frac{v + m_j}{2R_A} \right)
$$

$$
= 0, \tag{23}
$$

which leads to the expected bound $2N - 3 \geq N_f$.

Note that the limits on N_f for orthogonal and symplectic groups can be interpreted from the brane configuration in the same way as in the four dimensional case if one takes into account that an $O5$ plane now has (minus) the charge of a $D5$ plane, so that by the combined arguments of Refs. [20,25,21] one needs to add twice as many semi-infinite (SO) or finite (Sp) D5 branes at the position of the orientifold plane.

6. Summary

The M-theory approach can be extended to discuss type IIB configurations and their corresponding (compactified) 5D field theories. From the M-theory point of view we still have just one M5 brane embedded in $\mathbf{R}^{3,1} \times \mathbf{R}^2 \times \mathbf{T}^2$ where the torus corresponds the (x^4, x^{10}) subspace. We have constructed the curves which account for $SU(N)$, $SO(N)$ and $Sp(2N)$ 5D supersymmetric gauge theories compactified on a circle. In particular, for the $SU(N)$ case we account for all 5D theories with $N_f \leq 2N$ and identify the parameter in the curve corresponding to $c_{\rm cl}$. Recall that from the field theory point of view it is $c_{\rm cl}$ that characterizes the theory [6]. Here we find the curves associated with these theories. The bound $N_f \leq 2N$ is correct for $N \geq 3$. However for $N = 2$ it is known [8] that there exist theories for $N_f = 5, 6, 7$ which lead to non-trivial fixed points with exceptional global symmetries $E_{6,7,8}$. We note that for these theories also no brane construction is known (but they can be realized using branes as probes [8]). It seems that whenever there are theories with excep-

tional symmetries it is difficult to get them using flat brane constructions or in the related M-theory approach. These theories can however be constructed within the geometric engineering approach in which a compactification on some curved (non-compact) manifold is considered [22].

The curves which we have found are intimately related to the curves which were considered in the discussion of integrable 5D models compactified on a circle [7]. In our analysis we have considered only curves leading to theories with classical gauge groups and N_f flavors. We have found in all cases the known bounds on N_f. In the analysis of the different inequivalent curves we have discarded curves which do not lead to such theories. It corresponds in the brane picture to brane configurations with N_f outside this bound $(N_f \geq 2N$ for the $SU(N)$ case) which necessarily involve more intersections of the five-branes than the ones which exist on the Coulomb branch within the bound. It would be important to further investigate these curves and see whether they correspond to new interesting superconformal theories in the IR limit as was conjectured in [2].

We want to close with a comment on an alternative way of introducing matter in the five dimensional systems. For the brane description of three and four dimensional gauge theories this was also possible via infinite D5 and D6 branes, respectively. This way of introducing fundamental matter multiplets allows for the discussion of the Higgs branches of these theories. One might now try to extend this to the five-dimensional theories discussed in this letter and arrive at the appropriate brane configuration via T duality. This is however not straightforward, for the following reasons. First we recall that the type IIB brane configurations discussed here are not the naive T duals of the ones discussed in $d = 3$ and $d = 4$, since T duality would not automatically lead to the polymeric brane configurations, but rather to a network of straight branes. Also, the five and six branes necessary to discuss the Higgs branches in $d = 3$ and $d = 4$ would, under T duality, turn into D7 branes, which, since their transverse space-time is $2 + 1$ dimensional, would be expected to lead to a deficit angle. Indeed, the 7 branes constructed in [23] give a *constant* deficit angle of $\frac{\pi}{6}$. However, they are not related to lower dimensional D branes via T-duality. For IIB theory compactified on a circle,

there exists a seven brane solution which is T-dual to the six brane of IIA [24]. It reduces, however, in the decompactification limit to flat 10 dimensional Minkowski space-time, i.e. there is no deficit angle. This issue needs further study.

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