MASS INFLATION INSIDE NON–ABELIAN BLACK HOLES [a]

P. BREITENLOHNER [§], G. LAVRELASHVILI [‡ b], and D. MAISON [§]

Max-Planck-Institut für Physik, Föhringer Ring 6 [§]

80805 Munich, Germany

Institute for Theoretical Physics, University of Bern, Sidlerstrasse 5 [‡]

CH-3012 Bern, Switzerland

The interior geometry of static, spherically symmetric black holes of the Einstein-Yang-Mills-Higgs theory is analyzed. It is found that in contrast to the Abelian case generically no inner (Cauchy) horizon is formed inside non-Abelian black holes. Instead the solutions come close to a Cauchy horizon but then undergo an enormous growth of the mass function, a phenomenon which can be termed 'mass inflation' in analogy to what is observed for perturbations of the Reissner-Nordstrøm solution. A significant difference between the theories with and without a Higgs field is observed. Without a Higgs field the YM field induces repeated cycles of mass inflation – taking the form of violent 'explosions' – interrupted by quiescent periods and subsequent approaches to an almost Cauchy horizon. With the Higgs field no such cycles occur. Besides the generic solutions there are non-generic families with a Schwarzschild, Reissner-Nordstrøm and a pseudo Reissner-Nordstrøm type singularity at $r = 0$.

arXiv:gr-qc/9711024v2 10 Nov 1997

The discovery of regular, static, spherically symmetric solutions of the Einstein–Yang–Mills (EYM) equations by Bartnik and McKinnon[1] has lead to many surprises (see e.g. [2] and ref. therein). Especially important is the discovery of 'coloured' black holes[3] which among other things serve as counter examples to the 'No-Hair' conjecture.

Until recently only the region outside the horizon of these black holes was studied, but now also their interior structure is under investigation [4,5] and new surprises were found.

Figure 1: First two cycles of the generic solution in the EYM theory with parameter values r_h=0.97 and $W_h = 0.2$. For the second cycle a suitably stretched coordinate x is used. Different curves represent: the gauge field amplitude W, its derivative with respect to the Schwarzschild radial coordinate $W' = dW/dr$ and logarithm of the local mass function m.

The generic black hole solution of the EYM theory was found to be oscillatory inside the horizon [4,5]. Our numerical results for this case are illustrated in the Fig. 1. As one performs numerical integration starting at the horizon and integrates towards $r = 0$ one observes a sudden steep rise of W' and a subsequent exponential growth of the mass function $m(r)$ (parametrizing the g^{rr}-component of the metric via $g^{rr} = 1 - 2m(r)/r$). This phenomenon can be understood as manifestation of the instability of a possible inner (Cauchy) horizon and is closely related to the 'usual' mass inflation phenomenon observed for perturbations of the RN and Kerr black holes [6,7]. Within a short interval of r the mass function reaches a plateau and stays constant for a 'while' until it starts to decrease again. When the solution comes close to an inner horizon, the same inflationary process repeats itself with an even more violent next 'explosion'.

By a suitable fine tuning of the initial data at the horizon it is possible to obtain different special solutions. Black holes with an inner (Cauchy) horizon – non-Abelian analogues of the Reissner-Nordstrøm (RN) solution (NARN) and two types of solutions without inner horizon – generalization of the Schwarzschild solution (NAS) and non-Abelian solutions with a pseudo-RN type singularity (NAPRN) were found.

A qualitative understanding of the behaviour of the generic solutions was gained using a simplified dynamical system described in[4,5,8]. The generic behaviour is ruled

by a fixed point of this system, which is a repulsive focal point around which the solutions spiral with growing amplitude. It is possible to obtain an approximate solution leading to a 'plateau – to – plateau formula'[5] relating quantities at one plateau (before the 'explosion') to those on the next plateau (after the 'explosion').

Figure 2: Two characteristic types of interior solutions with Higgs fields.

In order to understand the model dependance of these results a theory with an additional Higgs field was investigated[5]. It was found that after adding the Higgs field no more oscillations occur inside the horizon[c]. This change in the behaviour of the generic solution is due to the different fixed points of the corresponding simplified dynamical system. The focal point disappears and the inflationary behaviour is now ruled by a a stable attractor leading asymptotically to a linear growth of $\ln(m)$ with $\ln(r)$.

$$\ln(m) = \ln(m_0) - z_0^2 \ln(r), \tag{1}$$

where m_0 and $z_0^2 > 1$ are some constants. This behaviour is obviously supported by our numerical results shown in the Fig. 2.

The main conclusion is that no inner (Cauchy) horizon is formed inside non–Abelian black holes in the generic case, instead one obtains a kind of mass inflation. Without a Higgs field, i.e. for the EYM theory, this mass inflation repeats itself in cycles of ever more violent growth. Mass inflation is exponential and is assotiated to the instability of the inner horizon. With the Higgs field there are no such cycles and the mass function diverges according to a power law.

Acknowledgments

[c] It is interesting to note that the similar results were obtained in the Abelian case in the homogeneous mass inflation model[9].

References

1. R. Bartnik and J. McKinnon, *Phys. Rev. Lett.* **61**, 141 (1988).

2. G. Lavrelashvili, *Non-Abelian Surprises in Gravity*, gr-qc/9701049.

3. H.P. Künzle and A.K.M. Masood-ul-Alam, *J. Math. Phys.* **31,** 928 (1990);
   M.S. Volkov and D.V. Gal'tsov, *JETP Lett.* **50,** 345 (1990);
   P. Bizon, *Phys. Rev. Lett.* **61**, 284 (1990);
   P. Breitenlohner, P. Forgacs, and D. Maison, *Nucl. Phys.* B **383**, 357 (1992);
   K. Lee, V.P. Nair, and E.J. Weinberg, *Phys. Rev.* D **45**, 2751 (1992));
   G. Lavrelashvili and D. Maison, *Nucl. Phys.* B **410**, 407 (1993);
   B.R. Greene, S.D. Mathur, and C.M. O'Neill, *Phys. Rev.* D **47**, 2242 (1993).

4. E.E. Donets, D.V. Gal'tsov, M.Yu. Zotov, gr-qc/9612067, versions 1-4 and *Phys. Rev.* D **56**, 3459 (1997).

5. P. Breitenlohner, G. Lavrelashvili, and D. Maison, *Mass inflation and chaotic behaviour inside hairy black holes*, gr-qc/9703047.

6. E. Poisson and W. Israel, *Phys. Rev.* D **41**, 1796 (1990).

7. A.Ori, *Phys. Rev. Lett.* **67**, 789 (1991); *Phys. Rev. Lett.* **68**, 2117 (1992).

8. P. Breitenlohner, G. Lavrelashvili, and D. Maison, *Non–Abelian black holes: The inside story*, gr-qc/9708036.

9. D.N. Page, *Black–Hole Thermodynamics, Mass-Inflation, and Evaporation*, in: Black Hole Physics, V. De Sabbata and Z. Zhang (eds.), (Kluwer, 1992), p. 185–224;
   A.Ori, 1991, (unpublished).