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White-light cavities, atomic phase coherence, and gravitational wave detectors

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Abstract

We propose a new concept to realize optical cavities with large buildup but broadband response (*white-light cavities*) using atomic phase coherence. We demonstrate that strongly driven double-Λ systems can show negative dispersion without absorption, which is needed in order to compensate for the variation of the wavelength with frequency. Internal buildup profiles and the cavity bandwidth of standard devices and *white-light cavities* will be briefly compared. These devices may be useful to improve the bandwidth and sensitivity of future generations of laser interferometric gravitational wave detectors.

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Several large-scale interferometric gravitational wave observatories with armlengths of 3 km (VIRGO [1]) and 4 km (LIGO [2]) are expected to go into operation before the end of this decade. Various concepts of interferometric gravitational wave detectors may utilize the enhancement of amplitude of electromagnetic fields inside a Fabry-Perot cavity. These are techniques to increase the (shot-noise limited) sensitivity of such devices, techniques known as power and signal recycling [3,4]. The medium size GEO 600 detector [5], also presently under construction, is based on such a design.

A detailed discussion of the concept of laser interferometric gravitational wave detectors (GWDs) will be omitted here. For this purpose the reader is referred to the literature, for example Refs. [3,4,6]. The concept is mainly based on the fact, that a gravitational wave (GW) interacting with an interferometer (the GWD) will give rise to two sidebands (SBs) superimposed on the carrier frequency (that is: the laser field) at the output of the interferometer. The frequency spacing between the carrier and the SBs is given by the frequency of the GW and will be on the order of 1 kHz for the GEO 600 detector and

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other earthbound devices. The amplitude of the SBs is proportional to the amplitude of the GW and to the laser amplitude at the carrier frequency. Due to the small amplitudes of the GWs the amplitude of the GW-induced SB will be about 20 orders of magnitude smaller than the amplitude at the carrier frequency. In order to improve the shot noise limited sensitivity at the SB frequencies, the SB amplitudes can be resonantly enhanced by a signal recycling mirror, which, together with the interferometer, forms a cavity with roundtrip loss V_{SR}. Here this loss is assumed to be due only to transmission of the recycling mirror. So, in principle, the shot-noise limited sensitivity could be drastically improved by increasing the buildup in the signal recycling cavity, but this would come at the cost of diminishing bandwidth. Therefore, it would be desirable to increase the bandwidth of a standard cavity by orders of magnitude, while maintaining the high internal buildup ($\sim 1/V_{SR}$). Since the SB amplitude is also proportional to the carrier amplitude, all detectors presently under construction will resonantly enhance the carrier amplitude in a power recycling cavity [3] with a very small bandwidth. In a standard Michelson-type detector this is not a problem, because the laser source has to be extremely monochromatic also for other reasons.

A GWD based on a Sagnac-type interferometer has been proposed by Drever in 1982 [7]. Recently, Ke-Xun Sun et al. [8] have investigated an interferometer of this type. A detector based on this design will be possible to be operated with a broadband, multiaxial or even superfluorescent laser mode. While this greatly simplifies laser design and interferometer operation, it makes the Sagnac-design incompatible with power recycling. In the case of a Sagnac-type GWD the power enhancement cavity will have to provide both broadband response and high internal buildup in order to increase the circulating laser power and to improve the strain sensitivity of the device. From the things mentioned above it is clear, that there is strong interest in the realization of cavities with broadband response but nevertheless high internal buildup.

In this Letter a new concept is proposed to realize an optical cavity with high buildup, yet broadband response, which can be named a *white-light cavity*. The main idea is to place a suitably prepared medium, showing nonclassical optical-properties, between the two mirrors to cancel the variation of wavelength with frequency. This would make the cavity simultaneously resonant for all frequencies. Such a medium could be introduced, for example, into the signal recycling cavity at the output of the interferometer just in front of the signal recycling mirror or inside a power recycling cavity of a Sagnac-type GWD. This way, a very high buildup could be used to improve the sensitivity of GWDs without the corresponding inverse loss of bandwidth.

However, a number of troubling problems have to be solved, before a white-light cavity may be realized or even be used to enhance the performance of a GWD. Depending on the application one will have to set up high-power laser systems featuring very small power and frequency fluctuations. In addition, especially for the application in GWDs, a convenient medium showing the desired optical properties uniformly in a volume of several cm^3 has to be developed.

Nevertheless it is not the aim of this paper to give a detailed analysis of all these problems. We rather will give some estimates about the problems concerned with the system under discussion: the influence of the Doppler-effect and of fluctuations of the system parameters onto the optical properties of the medium, which is used to realize a white-light cavity.

In the following the optical features of the atomic system used for our calculations are briefly drawn out. Taking into account the results of this discussion, we focus on the optical properties of the white-light cavity. First we will have a look at an atomic system which provides the possibility to realize this new kind of cavities with high internal buildup but yet broadband response. Much work has been done on the theory of atomic three and four level systems in the past years [9–15], revealing interesting nonclassical features as, for example, coherent population trapping, an ultra-high index of refraction or lasing without inversion, which has just been demonstrated experimentally [11,12]. In particular, it has been shown theoretically that for certain atomic systems under suitable experimental conditions, high positive dispersion at a point of zero absorption can be maintained, leading to the concept of an optical magnetometer based on atomic phase coherence as proposed by Scully and Fleischhauer [15]. For our concept, a coherently prepared medium with just the right amount of negative (anomalous) dispersion at a point of zero absorption is needed to cancel the classical variation of wavelength with frequency. We would like to emphasize, that, to our knowledge, so far it has not been noticed, that there are applications for media showing strong dispersion at a point of vanishing absorption.

For our calculations the double-Λ scheme of Fig. 1 has been used. This system can be described by the density matrix equations taken from Fleischhauer et al. [14]. We worked out the analytical solution for the following "typical" decay rates: $\gamma = \gamma' = \gamma_c = \gamma_c' = 100$ MHz, $\gamma_b = \gamma_b' = 10^{-3}\gamma$. It was checked, that for atomic densities as used in our discussion, the dephasing of the ground state coherence due to collisions and radiation trapping is taken into account by the introduction of the coherence decay rate $\gamma_{bb'} = \frac{1}{2}(\gamma_b + \gamma_b') = 10^{-3}\gamma$ [16,17]. We did not consider the influence of the Doppler-effect. The splitting between the two lower lying levels is set to $\omega_{bb'} = \gamma$. The coupling field is taken to be "resonant": $\omega_k = \frac{1}{2}(\omega_{cb} + \omega_{cb'})$ and to have a field strength which is described by the Rabi frequency Ω_k. The probing field Rabi frequency is chosen to be $\Omega_p = 10^{-6}\gamma$ to achieve a linear response. The wavelength of the probe transition is assigned to $\lambda_p = 500$ nm. The only parameters not specified so far are the atomic density N, the indirect pumping rates $\tilde{r} = \tilde{r}'$, the coupling field strength Ω_k, and the probe detuning $\Delta_p = \omega_m - \omega_p$, where $\omega_m = \frac{1}{2}(\omega_{ab} + \omega_{ab'})$.

From the density matrix we obtain the complex index of refraction $n_c = n + ik$, where k is related to the absorption coefficient α via $\alpha = \omega k/c$. For further algebraic manipulation the coefficients n and k are expanded in terms of Δ_p and $\tilde{r}$ yielding

$$n = n^{(1)}\Delta_p + n^{(3)}\Delta_p^3, \quad \text{where } n^{(i)} = n^{(i)}_{\tilde{r},0} + n^{(i)}_{\tilde{r},1}\tilde{r},$$
$$k = k^{(0)} + k^{(2)}\Delta_p^2 + k^{(4)}\Delta_p^4, \quad \text{where } k^{(i)} = k^{(i)}_{\tilde{r},0} + k^{(i)}_{\tilde{r},1}\tilde{r}. \qquad (1)$$

For the range of parameters N, Ω_k and $\tilde{r}$ discussed here, this expansion was checked against the analytical solution. The relative deviation for n and k was found to be much less than 10% for $|\Delta_p| \leqslant 0.5\gamma$, so that good qualitative agreement was achieved.

As will be shown later on, an experimental situation leading to vanishing absorption at resonance ($\alpha(\Delta_p = 0) = 0$) and an anomalous (negative) dispersion of $(\partial n/\partial\omega_p)(\Delta_p = 0) = -1/\omega_m$ is of great interest for our purposes. In the following, the first condition will be referred to as *zero gain at resonance* whereas the second will be termed *λ-compensation* for reasons which will become clear in what follows. The absorption coefficient α and the refractive index n for a situation where both conditions are satisfied, are shown in Fig. 2. For $|\Delta_p| \leqslant 0.1\gamma$ the absolute value of the absorption coefficient is reduced much below 0.01 m^{-1} whereas the dispersion $\partial n/\partial\omega_p = -\partial n/\partial\Delta_p$ turns out to be of the order of $\partial n/\partial\omega_p = -3 \times 10^{-10}$ MHz^{-1}, so *nega-*

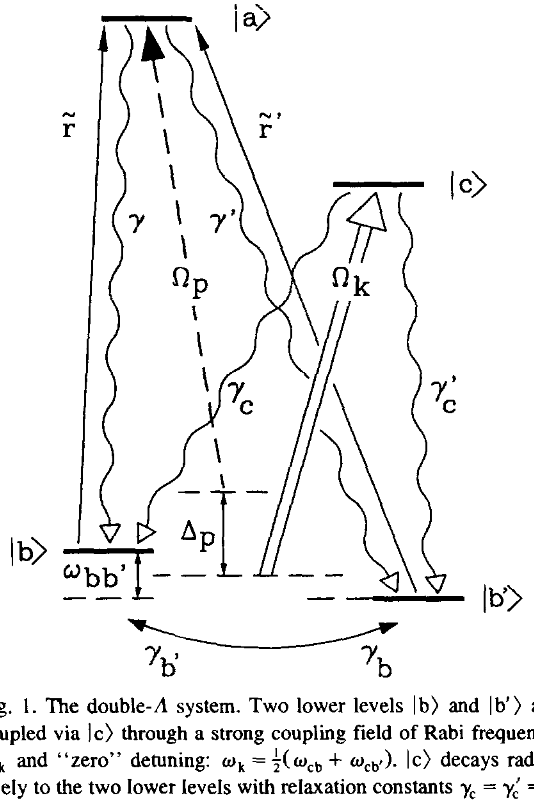

Fig. 1. The double-Λ system. Two lower levels $|b\rangle$ and $|b'\rangle$ are coupled via $|c\rangle$ through a strong coupling field of Rabi frequency Ω_k and "zero" detuning: $\omega_k = \frac{1}{2}(\omega_{cb} + \omega_{cb'})$. $|c\rangle$ decays radiatively to the two lower levels with relaxation constants $\gamma_c = \gamma_c' = \gamma$ where $\gamma = \gamma'$ are the radiatively damping constants of the upper probing level $|a\rangle$. The probing field is assumed to be weak ($\Omega_p = 10^{-6}\gamma$) and to be detuned by $\Delta_p = \frac{1}{2}(\omega_{ab} + \omega_{ab'}) - \omega_p$. Indirect pumping rates $\tilde{r} = \tilde{r}'$ are applied without establishing an additional coherence between $|a\rangle$ and $|b\rangle$ or $|b'\rangle$. The two lower levels are coupled via small longitudinal relaxation constants $\gamma_b = \gamma_{b'} = 10^{-3}\gamma$. The level spacing of the two lower levels is $\omega_{bb'} = \gamma$.

tive dispersion of the right amount without absorption could be realized theoretically.

Taking into account the absorption and index profiles as plotted in Fig. 2 the optical properties of a white-light cavity can be analyzed. The mirrors are assumed to be ideal in the way that they exhibit no additional phase shifts or any non-transmissive losses. The medium is taken to fill completely the space inside the cavity. The amplitude coefficient for single path transmission of the medium is described by $r \times \exp(i\Phi_r)$ where r contains the absorption and ϕ_r the phase shift of the electric field due to the medium. For the reflected (E^r) and transmitted (E^t) complex field amplitude we obtain

$$E^r = E^i \frac{1}{\sqrt{R_1}} \left(R_1 - (1 - R_1) \frac{\tilde{\rho}\, e^{i\Delta}}{1 - \tilde{\rho}\, e^{i\Delta}} \right),$$

$$E^t = E^i \tilde{\tau} \frac{e^{i\Delta/2}}{1 - \tilde{\rho}\, e^{i\Delta}}, \tag{2}$$

where $\tilde{\rho} = [R_1 R_2 r^4(\Delta_p)]^{1/2}$ is the amplitude coefficient for one round trip and $\tilde{\tau} = [(1 - R_1)(1 - R_2) r^2(\Delta_p)]^{1/2}$ for single path transmission. Here the power reflection coefficients of the two mirrors are denoted by R_1 and R_2, respectively. The phase Δ is given by $\Delta = 2\phi_r$, where ϕ_r is given by Eq. (3). Note, that reflection $\tilde{\rho}$ and transmission $\tilde{\tau}$ are functions of the probe laser detuning Δ_p. In the following, impedance matching ($E^r(\Delta_p = 0) = 0$) is assumed, implying $R_1 = R_2 r^2(\Delta_p = 0)$.

Now r and ϕ_r have to be calculated from the optical properties of the medium. Having assumed the cavity to be resonant with the probing field in the case of zero detuning $\Delta_p = 0$, that is $\omega_m / \Delta\nu = 2\pi M$, where $\Delta\nu$ is the free spectral range of the cavity and M is an integer, we find

$$\exp(i\phi_r) = \exp\left(\frac{i}{2\Delta\nu} \left[\left(\omega_m n^{(1)} - 1 \right) \Delta_p + \omega_m n^{(3)} \Delta_p^3 + O\left(\Delta_p^4 \right) \right] \right). \tag{3}$$

From Eq. (3) it is clear that $\partial n / \partial \Delta_p = 1/\omega_m$ leads to a phase ϕ_r independent of the laser frequency up to the first order. This physically means that the frequency dependence of the wavelength is compensated by the dispersion of the medium (*λ-compensation*). Hence the resonance condition of the Fabry-

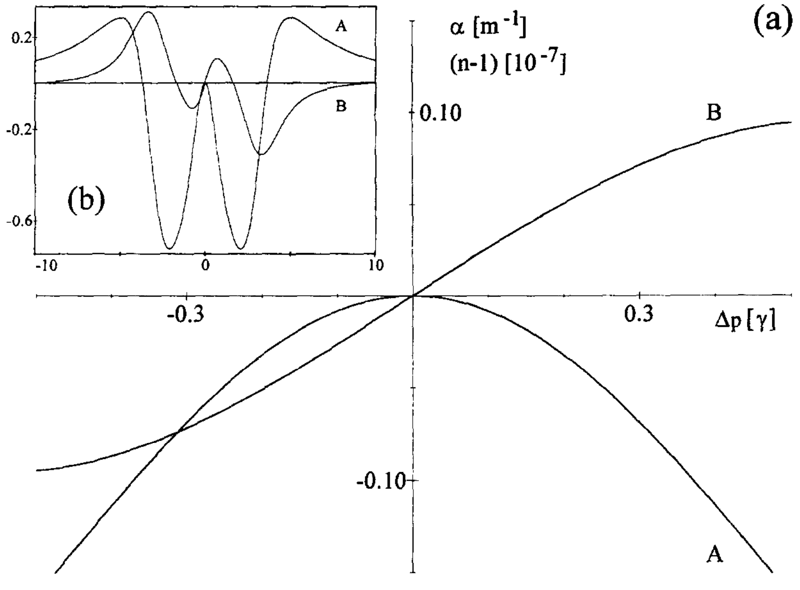

Fig. 2. Specific absorption coefficient $\alpha(\mathrm{m}^{-1})$ ('A') and variation of the refractive index $(n - 1)$ (10^{-7}) ('B') versus probe laser detuning Δ_p for the parameters $N = 2 \times 10^{16}$ m^{-3}, $\Omega_k = 2.216\gamma$ and $\tilde{r} = 6.337 \times 10^{-2}\gamma$. Part (b) shows the global range characteristics. It should be noted, that $\alpha < 0$ means gain. The region of $\Delta_p \approx 0$ shows strong negative dispersion (positive slope!) while the absorption is greatly reduced. Part (a) shows the region of $|\Delta_p| \leqslant \frac{1}{2}\gamma$ in greater detail.

Perot interferometer becomes independent of the laser frequency (white-light cavity).

The real amplitude coefficient r turns out to be

$$r=\exp\left(-\frac{1}{2}\frac{\omega_m}{\Delta\nu}\left[k^{(0)}+k^{(2)}\Delta_p^2+k^{(4)}\Delta_p^4+O\left(\Delta_p^6\right)\right]\right). \tag{4}$$

In order to affect the incoming field as little as possible (e.g. preventing the resonator–amplifier system from starting to oscillate on one hand, and to preserve the high buildup of the empty resonator on the other) $k^{(0)}$ has to be chosen to be $k^{(0)}=0$, that is *zero absorption at resonance*, implying $r(\Delta_p=0)=1$. In this case the impedance matching condition is reduced to the ordinary impedance matching condition $R_1=R_2$.

The problem now is how to obtain zero gain at resonance and λ-compensation simultaneously. From Eq. (1) the indirect pumping rate $\tilde{r}_\lambda$, at which λ-compensation is achieved, is found to be

$$\tilde{r}_\lambda=\left(\frac{1}{\omega_m}-n^{(1)}_{\tilde{r},0}\right)\frac{1}{n^{(1)}_{\tilde{r},1}}. \tag{5}$$

Note that the coefficients $n^{(1)}_{\tilde{r},i}$ are functions of the coupling field strength Ω_k and that they especially are linear in the atomic density N. So after fixing N to the value of $N=2\times10^{10}$ cm^{-3}, the Rabi frequency Ω_k can be varied to achieve zero gain at resonance while maintaining λ-compensation. This is the way the parameters $N=2\times10^{10}$ cm^{-3}, $\Omega_k=2.216\gamma$ and $\tilde{r}_\lambda=6.337\times10^{-2}\gamma$ were found, which are used for our discussion. An analysis of the internal buildup profile $I_{\text{intern}}/I_{\text{in}}(\Delta_p)$ shows off-resonant "gain peaks" due to the off-resonant gain of the medium at detunings, where the resonance condition still is maintained. By dropping the requirement of perfect λ-compensation, quadratically flat internal buildup profiles

$$\frac{\partial^2 I_{\text{int}}}{\partial\Delta_p^2}(\Delta_p=0)=0 \tag{6}$$

can be achieved. This is done by taking into account that the absorption and the dispersion are proportional to the atomic density N. Therefore, when Ω_k, $\tilde{r}$ and N are chosen to yield λ-compensation and zero gain at resonance simultaneously (then denoted by $\Omega_{k,\lambda}$, $\tilde{r}_\lambda$, N_λ) a variation of N modifies the dispersion while the absorption at resonance still is vanishing. We denote the desired atomic density for quadratically flat profiles by N_{flat} and define δN_{flat} by $N_{\text{flat}}=\delta N_{\text{flat}}N_\lambda$. Using Eq. (6) and Eq. (2) one obtains

$$\delta N_{\text{flat}}=\epsilon-\sqrt{\epsilon^2-1},$$

where

$$\epsilon=1-\tfrac{1}{2}\omega_m k_\lambda^{(2)}\Delta\nu\frac{(1-\tilde{\rho}_0)(1+\tilde{\rho}_0)}{\tilde{\rho}_0}, \tag{7}$$

and $\tilde{\rho}_0=\tilde{\rho}(\Delta_p=0)=(R_1R_2)^{1/2}$, which equals the reflectivity of one of the mirrors in the case of impedance matching. For sake of simplicity this is assumed for the following discussion. The coefficient $k_\lambda^{(2)}$ is given by $k^{(2)}(\Omega_{k,\lambda},\tilde{r}_\lambda,N_\lambda)$. Fig. 3a shows the internal buildup of a quadratically flat cavity for $R_1=R_2=0.99$ and $\Delta\nu=5/(2\pi)\gamma$. The atomic parameters are the same as before and δN_{flat} is selected according to Eq. (7). As can be seen, the bandwidth of the cavity is enhanced by a factor of 11 compared to the empty cavity. Fig. 3b shows the bandwitdh (HWHM) of a white-light cavity (curve 'B') obeying Eq. (7) and of an ordinary empty cavity (curve 'A'), respectively. For each mirror reflectivity $R_1=R_2=R=1-10^{-\kappa}$, where κ describes the loss of one of the mirrors and equals the logarithm of the cavity-buildup, the value of δN_{flat} was calculated. The result points out that the relative bandwidth $(\Delta_p^{1/2,\text{white-light}}/\Delta_p^{1/2,\text{ordinary}})$ increases with increasing buildup reaching the order of 320 for an internal buildup of $Q=10^5$. This is a value corresponding to a reflection loss per mirror of 10 ppm, a value which is not too unrealistic. So this analysis has demonstrated that broadband high-Q cavities can be realized by the aid of this new idea.

At the end of this paper we would like to give some estimations about the influence of the most serious effects onto the performance of the white-light cavity. The influence of the Doppler-effect as well as of the residual noise of the driving field power and frequency, of the indirect pumping rate and of the atomic density will be discussed.

One of the most crucial aspects is the influence of the Doppler-effect onto the optical properties of the medium. We have done a numerical analysis for the atomic parameters being the same as for Fig. 2. The result is shown in Fig. 4. For collinear driving and probing fields the dispersion (Fig. 4a) decreases drastically as the "Doppler-width" $\omega_{ab} \times v_w/c$ reaches γ. Here v_w is the most probable velocity of the ensemble. The influence onto the absorption is much less pronounced, as can be seen from Fig. 4b. From Fig. 4a it follows, that the "Doppler-width" has to be reduced below the natural linewidth. Of course, this is equivalent to cancelling the Doppler-effect completely. Although this result has been derived for a special set of parameters and for the case

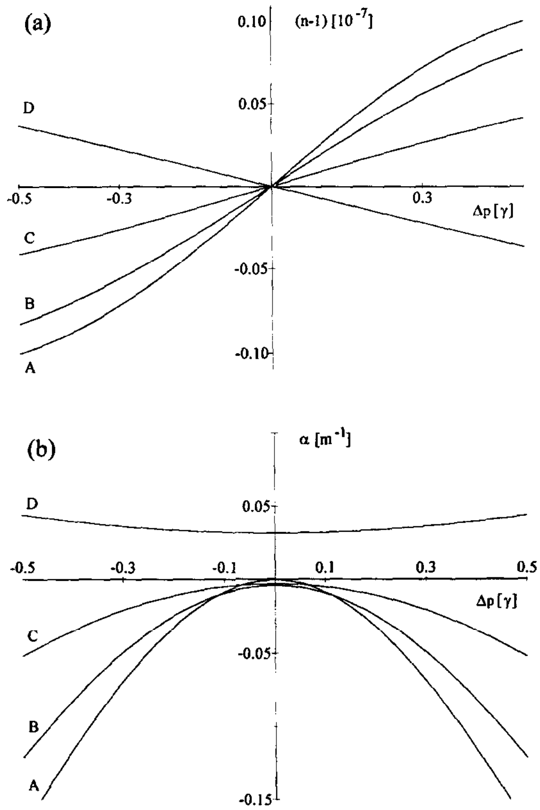

Fig. 4. Influence of the Doppler-effect onto the absorption and the index of refraction of the medium for the parameters being the same as used for Fig. 2. A collinear setup of both of the fields is assumed. Part (a) shows the index of refraction and part (b) the specific absorption coefficient versus the probe frequency for different values of the "Doppler-width" $(\omega_{ab} \times v_w/c)/\gamma$, where v_w is the most probable velocity of the atoms and c the speed of light. ('A') $(\omega_{ab} \times v_w/c)/\gamma = 0$, ('B') $(\omega_{ab} \times v_w/c)/\gamma = 0.5$, ('C') $(\omega_{ab} \times v_w/c)/\gamma = 1.0$, ('D') $(\omega_{ab} \times v_w/c)/\gamma = 2.0$.

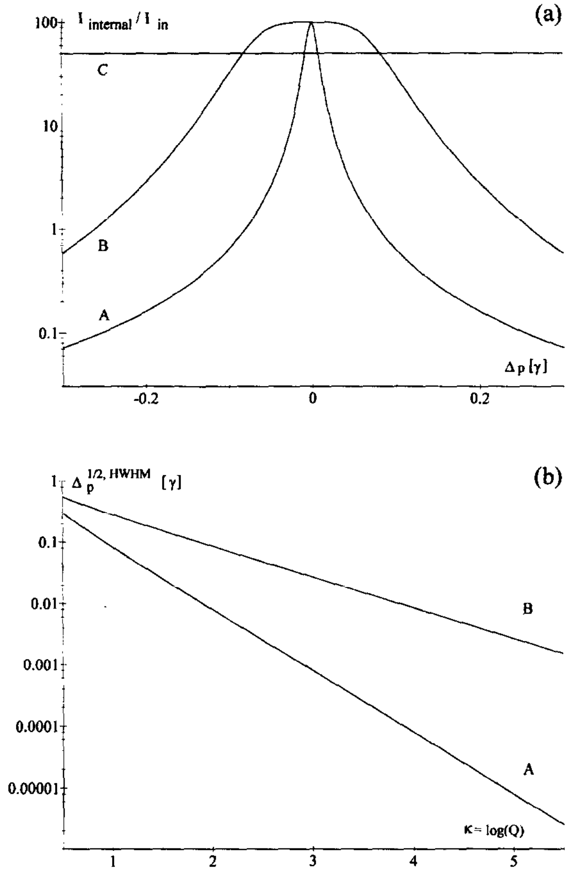

Fig. 3. (a) Internal buildup $I_{internal}/I_{in}$ versus probe laser detuning Δ_p for the same atomic parameters as for Fig. 2 and $R_1 = R_2 = 0.99$, $\Delta\nu = 5/(2\pi)\gamma$. 'A' denotes the standard cavity, 'B' shows the white light cavity with quadratically flat profiles and 'C' gives the half maximum internal buildup. (b) HWHM-bandwidth $\Delta_p^{1/2}$ of the standard cavity ('A') and the quadratically flat white-light cavity ('B') versus the logarithm of the cavity-buildup $Q = 10^{-\kappa}$. Note that the ratio of the bandwidths increases with increasing Q.

of a double-Λ system only, it seems to be valid in general. All of the atomic systems known so far to show negative dispersion at a point of vanishing absorption [14], have a common characteristic: negative dispersion in regions of nearly vanishing absorption is realized only in a frequency range of width of order γ centered at the corresponding resonance [1]. For some of these systems (Λ and double-Λ sys-

[1] For two-level systems see, for example, Ref. [18], or a more recent publication [19]. For three- and four-level systems see, for example, Ref. [20] and references therein.

tems) the coherent coupling of the corresponding states may even be relatively insensitive to the Doppler-effect due to Doppler-compensation. With Doppler-compensation we describe the fact, that for a two-photon (or higher order) transition governed by the two-photon resonance condition, the influence of the Doppler-effect can be strongly reduced or even cancelled completely, if the starting and the final level of the transition are closely spaced and a collinear setup for both of the fields is chosen. However, in the weak probing field limit, the absorptive and dispersive properties of all of these systems are due to first order (with respect to probing power) processes. Therefore the probing field never may benefit from Doppler-compensation. In conclusion, this discussion suggests, that for the realization of negative dispersion without absorption in general the Doppler-effect has to be avoided.

Another group of serious problems is due to the fluctuations of the system parameters, that are variations of the atomic density, of the driving field strength and frequency, and of the indirect pumping rate. In general, these will give rise to fluctuations of the absorption, the index of refraction and the dispersion. We will first give an estimate about the order of magnitude of these effects and will then discuss the resulting influence onto the performance of the white-light cavity.

For sake of simplicity let us assume, that fluctuations of the total number of atoms N_{total} are Poissonian so that $\Delta N_{total}/N_{total} = 1/\sqrt{N_{total}}$. The total number of atoms may be estimated by assuming a cavity laser mode of beam waist $w_0 = 0.5$ mm and of length $L = 1$ cm resulting in a total volume of about $V \simeq 10^{-2}$ cm^3. The atomic density used in our calculation (Fig. 2) is $N = 2 \times 10^{10}$ cm^{-3} so that the total number of atoms is $N_{total} \simeq 10^8$. Therefore, fluctuations of the total number of atoms will be on the order of $\Delta N_{total}/N_{total} \simeq 10^{-4}$. Since the absorption and the refractive index both are directly proportional to N_{total}, any relative change $\Delta N_{total}/N_{total}$ will introduce $\Delta\alpha/\alpha = \Delta n/n = \Delta N_{total}/N_{total}$. Please note, that $n(\Delta_p = 0) - 1 = 0$, so that fluctuations ΔN_{total} may be understood to give rise to variations of the dispersion rather than to change the index of refraction. This is worthwhile to mention because, as a result, fluctuations ΔN_{total} do not affect the resonance frequency of the white-light cavity. This, of course, is not the case for a standard cavity. For the dispersion as well as for the absorption and the refractive index it is found, that $\Delta(\partial_\omega n)/\partial_\omega n = \Delta N_{total}/N_{total}$. We now may conclude, that relative fluctuations of the atomic density may give rise to relative fluctuations of the absorption and dispersion not exceeding 10^{-4}.

The next aspect we will have a look at, is the residual noise of the indirect pumping rate $\tilde{r}$. A numerical analysis for the parameters used in our discussion (Fig. 2) reveals the fact, that relative changes $\Delta\tilde{r}/\tilde{r}$ will introduce $\Delta(\partial_\omega n)/\partial_\omega n \simeq 0.6 \times \Delta\tilde{r}/\tilde{r}$. In addition, any deviation $\Delta\tilde{r}$ will induce an absorption/gain of $\alpha \simeq -1.8$ m^{-1} $\Delta\tilde{r}/\tilde{r}$. Again, as for fluctuations of the atomic density, the changes of the indirect pumping rate give rise to variations of the dispersion rather than of the index of refraction. It is not too unrealistic to assume stable indirect pumping rates with $\Delta\tilde{r}/\tilde{r} < 10^{-4}$. Then again, relative fluctuations of the dispersion will not exceed 10^{-4} and the fluctuation induced absorption/gain will be less than $\approx 10^{-4}$ m^{-1}.

We now will be concerned with variations of the pumping field power. The numerical analysis points out, that for the parameters used in our discussion (Fig. 2) $\Delta(\partial_\omega n)/\partial_\omega n \simeq 0.8 \times \Delta I_k/I_k$ and $\alpha \simeq -2.4$ m$^{-1} \times \Delta I_k/I_k$, where I_k is the driving field laser power. Of course, it is no problem to have well stabilized coherent radiation sources with $\Delta I_k/I_k \ll 10^{-4}$ [21], so that again relative variations of the dispersion will not exceed 10^{-4} and the fluctuation induced absorption will be less than 10^{-4} m^{-1}. We also would like to outline that, as was mentioned before, variations ΔI_k do affect the dispersion rather than change the index of refraction.

The last aspect we would like to deal with is residual noise of the driving field laser frequency. The numerical analysis for the parameters used by us (Fig. 2) shows, that noise of the driving field frequency $\Delta(\Delta_k)$ to first order does not affect the dispersion and the absorption. In contrast to the results presented above, these fluctuations actually introduce first order corrections to the index of refraction (at resonance $\Delta_p = 0$). To be more pictorial, the index profile as a whole (in the range of $|\Delta_p| < 0.5\gamma$) is shifted up and down when Δ_k changes from zero to positive or negative values. The first order corrections are given by $\Delta n = 4.1 \times 10^{-8}$ $\Delta(\Delta_k)/\gamma$.

[redacted]

Of course, it is not too unrealistic to have $\Delta(\Delta_k)/\gamma < 10^{-5}$ [21]. Therefore it should be possible to have $\Delta n < 4 \times 10^{-13}$, which, for a wide range of applications, should be small enough.

Taking into account these results we now may question for the influence of the fluctuations of the optical properties onto the performance of the white-light cavity. The maximum bandwidth of the white-light cavity is of order γ due to the refractive index profile of the medium (Fig. 2). Of course, this defines the maximum bandwidth enhancement but nevertheless, in general the bandwidth enhancement will be much smaller (see Fig. 3). In the following, we will describe this enhancement by the factor $g = \gamma_{\mathrm{wlc}}/\gamma_{\mathrm{standard}}$, where $\gamma_{\mathrm{standard}}$ is the bandwidth of the standard cavity and γ_{wlc} the corresponding bandwidth of the same cavity when operated as a white-light cavity. We now allow for some small fluctuations of the dispersion $\partial_\omega n$ due to some noise of the driving field power and frequency, the atomic density and the indirect pumping rate. Then $g^{\max} = \partial_\omega n / \Delta(\partial_\omega n)$ will be the maximum possible bandwidth enhancement factor. Therefore, as far as $g^{\max} > g$, all fluctuations may be neglected. From Fig. 2 we can see, that fluctuation induced absorption being on the order of $\sim 10^{-4}\ \mathrm{m}^{-1}$ may be neglected as well.

In conclusion, $\Delta(\partial_\omega n)/\partial_\omega n < 10^{-4}$ should be possible. Therefore, the maximum bandwidth enhancement factor is of order 10^4. This is about one order of magnitude more than we have predicted above (Fig. 3). Therefore, the consideration of the noise of the driving field power and frequency, the atomic density and the indirect pumping rate seems to be not against the realization of the white-light cavity. The influence of the Doppler-effect was found to be crucial but may be avoided by using an atomic beam.

In summary, in this letter we have proposed a new concept for cavities with large internal buildup, but nevertheless broadband response, which can be used to enhance the sensitivity of interferometric gravitational wave detectors. We have shown that an optically pumped double-Λ system can exhibit the right amount of (strong) negative dispersion without absorption due to the coherent coupling of the two lower states. These non-classical properties of a medium inside a cavity permit the realization of what we have called a *white-light* cavity. If introduced into the signal recycling cavity of a GWD at the output of the interferometer just in front of the signal recycling mirror, such a medium would increase the bandwidth of a signal recycled detector with 10 ppm loss by a factor of 320. Or, implemented in the power recycling cavity of a Sagnac-type receiver just in front of the the power recycling mirror, it would permit using higher reflectivity mirrors in the power recycling cavity for a power buildup 320 times higher than otherwise, corresponding to a gravitational wave amplitude sensitivity which is $\sqrt{320} \approx 18$ times better than that of a normal Sagnac interferometer. Fortunately, this enhancement increases as the cavity-Q increases so that this new concept becomes interesting especially for high-Q cavities. Finally, it should be noted that this technique is only concerned with the shot-noise limited sensitivity, it does not address the other fundamental noise sources in a gravitational wave detector, in particular thermal noise and the uncertainty principle, which necessitate a very long armlength for the interferometer.

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