WORLD-SHEET VERSUS SPECTRUM SYMMETRIES IN HETEROTIC AND TYPE II SUPERSTRINGS

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We discuss the relation between world-sheet symmetries of general four-dimensional super-string theories and the classification of their massless and massive excitations. This follows from the connection between extended superconformal algebras on the world-sheet and extended space-time supersymmetry algebras. In particular we show that the relevant Kac–Moody symmetries are extended to the spectrum symmetries of the supermultiplets of states. This analysis reproduces results for the field theory limit of superstring compactifications.

1. Introduction

Recently some progress [1–8] has been made in understanding the geometrical properties of the moduli spaces [9] of four-dimensional string theories [1, 10–16]. Many of these results are obtained using arguments based on the symmetries of the low energy effective lagrangians. This method proves especially powerful for those string vacua possessing space-time supersymmetry such as compactification on Calabi–Yau manifolds [10] which are thought to be stable to all finite orders in string pertubation theory. In order to determine the structure of the moduli space of the string compactification one has to know the invariances of the string theory, i.e. the symmetries which leave the string spectrum invariant. Therefore, it is important to understand the relation between the space-time properties of these string vacua and the properties of the underlying superconformal field theory.

The aim of the present paper is to clarify the connection between the chiral Kac–Moody symmetry currents on the string world-sheet and the symmetries which

are known to classify the spectrum of any field theory with (extended) space-time supersymmetry. For the massless string modes in the heterotic string theories the internal Kac–Moody symmetries are identical to the symmetries which follow from the space-time supersymmetry algebra. This is implied by the existence of auxiliary (unphysical) gauge fields for the corresponding Kac–Moody symmetries [17,18] such that the kinetic terms in the effective lagrangian of the massless modes are invariant under these symmetries. Furthermore this fact became also clear from the relation [13,18–23] between the exceptional groups and the supersymmetric string theories. The exceptional groups always contain the internal Kac–Moody group as a subgroup; the auxiliary gauge fields directly follow from the representations of the exceptional groups. However the classification of the massive string spectrum by the world-sheet Kac–Moody symmetries is, except for local gauge symmetries, meaningless from the space-time point of view because the Kac–Moody symmetry generators do not commute with the little group SO(3). It means that different helicity components of a single massive state are in different representations of the world-sheet Kac–Moody symmetry group. We show that the symmetry generators which classify all states in a Lorentz-invariant fashion are in general linear combinations of the Kac–Moody currents and additional world-sheet currents which are however not of the Kac–Moody type.

The determination of the space-time symmetries is especially interesting for the type II superstring theories since additional constraints on the moduli space of the superconformal field theory, when used for compactification of type II theories, are obtained due to the enhanced space-time supersymmetry with respect to the heterotic theory [3,4]. Simultaneously, the enlarged space-time supersymmetry algebra also extends the world-sheet Kac–Moody symmetries to a larger group. For example, in the case of (2,2) compactification on a Calabi–Yau manifold the $N = 2$ supersymmetry algebra leads to a SU(2) × U(1) symmetry which classifies all states. The corresponding (commuting) generators are built from the holomorphic and antiholomorphic internal U(1) Kac–Moody as well as the holomorphic and anti-holomorphic space-time SO(2) helicity currents. In this way, deriving the SU(2) generators directly from the string world-sheet degrees of freedom, we can determine the SU(2) transformation properties of any state. E.g. the scalars of the universal sector of the $N = 2$ theory which contain the space-time dilaton transform as a SU(2) doublet and must therefore be part of a complex hypermultiplet which parametrizes the quaternionic manifold (SU(2,1)/SU(2) × U(1)). This proves the result of ref. [4] which was obtained by use of the hidden symmetries of the $N = 8$ supergravity lagrangian. More generally, we are able to derive the linearly realized H part of the supergravity non-compact scalar manifolds G/H (especially for the scalars of the universal sector) directly from the string world-sheet symmetries. The hidden non-compact symmetries G are reproduced knowing the H transformation properties of all massless scalars since they are just the coset representatives of the corresponding non-linear σ-models.

The paper is organized as follows. In sect. 2 we explain how one obtains a Lorentz-invariant classification of the string spectrum from the underlying world-sheet Kac–Moody symmetries. In sect. 3 this discussion will be the basis for the derivation of the relation between the space-time symmetries, which are implied by the space-time supersymmetry algebra, and the world-sheet symmetry currents of the underlying superconformal field theory. We emphasize the difference between the heterotic and type II superstrings and also the different classification of massive states due to the presence of central charges in the supersymmetry algebra. Finally in sect. 4 these results are applied to some models to derive the transformation properties of the massless scalars under the relevant symmetry groups and to reproduce the known coset manifolds which are parametrized by the massless scalars.

2. General aspects of symmetry currents in four-dimensional fermionic string theories

In this section we want to emphasize some issues concerning symmetries which classify massless and massive states in four-dimensional fermionic string theories (see also the discussion of Banks and Dixon [24]). The holomorphic world-sheet degrees of freedom of the fermionic string in four uncompactified space-time dimensions consist of four free bosons $X^\mu(z)$ and fermions $\psi^\mu(z)$ ($\mu = 1, \ldots, 4$) together with the conformal and superconformal ghosts $b(z)$, $c(z)$, $\beta(z)$ and $\gamma(z)$. Via bosonization one can consider instead of $\psi^\mu(z)$ two bosonic fields $\phi_i(z)$ ($i = 1, 2$) and instead of $\beta(z)$, $\gamma(z)$ a scalar field $\phi(z)$ [25]. These fields provide -9 units to the central charge of the Virasoro algebra. Thus, the so far unspecified internal $n = 1$ superconformal field theory of the fermionic string must have $c = 9$ to cancel the conformal anomaly. The local $n = 1$ world-sheet supersymmetry is generated by the supercurrent $T_F(z)$ which is a sum of the space-time supercurrent $T_F^{\text{s.t.}}(z) = -\frac{1}{2}\psi_\mu \partial X^\mu(z)$ and the internal supercurrent $T_F^{\text{int}}(z)$ which has to satisfy the internal superconformal algebra

$$T_F^{\text{int}}(z) T_F^{\text{int}}(w) = \frac{\frac{3}{2}}{(z-w)^3} + \frac{\frac{1}{2}T^{\text{int}}(w)}{(z-w)} + \cdots$$

$$T^{\text{int}}(z) T_F^{\text{int}}(w) = \frac{\frac{3}{2}T_F^{\text{int}}(w)}{(z-w)^2} + \frac{\partial T_F^{\text{int}}(w)}{(z-w)} + \cdots . \tag{2.1}$$

The covariant vertex operators of the fermionic string can be written as

$$V_w(z) = \left(\partial^n X^\mu(z)\right)^N \left(\partial^m \phi_i(z)\right)^M e^{iw \cdot \Phi(z)} V_{\text{int}}(z),$$

$$w = (\lambda_1, \lambda_2; q), \quad \Phi = (\phi_1, \phi_2; -i\phi). \tag{2.2}$$

The $V_{int}(z)$ are operators of the internal $c = 9$ superconformal field theory. The conformal weight of $V_w(z)$ is given by

$$h = nN + mM + \tfrac{1}{2}\lambda^2 - \tfrac{1}{2}q^2 - q + h_{int}, \qquad (2.3)$$

where h_{int} is the conformal weight of $V_{int}(z)$. The w's are lattice vectors of the lorentzian "covariant" lattice $D_{2,1}$ [13] with conjugacy classes 0, S, C, V (for a review with more details and references on covariant lattices see ref. [26]). Decomposing $D_{2,1}$ to $D_2 \otimes D_1$ the factor D_2 corresponds to the $SO(4)_{Lorentz}$ level one Kac–Moody algebra with lattice vectors $\lambda \in D_2$. λ describes the transformation properties under the space-time Lorentz group $SO(4)_{Lorentz}$ and $q \in D_1$ is the ghost charge. For space-time fermions λ is a weight vector of one of the two spinor conjugacy classes S, C of D_2 and q is half-integer. Space-time bosons have $\lambda \in 0$, V and q integer. A unique identification of physical, BRST invariant states is given if one considers only the canonical ghost charge $q = -1, -\tfrac{1}{2}$ and counts only the transverse degrees of freedom of any state. This amounts in this formalism to decompose $D_{2,1}$ to $D_1^{transverse} \otimes D_{1,1}$. Then physical light-cone states are characterized by their helicity λ_1 and have fixed entries in $D_{1,1}$. On the other hand, states in different ghost pictures are only physical if they can be obtained from physical states in the canonical ghost picture via the so-called picture changing operation, which is basically a two-dimensional supersymmetry transformation:

$$V_{q+1}(w) = \lim_{z \to w} P_F(z) V_q(w), \qquad P_F(z) = e^{\phi(z)} T_F(z). \qquad (2.4)$$

Let us briefly discuss the symmetries which classify the massless states in the theory. Any massless state is characterized by its transformation properties under the transverse Lorentz group $SO(2)$, i.e. by its helicity component $\lambda_1 \in D_1$. For the state to be massless h must be one and therefore λ_1 can be $0, \pm 1, \pm \tfrac{1}{2}$ with corresponding $h_{int} = \tfrac{1}{2}, 0, \tfrac{3}{8}$. In addition, massless states in the canonical ghost picture cannot have oscillator excitations. The generator of the transverse Lorentz group with the correct action on the massless states in the canonical ghost picture is given by the $SO(2)$ Kac–Moody current:

$$L^3 \equiv L^{12} = \oint \frac{dz}{2\pi i} L^3(z), \qquad L^3(z) = \psi^1\psi^2(z) = i\,\partial\phi_1(z). \qquad (2.5)$$

The transverse Lorentz symmetry is of course not a gauge symmetry since $L^3(z)$ cannot provide a super-BRST invariant gauge boson vertex operator, i.e. $L^3(z)$ is not the highest component of a two-dimensional superfield and therefore cannot be obtained via picture changing from a physical vertex operator in the -1 ghost picture.

Now assume that there is an internal Kac–Moody algebra g generated by dimension one currents $j^a(z)$. It is clear that all massless states automatically build

representations of g; $SO(2)_{\text{helicity}}$ trivially commutes with g. For g being a local gauge symmetry with corresponding BRST invariant gauge boson vertex operator

$$V(\bar{z}, z) = \partial X^{\mu}(\bar{z}) j^{a}(z) e^{ik_{r}X^{r}(\bar{z}, z)}, \tag{2.6}$$

the currents $j^{a}(z)$ must be the upper components of a dimension $\frac{1}{2}$ world-sheet superfield:

$$T_{F}^{\text{int}}(z) j^{a}(w) = \frac{1}{(z-w)^{2}} \hat{j}^{a}(w) + \frac{\partial \hat{j}^{a}(w)/2}{z-w} + \cdots,$$

$$T_{F}^{\text{int}}(z) \hat{j}^{a}(w) = \frac{j^{a}(w)/2}{z-w} + \cdots. \tag{2.7}$$

Then in the canonical ghost picture the gauge bosons of g have the form

$$V(\bar{z}, z) = \partial X^{\mu}(\bar{z}) \hat{j}^{a}(z) e^{-\phi(z)} e^{ik_{\mu}x^{\mu}(\bar{z}, z)}. \tag{2.8}$$

If eq. (2.7) is not satisfied g cannot be a local gauge symmetry but only a symmetry which classifies all massless states. (Then the vertex operator (2.6) corresponds to auxiliary (non-physical) gauge fields.)

As a simple example consider the holomorphic torus compactification of the fermionic string from ten to four dimensions. The internal free fermions and bosons $\psi^{M}(z)$, $\partial Y^{M}(z)$ $(M = 1, \ldots, 6)$ generate an internal Kac–Moody algebra

$$g = SO(6) \times [U(1)]^{6} \tag{2.9}$$

with dimension one currents

$$j_{SO(6)}^{MN}(z) = \psi^{M}\psi^{N}(z), \qquad j_{U(1)}^{M}(z) = i \, \partial Y^{M}(z). \tag{2.10}$$

The internal supercurrent is simply

$$T_{F}^{\text{int}}(z) = -\frac{1}{2} \sum_{M=1}^{6} \psi^{M} \partial Y^{M}(z). \tag{2.11}$$

It now follows immediately that the SO(6) currents do not lead to a SO(6) gauge symmetry. On the other hand, the $\partial Y^{M}(z)$ are the two-dimensional superpartners to $\psi^{M}(z)$ such that $[U(1)]^{6}$ is a local gauge symmetry of the theory. The six physical gauge bosons are the six right-moving graviphotons which arise in any compactification on a six-torus.

Let us now turn to the discussion of the massive states. We will go to their rest frame where their momentum is $k^{\mu} = (M, 0, 0, 0)$ with M being their mass. The

SO(2) helicity generator gets an additional contribution which contains the transverse bosonic coordinates $X^i(z)$ $(i = 1, 2)$ and acts only on massive states with X-oscillator excitations:

$$L^3(z) = \psi^1 \psi^2(z) - \tfrac{1}{2}(X^+ \, \partial X^-(z) - X^- \, \partial X^+(z)), \qquad (2.12)$$

where $X^\pm = (X^1 \pm iX^2)/\sqrt{2}$. Note that we have defined the holomorphic objects $X^i(z)$ as integrals over the dimension one fields $\partial X^i(z)$, $X^i(z) = \int^z dz' \, \partial X^i(z')$. In the rest frame we will not encounter logarithmic z-dependences in the operator product between $L^3(z)$ and physical state vertex operators since $L^3(z) e^{ik \cdot X(w)} =$ finite. In this way, L^3 correctly measures the helicity eigenvalue ± 1 of $\partial X^\pm(z)$. Also, eq. (2.12) leads to the correct oscillator expression.

However massive states must build representations of the little group $SO(3)_{\text{Lorentz}}$ and not of $SO(2)_{\text{helicity}}$. In the light-cone formalism we are using this means that acting with the raising and lowering operators $L^\pm$ of $SO(3)$ the different helicity components of a single massive state are obtained from each other. $L^\pm$ has a rather complicated form in the light-cone formalism. We are only interested in the part without purely bosonic transverse oscillator excitations:

$$L^\pm = \oint \frac{dz}{2\pi i} \, z L^\pm(z), \qquad L^\pm(z) \sim \psi^\pm(z) T_F'(z), \qquad (2.13)$$

where

$$T_F'(z) = -\tfrac{1}{2}(\partial X^+ \psi^-(z) + \partial X^- \psi^+(z)) + T_F^{\text{int}}(z).$$

Note that $L^\pm(z)$ acts very similar to a picture changing, resp. two-dimensional supersymmetry transformation. Therefore we can understand the appearance of the second term in the SO(2) helicity generator (2.12) from a slightly different, but equivalent point of view. We know that the picture changing operation always provides a second version of any state. E.g. a massless holomorphic vector $\psi^\pm(z) e^{-\phi(z)}$ becomes in the zero ghost picture simply $\partial X^\pm(z)$. Thus the second term in eq. (2.12) ensures that also the picture changed version has helicity ± 1.

Now let us turn to the internal Kac–Moody symmetries. It is clear that all states, massless and massive, build representations of the Kac–Moody symmetry g, i.e. g classifies the whole string spectrum since it is a symmetry of the string theory. The non-trivial question we are interested in is whether g is also a good symmetry from the space-time point of view, i.e. whether it classifies complete massive multiplets and commutes with the Lorentz group SO(3). This is not always the case as we shall see. Specifically, for massive multiplets to build representations under the internal symmetry group g one must ensure that $L^\pm$ commutes with the symmetry currents j^a, i.e.

$$\oint \frac{dz}{2\pi i} \, L^\pm(z) j^a(w) = 0 \text{ up to total derivatives}. \qquad (2.14)$$

Again we consider two cases. First, if the dimension one currents $j^a(z)$ are the upper components of a dimension $\frac{1}{2}$ superfield such that it leads to a local gauge symmetry, eq. (2.14) is automatically satisfied. However if $j^a(z)$ corresponds to a BRST variant "global" current it does not commute with $L^\pm$. Then the symmetry currents, similar to L^3, need an additional piece which does not act on the massless states in the canonical ghost picture.

As an example take again the toroidal compactification from ten to four dimensions. It is easily shown that the $[U(1)]^6$ gauge currents ∂Y^M commute with $L^\pm$. However in order to obtain the correct SO(6) currents which classify all massive states one has to replace eq. (2.10) by

$$j^{MN}_{SO(6)}(z) = \psi^M \psi^N(z) + \tfrac{1}{2} i \left(Y^M(z) \, \partial Y^N(z) - Y^N(z) \, \partial Y^M(z) \right). \qquad (2.15)$$

Note that the second part in this equation is not a generator of an internal Kac–Moody symmetry. The SO(6) which commutes with $SO(3)_{Lorentz}$ is the sum of the internal $SO(6)_{Kac-Moody}$ currents, generated by the fermions ψ^M, and the SO(6) generated by the internal bosons Y^M. This behavior originates from the fact that the internal supercurrent (2.11) defines a map from the $c = 3$, SO(6) Kac–Moody algebra to the internal $c = 6$ conformal field theory. The image of the SO(6) vector ψ^M is nothing other than the $[U(1)]^6$ gauge currents ∂Y^M. Thus, the second part in eq. (2.15) must be included. We will encounter this phenomenon in all supersymmetric string theories as described in sect. 3. However it is not true that all massive multiplets fall into SO(6) representations. Remember that in the rest frame of massive states their transverse momenta vanish and the zero modes of X^i decouple. As far as the internal coordinates $Y^M(z)$ (which we define as $Y^M(z) = \int^z dz' \, \partial Y^M(z')$) are concerned, their zero modes break the SO(6) symmetry. Only those states which carry no internal momentum p^M, i.e. which do not have $e^{ip^M Y^M(z)}$ as part of their vertex operator, build SO(6) representations. We will discuss this phenomenon more carefully in the context of central charges of the supersymmetry algebra.

3. Symmetry groups in supersymmetric string theories

3.1. WORLD-SHEET SYMMETRIES

Let us consider N holomorphic (i.e. right-moving) space-time supercharges Q^A_α ($A = 1, \ldots, N$):

$$Q^A_\alpha = \oint \frac{dz}{2\pi i} \, Q^A_\alpha(z). \qquad (3.1)$$

In the covariant formalism $Q^A_\alpha(z)$ are conformal fields of dimension one and are given by

$$Q^A_\alpha(z) = e^{i\lambda_\alpha \cdot \phi(z)} e^{-\phi(z)/2} \Sigma^A(z), \qquad (3.2)$$

where λ_α are the two spinor weights of $SO(4)_{\text{Lorentz}}$ and $\Sigma^A(z)$ are conformal fields of dimension $h = 3/8$ of the internal $c = 9$ superconformal field theory. The supercharges satisfy the N-extended supersymmetry algebra

$$\left\{ Q_\alpha^A, Q_{\beta B}^* \right\} = 2i\, \delta_B^A (\gamma^\mu)_{\alpha\beta} P_\mu,$$

$$\left\{ Q_\alpha^A, Q_\beta^B \right\} = 2C_{\alpha\beta} Z^{AB}. \tag{3.3}$$

For $N > 1$, the central charges $Z^{AB} = \oint (dz/2\pi i)\, Z^{AB}(z)$ arise due to poles in the operator product expansion of $Q_\alpha^A(z)$ and $Q_\beta^B(w)$.

It is by now well known [18, 27–29] that the existence of the holomorphic supercharges implies an n-extended internal superconformal algebra with corresponding Kac–Moody symmetry g. Its currents can always be realized by free bosons or free fermions since g is at level one. Specifically, for $N = 1$ one deals with a system with $c = 9$, $n = 2$ and g $= U(1)$, for $N = 2$ one obtains a system with $c = 6$, $n = 4$ and g $= SU(2)$ plus a system with $c = 3$, $n = 2$ and g $= SO(2)$ where the second part corresponds to a holomorphic torus compactification of two internal coordinates, and finally for $N = 4$ there are three systems with $c = 3$, $n = 2$, and g $= SO(6)$ which corresponds to a holomorphic torus compactification of all six internal coordinates. The existence of the central charges in eq. (3.3) reflects the fact that N-extended theories are holomorphic torus compactifications. The central charge currents $Z^{AB}(z)$ are nothing else than the compactified coordinates $\partial Y^M(z)$ [18]. Thus the central charge of any state is given by its internal momentum on the torus.

It is also known [13, 18–22] that the weight lattice of $D_2^{\text{Lorentz}} \otimes D_1^{\text{ghost}} \otimes$ g, when demanding the necessary charge quantization conditions, is extended to the lorentzian lattices $E_{3,1}$, $E_{4,1}$, $E_{5,1}$ for $N = 1, 2, 4$. These lattices are the lorentzian analogs of the euclidian weight lattices of the exceptional groups E_6, E_7 and E_8 which are obtained by replacing the negative metric part D_1^{ghost} by a positive metric lattice D_3. This fact is very important when constructing the spectrum of the supersymmetric heterotic or type II theories since the representations of the exceptional groups contain all information about the supermultiplet structure of the corresponding supergravity theory.

Let us briefly discuss the different cases. For $N = 1$ the relevant $U(1)$ Kac–Moody current is given by

$$j(z) = i\sqrt{3}\, \partial H(z), \tag{3.4}$$

where $H(z)$ is an internal free boson. The $n = 2$ superconformal algebra

$$T_F^+(z) T_F^-(w) = \frac{3/4}{(z-w)^3} + \frac{j(w)/4}{(z-w)^2} + \frac{\partial j(w)/8 + T_{\text{int}}(w)/4}{(z-w)} \tag{3.5}$$

is generated by the two supercurrents

$$T_F^{\pm}(z) = e^{\pm i(\sqrt{3}/3)H(z)}\tilde{T}_F^{\pm}(z), \qquad (3.6)$$

where $\tilde{T}_F^{\pm}(z)$ are operators in the remaining conformal field theory with $c = 8$. According to the discussion in sect. 2 the U(1) current (3.4) is not a gauge current and acts properly only on the massless states in the canonical ghost picture since $j(z)$ is not the highest component of a two-dimensional dimension $\frac{1}{2}$ superfield. From eqs. (2.13) and (3.6) it is evident that $j(z)$ as given in eq. (3.4) does not commute with SO(3)$_{\text{Lorentz}}$. We have to complete it by a dimension one operator $\tilde{j}(z)$ which satisfies $\tilde{j}(z)\tilde{T}_F^{\pm}(w) \sim \mp \tilde{T}_F^{\pm}/(z - w)$. In this way the U(1) Kac–Moody symmetry finds its image in the internal $c = 8$ part of the theory. In order to indicate how $\tilde{j}(z)$ can be constructed, let us consider a situation where $\tilde{T}_F^{\pm}(z)$ can be written as

$$T_F^{\pm}(z) \sim \sum_{M=1}^{k} G_M^{\pm}(z)\tilde{O}_M^{\pm}(z) = e^{\pm i(\sqrt{3}/3)H(z)}\sum_{M=1}^{k}\tilde{G}_M^{\pm}(z)\tilde{O}_M^{\pm}(z), \qquad (3.7)$$

where the operators $\tilde{G}_M^{\pm}(z)$ and $O_M^{\pm}(z)$ have conformal dimensions $\frac{1}{3}$ and 1 respectively. $G_M^{\pm}$ and $O_M^{\pm}$ are the two components of dimension $\frac{1}{2}$ superfields with operator products

$$\tilde{O}_M^{+}(z)\tilde{O}_N^{-}(w) \sim \delta^{MN}(z - w)^{-2},$$

$$G_M^{+}(z)G_N^{-}(w) \sim \delta^{MN}(z - w)^{-1}. \qquad (3.8)$$

Assuming the existence of k operators $\tilde{G}_M^{\pm}(z)$ immediately leads to k holomorphic vertex operators for k massless chiral multiplets:

$$V_{\text{scalar}}(z) = e^{\pm i(\sqrt{3}/3)H(z)}\tilde{G}_M^{\pm}(z)e^{-\phi(z)},$$

$$V_{\text{fermion}}(z) = e^{i\lambda_\alpha \cdot \phi(z)}e^{\mp i(\sqrt{3}/6)H(z)}\tilde{G}_M^{\pm}(z)e^{-\phi(z)/2}. \qquad (3.9)$$

(λ_α is a spinor weight of SO(4).) The (internal) picture changed versions of these operators are

$$V_{\text{scalar}}(z) = \tilde{O}_M^{\mp}(z), \qquad V_{\text{fermion}}(z) = e^{i\lambda_\alpha \cdot \phi(z)}e^{\mp i(\sqrt{3}/2)H(z)}\tilde{O}_M^{\mp}(z)e^{\phi(z)/2}. \qquad (3.10)$$

(One of these operators is present in any symmetric (2, 2) Calabi–Yau compactification. It leads, after combining it with the analogous left-moving piece, to an SU(3) invariant state – the corresponding complex scalar field is associated with the harmonic (1,1)-form which corresponds to the overall scale deformations of the

six-dimensional compact manifold.) Then the U(1) current which commutes with $L^{\pm}$ is given by

$$j(z) = i\sqrt{3}\,\partial H(z) - \tfrac{1}{2}\left\{\sum_{M=1}^{k}\left(\int^{z}\mathrm{d}z'\,\tilde{O}_{M}^{+}(z')\tilde{O}_{M}^{-}(z) - \int^{z}\mathrm{d}z'\,\tilde{O}_{M}^{-}(z')\tilde{O}_{M}^{+}(z)\right)\right\}.$$

(3.11)

Adding the second term in the above equation it is ensured that $\tilde{T}_{F}^{\pm}(z)$ has U(1) charge ∓ 1 and also that the scalar and fermion operators (3.9) and (3.10) have the same U(1) charge in both ghost pictures.

To illustrate this procedure which might seem ad hoc, consider the symmetric Z_3 orbifold with internal complex bosonic coordinates $\partial Y_{M}^{\pm}(z)$ ($M = 1,\ldots,3$) and their superpartners $\psi_{M}^{\pm}(z)$. We can write $\psi_{M}^{\pm}(z)$ as

$$\psi_{M}^{\pm}(z) = \mathrm{e}^{\pm i(\sqrt{3}/3)H(z)}\tilde{G}_{M}^{\pm}(z),$$

$$\tilde{G}_{M}^{\pm}(z) = \mathrm{e}^{\pm i w_{M}\cdot\tilde{H}(z)},$$

$$i\sqrt{3}\,\partial H(z) = \sum_{M=1}^{3}\psi_{M}^{+}\psi_{M}^{-}(z),$$

(3.12)

where the w_{M} are the three weights of the fundamental representation of SU(3) and $(H,\tilde{H})$ are the three free bosons obtained from bosonizing $\psi_{M}^{\pm}$. (Combining with the corresponding left-moving vertex operators, $\psi_{M}^{\pm}(z)$ resp. $\partial Y_{M}^{\pm}(z)$ leads to the nine complex untwisted moduli of the Z_3 orbifold.) The two internal supercurrents take the form

$$T_{F}^{\pm}(z) = -\tfrac{1}{2}\sum_{M=1}^{3}\psi_{M}^{\pm}(z)\,\partial Y_{M}^{\pm}(z) = -\tfrac{1}{2}\mathrm{e}^{\pm i(\sqrt{3}/3)H(z)}\sum_{M=1}^{3}\tilde{G}_{M}^{\pm}(z)\,\partial Y_{M}^{\pm}(z).$$

(3.13)

Clearly, the operators $\tilde{O}_{M}^{\pm}(z)$ have to be identified with $\partial Y_{M}^{\pm}(z)$.

The case $N = 2$ works out quite similarly. Here the $c = 6$, $n = 4$ superconformal algebra contains the SU(2) currents

$$j^{3}(z) = i\sqrt{\tfrac{1}{2}}\,\partial H(z), \qquad j^{\pm}(z) = \sqrt{\tfrac{1}{2}}\,\mathrm{e}^{\pm i\sqrt{2}H(z)},$$

(3.14)

and the four internal supercurrents are given by

$$T_{F}^{a}(z) = \mathrm{e}^{\pm i(\sqrt{2}/2)H(z)}\tilde{T}_{F}^{\pm}(z) \quad (a = ++, +-, -+, --),$$

(3.15)

where $\tilde{T}_F^{\pm}$ are two complex conjugate fields of dimension $\frac{5}{4}$. They can be constructed analogously to the previous case. Now there is a map from the SU(2) Kac–Moody to a second SU(2) in the $c = 5$ part of the $n = 4$ superconformal theory:

$$\tilde{T}_F^{\pm}(z) = \sum_{M=1}^{k} \left(\tilde{G}_M^+(z)\tilde{O}_M^{\pm +}(z) + \tilde{G}_M^-(z)\tilde{O}_M^{\pm -}(z) \right) \tag{3.16}$$

(the fields $\tilde{G}_M^{\pm}(z)$ now have conformal dimension $\frac{1}{4}$). Then the Lorentz-invariant SU(2) currents are given by

$$j^3(z) = i\sqrt{\tfrac{1}{2}}\; \partial H(z) - \tfrac{1}{2}\Bigg\{ \sum_{M=1}^{k} \left(\int^z \mathrm{d}z'\, \tilde{O}_M^{++}(z')\tilde{O}_M^{--}(z) + \int^z \mathrm{d}z'\, \tilde{O}_M^{+-}(z')\tilde{O}_M^{-+}(z) \right.$$

$$\left. - \int^z \mathrm{d}z'\, \tilde{O}_M^{--}(z')\tilde{O}_M^{++}(z) - \int^z \mathrm{d}z'\, \tilde{O}_M^{-+}(z')\tilde{O}_M^{+-}(z) \right) \Bigg\},$$

$$j^{\pm}(z) = \sqrt{\tfrac{1}{2}}\; \mathrm{e}^{\pm i\sqrt{2}\,H(z)} - \tfrac{1}{2}\Bigg\{ \sum_{M=1}^{k} \left(\int^z \mathrm{d}z'\, \tilde{O}_M^{\pm +}(z')\tilde{O}_M^{\pm -}(z) - \int^z \mathrm{d}z'\, \tilde{O}_M^{\pm -}(z')\tilde{O}_M^{\pm +}(z) \right) \Bigg\}.$$

$$\tag{3.17}$$

The second part of the internal conformal field theory with $c = 3$, $n = 2$ and $g = SO(2) \times [U(1)]^2$ is very simple since it corresponds to a compactification of two internal coordinates and the global SO(2) as well as the local $[U(1)]^2$ currents are expressed by the complex internal fields $\psi^{\pm}(z) = \mathrm{e}^{\pm iH'(z)}$ and $\partial Y^{\pm}(z)$:

$$j_{SO(2)}(z) = i\,\partial H'(z) - \tfrac{1}{2}\left(Y^+(z)\,\partial Y^-(z) - Y^-(z)\,\partial Y^+(z) \right),$$

$$j_{U(1)}^{\pm} = i\,\partial Y^{\pm}(z). \tag{3.18}$$

Finally the case $N = 4$ with $g = SO(6) \times [U(1)]^6$ corresponds to a holomorphic torus compactification of all six internal coordinates and was already discussed in sect. 2.

The existence of these kinds of symmetries is not in contradiction to the statement [24] that there are no global continuous symmetries in string theory. The currents we have considered correspond only to symmetries in the sense that they classify the string spectrum, but they will not be obeyed by the effective action of the string states (except for massless states with vanishing potential, i.e. the moduli of the superconformal field theory).

3.2. CLASSIFICATION OF MASSLESS STATES

So far we have investigated the space-time helicity and the internal Kac–Moody currents which reflect the world-sheet symmetries of the underlying superconformal field theories. Our aim is to show the relation of these symmetries to those which are known to follow from the representation theory of field theories with extended space-time supersymmetries (see e.g. ref. [30]). In other words, we will relate the representations of the world-sheet Kac–Moody algebras to those of the space-time supersymmetry algebra. Specifically we will show how the spectrum symmetries are built up from the external and internal world-sheet currents. Consider first the case of massless states in the canonical ghost picture. Choosing the frame where the momenta are $k^\mu = (E, 0, 0, E)$ where E is the energy of the state in this frame, the supercharges are ($A = 1, \ldots, N$):

$$Q_1^A \equiv Q^A, \qquad Q_A^{*1} \equiv Q_A^*,$$

$$Q_2^A = 0, \qquad Q_A^{*2} = 0. \tag{3.19}$$

In terms of Q^A and Q_A^* the supersymmetry algebra takes the form

$$\{Q^A, Q_B^*\} = \delta_B^A,$$

$$\{Q^A, Q^B\} = \{Q_A^*, Q_B^*\} = 0, \tag{3.20}$$

where we have rescaled the supersymmetry charges by $\sqrt{E}$. The $2N$ supercharges Q^A, Q_A^* build a SO($2N$) Clifford algebra

$$\Gamma_{2A-1} = Q^A + Q_A^*, \qquad \Gamma_{2A} = i(Q^A - Q_A^*),$$

$$\{\Gamma_i, \Gamma_j\} = 2\,\delta_{ij}, \quad i, j = 1, \ldots, 2N. \tag{3.21}$$

The SO($2N$) generators are

$$\Lambda_{ij} = \frac{1}{4i}[\Gamma_i, \Gamma_j]. \tag{3.22}$$

Consider the SU(N) × U(1) subgroup of SO($2N$) specified by the following generators:

$$\Lambda_B^A = \tfrac{1}{2}[Q^A, Q_B^*] - \frac{1}{2N}\delta_B^A[Q^C, Q_C^*] \quad \text{for SU}(N),$$

$$\Lambda = \tfrac{1}{4}[Q^A, Q_A^*] \quad \text{for U}(1). \tag{3.23}$$

This SU(N) commutes with SO(2)$_{\text{helicity}}$ and classifies massless states. The eigen-value of the supercharge under the U(1), which is called intrinsic helicity, is the same as that under the space-time helicity:

$$\left[L^3, Q^A\right] = \left[\Lambda, Q^A\right] = \tfrac{1}{2}Q^A. \tag{3.24}$$

This shows that one can construct a new generator Λ' called superhelicity,

$$\Lambda' = L^3 - \Lambda, \tag{3.25}$$

which commutes with Q^A and therefore is a constant on supermultiplets.

To derive these generators from the superstring theories we first have to specify the supercharges Q^A and Q_A^*. In the light-cone gauge the longitudinal and ghost parts in eq. (3.2) are irrelevant. We use instead the Green–Schwarz supercharges (see ref. [31]) which are the zero modes of the remaining transverse and internal parts:

$$Q^A = \oint \frac{dz}{2\pi i} z^{-1/2} e^{i\phi_1(z)/2} \Sigma^A(z) \equiv \oint \frac{dz}{2\pi i} z^{-1/2} Q^A(z),$$

$$Q_A^* = \oint \frac{dz}{2\pi i} z^{-1/2} e^{-i\phi_1(z)/2} \Sigma_A^*(z) \equiv \oint \frac{dz}{2\pi i} z^{-1/2} Q_A^*(z). \tag{3.26}$$

Then the supersymmetry algebra (3.20) follows immediately from the operator product expansion of $\Sigma^A(z)$ and $\Sigma_A^*(w)$.

Let us first investigate the heterotic string theories. For $N = 1$ only the intrinsic helicity current is present and $\Sigma^A = e^{i(\sqrt{3}/2)H}$. The operator product expansion between $Q(z) = e^{i\phi_1(z)/2} e^{i(\sqrt{3}/2)H(z)}$ and $Q^*(w)$ has the following form:

$$Q(z)Q^*(w) = \frac{1}{z-w} + \tfrac{1}{2}i\,\partial\phi_1(z) + \tfrac{1}{2}\sqrt{3}\,i\,\partial H(z) + \cdots. \tag{3.27}$$

The singular term leads to the correct anticommutator between Q and Q^*. The first non-singular term is the normal ordered product $:Q(z)Q^*(z):$ which we identify with the U(1) current. Thus, the commutator of two supercharges in field theory corresponds to the zero mode of the normal ordered product of two supercharge currents in string theory which means $\Lambda_B^A \sim Q^A Q^*_B$. This procedure is analogous to the one defining the SO($2n$) Kac–Moody currents as the normal ordered product of two free anticommuting world-sheet fermions. Then for the $N = 1$ theory the intrinsic helicity U(1) current is given by

$$\Lambda(z) = \tfrac{1}{2}:Q(z)Q^*(z): = \tfrac{1}{4}i\,\partial\phi_1(z) + \tfrac{1}{4}i\sqrt{3}\,\partial H(z), \tag{3.28}$$

where the corresponding U(1) charge is $\Lambda = \oint(dz/2\pi i)\Lambda(z)$. One easily verifies that the intrinsic helicity of the supercharge is $\tfrac{1}{2}$. Note that the intrinsic helicity

current is given as a linear combination of the space-time helicity and the internal U(1) Kac–Moody current.

Next we discuss the $N = 2$ heterotic string theory with two supercharges:

$$Q^{1,2}(z) = e^{i\phi_1(z)/2} e^{\pm i(\sqrt{2}/2)H(z)} e^{iH'(z)/2}. \tag{3.29}$$

Then we derive the three SU(2) currents as

$$\Lambda^3(z) = \tfrac{1}{2}\big(:Q^1(z)Q_1^*(z): - :Q^2(z)Q_2^*(z):\big) = i\sqrt{\tfrac{1}{2}}\,\partial H(z),$$

$$\Lambda^+(z) = \sqrt{\tfrac{1}{2}}\,:Q^1(z)Q_2^*(z): \, = \tfrac{1}{2}\sqrt{2}\,e^{i\sqrt{2}H(z)},$$

$$\Lambda^-(z) = \sqrt{\tfrac{1}{2}}\,:Q^2(z)Q_1^*(z): \, = \tfrac{1}{2}\sqrt{2}\,e^{-i\sqrt{2}H(z)}. \tag{3.30}$$

We recognize that these currents are identical to the SU(2) currents of the internal superconformal algebra (3.14). The intrinsic helicity U(1) current is obtained as

$$\Lambda(z) = \tfrac{1}{2}\big(:Q^1(z)Q_1^*(z): + :Q^2(z)Q_2^*(z):\big) = \tfrac{1}{2}i\,\partial\phi_1(z) + \tfrac{1}{2}i\,\partial H'(z). \tag{3.31}$$

Finally for the $N = 4$ heterotic case, the 15 SO(6) currents are again identical to the internal SO(6) world-sheet currents given in eq. (2.10).

Let us now turn to the four-dimensional type II theories which were recently investigated in refs. [3, 4, 23, 32]. There are more possibilities for extended space-time supersymmetries since we now have left-moving and right-moving supercharges. One can obtain theories with $N = 1, 2, 3, 4, 5, 6, 8$ space-time supersymmetry. Their supermultiplet structure follows from the product of the corresponding left- and right-moving exceptional groups [23] and some of the theories were constructed explicitly [32]. The supersymmetry algebra is verified in analogy to the heterotic case. The symmetry currents $\Lambda(z)$ will now contain linear combinations of internal left- and right-moving Kac–Moody currents and also the left- and right-moving space-time helicity generators. This is so because the symmetry generators must transform the left-moving and right-moving supercharges into each other and therefore the right-moving Ramond sector into the left-moving Ramond sector and vice versa. Therefore the non-commuting generators necessarily involve the internal and also the space-time spin fields and some of the commuting generators must measure the left- and right-moving space-time helicities.

Let us consider specific cases. First for $N = 2$ type II (specifically, we are considering type II B) there is one left-moving and one right-moving supercharge each:

$$Q^1 = \oint \frac{dz}{2\pi i}\, z^{-1/2}Q^1(z) = \oint \frac{dz}{2\pi i}\, z^{-1/2} e^{i\phi_1(z)/2} e^{i(\sqrt{3}/2)H(z)},$$

$$Q^2 = \oint \frac{d\bar{z}}{2\pi i}\, \bar{z}^{-1/2}Q^2(\bar{z}) = \oint \frac{d\bar{z}}{2\pi i}\, \bar{z}^{-1/2} e^{i\phi_1(\bar{z})/2} e^{i(\sqrt{3}/2)H(\bar{z})}. \tag{3.32}$$

Then the corresponding SU(2) generators are derived as follows:

$$\Lambda^3 = \oint \frac{\mathrm{d}z}{2\pi i} \frac{1}{2} :Q^1(z)Q_1^*(z): - \oint \frac{\mathrm{d}\bar z}{2\pi i} \frac{1}{2} :Q^2(\bar z)Q_2^*(\bar z):$$

$$= \oint \frac{\mathrm{d}z}{2\pi i} \left\{ \tfrac14 i\, \partial\phi_1(z) + \tfrac14\sqrt{3}\, i\, \partial H(z) \right\} - \oint \frac{\mathrm{d}\bar z}{2\pi i} \left\{ \tfrac14 i\, \bar\partial\phi_1(\bar z) + \tfrac14\sqrt{3}\, i\, \bar\partial H(\bar z) \right\},$$

$$\Lambda^+ = \sqrt{\tfrac12}\, Q^1 Q_2^* = \sqrt{\tfrac12} \left\{ \oint \frac{\mathrm{d}z}{2\pi i} z^{-1/2} Q^1(z) \right\} \left\{ \oint \frac{\mathrm{d}\bar z}{2\pi i} \bar z^{-1/2} Q_2^*(\bar z) \right\},$$

$$\Lambda^- = \sqrt{\tfrac12}\, Q_1^* Q^2 = \sqrt{\tfrac12} \left\{ \oint \frac{\mathrm{d}z}{2\pi i} z^{-1/2} Q_1^*(z) \right\} \left\{ \oint \frac{\mathrm{d}\bar z}{2\pi i} \bar z^{-1/2} Q^2(\bar z) \right\}. \tag{3.33}$$

We see that Λ^3 contains the internal $U(1)_L$, $U(1)_R$ Kac–Moody currents as well as the space-time helicity currents. So the isospin of every massless state in the canonical ghost picture is given by $\tfrac14(\lambda_1 + Q - \bar\lambda_1 - \bar Q)$ where λ_1 ($\bar\lambda_1$) are the left (right) helicities and Q ($\bar Q$) the left (right)-moving internal $U(1)$ charges. The non-commuting SU(2) generators $\Lambda^\pm$ are given as the product of the left- and right-moving supercharges which act independently on the holomorphic and anti-holomorphic sectors of the theory. $\Lambda^\pm$ do not originate from a Kac–Moody algebra in the usual sense. There is no holomorphic mode expansion for these currents where the modes generate an infinite dimensional Kac–Moody algebra. This behavior is very different from the heterotic case. Analogously, the intrinsic helicity generator is given by

$$\Lambda(\bar z, z) = \tfrac14 i\, \partial\phi_1(z) + \tfrac14\sqrt{3}\, i\, \partial H(z) + \tfrac14 i\, \bar\partial\phi_1(\bar z) + \tfrac14\sqrt{3}\, i\, \bar\partial H(\bar z) \tag{3.34}$$

and it is trivial to check that the supercharges $Q^{1,2}$ have intrinsic helicity $\tfrac12$.

We can now construct the SO(6) symmetry currents for the $N = 4$, type II theory where two supercharges are left-moving and two are right-moving in exactly the same way. The result for the three commuting currents is

$$\Lambda^1(\bar z, z) = \tfrac12 i\, \partial\phi_1(z) + \tfrac12 i\, \partial H'(z) - \tfrac12 i\, \bar\partial\phi_1(\bar z) - \tfrac12 i\, \bar\partial H'(\bar z),$$

$$\Lambda^2(\bar z, z) = i\tfrac12\sqrt{2}\, \partial H(z) + i\tfrac12\sqrt{2}\, \bar\partial H(\bar z),$$

$$\Lambda^3(\bar z, z) = i\tfrac12\sqrt{2}\, \partial H(z) - i\tfrac12\sqrt{2}\, \bar\partial H(\bar z). \tag{3.35}$$

The non-commuting generators are given by the generators of $SU(2)_L$ and $SU(2)_R$

and products of left- and right-moving supercharges like

$$\Lambda^{13} = \frac{1}{2}\left\{ \oint \frac{dz}{2\pi i}\, z^{-1/2} Q^1(z) \right\}\left\{ \oint \frac{d\bar{z}}{2\pi i}\, \bar{z}^{-1/2} Q_3^*(\bar{z}) \right\}. \tag{3.36}$$

We see that this SO(6) contains SO(4) = SU(2)$_\mathrm{L}$ × SU(2)$_\mathrm{R}$ as regular subgroup.
  Similarly, all the other models with $N = 3, 5, 6, 8$ can be treated.

3.3. CLASSIFICATION OF MASSIVE STATES AND CENTRAL CHARGES

  The generators Λ constructed so far do not act properly on the massive states – they do not commute with $L^{\pm}$. One must add an additional part which has only action on the massive states. To do so let us again consider the supercharges. Now also the second helicity components are relevant and perform a supersymmetry transformation on the massive states. Let us use the following notation:

$$Q_2^A = \tilde{Q}^A, \qquad Q_A^{*2} = Q_A^*. \tag{3.37}$$

The supersymmetry algebra between the $\tilde{Q}$ looks like

$$\left\{ \tilde{Q}^A, \tilde{Q}_B^* \right\} = M^2 \delta_B^A, \qquad \left\{ \tilde{Q}^A, \tilde{Q}^B \right\} = 0, \tag{3.38}$$

where M^2 is the invariant mass of any state. Because of the existence of the $\tilde{Q}^A$, the Clifford algebra is now enlarged; Q^A and $\tilde{Q}^A$ build an SO(4N) Clifford algebra (on states without central charges – see next paragraph). The maximal subalgebra which commutes with SO(3)$_\mathrm{Lorentz}$ is U Sp(2N). Therefore U Sp(2N) classifies all massive states (without central charge). Consider again the SU(N) × U(1) subgroup of U Sp(2N) with generators $\Lambda_\mathrm{tot} = \Lambda + \tilde{\Lambda}$. The $\tilde{\Lambda}$ are obtained by replacing Q by $\tilde{Q}$ in eq. (3.23). To get an idea what these currents look like we need the expression for $\tilde{Q}^A$. It is clear from our previous discussion that the $\tilde{Q}^A$ correspond to the second versions of the supercharges obtained from Q^A by the picture changing operation with the space-time and internal supercurrent. Thus we are using the following expression:

$$\tilde{Q}^A = \oint \frac{dz}{2\pi i}\, z^{1/2}\left\{ -\tfrac{1}{2} e^{i\phi_1(z)/2} \Sigma^A(z)\, \partial X^-(z) + e^{-i\phi_1(z)/2} \tilde{\Sigma}^A(z) \right\}. \tag{3.39}$$

The first term originates from the action of the space-time supercurrent, the second term arises due to the action of the internal supercurrent. $\tilde{\Sigma}^A(z)$ is defined as

$$T_F^\mathrm{int}(z)\Sigma^A(w) = \frac{1}{(z-w)^{1/2}} \tilde{\Sigma}^A(w) + \cdots. \tag{3.40}$$

Consider first the simplest case of $N = 1$ supersymmetry. Here we derive from eq. (3.40) that $\tilde{\Sigma}^A(z) = e^{i(\sqrt{3}/6)H(z)}\tilde{T}_F^-(z)$. Then using the $n = 2$ superconformal algebra (3.5) we derive that

$$
\tilde{Q}(z)\tilde{Q}^*(w) = \frac{1}{(z-w)^3} + \frac{1}{(z-w)^2}\left\{ -\tfrac{1}{4}i\,\partial\phi_1(w) + i\tfrac{1}{4}\sqrt{3}\,\partial H(w))\right\}
$$

$$
+ \frac{1}{(z-w)}\left\{ -\tfrac{1}{8}\,\partial^2\phi_1(w) + \tfrac{1}{8}\sqrt{3}\,\partial^2 H(w)\right.
$$

$$
\left. +\tfrac{1}{4}\left(-\partial X^+\,\partial X^-(w) - \tfrac{1}{2}\,\partial\phi_1^2(w) + T^{\text{int}}(w)\right)\right\}. \quad (3.41)
$$

Upon contour integration over z and w one obtains the supersymmetry algebra with $M^2 \sim \tfrac{1}{4}(L_0 - \tfrac{1}{2})$.

To construct the U(1) generator $\tilde{\Lambda}$, Λ_{tot} must act on $\tilde{Q}$ in such a way that $\tilde{Q}$ and Q have the same U(1) charge. Inspection of eq. (3.39) shows that $\tilde{\Sigma}(z)$ must carry $-\tfrac{5}{8}$ units of U(1) charge which implies that $\tilde{T}_F^-(z)$ must have U(1) charge $-\tfrac{1}{2}$. Now write $\tilde{T}_F^\pm(z)$ as in sect. 2 and the current $\tilde{\Lambda}(z)$ can be defined as

$$
\tilde{\Lambda}(z) = \tfrac{1}{4}\sum_{M=1}^{k}\left\{\int^z \mathrm{d}z'\,\tilde{O}_M^+(z')\tilde{O}_M^-(z) - \int^z \mathrm{d}z'\,\tilde{O}_M^-(z')\tilde{O}_M^+(z)\right\}. \quad (3.42)
$$

This operator measures the U(1) charge of $\tilde{T}_F^\pm(z)$ and therefore also of $\tilde{Q}$ in the correct way.

For the case of heterotic strings with $N = 2,4$ space-time supersymmetry, an analogous construction immediately leads to the Lorentz-invariant SU(2) resp. SO(6) currents already shown in eqs. (3.17) and (2.15). Furthermore the U(1) currents are similar to the one in eq. (3.42).

Finally let us briefly consider the Lorentz-invariant completion of the symmetry currents in the type II theories. Arguments analogous to the heterotic case lead to the following SU(2) currents for $N = 2$:

$$
\tilde{\Lambda}^3 = \oint\frac{\mathrm{d}z}{2\pi i}\,\tfrac{1}{4}i\sum_{M=1}^{k}\left\{\int^z \mathrm{d}z'\,\tilde{O}_M^+(z')\tilde{O}_M^-(z) - \int^z \mathrm{d}z'\,\tilde{O}_M^-(z')\tilde{O}_M^+(z)\right\}
$$

$$
-\oint\frac{\mathrm{d}\bar{z}}{2\pi i}\,\tfrac{1}{4}i\sum_{M=1}^{k}\left\{\int^{\bar{z}} \mathrm{d}\bar{z}'\,\tilde{O}_M^+(\bar{z}')\tilde{O}_M^-(\bar{z}) - \int^{\bar{z}} \mathrm{d}\bar{z}'\,\tilde{O}_M^-(\bar{z}')\tilde{O}_M^+(\bar{z})\right\},
$$

$$
\tilde{\Lambda}^+ = \left\{\oint\frac{\mathrm{d}z}{2\pi i}\,z^{1/2}\tilde{Q}^1(z)\right\}\left\{\oint\frac{\mathrm{d}\bar{z}}{2\pi i}\,\bar{z}^{1/2}\tilde{Q}_2^*(\bar{z})\right\},
$$

$$
\tilde{\Lambda}^- = \left\{\oint\frac{\mathrm{d}z}{2\pi i}\,z^{1/2}\tilde{Q}_1^*(z)\right\}\left\{\oint\frac{\mathrm{d}\bar{z}}{2\pi i}\,\bar{z}^{1/2}\tilde{Q}^2(\bar{z})\right\}. \quad (3.43)
$$

Similar expressions are obtained for $N = 4$ type II.

So far we have discussed the supersymmetry algebra as its acts on massive states without central charge. However in the presence of central charge operators Q^A and $\tilde{Q}^A$ generate a smaller SO(2N) Clifford algebra whose maximal subalgebra is Sp(N) × SO(3)$_{\text{Lorentz}}$. Therefore Sp(N) classifies all states which carry non-vanishing central charge. As already discussed, the dimension one central charge operators $Z^{AB}(z)$ are nothing else than the internal compactified momenta $\partial Y_M(z)$ – the central charge of a massive state is just its momentum eigenvalue in the compactified torus direction. Since the existence of central charges originates from a holomorphic torus compactification and is based on a non-vanishing operator product expansion between the supercharge currents, it is clear that heterotic and type II strings with the same number of supersymmetries have a different number of central charges and therefore also a totally different organization of the massive supermultiplets. As a first example consider $N = 2$ heterotic versus $N = 2$ type II theories. For the heterotic models Z^{AB} is given by two internal bosonic field $\partial Y_{1,2}(z)$ which corresponds to the holomorphic torus compactification of two dimensions. It follows that the symmetry group SO(2) gets broken for massive states with internal momenta which are of the form $|p\rangle = e^{ip \cdot Y(z)}|0\rangle$. To see this consider the SO(2) generator Λ^{MN},

$$\Lambda^{MN} \sim \oint \frac{dz}{2\pi i} \left\{ \tfrac{1}{2} Y_M(z)\, \partial Y_N(z) - \tfrac{1}{2} Y_N(z)\, \partial Y_M(z) \right\}. \tag{3.44}$$

Acting with this operator on the state $|p\rangle$ we will encounter a logarithmic z-dependence in the operator product between $\Lambda^{MN}(z)$ and $e^{ip \cdot Y(w)}$. This implies that this state does not transform as a SO(2) representation and shows that SO(2) is broken since $|p\rangle$ defines a specific direction in SO(2); it acts like a symmetry breaking vacuum expectation value. On the other hand the type II, $N = 2$ theories do not involve any torus compactification and there is no central charge operator since Q^1 and Q^2 trivially commute.

Next consider the $N = 4$ heterotic theory. Here there are six central charge operators corresponding to six holomorphic coordinates on a torus. Now again, a state with internal momentum, $|p\rangle = e^{ip \cdot Y}|0\rangle$ creates logarithms in the operator product with the SO(6) generators in eq. (2.15). Therefore these generators do not classify states with non-vanishing central charge and SO(6) gets broken. However, the state $|p\rangle$ can always be rotated into the state $|p_1, 0^5\rangle$ by a SO(6) transformation. It implies that SO(6) is broken to SO(5) ~ Sp(4). However for massive states with different central charges the SO(6) vectors $|p\rangle$ define different directions in SO(6) leading to an in general different embedding of SO(5) into SO(6). In other words, there is no unique SO(5) which classifies all massive states. However remember that from the world-sheet point of view the SO(6) Kac–Moody symmetry still classifies the whole string spectrum. On the other hand, this classification is meaningless from the space-time point of view because it is not Lorentz-invariant.

4. Massless supermultiplets and manifolds of massless scalars

In this section we will consider the transformation properties of the massless states – especially of the massless scalars – under the derived symmetries (therefore only the generators of subsect. 3.2 are now relevant). This consideration provides some important information about the effective action of these fields, i.e. about the manifold which is parametrized by the massless scalars. Therefore the discussion is also relevant for the structure of the moduli space of the Calabi–Yau and K_3 manifolds. We are here dealing with examples where the local structure of the manifold of the scalar fields is described by a non-compact coset manifold G/H. The linearly realized symmetry group H (H transformations leave the spectrum invariant) describes the invariance group of the string spectrum. Therefore H always contains the world-sheet Kac–Moody symmetries of the string theory and also the replication symmetry of identical massless supermultiplets. Furthermore, as discussed in subsects. 3.2 and 3.3 H can be enlarged for type II theories because of the enlarged space-time supersymmetry algebra. We will see how for the type II cases the manifolds of the Ramond–Ramond (R–R) scalars and the manifolds of the Neveu–Schwarz–Neveu–Schwarz (NS–NS) scalars are contained in G/H. The latter fields are the moduli of the spatial manifold the string is compactified on; they are also present in the corresponding heterotic string theory which is compactified on the same internal manifold. It follows that the isotropy group of the NS–NS) field's coset space is entirely built by the Kac–Moody symmetries plus the family replication symmetry of the corresponding string theory.

Let us start to consider the manifold of the four-dimensional dilaton D and antisymmetric tensor field $B_{\mu\nu}$ which build after duality transforming $B_{\mu\nu}$ a complex scalar field commonly denoted by S. This field parametrizes a Kähler manifold of complex dimension one (which can however be embedded into a larger space as we will see later). It builds the universal sector of any string theory with $N = 0, 1$ supersymmetry. The manifold of the S-field can be derived by considering a torus compactification from four to two dimensions. Now the graviton, dilaton and antisymmetric tensor field are internal fields and parametrize the manifold

$$\mathbf{M} = \frac{SO(2,2)}{SO(2)_L \times SO(2)_R} \,, \tag{4.1}$$

where for the case of type II strings the left- and right-moving SO(2) currents are given by $\partial\phi_1(z)$, $\bar{\partial}\phi_1(\bar{z})$. Now it is important to realize that M is in fact a product manifold,

$$\mathbf{M} = \frac{SU(1,1)}{U(1)_{L+R}} \times \frac{SU(1,1)}{U(1)_{L-R}} \,, \tag{4.2}$$

where $U(1)_{L+R}$ is the space-time helicity with current $j_{L+R}(\bar{z}, z) = \partial\phi_1(z) + \bar{\partial}\phi_1(\bar{z})$

and $U(1)_{L-R}$ the "anti-helicity" with $j_{L-R}(\bar{z}, z) = \partial\phi_1(z) - \bar{\partial}\phi_1(\bar{z})$. Clearly, the graviton lives on the first part of eq. (4.2) whereas S parametrizes the second part. $B_{\mu\nu}$ and D are charged only under the anti-helicity current. From this point of view it should be clear that some of the symmetry generators which classify the massless (and also the massive) states contain the space-time anti-helicity currents, especially in those cases where the dilaton and $B_{\mu\nu}$ are in common representations with other (R–R) scalars. If on the other hand D and $B_{\mu\nu}$ decouple from the remaining massless spectrum as e.g. in the $N = 1$ heterotic case, the symmetry generators contain only the internal currents.

Let us now consider the $N = 2$, type II theories with one left-moving and one right-moving supercharge. The corresponding internal degrees of freedom are given by a $(2, 2)$ superconformal field theory which can be thought of as a compactification on a six-dimensional Calabi–Yau manifold. The universal sector (the universal sector always corresponds to the identity operator of the conformal field theory) consists of a supergravity multiplet

$$\left(g_{\mu\nu}, \psi_\mu^i, A_\mu\right) \sim (\underline{1}, \underline{2}, \underline{1}) \tag{4.3}$$

and a complex hypermultiplet

$$(\lambda, \phi^i) \sim (\underline{1}, \underline{2}), \tag{4.4}$$

where we have given the SU(2) quantum numbers with respect to the generators (3.33). (The spectrum and the SU(2) charges can for example be derived using the exceptional group decomposition [23].) The four scalar fields in the complex hypermultiplet are the S-field and two (R–R) scalars which build a complex SU(2) doublet. Therefore the SU(2) currents (3.33) contain the space-time helicity current. The coset manifold which is parametrized by these four scalars contains SU(2) × U(1) as isotropy group and is given by the quaternionic manifold [4]

$$M = \frac{SU(2, 1)}{SU(2) \times U(1)} . \tag{4.5}$$

M contains as submanifold $(SU(1, 1)/U(1))_S$ and also a different $SU(1, 1)/U(1)$ which is parametrized by the (R–R) scalars (M however does not factorize into these submanifolds). Seen from a slightly different point of view, setting the (R–R) fields to zero breaks $N = 2$ supersymmetry to $N = 1$ and SU(2) to U(1) which just corresponds to the heterotic string compactification on the Calabi–Yau manifold. We can also add n additional hypermultiplets of the form (4.4) building the matter sector. In type II A (B) theories these are associated to the harmonic $(2, 1)$ $((1, 1))$ forms on the Calabi–Yau manifolds. The space M which is parametrized by the $4n + 4$ scalars (we also count the four scalars of the universal sector) has in general

a complicated form and depends on the couplings between the various fields which is described by a holomorphic function $f(z_i)$ $(i = 1, \ldots, n)$ [8]. However note that the coset (4.5) is always a submanifold of the general space M. Only for very special choices of the holomorphic function $f(z_i)$ the space M has still a coset structure [4]. For example, take as the simplest case the minimal coupling for the $(2, 1)$ forms in the type II A theories: $f(z_i) = 1 - \sum_{i=1}^{n} z_i^2$. Then the corresponding quaternionic manifold is

$$ M = \frac{SU(2, n + 1)}{SU(2) \times SU(n + 1) \times U(1)} . \tag{4.6} $$

Setting again all (R–R) fields to zero the remaining (NS–NS) fields parametrize the Kähler manifold

$$ M = \frac{SU(1, n)}{U(1) \times SU(n)} \times \left(\frac{SU(1, 1)}{U(1)} \right)_S . \tag{4.7} $$

This is exactly what the so-called c-map achieves [4]. It maps the quaternionic manifold of the hypermultiplets in the type II A theory onto the Kähler manifold of the vector multiplets in the type II B theory respectively of the gauge singlet chiral multiplets in the heterotic theory by setting all (R–R) fields to zero. The first part in eq. (4.7) is just the moduli space of the underlying Calabi–Yau manifold which is associated to the deformations of the complex structure. The U(1) factor in the first part of eq. (4.7) is a linear combination of the internal $U(1)_L \times U(1)_R$ Kac–Moody currents. For the general case, where the holomorphic function is not of that simple form and the associated space has no coset structure, all moduli are nevertheless characterized either by the vector-like or axial linear combination of $U(1)_L \times U(1)_R$ since the left- and right-moving U(1) charges must be either equal or opposite (depending on whether we deal with type II B or A). Therefore this specific U(1) combination appears in the moduli space of every Calabi–Yau manifold and is always contained in the holonomy group of the moduli space even for those cases which do not have a coset structure. Furthermore the c-map still applies in the general case – it maps the quaternionic space of the hypermultiplets to the kählerian subspace which is associated to the (NS–NS) scalar fields. The reason is that the existence of the c-map has its origin in the enlarged $N = 2$ space-time supersymmetry which in turn enlarges the world-sheet symmetries to SU(2).

Now let us switch to the case of heterotic $N = 2$ theories assuming that we are dealing with a compactification on the K_3 surface with $(4, 4)$ world-sheet supersymmetry times a two-dimensional torus T_2. Now the universal sector consists of a gravity multiplet, already shown in eq. (4.3), and a vector multiplet

$$ \left(A_\mu, \lambda^i, S \right) \sim (\underline{1}, \underline{2}, \underline{1}), \tag{4.8} $$

where the SU(2) quantum numbers are now with respect to the generators (3.30). The two vectors in the gravity and vector multiplet originate from the holomorphic torus compactification of two dimensions. The dilaton and $B_{\mu\nu}$ are SU(2) singlets and decouple from the states which transform non-trivially under SU(2). Therefore they parametrize the manifold SU(1,1)/U(1). In addition to these multiplets there are also matter multiplets which are in general model dependent. For compactification on $K_3 \times T_2$ there are two further vector multiplets of the form (4.8). The two vectors are two missing U(1) gauge bosons which one expects from the compactification on the two-dimensional torus T_2. The four appearing scalar fields are just the internal metric degrees of freedom of T_2 plus the internal dilaton and antisymmetric tensor field which are called T. These four scalars parametrize the moduli space of T_2,

$$M = \frac{SO(2,2)}{SO(2) \times SO(2)},\qquad (4.9)$$

where the SO(2) currents are given by $\partial H'(z)$ and $\bar{\partial} H'(\bar{z})$. Finally there arise 20 complex $N = 2$ hypermultiplets of the form of eq. (4.4). Their 80 scalar fields parametrize the moduli space of K_3 [3]:

$$M = \frac{SO(20,4)}{SO(20) \times SO(4)}.\qquad (4.10)$$

We see that all the states are classified under $SO(4) = SU(2)_L \times SU(2)_R$ where the $SU(2)_R$ is unavoidable because of $N = 2$ space-time supersymmetry. However $SU(2)_L$ only arises because we are considering a symmetric $(4,4)$ compactification. In summary, the complete manifold of all the scalars is given by

$$M = \left(\frac{SO(20,4)}{SO(20) \times SO(4)} \right)_{K_3} \times \left(\frac{SU(1,1)}{U(1)} \right)_{g_{ij}} \times \left(\frac{SU(1,1)}{U(1)} \right)_T \times \left(\frac{SU(1,1)}{U(1)} \right)_S.$$

$$(4.11)$$

Now let us turn to the $N = 4$, type II theory, again compactified on $K_3 \times T_2$. The relevant $SO(6) \times SO(2)$ symmetry currents are given in eqs. (3.35) and (3.36). The universal sector is built by a gravity multiplet

$$\left(g_{\mu\nu}, A_{\mu\nu}, \psi_\mu^i, A_\mu^{[ij]}, \lambda_i, T \right) \sim (\underline{1}, \underline{1}, \underline{4}, \underline{6}, \underline{4}, \underline{1}),\qquad (4.12)$$

where the numbers in parenthesis are the $SU(4) \simeq SO(6)$ quantum numbers, plus two vector multiplets

$$2\left(A_\mu, \lambda^i, \phi^{[ij]} \right) \sim 2(\underline{1}, \underline{4}, \underline{6}).\qquad (4.13)$$

The two vector multiplets contain four (NS–NS) scalars, namely the S-field and the internal metric of T_2 plus eight (R–R) scalars. It is perhaps surprising that the T-field (the internal dilaton and antisymmetric tensor) appears in the gravity multiplet and parametrize the coset $SU(1,1)/U(1)$ (they are $SO(6)$ singlets), where on the other hand the space-time dilaton and $B_{\mu\nu}$, the S-field, mix with the (R–R) scalars under $SO(6)$ transformations and therefore is a member of the vector multiplets. Altogether the 12 scalars of the vector multiplets are coordinates on the coset space

$$M = \frac{SO(6,2)}{SO(6) \times SO(2)} . \tag{4.14}$$

We can also determine the transformation properties of the various fields under the $SO(4) = SU(2)_L \times SU(2)_R$ subgroup of $SO(6)$. The four (NS–NS) scalars are singlets under $SO(4)$ where the eight (R–R) fields transform like $\underline{2} \sim (\underline{2},\underline{1})$ resp. $\bar{\underline{2}} \sim (\underline{1},\underline{2})$. It follows that the (NS–NS) fields themselves build the manifold $(SU(1,1)/U(1)) \times (SU(1,1)/U(1))$ where the (R–R) fields lie on $SU(2,2)/(SU(2) \times SU(2) \times U(1))$. Both of these spaces are submanifolds of (4.14). However note that eq. (4.14) is of course not a product manifold of the (NS–NS) and (R–R) spaces.

We can also consider the complete theory including also the matter sector. It consist of 20 vector multiplets of the form of eq. (4.13). Thus combining the universal with the matter multiplets the total number of 132 scalars parametrize

$$M = \frac{SO(22,6)}{SO(22) \times SO(6)} . \tag{4.15}$$

However there is an important difference between the scalars in the universal and the matter sector. For the latter case, each vector multiplet contains four (NS–NS) scalars which transform like $\underline{4} \sim (\underline{2},\underline{2})$ under $SO(4) = SU(2)_L \times SU(2)_R$ plus two (R–R) scalars which are $SO(4)$ singlets. From this fact we derive that 84 (NS–NS) scalars parametrize

$$M = \left(\frac{SO(20,4)}{SO(20) \times SO(4)} \right)_{K_3} \times \left(\frac{SU(1,1)}{U(1)} \right)_S \times \left(\frac{SU(1,1)}{U(1)} \right)_{g_{ij}} . \tag{4.16}$$

This, together with the manifold of the T-fields and neglecting the S-field is just the moduli space of $K_3 \times T_2$. On the other hand the (R–R) fields are coordinates on

$$M = \frac{SO(20,2)}{SO(20) \times SO(2)} \times \frac{SO(2,4)}{SO(2) \times SO(4)} . \tag{4.17}$$

Now we clearly recognize the relation between the $N = 2$ heterotic string and the $N = 4$ type II string both obtained by compactification on $K_3 \times T_2$. Setting all (R–R) fields of the type II theory to zero one breaks not only $SO(6)$ to $SO(4) = SU(2)_L \times SO(2)_R$ but also $N = 4$ supersymmetry to $N = 2$. The (NS–NS) scalars are

common to both types of theories. From the field theory point of view we understand that the appearance of the SO(4) factor in the moduli space of the heterotic theory, although not being enforced by the $N = 2$ supersymmetry algebra, has its origin in the fact that the same background also allows for an $N = 4$ supergravity theory which inevitably implies the SO(4) symmetry.

Finally, turn to the $N = 4$ heterotic theory obtained as compactification on a six-dimensional torus. The universal sector contains just the gravity multiplet

$$\left(G_{\mu\nu}, A_{\mu\nu}, \psi_\mu^i, A_\mu^{[ij]}, \lambda_i, S\right) \sim (\underline{1}, \underline{1}, \underline{4}, \underline{6}, \underline{4}, \underline{1}). \tag{4.18}$$

We see that in contrast to the $N = 4$ type II theory the space-time dilaton and $B_{\mu\nu}$ field is in the universal gravity multiplet and is inert under SO(6) transformations parametrizing the coset SU(1,1)/U(1). The matter sector is described by 22 vector multiplets with 132 scalars parametrizing the moduli space of the torus compactification (4.15). Comparing the $N = 4$ heterotic and type II theory we learn that the S and T fields are exactly interchanged. It follows that in the heterotic models the SO(6) singlet S couples in the field theory lagrangian to all other vector multiplets whereas in the type II case the T field has non-vanishing coupling to all the vectors.

At the end let us briefly mention the $N = 8$ type II string theory which is just a trivial torus compactification from ten dimensions. The SU(8) symmetry currents can be easily constructed by the methods of subsect. 3.2. This theory contains 38 (NS–NS) scalars including the S-field and 32 (R–R) scalars in the universal gravity multiplet. Altogether these fields parametrize the coset space [33]

$$M = E_{7(+7)}/SU(8). \tag{4.19}$$

This space contains as (NS–NS) submanifold the moduli space of the six-torus times the S-field manifold,

$$M = \frac{SO(6,6)}{SO(6) \times SO(6)} \times \left(\frac{SU(1,1)}{U(1)}\right)_S \tag{4.20}$$

and for the 32 (R–R) fields the coset space [32]

$$M = \frac{SU(4,4)}{SU(4) \times SU(4) \times U(1)}. \tag{4.21}$$

5. Summary

This paper explains the connection between the string world-sheet symmetries and the space-time symmetries which classify the string spectrum in theories with (extended) space-time supersymmetry. This relation is true for the whole string theory and not only for its low energy sector. Specifically, we have identified those symmetry generators which transform the space-time supersymmetry charges into

each other. This leads to the determination of the coset manifolds which are parametrized by the scalars of the universal sector of the string theory. Furthermore, it would also be interesting to explicitly derive those symmetry generators from the string world-sheet degrees of freedom which are connected to the replication symmetry of the matter multiplets which contain the moduli of the underlying superconformal field theory. Then the complete linear realized symmetry H, which corresponds in general to the holonomy group of the moduli space, would be shown to have its direct origin in the two-dimensional superconformal field theory.

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