SUPERSTRING PARTITION FUNCTIONS
AND THE CHARACTERS OF EXCEPTIONAL GROUPS

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We study the relation between the one-loop partition function of supersymmetric four-dimensional heterotic string theories and the level-one characters of exceptional groups. This is used to derive identities among θ-functions which show that the one-loop partition functions vanish identically. It is also shown that the exceptional groups provide the Kač–Moody characters of the underlying conformal field theories.

The appearance of ($N=1, 2, 4$) space–time supersymmetries in four-dimensional heterotic string theories [1–7] requires [8–12] the existence of internal $N=2, 4$ superconformal field theories [13]. These contain, besides the super-Virasoro algebra, also the infinite dimensional U(1) and SU(2) Kač–Moody algebras respectively, which are essential for the construction of the highest weight states within the supersymmetric string theories.

As shown in refs. [5,14,12], the relevant Kač–Moody currents of the superconformal algebras are generated by the exceptional groups E_6, E_7, E_8 which, in addition, provide a link between the external Lorentz transformation properties of any string state and the unitary representations of the internal superconformal algebra in a well defined way, such that the resulting spectrum is space–time supersymmetric. Thus, the weights of these exceptional groups appear in any four-dimensional supersymmetric string construction as right-moving bosonic momenta building a euclidian covariant lattice [15–17,5]. It follows also that E_6, E_7 and E_8 play the role of the left-moving gauge group when considering models with identical left-moving world-sheet supersymmetries.

In this letter we show that the one-loop partition function of heterotic strings contains the (modified) level-one Kač–Moody characters of the exceptional groups. The connection between the partition functions and the characters of the exceptional groups were also independently formulated in refs. [18,19]. In ref. [19] the vanishing of the one-loop partition functions was shown using group theoretical properties of the exceptional groups namely the inner automorphism of the exceptional algebras, called "ghost triality". The analysis in ref. [19] extends, under some assumption, to the partition function at arbitrary genus g. We prove directly, using θ-function identities or results from the theory of modular forms, that the (modified) level-one characters of the exceptional groups identically vanish. In addition, we further analyze the relation between the characters of the exceptional groups and the characters of the $N=2$ superconformal algebra showing in particular how the U(1) part, contained in the decomposition of the E_6 characters, is related to the characters of the $N=2$ superconformal algebra. This will elucidate that the U(1) Kač–Moody subalgebra, which is contained in the internal $N=2$, $c=9$ superconformal field theory, builds by itself an $N=2$, $c=1$ superconformal theory in the discrete series, after projecting onto an ($N=1$) space–time supersymmetric spectrum. The corresponding $c=1$ characters are given by level-six classical theta functions.

Let us recall te main aspects of the relation of the exceptional groups E_6, E_7 and E_8 and $N=1, 2, 4$ space–time supersymmetry [5,20,14,12]. First consider only the right-moving degrees of freedom of a four-dimensional heterotic string theory which consist of external free world-sheet fermions $\psi^\mu(z)$ (μ is a four-dimensional space–time index), the superconformal ghost system and an internal superconformal field theory with central charge

$c = 9$. Let us decompose the exceptional groups into D_5 times its centralizer in E_6, E_7, E_8:

$$E_6 \subset D_5 \otimes U(1) , \quad E_7 \subset D_5 \otimes A_1 \otimes U(1) , \quad E_8 \subset D_5 \otimes D_3 . \tag{1}$$

D_5 describes the external D_2-Lorentz and superconformal ghost properties. The second part is identical to the Kač–Moody currents of the internal $c = 9$ superconformal field theory. Specifically, for $N = 1$ space–time supersymmetry, the $U(1)$ factor belongs to the Kač–Moody current of the $c = 9$, $N = 2$ superconformal algebra generated by

$$J(z) = -i\sqrt{3}\partial H(z) , \quad T_F^{\pm}(z) = \exp[\pm i\tfrac{1}{3}\sqrt{3}H(z)]\tilde{T}_F^{\pm}(z) , \quad T(z) = -\tfrac{1}{2}[\partial H(z)]^2 + \tilde{T}(z) , \tag{2}$$

where the tilded fields are conformal fields of a $c = 8$ conformal field theory. Similarly, E_7 leads to a $c = 3$, $N = 2$ superconformal algebra with a $U(1)$ Kač–Moody current at level two plus an $N = 4$, $c = 6$ superconformal algebra with level one $SU(2)$ Kač–Moody current (see ref. [12] for details). Finally, E_8 leads to internal $SO(6)$ currents. The Cartan subalgebra of $SO(6)$ corresponds to three abelian $U(1)$ currents of three $N = 2$, $c = 3$ superconformal algebras.

The importance of the exceptional groups consists not only in providing the relevant world-sheet Kač–Moody currents, but the quantized charges of the superconformal fields under these symmetries, as required for a supersymmetric spectrum, are determined by the representations of the exceptional groups. In addition, they link the internal fields to the representations of the external Lorentz group in a well defined way. This entails that the representations and partition functions of the superconformal field theories inherit important properties from the level-one characters of the exceptional groups; this is what we will demonstrate in the following.

To obtain the physical light-cone states, which have fixed superconformal ghost charge and also fixed longitudinal components in the Lorentz group D_2, the following procedure [21,17,5] is in order. First decompose D_5 to $D_1 \otimes D_4$ where D_1 stands for the physical transverse degrees of freedom and D_4 for the unphysical longitudinal and ghost degrees of freedom. Then states which are characterized by D_5 weight vectors $\lambda = (\lambda^{(1)}, \lambda^{(4)})$ are physical if $\lambda^{(4)} = (\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, -\tfrac{1}{2})$ for space–time fermions and $\lambda^{(4)} = (0, 0, 0, -1)$ for space–time bosons. The truncation to the physical degrees of freedom $D_5 \to D_1$ becomes, in terms of D_5 characters,

$$D_5 \to D_1 : \left(\frac{\theta_i(0|\tau)}{\eta(t)}\right)^5 \to -(-1)^i \frac{\theta_i(0|\tau)}{\eta(\tau)} , \tag{3}$$

where we have taken the statistics of space–time fermions into account. It is important to note that this replacement does not change the modular transformation properties. Performing this truncation on the level-one characters of the exceptional groups [22] we obtain

$$P_{(E_6)0}^{c=2} = \frac{1}{\eta^2(\tau)} [\theta_3(0|3\tau)\theta_3(0|\tau) - \theta_4(0|3\tau)\theta_4(0|\tau) - \theta_2(0|3\tau)\theta_2(0|\tau)] ,$$

$$P_{(E_6)1}^{c=2} = \frac{1}{\eta^2(\tau)} \left\{ -\theta\begin{bmatrix} 1/6 \\ 0 \end{bmatrix}(0|3\tau)\theta(0|\tau) + \theta\begin{bmatrix} 2/3 \\ 0 \end{bmatrix}(0|3\tau)\theta_3(0|\tau) - \exp(-2\pi i/3)\theta\begin{bmatrix} 2/3 \\ 1/2 \end{bmatrix}(0|3\tau)\theta_4(0|\tau) \right\} ,$$

$$P_{(E_6)\bar{1}}^{c=2} = \frac{1}{\eta^2(\tau)} \left\{ -\theta\begin{bmatrix} 5/6 \\ 0 \end{bmatrix}(0|3\tau)\theta_2(0|\tau) + \theta\begin{bmatrix} 1/3 \\ 0 \end{bmatrix}(0|3\tau)\theta_3(0|\tau) + \exp(-\pi i/3)\theta\begin{bmatrix} 1/3 \\ 1/2 \end{bmatrix}(0|3\tau)\theta_4(0|\tau) \right\} ,$$

$$P_{(E_7)0}^{c=3} = \frac{1}{\eta^3(\tau)} \left\{ -\theta_2(0|2\tau)[\theta_2(0|\tau)]^2 + \theta_3(0|2\tau)([\theta_3(0|\tau)]^2 - [\theta_4(0|\tau)]^2) \right\} ,$$

$$P_{(E_7)1}^{c=3} = \frac{1}{\eta^3(\tau)} \left\{ -\theta_3(0|2\tau)[\theta_2(0|\tau)]^2 + \theta_2(0|2\tau)([\theta_3(0|\tau)]^2 + [\theta_4(0|\tau)]^2) \right\} , \tag{4}$$

$$P_{(E_8)0}^{c=4} = \frac{1}{\eta^4(\tau)} \{ -[\theta_2(0|\tau)]^4 + [\theta_3(0|\tau)]^4 - [\theta_4(0|\tau)]^4 \}, \qquad (4 \text{ cont'd})$$

where the superscripts correspond to the central charges of the associated Virasoro algebras and the subscript distinguishes between different conjugacy classes. Now, because of space–time supersymmetry, these characters are supposed to vanish identically. For E_7 and E_8 this is easily seen to vanish using well-known θ-function identities. To show that the $E_6^{(c=2)}$ characters vanish is more involved. Let us first notice that the characters of a given conjugacy class of any group are given by $\sum q^{\lambda^2}$ where the sum extends over all weights in the conjugacy class. Since the weight vectors of the conjugacy classes of two complex conjugate representations are related by $\lambda \leftrightarrow -\lambda$ their characters are equal. In particular $P_{(E_6)1}^{c=2} = P_{(E_6)\bar{1}}^{c=2}$. We will now show that $F_0(\tau) \equiv \eta^2(\tau) P_{(E_6)0}^{c=2}(\tau)$ and $F_1(\tau) \equiv \eta^2(\tau) [P_{(E_6)1}^{c=2}(\tau) + P_{(E_6)\bar{1}}^{c=2}(\tau)]$ vanish. We will follow ref. [23]. Consider the subgroup $\Gamma_0(3)$ of the modular group, defined by $\Gamma_0(3) = \{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix} \right) | c \equiv 0 \bmod 3 \}$. It is a subgroup of index 3, generated by $T = \left(\begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix} \right)$ and $W^3 = \left(\begin{smallmatrix} 1 & 0 \\ 3 & 1 \end{smallmatrix} \right)$ (note that $W = TST$ where $S = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix} \right)$). Also, $3W^3(\tau) = W(3\tau) = 3\tau/(3\tau+1)$. Then, with the help of the modular transformation rules of the θ-functions we easily show that

$$F_{0,1}(T\tau) = v(T)F_{0,1}(\tau), \quad F_{0,1}(W^3\tau) = v(W^3)(3\tau+1)F_{0,1}(\tau),$$

where $v(T) \equiv \exp(2\pi i \kappa) = -1 \ (0 \leqslant \kappa < 1)$, $v(W^3) = -1$; i.e. $F_{0,1}$ are modular forms of $\Gamma_0(3)$ of weight 1 with multiplier system $v(T)$ and $v(W^3)$. If the space of forms of weight k of a subgroup Γ of index μ of the modular group with multiplier system v is denoted by $\{\Gamma, k, v\}$, then one can show (cf. ref. [23]) that $\dim\{\Gamma, k, v\} \leqslant \max\{0, 1 - \xi + k\mu/12\}$ where for our purposes it suffices to know that $\xi \geqslant \kappa = \frac{1}{2}$. We then easily find that $\dim\{\Gamma_0(3), 1, v\} = 0$, i.e. $F_0(\tau) = F_1(\tau) = 0$.

In the following we will concentrate on the $N = 1$ supersymmetric heterotic string theories which are the most interesting from a phenomenological point of view.

The most general partition function of an $N = 1$, $d = 4$ supersymmetric heterotic string theory has the following structure:

$$\chi(\tau, \bar{\tau}) \sim \frac{1}{\text{Im} \, \tau} \frac{1}{|\eta(\tau)|^4} P_{(D_1)i}^{c=1}(\tau) P_{(N=2)j}^{c=9}(\tau) P_l^{\bar{c}=22}(\bar{\tau}) a_{ijl}, \qquad (5)$$

with $P_{(D_1)i}^{c=1}$ ($i = 0, V, S, C$) being the characters of the transverse Lorentz group SO(2), $P_{(N=2)j}^{c=9}$ the characters of the internal $N = 2$, $c = 9$ superconformal field theory, $P_l^{\bar{c}=22}$ the characters of the left-moving part of the heterotic string and a_{ijl} some integers satisfying the constraints of modular invariance and spin-statistics. Many models are known for which the a_{ijk} can be given explicitly. As was shown in refs. [14,12], locality of the operator product with the gravitino vertex restricts the physical states to fall into one of the conjugacy classes of the exceptional groups. Furthermore, starting from one $D_1 \times U(1)$ conjugacy class within a specific E_6 conjugacy class we can reach all the other conjugacy classes by the action of the supercharge (gravitino vertex operator) which is equivalent to the action of the spectral flow. Using this information, the partition function can now be rewritten in terms of the E_6 characters:

$$\chi(\bar{\tau}, \tau) \sim \frac{1}{\text{Im} \, \tau} \frac{1}{|\eta(\tau)|^4} P_{(E_6)i}^{c=2}(\tau) P_{(N=2)j}^{c=8}(\tau) P_l^{\bar{c}=22}(\bar{\tau}) a'_{ijl} \qquad (6)$$

($i = 0, 1, \bar{1}$), where $P_{(N=2)j}^{c=8}$ are the characters of the $N = 2$ superconformal field theory without the free boson H.

The E_6 characters build a representation of the modular group with the following transformation properties. For the T transformation ($\tau \to \tau + 1$), $P_{(E_6)i}^{c=2} \to T_{ij} P_{(E_6)j}^{c=2}$ and the S transformation ($\tau \to -1/\tau$), $P_{(E_6)i}^{c=2} \to S_{ij} P_{(E_6)j}^{c=2}$ we obtain

$$T = \exp(\pi i/6) \begin{pmatrix} 1 & 0 & 0 \\ 0 & \exp(\pi i/3) & 0 \\ 0 & 0 & \exp(\pi i/3) \end{pmatrix}, \quad S = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \exp(2\pi i/3) & \exp(-2\pi i/3) \\ 1 & \exp(-2\pi i/3) & \exp(2\pi i/3) \end{pmatrix}. \qquad (7)$$

Note that these transformations are (except for the dependence on the central charge) identical to the modular transformations of the level-one representations of SU(3). Therefore, tensoring $P^{c=2}_{(E_6)i}$ and $P^{c=2}_{(SU(3))i}$ in a diagonal way one gets a modular invariant expression, namely nothing else than $P^{c=4}_{(E_8)0}$.

Let us define the classical level-m θ-functions

$$\theta_{n,m}(\tau, z, u) = \exp(-2\pi i m u) \sum_{j \in \mathbb{Z} + n/2m} \exp(2\pi i \tau m j^2 + 2\pi i j z) . \tag{8}$$

They are the characters of the $A_1^{(1)}$ Kač–Moody algebra at level m. With this definition we can rewrite the E_6 characters in the following way:

$$P^{c=2}_{(E_6)0} = \frac{2}{\eta^2(\tau)} \left[\theta_{0,6}(\tau)\theta_V^{(1)}(\tau) + \theta_{6,6}(\tau)\theta_0^{(1)}(\tau) - \theta_{3,6}(\tau)\theta_S^{(1)}(\tau) - \theta_{9,6}(\tau)\theta_C^{(1)}(\tau) \right] ,$$

$$P^{c=2}_{(E_6)1} = \frac{2}{\eta^2(\tau)} \left[\theta_{4,6}(\tau)\theta_V^{(1)}(\tau) + \theta_{10,6}(\tau)\theta_0^{(1)}(\tau) - \theta_{7,6}(\tau)\theta_S^{(1)}(\tau) - \theta_{1,6}(\tau)\theta_C^{(1)}(\tau) \right] ,$$

$$P^{c=2}_{(E_6)\bar{1}} = \frac{2}{\eta^2(\tau)} \left[\theta_{8,6}(\tau)\theta_V^{(1)}(\tau) + \theta_{2,6}(\tau)\theta_0^{(1)}(\tau) - \theta_{11,6}(\tau)\theta_S^{(1)}(\tau) - \theta_{5,6}(\tau)\theta_C^{(1)}(\tau) \right] , \tag{9}$$

where all theta functions are evaluated at $z = u = 0$. We see that each term is the product of a level-six theta function referring to the U(1) part and a theta function corresponding to one of the four D_1 conjugacy classes.

Let us now analyze the representations and the superconformal structure of E_6 more carefully. $E_6^{c=2}$ decomposes into $D_1^{transverse} \otimes U(1)$ conjugacy classes as follows:

$$0 \to (0, \sqrt{3}) + (V, 0) + (S, \tfrac{1}{2}\sqrt{3}) + (C, -\tfrac{1}{2}\sqrt{3}) . \tag{10}$$

The second entry, α, is related to the U(1) charge Q of any state by $Q = \sqrt{3}\alpha$. Specifically, the $(0, \sqrt{3})$ conjugacy class contains the massive holomorphic three-form field which has to be present in any $N = 1$ supersymmetric string theory just like the gravitino field which belongs to $(S, \tfrac{1}{2}\sqrt{3})$, $(C, -\tfrac{1}{2}\sqrt{3})$ and the graviton, dilaton and antisymmetric tensor field which are contained in $(V, 0)$ (see ref. [14] for details). Secondly we have

$$1 \to (0, -\tfrac{1}{3}\sqrt{3}) + (V, +\tfrac{2}{3}\sqrt{3}) + (S, -\tfrac{5}{6}\sqrt{3}) + (C, \tfrac{1}{6}\sqrt{3}) , \tag{11}$$

and its complex conjugate. These conjugacy classes contain the massless chiral matter fermions with $Q = -\tfrac{1}{2}$ and massless matter scalars with $Q = 1$. The remaining two conjugacy classes contain only massive vectors and spinors.

We note that for states in the R sector (space–time fermions) Q is a half integer whereas it is integer for NS states (space–time bosons). Inspection of the last two equations shows that $2Q$ is determined modulo 12, i.e. U(1) possesses 12 conjugacy classes which build a representation of the modular group. If we define $\alpha = \tfrac{1}{6}\sqrt{3}(q + 12k)$ $(k \in \mathbb{Z}, q = 0, ..., 11)$ the characters are

$$P^{c=1}_{(U(1))q} = \mathrm{Tr} \exp[2\pi i \tau (L_0 - \tfrac{1}{24}) + 2\pi i z J_0] , \tag{12}$$

where the eigenvalues of L_0 and J_0 are $(h, Q) = (\tfrac{1}{2}\alpha^2, \sqrt{3}\alpha)$. With the definition of the θ-functions in eq. (8) we easily get

$$P^{c=1}_{(U(1))q} = \frac{1}{\eta(\tau)} \theta_{q,6}(\tau, 6z, 0) . \tag{13}$$

This means that the U(1) characters are level-six theta functions. ($\eta(\tau)$ is the contribution from the oscillators.) This result was already anticipated by eq. (9) and our discussion at the beginning of this section.

These θ-functions are also the characters of the $N = 2$, $c = 1$ superconformal field theory which is in the unitary discrete series with $c < 3$. To see this, let us demonstrate that the U(1) part of E_6 generates the complete $c = 1$,

$N = 2$ superconformal field theory [24] The energy–momentum tensor is given by $T(z) = -\frac{1}{2}[\partial H(z)]^2$, the two supercurrents are $G^\pm(z) = (1/2\sqrt{3})\exp[\pm i\sqrt{3}H(z)]$ and the U(1) current is given by $J'(z) = -i\sqrt{\frac{1}{3}}\partial H(z)$. The highest weight states (primary conformal fields) have the form $\phi(z) = \exp[i\alpha H(z)]$ with conformal dimension $h = \alpha^2/2$ and U(1) charge $\alpha/\sqrt{3}$. Unitarity constrains α to be from a finite set of discrete numbers. The descendant fields are obtained by acting with $J'(z)$, $T(z)$ and $G^\pm(z)$ on $\phi(z)$ which means in particular that fields whose values of α differ by $\sqrt{3}$ belong to the same conjugacy class.

Specifically, for the highest weight states in the NS sector the allowed values for (h, α) are $(0, 0)$, $(\frac{1}{6}, \pm\frac{1}{3}\sqrt{3})$ and in the R sector they are $(\frac{1}{24}, \pm\frac{1}{6}\sqrt{3})$, $(\frac{3}{8}, \frac{1}{2}\sqrt{3})$. For a general state α is

$$\alpha = \frac{1}{6}\sqrt{3}(q' + 6k) \quad (q' = 0, ..., 5, \quad k \in \mathbb{Z}) . \tag{14}$$

The corresponding conformal characters $P^{c=1}_{(N=2)q'}$ have the following form:

$$P^{c=1}_{(N=2)q'} = \frac{1}{\eta(\tau)} \sum_{j \in \sqrt{3}(q'+6k)/6} \exp[2\pi i\tau\tfrac{1}{2}j^2 + 2\pi iz(1/\sqrt{3})j] = \frac{1}{\eta(\tau)}\theta_{q'/2,3/2}(\tau, z, 0) . \tag{15}$$

Clearly, these are not identical to the characters coming from E_6 decomposition (see eq. (13)). The reason for this is that the characters eq. (15) do not transform into each other under modular transformation, i.e. they do not form a representation of the modular group.

To cure this defect one has to introduce a fermion number projection $(-1)^F$ such that the supercurrent $G^\pm$ has $F = -1$. This projection splits each conformal block into two sectors. Specifically, define the following characters:

$$\tilde{P}^{c=1}_{(N=2)q'} = \frac{1}{\eta(\tau)}\tilde{\theta}_{q'/2,3/2}(\tau, z, 0) , \quad \tilde{\theta}_{n,m}(\tau, z, 0) = \sum_{j \in \mathbb{Z}} \exp[2\pi im\tau(j+n/2m)^2 + 2\pi izj]\exp(\pi ij) . \tag{16}$$

Then one obtains the partition functions which represent the trace over states with either even or odd fermion number as

$$P^{c=1}_{(N=2)e} = \tfrac{1}{2}(P^{c=1}_{(N=2)q'} + \tilde{P}^{c=1}_{(N=2)q'}) = \frac{1}{\eta(\tau)}\theta_{q',6}(\tau, 2z, 0) ,$$

$$P^{c=1}_{(N=2)o} = \tfrac{1}{2}(P^{c=1}_{(N=2)q'} - \tilde{P}^{c=1}_{(N=2)q'}) = \frac{1}{\eta(\tau)}\theta_{q'+6,6}(\tau, 2z, 0) . \tag{17}$$

Now, these characters build indeed a representation of the modular group and moreover are identical to those which arise from the U(1) subgroup of E_6 (after rescaling of the U(1) charge Q).

In conclusion, any $c = 9$, $N = 2$ superconformal algebra contains, after performing the necessary projections which lead to a space–time supersymmetric spectrum, a $c = 1$, $N = 2$ superconformal subalgebra with the U(1) Kač–Moody current generated by the exceptional group E_6. The physical reason is that due to space–time supersymmetry the supercurrents $G^\pm(z)$ of the $c = 1$ superconformal subalgebra are always present as conformal fields since they correspond to the holomorphic three-form fields. $G^\pm(z)$ are of course not identical to the supercurrents $T^\pm_F(z)$ of the internal $N = 2$, $c = 9$ superconformal field theory. It is obvious from our discussion that the organization of states under the $c = 1$, $N = 2$ superconformal algebra is implicit in the decomposition of the E_6 conjugacy classes to $D_1 \times U(1)$. However, the E_6 decomposition contains more information since it correlates the space–time and internal charges of any state.

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