THE INTEGRABILITY OF $N=16$ SUPERGRAVITY ☆

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For the maximally extended $N=16$ supergravity theory in two dimensions, we explicitly construct a linear system whose integrability conditions are equivalent to the full nonlinear field equations of this theory. All the (on-shell) information contained in it can thus be encoded into a single E_8 matrix and its dependence on a spectral parameter; the invariance of the equations of motion under E_9 is manifest. Possible consequences and further developments are briefly discussed.

The recent surge of interest in two-dimensional field theories has been largely confined to (super)conformally invariant theories because of their direct relevance to (super)string theories. By contrast, very little attention has been devoted to the maximally extended $N=16$ supergravity theory in two dimensions although this theory has been conjectured to possess rather special properties (especially, a hidden E_9-invariance) [1]. Unlike the conformal supergravities, which in two dimensions only exist for $N\leqslant 4$, the $N=16$ theory does not admit a gauge choice where the physical degrees of freedom are governed by free wave equations. Nonetheless, it comes very close to being exactly solvable as will be demonstrated in this paper: there exists a linear system whose integrability conditions are equivalent to the full nonlinear field equations of $N=16$ supergravity [#1]. The derivation of this result relies on recent progress in understanding the dimensional reduction of bosonic coset-space σ-models and the emergence of infinite dimensional symmetries in two dimensions [2]. While one expects integrability of the bosonic sector on the basis of these results, we here find that, quite remarkably, this approach also furnishes a natural explanation for the fermionic nonlinearities. Some of the special (and, in fact, unique) features of $N=16$ supergravity can now be understood very explicitly. There are intriguing possibilities for further development but, for lack of space, we can only briefly mention these at the end of this paper.

$N=16$ supergravity in two dimensions may be derived from the corresponding theory in three dimensions by dimensional reduction. The $d=3$, $N=16$ theory has been constructed in ref. [4] whose notation and conventions we will adhere to in this paper with the following exceptions and additional conventions. Three-dimensional curved and flat indices are denoted by $m, n, \ldots$ and $a, b, \ldots$, respectively, whereas we reserve Greek letters $\mu, \nu, \ldots$ and $\alpha, \beta, \ldots$ for curved and flate two-dimensional indices. We define $\gamma^3 \equiv \mathrm{i}\gamma^2 = -\mathrm{i}\gamma_2$ and $\epsilon^{\alpha\beta} = \gamma^{\alpha\beta}\gamma^3$. The physical states of the $d=3$, $N=16$ theory constitute an irreducible $N=16$ supermultiplet with 128 bosons and 128 fermions transforming as inequivalent fundamental spinor representations of SO(16). The theory has a rigid $E_{8(+8)}$ invariance which is linearized in the usual way by introducing a local SO(16) invariance. Consequently, the scalars $\mathscr{V}_0(x)$ are properly described as elements of the coset space $E_{8(+8)}/SO(16)$, and the "composite" SO(16) gauge field Q_m is obtained from the E_8 Lie algebra decomposition

$$\begin{aligned}\mathscr{V}_0^{-1}\partial_m\mathscr{V}_0 &= Q_m + P_m \\ &= \tfrac{1}{2}Q_m^{IJ}X^{IJ} + P_m{}^A Y^A \,. \qquad (1)\end{aligned}$$

☆ this work is dedicated to my father.

[#1] We employ the notion of "integrability" in the same sense as ref. [2]. For a more thorough discussion, see refs. [1,2] and references therein. There is ample literature about integrable systems, see ref. [3] and references therein.

Here the indices $I, J, ... = 1, ..., 16$ and $A, B, ... = 1, ..., 128$ (or $\dot{A}, \dot{B}, ... = 1, ..., 128$) label the vector representation and the fundamental spinor (or conjugate spinor) representation of SO(16), respectively. The E_8 generators $X^{IJ} = -X_{IJ}$ and Y^A obey the commutation relations

$$[X^{IJ}, X^{KL}] = \delta^{IL} X^{JK} + \delta^{JK} X^{IL} - \delta^{IK} X^{JL} - \delta^{JL} X^{IK} ,$$

$$[X^{IJ}, Y^A] = -\tfrac{1}{2} \Gamma^{IJ}_{AB} Y^R ,$$

$$[Y^A, Y^B] = \tfrac{1}{4} \Gamma^{IJ}_{AB} X^{IJ} . \tag{2}$$

Besides the physical boson and fermion fields φ^A and $\chi^{\dot{A}}$, the $N=16$ theory contains a dreibein $e_m{}^a$ and a gravitino $\psi_m{}^I$ transforming as the 16-dimensional vector representation of SO(16). These fields do not correspond to any phyaiscal degrees of freedom, but are nevertheless indispensable for the formulation of the theory. In 1.5 order formalism, the complete lagrangian of $d=3$, $N=16$ supergravity reads [4][#2]

$$\mathscr{L} = -\tfrac{1}{4} eR + \tfrac{1}{2} \epsilon^{mnp} \bar{\psi}^I_m D_n \psi^I_p + \tfrac{1}{4} e g^{mn} P^A_m P^A_n$$
$$- \tfrac{1}{2} i e \bar{\chi}^{\dot{A}} \gamma^m D_m \chi^{\dot{A}} - \tfrac{1}{2} e \bar{\chi}^{\dot{A}} \gamma^n \gamma^m \psi^I_n \Gamma^I_{A\dot{A}} P^A_m$$
$$- \tfrac{1}{8} e [\bar{\chi}^{\dot{A}} \gamma_p \Gamma^{IJ}_{\dot{A}\dot{B}} \chi^{\dot{B}} (\bar{\psi}^I_m \gamma^{mnp} \psi^J_n - \bar{\psi}^I_m \gamma^p \psi^{mJ})$$
$$+ \bar{\chi}^{\dot{A}} \chi^{\dot{A}} \bar{\chi}^I_m \gamma^n \gamma^m \psi^I_n]$$
$$+ e [\tfrac{1}{8} (\bar{\chi}^{\dot{A}} \chi^{\dot{A}})^2 - \tfrac{1}{96} (\bar{\chi}^{\dot{A}} \gamma^m \Gamma^{IJ}_{\dot{A}\dot{B}} \chi^{\dot{B}})^2] . \tag{3}$$

As shown in ref. [4], this lagrangian is invariant under $d=3$ general coordinate transformations, local Lorentz ($=$SO(1, 2)) and local $N=16$ supersymmetry transformations. Since these results are described in great detail in ref. [4], we refrain from giving further formulas here.

The dimensional reduction of the lagrangian (3) to two dimensions involves some novel features in comparison with the dimensional reduction to dimensions higher than two. One first drops all dependence on the third coordinate and then tries to simplify the field equations as much as possible by suitable gauge conditions. For the dreibein, a natural choice is

$$e_m{}^a \begin{pmatrix} \lambda \delta^\alpha_\mu & \rho B_\mu \\ 0 & \rho \end{pmatrix} , \tag{4}$$

where we have exploited local SO(1, 2) invariance and $d=2$ diffeomorphism invariance to bring $e_m{}^a$ into triangular form and to diagonalize the zweibein $e_\mu{}^\alpha$ (a trick well-known to string aficionados). The field B_μ is auxiliary in two dimensions and appears only through its invariant field strength

$$F \equiv \rho \lambda^{-1} \epsilon^{\alpha\beta} \partial_\alpha B_\beta . \tag{5}$$

Its elimination leads to further quartic spinorial terms, see below. Finally, we make use of the local $N=16$ supersymmetry to eliminate part of the $d=3$ gravitino ψ^I_m. The following gauge choice is associated with the diagonality of the zweibein in (4) (for flat $d=2$ indices)

$$\gamma^\beta \gamma^\alpha \psi^I_\beta = 0 . \tag{6}$$

At this point it turns our that, fortunately, a further simplification is possible because the above gauge choice admits residual invariances. Namely, the diagonal form of the zweibein is preserved by (anti)holomorphic diffeomorphisms, and one can verify that the gauge (6) is left invariant under local $N=16$ supersymmetry transformations with parameters satisfying $\gamma^\beta \gamma^\alpha D_\beta \epsilon^I = 0$ (thus, in the absence of scalar couplings, the chiral components of the parameter ϵ^I are homomorphic or antiholomorphic, respectively). This residual freedom can be used to identify the function ρ in (4) with one of the two-dimensional coordinates (see e.g. ref. [2]) and to get rid of one more gravitino component such that

$$\psi^I_a = (\psi^I_\alpha, \psi^I_2) = (\gamma_\alpha \psi^I, 0) \tag{7}$$

is the equations of motion [#3]. In the unphysical sector, we are thus left with the conformal factor λ and its $N=16$ "superpartner" ψ^I. In contradistinction to conformally invariant theories, these fields do not decouple but play an important role here. For instance, the central extension of E_9 acts nontrivially on λ [1,2].

[#2] In the last term $(\bar{\chi}\gamma^m \Gamma^{IJ} \chi)^2$, we differ by a factor 1/3 from ref. [4].

[#3] Alternatively, one can pick the gauge $\psi^I_2 = 0$ and prove that $\psi^I_\alpha = \gamma_\alpha \psi^I$ from the equations of motion.

The above gauge choices entail dramatic simplifications in the dimensionally reduced field equations. After redefining the fermionic fields according to $\psi^I \to \lambda^{1/2}\psi^I$, $\chi^{\dot A} \to \lambda^{1/2}\chi^{\dot A}$, the physical fields decouple entirely from the unphysical ones in that their equations of motion no longer contain λ and ψ^I. For the scalar fields, one obtains

$$\rho^{-1} D^\alpha(\rho P_\alpha{}^A) = \tfrac{1}{16}\Gamma^{IJ}_{AB} P^B_\alpha \bar\chi^{\dot C}\gamma^\alpha \Gamma^{IJ}_{\dot C\dot A}\chi^{\dot D} , \quad (8)$$

where, in accordance with the gauge (4), all index contractions are with respect to the flat (=Minkowski) metric $\eta_{\alpha\beta}$. D_α is only covariant with respect to local SO(16); all Lorentz-covariantizations have been written out explicitly and are properly accounted for by the conformal rescalings and the extra factors of ρ in (8) and $\rho^{1/2}$ in (11) below. Note that there is no $\bar\psi\psi$ term on the right hand side of (8). The simplifications among the fermionic quartic terms are even more spectacular. Elimination of the auxiliary field F (see (5)), leads to

$$\mathscr{L}_{\text{aux}} = e[-\tfrac{1}{32}(\bar\chi\chi)^2 - \tfrac{1}{8}(\bar\psi\psi)(\bar\chi\chi) + \tfrac{1}{8}(\bar\psi\psi)^2] , \quad (9)$$

where we have omitted terms containing ψ^I_2 which do not contribute in the equations of motion because of (7). In addition, we get the following terms in second-order formalism (again dropping terms with ψ^I_2):

$$\mathscr{L}_{\text{torsion}} = e[-\tfrac{3}{32}(\bar\chi\chi)^2 + \tfrac{1}{8}(\bar\psi\psi)(\bar\chi\chi) - \tfrac{1}{8}(\bar\psi\psi)^2] . \quad (10)$$

Adding (9) and (10) to (3) and substituting the gauge condition (7) into the equations of motion, we see that al cubic spinorial terms cancel with the exception of those coming from $\bar\chi\gamma^a\Gamma^{IJ}\chi)^2$. Consequently, the full Dirac equation takes the form (remember that $\alpha = 0, 1$ whereas $a = 0, 1, 3$)

$$-i\rho^{-1/2}\gamma^\alpha D_\alpha(\rho^{1/2}\chi^{\dot A}) = \tfrac{1}{24}\gamma^a \Gamma^{KL}_{\dot A\dot B}\chi^{\dot B}\bar\chi^{\dot C}\gamma_a \Gamma^{KL}_{\dot C\dot D}\chi^{\dot D} . \quad (11)$$

Observe that the usual Noether term is also absent from (11).

The only equations remaining are those of the unphysical fields. They are

$$\rho^{-1}\partial_{(\alpha}\rho\lambda^{-1}\partial_{\beta)}\lambda - \tfrac{1}{2}\eta_{\alpha\beta}\rho^{-1}\partial^\gamma\rho\lambda^{-1}\partial_\gamma\lambda = T_{\alpha\beta} - \tfrac{1}{2}\eta_{\alpha\beta}T^\gamma{}_\gamma \quad (12)$$

with the energy–momentum tensor

$$T_{\alpha\beta} \equiv \tfrac{1}{2}P^A_\alpha P^A_\beta - \tfrac{1}{2}i\bar\chi^{\dot A}\gamma_{(\alpha}D_{\beta)}\chi^{\dot A} , \quad (13)$$

and

$$\rho^{-1}\partial_\beta\rho\gamma^\beta\gamma^\alpha\psi^I = i\Gamma^I_{A\dot A}\gamma^\beta\gamma^\alpha\chi^{\dot A}P^A_\beta . \quad (14)$$

Again, all cubic spinorial terms have disappeared in (14). Making use of the identification $\rho = x^1$ and introducing light-cone notation $\partial_\pm \equiv \partial_0 \pm \partial_1$, $P^A_\pm \equiv P^A_0 \pm P^A_1$, etc. as well as chiral spinors $\psi^I_\pm \equiv \tfrac{1}{2}(1 \pm \gamma^3)\psi^I$, etc., we can write (12) and (14) more succinctly as

$$\partial_+ \ln\lambda = \rho T_{++}, \quad \partial_- \ln\lambda = -\rho T_{--} , \quad (12')$$

and

$$\psi^I_+ = -i\rho\Gamma^I_{A\dot A}P^A_-\chi^{\dot A}_+ ,$$
$$\psi^I_- = i\rho\Gamma^I_{A\dot A}P^A_+\chi^{\dot A}_- . \quad (14')$$

This confirms that the fields λ and ψ^I carry no degrees of freedom of their own but can be explicitly expressed in terms of the physical fields.

To proceed further, we must briefly recall the essential results of ref. [2]. The decomposition (1) is, of course, valid for arbitrary Lie algebras $\mathbb{G} = \mathbb{H} \oplus \mathbb{K}$ so that $Q_\mu \in \mathbb{H}$, $P_\mu \in \mathbb{K}$, and the scalar fields $\mathscr{V}_0$ live on the coset space G/H. As is well known, the "composite" fields Q_μ and P_μ obey the integrability constraints (we now specialize to the case of two dimensions)

$$\partial_\mu Q_\nu - \partial_\nu Q_\mu + [Q_\mu, Q_\nu] + [P_\mu, P_\nu] = 0 ,$$
$$D_\mu P_\nu - D_\nu P_\mu = 0 \quad (15)$$

(D_μ is the H covariant derivative). The crucial observation is now that, in two dimensions, one can generalize (1) in such a manner that the scalar field equation $\rho^{-1}D^\alpha(\rho P_\alpha) = 0$ (which coincides with (8) for $\chi = 0$) also follows from the integrability constraint. To do so, one replaces $\mathscr{V}_0(x)$ by an element $\mathscr{V}(x, t)$ of the affine (= Kac Moody) extension G^∞ of G. Here, t is the "spectral parameter" by means of which the affine algebra $\mathbb{G}^\infty$ can be represented in the form $\sum t^n \otimes \mathbb{G}$ (plus central extension). Owing to the nontrivial ρ-dependence of the scalar field equation, the spectral parameter is itself a nontrivial function $t(x, w)$ (w is an integration constant) obeying [#4]

$$\epsilon_{\alpha\beta}\partial^{\beta}\rho = -\partial_{\alpha}[\tfrac{1}{2}\rho(t+1/t)] , \tag{16}$$

or, alternatively

$$t(1-t^2)\rho^{-1}\partial_{\alpha}\rho$$
$$=(1+t^2)\partial_{\alpha}t - 2t\epsilon_{\alpha\beta}\partial^{\beta}t . \tag{17}$$

Generalizing (1) to

$$\mathcal{V}^{-1}\partial_{\mu}\mathcal{V}$$
$$=Q_{\mu}+\frac{1+t^2}{1-t^2}P_{\mu}+\frac{2t}{1-t^2}\epsilon_{\mu\nu}P^{\nu} , \tag{18}$$

one can show that the new integrability condition for (18) is equivalent to (15) *and* the scalar field equation. As emphasized in ref. [2], the right-hand side of (18) is invariant under the transformation

$$\tau^{\infty}:\ \mathcal{V}(x,t)\to\tau\mathcal{V}(x_1, 1/t)\equiv(\mathcal{V}^{\mathrm{T}})^{-1}(x, 1/t) , \tag{19}$$

which induces the usual automorphism $\tau(\mathbb{H}, \mathbb{K})=(\mathbb{H}, -\mathbb{K})$ on the Lie-algebra. The elements $\mathcal{V}\in \mathrm{G}^{\infty}$ invariant under τ^{∞} constitute an infinite-dimensional subgroup H^{∞} of G^{∞} which, however, does not coincide and should not be confused with the affine extension of H. Note that H^{∞} is not of finite codimension in G^{∞}.

Specializing to the case at hand, i.e. $\mathrm{G}=\mathrm{E}_8$ and $\mathrm{H}=\mathrm{SO}(16)$, we now propose to further extend (18) such that the full $N=16$ field equations (8) and (11) can be derived from an integrability constraint. This evidently requires adding new terms depending on the fermions to the right-hand side of (18). Since ψ^I does not appear in either (8) or (11), we proceed on the assumption that the modification of (18) is also independent of ψ^I. This automatically excludes additional terms in P_{μ}^{A} (which would have to be proportional to $\Gamma^I_{A\dot{A}}\bar{\chi}^{\dot{A}}\gamma_{\mu}\psi^I$) and leaves us with the following possibility for the modification of Q_{μ}:

$$\hat{Q}_{\mu}^{IJ}=Q_{\mu}^{IJ}+f(t)\bar{\chi}\gamma_{\mu}\Gamma^{IJ}\chi+g(t)\bar{\chi}\gamma^3\gamma_{\mu}\Gamma^{IJ}\chi . \tag{20}$$

The next step is the investigation of the extra terms in the integrability constraint arising from the fermionic bilinears in (20). Requiring absence of terms proportional to $P_{\alpha}\bar{\chi}\gamma^3\gamma^{\alpha}\chi$, we find

$$g(t)=\frac{1+t^2}{2t}f(t) . \tag{21}$$

Next, we demand

$$0\overset{!}{=}\partial_{[\mu}(\rho^{-1}f)\rho\bar{\chi}\gamma_{\nu]}\Gamma^{IJ}\chi$$
$$+\partial_{[\mu}(\rho^{-1}g)\rho\bar{\chi}\gamma^3\gamma_{\nu]}\Gamma^{IJ}\chi$$
$$=\tfrac{1}{2}\epsilon_{\mu\nu}\rho\bar{\chi}\gamma^{\alpha}\Gamma^{IJ}\chi$$
$$\times[\epsilon_{\alpha\beta}\partial^{\beta}(\rho^{-1}f)-\partial_{\alpha}(\rho^{-1}g)] \tag{22}$$

(the factor ρ must be pulled out because of the extra $\rho^{1/2}$ in the Dirac equation (11)). The differential equation (22) can be solved by use of (16), (17) and (21); the result is

$$f(t)=2\mathrm{i}\frac{t^2}{(1-t^2)^2} , \tag{23}$$

where the prefactor is determined by matching the term proportional to $P_{\alpha}\bar{\chi}\gamma^{\alpha}\chi$ to the scalar field equation (8). At this point, all available coefficients have been fixed, and the rest of the calculation only checks the consistency of the ansatz (20). The action of D_{α} on χ in (20) yields the Dirac equation twice. It is crucial here that the right-hand side of (11) vanishes when multiplied by $\bar{\chi}\Gamma^{IJ}$ but *not* when multiplied by $\bar{\chi}\gamma^3\Gamma^{IJ}$. The cubic term, then, is correctly reproduced if one makes use of the Fierz identity

$$\bar{\chi}\gamma^3\gamma^{\alpha}\Gamma^{K[I}\chi\bar{\chi}\gamma_{\alpha}\Gamma^{J]K}\chi$$
$$=-\tfrac{1}{6}\bar{\chi}\gamma^3\gamma^{a}\Gamma^{IJ}\Gamma^{KL}\chi\bar{\chi}\gamma_{a}\Gamma^{KL}\chi , \tag{24}$$

which may be deduced from the identities given in the appendix of ref. [4].

To summarize, we have shown that

$$\hat{\mathcal{V}}^{-1}\partial_{\mu}\hat{\mathcal{V}}=\hat{Q}_{\mu}+\frac{1+t^2}{1-t^2}P_{\mu}+\frac{2t}{1-t^2}\epsilon_{\mu\nu}P^{\nu} , \tag{25}$$

with

$$\hat{Q}_{\mu}^{IJ}=Q_{\mu}^{IJ}+2\frac{t^2}{(1-t^2)^2}\mathrm{i}\bar{\chi}\gamma_{\mu}\Gamma^{IJ}\chi$$
$$+\frac{t(1+t^2)}{(1-t^2)^2}\mathrm{i}\bar{\chi}\gamma^3\gamma_{\mu}\Gamma^{IJ}\chi , \tag{26}$$

[#4] The occurrence of $t+1/t$ rather than $t-1/t$ in our expressions is due to the indefiniteness of our metric (in ref. [2] a euclidean metric is used.

satisfies an integrability constraint if and only if (8), (11) and (15) are satisfied. This, in turn, ensures that there exists a t-dependent E_8 matrix $\hat{\mathcal{V}}$ such that (25) holds. This matrix contains all the (on-shell) information about $d=2$, $N=16$ supergravity. It is remarkable that our construction also supplies a natural raison d'etre for the quartic fermionic terms in (maximally extended) supergravities!

The above construction effectively amounts to a bosonization of the 128 fermions of $N=16$ supergravity. Of course, bosonization makes sense only in the quantum theory, and we here assume that the $N=16$ theory is sufficiently benign so that all our manipulations survive quantization. The formulas (25) and (26) are reminiscent of the non-abelian bosonization formulas of refs. [5,6] (where it was suggested that 128 fermions belonging to the coset space $E_8/SO(16)$ can be bosonized in terms of 120 bosons), but there are important differences. The presence of an x-dependent spectral parameter is a complication not encountered in refs. [5,6] and, unlike refs. [5,6] which are concerned with free fermions, we are dealing with a system of fermions and bosons in interaction. Also, it is not immediately obvious in our case how the E_9 symmetry acts on the fields. In this context, it is significant that the extra fermionic terms in (26) are still invariant under the automorphism (19). This means that, following [2], one can construct a "scattering matrix"

$$M(x,t)=\hat{\mathcal{V}}\tau^{\infty}(\hat{\mathcal{V}}^{-1})(x,t) \tag{27}$$

and prove that M is actually x-independent, i.e. $M(x, t(x,w))=M(w)$. Under the action of $g(w)\in E_9$, we have

$$\hat{\mathcal{V}}(x,t)\to g(w)^{-1}\,\hat{\mathcal{V}}(x,t)h(x,t)\,, \tag{28}$$

where $h\in H^{\infty}$ is a compensating transformation which renders $\hat{\mathcal{V}}$ "triangular" (in the sense of ref. [2]). One can also study the E_9 transformations before bosonization and explicitly derive their action on the fields χ^A and ψ^I.

By encoding the whole theory into a single (bosonic) matrix $\hat{\mathcal{V}}$, we have succeeded in fusing SO(16) symmetry and $N=16$ supersymmetry such that they are now part of E_9. However, this "bosonisation" of $N=16$ supersymmetry certainly requires further elucidation. One of the more futuristic aspects of this work is a possible "stringy" (or rather "membrany" as $\hat{\mathcal{V}}$ really depends on three variables) interpretation of $N=16$ supergravity. If this theory has soliton solutions, as is suggested by the existence of such solutions for other integrable models (see e.g. ref. [3]), these might be converted ito higher-dimensional particle-like excitations in the same way as the "solitons" of string theory (i.e. the free vibrational modes of the string). It is difficult to see how the "zero-slope-limit" of such a theory could not be related to $d=11$ supergravity [7]. In refs. [8,9], it has been shown that the "hidden" symmetries of the maximally extended supergravities in four and three dimensions can be "lifted" to eleven dimensions. In this process, internal symmetries of the dimensionally reduced theories are elevated to space–time symmetries. If simular results hold for the case considered here, there would be a much more stringent relationship between two-dimensional symmetries and "target space" symmetries than in string theories. This line of argument suggests a sort of "bootstrap" whereby $d=11$ supergravity via dimensional reduction gives rise to a theory that contains the on-shell states of $d=11$ supergravity in its soliton spectrum and thus regenerates its own ancestor (the on-shell states of SO(16) invariant $d=11$ supergravity [9] transform as the 128_L and 128_R spinor representations of SO(16)). Finally, it has been pointed out in ref. [1] that E_{10} is a natural successor to E_9, and perhaps the results described here will help us towards a better understanding of these issues. A more daring speculation is the possible realization of E_{10} as a symmetry of the multiparticle states in a "stringy" interpretation of $N=16$ supergravity.

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