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HIDDEN CONSTANTS: THE θ PARAMETER OF QCD AND THE COSMOLOGICAL CONSTANT OF $N = 8$ SUPERGRAVITY

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For field theories that include the abelian gauge field $A_{\mu\nu\rho}$ the field equations allow an arbitrary integration constant, which does not appear in the lagrangian but which does affect the physics. We present two applications: (i) the θ parameter of effective lagrangians for chiral symmetry breaking in QCD, and (ii) the cosmological constant in $N = 8$ supergravity, which does not require a gauging of the O(8) symmetry, but is rather due to a spontaneous breakdown of supersymmetry.

1. Introduction

The solution of a field theory can depend on more parameters than actually appear in the lagrangian. A well-known example is the θ parameter of QCD. This parameter *can* be introduced into the lagrangian but only as the coefficient of a total derivative, the topological charge density $F^a_{\mu\nu}F^a_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}$. Within perturbation theory θ dependence of physical quantities cannot arise, but it is plausibly conjectured that θ dependence does arise in leading order of the $1/N$ expansion [1]. The fact that θ dependence does not arise in perturbation theory in the coupling constant is due not just to the fact that $F^a_{\mu\nu}F^a_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma}$ is a total derivative, but also to the fact that it is multilinear in the fundamental field A^a_μ. In the massive Schwinger model, for example, the topological charge density $\varepsilon^{\mu\nu}\partial_\mu A_\nu$ is linear in the photon field A_ν and the addition of a term $\theta\varepsilon^{\mu\nu}\partial_\mu A_\nu$ to the lagrangian causes physical quantities to be θ dependent in perturbation theory. There is another way to see how the θ dependence arises in this two-dimensional model. In two dimensions the photon field A_μ propagates no physical degrees of freedom and can be eliminated. But the solution of its field equation allows an arbitrary *integration constant* which can be identified as proportional to θ [2]. Although the QCD topological charge

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density is multilinear in fields it can be written in the form

$$F^a_{\mu\nu}F^a_{\rho\sigma}\varepsilon^{\mu\nu\rho\sigma} \equiv \lambda^2 F_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu\rho\sigma},$$

$$F_{\mu\nu\rho\sigma} = 4\partial_{[\mu}A_{\nu\rho\sigma]}, \tag{1.1}$$

where $A_{\mu\nu\rho}$ is an antisymmetric abelian gauge field transforming as $\partial A_{\mu\nu\rho} = 3\partial_{[\mu}\Lambda_{\nu\rho]}$ under non-abelian gauge transformations of A^a_μ [3], and [ ] indicates antisymmetrization with strength 1. λ is a constant with dimensions of mass, so that $A_{\mu\nu\rho}$ has dimensions of mass also. $A_{\mu\nu\rho}$ is a composite field in QCD with no one-particle pole in perturbation theory, but it is attractive to suppose that it does develop a one-particle pole to leading order in the $1/N$ expansion [3]. The effective lagrangian for QCD in the large-N limit will then contain $A_{\mu\nu\rho}$ as a fundamental field. Then, just as for the Schwinger model, θ dependence can arise in perturbation theory. The "topological gauge field" $A_{\mu\nu\rho}$ plays the same role in this effective four-dimensional theory as A_μ in the two-dimensional Schwinger model [4]; it propagates no physical degrees of freedom and can be eliminated. But again its field equation allows an arbitrary integration constant [5], which can be interpreted as the parameter θ, up to a proportionality factor.

Because of the fact that, in general, the background energy density is θ dependent we might also have interpreted this integration constant as a cosmological constant. Since we are not usually interested in gravity when discussing QCD and since there is no symmetry that forbids us to add another arbitrary constant to the energy density, this interpretation would not be so useful. However, there are certain supergravity theories which include the gauge field $A_{\mu\nu\rho}$ where this identification is appropriate. In the presence of a gravitational field a contribution to the background energy density is a contribution to the cosmological constant. Already in the simplest, $N = 1$, supergravity the addition of such a term to the lagrangian requires certain mass-like terms for the gravitino to be added, and in the case of $N = 2, 3, 4$ extended supergravity the O(N) symmetry must be gauged [6]. Above $N = 4$ it is not known if a cosmological term can be consistently included in this way. Even in $N = 1$ supergravity a superspace formulation does not allow the direct addition of a cosmological constant, but as Ogievetsky and Sokatchev have recently shown it can nevertheless appear as an *integration constant* of the equations of motion [7]. This can be related to the appearance of the gauge field $A_{\mu\nu\rho} \equiv \varepsilon_{\mu\nu\rho\sigma}S^\sigma$ in their lagrangian.

Another theory in which $A_{\mu\nu\rho}$ appears quite naturally is 11-dimensional supergravity [8], whose field constant is an elfbein, e^A_M, a Majorana gravitino, ψ_M, and the 11-dimensional gauge field A_{MNP}. In 11-dimensions A_{MNP} propagates physical modes but on reduction to four dimensions this theory becomes $N = 8$ supergravity [9] theory and A_{MNP} reduces into scalars, vectors and antisymmetric tensors which propagate some of the spin 0 and spin 1 degrees of freedom, plus a *four-dimensional*

$A_{\mu\nu\rho}$ which appears in the lagrangian only through its field strength $F_{\mu\nu\rho\sigma}$. Again, the elimination of $A_{\mu\nu\rho}$ introduces an integration constant which can be identified as a cosmological constant.

The outline of this paper, then, is as follows: in sect. 2 we discuss a simple effective lagrangian for chiral symmetry breaking [10] in four dimensions in which the θ dependence appears in the same way as the two-dimensional Schwinger model. As this analysis is applicable equally to the more realistic effective lagrangians proposed recently [11] it constitutes a more satisfactory way of deriving the θ dependence of these theories. In sect. 3 we go on to discuss the introduction of a cosmological constant into $N=8$ supergravity. This is the chief new result of the paper and our solution to this problem exhibits some unusual features. For example, we find that there is no gauging of the O(8) symmetry but that rather there is a spontaneous breaking of the supersymmetry.

Before continuing with these results we would like to make a few more detailed remarks on the general idea that connects the two very different applications of this paper. The central feature is the presence of the gauge field $A_{\mu\nu\rho}$, with field strength $F_{\mu\nu\rho\sigma}=4\partial_{[\mu}A_{\nu\rho\sigma]}$, and Maxwell-like lagrangian

$$\mathcal{L}=-\tfrac{1}{48}eF_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}, \tag{1.2}$$

where e is the vierbein determinant, added for general coordinate invariance. This has the equivalent first-order form

$$\mathcal{L}=\frac{1}{4!}\rho\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}-\tfrac{1}{2}e\rho^2, \tag{1.3}$$

with ρ an independent field and $\varepsilon^{\mu\nu\rho\sigma}$ the usual constant tensor density. The ρ field equation is

$$\rho=\frac{1}{4!}e^{-1}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \tag{1.4}$$

and substitution of (1.4) into (1.3) gives back (1.2). From (1.3) we see easily that the $A_{\mu\nu\rho}$ field equation is

$$\partial_\mu\rho=0\Rightarrow\rho=a=\text{constant}. \tag{1.5}$$

It is also clear from (1.3) that this constant will appear in the right-hand side of the Einstein field equation and can therefore be interpreted as a cosmological constant.

The reader will notice that in the above discussion we have stressed the field equations, and in particular the $A_{\mu\nu\rho}$ field equation whose solution introduces an integration constant. Usually the substitution of the solution of a field equation back into the lagrangian is legitimate if the field appears only quadratically, because this amounts to performing the gaussian path integral over this field in the functional integral of the quantum theory. In this case, however, there is a subtlety. If the functional integral is well defined, integration over a field *cannot* produce an *arbitrary* constant so that for the case of $A_{\mu\nu\rho}$, substitution of its field equation cannot be equivalent to functional integration. Where then does this constant appear in the functional integral formulation of the theory? The answer is that it appears in the form of a possible total derivative $\theta\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$ in the lagrangian. It is easy to see that because the $A_{\mu\nu\rho}$ propagator has a pole at $p^2=0$ the extra θ dependent vertex represented by this additional term in the lagrangian does make a difference to the physics even in perturbation theory. For example, the Feynman diagram in which a zero momentum $A_{\mu\nu\rho}$ is produced at a θ vertex and subsequently absorbed at a θ vertex gives a θ^2 contribution to the vacuum energy density. In general it is not difficult to check that the contribution of the θ vertex to tree diagrams produces the same θ dependence of physical quantities as one would deduce from the equation of motion. The presence of an integration constant in the equations of motion and the possibility of adding a "topological invariant" to the action are therefore essentially equivalent. In the title of this paper we have called constants that appear in this way, "hidden", to distinguish them from the usual mass and coupling constants that appear in a lagrangian in the conventional way.

One final remark here concerns the nomenclature "topological gauge field" for $A_{\mu\nu\rho}$. This is motivated by its appearance as a composite field in the QCD topological charge density and by analogy with the Schwinger model where $\int\varepsilon^{\mu\nu}\partial_\mu A_\nu \,\mathrm{d}^2_x$ takes integral values for boundary conditions such that A_μ vanishes sufficiently rapidly at infinity. As the Schwinger model suggests, however, there is no reason in general to suppose these boundary conditions to apply, and in the examples we discuss $\int\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,\mathrm{d}^4x$ is certainly not a topological invariant in the conventional sense. For example, on equating (1.4) and (1.5) we find that the equation

$$a^2\int e\,\mathrm{d}^4x=\frac{1}{4!}a\int\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}\,\mathrm{d}^4x \tag{1.6}$$

is a consequence of the $A_{\mu\nu\rho}$ field equation. The right-hand side is our "topological invariant", it can be calculated from a knowledge of $A_{\mu\nu\rho}$ on the boundary only; while the left-hand side is an ordinary cosmological term!

2. θ dependence and chiral symmetry breaking

The four-dimensional generalization of the bosonized massless Schwinger model is the $\mu\to 0$ limit of the following lagrangian [10]:

$$\begin{aligned}\mathcal{L}= &-\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\mu^2\phi^2+\frac{\lambda}{3!}\partial_\mu\phi\varepsilon^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}\\ &+\tfrac{1}{4}\rho\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}-\tfrac{1}{2}\rho^2.\end{aligned} \tag{2.1}$$

As in sect. 1 we have written the kinetic term for $A_{\mu\nu\rho}$ in first-order form. In the limit $\mu \to 0$ this lagrangian possesses a non-linearly realized chiral symmetry $\delta\phi = \omega$; chiral because ϕ is pseudoscalar. The corresponding conserved Noether current is $J^{\mu 5} = \partial^{\mu}\phi + (\lambda/3!)\varepsilon^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}$, which is gauge variant. The gauge-invariant current $j^5_\mu = \partial_\mu\phi$ has the divergence

$$\partial^\mu j^5_\mu = -\mu^2\phi - \frac{\lambda}{4!}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \tag{2.2}$$

so that even in the chiral symmetry limit $\mu^2 \to 0$ this current is not conserved because of the "anomaly" $\lambda\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$. The coefficient of this anomaly is proportional to the dimensional constant λ, as in the coefficient of the ϕ-$A_{\mu\nu\rho}$ mixing. Without this mixing the Goldstone field ϕ would be massless.

The field equations for ρ and $A_{\mu\nu\rho}$ are respectively

$$\rho = \frac{1}{4!}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}, \tag{2.3}$$

$$\partial_\mu(\rho - \lambda\phi) = 0 \Rightarrow \rho = \lambda\phi + a, \qquad a \equiv \lambda^2\theta. \tag{2.4}$$

Again, the $A_{\mu\nu\rho}$ field equation introduces an arbitrary constant, a, which can be traded for the dimensionless constant θ. Using (2.3) and (2.4) the ϕ field equation can be written as

$$\Box\phi - (\mu^2 + \lambda^2)\phi - \lambda^3\theta. \tag{2.5}$$

To remove the apparent source term $\lambda^3\theta$ we must make the change of variables $\phi = \phi' - \lambda^3\theta/(\mu^2 + \lambda^2)$. Keeping track also of the vacuum energy density we can write down a reduced lagrangian for ϕ' alone

$$\mathcal{L}_{\text{red}} = -\tfrac{1}{2}(\partial_\mu\phi')^2 - \tfrac{1}{2}(\mu^2 + \lambda^2)\phi'^2 - \frac{1}{2}\frac{\lambda^4\mu^2}{\mu^2 + \lambda^2}\theta^2. \tag{2.6}$$

(One way to keep track of the energy density is to include a coupling to gravity and to read off the vacuum energy density from the right-hand side of Einstein's field equation.)

Notice first that the particle spectrum is one *massive* spin 0 pseudoscalar boson, thus evading the Goldstone theorem [10–12]. Also, comparing the energy density with and without the ϕ field we see that

$$E(\theta)^{\text{with }\phi} = \frac{1}{2}\frac{\lambda^4\mu^2}{\mu^2 + \lambda^2}\theta^2,$$

$$E(\theta)^{\text{without }\phi} = \tfrac{1}{2}\lambda^4\theta^2. \tag{2.7}$$

The parameter θ arose as an integration constant of the $A_{\mu\nu\rho}$ field equation: what is its significance? Recall [10] that the lagrangian (2.1) can be considered as a prototype of an effective lagrangian for QCD in the large-N limit, with only one flavour for the quarks, and only one bound state, the meson η'. The chiral symmetry breaking mass μ is assumed to be proportional to the mass of the quark. In the theory with the quark, i.e., with ϕ, $E(\theta)\rightarrow 0$ as $\mu\rightarrow 0$; the θ dependence vanishes with vanishing quark mass. [Also in the $\mu\rightarrow 0$ limit, we have on dimensional grounds that $m_{\eta'}\propto f_{\eta'}\propto \lambda$ and therefore

$$\frac{1}{f_{\eta'}^2}\left(\frac{\mathrm{d}^2E}{\mathrm{d}\theta^2}\right)_{\theta=0}^{\text{no quarks}}\propto m_{\eta'}^2, \tag{2.8}$$

which we recognize as Witten's [12] current algebra formula for the mass of the η'.] From these remarks it is clear that θ is the usual CP-violating parameter, up to a normalization factor.

The point of this exercise has been to show that effective lagrangians for chiral symmetry breaking that include the abelian gauge field $A_{\mu\nu\rho}$ contain a hidden parameter, an integration constant in the equations of motion, that can be identified as the usual θ parameter*. According to our discussion in sect. 1, the appearance of an integration constant in the classical equations of motion of (2.1) is equivalent to the statement that one can add the "topological invariant" $\theta\lambda^2\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$ to the lagrangian in the path integral, and that the physics is, in general, θ dependent in perturbation theory. The easiest way to deduce this θ dependence is to treat $\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$ as an independent variable and eliminate it by performing the gaussian functional integral, as advocated by Di Vecchia and Veneziano [11]. This gives the correct result but needs some justification. In ref. [11] the authors justified it by showing that the results were such that the chiral symmetry Ward identities were satisfied. From our point of view it is justified by the fact that it gives the same results as obtained by using the $A_{\mu\nu\rho}$ field equation in the other field equations and then constructing the reduced lagrangian for the reduced set of field equations.

There is another point we should mention. $E(\theta)$ in (2.9) is not periodic in θ. This is because all interactions have been omitted and the chiral symmetry is $E(1)$, rather than U(1). This inclusion of interactions as in the more realistic effective lagrangians will force θ to be an angular variable. Also, by analogy with the Schwinger model [2], we would expect the introduction of external membrane sources [5] for $A_{\mu\nu\rho}$ to produce non-analytic θ dependence at periodic intervals, forcing the physics to be periodic in θ.

* We should point out that Rosenzweig, Schechter and Trahern [11] did notice the existence of this integration constant, but set it to zero.

3. The cosmological constant in $N = 8$ supergravity

An elegant way to obtain the $N = 8$ supergravity theory is by dimensional reduction of $N = 1$ supergravity in 11 dimensions [9]. The lagrangian of the latter is

$$\kappa^2 \mathcal{L}_{11} = -\tfrac{1}{4} e_{11} R(\omega) - \tfrac{1}{48} e_{11} F_{MNPQ} F^{MNPQ} - \tfrac{1}{2} i e_{11} \bar{\psi}_M \Gamma^{MNP} D_N\left(\frac{\omega + \hat{\omega}}{2}\right) \psi_P$$
$$+ \tfrac{1}{192} e_{11} \left(\bar{\psi}_M \Gamma^{MNWXYZ} \psi_N + 12 \bar{\psi}^W \Gamma^{YZ} \psi^X \right) \left(F_{WXYZ} + \hat{F}_{WXYZ} \right)$$
$$+ \frac{2}{(12)^4} \varepsilon^{M_1 \dots M_{11}} F_{M_1 M_2 M_3 M_4} F_{M_5 M_6 M_7 M_8} A_{M_9 M_{10} M_{11}} . \tag{3.1}$$

Our notations and conventions are those of Cremmer and Julia [9], except by e_{11} we denote $\det(e_M{}^A)$. F_{WXYZ} is the field strength of the abelian gauge field A_{XYZ}, and ω_{MAB} is the usual spin connection with torsion. $\hat{F}$ and $\hat{\omega}$ are the same quantities covariantized with respect to the local supersymmetry transformations

$$\delta e_M{}^A = -i \bar{\varepsilon} \Gamma^A \psi_M ,$$
$$\delta A_{MNP} = \tfrac{3}{2} \bar{\varepsilon} \Gamma_{[MN} \psi_{P]} ,$$
$$\delta \psi_M = D_M(\hat{\omega}) \varepsilon + \tfrac{1}{144} i \left(\Gamma_M{}^{NPQR} - 8 \Gamma^{PQR} \delta_M^N \right) \varepsilon \hat{F}_{NPQR} . \tag{3.2}$$

These are the only independent fields of the theory. Although this 11-dimensional theory is remarkably simple, the reduction of it to four dimensions is quite complicated. What is of interest to us here is the reduction of F_{WXYZ} to $F_{\mu\nu\rho\sigma}$, the four-dimensional field strength for the four-dimensional gauge field $A_{\mu\nu\rho}$.

Let us start according to ref. [9] by making the partial gauge choice for $e_M{}^A$

$$e_M{}^A = \begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^a \\ 0 & e_m{}^a \end{pmatrix} , \tag{3.3}$$

i.e., $e_m{}^\alpha = 0$. With this choice the relation between the world and tangent space indices of the 11-dimensional Levi-Civita symbol,

$$\varepsilon^{M_1 \dots M_{11}} = e_{11} e_{A_1}{}^{M_1} \dots e_{A_{11}}{}^{M_{11}} \varepsilon^{A_1 \dots A_{11}} , \tag{3.4}$$

is consistent with the similar definitions for the reduced Levi-Civita symbols

$$\varepsilon^{\mu\nu\rho\sigma} = e e_\alpha{}^\mu e_\beta{}^\nu e_\gamma{}^\rho e_\delta{}^\sigma \varepsilon^{\alpha\beta\gamma\delta} ,$$
$$\varepsilon^{ijklmno} = \Delta^{1/2} e_a{}^i \dots e_g{}^o \varepsilon^{abcdefg} , \tag{3.5}$$

where also

$$e_{11} = \det e_M{}^A = e\Delta^{1/2},$$

$$e = \det e_\mu{}^\alpha, \qquad \Delta^{1/2} = \det e_m{}^a. \tag{3.6}$$

Now we define the quantity $F_{\alpha\beta\gamma\delta}$ with four-dimensional tangent space indices by

$$\begin{aligned} F_{\alpha\beta\gamma\delta} &= e_\alpha{}^M e_\beta{}^N e_\gamma{}^P e_\delta{}^Q F_{MNPQ} \\ &= e_{[\alpha}{}^\mu e_\beta{}^\nu e_\gamma{}^\rho e_{\delta]}{}^\sigma \{ F_{\mu\nu\rho\sigma} - 4B_\sigma{}^k F_{\mu\nu\rho k} \\ &\qquad + 6B_\rho{}^j B_\sigma{}^k F_{\mu\nu jk} - 4B_\nu{}^i B_\rho{}^j B_\sigma{}^k F_{\mu ijk} \} . \end{aligned} \tag{3.7}$$

The last line of (3.7) follows from the gauge choice (3.3) and the definitions

$$B_\rho{}^i = B_\rho{}^a e_a{}^i, \qquad e_a{}^i e_i{}^b = \delta_a^b. \tag{3.8}$$

$B_\rho{}^i$ transforms under 11-dimensional general coordinate transformations such that after the gauge choice (3.3) and after dimensional reduction this transformation becomes the ordinary gauge transformation $\delta B_\rho{}^i = \partial_\rho \xi^i$. But $A_{\mu ij}$ and $A_{\mu\nu i}$ also transforms under this symmetry. Therefore we construct the new fields $A'_{\mu ij}$ and $A'_{\mu\nu i}$ which are invariant under this symmetry [9],

$$A'_{\mu ij} = A_{\mu_{ij}} - B_\mu{}^k A_{kij},$$

$$A'_{\mu\nu i} = A_{\mu\nu i} - B_\mu{}^i A_{j\nu i} - B_\nu{}^j A_{\mu ji} + B_\mu{}^j B_\nu{}^k A_{jki}, \tag{3.9}$$

and define $F'_{\mu\nu ij}$ and $F'_{\mu\nu\rho i}$ to be the corresponding field strengths. Writing (3.7) in terms of the primed fields, and multiplying by $\varepsilon^{\alpha\beta\gamma\delta}$ we find,

$$\varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} = e\varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta} + 4\varepsilon^{\mu\nu\rho\sigma} B_\sigma{}^k F'_{\mu\nu\rho k}, \tag{3.10}$$

which will be useful later.

Now we proceed to pick out those contributions to the dimensionally reduced lagrangian that contain $F_{\alpha\beta\gamma\delta}$. These are

$$\begin{aligned} \mathcal{L}_F = &-\tfrac{1}{48} e\Delta^{1/2} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \\ &+ \tfrac{1}{96} e\Delta^{1/2} \left(\bar\psi_M \Gamma^{MN\alpha\beta\gamma\delta} \psi_N + 12 \bar\psi^\alpha \Gamma^{\gamma\delta} \psi^\beta \right) F_{\alpha\beta\gamma\delta} . \end{aligned} \tag{3.11}$$

At this point we could proceed according to ref. [9] by taking $F_{\alpha\beta\gamma\delta}$ to be an independent field and eliminating it by its field equation. This would produce various quartic fermion terms. However, the independent field is $A_{\mu\nu\rho}$ not $F_{\mu\nu\rho\sigma}$ (or $F_{\alpha\beta\gamma\delta}$) and its elimination will introduce an arbitrary integration constant. As we explained in the previous sections, this is equivalent to adding a total derivative,

$$\mathcal{L}_a = \frac{1}{4!} a \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}, \tag{3.12}$$

to the lagrangian. Then, if we consider $F_{\mu\nu\rho\sigma}$ (or $F_{\alpha\beta\gamma\delta}$) as the independent field its elimination will produce additional a and a^2 dependent terms in the lagrangian. We have explained previously why this replacement is justified, (i.e., it gives the same a dependence as the integration constant in the equations of motion), but there is another justification in this case. Since $F_{\mu\nu\rho\sigma}$ is a total derivative it must transform under supersymmetry (and all other symmetries) as a total derivative. Since $A_{\mu\nu\rho}$ appears only through its field strength $F_{\mu\nu\rho\sigma}$, it makes no difference to the supersymmetry whether $F_{\mu\nu\rho\sigma}$ is defined in terms of $A_{\mu\nu\rho}$ or is an independent field, because in either case it transforms in the same way. The reader may be suspicious of this argument because it would appear to apply equally to a theory like $N = 2$ supergravity [13] where the photon field A_μ appears only through its field strength $F_{\mu\nu}$. But in that case supersymmetry requires that $F_{\mu\nu}$ satisfy $(\partial_\mu F_{\nu\rho} + \partial_\rho F_{\mu\nu} + \partial_\nu F_{\rho\mu}) = 0$, which implies that $F_{\mu\nu}$ is the curl of A_μ, i.e., we must use the Bianchi identity in the proof of supersymmetry. This is to be contrasted with the Bianchi identity for $F_{\mu\nu\rho\sigma}$, $\partial_{[\mu} F_{\nu\rho\sigma\lambda]} \equiv 0$, which imposes no restriction on $F_{\mu\nu\rho\sigma}$ because a totally antisymmetric tensor with five indices vanishes in four dimensions.

To continue, we add (3.12) to (3.11) using (3.10) to replace $F_{\mu\nu\rho\sigma}$ by $F_{\alpha\beta\gamma\delta}$. Then we make the definitions [9],

$$\chi_\alpha = \psi_M e_\alpha{}^M, \qquad \chi_a = \psi_M e_a{}^M,$$
$$f = \frac{1}{4!} \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta\gamma\delta}, \tag{3.13}$$

and use the following representation for the 11-dimensional matrices,

$$\gamma^\alpha \otimes \mathbb{1}_A{}^B, \qquad \gamma^5 \otimes (\Gamma^a)_A{}^B, \tag{3.14}$$

where the Γ^a are the 8×8 matrices of the SO(7) Clifford algebra, to obtain,

$$\mathcal{L}_a = \tfrac{1}{2} e \Delta^{1/2} f^2 + e \Delta^{1/2} \{ -\tfrac{1}{4} \bar{\chi}_a^A (\Gamma^{ab})_A{}^B i \gamma^5 \chi_{bB}$$
$$+ \tfrac{1}{2} \bar{\chi}^{\alpha A} i \gamma^5 \sigma_{\alpha\beta} \chi_A^\beta \} f + aef + \tfrac{1}{6} a \varepsilon^{\mu\nu\rho\sigma} B_\sigma{}^k F'_{\mu\nu\rho k} \tag{3.15}$$

($\sigma^{\alpha\beta}=\frac{1}{2}\Gamma^{\alpha\beta}=\frac{1}{4}[\Gamma^{\alpha},\Gamma^{\beta}]$). The fields appearing in (3.14) are not yet those of the final lagrangian. We must first perform some field redefinitions [9]; namely,

$$e_{4\mu}{}^{\alpha}=e_{\mu}{}^{\alpha}\Delta^{1/4}\Rightarrow e_4=e\Delta,$$

$$\chi_{\alpha}=\chi_{4\alpha}\Delta^{1/8}, \qquad \chi_a=\chi_{4a}\Delta^{1/8}, \tag{3.16}$$

followed by

$$\chi_{4\alpha}=e_{4\alpha}{}^{\mu}\psi_{4\mu}+\tfrac{1}{2}\gamma_5\gamma_{\alpha}\Gamma^{a}\chi_{4a}, \tag{3.17}$$

and then again followed by

$$\psi_4{}^{\mu}=(i\gamma^5)^{-1/2}\hat{\psi}^{\mu}, \qquad \chi_{4a}=(i\gamma^5)^{-1/2}\hat{\chi}_a. \tag{3.18}$$

After these redefinitions (3.15) becomes

$$\begin{aligned}\mathcal{L}_a=\tfrac{1}{2}e_4\Delta^{-1/2}f^2+e_4\Delta^{-1/4}\Big\{&-\tfrac{1}{4}\hat{\bar{\chi}}{}_a^A\Gamma^{ab}_{AB}\hat{\chi}_b^B+\tfrac{1}{2}\hat{\bar{\psi}}{}_\mu^A\sigma^{\mu\nu}\hat{\psi}_{\nu A}\\ &-\tfrac{3}{4}\hat{\bar{\chi}}{}_a^A(\Gamma^a\Gamma^b)_{AB}\hat{\chi}_b^B+\tfrac{3}{4}i\hat{\bar{\psi}}{}_\mu^A\gamma^\mu\Gamma^a_{AB}\hat{\chi}_a^B\Big\}f\\ &+e_4\Delta^{-1}af+\tfrac{1}{6}a\varepsilon^{\mu\nu\rho\sigma}B_\sigma{}^kF'_{\mu\nu\rho k}.\end{aligned} \tag{3.19}$$

In the curly bracket above, γ^{μ} and $\sigma^{\mu\nu}$ are the ordinary four-dimensional Dirac matrices, defined relative to the constant matrices γ^{α}, $\sigma^{\alpha\beta}$ by the vierbein $e_{4\mu}{}^{\alpha}$. But the Majorana restriction on the fermions is still with respect to the 11-dimensional charge conjugation matrix $\mathcal{C}_{11}=\mathcal{C}_4\otimes\mathcal{C}_7$. $\mathcal{C}_4$ is the usual four-dimensional charge conjugation matrix while $\mathcal{C}_7$ is a *symmetric* 8×8 matrix satisfying $\mathcal{C}_7\Gamma^a\mathcal{C}_7^{-1}=-\Gamma^{aT}$ [14] (e.g., $\mathcal{C}_7=1$ in the representation where the Γ^a are all antisymmetric). There is one other field redefinition made by Cremmer and Julia by which $\hat{\chi}_{aA}$ is replaced by λ_{ABC}, but we omit this.

To find the new terms in the lagrangian we eliminate f from (3.19) keeping only the a-dependent terms:

$$\begin{aligned}\mathcal{L}_a=-\tfrac{1}{2}a^2e_4\Delta^{-3/2}-ae_4\Delta^{-3/4}\Big\{&-\tfrac{1}{4}\hat{\bar{\chi}}{}_a^A\Gamma^{ab}_{AB}\hat{\chi}_b^B+\tfrac{1}{2}\hat{\bar{\psi}}{}_\mu^A\sigma^{\mu\nu}\hat{\psi}_{\nu A}-\tfrac{3}{4}\hat{\bar{\chi}}{}_a^A(\Gamma^a\Gamma^b)_{AB}\hat{\chi}_b^B\\ &+\tfrac{3}{4}i\hat{\bar{\psi}}{}_\mu^A\gamma^\mu\Gamma^a_{AB}\hat{\chi}_a^B\Big\}+\tfrac{1}{6}a\varepsilon^{\mu\nu\rho\sigma}B_\sigma{}^kF'_{\mu\nu\rho k}.\end{aligned} \tag{3.20}$$

This is the result that we have been working towards. It is a generalized cosmological term plus various generalized mass terms, familiar from the $N=4$ supergravity theory [15]. But apart from these similarities there are several important differences which we should like to emphasize:

(i) the *sign* of the cosmological term is opposite to what one expects from a gauging of the O(N) symmetry;

(ii) there is *no* gauging of the O(N) symmetry;

(iii) there is a spin $\frac{1}{2}$/spin $\frac{1}{2}$ generalized mass term;

(iv) there is a mixing between the vectors and the scalars (in the form of antisymmetric tensors);

(v) the transformation laws are modified (as we shall see below) such that not only do we get a $\delta\psi_\mu \sim a\gamma_\mu\varepsilon$ term but *also* a constant shift for the spin $\frac{1}{2}$ fields $\delta\chi_a \sim a\Gamma_a\varepsilon$.

The last remark, (v) suggests that what we have constructed is a version of the $N=8$ theory in which *the* 8 *supersymmetries are spontaneously broken*, with the eight spinor fields $(\Gamma^a\chi_a)_A$ transforming as the eight Goldstone fields of supersymmetry. Because the symmetry is local these Goldstone fermions can be gauged away. This interpretation is perhaps not too surprising when one realizes that our method of construction is rather similar to the addition of the Fayet-Iliopoulos term in super-QED [16], which also causes a spontaneous breaking of supersymmetry. In both cases one adds to the lagrangian a term linear in a field that transforms as a total derivative under supersymmetry. Further evidence for this interpretation is that with the sign of the cosmological constant as in (3.20) one (presumably) cannot argue that the spin $\frac{3}{2}$ mass is fictitious, as is possible with the other sign [17]. Note also that this difference in sign of the cosmological constant is only possible because of the spin$\frac{3}{2}$/spin$\frac{1}{2}$ mixing and the constant shift of $\Gamma^a\chi_a$ under supersymmetry; this produces an additional $\bar{\varepsilon}\gamma\cdot\psi$ term in $\delta\mathcal{L}$. The absence of this additional term would force the cosmological constant to take the customary sign in order that the $\bar{\varepsilon}\gamma\cdot\psi$ contributions to $\delta\mathcal{L}$ cancel. Therefore, the sign of the cosmological constant and the constant shift of $\Gamma^a\chi_a$ under supersymmetry are related.

Remark (iv) above suggests also that the seven U(1) symmetries gauged by the $B_\sigma{}^i$ are spontaneously broken. This is because the coupling between the seven $B_\sigma{}^i$ and the seven $A'_{\mu\nu i}$ constitutes a description of seven *massive* spin 1 particles [18]; i.e., the seven vectors have eaten the seven spin 0 particles described by the antisymmetric tensors. (Actually this is again a generalized mass, i.e., multiplied by a function of scalars, because the kinetic term for $A'_{\mu\nu i}$ is multiplied by a factor of Δ). The internal O(8) symmetry is also broken because it is obvious that the $B_\sigma{}^i$ and $A'_{\mu ij}$ can no longer be put together in the antisymmetric tensor representation of O(8), (after a duality transformation on $A'_{\mu ij}$). The same remark applies to the scalars. The surviving internal symmetry appears to be O(7).

One disturbing feature of this model is that the potential (generalized cosmological term),

$$V = \tfrac{1}{2}a^2(\det e_m{}^a)^{-3/2} \tag{3.21}$$

has no minimum, which is a feature in common with the gauged $N=4$ theory [15]. So it is not clear how we are to choose the vacuum values of the scalar fields.

Finally we shall check by direct calculation the validity of our results. This is most easily done before all the field redefinitions. Accordingly, we return to (3.11) and follow the path to (3.15) but keeping the field ψ_α, ψ_a as directly reduced from ψ_A. We eliminate f by its equation of motion

$$f=-\tfrac{1}{4}\left[\bar{\psi}_a\Gamma^{ab}i\gamma^5\psi_b-\bar{\psi}_\alpha\Gamma^{\alpha\beta}i\gamma^5\psi_\beta\right]-a\Delta^{-1/2}, \tag{3.22}$$

to get the following addition to the lagrangian:

$$\mathcal{L}_{\text{new}}=-\tfrac{1}{2}a^2e\Delta^{-1/2}+\tfrac{1}{4}ae\left[\bar{\psi}_a i\gamma^5\Gamma^{ab}\psi_b-\bar{\psi}_\alpha i\gamma^5\Gamma^{\alpha\beta}\psi_\beta\right]$$
$$+\tfrac{1}{6}a\varepsilon^{\mu\nu\rho\sigma}B_\sigma{}^kF'_{\mu\nu\rho\kappa}. \tag{3.23}$$

Now the transformation law for ψ_M contains $F_{\alpha\beta\gamma\delta}=-\varepsilon_{\alpha\beta\gamma\delta}f$ and so picks up new a-dependent terms,

$$\delta_{\text{new}}\psi^A_\mu=\tfrac{1}{3}a\Delta^{-1/2}\gamma_\mu\gamma_5\varepsilon^A,$$
$$\delta_{\text{new}}\psi^A_a=-\tfrac{1}{6}a\Delta^{-1/2}(\Gamma_a)^{AB}\varepsilon_B \tag{3.24}$$

Now we must check that all a-dependent terms in $\delta\mathcal{L}$ cancel. The a^2 terms cancel in the usual way; the old variation of e and Δ in the cosmological term cancels against the new variations of the fermions in the mass terms. Similarly the new fermion variations in the kinetic terms cancel against the $\delta\psi_\mu=\partial_\mu\varepsilon$ variation in the spin $\frac{3}{2}$ mass term, which is possible only because, *before* the field redefinitions, there is a mixing of spin $\frac{3}{2}$ and spin $\frac{1}{2}$ in the kinetic terms. Now comes the crucial test; the cancellation of the "$\psi\varepsilon F$" terms, in particular those with $F_{\mu\nu ij}$, the field strength of the vectors $A_{\mu ij}$. For us there are two possible sources of such a term in δ:

(i) $\delta\psi\sim a\varepsilon$ in "$\bar{\psi}\psi F$",

(ii) $\delta\psi\sim F\varepsilon$ in a"$\bar{\psi}\psi$".

If there were also,

(iii) a $\delta\psi\sim A\varepsilon$ with a new term $\bar{\psi}\psi A$ in the lagrangian, then this would produce a third contribution. For $N=2, 3, 4$ supergravity the contributions from (i) and (ii) do not cancel and therefore (iii) is also required, and this implies a gauging of the $O(N)$ symmetry [19]. In our case (iii) is not available and we must hope that (i) and (ii) cancel. It is straightforward to check that all $F_{\mu ijk}$ and $F_{\mu\nu ij}$ contributions to $\delta\mathcal{L}\sim\psi\varepsilon F$ cancel and that only the following $F_{\mu\nu\rho i}$ contribution remains:

$$\tfrac{1}{6}ia\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\Gamma^k\varepsilon F_{\nu\rho\sigma k}. \tag{3.25}$$

But there is one more source of this variation that we did not yet consider. This is the $\delta B_\mu{}^i$ variation in the last term of (3.23), $+\frac{1}{6}a\varepsilon^{\mu\nu\rho\sigma}B_\sigma{}^i F'_{\mu\nu\rho i}$. For our purposes $F_{\mu\nu\rho i}$ and $F'_{\mu\nu\rho i}$ are equivalent as the difference does not contribute to the terms in $\delta\mathcal{L}$ that we are discussing. Then it is easily checked that the $\delta B_\mu{}^i$ variation in this term cancels (3.25). Thus all terms in $\delta\mathcal{L}$ of the form "$\psi\varepsilon F$" cancel.

One last point about (3.20) or (3.23) that may be puzzling is that under the transformation $\delta A'_{\mu ij} = \partial_\mu \xi_{ij}$, $F'_{\mu\nu\rho i}$ is not invariant [9]. It transforms as

$$\delta F'_{\mu\nu\rho i} = -3G_{[\mu\nu}{}^k \partial_{\rho]}\xi_{ik},$$

$$G_{\mu\nu}{}^k \equiv \partial_\mu B_\nu{}^k - \partial_\nu B_\mu{}^k. \tag{3.26}$$

But under this transformation the new term, $\frac{1}{6}a\varepsilon^{\mu\nu\rho\sigma}B_\sigma{}^i F'_{\mu\nu\rho i}$ transforms as a total derivative, so the action remains invariant.

4. Conclusions

The free abelian gauge field $A_{\mu\nu\rho}$ propagates no physical particles. It is surprising then that it should be so useful. It can be used to derive the θ dependence of effective lagrangians for chiral symmetry breaking, and it can be used to find an extension of the $N=8$ supergravity theory with a cosmological constant. The first of these uses is a rederivation of a known result, but the second is quite new. In principle we are not forced to use the field $A_{\mu\nu\rho}$ to formulate these results because, in the end, it is eliminated, but we think that its initial inclusion provides an elegant derivation of them. In particular, it was through an appreciation of the role of the $A_{\mu\nu\rho}$ field that we were led to the cosmological constant in $N=8$ supergravity.

The introduction of the cosmological constant in this way seems to be quite different from the previous constructions in $N=1, 2, 3, 4$ supergravity, chiefly because the O(N) symmetry is not gauged. The most natural interpretation of our results is that we have constructed a model in which the supersymmetry is spontaneously broken, i.e., a local version of the Fayet-Iliopoulos mechanism. The O(8) symmetry is also broken down to O(7). We do not know yet what happens to the local SU(8) and global E(7) symmetries discovered by Cremmer and Julia. Nor do we know really how to deal with a potential without a minimum. These problems remain to be investigated.

One reason why our results may be important concerns the quantum theory. The cosmological constant in the O(N) supergravities with $N>4$ is only finitely renormalized at one loop [20]. This remarkable result would be considerably less interesting if it happened that it was impossible to construct models with a cosmological constant for $N>4$. Our results show that it can be done, but as this involves a spontaneous breaking of the supersymmetry it is not clear if they are relevant to this question.

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Note added in proof

In a recent work by M. J. Duff and P. van Nieuwenhuizen, Phys. Lett. 94B (1980) 179, it has been shown that the $A_{\mu\nu\rho}$ field contributes to the one-loop topological counterterm of quantum gravity. They have also noted that, when coupled to gravity, the $A_{\mu\nu\rho}$ field leads to a cosmological constant.

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