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# SUPERSYMMETRY AND FUNCTIONAL INTEGRATION MEASURES

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We complete the proof of a recently proposed new characterization of scalar supersymmetric theories and extend this result to "non-scalar" models such as supersymmetric gauge theories. The new characterization does not make use of anticommuting variables since supersymmetry can now be directly understood as a property of certain purely bosonic functional integration measures where all fermionic variables have been "integrated out".

## 1. Introduction

In a recent letter [1], we proposed to characterize scalar supersymmetric theories by the following statement: there exists a transformation of the bosonic fields which rotates the full (interacting) functional integration measure into a free measure and whose Jacobi determinant equals the Matthews–Salam–Seiler (MSS) determinant [2] of the theory. The unusual and novel feature of this approach is that it enables us to avoid the use of abstractly defined anticommuting objects, a tool which had been indispensable so far in formulating supersymmetric theories and exploring their properties. Apart from its intrinsic interest, this result deserves further attention and study for a variety of reasons; for example, one may hope for easier constructibility of supersymmetric models as compared to non-supersymmetric ones. The vanishing of the vacuum energy in supersymmetric theories is explained quite naturally if one turns around the argument to reconstruct supersymmetric models. Furthermore, it appears certainly worthwhile to try to do without quantities which are very convenient algebraically (e.g., in superfield perturbation theory) but, so far, have defied analytic treatment (no positivity properties and the like, only formal definability of functional integrals over superfields, etc.).

It is the aim of this paper to extend in several directions the results that have been obtained previously. First of all, we close a gap that has inadvertently occurred in the proof of the main theorem of ref. [1]. We then proceed to prove *global* invertibility of the field transformation in some cases, verifying a conjecture made in ref. [1]. Thirdly, we generalize our result which, in ref. [1], had been derived for "scalar" supersymmetry only, to more sophisticated and physically more interesting supersymmetric models, such as supersymmetric gauge theories [3]. These had not been treated in ref. [1] because the additional gauge symmetry necessitates a gauge-fixing procedure [4] which either explicitly violates supersymmetry [5] or, through additional ghost multiplets, renders the theory considerably more complicated [6]. Even though, in the latter case, we expect our theorem to remain valid after a suitable reinterpretation, the statement loses much of its transparency and we will therefore disregard this possibility. We will rather rely on the non-supersymmetric gauge fixing procedure because the main ingredient of our proof, namely the vanishing of the vacuum energy in supersymmetric theories [7], fortunately remains unaffected by how we fix the gauge. The main theorem then carries over unchanged except that now the Jacobi determinant of the field transformation equals the product of the MSS determinant and the Faddeev–Popov determinant. The proof is contained in sect. 3 of this paper where also an explicit example is treated in some detail.

## 2. The proof of the main theorem completed

We will adopt the same conventions (euclidean metric, etc.) and notations as in ref. [1]. In particular, if $\psi$ denotes the (Majorana) spinors of the model, we write the part of the action containing these as

$$\begin{aligned}\tfrac{1}{2}\bar{\psi}M(A)\psi &\equiv \tfrac{1}{2}\bar{\psi}M(\lambda\,;A)\psi\\ &\equiv \tfrac{1}{2}\int \bar{\psi}_\alpha(x)M_{\alpha\beta}(x,y,\lambda\,;A)\psi_\beta(y)\,\mathrm{d}x\,\mathrm{d}y\,,\end{aligned} \tag{2.1}$$

where, as in ref. [1], $\lambda$ stands for the various coupling parameters and will be occasionally omitted; the bosonic fields are compactly denoted by $A$. We will make repeated use of the fact that the fermions can be "integrated out" [8], for instance

$$\int \mathrm{d}\psi\, \exp\left[-\tfrac{1}{2}\bar{\psi}M(\lambda\,;A)\psi\right] = \det M(\lambda\,;A)^{1/2} \equiv (\det M(\lambda=0;A))^{1/2}\cdot D(\lambda\,;A)\,. \tag{2.2}$$

$D(\lambda\,;A)$ is the MSS determinant in the notation of ref. [1]. At this point, it is inessential that the action is quadratic in the fermions since, for non-quadratic actions, Berezin's integral [8] serves as well to eliminate the fermionic variables in which case, however, $D(\lambda\,;A)$ is no longer the square root of a determinant but some less familiar function instead. For the bosonic part of the action we will simply write $S(A)\equiv S(\lambda\,;A)$; $S_0(A)\equiv S(0,A)$ represents the free action.

It was proved in ref. [1] that for supersymmetric theories there exists a transformation $A(x)\to A'(x,\lambda\,;A)$ which "rotates away" the interaction in the functional measure which formally defines the Schwinger functions, that is

$$\mathrm{e}^{-S(\lambda;A)}D(\lambda\,;A)\,\mathrm{d}A = \mathrm{e}^{-S_0(A'(\lambda;A))}\,\mathrm{d}A'(\lambda\,;A)\,. \tag{2.3}$$

This, as has been demonstrated in ref. [1], is a consequence of the vanishing of the vacuum energy in supersymmetric theories. From (2.3), we infer

$$S(\lambda\,;A) = S_0(A') + K(\lambda\,;A')\,, \tag{2.4}$$

$$D(\lambda; A) = \mathrm{e}^{K(\lambda; A')} \det \frac{\delta A'(x, \lambda; A)}{\delta A(y)} . \tag{2.5}$$

It was claimed in ref. [1] and will be proved now that, in addition to (2.4) and (2.5), supersymmetry also implies*

$$K(\lambda; A) = \sum_{n=1}^{\infty} \lambda^n K_n(A) = 0 \tag{2.6}$$

at least in the sense of formal power series (which is all we are concerned with here).

For the proof of eq. (2.6) we split the full supersymmetric action in two parts as follows:

$$S(A, F, \psi) = S(A, F) + \tfrac{1}{2}\bar{\psi} M(A)\psi , \tag{2.7}$$

where, in two dimensions,

$$S(A, F) = \int [\tfrac{1}{2}(\partial_\mu A)^2 + \tfrac{1}{2}F^2 + iFp(A)] \, \mathrm{d}x , \tag{2.8}$$

and

$$M_{\alpha\beta}(x, y; A) = \{\gamma^\mu_{\alpha\beta}\partial^\mu + \delta_{\alpha\beta} p'(A(x))\}\delta(x-y) . \tag{2.9}$$

$F$ is an auxiliary field and $p(A)$ an arbitrary polynomial in $A$. Even though (2.8), (2.9) are special to two space-time dimensions, there is no difficulty in generalizing to other cases, and all steps in the arguments below extend naturally to other supersymmetric models. All we need is that the action is quadratic in the fermions and the auxiliary fields. Our main input is the infinite set of identities ($n \in \mathbb{N}$)

$$\int S(A, F, \psi)^n \exp[-S(A, F, \psi)] \, \mathrm{d}A \, \mathrm{d}F \, \mathrm{d}\psi = 0 , \tag{2.10}$$

valid for all values of the coupling parameters. These identities are generated from the supersymmetry relation

$$\int \exp[-(1+\alpha)S(A, F, \psi)] \, \mathrm{d}A \, \mathrm{d}F \, \mathrm{d}\psi = \text{const} \tag{2.11}$$

by differentiation with respect to $\alpha$. Although, for a non-supersymmetric theory, the vacuum energy may always be made to vanish by adding a suitable function $f(\alpha)$ to the action, the identities (2.10) are only obtained if $f'(\alpha) = 0$ and therefore $f(\alpha) = 0$ since $f(0) = 0$ which is only possible in supersymmetric theories (I am indebted to E. Seiler for raising this point.) Introducing a shifted field $\tilde{F}(x) = F(x) + ip(A(x))$ and replacing $F$ by $\tilde{F}$ in (2.8), we obtain

$$S(A, F) = S(A) + \tfrac{1}{2} \int \tilde{F}(x)^2 \, \mathrm{d}x , \tag{2.12}$$

* For simplicity, we assume that there is only one coupling constant $\lambda$.

with

$$S(A)=\frac{1}{2}\int[(\partial_\mu A)^2+p(A)^2]\,\mathrm{d}x\,. \tag{2.13}$$

Thus, (2.10) becomes

$$0=\int \mathrm{d}A\,\mathrm{d}\tilde{F}\,\mathrm{d}\psi\Big(S(A)+\frac{1}{2}\int\tilde{F}^2\,\mathrm{d}x+\tfrac{1}{2}\bar{\psi}M(A)\psi\Big)^n$$
$$\times\exp\Big[-S(A)-\frac{1}{2}\int\tilde{F}^2\,\mathrm{d}x-\tfrac{1}{2}\bar{\psi}M(A)\psi\Big]$$
$$=\sum_{k=0}^{n}\binom{n}{k}c_k\int S(A)^{n-k}\exp[-S(A)]\det M(A)^{1/2}\,\mathrm{d}A\,. \tag{2.14}$$

The crucial observation is now that the numerical coefficients $c_k$ do *not* depend on the various couplings* of the model as may be seen by making the substitution

$$\psi'=M(\lambda;A)^{1/2}\psi\,,\qquad \mathrm{d}\psi'=\det M(\lambda;A)^{-1/2}\,\mathrm{d}\psi \tag{2.15}$$

in the Berezin integral. Therefore, (2.14) is also fulfilled for the free theory

$$\sum_{k=0}^{n}\binom{n}{k}c_k\int S_0(A)^{n-k}\exp[-S_0(A)]\,\mathrm{d}A=0\,. \tag{2.16}$$

In the functional integral (2.14), we now substitute $A\to A'$, so, by (2.3), the functional measure becomes a free measure. Eq. (2.4) tells us that (2.14) is equivalent to

$$\sum_{k=0}^{n}\binom{n}{k}c_k\int(S_0(A)+K(\lambda;A))^{n-k}\exp[-S_0(A)]\,\mathrm{d}A=0 \tag{2.17}$$

(we have dropped the primes). From this identity, from (2.16) and from the fact that $c_0\neq 0$, we get

$$\int(S_0(A)+K(\lambda;A))^n\,\mathrm{e}^{-S_0(A)}\,\mathrm{d}A=\int S_0(A)^n\,\mathrm{e}^{-S_0(A)}\,\mathrm{d}A\,, \tag{2.18}$$

for all $n$ by induction. Upon inserting the asymptotic expansion (2.6) into (2.18) the left-hand side becomes

$$\int\Big(S_0(A)+\sum_{\nu=1}^{\infty}\lambda^\nu K_\nu(A)\Big)^n\exp[-S_0(A)]\,\mathrm{d}A\,. \tag{2.19}$$

Differentiating this expression with respect to $\lambda$ and setting $\lambda=0$, we obtain

$$\sum_{k=1}^{n}\binom{n}{k}\sum_{\nu_1+\cdots+\nu_k=m}\langle S_0(A)^{n-k}K_{\nu_1}(A)\dots K_{\nu_k}(A)\rangle_0=0\,. \tag{2.20}$$

* It is here that we need the action to be quadratic in $\psi$ and $\tilde{F}$.

For every $m \in \mathbb{N}$, these identities impose infinitely many constraints on the functionals $K_1(A), \ldots, K_m(A)$. Taking into account that the polynomiality of $p(A)$ entails that each $K_n(A)$ is also polynomial in the bosonic fields, we conclude that all $K_n(A)$ vanish identically which completes the proof of (2.6) and thus of the main theorem of ref. [1].

An interesting consequence of the equality of the MSS determinant and the Jacobi determinant of the transformation is that whenever we are able to show that

$$\det \frac{\delta A'(x, \lambda; A)}{\delta A(y)} = D(\lambda; A) > 0 \tag{2.21}$$

the transformation $A \to A'$ is locally invertible everywhere. But then we can also prove that it has a *global* inverse because, by supersymmetry,

$$\int \exp[-S(\lambda; A)] D(\lambda; A)\, \mathrm{d}A = \text{const}. \tag{2.22}$$

Taking the limit $\lambda \to 0$ and assuming continuity at $\lambda = 0$, we find that the winding number of the transformation equals one which, together with local invertibility, implies global invertibility. Since for Majorana spinors $D(\lambda; A)$ is the square root of a determinant, (2.21) is usually more difficult to verify than for Dirac spinors where $D(\lambda; A)^2$ would be the relevant quantity; so, for example, the inequality $D(\lambda; A)^2 \geqslant 0$, first established in the third paper of ref. [2], is insufficient for (2.21). In the two-dimensional case, we find, employing the methods of ref. [9], that (2.21) is satisfied if $p'(A) > 0$ in agreement with the conjecture made in ref. [1]; but $p'(A) > 0$ (for all $\lambda$) is also the condition that ensures continuity at $\lambda = 0$ in (2.22). In four dimensions, the simplest model [10] already contains both Yukawa and pseudo-Yukawa interactions and it is not known whether (2.21) is true or not. Let us also briefly comment on what happens when $p'(A)$ (or the corresponding Jacobi determinant in more complicated cases) is not strictly positive. Then, the potential has at least two minima and translation invariance and thus supersymmetry must be broken explicitly to lift the degeneracy and pick a physical vacuum for the theory [11]. If we turn on the interaction in a compact volume $\Lambda \subset \mathbb{R}^d$ only, the transformation itself becomes $\Lambda$-dependent and our main theorem has to be modified because the vacuum energy no longer vanishes [7]. Although the transformation is not necessarily locally invertible everywhere, it is a continuous deformation of the identity and, therefore, its winding number is one. If the limit $\Lambda \nearrow \mathbb{R}^d$ exists, the winding number stays at that value which would be impossible if the symmetry had not been broken explicitly. Thus, a discontinuity similar to the one that occurs in systems with spontaneous magnetization will arise.

## 3. Supersymmetric gauge theories

Up to this point, we have not dealt with supersymmetric gauge theories for the reasons already explained in sect. 1. However, as pointed out there, the salient feature of supersymmetric theories, namely the vanishing of the vacuum energy, survives the gauge-fixing procedure (which merely amounts to factorizing out the group volume), and so, it comes as no surprise that *mutatis mutandis* the theorem of ref. [1] extends to this case.

*Theorem.* Supersymmetric gauge theories are characterized by the existence of a transformation $T$ of the bosonic fields $A_i$ (where now, the index may be internal, vectorial or both)

$$T: A_i(x) \to A_i'(x; A) \tag{3.1}$$

with the following properties:

(i) $T$ is invertible in the sense of formal power series;

(ii) $S(A) = S_0(A'(A))$, where $S$ denotes the full bosonic part of the action *including gauge-fixing terms* and $S_0$ its quadratic part;

(iii) the Jacobi determinant of the transformation equals the product of the MSS determinant and the Faddeev–Popov determinant.

To keep the notation at a reasonably simple level and to have something definite in mind, we will give the proof only in the simplest non-trivial case [3]; the generalization to, say, supersymmetric gauge multiplets in interaction with matter multiplets is straightforward. In a *euclidean* space-time, the relevant lagrangian reads [3]*

$$\mathcal{L} = \tfrac{1}{4} F^a_{\mu\nu}(A)^2 + \tfrac{1}{2}\bar{\psi}^a \not{D} \psi^a + \tfrac{1}{2} D^a D^a . \tag{3.2}$$

Of course,

$$F^a_{\mu\nu}(A) = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \tag{3.3}$$

and

$$\not{D}\psi^a = \gamma\mu \partial_\mu \psi^a + g f^{abc} \gamma_\mu A^b_\mu \psi^c . \tag{3.4}$$

(3.2) is invariant (up to a total derivative) with respect to the supersymmetry transformations [3]

$$\begin{gathered} \delta A^a_\mu = -\bar{\varepsilon}\gamma_\mu \psi^a , \qquad \delta D^a = \bar{\varepsilon}\gamma_5 \not{D}\psi^a , \\ \delta\psi^a = (\sigma^a_{\alpha\beta} F^a_{\alpha\beta} - \gamma^5 D^a)\varepsilon . \end{gathered} \tag{3.5}$$

We now fix the gauge by adding a term $\frac{1}{2}(f^a(A)^2)$ to the lagrangian (3.2) [4]. The function

$$f^a(x, \lambda; A) = \frac{1}{\sqrt{\alpha}} \partial_\mu A^a_\mu(x) + \lambda h^a(x; A) \tag{3.6}$$

* The euclideanization procedure of ref. [12] applies equally well to supersymmetric gauge theories which accounts for the perhaps unusual factors in (3.5). We also remind the reader that in euclidean field theory, complex conjugation for the fermions is replaced by "Osterwalder–Schrader conjugation" $\psi \to \theta\psi\theta^{-1} = \mathscr{C}\gamma^0\psi$. It is an involutive map and, as such, has all the required properties.

may also contain non-linear pieces which we absorb into $h^a(x; A)$. In analogy with (2.7), the action is decomposed into a bosonic and a fermionic part

$$S(g, \lambda; A, \psi) = S(g, \lambda; A) + \tfrac{1}{2}\bar{\psi}M(g; A)\psi\,, \tag{3.7}$$

where

$$\begin{aligned} S(g, \lambda; A) &= \frac{1}{4}\int F^a_{\mu\nu}(g; A)^2\,\mathrm{d}x + \frac{1}{2}\int f^a(\lambda; A)^2\,\mathrm{d}x \\ &\equiv S_1(g; A) + S_2(\lambda; A) \end{aligned} \tag{3.8}$$

(we dropped the auxiliary fields) and

$$M^{ac}_{\alpha\beta}(x, y, g; A) = \{\delta^{ac}\gamma^\mu_{\alpha\beta}\partial^\mu + gf^{abc}\gamma^\mu_{\alpha\beta}A^{b\mu}(x)\}\delta(x-y)\,. \tag{3.9}$$

The MSS determinant $D(g; A)$ is defined as before:

$$\det M(g; A)^{1/2} \equiv \det M(0; A)^{1/2} D(g; A)\,. \tag{3.10}$$

To compensate for the explicit breaking of gauge invariance, the functional measure must be weighted with the Faddeev–Popov determinant [4]*

$$\Delta_{\mathrm{f}}(g, \lambda; A) = \det\frac{\delta f^a(x, \lambda; A)}{\delta\omega^b(y)} \equiv \det\frac{\delta f}{\delta\omega}\,. \tag{3.11}$$

Hence, the functional measure is

$$\exp[-S(g, \lambda; A)]D(g, A)\Delta_{\mathrm{f}}(g, \lambda; A)\,\mathrm{d}A\,, \tag{3.12}$$

and, by invoking the arguments of ref. [1], we conclude that there exists a transformation

$$T_{g,\lambda}: A^a_\mu(x) \to A'^a_\mu(x, g, \lambda; A) \tag{3.13}$$

which reduces (3.12) to a gaussian measure, viz.,

$$(3.12) = \exp[-S(0, 0; A'(g, \lambda; A))]\,\mathrm{d}A'(g, \lambda; A)\,. \tag{3.14}$$

It remains to be shown that

$$\det\frac{\delta A'^a_\mu(x, g, \lambda; A)}{\delta A^b_\nu(y)} = D(g; A)\Delta_{\mathrm{f}}(g, \lambda; A)\,. \tag{3.15}$$

For this purpose, we re-express the Faddeev–Popov determinant as a functional integral over the ghost fields $c^a(x)$, $\bar{c}^a(x)$ [4] and consider the function

$$\begin{aligned} H(\xi, g, \lambda) \equiv \int &\exp\left(-\xi\left[S(g, \lambda; A) + \tfrac{1}{2}\bar{\psi}M(g; A)\psi + \bar{c}\frac{\delta f}{\delta\omega}(g, \lambda; A)C\right]\right) \\ &\times \mathrm{d}A\,\mathrm{d}\psi\,\mathrm{d}c\,\mathrm{d}\bar{c}\,. \end{aligned} \tag{3.16}$$

* $\omega$ is a gauge transformation parameter. Note also, that we do not explicitly distinguish between $\Delta_{\mathrm{f}}(g, \lambda; A)$ and $\Delta_{\mathrm{f}}(g, \lambda; A)\Delta_{\mathrm{f}}^{-1}(0, 0; A)$.

By supersymmetry, $H(\xi, g, \lambda)$ is independent of $g$; moreover, the vacuum energy does not depend on the choice of the gauge parameter $\lambda$. Therefore, $H(\xi, g, \lambda)$ is actually only a function of $\xi$ alone:

$$H(\xi, g, \lambda) = H(\xi, 0, 0)\,. \tag{3.17}$$

Differentiating $n$ times with respect to $\xi$, we obtain the identities

$$\begin{aligned}
&\int \left(S(g, \lambda; A) + \tfrac{1}{2}\bar{\psi}M(g; A)\psi + \bar{c}\frac{\delta f}{\delta\omega}(g, \lambda; A)c\right)^n \\
&\quad\times \exp\left(-\left[S(g, \lambda; A) + \tfrac{1}{2}\bar{\psi}M(g; A)\psi + \bar{c}\frac{\delta f}{\delta\omega}(g, \lambda; A)c\right]\right) \mathrm{d}A\, \mathrm{d}\psi\, \mathrm{d}c\, \mathrm{d}\bar{c} \\
&= \int \left(S(0, 0; A) + \tfrac{1}{2}\bar{\psi}M(0; A)\psi + \bar{c}\frac{\delta f}{\delta\omega}(0, 0; A)c\right)^n \\
&\quad\times \exp\left(-\left[S(0, 0; A) + \tfrac{1}{2}\bar{\psi}M(0; A) + \bar{c}\frac{\delta f}{\delta\omega}(0, 0; A)\right]\right) \mathrm{d}A\, \mathrm{d}\psi\, \mathrm{d}c\, \mathrm{d}\bar{c}\,,
\end{aligned} \tag{3.18}$$

which constitute the analogue of the identities (2.14). Substituting

$$\psi' = M(g; A)^{1/2}\psi\,, \qquad c' = \frac{\delta f}{\delta\omega}(g, \lambda; A)c\,, \qquad \bar{c}' = \bar{c}\,, \tag{3.19}$$

and setting

$$S(g, \lambda; A) = S(0, 0; A') + K(g, \lambda; A')\,, \tag{3.20}$$

we now repeat the arguments of sect. 2 to show that $K(g, \lambda; A')$ vanishes indeed.

As an example, we take $f^a(x; A) = \partial_\mu A^a_\mu(x)$, so the purely bosonic part of the action becomes*

$$\frac{1}{4}\int F^a_{\mu\nu}(A)^2\, \mathrm{d}x + \frac{1}{2}\int (\partial_\mu A^a_\mu)^2\, \mathrm{d}x = \frac{1}{2}\int A^a_\mu(-\delta_{\mu\nu}\delta^{ab}\Delta)A^b_\nu\, \mathrm{d}x + \mathrm{O}(g)\,. \tag{3.21}$$

The Faddeev–Popov determinant for this choice of gauge assumes the form

$$(\det \delta^{ac}\Delta) \det\{\delta^{ac}\delta(x-y) - gf^{abc}\partial_\mu C(x-y)A^b_\mu(y)\}\,. \tag{3.22}$$

$C(x) = -\Delta^{-1}(x)$ denotes the usual propagator. After some calculation, we obtain for the product of the MSS determinant and the Faddeev–Popov determinant

$$\begin{aligned}
\exp\Big[ng^2 \int \mathrm{d}x\, \mathrm{d}y \{&\tfrac{3}{2}\partial_\mu C(x-y)A^a_\mu(y)\partial_\nu C(y-x)A^a_\nu(x) \\
&- \partial_\mu C(x-y)A^a_\nu(y)\partial_\mu C(y-x)A^a_\nu(x) \\
&+ \partial_\mu C(x-y)A^a_\nu(y)\partial_\nu C(y-x)A^a_\mu(x)\} + \mathrm{O}(g^3)\Big]\,,
\end{aligned} \tag{3.23}$$

* $\Delta$, the laplacian, should not be confused with $\Delta_f$, the Faddeev–Popov determinant.

where we made use of the relation $f^{abc}f^{a'bc}=n\delta^{aa'}$ and omitted a trivial factor $\det\delta^{ac}\Delta$. The field transformation we are looking for is found to be

$$\begin{aligned}
A'^a_\mu(x,g;A) &= A^a_\mu(x)+gf^{abc}\int \mathrm{d}y\,\partial_\lambda C(x-y)A^b_\mu(y)A^c_\lambda(y)\\
&\quad+\tfrac{1}{2}g^2 f^{abc}f^{bde}\int \mathrm{d}y\,\mathrm{d}z\,\partial_\rho C(x-y)A^c_\lambda(y)\\
&\quad\times\{\partial_\rho C(y-z)A^d_\mu(z)A^e_\lambda(z)+\partial_\lambda C(y-z)A^d_\rho(z)A^e_\mu(z)\\
&\quad+\partial_\mu C(y-z)A^d_\lambda(z)A^e_\rho(z)\}+\mathrm{O}(g^3)\,. \qquad (3.24)
\end{aligned}$$

It satisfies both

$$\begin{aligned}
&\frac{1}{2}\int A'^a_\mu(x,g;A)(-\delta_{\mu\nu}\delta^{ab}\Delta)A'^b_\nu(x,g;A)\,\mathrm{d}x\\
&\quad=\frac{1}{4}\int F^a_{\mu\nu}(g;A)^2\,\mathrm{d}x+\frac{1}{2}\int(\partial_\mu A^a_\mu)^2\,\mathrm{d}x+\mathrm{O}(g^3)\,, \qquad (3.25)
\end{aligned}$$

and

$$\det\frac{\delta A'^a_\mu(x,g;A)}{\delta A^b_\nu(y)}=(3.23)+\mathrm{O}(g^3)\,. \qquad (3.26)$$

We note that in contradistinction to the "scalar" supersymmetric case the problem of making (3.24) mathematically acceptable for distribution valued $A^a_\mu(x)$ has not yet been solved: there exists up to date no *non-perturbative* regularization prescription which respects both supersymmetry and gauge invariance.

## 4. Outlook

In this paper, the new characterization of supersymmetry proposed in ref. [1] has been extended to a large class of supersymmetric theories. However, there remain some problems which have not been tackled in this paper and it may be useful to conclude with a few pertinent remarks. We have not considered locally supersymmetric theories or models with quartic fermion couplings for quite obvious reasons: even in ordinary supergravity [13], the vanishing of the vacuum energy in perturbation theory has not been checked explicitly* although it would be rather astonishing if the calculation yielded a non-zero result. As for quartic fermion couplings, some modification of our theorem will be unavoidable since the proof given in the preceding sections heavily depended on the fact that the action was at most quadratic in the fermions.

* At least to the author's knowledge. However, for an interesting discussion of induced cosmological constants in extended supergravity theories and their possible finiteness in perturbation theory, the reader may consult ref. [15].

Another question of interest concerns the possible relation between the field transformation constructed in ref. [1] and the perturbative solution of the Yang–Feldman equations (see, e.g., ref. [14]). So far, we have not been able to establish any such relationship, but it would be clearly advantageous if one could understand the transformation in terms of an integral equation. This would provide us not only with some hints as to the convergence properties of the perturbation series by means of which the transformation has been defined in ref. [1] but also with a possible clue to the non-perturbative construction of supersymmetry theories.

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## References

[1] H. Nicolai, Phys. Lett. 89B (1980) 341
[2] T. Matthews and A. Salam, Nuovo Cim. 12 (1954) 563; 2 (1955) 120;
E. Seiler, Comm. Math. Phys. 42 (1975) 163
[3] J. Wess and B. Zumino, Nucl. Phys. B78 (1974) 353;
R. Delbourgo, A. Salam and J. Strathdee, Phys. Lett. 51B (1974) 475;
S. Ferrara and B. Zumino, Nucl. Phys. B79 (1974) 413
[4] L.D. Faddeev and V.N. Popov, Phys. Lett. 25B (1967) 29;
G. 't Hooft, Nucl. Phys. B33 (1971) 173
[5] B. de Wit and D.Z. Freedman, Phys. Rev. D12 (1975) 2286
[6] B. de Wit, Phys. Rev. D12 (1975) 1268;
S. Ferrara and O. Piguet, Nucl. Phys. B93 (1975) 361;
F. Honerkamp, F. Krause, M. Scheunert and M. Schlindwein, Nucl. Phys. B95 (1975) 397
[7] B. Zumino, Nucl. Phys. B89 (1975) 535
[8] F.A. Berezin, The method of second quantization (Academic Press, New York, 1966)
[9] H. Nicolai, Comm. Math. Phys. 59 (1978) 71
[10] J. Wess and B. Zumino, Nucl. Phys. B70 (1974) 39; Phys. Lett. 79B (1974) 52
[11] J. Glimm, A. Jaffe and T. Spencer, Ann. of Phys. 104 (1976) 610
[12] H. Nicolai, Nucl. Phys. B140 (1978) 294
[13] D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys. Rev. D13 (1976) 3214;
S. Deser and B. Zumino, Phys. Lett. 62B (1976) 335
[14] K. Nishijima, Fields and Particles (Benjamin, New York, Amsterdam, 1969)
[15] J. Scherk and J. Schwarz, Phys. Lett. 82B (1979) 60;
E. Cremmer, J. Scherk and J. Schwarz, Phys. Lett. 84B (1979) 83