

Stabilization of moduli by fluxes

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In order to fix the moduli, non-trivial fluxes are an essential input. In this talk I summarize different aspects of compactifications in the presence of fluxes, as there is the relation to generalized Scherk-Schwarz reductions and gauged supergravity, but also the description of flux-deformed geometries in terms of G -structures and intrinsic torsion.

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1 Introduction

One of the major problems appearing in compactifications of string theory, is the emergence of a continuous moduli space of string vacua. In order to get contact, not only to the standard model of particle physics, but also to (inflationary) cosmology, these moduli have to be fixed and if supersymmetry is broken only at fairly low energies, we have to understand the mechanism while preserving at least some supersymmetry. Moduli appear in two guises: the closed string or geometrical moduli, related to deformations of size and shape of cycles of the internal manifold and open string moduli, related to un-fixed positions of wrapped branes. Fluxes seem to provide a mechanism to lift both moduli, the geometric moduli, because they do not allow continuous deformations of the corresponding cycle and open string moduli, because fluxes couple to the Born-Infeld action and hence produce a potential also for these moduli. Since the open string moduli are compact, any potential has an extremum.

Typically, one refers to fluxes as non-zero expectation values of the RR- and NS-fields in the vacuum and there is a growing literature on this subject [1–8]. Branes are also sources for RR-fields or NS-fields, but the corresponding fluxes are not closed and are supported by δ -function singularities and one often refers to fluxes only to closed RR/NS-forms. On the other hand, metric fluxes are also possible and known as twisting. All fluxes can be understood as appearing from generalised Scherk-Schwarz reduction [9, 10], which one can apply to any global symmetry of the vacuum. A subclass are shift symmetries due to the gauge symmetries of RR- and/or NS-form fields and the corresponding generalised Scherk-Schwarz reductions are known to preserve supersymmetry. Since these reductions are related to gauged supergravity [11], we have a tool at hand to calculate (at least in principle) the Kaluza-Klein spectrum. It is also known that Scherk-Schwarz reductions allow for a consistent truncation to the massless Kaluza-Klein spectrum; explicit examples have been explored in [12–14].

Moreover, as long as we consider only the closed string sector and do not take into account any branes or orientifolds, supersymmetry requires that the potentials have to fit into gauged supergravity. This relation provides a powerful tool to get the explicit form of the potential as function of the different scalar fields and one can address the question of the number of vacua and the number of flat directions, ie. the number moduli that are still free. In the next section, we will discuss in more detail how one can address these issues

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in gauged supergravity and in the last section, we will comment on the possibility to discuss the moduli fixing by using G-structures.

2 Moduli fixing in gauged supergravity

One starts with the low-energy effective Lagrangian that is obtained by standard Calabi-Yau compactification, where the scalar fields (z^A, q^u) enter vector and hyper multiplets and the corresponding moduli space is a direct product: $\mathcal{M}_V \otimes \mathcal{M}_H$. Fluxes (3-form flux on the IIB side e.g.) correspond to a gauged isometry of the quaternionic space \mathcal{M}_H and give a mass to scalars of hyper multiplets [15]. The resulting graviton mass term which becomes the superpotential upon truncation $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ is SU(2)-valued and has the structure

$$W^x \sim X^I \mathcal{P}_I^x \quad (1)$$

where $X^I = X^I(z)$ is the “electric” part of the symplectic section $V = (X^I, F_I)$ and the Killing prepotentials are denoted by \mathcal{P}_I^x are related to a set of Killing vectors k_I by: $\mathcal{P}_I^x = K_{uv}^x \partial^u k_I^v$, where K^x is the triplet of Kahler 2-forms defining the quaternionic space. It is a known problem, that gauged supergravity prefers the electric part and does not produce the magnetic part of the superpotential. But by taking into account also (massive) tensor multiplets, one can promote it to a manifestly symplectic invariant expression [16, 17]. An important property of this setup is however, that by a symplectic transformation one can always go into a strictly perturbative regime where all magnetic charges vanish and hence we can always consider a potential as in (1). This property that the electric and magnetic charges are mutually local is a consequence of supersymmetric Ward identities [18].

We can now discuss the conditions of getting a complete lifting of the moduli space. A necessary condition for this is that the variations of the hyperino and gaugino vanish for constant scalars, ie. if

$$(i) \quad (\nabla_A X^I) \mathcal{P}_I^x = 0 \quad , \quad (ii) \quad X^I (\nabla_u \mathcal{P}_I^x) = K_{uv}^x (X^I k_I^v) = 0 \quad (2)$$

where: ∇_A denotes the Kahler covariant derivative with respect to the scalars z^A in vector multiplets, K^x is the SU(2) triplet of Kahler forms of the quaternionic Kahler space and $X^I k_I = k$ is the combination of Killing vectors k_I that has been gauged. The condition (ii) is equivalent to the existence of a fixed point for the Killing vector k and the *complete* hyper multiplet moduli space is lifted if k has a NUT fixed point, ie. if it represents a *point* on \mathcal{M}_H . This excludes by the way, axionic shift symmetries and requires a *compact* isometry [19]. But one can give even a criteria, when this is the case, because the fixed point set of a Killing vector field is always of even co-dimension which is related to the rank of the 2-form dk calculated on the fixed point set. If the rank is maximal, ie. $\det(dk) \neq 0$, the fixed point set is in fact a point on the manifold and dk parameterises a rotation around the fixed point. If on the other hand, $\det(dk) = 0$, some “flat directions” related to translational symmetries of the fixed point set exist. Therefore, we get the following two conditions for lifting the hyper multiplet moduli space

$$|k| = 0 \quad , \quad \text{with : } \det(dk) \neq 0 . \quad (3)$$

If we can find a Killing vector that satisfies both conditions, the hyper multiplet moduli space will be lifted in the vacuum. But we should place a warning here. Although, the isometries on the classical level are well understood it is unclear whether the full quantum corrected moduli space has isometries at all and this would make the issue of fixing the moduli obscure. But we do not want to speculate here about the quantum moduli space for hyper multiplets and let us instead continue with the discussion of the second condition (i) in (2). If the hyper scalars are fixed, the Killing prepotentials are some fixed function of the scalars of the vector multiplet, ie. $\mathcal{P} = \mathcal{P}(q(z))$ and hence they vary over \mathcal{M}_V . If \mathcal{P} would be constant, only one vacuum can occur, namely at the point where this constant symplectic vector is a *normal vector* on \mathcal{M}_V [20, 21], which is exactly implied by the constraint (i) in (2). But since \mathcal{P} varies now, it might become normal at different

points, related to the appearance of multiple critical points as eg. the ones discussed in [19]. If we calculate the second covariant derivatives on \mathcal{M}_V at this fixed point and use relations from special geometry¹ we find that all these critical points are isolated – at least as long as the metric does not degenerate. Therefore, there are no further constraints from the vector multiplet moduli space and the crucial relations that have to be realized are the ones in (3).

In a generic situation, the potential at the fixed point will be non-zero and hence the supersymmetric vacuum will be anti de Sitter. If one is however interested in a flat space vacuum, additional input is required. To understand this issue, note that the (negative) cosmological constant is given by the absolute square of (1), which vanishes iff: $W^x = 0$. Since $\mathcal{P}_I^x = K_{uv}^x \partial^u k_I^v$, this is the case if the 2-form dk on the quaternionic space is perpendicular to the $SU(2)$ curvatures K^x . For the simplest case of a 4-dimensional quaternionic space, K^x are anti-selfdual 2-forms and hence dk must be self-dual to yield a flat space vacuum. In general, the rotations parameterised by dk (calculated at the fixed point) are inside the holonomy and an n -dimensional quaternionic space has the holonomy $SU(2) \otimes Sp(2n)$. Hence a flat space vacuum corresponds to an Killing vector so that dk can be expanded only with respect to the $Sp(2n)$ curvatures, but has no components along the $SU(2)$. These statements here refer only to the case without flat directions, ie. a NUT fixed points where the 2-form dk is not degenerate. If there are flat directions, it is well known that flat space vacua appear if the potential has the no-scale structure. But we do not want to discuss this case here.

In gauged supergravity the problem of fixing the moduli is rather well formulated. The question is however, to embed these models into string or M-theory, ie. to obtain the potential by a suitable dimensional reduction. In addition, although these reductions may lift the moduli space, it is not granted that we will obtain a unique vacuum and we can end up with a landscape of string vacua [22]. Although this conclusion was reached only for specific fluxes and it is not inevitable for (most) general fluxes, it may happen that we have to rely – at least to a certain extent, on an anthropic selection for choosing the vacuum in which we (can) live [23]. Adopting this philosophy, the problem with the cosmological constant disappears. In gauged supergravity this vacuum ambiguity is related to the different ways of gauging a given isometry with different $U(1)$ gauge fields. On the other hand, if one takes into account general fluxes on the type IIA side, the vacuum is rather unique [24, 25]. This brings us to another approach to address the consequences of fluxes and which allows directly to determine the resulting deformed geometry.

3 G-structure and fixed moduli

Recall, the advantage of gauged supergravity was that it yields an explicit form of the potential as function of the scalar fields and hence one can directly address the issue of stability if supersymmetry will be broken. On the other hand, we do not get the deformed geometry due to the back reaction of the fluxes. The new geometry can be derived by solving directly the 10-dimensional equations, that on top uncover also the explicit embedding of the fluxes in the internal space. A draw-back of this approach is however, that the Kaluza-Klein reduction becomes less clear. Because for general fluxes, ie. even and odd field strengths, the Chern-Simon terms imply that the field strengths are not given by closed forms anymore, it is unclear in which basis of forms one should expand these fields in the Kaluza-Klein reduction. A discussion on this issue can be found in [26, 27].

In solving the 10-dimensional (Killing spinor) equations, one re-writes the fluxes as torsion components of the internal space [28, 29] and this approach shows, that only specific fluxes can be non-zero while still preserving a Calabi-Yau internal space [5, 30]. Note, finally a deformation of the internal space is necessary if we want to lift the moduli space – because any Calabi-Yau compactification will yield some moduli upon compactification. In this approach, the vacua are classified with respect to the G -structure, ie. the group structure which leaves invariant the Killing spinor. More or less good understood is the situation with $SU(3)$ structures, where the internal spinor is a singlet under the $SU(3)$ structure group. On the IIA side there are only three vacua known: (i) where all RR fields are trivial, which is basically the solution of the common

¹ Because: $\nabla_{\bar{A}} \nabla_B V \sim g_{\bar{A}B} V$.

sector of all string theory [1] (ii) the example that lift to a G_2 holonomy space in M-theory, ie. with trivial H - and 4-form flux [31], (iii) where all fluxes and the mass parameter are non-zero. This last example has been discussed in [24,25] where the internal space has weak $SU(3)$ holonomy (or nearly Kahler) and examples are S^6 , CP_3 or the flag manifold. Here only the $SU(3)$ singlet components of the fluxes are non-zero and in [32] was an additional 2-form flux added with the consequence that the internal space is not nearly Kahler anymore.

Let us add some more comments about the nearly Kahler example, which is an interesting example since it seems to lift the whole moduli space – at least the regular examples are known to have no (closed string) moduli which preserve the nearly Kahler structure. What is a nearly Kahler spaces? These spaces are Einstein spaces and if they have positive curvature, they can be defined by building a cone of them yielding a 7-dimensional space of G_2 -holonomy [33]. But is there a physical interpretation of this 7-dimensional space? It cannot be related to a M-theory compactification, because for massive type IIA supergravity there is no standard M-theory lift known. But instead, the 7th coordinate can be identified with the radial coordinate of the external anti deSitter space. Now, the question whether all moduli are lifted translates to question of the co-dimension of the conifold singularity in the moduli space of G_2 holonomy spaces. For the known regular examples (ie. S^6 , $S^3 \times S^3$, CP_3 or the flag manifold) the conifold is in fact a point in the moduli space and therefore, there are no moduli for these spaces. On the other hand, there are also singular spaces constructed e.g. as twistor spaces over selfdual Einstein spaces and these spaces have in fact some moduli. But also for these spaces, we have (quantised) fluxes along the associated 3-form of the G_2 -holonomy space which do not allow for continuous deformations and thus the corresponding moduli are lifted. Of course, if we wrap branes, there can still be open string moduli related to the exact position of the branes in the internal space. But this question cannot be addressed in this framework, ie. by solving the Killing spinor equations.

On the IIB side, the general solution related to $SU(3)$ structures has not yet been found, but the constraints on the fluxes have been discussed in [34,35]. Interestingly, on the IIB side $SU(3)$ structures require that the external space is flat [36], ie. in order to generate a (negative) cosmological constant, one has to consider $SU(2)$ structures. The fact that all supersymmetric vacua are flat, points towards a no-scale structure, which has always at least one flat direction in the potential. Ie. in order to fix all moduli, we may have to consider $SU(2)$ structures as it has been done in [37]. But in this case the spinor is only a singlet under $SU(2)$ rotations and hence it cannot be invariant under orbifolds/orientifolds which have point like fixed points in the internal space. Therefore, there might appear an issue in obtaining chiral fermions in this case, but more work is necessary to clear this issue.

Let us end with one important remark. It is not enough to fix the moduli, one has also to ensure the stability of the vacuum. Although this is not an issue as long as some supersymmetries are unbroken, it becomes urgent if supersymmetry is broken. Unfortunately, the vacua, obtained in gauged supergravity, have some instable or flat directions. An instable direction may not be a problem for an anti de Sitter vacuum if it still obeys the Breitenlohner/Freedman bound. But finally, we have to lift the vacuum to a de Sitter space, which has to be a local minimum. The authors of [38] were in fact able to construct an interesting model with a de Sitter vacuum, but in [39] possible instabilities of this model have been addressed. At the end, the issue of stability is a question of how we break supersymmetry and what are the (quantum) corrections to the potential in the non-supersymmetric vacuum. Every progress in this long-standing problem is helpful.

References

- [1] A. Strominger, Nucl. Phys. B **274**, 253 (1986).
- [2] J. Polchinski and A. Strominger, “New vacua for type II string theory,” Phys. Lett.B **388**, 736–742 (1996) [hep-th/9510227].
- [3] J. Michelson, Nucl. Phys.B **495**, 127–148 (1997) [hep-th/9610151].
- [4] T.R. Taylor and C. Vafa, Phys. Lett.B **474**, 130–137 (2000) [hep-th/9912152].
- [5] M. Grana and J. Polchinski, Phys. Rev.D **63**, 026001 (2001) [hep-th/0009211].

- [6] M. Grana and J. Polchinski, Phys. Rev.D **65**, 126005 (2002) [hep-th/0106014].
- [7] S. Kachru, M. B. Schulz, and S. Trivedi, JHEP **10**, 007 (2003) [hep-th/0201028].
- [8] S. Kachru, M. B. Schulz, P. K. Tripathy, and S. P. Trivedi, JHEP **03**, 061 (2003) [hep-th/0211182].
- [9] J. Scherk and J. H. Schwarz, Phys. Lett. B **82**, 60 (1979).
- [10] P. M. Cowdall, H. Lu, C. N. Pope, K. S. Stelle, and P. K. Townsend, Nucl. Phys. B **486**, 49–76 (1997) [hep-th/9608173].
- [11] E. Bergshoeff, M. de Roo, and E. Eyraas, Phys. Lett. B **413**, 70–78 (1997) [hep-th/9707130].
- [12] E. Bergshoeff, M. de Roo, M. B. Green, G. Papadopoulos, and P. K. Townsend, Nucl. Phys. B **470**, 113–135 (1996) [hep-th/9601150].
- [13] N. Kaloper and R. C. Myers, JHEP **05**, 010 (1999) [hep-th/9901045].
- [14] K. Behrndt, E. Bergshoeff, D. Roest, and P. Sundell, Class. Quant. Grav. **19**, 2171–2200 (2002) [hep-th/0112071].
- [15] J. Louis and A. Micu, Nucl. Phys. B **635**, 395–431 (2002) [hep-th/0202168].
- [16] G. Dall’Agata, R. D’Auria, L. Sommovigo, and S. Vaula, Nucl. Phys. B **682**, 243–264 (2004) [hep-th/0312210].
- [17] R. D’Auria, L. Sommovigo, and S. Vaula, JHEP **11**, 028 (2004) [hep-th/0409097].
- [18] L. Sommovigo and S. Vaula, Phys. Lett. B **602**, 130–136 (2004) [hep-th/0407205].
- [19] A. Ceresole, G. Dall’Agata, R. Kallosh, and A. Van Proeyen, Phys. Rev.D **64**, 104006 (2001) [hep-th/0104056].
- [20] K. Behrndt and S. Gukov, Nucl. Phys. B **580**, 225–242 (2000) [hep-th/0001082].
- [21] K. Behrndt and M. Cvetič, Phys. Rev.D **61**, 101901 (2000) [hep-th/0001159].
- [22] S. Ashok and M. R. Douglas, JHEP **01**, 060 (2004) [hep-th/0307049].
- [23] L. Susskind, The anthropic landscape of string theory, hep-th/0302219.
- [24] K. Behrndt and M. Cvetič, General $N = 1$ supersymmetric flux vacua of (massive) type IIA string theory, hep-th/0403049.
- [25] K. Behrndt and M. Cvetič, General $n = 1$ supersymmetric fluxes in massive type IIA string theory, hep-th/0407263.
- [26] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, Nucl. Phys. B **654**, 61–113 (2003) [hep-th/0211102].
- [27] T. House and A. Micu, M-theory compactifications on manifolds with $G(2)$ structure, hep-th/0412006.
- [28] J. P. Gauntlett, D. Martelli, S. Pakis, and D. Waldram, Commun. Math. Phys. **247**, 421–445 (2004) [hep-th/0205050].
- [29] J. P. Gauntlett, D. Martelli, and D. Waldram, Phys. Rev.D **69**, 086002 (2004) [hep-th/0302158].
- [30] S. Kachru and A.-K. Kashani-Poor, Moduli potentials in type IIA compactifications with RR and NS flux, hep-th/0412279.
- [31] P. Kaste, R. Minasian, M. Petrini, and A. Tomasiello, JHEP **09**, 033 (2002) [hep-th/0206213].
- [32] D. Lust and D. Tsimpis, JHEP **0502**, 027 (2005) [hep-th/0412250].
- [33] A. Gray, J. Diff. Geom. **4**, 283 (1970).
- [34] A. R. Frey and M. Grana, Phys. Rev.D **68**, 106002 (2003) [hep-th/0307142].
- [35] M. Grana, R. Minasian, M. Petrini, and A. Tomasiello, JHEP **08**, 046 (2004) [hep-th/0406137].
- [36] K. Behrndt, M. Cvetič, and G. Peng, in preparation.
- [37] G. Dall’Agata, Nucl. Phys. B **695**, 243–266 (2004) [hep-th/0403220].
- [38] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev.D **68**, 046005 (2003) [hep-th/0301240].
- [39] K. Choi, H. Falkowski, A. Nilles, M. Olechowski, and S. Pokorski, JHEP **11**, 076 (2004) [hep-th/0411066].