DFTT-05/2008
AEI-2008-014

The Supermembrane with Central Charges on a G2 Manifold

A. Belhaj[1], **M.P. Garcia del Moral**[2], **A. Restuccia**[3,4], **A. Segui**[1], **J.P. Veiro**[5*]

[1] Departamento de Física Teórica, Universidad de Zaragoza
12, Pedro Cerbuna, 50009-Zaragoza, Spain

[2] Dipartimento di Fisica Teorica, Università di Torino
and INFN - Sezione di Torino; Via P. Giuria 1; I-10125 Torino, Italy

[3] Max-Planck-Institut für Gravitationphysik, Albert-Einstein-Institut
and Mülenberg 1, D-14476 Potsdam, Germany

[4] Departamento de Física, Universidad Simón Bolívar
Apartado 89000, Caracas 1080-A, Venezuela

[5] Departamento de Matemáticas Puras y Aplicadas, Universidad Simón Bolívar
Apartado 89000, Caracas 1080-A, Venezuela

March 28, 2008

Abstract

We construct the 11D supermembrane with topological central charges induced through an irreducible winding on a G2 manifold realized from the T^7/Z_2^3 orbifold construction. The hamiltonian H of the theory on a T^7 target has a discrete spectrum. Within the discrete symmetries of H associated to large diffeomorphisms, the $Z_2 \times Z_2 \times Z_2$ group of automorphisms of the quaternionic subspaces preserving the octonionic structure is relevant. By performing the corresponding identification on the target space, the supermembrane may be formulated on a G2 manifold, preserving the discretness of the spectrum. The corresponding 4D low energy effective field theory has $N = 1$ supersymmetry.

Keywords: Supermembrane with central charges, minimal immersions, G2 manifolds.

*belhaj@unizar.es; garcia@to.infn.it; restucci@aei.mpg.de, arestu@usb.ve; segui@unizar.es; pierre@ma.usb.ve

Contents

1 Introduction

Compactifications of the low energy limit of M-theory to four dimensions (4D) have received much attention during the past years. Special interest has been given to the compactification over real manifolds of dimension seven, X7, with non trivial holonomy. This interest is due to the fact that these manifolds provide a potential point of contact with low energy semi-realistic physics from M-theory [1, 2]. In particular, one can obtain 4D $N = 1$ supersymmetry by compactifying M-theory on X7 with G2 holonomy group [3, 4, 5]. In this regard, the 4D $N = 1$ resulting models depend on geometric properties of X7. For instance, if X7 is smooth, the low energy theory contains, in addition to $N = 1$ supergravity, only abelian gauge groups and neutral chiral multiplets. However, non abelian gauge symmetries with charged chiral fermions can be obtained by considering limits where X7 develops singularities [7, 8, 9, 10, 6]. For a review see for example [11].

Besides the ordinary compactification on G2 manifolds, Calabi-Yau flux compactifications and twisted toroidal compactifications have been also studied intensively, see for example, [12]- [21]. Indeed their respectively phenomenological predictions with different signatures on the LHC have also been considered, see [22, 23] for G2 compactifications, and [24] for large volume approach in Calabi-Yau compactifications. They have also been considered as particular cases of non-geometric compactifications. Most of these approaches follow a bottom-up pattern by studying the $N = 1$ gauged supergravity potentials in 4D and trying to perform the uplift to M-theory. Other compactifications from 11D supergravity with fluxes have also been done in a top-down approach [20, 21].

Recently new types of compactifications have appeared involving twisted boundary conditions or non-trivial fiber bundles over some compact manifolds (with or without singularities), T- foldings [25]. In this way, the metric and the gauge field forms get generically entangled. This kind of compactifications is called non-geometric [26, 27]. Some of these non-geometric compactifications are related with the ordinary ones by dualities. The non-triviality of the fiber bundle guarantees the existence of a monodromy, but usually due to the lack of 1-cycles inside a Calabi-Yau makes it necessary to include singularities. A simple example of these T-foldings is the twisted tori. It is a Scherk-Schwarz compactification of the 11D supergravity theory with twisted boundary conditions that allows to have a nontrivial monodromy, see for example in connection with G2 compactifications [28]. When the base space is a torus it is no longer necessary to include singularities in order to have a nontrivial monodromy [26, 27]. These twisted compactifications can have a geometrical dual which corresponds to an orbifold plus a shift, also known as asymmetric orbifold [29].

The compactification with a duality twist is more general than the orbifold compactification because it can be carried without restricting the moduli to special variations. The moduli can have nontrivial variation along the circle in the spacetime. However, the orbifold is possible for special values of the moduli where the lattice admits a symmetry and the class of allowed rotations is finite. All of the lattices admit a Z_2 symmetry as the discrete subgroup of the $SL(2,Z)$ of the torus, and for those cases the geometrical dual exists [30].

The 11D supermembrane is one of the basic elements of M-theory [31, 32]. Classically, it is unstable due to the existence of string-like spikes that leave the energy unchanged. At the quantum level, its supersymmetric spectrum is continuous and the theory was interpreted as a second quantized theory [33, 34]. Compactification on S^1 has been explored in order to see if the continuity of the spectrum is broken by the winding. It has been argued not to be the case [35] due to the presence of string-like spikes in the spectrum. In [36, 37, 38, 39, 40, 42] the minimally immersed supermembrane compactified on a torus associated to the existence of irreducible winding (MIM2) has been found. It is associated to nontrivial fiber bundles defined on Riemann surfaces. This MIM2 is classically stable since there are no singular configurations with zero energy. The quantum spectrum of the theory is purely discrete with finite multiplicity [38, 39, 40, 43, 41]. The theory of the supermembrane minimally immersed in a 7 torus has recently been found in [45]. It has a $N = 1$ supersymmetry in 4D. A natural question is to look for a connection with a compactification of the supermembrane in a nontrivial background of type of G2 manifold. In this paper we will be concerned with a full-flegged sector of M-theory which is the quantum supermembrane theory minimally immersed MIM2 on a $T^6 \times S^1$. This type of compactifications contains nontrivial discrete twists on the fibers as remanent discrete symmetries of the hamiltonian. We will show by identifying those symmetries on the target that the MIM2 can admit a compactification on a G2 manifold.

The paper is structured as follows: In Section 2 we introduce the supermembrane with central charges minimally immersed (M2MI) on a $T^6 \times S^1$. We summarize its main spectral properties and symmetries. In Section 3 we recall the main properties of the compactification on G2 manifolds. In Section 4 we construct the MIM2 on a $\frac{T^7}{Z_2^3}$, by studying the minimal immersions of the MIM2 on that target, finding the configuration space of states: the untwisted and twisted sectors of the theory. We also study its connections with Calabi-Yau compactifications. In Section 5 we present our discussions and final conclusions.

2 M2 with central charges associated to an irreducible winding

We start this section by recalling that the hamiltonian of the $D = 11$ supermembrane [31] in the light cone gauge (LCG) reads as

$$\int_\Sigma \sqrt{W}\left(\frac{1}{2}\left(\frac{P_M}{\sqrt{W}}\right)^2 + \frac{1}{4}\{X^M, X^N\}^2 + \text{ Fermionic terms }\right). \tag{1}$$

M runs for $M = 1, \ldots, 9$ corresponding to the transverse coordinates of the base manifold $R \times \Sigma$. Σ is a Riemann surface of genus g. The term $\frac{P_M}{\sqrt{W}}$ is the canonical momentum density and $\{X^M, X^N\}$ is given by

$$\{X^M, X^N\} = \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}}\partial_a X^M \partial_b X^N, \tag{2}$$

where $a, b = 1, 2$ and σ^a are local coordinates over Σ. $W(\sigma)$ is a scalar density introduced in the LCG fixing procedure. The former hamiltonian is subject to the two following constraints

$$\phi_1 := d(\frac{P_M}{\sqrt{W}}dX^M) = 0 \tag{3}$$

$$\phi_2 := \oint_{C_s} \frac{P_M}{\sqrt{W}} \quad dX^M = 0, \tag{4}$$

where $C_s, s = 1, \ldots, 2g$ is a basis of 1-dimensional cycles on Σ. ϕ_1 and ϕ_2 are generators of area preserving diffeomorphisms. When the target manifold is simply connected, the one-forms dX^M are exact.

The $SU(N)$ regularized model obtained from (1) was shown to have continuous spectrum from $[0, \infty)$, [33, 34, 32]. This property of the theory relies on two basic facts: supersymmetry and the presence of classical singular configurations. The latter is related to string-like spikes which appear with zero cost energy. These spikes do not preserve neither the topology of the world-volume nor the number of particles. These properties do not disappear when the theory is compactified and the spectrum remains continuous [35].

To get a 4 dimensional model, we need a target space as $M_4 \times T^6 \times S^1$. In this way, the configuration maps satisfy the following condition on T^6

$$\oint_{c_s} dX^r = 2\pi S^r_s R^r \quad r, s = 1, \ldots, 6. \tag{5}$$

On the circle, we have the constraint

$$\oint_{c_s} dX^7 = 2\pi L_s R_7 \tag{6}$$

while for non compact directions, we have

$$\oint_{c_s} dX^m = 0 \quad m = 8, 9. \tag{7}$$

$S^r_s, L_s \in Z$ and R_r, R_7 represent respectively the radii of the 6-torus and the radius of the circle. We shall now impose a topological irreducible wrapping condition to be satisfied by all configurations in the above model. This generates a non-trivial central charge in the 11D supersymmetric algebra. The topological condition is

$$I^{rs} \equiv \int_\Sigma dX^r \wedge dX^s = n(2\pi R^r R^s)\omega^{rs} \tag{8}$$

where ω^{rs} is a symplectic matrix on the T^6 which can be taken as

$$\omega^{rs} = \begin{pmatrix} 0 & 1 & & & & \\ -1 & 0 & & & & \\ & & 0 & 1 & & \\ & & -1 & 0 & & \\ & & & & 0 & 1 \\ & & & & -1 & 0 \end{pmatrix}. \tag{9}$$

Each block $M = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ defines a sympletic geometry on a T^2. It also describes the intersection matrix of the homology basis. If we denote by a and b the two elements of the basis of T^2, M_{ab} is defined by the following intersection numbers: $a.b = -b.a = 1$ and $a.a = b.b = 0$. For simplicity on our analysis we will take $n = 1$, the general case only involve some technical additional details.

The above topological condition leads to a $D = 11$ supermembrane with non-trivial central charges generated by its wrapping on the compact part of the target space. Since the topological constraint commutes with the rest of the constraints, it represents a sector of the full theory characterized by an integer $n = det\omega^{rs}$, see [38] for a more general discussion. Under such correspondence there exists a minimal holomorphic immersion from the base to the target manifold. The image of Σ under that map is a calibrated submanifold of T^6. The spectrum of the theory changes dramatically since it has a pure discrete spectrum at the classical and the quantum level [38, 39, 40, 41, 43]; see also [45, 42] [1].

The model that we study here involves additional symmetries beyond the original ones [41] which will be crucial in our coming discussion. In the following the minimally immersed M2 associated to this sector of the theory will be denoted by MIM2 to distinguish it from the usual one.

We notice that the condition in (8) only restricts the values of S^r_s. From equation (5) we can see that these values should be integral numbers. The condition in (8) can be solved by

$$dX^r = M^r_s d\widehat{X}^s + dA^r \tag{10}$$

where we have decomposed the closed one-forms dX^r into their harmonic plus exact parts. Note that $d\widehat{X}^s, s = 1, \dots, 2g$ is a basis for harmonic one-forms over Σ. They may be normalized with respect to the associated canonical basis of homology,

$$\oint_{c_s} d\widehat{X}^r = \delta^r_s. \tag{11}$$

[1]The geometrical interpretation of this condition has been discussed in previous work [36],[37]

We have now considered a Riemann surface with a class of an equivalent canonical basis. The condition in (5) leads to

$$M^r_s = 2\pi R^r S^r_s. \tag{12}$$

Imposing the condition in (8), we get

$$S^r_t \omega^{tu} S^s_u = \omega^{rs}, \tag{13}$$

which says that $S \in Sp(2g, Z)$. This is the most general map satisfying (8).

A sufficient condition in order to have a consistent global construction of the theory, subject to the topological constraint, is to have a surface Σ of genus g such that the space of holomorphic one-forms is of the same complex dimension as the flat torus in the target space. This condition ensures the existence of a holomorphic immersion, and so minimal, from Σ to T^{2g} [42]. In [45] we analyzed the theory for genus 3 and the breaking of the SUSY by the ground state (the holomorphic immersion) for genus $1, 2, 3$. It was also emphasized there that in order to consider the MIM2 from Σ to a given target space one should consider all possible immersions, in particular all holomorphic immersions. This consideration will become important in the following sections when we analyse a $\frac{T^7}{Z_2^3}$ target space.

The theory is invariant not only under the diffeomorphisms generated by ϕ_1 and ϕ_2 but also under the diffeomorphisms, which are biholomorphic maps, changing the canonical basis of homology by a modular transformation.

We may always consider a canonical basis such that

$$dX^r = 2\pi R^r d\widehat{X^r} + dA^r. \tag{14}$$

In this manner, the corresponding degrees of freedom are described exactly by the single-valued fields A^r. By using the condition in (6), we perform a similar decomposition with the remaining 1-form associated to the compactification on S^1

$$dX^7 = 2\pi R L_s d\widehat{X}^s + d\widehat{\phi} \tag{15}$$

where $d\widehat{\phi}$ is a new exact 1-form and $d\widehat{X}^s$ are the basis of harmonic forms as before. The final expression of the hamiltonian of the MIM2 wrapped in an irreducible way on $T^6 \times S^1$ [45] is

$$H_d = \int \sqrt{w} d\sigma^1 \wedge d\sigma^2 [\frac{1}{2}(\frac{P_m}{\sqrt{W}})^2 + \frac{1}{2}(\frac{\Pi^r}{\sqrt{W}})^2 + \frac{1}{4}\{X^m, X^n\}^2 + \frac{1}{2}(\mathcal{D}_r X^m)^2 + \frac{1}{4}(\mathcal{F}_{rs})^2 + \frac{1}{2}(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})^2 + \frac{1}{8}(\frac{\Pi^c}{\sqrt{W}}\partial_c X^m)^2 + \frac{1}{8}[\Pi^c \partial_c(\widehat{X}_r + A_r)]^2] + \Lambda(\{\frac{P_m}{\sqrt{W}}, X^m\} - \mathcal{D}_r \Pi^r - \frac{1}{2}\Pi^c \partial_c(F_{ab}\frac{\epsilon^{ab}}{\sqrt{W}})) + \lambda \partial_c \Pi^c]$$

where $\mathcal{D}_r X^m = D_r X^m + \{A_r, X^m\}$, $\mathcal{F}_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\}$, $D_r = 2\pi R^r \frac{\epsilon^{ab}}{\sqrt{W}} \partial_a \widehat{X}^r \partial_b$. P_m and Π_r are the conjugate momenta to X^m and A_r respectively. $\mathcal{D}_r$ and $\mathcal{F}_{rs}$ are the covariant derivative and curvature of a symplectic noncommutative theory [37, 39], constructed from the symplectic structure $\frac{\epsilon^{ab}}{\sqrt{W}}$ introduced by the central charge. The physical degrees of the theory are then described by X^m, A_r, and the corresponding spinorial ones Ψ_α. They are single valued fields on Σ.

At this level, one might naturally ask the following question. Does there exist a MIM2 compactified on a seven dimensional manifold with G2 holonomy group? In what follows we address this question using a recent result from algebraic geometry of toroidal compactification in the presence of discrete symmetries.

3 G2 compactification in M-theory

As we mentioned in the introduction, a possible way to get four dimensional models with four supercharges is to consider the compactification of M-theory on seven dimensional manifolds with G2 holonomy group[2] [6, 47, 48, 49]. We will refer to them as G2 manifolds. In this manner, different $N = 1$ models in four dimensions depend on the geometric realization of the G2 manifold. As for the Calabi-Yau case, there are many geometric realizations. In what follows we quote some of them [50].

[2] G2 is a group of dimension 14 and rank 2.

3.1 G2 manifolds

Let us consider R^7 parametrized by $(x_1, x_2, ..., x_7)$. On this space, one can define the metric as $g = dx_1^2 + ... + dx_7^2$. Reducing the group $SO(7)$ to G2, there is a special real three-form

$$\Psi = dx_{127} + dx_{135} - dx_{146} - dx_{236} - dx_{245} + dx_{347} + dx_{567} \qquad (16)$$

where dx_{ijk} denotes the exterior form $dx_i \wedge dx_j \wedge dx_k$. This expression for Ψ arises from the fact that G2 is the group of automorphisms for the octonionic algebra structure given by

$$t_i t_j = -\delta_{ij} + f_{kij} t_k \qquad (17)$$

which yields the correspondence

$$f_{kij} \rightarrow dx_{kij}. \qquad (18)$$

In general if a seven Riemanian metric admits a covariant constant spinor the holonomy group is G2 and there is exactly one. In such manifolds there exists an orthogonal frame, $\widehat{e}^i$, in which the octonionic three form $\phi = f_{ijk} \widehat{e}^i \wedge \widehat{e}^j \wedge \widehat{e}^k$ and its dual are closed. ϕ is G2 invariant. It turns out that the simplest example of G2 manifolds, which we are interested in here, is the orbifold realization. Let us consider a 7-tori $T^7 = R^7/Z^7$, where now x parameterizes R/Z. A G2 manifold can be constructed from an orbifold action T^7/Γ where Γ a discrete subgroup of G2, and hence leaving the above three-form Ψ invariant. A possible choice is given by

$$\Gamma = Z_2 \times Z_2 \times Z_2 \qquad (19)$$

to be defined in the next section.

3.2 $Z_2 \times Z_2 \times Z_2$ symmetries of the G2 structure

The Z_2 symmetries leaving invariant the 3-form (16), which we will consider, change signs on certain elements of the basis for the octonions. A change of sign for one element of the basis condemns the same for other elements. These combinations are given by the multiplication table. For convenience in further identifications we have chosen the multiplication table represented in Figure 1, where the e_i are the elements from the basis of the octonions. The result for the multiplication of two elements in the basis is the only other element that shares the line passing through the first two, and the sign is given by the arrows. For example, $e_6e_7 = e_5$ while $e_5e_2 = -e_4$.

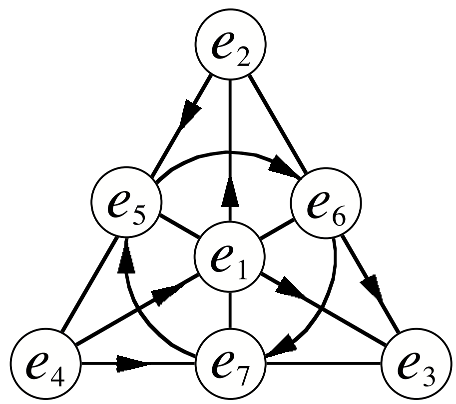

Figure 1: Fano plane representing the multiplication table for the octonions used throughout this paper.

A very quick way to determine such subsets of elements is by considering the canonical quaternionic subspaces of the octonions.

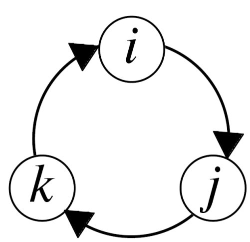

Figure 2: quaternionic diagram.

Changing signs for the elements in these subsets or their complements each preserve the octonionic structure. The former maps $\Psi \rightarrow -\Psi$ while the latter leaves Ψ completely unchanged. According to the multiplication table we have chosen, the indexes of the elements from the basis corresponding to these sets are given as follows:

$\Psi \to -\Psi$	1,2,7	1,3,5	1,4,6	2,3,6	2,4,5	3,4,7	5,6,7
$\Psi \to \Psi$	3,4,5,6	2,4,6,7	2,3,5,7	1,4,5,7	1,3,6,7	1,2,5,6	1,2,3,4

Table 1.

These seven transformations obtained by changing signs for the elements on the second file, together with the identity, form a commutative group with eight elements of order two. This group is $Z_2 \times Z_2 \times Z_2 \cong Z_2^3$. There is a nice geometric interpretation for the operation in this group. Given two transformations, they correspond to two quaternionic subspaces of the multiplication table for the octonions and share only one element -see the first row in the previous table. The composition of these transformations is the one related to the only other quaternionic subspace that shares this element in common. Using the same labeling for the multiplication table of the octonions as the one that determines (16) we can list all the Z_2 symmetries that leave invariant the 3-form Ψ as follows,

elements that change sign		the element in Z_2^3
x_3, x_4, x_5, x_6	$\longleftrightarrow$	(0,1,1)
x_2, x_4, x_6, x_7	$\longleftrightarrow$	(1,1,1)
x_2, x_3, x_5, x_7	$\longleftrightarrow$	(1,0,0)
x_1, x_4, x_5, x_7	$\longleftrightarrow$	(0,1,0)
x_1, x_3, x_6, x_7	$\longleftrightarrow$	(0,0,1)
x_1, x_2, x_5, x_6	$\longleftrightarrow$	(1,0,1)
x_1, x_2, x_3, x_4	$\longleftrightarrow$	(1,1,0)

Table 2.

and, naturally, the identity transformation is in correspondence with (0,0,0). Aiming towards a $T^2 \times T^2 \times T^2 \times S^1$ compact space, we shall identify the coordinates $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ with $(z_1, z_2, z_3, x_7) \in \mathbb{C} \times \mathbb{C} \times \mathbb{C} \times \mathbb{R}$ writing $z_k = x_{2k-1} + ix_{2k}$ for $k = 1, 2, 3$. The transformations given in the previous table are then expressed as follows.

symmetry transformation	the element in Z_2^3
$(z_1, z_2, z_3, x_7) \to (z_1, -z_2, -z_3, x_7)$	(0,1,1)
$(z_1, z_2, z_3, x_7) \to (\bar{z}_1, \bar{z}_2, \bar{z}_3, -x_7)$	(1,1,1)
$(z_1, z_2, z_3, x_7) \to (\bar{z}_1, -\bar{z}_2, -\bar{z}_3, -x_7)$	(1,0,0)
$(z_1, z_2, z_3, x_7) \to (-\bar{z}_1, \bar{z}_2, -\bar{z}_3, -x_7)$	(0,1,0)
$(z_1, z_2, z_3, x_7) \to (-\bar{z}_1, -\bar{z}_2, \bar{z}_3, -x_7)$	(0,0,1)
$(z_1, z_2, z_3, x_7) \to (-z_1, z_2, -z_3, x_7)$	(1,0,1)
$(z_1, z_2, z_3, x_7) \to (-z_1, -z_2, z_3, x_7)$	(1,1,0)

Table 3.

All these symmetries can be obtained as composition of the three canonical generators, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$, for Z_2^3. Nevertheless, there are 28 different subsets of generators for Z_2^3 but all geometrically equivalent.

4 MIM2 on a G2 manifold

In this section we will consider the construction of a MIM2 on a G_2 manifold. We start from the MIM2 on a seven torus T^7 and we will perform the identification of the $Z_2 \times Z_2 \times Z_2$ group, described in Section 3, on the target space.

The MIM2 theory on T^7 is invariant under the area preserving diffeomorphisms. The ones homotopic to the identity are generated by the area preserving constraints (3) and (4). The theory is also invariant under large area preserving diffeomorphisms, non-homotopic to the identity, associated to $Sp(6,Z)$ acting on a Teichmüler space of the moduli space of $g=3$ Riemann surfaces as explained in Section 2. We will now show that the $Z_2 \times Z_2 \times Z_2$ automorphisms of quaternionic subspaces of the octonionic algebra described in Section 3.2 are also symmetries of the hamiltonian of the MIM2 on T^7. Moreover, those are the maximal identifications we can perform on the target space preserving $N=1$ susy. We will see that the remaining symmetries of the fiber become spureous whenever the orbifold action on the states is considered.

4.1 Minimal immersions on the target space

The maps (14,15) from the base Σ (g=3) to the compact sector of the target space T^7. The requirement introduced in [45], to consider all possible immersions from the base manifold to the target space. It has a natural interpretation in terms of the existence of fluxes on the compact sector of the target space. In fact, the existence of fluxes is equivalent to the existence of a bundle gerbe or higher order bundle on the target space[54], [55][56], and [57],[58], [59],[60]. Given a closed p-form W_p satisfying the quantization condition

$$\int_{\Sigma_p} W_p = 2\pi n \tag{20}$$

for any Σ_p submanifold there always exists a bundle gerbe or higher order bundle with its corresponding transition functions on $p-1,\ldots,1$ forms such that W_p is the field strength of a generalized connection. The consistency condition on the transition functions is now satisfied on the overlapping of $p+1$ open sets of an atlas. For the case $p=2$, it is a $U(1)$ principle bundle and the quantization condition ensures the existence of a connection on it such that W_2 is its curvature. In the case of the MIM2 on a T^7 target we should then consider all possible immersions and impose for each of them the topological or central charge condition. This is a geometrical argument emphasizing that we should consider the summation of all possible immersions from Σ to the target, see also [61].

We now proceed to consider all possible immersions from Σ, a genus 3 Riemann surface to $T^7 = S^1 \times \cdots \times S^1$. The reason to consider a genus 3 surface was explained in Section 2, they are the relevant ones when considering the wrapping of a supermembrane on a T^6 target. We consider all decompositions of T^7 into $T^6 \times S^1$, by changing the S^1 we obtain the complete set of seven sectors. On each sector the supermembrane wraps in an irreducible way to the T^6, we ensure this by imposing the topological condition on all configurations of the supermembrane on that sector. We distinguish each sector by an integer $i = 1, \ldots, 7$ and denote the corresponding maps to the T^6 by X_i^r, $r = 1, \ldots, 6$

$$dX_i^r = 2\pi R S_{is}^r d\widehat{X}^s + dA_i^r \tag{21}$$

while the remaining one to the S^1 by X_i

$$dX_i = 2\pi R m_{ir} d\widehat{X}^r + dA_i \tag{22}$$

where $S_{is}^r \in Sp(6, Z)$ for each i=1,. . . ,7, dA_i^r and dA_i are exact one-forms. These ones are completely general without restrictions and as well as the spinor fields on the target which are also scalars on the worldvolume. They carry the local degrees of freedom of the supermembrane. For each T^6 we provide a symplectic structure in order to define the topological condition on Section 2, we will give them in the following analysis. The hamiltonian of the MIM2 supermembrane is invariant under area-preserving diffeomorphisms homotopic to the identity generated by the constraints in Section 2 and by the large diffeomorphisms associated to the change of the basis of the harmonic one-forms by a $Sp(6, Z)$ matrix. Under these transformations the fields A^r, X^m, Ψ transforms as scalars. The harmonic part of dX^r transforms as a one-form under diffeomorphisms and in addition

$$dX_h^{\prime r}(\sigma) = S_s^r dX_h^r(\sigma) \tag{23}$$

$S^r_s \in Sp(6, Z)$. The transformation in the harmonic part is only through a constant $Sp(6, Z)$ matrix. The MIM2 theory is also invariant under $SO(7)$ transformations, acting on the target indices, which preserve the lattice and the topological condition.

Among these symmetries the ones transforming the quaternionic subspaces of Table 1 are relevant. We will denote by $\Gamma \equiv Z_2^3$ the discrete group whose elements change the sign of the maps from Σ to T^7 according to the second row in Table 1. We will denote by $\Lambda \equiv Z_2^4$ the discrete group whose elements change the sign of the maps from Σ to T^7 according to the complete Table 1. Γ is a discrete subgroup of G2 and Λ. For each sector i identified by the elements of the first row of Λ associated to the triplets, take for example $i = 7$: $(1, 2, 7), (4, 3, 7), (5, 6, 7)$ there is a $Z_2 \times Z_2$ subgroup of Γ, whose corresponding elements are $(3, 4, 5, 6), (1, 2, 5, 6), (1, 2, 3, 4)$ respectively. The transformations on this $Z_2 \times Z_2$ subgroup map the sector into itself, since they belong to the $Sp(6, Z)$ associated to the sector.

Computation of m_{ir} We will now show that the other elements in Γ transform the admissible maps (the ones satisfying the topological constraint) of one sector into the admissible maps of another one. The integers m_{ir} become determined in the procedure.

We start with the most general expression (21), (22) in sector $i = 7$, by performing a change on the homology basis and in the corresponding normalized basis of one-forms, it can always be reduced to,

$$d\widehat{X}^1, d\widehat{X}^2, d\widehat{X}^3, d\widehat{X}^4, d\widehat{X}^5, d\widehat{X}^6, m_{7r}d\widehat{X}^7 \tag{24}$$

where from now on we denote in a file the harmonic part of dX^i, for each i ordered from 1 to 7. To symplify the notation we do not write explicitly the $2\pi R$ factors. The exact part is not relevant in the determination of the admissibility of a map, and may be added at any stage of the argument. If we now apply transformation $(2, 4, 6, 7)$ the new map

$$d\widehat{X}^1, -d\widehat{X}^2, d\widehat{X}^3, -d\widehat{X}^4, d\widehat{X}^5, -d\widehat{X}^6, -m_{7r}d\widehat{X}^7 \tag{25}$$

is not admissible in sector 7 but it is in the other sectors. For example if we take the sector 1, with symplectic structure given in Table 4 below, it is admissible if

$$m_{7r}d\widehat{X}^r = d\widehat{X}^1 + m_{72}d\widehat{X}^2 \tag{26}$$

for any integer m_{72}.

If we now consider the transformation $(2,3,5,7)$ of Γ, (25) transforms into

$$d\widehat{X}^1, -d\widehat{X}^2, -d\widehat{X}^3, d\widehat{X}^4, -d\widehat{X}^5, d\widehat{X}^6, -m_{7r}d\widehat{X}^7 \tag{27}$$

which is admissible in sector 1 for any m_{72}. Under $(1,4,5,7)$ (25) transforms into

$$-d\widehat{X}^1, d\widehat{X}^2, d\widehat{X}^3, -d\widehat{X}^4, -d\widehat{X}^5, d\widehat{X}^6, -d\widehat{x}^1 - m_{7r}d\widehat{X}^7 \tag{28}$$

it is admissible only in sector 2 with $m_{72} = 1$. Finally under $(1,3,6,7)$ (25) transforms into

$$-d\widehat{X}^1, d\widehat{X}^2, -d\widehat{X}^3, d\widehat{X}^4, d\widehat{X}^5, -d\widehat{X}^6, -d\widehat{x}^1 - d\widehat{X}^2 m_{7r} \tag{29}$$

which is also admissible in sector 2. The general values of m_{7r} in order to have the full Γ as a symmetry on the admissible set of maps is

$$m_{7r}d\widehat{X}^r = \begin{cases} \pm(d\widehat{X}^1 + d\widehat{X}^2) \\ \pm(d\widehat{X}^3 + d\widehat{X}^4) \\ \pm(d\widehat{X}^5 + d\widehat{X}^6) \end{cases} \tag{30}$$

The general expression for m_{ir} is obtained from m_{7r} by applying the elements of Γ.

We conclude then that given a general admissible map on any sector there always exists another admissible map which is the transformed under Γ of the original one. The integers m_r take some particular value in the procedure. In other words, for that particular values of m_r, the set of admissible maps is preserved under the action of Γ. Moreover, the hamiltonian as a map from the space of configurations to the reals is invariant under Γ. The same properties are valid for the discrete group $\Lambda = Z_2^4$, with the same values of m_r. All other discrete symmetries of the hamiltonian whose bosonic part is quartic but quadratic on each map, are not symmetries of the admissible set.

The symplectic structure on each sector we have used is given in Table 4. We notice that there is no loss of generality by using it, since on any other election of the symplectic matrices the above properties of the admissible set are also valid. The only change is on the explicit realization of the maps,

ω_7	$=$	$dX^1 \wedge dX^2 + dX^3 \wedge dX^4 + dX^5 \wedge dX^6$
ω_6	$=$	$dX^2 \wedge dX^1 + dX^4 \wedge dX^3 + dX^5 \wedge dX^7$
ω_5	$=$	$dX^2 \wedge dX^1 + dX^4 \wedge dX^3 + dX^7 \wedge dX^6$
ω_4	$=$	$dX^2 \wedge dX^1 + dX^3 \wedge dX^7 + dX^6 \wedge dX^5$
ω_3	$=$	$dX^2 \wedge dX^1 + dX^7 \wedge dX^4 + dX^6 \wedge dX^5$
ω_2	$=$	$dX^1 \wedge dX^7 + dX^4 \wedge dX^3 + dX^6 \wedge dX^5$
ω_1	$=$	$dX^7 \wedge dX^2 + dX^4 \wedge dX^3 + dX^6 \wedge dX^5$

Table 4.

Remark 1 We are considering the wrapping of the MIM2 on an oriented T^7 hence the group $\Lambda = (Z_2^4)$ reduces to $\Gamma = (Z_2^3)$, since the transformations on the first row of Table 1 do not preserve the orientation of T^7. We also mentioned that the transformations which changes sign to only two coordinates are not symmetries of the admissible set. We are then left with only the group of discrete symmetries $\Gamma = (Z_2^3)$. They are also symmetries of the hamiltonian of the MIM2.

Remark 2 It is important to emphasize the relation between Γ and the $Sp(6, Z)$ group of large area preserving diffeomorphisms. The space of admissible maps is invariant under the full group $Sp(6, Z)$. It transforms admissible maps of one sector into admissible maps of the same sector. Its action on the harmonic sector of the maps shares in common with Γ a subgroup $Z_2 \times Z_2$ provided the maps are restricted to that sector. When the $Z_2 \times Z_2$ is lifted to the whole admissible set it is not anymore a transformation generated by an area preserving diffeomorphism. However it is a symmetry, as we have shown, of the admissible set. Γ is then the unique discrete symmetry relating the different sectors of the admissible set; the $Sp(6, Z)$ acts only on each sector. We notice that these sectors arise from the different possible wrappings of the MIM2 on T^7 and their origin is not related to the twisted or untwisted sectors of the MIM2 when the identification on an orbifold is performed.

4.2 Configuration space

We will now define a MIM2 on the $G2$ orbifold T^7/Γ constructed by Joyce [50]. The group of transformations $\Gamma = Z_2^3$ introduced by Joyce has additional shifts with respect to the transformations in Section 3.2. Those shifts are irrelevant concerning the action of the group in the MIM2 theory since the maps only enter in terms of one-forms and hence the shifts disappear. However they are important in the construction of the orbifold. These shifts can be generated in the MIM2 theory by the constraint (4). It generates area preserving diffeomorphisms homotopic to the identity with infinitesimal parameters which enter as harmonic one-forms on Σ. The transformation for the maps are

$$\delta X^i = \{x^i, \xi\} \tag{31}$$

with $\xi = \xi_r d\widehat{X}^r$, $\xi_r \quad r = 1, \ldots, 6$ constants. dX^i may be decomposed into its harmonic parts. The harmonic part is invariant under diffeomorphisms homotopic to the identity. We then have

$$\int_\Sigma \delta A^r \sqrt{w} d\sigma^1 \wedge d\sigma^2 = \omega^{rs} \xi_s n Area_\Sigma. \tag{32}$$

We may fix six shifts, corresponding to the mean value of the map over Σ. In the notation of Section 3.2 the generators of the Joyce Z_2^3 are: $\alpha = (3,4,5,6), \beta = (1,2,5,6), \gamma = (2,4,6,7)$ with the same shifts of value 1/2. That is,

$$\begin{aligned} \alpha : (x^1, \ldots, x^7) &\rightarrow (x^1, x^2, -x^3, -x^4, -x^5, -x^6, x^7) \\ \beta : (x^1, \ldots, x^7) &\rightarrow (-x^1, -x^2, x^3, x^4, 1/2 - x^5, -x^6, x^7) \\ \gamma : (x^1, \ldots, x^7) &\rightarrow (x^1, x^2, -x^3, 1/2 - x^4, -x^5, 1/2 - x^6, x^7). \end{aligned} \tag{33}$$

The elements of the group Γ are isometries of T^7, preserving its flat G_2-structure. The fixed points of α, β, γ are each 16 copies of T^3. The singular set S of $\frac{T^7}{\Gamma}$ is a disjoint union of 12 copies of T^3. The singularity on each component of S is of the form $T^3 \times \frac{\mathbb{C}}{\pm 1}$. The singularities of T^7/Γ can be resolved and a metric with holonomy G2 on a compact 7 manifold may be obtained [50].

We may now consider the construction of the untwisted sector of the MIM2 on the G2 orbifold T^7/Γ. We start from the general space of configurations satisfying the topological constraint ensuring the irreducible wrapping of all configurations of the membrane. We then consider the subspace of configurations invariant under Γ. This was constructed in the previous section. The maps are of the form (21) (22) with the restrictions on the values of

m_{ir} (30) (on that particular basis of harmonic one-forms). The part of the map associated to the wrapping of the MIM2 has no additional restrictions beyond the topological constraint defining the wrapping. On the space of configurations we construct classes, two elements of a class are related by a transformation of Γ. The hamiltonian, as mentioned before has the same value on each element of the class. The untwisted sector of the theory is now defined on the space of classes. Each class represents now the map from Σ to the orbifold. This construction may be implemented directly in the functional integral of the supermembrane, which in the case of the MIM2 has a well defined gaussian measure. We now proceed to define the twisted sectors of the theory[3]. We have to consider those configurations defined on the Riemann surface Σ satisfying

$$X(2\pi) = \Gamma X(0) + 2\pi n \tag{34}$$

when we go around one element of the basis of homology on Σ. If we take for example $\alpha \in \Gamma$ as defined before the configurations correspond to sections of the spin bundle $Spin(4) = Sp(1) \times Sp(1)$ obtained by lifting a vector bundle of rank 4. If this were the only element of Γ we then have to sum over all spin structures for that vector bundle. However we have to consider all elements of Γ. For each of them we have the corresponding sections of $Spin(4)$. The extended spin structure is now constructed with the assignement to each element of the basis of homology a $+$ or $-$ and looking for the Γ element which is associated to the $-$ contribution. We then have to sum over all spin structures defined with the above data. Each twisted sector correspond to each spin structure.

It is important to observe that the twisted sectors are only defined in terms of spinor fields, it is a $2+1$ theory, on the worldvolume. It is the only way to have a global construction with the required periodicity or antiperiodicity conditions. However the maps X and the spinors Ψ of the supermembrane action are scalars over the worldvolume. That means that the construction of the twisted sectors in the light cone gauge require the lifting of the supermembrane action to include square root bundles among its configurations. the untwisted sector is constructed directly in terms of these scalar fields. there is then a geometrical distinction between the sectors, since the theory is still invariant under diffeomorphisms homotopic to the identity generated

[3]A former study of the twisted states of an extended membrane in the case of M theory on an orbifold $\frac{S^1}{Z_2}$ was considered in [62].

by constraints (3), (4). The untiwsted sector we have constructed breaks SUSY to $N = 1$ and it is directly related to the analysis in [45]. The precise global cosntruction of the twisted sectors will be discussed elsewhere.

Remark The $Sp(6, Z)$ symmetry on the admissible set is broken after identifying the points on T^7 by Γ. On each sector of the admissible set one is left with a $Z_2 \times Z_2$ symmetry.

4.3 Connection with Calabi-Yau compactifications

It is very well known that the G2 manifold can be also built using a partial complex structure coordinate [48]. The above 3-form can be re-expressed as

$$\Psi = Re(\Omega) + w \wedge dx_7. \tag{35}$$

In this equation $\Omega = dz_1 \wedge dz_2 \wedge dz_3$ is the complex holomorphic form of C^3 and $w = \frac{i}{2}(dz_1 \wedge d\bar{z}_1 + dz_2 \wedge d\bar{z}_2 + dz_3 \wedge d\bar{z}_3)$ is the Kahler form. Since SU(3) is a subgroup of G2, one can identify the C^3 factor with a local Calabi-Yau threefold (CY3) used in two dimensional $N = 2$ sigma model [51, 52, 53]. In this realization, the above three-form (35) is invariant under the symmetry

$$z_i \rightarrow \overline{z_i} \quad x_7 \rightarrow -x_7. \tag{36}$$

which is needed to ensure $N = 1$ in 4D. We will try to show that this transformation can be related to the above $Z_2 \times Z_2 \times Z_2$ symmetry used in the orbifold construction. This can be done by imposing certain constraints depending on the precise Z_2 action. Indeed, the CY3 could be taken as $T^2 \times T^2 \times T^2$ quotiented by $Z_2 \times Z_2$. Since the CY condition requires the use of only two Z_2's ($Z_2^1 \times Z_2^2$), we need to single out the third Z_2^3 factor. $Z_2^1 \times Z_2^2$ acts on the six-torus structure, producing as a result a CY3, and trivially on the circle S^1. The third Z_2^3 acts on both, the CY3 and the circle leading to the G2 structure manifold. In this way, one can identify the last action with the transformation given in (36).

The singularities of this orbifold can be identified with its fixed points. In the three dimensional complex factor, the fixed locus of this G2 manifold is a Lagrangian submanifold. Its volume form is defined by the real part of Ψ. Since the circle has two fixed points, the total singular geometry then consists of two copies of such a lagrangian submanifold. The singularities can have an interpretation in the MIM2 picture as critical points. However this does not mean that there is a degenerate locus of extremal points. On the contrary, the quantum analysis reveals that there is an absolute minimum for the hamiltonian of the supermembrane. There are no flat directions in the potential. This fact can be understood from the fact that the dual of the gauge symmetries correspond to different backgrounds and not a unique one.

Locally each singular point should be resolved like $R^3 \times X$, where X is an ALE CalabiYau 2-fold asymptotic to C^2/Z_2, is known as ALE space with A_1 singularity. The ALE space with A_1 singularity is described by

$$z_1^2 + z_2^2 + z_3^2 = 0. \tag{37}$$

Using a simple change of variables, this is equivalent to

$$xy = z^2 \tag{38}$$

where x, y and z are complex coordinates. As usual, this singularity can be removed either by deforming the complex structure or by a blow-up procedure. Geometrically, this corresponds to replacing the singular point ($x = y = z = 0$) by a $CP^1 \sim S^2$. As previously explained the (APD) connected with the identity deform the shape of each T^2 and they produce translation on the orbifold side. They serve to blow up the corresponding orbifold singularities leading to a compactification on a true G2 manifold.

5 Discussion and conclusions

In this paper we have shown, in a top-down approach, that the 11D supermembrane theory restricted by a topological condition can be compactified on a $\frac{T^7}{Z_2^3}$ orbifold preserving its quantum stability properties. The resulting theory can be interpreted as a compactification on a G2 manifold. Indeed, the symmetries of the theory produce a holonomy bundle that corresponds exactly to those associated to the Riemanian holonomy of a G2 manifold. By performing the identification on the target space of the $SO(7)$ discrete symmetries preserving the topological condition, only those symmetries associated to the G2 orbifold space are possible, neither the configuration states nor the minimal immersions are invariant under the spureous symmetries that would break the supersymmetry to $N = 0$. One can see that the holonomy bundle associated to the compactification to 5D is related with the Klein subgroup. When this is further compactified to the remaining S^1 there exist seven possible immersions of the M2-brane on the target space of the T^7 that allow to make exactly the identifications with the G2 holonomy group. The singularities of this G2 orbifold may be resolved as shown by Joyce leading to a true G2 manifold. The shifts have their origin in the diffeomorphisms homotopic to the identity of the MIM2. Moreover, this result can also be seen in terms of a $\frac{CY_3 \times S^1}{Z_2}$. It has been pointed out in [11] the phenomenological interest of G2 compactifications that admit an expression in terms of CY compactification since for those manifolds explicit metric can be obtained and ALE resolutions of the singularities may lead to interesting phenomenological properties as chirality and nonabelian gauge groups. In that sense it would be interesting to compute explicitly the corresponding metric and study its phenomenological properties. Other aspects of interest like confinement from G2-manifolds [63] (considered mainly in G2 manifolds with ALE singularities) emerge naturally in our case since the spectral properties of the MIM2 have not changed when we have performed the identification in the target space. In [41], it was argued how the MIM2 theory could reproduce the strong coupling regime of susy QCD since there are present glueballs and it posseses a discrete spectrum with a mass gap. Indeed it corresponds exactly to a symplectic Super Yang-Mills in 4d coupled to several scalar fields. In [41], an interpretation of the M2 theory in terms of SUSY QCD was proposed: the confined phase of the theory corresponds to the MIM2 and the quark-gluon plasma phase to the ordinary M2 compactified in a 7-torus. Both phases are connected through a topological phase transition that breaks the center of the group. Since the theory of MIM2 on a G2 manifold do not change its quantum spectral properties, those previous properties would apply and it could also described the confined phase of the theory. Regarding moduli stabilization aspects, assuming the target torus is fixed to be isotropic, the moduli parametrizing the position of the MIM2 on a 7-torus as well as the overall moduli parametrizing the size of the manifold is fixed [45]. When the MIM2 is compactified on the G2 orbifold the singularities are resolved through a backreaction effect due to the wrapping, then the moduli associated to those singularities are also fixed. We have then obtained the 11D supermembrane minimally immersed on a particular G2 manifold. An interesting question is the comparison between the MIM2 compactified on a T^7 and the MIM2 compactified on a G2 manifold. Since the theory posseses all of the symmetries that can be identified in the target space, there is no breaking of any symmetry or lack of consistency. The untwisted sector of the

G2 orbifold is the Hilbert space of invariant states under the corresponding discrete symmetries of the hamiltonian. The twisted sector are states invariant under the action of the discrete symmetries around the singularities, as usual. In our case the identification in the target space a priori (since there is not so far an explicit computation) enlarges the configuration space with respect to the one of MIM2 compactified on T^7 and fixes the integers m_{7r} to a particular value. However, the number of supersymmetries in 4D is the same in both cases. The reason for that lies in the minimal immersion maps inducing flux charges on the target space. In the case of the T^7, it breaks supersymetries not only in the gauge sector but also in the gravitational sector. This is an important difference with respect to string theory. In MIM2 case, the gauge and gravity sectors are in no way decoupled.

Acknowledgements

We thank L. J. Boya, G. Bonelli, C. Hull, A. Lerda, H. Nicolai, M. Petropoulos, N. Prezas, J. Roseel, V. N. Suryanarayana, I. Stavrov, A. Uranga, for helpful discussions. MPGM gives thanks to Departamento de Fisica, Universidad de Zaragoza, Spain, for kind invitation and hospitality while part of this work was done. AB would like to thank UFR-Lab/ PHE, Rabat for hospitality where a part of this work was done, and the project, between Universidad de Zaragoza (Spain) and Faculte des Sciences de Rabat (Morocco), *Fisica de altas energias: Particulas, Cuerdas y Cosmologia* (grant A9335/07). MPGM is partially supported by Dipartimento di Fisica di Universita di Torino under European Comunity's Human Potential Programme and by the Italian MUR under contracts PRIN-2005023102 and PRIN-2005024045. The work of AR is partially supported by a grant from MPG, Albert Einstein Institute, Germany and by PROSUL, under contract CNPq 490134/2006-08. The work of AB and AS has been supported by CICYT (grant FPA-2006-02315) and DGIID-DGA (grant 2007-E24/2), Spain.

References

[1] P. K. Townsend and G. Papadopoulos, *Compactification of D=11 supergravity on spaces of exceptional holonomy*, Phys. Lett **B 357** (1995)472, hep-th/9506150.

[2] B. Acharya, *On realizing N = 1 super Yang-Mills in M-theory*, hep-th/0011089.

[3] J. A. Harvey, G. Moore, *Superpotentials and Membrane Instantons*, hep-th/9907026.

[4] S. Kachru, J. McGreevy, *M-theory on Manifolds of G2 Holonomy and Type IIA Orientifolds*, **JHEP 0106** (2001) 027, hep-th/0103223.

[5] M. F. Atiyah and E. Witten, *M-theory dynamics on a manifold of G2 Holonomy*, hep-th/0107177.

[6] L. B. Anderson, A. B. Barrett, A. Lukas, M. Yamaguchi, *Four-dimensional Effective M-theory on a Singular G(2) Manifold*, Phys. Rev.**D74**:086008, 2006, hep-th/0606285.

[7] E. Witten, *Anomaly Cancellation On Manifolds Of G2 Holonomy*, hep-th/0108165.

[8] B. Acharya, E. Witten, *Chiral Fermions from Manifolds of G2 Holonomy*, hep-th/0109152.

[9] A. Belhaj, L.B. Drissi, J. Rasmussen, *On N=1 gauge models from geometric engineering in M-theory*, Class. Quant. Grav.**20** (2003) 4973-4982, hep-th/0304019.

[10] A. Belhaj, *On manifolds of G(2) holonomy in M-theory compactifications*, Afr.J. Math. Phys. **1**(2004) 61-68.

[11] B. S. Acharya, S. Gukov *M theory and Singularities of Exceptional Holonomy Manifolds* Phys.Rept. **392** (2004) 121-189, hep-th/0409191.

[12] M. Cicoli, J. P. Conlon, F. Quevedo, *Systematics of String Loop Corrections in Type IIB Calabi-Yau Flux Compactifications*, arXiv:0708.1873 [hep-th].

[13] V. Balasubramanian, P. Berglund, J. P. Conlon, F. Quevedo, *Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications*, **JHEP 0503** (2005) 007, hep-th/0502058.

[14] F. Denef, M. R. Douglas, S. Kachru, *Physics of String Flux Compactifications*, arXiv:hep-th/0701050.

[15] J.F.G. Cascales, M.P. Garcia del Moral, F. Quevedo, A. Uranga, *Realistic D-Brane Models on Warped Throats: Fluxes, Hierarchies and Moduli Stabilization*, JHEP **0402** (2004) 031, hep-th/0312051.

[16] J.F.G. Cascales, A.M. Uranga, *Chiral 4d string vacua with D-branes and NSNS and RR fluxes*, JHEP **0305** (2003) 011 hep-th/0303024.

[17] G. Curio, A. Klemm, D. Lust, S. Theisen, *On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H fluxes.* Nucl.Phys.**B 609** (2001), 3-45, hep-th/0012213.

[18] R. Blumenhagen, B. Kors, D. Lust, S. Stieberger, *Four-dimensional String Compactifications with D-Branes, Orientifolds and Fluxes*, hep-th/0610327.

[19] C. M. Hull, R. A. Reid-Edwards, Flux Compactifications of M-Theory on Twisted Tori, **JHEP 0610** (2006) 086, arXiv:hep-th/0603094.

[20] G. Dall'Agata, R. D'Auria, S. Ferrara, *Compactifications on twisted tori with fluxes and free differential algebras*, Phys. Lett. **B619** (2005) 149-154, hep-th/0503122.

[21] G. Dall'Agata, S. Ferrara, *Gauged supergravity algebras from twisted tori compactifications with fluxes*, Nucl. Phys. **B717** (2005) 223-245, arXiv:hep-th/0502066.

[22] B. Acharya, K. Bobkov, G. Kane, P. Kumar, D. Vaman, *An M theory Solution to the Hierarchy Problem*, Phys.Rev.Lett. **97** (2006) 191601, hep-th/0606262.

[23] B.S. Acharya, K. Bobkov, G.L. Kane, P. Kumar, J. Shao *The G_2-MSSM - An M Theory motivated model of Particle Physics* arXiv:0801.0478 [hep-th].

[24] J.P. Conlon, C.H. Kom, K. Suruliz, B.C. Allanach, F. Quevedo, *Sparticle Spectra and LHC Signatures for Large Volume String Compactifications.* JHEP **0708**:061,2007.arXiv:0704.3403 [hep-ph].

[25] C.M. Hull, *Doubled Geometry and T-Folds*, JHEP **0707** (2007)080, hep-th/0605149.

[26] A. Dabholkar, C. Hull, *Duality twists, orbifolds, and fluxes*, JHEP **0309** (2003) 054, `hep-th/0210209`.

[27] A. Flournoy, B. Wecht, B. Williams, *Constructing nongeometric vacua in string theory*, Nucl. Phys. **B 706** (2005) 127-149, `hep-th/0404217`.

[28] G. Dall'Agata, N. Prezas, *Scherk-Schwarz reduction of M-theory on G2-manifolds with fluxes*, JHEP **0510** (2005) 103, `hep-th/0509052`.

[29] R. Blumenhagen, L. Gorlich, B. Kors, D. Lust, *Asymmetric orbifolds, noncommutative geometry and type I string vacua*, Nucl. Phys. **B 582**(2000) 44-64, `hep-th/0003024`.

[30] C.M. Hull, *A Geometry for non-geometric string backgrounds* JHEP **0510** (2005) 065, hep-th/0406102.

[31] E. Bergshoeff, E. Sezgin, P.K. Townsend, *Supermembranes and eleven-dimensional supergravity*, Phys. Lett. **B189** (1987) 75-78.

[32] B. de Wit, J. Hoppe, H. Nicolai, *On the quantum mechanics of supermembranes*, Nucl. Phys. **B305** (1988) 545.

[33] B. de Wit, M. Luscher, H. Nicolai, *The supermembrane is unstable*, Nucl. Phys. **B320** (1989) 135.

[34] B. de Wit, U. Marquard, H. Nicolai, *Area preserving diffeomorphisms and supermembrane lorentz invariance*, Commun. Math. Phys. **128** (1990) 39-62.

[35] B. de Wit, K. Peeters, J. Plefka, *Supermembranes with winding*, Phys. Lett. **B409** (1997) 117-123, `hep-th/9705225`.

[36] I. Martin, A. Restuccia, R. S. Torrealba, *On the stability of compactified $D = 11$ supermembranes*, Nucl. Phys. **B521** (1998) 117-128, `hep-th/9706090`.

[37] I. Martin, J. Ovalle, A. Restuccia, *D-branes, symplectomorphisms and noncommutative gauge theories*, Nucl. Phys. Proc. Suppl. **102**(2001) 169-175. *Compactified $D = 11$ supermembranes and symplectic noncommutative gauge theories*, Phys. Rev. **D64** (2001) 046001, `hep-th/0101236`.

[38] M.P. Garcia del Moral, A. Restuccia, *On the spectrum of a noncommutative formulation of the D=11 supermembrane with winding*, Phys. Rev. **D66** (2002) 045023, hep-th/0103261.

[39] L. Boulton, M. P. Garcia del Moral, I. Martin, A. Restuccia *On the spectrum of a matrix model for the D=11 supermembrane compactified on a torus with non-trivial winding*, Class. Quant. Grav. **19** (2002) 2951, hep-th/0109153.

[40] L. Boulton, M.P. Garcia del Moral, A. Restuccia, *Discreteness of the spectrum of the compactified D=11 supermembrane with non-trivial winding*, Nucl. Phys. **B671** (2003) 343-358, hep-th/0211047.

[41] L. Boulton, M.P. Garcia del Moral, A. Restuccia, *The Supermembrane with central charges: (2+1)-D NCSYM, confinement and phase transition*, hep-th/0609054.

[42] J. Bellorin, A. Restuccia, *D=11 Supermembrane wrapped on calibrated submanifolds*, Nucl. Phys. **B737** 190-208, 2006, hep-th/0510259.

[43] L. Boulton and A. Restuccia, *The Heat kernel of the compactified D=11 supermembrane with non-trivial winding*, Nucl. Phys. **B724** (2005) 380-396, hep-th/0405216.

[44] B. de Wit, K. Peeters, J. Plefka, *Superspace Geometry for Supermembrane Backgrounds*, Nucl. Phys. **B532** (1998) 99-123, arXiv:hep-th/9803209.

[45] M.P. Garcia del Moral, J.M. Pena, A. Restuccia, *N=1 4D Supermembrane from 11D*, arXiv:0709.4632[hep-th].

[46] M.P. Garcia del Moral, I. Martin, A. Restuccia, *Nonperturbative SL(2,Z) (p,q)-strings manifestly realized on the quantum M2.* arXiv:0802.0573 [hep-th]

[47] A. Belhaj, M. P. Garcia del Moral, *On Non Commutative G2 structure*, J. Phys. **A38** (2005) 2773-2784, hep-th/0408011.

[48] R. Roiban, C. Romelsberger, J. Walcher, *Discret Torsion in Singular G_2-Manifolds and Real LG*, Adv. Theor. Math. Phys. **6** (2003) 207-278, hep-th/0203272.

[49] O.P. Santillan, *A Construction of G(2) holonomy spaces with torus symmetry*, Nucl.Phys. **B660** (2003)169-193, hep-th/0208190.

[50] D. Joyce, *Compact manifolds of Special Holonomy*, Oxford University Press, **2000**.

[51] E. Witten, *Phases of $N = 2$ theories in two dimensions*, Nucl. Phys. **B 403** (1993) 159, hep-th/9301042.

[52] M. Aganagic, C. Vafa, *Mirror symmetry, D-branes and counting holomorphic discs*, hep-th/0012041.

[53] A. Belhaj, *Mirror symmetry and Landau Ginzburg Calabi-Yau superpotentials in F-theory compactifications*, J. Phys. **A 35** (2002) 965, hep-th/0112005.

[54] J. Giraud, *Cohomologie non-abelienne*, volume 179 of Grundl. Springer Verlag, Berlin,1997.

[55] J.L. Brylinski, Prog. in Math, Vol, 107, *Loop Spaces Characteristic Clases and Geometric Quantization*, Birkhäuser, Boston (1993).

[56] M. Caicedo, I. Martín, A. Restuccia, *Gerbes and duality*, Annals Phys. **300** (2002), 32-53, arXiv:hep-th/0205002.

[57] J. C. Baez, U. Schreiber, *Higher gauge theory*, math/0511710.

[58] J. C. Baez, *Higher Yang-Mills theory*, hep-th/0206130.

[59] J.M. Isidro, *Gerbes, quantum mechanics and gravity*, hep-th/0510075.

[60] M. Mackaay, R. Picken, *The Holonomy of gerbes with connections*, math/0007053.

[61] T. Zehra Husain, *M2-branes wrapped on holomorphic curves*, JHEP **0312** (2003) 037, hep-th/0211030.

[62] P. Horava, E. Witten, *Heterotic and type I string dynamics from eleven-dimensions*, Nucl.Phys.**B460** (1996)506-524, hep-th/9510209.

[63] B.S. Acharya, *Confining Strings from G_2-holonomy spacetimes* arXiv:hep-th/0101206.