New Charged Black Holes with Conformal Scalar Hair.

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A general class of four dimensional, stationary solutions of the Einstein-Maxwell system with a conformally coupled scalar field is constructed in this paper. The stationary case is presented and shown to belong to the Plebanski-Demianski family which implies that the static metric has the form of the C-metric. It is shown that in the static, AdS case, a new family of Black Holes arises. They turn out to be cohomogeneity two, with horizons that are not Einstein neither homogenous manifolds. The usual conical singularities present in the C-metric are automatically removed from the spacetime due to the backreaction of the scalar field. The scalar field carries a continuous parameter that resembles the usual acceleration present in the C-metric. When this parameter vanishes the static family it is shown to contain either to the dyonic Bocharova-Bronnikov-Melnikov-Bekenstein solution or the dyonic extension of the Martinez-Troncoso-Zanelli black holes, depending on the value of the cosmological constant.

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I. INTRODUCTION, DISCUSSION AND CONCLUSIONS

The scalar hair have played an important rôle in our understanding of four dimensional black holes as fundamental objects, characterized by a small set of parameters (for a short review see [1]). Within all the possible hairs the conformal scalar hair is particularly interesting: it not only contains a well known family of $U(1)$ charged, stationary, black holes [2], [3], [5] , [6], [7], it also has the property that the asymptotically locally AdS solutions, when mapped to the Einstein frame, can be embedded in string theory [8] that they are stable against linear perturbations [9] and provides a relevant arena for the gravitational description of superconductors [10].

These interesting features are in contrast with the exiguous knowledge of exact solutions of this system. Moreover, the question on the existence of stationary axisymmetric solutions was already pointed out to be of relevance in one of the seminal papers of the subject [3] and, until now, there is no explicit construction of it. This is the first of a series of papers that will improve the situation on these regards. This will be done taking advantage of the well known fact that, the traceless of the energy momentum tensor for the conformally coupled scalar field implies that any space of constant Ricci scalar could in principle support its backreaction. Being the Plebanski-Demianski family of solutions [11] the most general Petrov type D, Einstein-Maxwell spacetime, it provides the natural starting point to further explore the space of solutions of the Einstein-Maxwell system and the conformally coupled scalar hair.

Thus, in section one the most general solution, within the Plebanski-Demianski family, of the Einstein-Maxwell-Conformally coupled scalar field is constructed. The addition of a quartic self interaction of the scalar field is necessary when the cosmological constant is included and it is also considered. In order to show that all the known solutions are included within this new family, section two is devoted to the the analysis of the static case. There is shown that the family of metric presented here, besides reproducing the known solutions provides the dyonic extension of [6].

The static solution, being of the form of the C-metric, is reanalyzed in the last section and a number of remarkable features are observed to occur. First, the conical singularities are removed due to the back reaction of the scalar field, this is not done at the expense of changing the asymptotic behavior of the spacetime (as oposite of the embedding of Ernst [12]); this seems to implies that the scalar field can be regarded as the source of the acceleration in the classical interpretation of this spacetime [13]. Second, as was pointed out in [14], the AdS C-metric can be interpreted as a single black hole in a certain range of the parameters. This new AdS black hole is geometrically characterized and it turns out to be a cohomogeneity two black hole whose event horizon is not an Einstein neither a homogenous manifold, resembling the five dimensional structure of the stationary black holes constructed in [15]. Third, this new family of static black holes are continuously connected with the maximally symmetric configuration, viz AdS. It turns out that in the limit where the spacetime is of constant curvature the scalar field develops a non-trivial vacuum expectation value: the energy momentum tensor vanishes but there is a explicit dependence on the spacetime point in the scalar field. These peculiar configurations were discovered in [16], however it was not known how they connect to non conformally flat spacetimes.

The elimination of conical singularities from the C-metric, due to the scalar field backreaction, is an interesting result and deserves some comments. The conical defects associated to the acceleration can be neatly described as follows: given the charged C-metric

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$$ds^2 = \frac{1}{A^2(q-p)^2}\left(\frac{dp^2}{X(p)} + X(p)d\sigma^2 + \frac{dq^2}{Y(q)} - Y(q)dt^2\right),$$

$$X(p) = (1-p^2)(1+Ar_+p)(1+Ar_-p), \qquad Y(q) = -X(q),$$

$$r_\pm = m \pm \sqrt{m^2 - Q^2} \qquad\qquad (1.1)$$

it is possible to compute the "surface gravity" (in the terminology of [17]), $k = \frac{g^{\mu\nu}\partial_\mu l^2 \partial_\nu l^2}{4l^2}$, of the angular killing vector

$$l = C\partial_\sigma = \partial_\chi \qquad\qquad (1.2)$$

at the degeration surfaces $p = \pm 1$:

$$k_{p=\pm 1} = C^2\left(A^2 Q^2 + 1 \pm 2Am\right)^2. \qquad (1.3)$$

From the previous expression it seems to be impossible, keeping the acceleration, to have the same normalization at each one of the degeneration surfaces (unless the mass vanishes in which case the charged C-metric metric represent a naked singularity). This is equivalent to say that there is a conical defect (or excess) in, at least, one of the "poles" of the compact, spacelike section, defined at constant q.

The situation completely changes in the presence of scalar fields. Slowly decaying scalar fields have a non-trivial contribution to the mass of the spacetime [18], [19]. Thus it is possible to eliminate the m parameter from the metric functions, and thus the conical singularities, and still have a spacetime with positive mass. Although this last point is not strictly proved below, it is supported due to the existence of Killing horizons of positive scalar curvature in the vanishing m limit.

The present construction could have many applications. One of the most interesting, in our view, is when the metric is locally asymptotically AdS but, due to the acceleration horizon, not asymptotically static. The explicit, and intrinsic, time dependence in the asymptotic region would allow to study the elusive out of equilibrium phenomena in the dual condense matter system. In an accompanying paper we will further discuss the rotating case, its thermodynamics and some of the physical interpretations of these spacetimes [20].

Our notations follows [21]. The conventions of curvature tensors are $[\nabla_\rho, \nabla_\sigma]V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$ and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$. The metric signature is taken to be mostly plus, greek letters are spacetime indices and we set $c = 1$.

Note added in proof: An abstract of part of this work was submitted on June 19 to the "Fifth Aegean Summer School" organized by one of the authors (E.P.) of [23], who informs us on some overlap with their still unpublished work. Although the line element in the static case coincide with the one of [23], their study is exclusively devoted to the case when the Q^2 term of (1.1) is positive. What makes the inclusion of a scalar field special is that it allows for this Q^2 term to not be the square of the electric charge. Actually it can be negative, which is the case analyzed in the last section.

II. THE STATIONARY SOLUTION

The Einstein-Maxwell-conformally coupled scalar field with quartic selfinteraction can be defined by the set of equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{\kappa}{4\pi}(F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\tau\lambda}F^{\tau\lambda}) + \kappa T^\phi_{\mu\nu}, \qquad R = 4\Lambda, \qquad F = dA, \qquad (2.1)$$

$$T^\phi_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \frac{1}{6}\left[g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu + G_{\mu\nu}\right]\phi^2 - \alpha g_{\mu\nu}\phi^4, \qquad (2.2)$$

where $\kappa := 8\pi G$ and Λ is a cosmological constant. The other field equations follows as consistency conditions of the previous ones. The above form of the equations will be useful at the time of solving them.

Given the Pleabanski-Demianski ansatz:

$$ds^2 = \frac{1}{(1-qp)^2}\left[(p^2+q^2)\left(\frac{dp^2}{X(p)} + \frac{dq^2}{Y(q)}\right) + \frac{X(p)}{p^2+q^2}\left(d\tau + q^2 d\sigma\right)^2 - \frac{Y(q)}{p^2+q^2}\left(d\tau - p^2 d\sigma\right)^2\right], \qquad (2.3)$$

it is possible to integrate the metric functions from the condition that the Ricci scalar is constant. Replacing it back in the full set of field equations we found that the most general solution with a non-trivial scalar field is given by:

$$X(p) = a_0 + a_2p^2 + a_4p^4, \qquad Y(q) = -a_4 - \frac{\Lambda}{3} - a_2q^2 - \left(a_0 + \frac{\Lambda}{3}\right)q^4, \tag{2.4}$$

$$A = \frac{c_1q + c_2p}{q^2 + p^2}d\tau + pq\frac{c_2q - c_1p}{q^2 + p^2}d\sigma, \qquad \phi = \pm\sqrt{B}\frac{1 - pq}{1 + pq}, \tag{2.5}$$

$$B = \frac{3\left(3\kappa\left(c_1^2 + c_2^2\right) + 24\pi\left(a_4 + a_0\right) + 8\pi\Lambda\right)}{4\kappa\pi\left(3a_0 + 3a_4 + \Lambda\right)}, \qquad \alpha = -\frac{\Lambda}{6B}. \tag{2.6}$$

In [20] it will be shown that the above spacetime has non-trivial angular momentum and that for certain values of the parameters it represent a black hole. In what follows the discussion will be focussed on the static limit.

Furthermore, in order to have non-singular limit we found that it is convenient to use another parametrization for the metric functions.

III. CONTRACTIONS TO THE KNOWN SOLUTIONS

In this section an acceleration parameter β will be introduced. This will allow us to recover the known solutions.

With the above remarks in mind we just state the result; the static limit is given by:

$$ds^2 = \frac{1}{(q - \beta p)^2}\left(\frac{dq^2}{Y(q)} - Y(q)d\tau^2 + \frac{dp^2}{X(p)} + X(p)d\sigma^2\right), \qquad A = c_1qd\tau + c_2pd\sigma, \tag{3.1}$$

$$X(p) = a_0 + \frac{a_1}{\beta}p + a_2p^2 + \beta a_3p^3 + \beta^2 a_4p^4, \qquad Y(q) = -\beta^2 a_0 - a_1q - a_2q^2 - a_3q^3 - a_4q^4 - \frac{\Lambda}{3}, \tag{3.2}$$

$$a_1 = a_3\frac{\left(4a_2a_4 - a_3^2\right)}{8a_4^2}, \qquad \phi = \pm\sqrt{B}\frac{\beta p - q}{\beta p + q + \frac{a_3}{2a_4}} \qquad B = \frac{3\left(\kappa c_1^2 + \kappa c_2^2 + 8\pi a_4\right)}{4\kappa\pi a_4}, \qquad \alpha = -\frac{\Lambda}{6B}. \tag{3.3}$$

The limit $\beta \to 0$ makes sense if $a_3 = 0$ or if $a_2 = \frac{a_3^2}{4a_4}$. When $a_3 = 0$ the limit implies a constant scalar field and the discussion of the last section of [6] applies. In the more interesting case, $a_2 = \frac{a_3^2}{4a_4}$ the change of coordinates $q = \frac{1}{r}$

and the parameters $a_4 = -M^2G^2$, $a_3 = 2MG$, $c_1 = e$, $c_2 = g$, $a_2 = -1$, $a_0 = 1$, provides the dyonic extension of the black hole [6]:

$$ds^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)} + r^2\left(\frac{dp^2}{1 - p^2} + \left(1 - p^2\right)d\phi^2\right), \qquad A = \frac{e}{r}d\tau + gpd\sigma, \tag{3.4}$$

$$\phi = \mp\sqrt{\frac{3}{4\pi}}\frac{\sqrt{M^2G - e^2 - g^2}}{r - MG}, \qquad f(r) = \left(1 - \frac{MG}{r}\right)^2 - \frac{\Lambda}{3}r^2, \qquad \frac{\left(e^2 + g^2\right)}{M^2} = G + \frac{2\Lambda}{9\alpha}G^2. \tag{3.5}$$

In the same way is possible to prove that all the known solutions of the relevant system are contained within this static family.

IV. NEW STATIC, COHOMOGENEITY TWO ADS BLACK HOLES

Now, let us study the above solution in all its generality. To this end we parameterize the solution in terms of the roots of

the metric functions, after a shift in the coordinates the static metric becomes [1]:

$$X(p) = b^2 \left(p^2 - \xi_1^2\right)\left(p^2 - \xi_2^2\right), \qquad (4.1)$$

$$Y(q) = -b^2 \left(q^2 - \xi_1^2\right)\left(q^2 - \xi_2^2\right) - \frac{\Lambda}{3} \qquad (4.2)$$

Let us set $\xi_1 < \xi_2$. The manifold associated to the coordinates (p, σ) is Euclidean and compact if $-\xi_1 \leq p \leq \xi_1$. The same condition that follows from the field equations, namely the fact that there are no linear neither cubic term in the metric functions implies that the spacetime is free of conical singularities. Indeed, from equation (1.2) we obtain

$$k_{p=\xi_1} = k_{p=-\xi_1} = C^2 b^4 \xi_1^2 \left(\xi_1^2 - \xi_2^2\right)^2. \qquad (4.3)$$

Infinity is located at $p = q$ and for vanishing cosmological constant there is an event horizon at $q = \xi_2$. When $\Lambda \neq 0$ the horizon is located at the largest root, q_H, of (4.2). In the case of positive and vanishing cosmological constant there is a further horizon and the asymptotic region is no longer static. The asymptotic region is static for $-\Lambda > 3b^2 \xi_1^2 \xi_2^2$. It is also interesting to note that acceleration horizon is extremal when $-\Lambda = 3b^2 \xi_1^2 \xi_2^2$. There is a curvature singularity at $q = \infty$. From the above discussion it follows that the allowed rank for q is $p < q < \infty$.

Although the geometry is regular the scalar field diverges outside the horizon. In this coordinates it is proportional to

$$\frac{q - p}{p + q}, \qquad (4.4)$$

so, although it goes to zero at infinity and is regular on the killing horizon it is divergent on the surface $p + q = 0$. From (3.3) it is possible to check that setting first $a_3 = c_1 = c_2 = 0$ and then $a_4 = 0$, the space becomes of constant curvature and the scalar field is nothing but an stealth field [16],[**?**]. Indeed, although it is non trivial,

$$\phi_S = \pm\sqrt{\frac{6}{\kappa}}\frac{q - p}{p + q}, \qquad (4.5)$$

its energy momentum tensor vanishes $T_{\mu\nu}\left(\phi = \phi_S\right) = 0$ **if and only if** the metric is of constant curvature.

The horizon metric is

$$ds_H^2 = \frac{1}{(q_H - p)^2}\left(\frac{dp^2}{X(p)} + X(p)d\sigma^2\right) \qquad (4.6)$$

which is not an Einstein neither an homogeneous manifold. In the case $\Lambda = 0$ its scalar curvature is

$$R_H = 2b^2(3p + \xi_2)\left(\xi_2 - p\right)^3. \qquad (4.7)$$

Note that suitable values of the roots can make the scalar curvature everywhere positive. To understand better the geometry of the horizon let us expand around the degeneration points, let us set a periodic coordinate $\chi \in [0, 2\pi)$, related with σ as $\sigma = C\chi$, where C is obtained requiring (4.3) equals one, and $p = -\xi_1 + \frac{x_1^2}{2C}$, $p = \xi_1 - \frac{x_2^2}{2C}$ Using these coordinates it is possible to show that the degeneration surface are smooth:

$$ds_H^2\big|_{x_1=0} \longrightarrow \frac{1}{(q_H + \xi_1)^2}\left(dx_1^2 + x_1^2 d\chi^2\right) \qquad (4.8)$$

$$ds_H^2\big|_{x_2=0} \longrightarrow \frac{1}{(q_H - \xi_1)^2}\left(dx_2^2 + x_2^2 d\chi^2\right) \qquad (4.9)$$

The horizon looks locally as a two sphere, in close resemblance with [15].

The metric is asymptotically locally AdS in the sense that

$$R^{\mu\nu}_{\cdot\cdot\lambda\rho}\Big|_{p=q} = \frac{\Lambda}{3}\left(\delta^\mu_\lambda \delta^\nu_\rho - \delta^\mu_\rho \delta^\nu_\lambda\right) \qquad (4.10)$$

Indeed, this is a local statement. In the AdS case exists acceleration horizons for certain values of the parameters. This allows to have a non-stationary behavior (because the explicit time dependence) of the asymptotic metric. In the case when the acceleration horizon is extremal (which only can occur when $\Lambda < 0$) it has been recently shown that the conformal structure of the C-metric is topologically $\Re^3$ [22]. A further study of the conformal structure in the case presented here is necessary to understand the properties of the theory induced in the boundary. The dual, condense matter description of these spacetimes, as well as the thermodynamics properties are better understood in the Einstein frame. Thus, this discussion will be postponed for a further work [20].

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