Dual-mode temperature compensation technique for laser stabilization to a crystalline whispering gallery mode resonator

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Abstract: Frequency stabilization of a diode laser locked to a whispering gallery mode (WGM) reference resonator made of a MgF_2 single crystal is demonstrated. The strong thermal dependence of the difference frequency between two orthogonally polarized TE an TM modes (dual-mode frequency) of the optically anisotropic crystal material allows sensitive measurement of the resonator's temperature within the optical mode volume. This dual-mode signal was used as feedback for self-referenced temperature stabilization to nanokelvin precision, resulting in frequency stability of 0.3 MHz/h at 972 nm, which was measured by comparing with an independent ultrastable laser.

OCIS codes: (120.4800) Optical standards and testing; (120.6810) Thermal effects; (140.3425) Laser stabilization.

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1. Introduction

Ultrastable lasers are typically realized by electronically locking the laser frequency to a resonance of an external reference cavity defined by two high-reflectivity mirrors attached to a low-thermal-expansion spacer. A different approach [1–7] has been developed in recent years, providing a reference for the laser's frequency by using whispering-gallery-mode resonators fabricated from single-crystal materials. These resonators can achieve very high quality factors ($Q > 10^{11}$) [8], are compact ($\lesssim 1\text{cm}^3$), and their monolithic nature allows their operation in noisy and space-constrained environments, they can be employed over a wide wavelength range, limited only by optical absorption in the host material. A relative optical frequency stability better than one part in 10^{13} in 1 s for a laser locked to the WGM resonators was predicted [9] and demonstrated recently [6]. This result was limited only by thermodynamical fluctuations of the resonator's temperature [6, 9, 10].

Suppression of *technical* temperature fluctuations is a precondition for achieving such performance, as both the radius and the refractive index of the resonator change with temperature, and so do its modes' resonance frequencies. This typically entails technically complex methods to shield the mode volume from the temperature instabilities of the environment. Requirements on the shielding may be less stringent if the relevant temperature of the resonator can be accurately measured, and its fluctuations compensated. To that end, it has been suggested to harness the differential frequency shift of two orthogonally polarized WGMs in an optically anisotropic crystal resonator upon a temperature change [5]. The shift of this "dual-mode frequency" can thus serve as a sensitive thermometer [11, 12]. This approach, often referred to as the dual-mode technique, could thus simplify laser frequency stabilization techniques and potentially lead to a "stability transformer" improving the stability of radio-frequency standards [13]. Note also that the dual-mode approach itself is already applied successfully in the radio frequency domain, improving the long term stability of quartz oscillators [14] and cryogenic sapphire oscillators [15–17].

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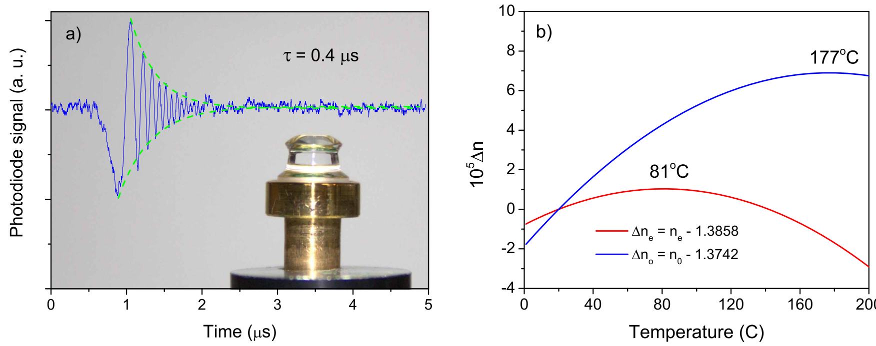

Fig. 1. a) MgF_2 resonator used for laser frequency stabilization. Ringing in the transmission signal observed in a fast laser frequency sweep across the resonance is shown by solid blue line. The envelope of the oscillations (dashed green line) indicates an exponential field decay from the WGM within 0.4 μs, corresponding to $Q \approx 3.9 \times 10^8$ and a linewidth of 800 kHz. Photograph of the resonator (with radius of 2 mm) on brass mount is shown in background. b) Temperature dependence of the thermorefractive deviation of the ordinary and extraordinary indexes of refraction of magnesium fluoride. The dependencies are obtained from measurement data [18] approximated for 972 nm.

A dual-mode temperature measurement and stabilization has been implemented recently [11, 12], achieving an *in loop* temperature stability on the order of several nK on 1000 s timescales, according to the residual error signal of the measured dual-mode frequency, and a projected frequency stability has been estimated from these temperature fluctuations. In this paper we report the first, to our knowledge, *direct* measurement of the optical frequency stability for a laser locked to the WGM resonator, whose temperature was stabilized by the dual-mode technique. This was achieved by comparing the frequency of the WGM-stabilized laser to an independent, ultrastable laser locked to a Fabry-Pérot-type cavity used for precision spectroscopy [19]. The dual-mode signal was used as an error signal to control the WGM resonator temperature by heating it with a light emitting diode (LED). In order to prove the concept we also measured the dual-mode frequency and the main WGM frequency at different temperatures. The results were compared with a theoretical estimate for 972 nm wavelength at room temperature.

2. Theoretical estimate

Let us consider a disk-shaped WGM resonator made from a z-cut magnesium difluoride (MgF_2) single crystal. The temperature dependence of the WGM resonance frequency is described by the thermorefractivity coefficients $\alpha_n^{(\mathrm{o,e})}$ and thermal expansion coefficient $\alpha_l^{(\mathrm{o})}$ as

$$\frac{df}{dT} = -f(\alpha_n^{(\mathrm{o,e})} + \alpha_l^{(\mathrm{o})}), \tag{1}$$

whereby horizontally—or ordinarily (o)—polarized laser light is resonant with the TE modes of such a resonator, and vertically—or extraordinarily (e)—polarized light with TM modes for a z-cut crystal [11]. The thermal expansion coefficient $\alpha_l^{(\mathrm{o})}$ is the same for both mode families, but the thermorefractivity coefficients $\alpha_n^{(\mathrm{o})}$ and $\alpha_n^{(\mathrm{e})}$ affect TE and TM modes in a different way (see Fig. 1(b)). At room temperature (20°C) and excitation wavelength $\lambda = 972$ nm, these coefficients can be found from experimental data [18, 20] as

$$\alpha_l^{(\mathrm{o})} = 9.3 \times 10^{-6}, \tag{2}$$

$$\alpha_n^{(\mathrm{o})} = 0.87 \times 10^{-6}, \tag{3}$$

$$\alpha_n^{(\mathrm{e})} = 0.33 \times 10^{-6}. \tag{4}$$

As $\alpha_l^{(\mathrm{o})} \gg \alpha_n^{(\mathrm{o,e})}$, the frequency drift in such a system is dominated by the thermal expansion:

$$\frac{df}{dT} \approx -f\alpha_l^{(\mathrm{o})} = -2.9 \text{ GHz/K}. \tag{5}$$

The difference, or "dual-mode" frequency between two orthogonal TE and TM modes $\Delta f = f_\mathrm{o} - f_\mathrm{e}$ depends on temperature as:

$$\frac{d}{dT}\left(f_\mathrm{o} - f_\mathrm{e}\right) \approx -f_\mathrm{o}(\alpha_n^{(\mathrm{o})} - \alpha_n^{(\mathrm{e})}) = -0.17 \text{ GHz/K}. \tag{6}$$

Large fluctuations due to the *thermorefractive noise* [6, 9, 10] limit the dual-mode frequency stability and, as a consequence, sensitivity of temperature monitoring at small sampling times. This noise originates from the well-known fact that the temperature of a body of volume V, density ρ and specific heat capacity C at an average temperature T intrinsically fluctuates as $\langle \delta T^2 \rangle = k_\mathrm{B} T^2/\rho C V$, where k_B is Boltzmanns constant. For the $\mathrm{MgF_2}$ resonator used in this study the estimated mode volume $V \approx 2 \times 10^{-12}$ m^3, the density $\rho = 3180$ kg/m^3, the specific heat capacity $C = 1020$ J/kg K lead to $\langle \delta T^2 \rangle = (400 \text{ nK})^2$. These temperature fluctuations modulate the thermorefractive indices of the WGM and its resonance ω_c as $\langle \delta\omega_c^2 \rangle = \omega_c^2 \alpha_n^{(\mathrm{o,e})2} \langle \delta T^2 \rangle$. If orthogonally polarized modes' volumes are perfectly overlapping the fluctuations of the reference mode (the one whose absolute frequency is not measured) are written onto the other mode (whose absolute frequency is measured), otherwise the fluctuations of the modes should be added in quadrature.

Equations (5) and (6) predict that the dual-mode frequency Δf is only 17 times less sensitive to temperature than any of the WGM modes. This is in stark contrast to the differential frequency shift of two neighboring modes (mode index i) of the same family $f_{i+1} - f_i$

$$\frac{d}{dT}\left(f_{i+1} - f_i\right) \approx -\left(f_{i+1} - f_i\right)\alpha_l^{(\mathrm{o})} \approx \frac{c}{2\pi n R}\alpha_l^{(\mathrm{o})} = -160 \text{ kHz/K}. \tag{7}$$

Therefore, the temperature inside the WGM resonator can be monitored with high sensitivity using the dual-mode technique, without using an external temperature sensor. Using feedback techniques, the temperature can therefore be stabilized as demonstrated in the following.

3. Experimental setup

The WGM resonator with a radius of 2 mm (see photograph in Fig. 1(a)) was made from a z-cut magnesium difluoride ultrapure crystalline material following earlier work [6, 21]. The WGMs were located in the rim of the resonator that was glued to a brass support, fixed in the prism-coupling setup.

A high-index prism (gadolinium gallium garnet) was used to couple the beam of an external cavity diode laser into the WGM resonator. The resonator was mounted into a setup shielded against vibrations and thermal fluctuations. This prism-coupling setup was the same as the one described in Ref. [6], but in this proof-of-concept experiment, our goal was not to get maximum short-term stability of the laser frequency. Therefore, we used for that moment available resonator with lower quality factor $Q \approx 3.9 \times 10^8$ compared to $Q \approx 2.0 \times 10^9$ in Ref. [6].

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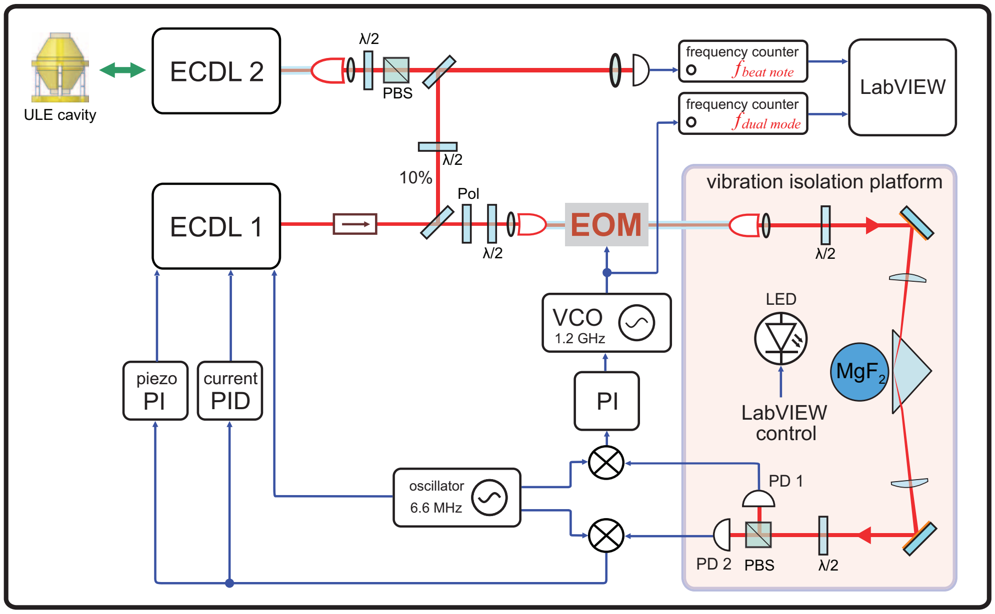

Fig. 2. The experimental setup for laser stabilization to a MgF_2 resonator using the PDH method, the dual-mode temperature control and comparison to an ultrastable laser locked to a mirror-based cavity on another optical table. ECDL: external cavity diode laser; EOM: electro-optic modulator; $\lambda/2$: half-wavelength retardation plate; PBS: polarizing beam splitter; PD: photodiode; PI(D): proportional-integral-(differential) feedback controller; ULE: ultralow-expansion glass; VCO: voltage controlled oscillator. LED is a one watt light emitting diode shining onto the WGM resonator and used in feedback loop for stabilization of the dual-mode frequency.

We measured the quality factor Q and the linewidth $\kappa = \omega/Q$ of a resonance by sweeping the laser frequency $\omega = 2\pi c/\lambda$ (c is the vacuum light speed, $\lambda = 972$ nm the laser wavelength) through the WGM resonance. A transient heterodyne beat in the transmitted power is observed when the circulating power in the WGM decayed during the ringdown time κ^{-1}, interfering with the laser field swept already away from the cavity resonance. The envelope of the beat decays with the cavity field amplitude ringdown time $\tau = 2\kappa^{-1}$ (see Figure 1(a)). An exponential fit of the envelope yields the linewidth $\kappa = 2\pi \times 800$ kHz for the WGM used in this work.

In order to suppress heat exchange through the gas as well as acoustic noise, and to avoid degradation of the quality factor Q of the crystalline resonator, the setup was placed in an aluminum chamber containing high vacuum ($p < 10^{-6}$ mbar). The chamber was located on a passive vibration isolation platform (Minus-K) to suppress the transmission of vibrations to the coupling setup. Two aluminum shields between the chamber's walls and the coupling setup (see Ref. [6]) reduced heat exchange via thermal radiation. The coupling setup was fixed to the chamber by three thermally insulating spacers. In addition, we used an active stabilization of the temperature of the outer shield to a value close to room temperature.

The beam of a home-made Littrow-type extended cavity diode laser with wavelength 972 nm was focused on the face of the coupling prism through antireflection coated windows in the vacuum chamber with input light polarization such as to excite both TE and TM mode families. To observe TE and TM modes, the out-coupled light was split by a polarizing beam splitter and registered by two amplified photodiodes. The laser was locked to a TM mode using the

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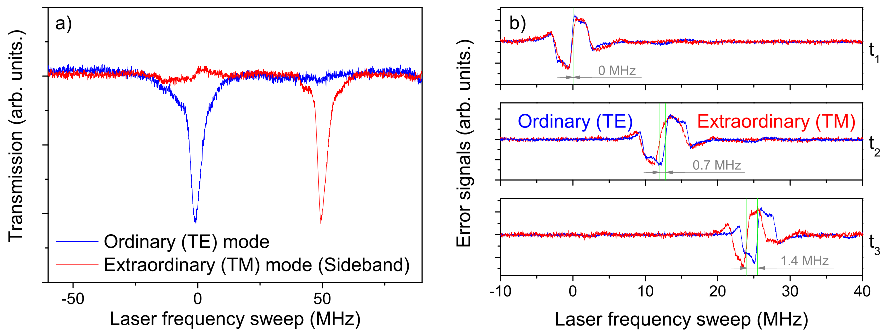

Fig. 3. a) Simultaneous measurement of the resonances of the ordinarily (blue) and extraordinarily (red) polarized WGM, probed with the laser carrier and a modulation sideband, respectively. Both linewidths are dominated by the unstabilized diode laser linewidth. b) Two error signals from these orthogonally polarized crystalline WGM modes are adjusted that signals overlap at time t_1 and drift away at later times due to cooling of the WGM resonator.

Pound-Drever-Hall (PDH) method [22] modulating the diode pump current at 6.6 MHz and demodulating the transmission signal at the same frequency (Fig. 2). This error signal was used as feedback to control both the grating tilt in the laser via a piezoactuator and the diode pump current. The signal in both branches was amplified by home-built proportional-integral (PI) controllers; the laser current branch had an additional phase advance (PID).

An independent mode with orthogonal polarization was found 1.2 GHz away from the TM mode used for laser stabilization. The difference frequency between those modes was applied to an external fiber-coupled electro-optical modulator (EOM) to create a sideband that excited the second mode. The laser sideband could then be locked to the TE mode, by demodulating the transmission signal of the corresponding light polarization with the 6.6 MHz modulation tone, amplifying and filtering this PDH error signal with a home-built proportional-integral (PI) controller, and feeding back the resulting correction signal to the input of the voltage-controlled-oscillator (VCO) driving the EOM. The error signals belonging to the orthogonally polarized modes were independent; the cross-mixing of the error signals was below 4% for properly aligned polarizers (see Figure 3(a)).

We studied how the modes' difference frequency (or dual-mode frequency) correlates to the WGM-stabilized absolute laser frequency, using a radio-frequency (rf) beat note with an independent diode laser locked to an ultrastable mirror-based resonator [19]. The dual-mode frequency and the beat note frequency were registered by rf counters.

In a last set of experiments, the dual-mode frequency was stabilized by an active LabVIEW-based feedback loop using proportional and integral gains. In this simple proof-of-principle experiment, a one watt white light emitting diode shining onto the WGM resonator through the vacuum window was used in the loop to counteract drifts in the dual-mode frequency by compensating the the WGM resonator temperature. The feedback time constant was 0.2 s.

In summary, we used three stabilization loops, which could all be activated separately: 1) the laser stabilized to one of the TM modes of the resonator; 2) Optical sideband generated by the EOM was locked to an orthogonally polarized TE mode, by feeding back on the EOM frequency; 3) The EOM's frequency was stabilized to a reference oscillator by feeding back on the resonator's temperature.

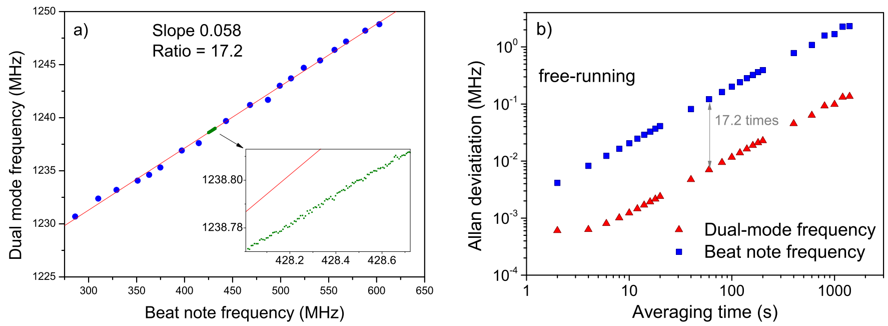

Fig. 4. a) Drift of the dual-mode frequency versus the simultaneously measured frequency of the beat note with an independently stabilized laser, while intentionally changing temperature of the vacuum chamber. Inset shows the same dependence measured during unstabilized temperature fluctuations. The ratio of the frequency drifts (inverse slope of the linear regression) is 17.2 ± 0.3. b) Absolute Allan deviations of the dual-mode frequency and the beat note frequency for the case of small temperature variations.

4. Results

Before implementing the dual-mode temperature compensation, we studied the dual-mode frequency and the main WGM frequency behavior at different temperatures. Results of a simple test, in which the unlocked laser frequency was scanned through both TE and TM modes are given in Fig. 3(b). In this experiment, the WGM resonator was heated by LED radiation, then the LED was turned off. The traces show the different drift of two orthogonal WGM modes of the MgF_2 resonator while the temperature of the resonator was cooling down. The resonance frequency of each TE and TM modes drifts ca. 17 times quicker than the frequency difference between them.

To prove the linear relation of the dual-mode frequency and the absolute frequency of the WGMs, both frequencies were simultaneously measured (Fig. 4(a)) while intentionally changing temperature of the vacuum chamber in the range from 21.08°C to 21.88°C. The true change of the resonator temperature should be smaller due to its effective temperature isolation. The ratio of the frequency drifts extracted from linear approximation of these data is 17.2 ± 0.3. The inset in Figure 4(a) shows the same dependence measured following smaller temperature fluctuations. In the case of the free-running temperature fluctuations the ratio varied from 16 to 18 in different measurements. These values are in good agreement with the values obtained from Eqs. (5) and (6).

In Fig. 4(b) the absolute Allan deviations of both the dual-mode frequency and the beat note frequency are depicted for the case of small temperature variations. For averaging times larger than 10 seconds their ratio is constant. For smaller averaging times the dual-mode frequency is noisier than expected from Eqs. (5) and (6). An explanation of such behavior could be that the dual-mode frequency's fluctuations come from the thermorefractive noise occurring in a small volume of the resonator close to its surface. In turn, the beat note frequency noise is dominated by the thermal fluctuations of the resonator radius depending on the average temperature of the whole resonator. The thermorefractive noise in MgF_2 doesn't affect the resonator radius significantly (see Ref. [5, 6]). While the ratio between the dual-mode frequency and the beat note frequency can be constant only for such thermal fluctuations that can affect whole volume of the WGM resonator, it implies the need for the dual-mode frequency to be reasonably averaged and can limit the application of the dual-mode technique for reduction of high frequency noises.

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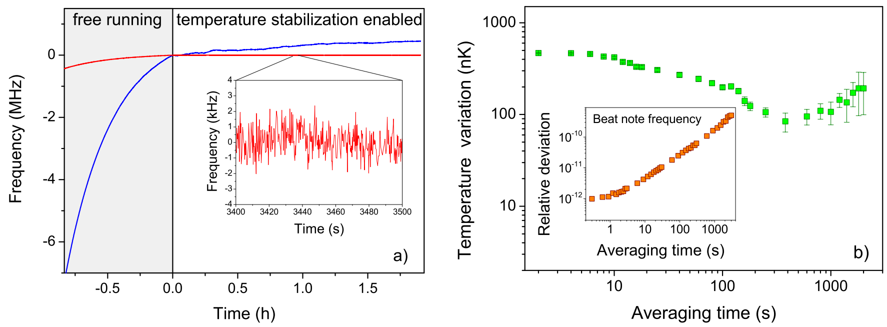

Fig. 5. a) Long-term WGM absolute frequency (blue solid line) and dual-mode frequency (red solid line) drifts are shown as functions of the observation time when the temperature was free-running ($t < 0$ h) and stabilized ($t > 0$ h) by the dual-mode technique. Without temperature feedback the WGM frequency drift is typically up to 10 MHz/h and with the feedback 0.3 MHz/h. Inset magnifies fluctuations of the dual-mode frequency. b) Allan deviation of the stabilized temperature in the WGM resonator according to the in-loop (dual-mode) frequency drift. Inset shows Allan deviation of the optical beat note normalized to the optical carrier at 308 THz while the dual-mode stabilization is enabled.

It is expected that feedback on the resonator temperature, nulling drifts of the dual-mode frequency, should suppress the drifts of the modes' (and therefore laser) frequency. As described above, such feedback is implemented here by heating the cavity with the light of a white LED. Figure 5(a) shows the result of the corresponding measurements. At 0 h the dual-mode temperature compensation is turned on and a drift of the dual-mode frequency cancelled. This evidently leads to an improvement of absolute frequency stability, but residual drift of ca 0.3 MHz/h has been observed in all measurements. Allan deviation of the dual-mode frequency converted to the temperature unit according to Eq. (6) is shown in Fig. 5(b). Accuracy of temperature measurement by the dual-mode technique increase from 480 nK at an integration time of 1 s to 80 nK at an integration time of 400 s. Inset in the same figure shows typical relative Allan deviation of 10^{-12} at an integration time of 1 s of the laser frequency while the dual-mode stabilization is enabled.

We explain the residual drift by particularity of the WGM resonator setup. The resonator was glued to the brass post (necessary for machining and cleaning), whose temperature was not stabilized by the dual-mode technique. Nevertheless, while the resonator's temperature was constant, its radius continued to covary with radius of its brass post. For frequency stabilization by the dual-mode compensation technique it is important to exclude any radial tension resulting from a temperature difference between a setup and a resonator itself. This problem could be alleviated by fixing the resonator on a pedestal with a properly chosen thermal expansion constant. More generally, an important challenge in this stabilization scheme is to properly counteract the temperature perturbations of the resonator, whose spatial distribution is not exactly known. Other feedback paths on local temperature, such as heating via the laser power absorbed in the WGM, may provide improved performance.

5. Conclusion

We studied the difference frequency between two orthogonally polarized modes (the dual-mode frequency) of a WGM resonator made from crystalline MgF_2 and its relation to frequency of one of the modes (the main frequency) by locking a 972 nm diode laser frequency to the WGM resonator and locking a sideband of the laser to an independent orthogonally polarized mode of

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the same WGM resonator. The ratio of the WGM frequency to the dual-mode frequency was estimated theoretically and approved experimentally.

We performed the first, to our knowledge, *direct* measurement of the optical frequency stability for a laser locked to a WGM resonator, whose temperature was kept constant by the dual-mode technique. This was achieved by comparing the frequency of the WGM-stabilized laser to an independent, ultrastable laser locked to a Fabry-Pérot-type cavity.

The resonator's temperature was stabilized to nanokelvin precision. We report the frequency drift reduction with enabled feedback. The problem of residual frequency drift was detected and possible solutions were proposed. This technique can be used to improve long-term frequency stability of a laser locked to a compact WGM resonator made from a birefringent material. It could provide an alternative to more complex and technically involved stabilization schemes such as the one provided by bigger reference cavities.

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