On the feasibility of employing solar-like oscillators as detectors for the stochastic background of gravitational waves

D.M. Siegel[1,⋆] and **M. Roth**[2]

[1] Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Am Mühlenberg 1, 14476 Potsdam-Golm, Germany
[2] Kiepenheuer-Institut für Sonnenphysik, Schöneckstr. 6, 79104 Freiburg, Germany

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We present a hydrodynamic model that describes excitation of linear stellar oscillations by a stochastic background of gravitational waves (SBGW) of astrophysical and cosmological origin. We find that this excitation mechanism is capable of generating solar g-mode amplitudes close to or comparable with values expected from excitation by turbulent convection, which is considered to be the main driving force for solar-like oscillations. A method is presented that places direct upper bounds on the SBGW in a frequency range, in which the SBGW is expected to contain rich astrophysical information. Employing estimates for solar g-mode amplitudes, the proposed method is demonstrated to have the potential to compete with sensitivities reached by gravitational wave experiments in other frequency ranges.

1 Introduction

One of the prime discoveries in cosmology has been the detection of the cosmic microwave background (CMB) by Penzias & Wilson (1965), which captures the state of the universe roughly 300 000 years after the Big Bang, when matter and radiation decoupled from each other and the universe became transparent to electromagnetic radiation. It is expected that the universe is also permeated by a stochastic background of gravitational radiation of astrophysical and cosmological origin (Maggiore 2000; Sathyaprakash & Schutz 2009). Proposed cosmological processes to generate the cosmological component include standard inflationary models, pre-Big-Bang models and cosmic strings (Maggiore 2000). The astrophysical component is thought to arise from the incoherent superposition of a large number of astrophysical sources that have an accelerated mass distribution with a quadrupole moment, like compact binary star systems, core collapse supernovae and rotating neutron stars (Schneider, Marassi & Ferrari 2010; Regimbau 2011). Since the universe has been transparent with respect to gravitational waves since the time they were first produced by cosmological processes in the very early universe, i.e., before 10^{-24} s after the Big Bang, they are ideal carriers of information on the cosmological and astrophysical processes that generated them and thus on the state of the universe at these times. This background encodes information that is not accessible to conventional observations based on electromagnetic waves and its amplitude has to be constrained and measured in a variety of frequency bands, in order to disentangle the signatures of the various contributions.

In the present paper, we show that asteroseismology can place upper bounds on the amplitude of a stochastic background of gravitational waves (SBGW) in the mHz and μHz frequency range, where the SBGW is likely to be dominated by the astrophysical component; in this frequency domain, the astrophysical component contains rich astrophysical information, e.g., on the physics of compact objects and on star formation history (Hils, Bender & Webbink 1990; Schneider et al. 2010; Regimbau 2011). However, it has proven to be difficult to probe the astrophysical background in the mHz range and it has thus remained essentially unconstrained until today. Many experiments are currently being proposed to explore the gravitational wave sky at mHz and μHz frequencies, such as the New Gravitational wave Observatory NGO (a.k.a. eLISA; Jennrich et al. 2011), the DECi-hertz Interferometer Gravitational wave Observatory DECIGO (Kawamura et al. 2011), or the torsion-bar antenna TOBA (Ishidoshiro et al. 2011).

The proposed method to constrain the SBGW at mHz and μHz frequencies is based on theoretical work showing that solar-like oscillations can be excited by gravitational waves (Siegel & Roth 2010, 2011). We elaborate on this topic in Sect. 2. In particular, we point out that in the case of the Sun, g modes are more sensitive to an SBGW than p modes and that solar g-mode amplitudes in the case of excitation by an SBGW can reach values comparable with values expected from excitation by turbulent convection, which is considered to be the main driving force for oscillations in solar-like stars. Section 3 is devoted to studying the inverse problem of constraining an SBGW, given asteroseismic data

⋆ Corresponding author: daniel.siegel@aei.mpg.de

on the surface velocities of stellar oscillations. In Sect. 4, the potential for an asteroseismic experiment to constrain the SBGW in the mHz and μHz frequency regime is discussed and conclusions are presented. Space missions like CoRoT (Baglin et al. 2006) and *Kepler* (Christensen-Dalsgaard et al. 2009), which record asteroseismic data for a large number of solar-like stars over a wide range of stellar parameters, but also ground-based telescope networks like SONG (Grundahl et al. 2009) and LCOGT (Shporer et al. 2011), offer a unique possibility to select stars with optimized sets of global stellar parameters, and to employ these stars as a large array of low-frequency antennas for the SBGW using the methods presented here.

2 Excitation of linear stellar oscillations by gravitational waves

2.1 General hydrodynamic model

In this Section, we discuss the theoretical model that describes excitation of solar-like oscillations by an SBGW (cf. Siegel & Roth 2010, 2011, for more details on the theoretical framework). The physical mechanism underlying this excitation is the fact that gravitational waves manifest themselves in oscillating tidal forces; they periodically stretch and compress the spatial dimensions orthogonal to the direction of propagation in a quadrupolar pattern, thus imposing stresses on the matter they pass through. Therefore, only quadrupolar ($l = 2$) stellar oscillations can be excited.

In order to formulate these considerations in a more mathematical framework, we start from the equations of energy and momentum conservation in general relativity[1],

$$\nabla_\nu T^{\mu\nu} = 0, \tag{1}$$

where $T^{\mu\nu}$ denotes the energy-momentum tensor of an ideal fluid and ∇_μ the covariant derivative. Using Fermi normal coordinates, applying the linearized theory assumption, assuming Newtonian internal motions, and applying the long wavelength approximation, we obtain from the spatial components of Eq. (1):

$$\frac{\partial \rho \boldsymbol{v}}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{v} \otimes \boldsymbol{v}) = -\nabla p + \boldsymbol{f}_{\mathrm{GW}}, \tag{2}$$

where t, ρ, p and $\boldsymbol{v}$ denote, respectively, time, density, pressure, and the internal velocity field of the star. This equation has an additional driving term with respect to the usual Euler equation in hydrodynamics, $\boldsymbol{f}_{\mathrm{GW}}(\boldsymbol{x}, t)$, with components

$$f^i_{\mathrm{GW}} = \frac{1}{2}\rho \ddot{h}^i{}_j x^j. \tag{3}$$

Here, $\boldsymbol{x}$ denotes the position in physical space and $\ddot{h}_{ij}$ is the second temporal derivative of the gravitational wave field $h_{ij}(\boldsymbol{x}, t)$. Perturbing Eq. (2) around the equilibrium state of the star in the usual way, retaining only terms up to linear order in the perturbations, decomposing the velocity field into a component $\boldsymbol{v}_{\mathrm{osc}}$ describing the velocity field due to oscillations and a component summarizing all other motions, together with the assumption of incompressible turbulence, we arrive at the equation of motion,

$$\rho\left(\frac{\partial^2}{\partial t^2} - \mathcal{L}\right)\boldsymbol{v}_{\mathrm{osc}} + \mathcal{D}(\boldsymbol{v}_{\mathrm{osc}}) = \frac{\partial}{\partial t}(\boldsymbol{f}_{\mathrm{Rey}} + \boldsymbol{f}_{\mathrm{Entr}} + \boldsymbol{f}_{\mathrm{GW}}). \tag{4}$$

The wave operator $\mathcal{L}$ is a linear differential operator, whose eigenvalues and eigenfunctions define the oscillation frequencies and eigenfunctions of the star (Siegel & Roth 2011; Unno et al. 1989). Furthermore, the linear damping operator $\mathcal{D}$ captures all the damping terms (Samadi & Goupil 2001; Siegel & Roth 2011). On the right-hand side of Eq. (4), we identify the usual Reynolds and entropy source terms (Samadi & Goupil 2001; Siegel & Roth 2011), which describe excitation of stellar oscillations by turbulent convection, and the aforementioned driving term due to gravitational waves, which is an immediate result from the general-relativistic framework and which is absent in fully Newtonian fluid dynamics. In other words, Eq. (4) can be seen as a first-order generalization of the usual Newtonian equation of motion for linear stellar oscillations (cf. Samadi & Goupil 2001) to general relativity.

For the time being we are solely interested in excitation by gravitational waves, and therefore we ignore the Reynolds and entropy source terms. In this case, Eq. (4) can be solved analytically (even when considering an arbitrary gravitational wave field), and in the case of an SBGW, we find the following expression for the mean-square amplitudes of the oscillations:

$$\left\langle |A_N|^2 \right\rangle = \frac{\pi^2}{25}\frac{\chi_N^2}{\eta_N \omega_N I_N^2} H_0^2\, \Omega_{\mathrm{GW}}(\omega_N). \tag{5}$$

Here, we made use of the fact that a stellar oscillation mode can be written as

$$\boldsymbol{\xi}_N(\boldsymbol{x}, t) = A_N(t)\boldsymbol{\xi}(\boldsymbol{x})\,\mathrm{e}^{-\mathrm{i}\omega_N t}, \tag{6}$$

where A_N is a time-dependent, complex amplitude, $N = (n, l, m)$ is an abridged index for the radial order n, harmonic degree l, and azimuthal order m of the mode,

$$\boldsymbol{\xi}_N(r, \Theta, \phi) = \left[\xi_{r,nl}(r)\boldsymbol{e}_r + \xi_{h,nl}(r) r \nabla\right] Y_{lm}(\Theta, \phi) \tag{7}$$

denotes the spatial eigenfunction of the mode, and ω_N its frequency. It is remarkable that the right-hand side of Eq. (5) is essentially given by the product of two factors, the first of which only depends on stellar properties, including the mode frequency ω_N, the damping rate η_N of the mode, the mode inertia I_N, and a quantity χ_N that measures the susceptibility of the mode with respect to excitation by gravitational waves. The second factor solely depends on the properties of the SBGW, which is entirely characterized by

[1] Greek indices take spacetime values 0, 1, 2, 3, whereas Latin indices take spatial values 1, 2, 3 only. Repeated indices are summed over.

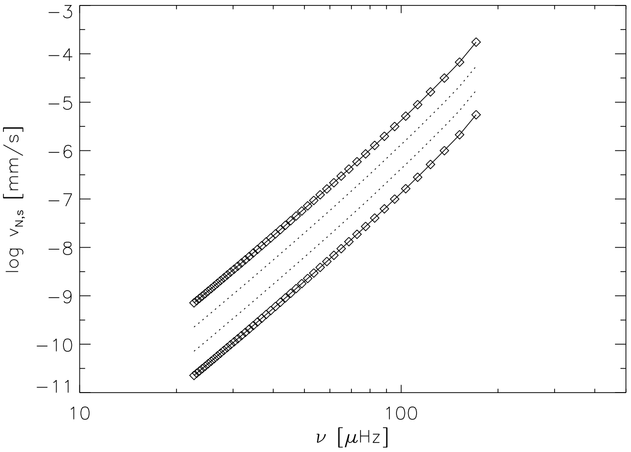

Fig. 1 Numerical results for intrinsic rms surface velocities of $l = 2$ solar g modes assuming a constant spectral energy density of $\Omega_{\rm GW} = 1\times10^5$ (top curve) and $\Omega_{\rm GW} = 1\times10^8$ (bottom curve) of the SBGW. The dotted curves indicate the cases $\Omega_{\rm GW} = 1\times10^6$ and $\Omega_{\rm GW} = 1 \times 10^7$ (cf. Siegel & Roth 2011).

the normalized, dimensionless function (Allen & Romano 1999; Maggiore 2000)

$$\Omega_{\rm GW}(\nu) = \frac{1}{\rho_{\rm c}}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln\nu}, \tag{8}$$

which measures the energy density of gravitational waves per unit logarithmic interval of frequency in units of the present critical energy density, $\rho_{\rm c}$, that is needed to close the universe,

$$\rho_{\rm c} = \frac{3c^2 H_0^2}{8\pi G}. \tag{9}$$

Here, $H_0 = 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$ denotes the present Hubble expansion rate (Komatsu et al. 2011), c the speed of light, and G the gravitational constant.

2.2 Numerical results for solar g modes

In Fig. 1, we show numerical results for the intrinsic root-mean square (rms) surface velocities of quadrupolar solar g modes when excited by an SBGW; as an example, we consider here an SBGW produced by cosmic strings (Siegel & Roth 2011). The upper and lower curve correspond to adopting an optimistic scenario ($\Omega_{\rm GW} = 1\times10^5$ in the considered frequency range) and a rather pessimistic scenario ($\Omega_{\rm GW} = 1\times10^8$ in the considered frequency range), respectively. Furthermore, dotted lines indicate the results from setting $\Omega_{\rm GW} = 1\times10^6$ and $\Omega_{\rm GW} = 1\times10^7$. The model parameters that produce these scenarios are entirely consistent with all observational constraints on SBGWs to date. Consequently, depending on model parameters we find maximal surface velocities of 10^{-5}–$10^{-3}\,{\rm mm\,s^{-1}}$.

We note that theoretical estimates for intrinsic rms surface velocities of $l = 1, 2, 3$ solar g modes in the case of stochastic excitation by turbulent convection differ from each other by orders of magnitude. These order of magnitude differences are predominantly due to the choice of the assumed turbulent eddy-time correlation function. The maximum values for quadrupolar modes lie in the range 10^{-3}–$1\,{\rm mm\,s^{-1}}$ (Appourchaux et al. 2010; Belkacem et al. 2009). Consequently, present models for an SBGW are capable of exciting quadrupolar solar g modes up to maximum amplitudes that lie very close to (or possibly even within) the presently expected range from excitation by turbulent convection.

3 The inverse problem: constraining an SBGW

In this section, we raise the following question: given observational data for rms surface velocities of quadrupolar modes, and given that there is no imprint of an SBGW in the oscillation data, what is the upper bound on an SBGW at these mode frequencies? If both excitation mechanisms (turbulent convection and the SBGW) produce rms surface velocities of the same order of magnitude, one can make use of the fact that an SBGW only excites quadrupolar modes and observational data for other harmonic degrees (e.g., $l = 1, 3$) can be employed to disentangle the contributions.

From Eq. (5), we derive an upper bound on the SBGW at the oscillation frequencies of the quadrupolar modes employed,

$$H_0^2\,\Omega_{\rm GW}(\omega_N) < \frac{50}{\pi^2}\frac{I_N^2}{\Psi_N^2(R)}\frac{\eta_N}{\omega_N}\frac{1}{\chi_N^2}v_{N,{\rm s}}^2, \tag{10}$$

where $v_{N,{\rm s}}$ denote the intrinsic rms surface velocities and $\Psi_N^2(R)$ is a quantity that essentially measures the amplitude of the eigenfunctions at the stellar surface (cf. Siegel & Roth 2011 for details on this quantity). In deriving Eq. (10), visibility effects have not been accounted for (e.g., limb darkening, geometrical effects, and height in the atmosphere where the velocity field is observed); this will be discussed in a forthcoming paper (Siegel & Roth 2012). However, Eq. (10) is accurate in terms of an order-of-magnitude estimate, and this is what we are interested in for the time being.

In order to investigate the significance of such an asteroseismic bound on the SBGW, we apply this formalism to the aforementioned case of quadrupolar solar g modes. The damping rates (as in Sect. 2.2) are obtained from non-adiabatic oscillation computations (Belkacem et al. 2009), which can be reliably calculated for asymptotic g modes due to the fact that radiative damping is the dominant damping mechanism for g modes in the asymptotic regime.

Assuming approximately constant rms surface velocities $v_{N,{\rm s}} \approx 3\times10^{-3}\,{\rm mm\,s^{-1}}$ for asymptotic solar g modes, which corresponds to the calculations of Kumar, Quataert & Bahcall (1996) and Belkacem et al. (2009) in the case of a Gaussian eddy-time correlation function, we obtain the results depicted in Fig. 2. The power law for the upper limit on $\Omega_{\rm GW}(\omega)$ that is evident from Fig. 2 is due to the power-law

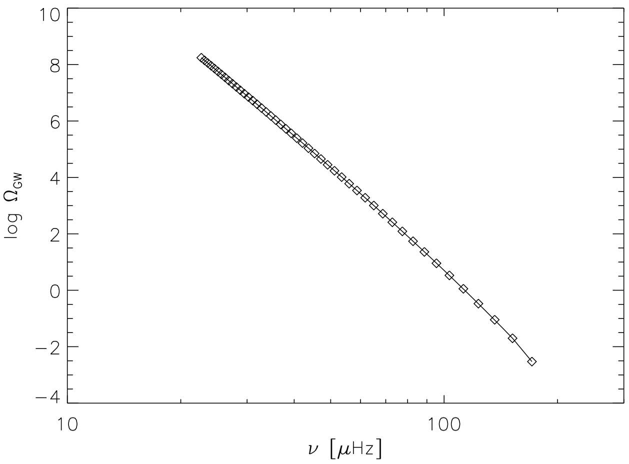

Fig. 2 Limits deduced from asymptotic, $l = 2$ solar g-mode amplitude estimates (diamonds indicate the limits from the individual modes) using the asteroseismic method reported in this paper (see the text for details).

behaviour of the damping rates for asymptotic solar g modes (Belkacem et al. 2009) and the power laws noticeable in the other quantities in Eq. (10), which also led to the power law for the rms surface velocities in Fig. 1. We find that the tightest upper limits can be deduced from low-order (high-frequency) g modes, with the most stringent limit being

$$\Omega_{\rm GW} \leq 3.0\times10^{-3} \quad \text{at } 0.17\,\text{mHz}. \tag{11}$$

We note that the upper bound given by Eq. (10) is very sensitive to even small changes in stellar properties and to the observed rms surface velocities $v_{N,\rm s}$. Assuming, for example, $v_{N,\rm s} \approx 1\,\text{mm}\,\text{s}^{-1}$ results in $\Omega_{\rm GW} \leq 329$ in Eq. (11).

4 Conclusions and prospects

In the present paper, a hydrodynamical model has been presented that describes excitation of linear stellar oscillations by an SBGW and it has been pointed out that current models for an SBGW are able to generate solar g-mode surface amplitudes very close to or comparable with values expected from excitation by turbulent convection.

A method has been presented that is able to directly constrain the energy density of an SBGW at mHz and μHz frequencies, given asteroseismic data on intrinsic rms surface velocities. This is a so far essentially unexplored frequency regime, where only indirect bounds on the cosmological component of the SBGW from CMB and Big Bang nucleosynthesis (BBN) data exist; however, the SBGW at these frequencies is likely dominated by the astrophysical component.

Comparing with limits at neighbouring frequencies, the deduced bound based on theoretical estimates for solar g-mode amplitudes would surpass the limits obtained from Doppler tracking of the Cassini spacecraft (10^{-6}–10^{-3} Hz; Armstrong et al. 2003), the limit from cross-correlation measurements between the Explorer and Nautilus cryogenic resonant bar detectors at 907.2 Hz (Astone et al. 1999), and the bound from the S1 LIGO science run around 100 Hz (Abbott et al. 2004) by orders of magnitude. Furthermore, it reaches sensitivities close to the CMB (Smith, Pierpaoli & Kamionkowski 2006), BBN (Cyburt et al. 2005), LIGO S3, S4 (Abbott et al. 2005, 2007), and the LIGO S5 (around 100 Hz; Abbott et al. 2009) bounds. It has thus been demonstrated that the method presented in this paper is potentially capable of competing with the sensitivity reached by gravitational wave experiments in other (adjacent) frequency bands. The upper bounds derived with this method are highly sensitive to local and global stellar properties and missions like *Kepler* and CoRoT offer a unique possibility to search for targets with an optimized set of stellar properties, in order to derive tight direct upper limits on an SBGW in the near future.

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