Fluctuating black hole horizons

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ABSTRACT: In this paper we treat the black hole horizon as a physical boundary to the spacetime and study its dynamics following from the Gibbons-Hawking-York boundary term. Using the Kerr black hole as an example we derive an effective action that describes, in the large wave number limit, a massless Klein-Gordon field living on the average location of the boundary. Complete solutions can be found in the small rotation limit of the black hole. The formulation suggests that the boundary can be treated in the same way as any other matter contributions. In particular, the angular momentum of the boundary matches exactly with that of the black hole, suggesting an interesting possibility that all charges (including the entropy) of the black hole are carried by the boundary. Using this as input, we derive predictions on the Planck scale properties of the boundary.

Contents

1   Introduction

By far our best understanding of black hole entropy has been based on the idea of holography [1–3]. In this framework one assumes that quantum gravity in a given black hole background has a dual description in terms of a field theory defined on the boundary of the space [4–7]. As far as black holes are concerned, all such dual descriptions either have been deduced in string theory [8] or have arisen from considerations of asymptotic symmetries [4, 9]. For more realistic black holes in General Relativity (GR), such dual descriptions are only known in bits and pieces [10–12] and a full understanding of the black hole entropy is still not available.

The idea of gauge/gravity duality is very profound and has had wide applications (see, e.g. [13–17]), but alternative methods to the black hole entropy should also be explored. In particular, even from the holography perspective one expects that both sides of a dual pair should be equally good (not considering the level of technical difficulty) in explaining the same physics. For a black hole, this means that one also expects to understand its entropy from the pure gravity side. Here a central problem is to identify the correct physical entity (entities) that is (are) responsible for the thermodynamical properties of the black hole.

In this respect, 't Hooft [18] has looked at point particles as possible carriers of the black hole charges. He noted that there should be a cut-off at a tiny distance away from outside the horizon, so that the density of states of the particles do not diverge. He then calculated the total energy and the entropy of the particles. In order for the entropy to match with that of the black hole, he noticed that the cutoff must be

$$\delta \sim \frac{\ell_p^2}{r_+} \,, \tag{1.1}$$

where $\ell_p = \sqrt{G_N} \approx 1.6 \times 10^{-35} m$ is the Planck length and r_+ is the radius of the black hole horizon. This cut off is referred to as the "brick wall". 't Hooft's calculation was

proposed as a toy model and there are obvious problems with it, including the fact that the final answer of the entropy depends on the number of available particle species living in the vicinity of the horizon. What's more, the physical origin of the "brick wall" is not entirely clear. An idea for addressing this later problem is recently proposed in [19], using an earlier idea from York [20]. York attempted to understand the black hole entropy by including back-reactions from the quantum fields to the black hole background, which leads to an oscillating metric [20].

In this paper, instead of looking at point particles, we want to study if extended objects can play any role in the statistical origin of black hole entropy. Extended objects like D-branes are well known in string theory and have played a crucial role in the first successful string calculation of black hole entropy [8]. Here we shall study in detail the possible connection between the dynamics of extended objects and the black hole entropy directly within the framework of GR.

For this purpose we need to look for brane-like objects which can be possibly related to the black hole thermodynamics. In fact, a candidate is readily available, which is nothing but the black hole horizon. The idea of viewing the black hole horizon as a membrane has been much explored in the membrane paradigm [21, 22].[1] The most direct reason for this possibility is that, if the horizon is a dynamical object, then its dynamics must be described by a field theory living on the world volume of the horizon. As a result, the entropy of the system must be proportional to the area of the horizon. Still, treating the black hole horizon as a physical entity may appear odd at the first glance. Let's discuss some further hint for this possibility.

Back in 1998 Carlip has shown that one can identify conformal symmetries on the (stretched) horizon, which can then be used to infer information about the entropy of the black hole [10]. This idea has been reinforced in recent years by the proposal of the Kerr/CFT correspondence, which showed how to identify the conformal symmetries by zooming in the near horizon region of an extremal Kerr black hole [11]. Later effort has further shown how conformal symmetries can be identified on the horizons of generic stationary and axisymmetric black holes in generic dimensions [25]–[30]. This suggests, in particular, that the quantum nature of a black hole might be captured by a dual conformal field theory (CFT) living on the horizon, and this should be a generic feature in all spacetime dimensions. If true, this can be the best explanation to the problem of "Universality" [31].

Another suggestive hint comes from the classic AdS_5/CFT_4 correspondence [5], where the boundary Super Yang-Mills theory is related to the dynamics of a stack of D-branes. By analogy, in the pure gravity case the dual field theory may also describe dynamics of extended objects like the branes. Since from the previous paragraph we know that the dual field theory of a black hole likely lives on the horizon, it is tempting to ask if the horizon itself is part of the objects described by the dual field theory. What's more, if this is true, we suggest that this could offer a way to obtain, instead of a full theory, an (or a partial contri-

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[1]The author thanks José Lemos for this point and for the related reference [23]. There has also been earlier effort in deriving boundary effective actions (see, e.g. [24]), as we will do in this paper. The author is indebted to the referee for this later point.

bution from the horizon to the) effective action of the dual field theory. The idea for doing this is based on the fact that the horizon acts as a boundary to the black hole spacetime.

After it was understood that the singularity at a black hole horizon can be removed by a coordinate transformation, it has been widely accepted that there is no true singularity on the horizon and one can pass through it without experiencing anything dramatic. This latter point, however, is being challenged recently [32, 33]. Despite this, everyone agrees that the horizon acts as a unidirectional membrane, i.e. a causal boundary of the spacetime. This is at the basis of trying to interpret the black hole entropy as an entanglement entropy (see, e.g. [34]).

With the conformal symmetries mentioned above, the horizon could be more than just a causal boundary. For a two dimensional statistical system, conformal symmetry usually arises at critical points of second order phase transitions. Since the conformal symmetries related to a black hole horizon are also those in two dimensions, this hints at the intriguing possibility that the horizon could also be a boundary separating two different phases of (the material that is making up) the spacetime. Inspired by this, we consider the possibility that the horizon is a *physical* boundary to the spacetime.

If treated as a brane with zero depth, a *physical* boundary contributes to the stress energy tensor with a delta function in the direction normal to the boundary, which then leads to a step function in the metric. This means that the metric inside the horizon is significantly different from that of a usual black hole. In the following, however, we will only need the part of the metric that is outside the horizon.

When there is a boundary to the spacetime, the Einstein-Hilbert action must be supplemented by the Gibbons-Hawking-York boundary term [35, 36] so that the variational principle can be well defined. In the presence of a physical boundary, one can view the boundary term as the contribution from the boundary to the total action. So it is reasonable to suggest that the dynamics of the boundary is governed by the boundary action.[2]

In the rest of the paper we study the dynamics of the horizon as predicted by the Gibbons-Hawking-York boundary term. Since the horizon is assumed to be a physical boundary to the spacetime, we will use the words "horizon" and "boundary" interchangeably.

In section 2, we collect all formulae related to the boundary action which we will need in later sections. In section 3, an effective action is derived from the Gibbons-Hawking-York boundary term in the background of a Kerr black hole. Instead of living precisely on the dynamical horizon, this effective action will be defined on the average location of the boundary. In section 4, all classical solutions to the effective action are found in the small rotation limit of the black hole. The system can then be quantized and the complete spectrum is found. In section 5, we study thermodynamical properties of the system. We show that the black hole angular momentum is fully accounted for by that of the boundary. This motivates us to assume that all charges of the black hole are carried by

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[2]If the bulk action is not Einstein-Hilbert, then the boundary term should be modified accordingly. If a black hole exists in this new theory, then the dynamics of the horizon should be governed by the new boundary action.

the horizon. With this assumption one can get predictions on the Planck scale properties of the boundary. The paper ends with a short summary in section 6.

2 Boundary action

Suppose that the boundary is defined by the function $B(x) = 0$, then the Gibbons-Hawking-York boundary term is given by

$$S_B = \frac{1}{16\pi\ell_p^2} \int_B (d^{n-1}x)_\mu N^\mu \sqrt{-g} \, K = \frac{1}{16\pi} \int_B d^{n-1}x\sqrt{-h} \, K \,, \tag{2.1}$$

where $(d^{n-1}x)_{\mu_1\cdots\mu_p} = \frac{1}{(n-p)!p!}\varepsilon_{\mu_1\cdots\mu_p\nu_1\cdots\nu_{n-p}}dx^{\nu_1} \wedge \cdots \wedge dx^{\nu_{n-p}}$, $|\varepsilon| = 1$, n is the dimension of the bulk spacetime, $N_\mu = \partial_\mu B/|\partial B|$, $|\partial B| = \sqrt{g^{\varrho\sigma}\partial_\varrho B\partial_\sigma B}$, g is the bulk metric, h is the induced metric on the boundary, and K is the extrinsic curvature

$$K = g^{\mu\nu}K_{\mu\nu}\,, \quad K_{\mu\nu} = \nabla_\mu N_\nu + \nabla_\nu N_\mu\,. \tag{2.2}$$

We will set $\ell_p = 1$ for most part of the calculation, but will restore it when needed. It is always possible to choose the bulk coordinates $x^\mu \in \{r, x^i \, (i = 1, \cdots, n-1)\}$ as such that the boundary function is of the form $B = r - f(x^i)$. One can then explicitly check that

$$\int_B (d^{n-1}x)_\mu = \int_B d^{n-1}x N_\mu|\partial B|\,, \quad \sqrt{-h} = |\partial B|\sqrt{-g}\,, \tag{2.3}$$

where the coordinates on $B = 0$ are taken to be x^i, and we have used the definition $h_{ij}dx^idx^j = g_{\mu\nu}dx^\mu dx^\nu|_{B=0}$. The boundary action (2.1) is determined up to a constant term, but which will not be relevant for our following calculations.

The total action is

$$S_{\text{tot}} = \frac{1}{16\pi} \int d^nx\sqrt{-g}\,(R - 2\Lambda) + S_B\,, \tag{2.4}$$

where R is the Ricci scalar in the bulk and Λ is the bulk cosmological constant, which can be zero. A variation of the bulk metric leads to

$$\delta S_{\text{tot}} = \frac{1}{16\pi} \int d^nx\sqrt{-g}\,\delta g^{\mu\nu}\left(R_{\mu\nu} - \frac{R - 2\Lambda}{2}g_{\mu\nu}\right) + \delta S_B'$$

$$\delta S_B' = \frac{1}{16\pi} \int_B d^{n-1}x\sqrt{-h}\left[\widetilde\nabla_\alpha(N_\beta\delta g^{\alpha\beta}) + \widetilde\nabla_\beta(N_\alpha\delta g^{\alpha\beta}) - \delta g^{\alpha\beta}\frac{K}{2}g_{\alpha\beta} - \mathcal{P}_{\alpha\beta}N^\mu\nabla_\mu\delta g^{\alpha\beta}\right]$$

$$+ \frac{1}{16\pi} \int_B d^{n-1}x\sqrt{-h}\,N_\mu\left(g_{\alpha\beta}\nabla^\mu\delta g^{\alpha\beta} - \nabla_\nu\delta g^{\mu\nu}\right)\,, \tag{2.5}$$

where $\mathcal{P}_{\alpha\beta} = g_{\alpha\beta} - N_\alpha N_\beta$ is the projector onto the boundary and $\widetilde\nabla_\alpha = \mathcal{P}_\alpha^\beta\nabla_\beta$. The first line of $\delta S_B'$ comes from δS_B, while the second line comes from varying the bulk action. Here one observes a big difference between the physics of a boundary and that of an isolated system, i.e. the stress energy tensor of a boundary also receives contributions from the bulk action. This is crucial to a consistent calculation in the following.

One can combine the two lines of (2.5) and find

$$\delta S'_B = \frac{1}{16\pi} \int_B d^{n-1}x \sqrt{-h} \left[\frac{\delta g^{\alpha\beta}}{2} \left(\widetilde{\nabla}_\alpha N_\beta + \widetilde{\nabla}_\beta N_\alpha - K\mathcal{P}_{\alpha\beta} \right) + \widetilde{\nabla}_\lambda (N_\beta \mathcal{P}^\lambda_\alpha \delta g^{\alpha\beta}) \right]$$

$$= \frac{1}{16\pi} \int_B d^{n-1}x \sqrt{-h} \frac{\delta g^{\alpha\beta}}{2} \mathcal{T}_{\alpha\beta} \, ,$$

$$\mathcal{T}_{\alpha\beta} = \widetilde{\nabla}_\alpha N_\beta + \widetilde{\nabla}_\beta N_\alpha - K\mathcal{P}_{\alpha\beta} + 2N_\beta \left(\frac{1}{2}\mathcal{P}^{\rho\sigma}\widetilde{\partial}_\alpha \mathcal{P}_{\rho\sigma} - \widetilde{\partial}_\alpha \ln\sqrt{-h} \right) , \tag{2.6}$$

where we have used $\int_B d^{n-1}x \widetilde{\partial}_\lambda f^\lambda = 0$ and[3]

$$\int_B d^{n-1}x \sqrt{-h} \, \widetilde{\nabla}_\lambda f^\lambda = \int_B dx^{n-1}x \sqrt{-h} \, f^\lambda \left(\frac{1}{2}\mathcal{P}^{\rho\sigma}\partial_\lambda \mathcal{P}_{\rho\sigma} - \widetilde{\partial}_\lambda \ln\sqrt{-h} \right) . \tag{2.7}$$

The last term in (2.6) can be shown to vanish and $\mathcal{T}_{\alpha\beta}$ becomes the quasi-local stress tensor by Brown and York [37]. The stress energy tensor of the boundary can be found as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S'_B}{\delta g^{\mu\nu}} = -\frac{\delta(B)|\partial B|}{16\pi}\mathcal{T}_{\mu\nu} \, , \tag{2.8}$$

where we have used

$$\int_B d^{n-1}x \sqrt{-h} = \int d^n x \delta(B) \sqrt{-h} = \int d^n x \sqrt{-g} \, \delta(B)|\partial B| \, . \tag{2.9}$$

The minus sign in the definition of $T_{\mu\nu}$ means that the boundary is treated as a matter contribution. As mentioned before, there is also a delta function in $T_{\mu\nu}$.

In a stationary and axisymmetric background, given the canonical time t and azimuthal angle ϕ, the energy and the angular momentum of the boundary are

$$H = -\frac{1}{16\pi} \int_B d^{n-1}x \sqrt{-h} \, \mathcal{T}^t_t \, , \quad J_\phi = -\frac{1}{16\pi} \int_B d^{n-1}x \sqrt{-h} \, \mathcal{T}^t_\phi \, . \tag{2.10}$$

3   Effective action in the Kerr background

Let's now study (2.1) in the background of a Kerr black hole. The metric is

$$ds^2 = f \left(\frac{dr^2}{\Delta} - \frac{\Delta}{v^2}dt^2 \right) + \frac{fdx^2}{1-x^2} + \frac{v^2(1-x^2)}{f}(d\phi - wdt)^2 \, ,$$

$$\Delta = (r-r_+)(r-r_-) \, , \quad w = \frac{r}{v^2}\sqrt{r_+ r_-} \, (r_+ + r_-) \, ,$$

$$f = r^2 + r_+ r_- x^2 \, , \qquad v^2 = (r^2 + r_+ r_-)^2 - \Delta r_+ r_-(1-x^2) \, , \tag{3.1}$$

where $r_\pm = M \pm \sqrt{M^2 - J^2/M^2}$ with M and J being the mass and angular momentum of the black hole, respectively. The outer (inner) horizon of the black hole is given by r_+ (r_-). The black hole temperature, angular velocity and entropy are

$$T = \frac{r_+ - r_-}{4\pi r_+(r_+ + r_-)} \, , \quad \Omega = \frac{\sqrt{r_- r_+}}{r_+(r_+ + r_-)} \, , \quad S = \pi r_+(r_+ + r_-) \, . \tag{3.2}$$

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[3]I thank Stefan Theisen for help in simplifying the result.

As the radius $r \to \infty$, the metric (3.1) approaches that of a Minkowski spacetime, where t is the time, $x \in [-1, 1]$ and $\phi = \phi + 2\pi$. We will refer to these as the canonical coordinates. The determinant of the metric (3.1) is $\sqrt{-g} = f$. In the background of (3.1), the extrinsic curvature for a surface with normal vector N_μ is

$$K = \frac{2}{f}\left\{\partial_r(\Delta N_r) + \partial_x[(1-x^2)N_x] + \frac{f^2\partial_\phi N_\phi}{v^2(1-x^2)} - \frac{v^2}{\Delta}(\partial_t + w\partial_\phi)(N_t + wN_\phi)\right\}. \quad (3.3)$$

As will be justified later, the horizon described by (2.1) fluctuates around an average location in the background of (3.1). The fluctuating horizon carries part of the black hole energy,[4] and this energy is distributed to each excitation of the boundary. So each excitation can be viewed as living in a background that has an energy slightly smaller than that of the full black hole. In canonical coordinates, this effectively means that the physically fluctuating horizon always lives *outside* the coordinate singularity of the background metric. For later convenience, let's call the horizon of the background metric, such as the r_+ in (3.1), the "background horizon", while the physically fluctuating horizon simply the "horizon" or the "boundary", interchangeably.

With the fixed background (3.1), one can then assume that the boundary is centered at $r_0 = r_+(1 + \epsilon)$ and is fluctuating with an amplitude $r_+\epsilon|\Phi(x, \phi, t)|$, where $\epsilon > 0$ is a small parameter. The configuration function of the fluctuating horizon is then given by

$$B = r - r_+\left\{1 + \epsilon\left[1 + \Phi(x, \phi, t)\right]\right\} = 0. \quad (3.4)$$

The unit normal vector is

$$N_r = \frac{1}{|\partial B|}, \quad N_i = -\epsilon r_+\frac{\partial_i\Phi}{|\partial B|}, \quad i = x, \phi, t,$$

$$|\partial B|^2 = \frac{\Delta}{f} + (\epsilon r_+)^2\left\{\frac{1-x^2}{f}(\partial_x\Phi)^2 + \frac{f(\partial_\phi\Phi)^2}{v^2(1-x^2)}\right.$$

$$\left. -\frac{v^2}{f\Delta}[(\partial_t + w\partial_\phi)\Phi]^2\right\}. \quad (3.5)$$

The induced metric on the boundary is

$$ds_H^2 = -\frac{f\Delta}{v^2}dt^2 + \frac{fdx^2}{1-x^2} + \frac{v^2(1-x^2)}{f}d\phi^2$$

$$+(\epsilon r_+)^2\frac{f}{\Delta}\left(\partial_x\Phi dx + \partial_\phi\Phi d\phi + \partial_t\Phi dt\right)^2, \quad (3.6)$$

which has the determinant $\sqrt{-h} = f|\partial B| = |\partial B|\sqrt{-g}$, as is expected.

By definition, one plugs (3.4)–(3.6) into (2.1) and then let $r = r_+\{1+\epsilon[1+\Phi(x, \phi, t)]\}$ to obtain the action on the boundary. As we will see later, ϵ is an extremely small parameter. So one can firstly expand the action (2.1) around ϵ and then look at the weak field limit $|\Phi| \to 0$. After doing this for (2.1), however, we find that

$$S_B = \frac{r_+ - r_-}{4} - \frac{\mu}{2}\int_B dxd\phi dt\left[\mathcal{L}_0 + \frac{2r_+(r_+ + r_-)^2\Phi(\partial_t + \Omega\partial_\phi)^2\Phi}{r_+ - r_-} + \mathcal{O}(\Phi^3)\right] + \mathcal{O}(\sqrt{\epsilon}),$$

$$\mathcal{L}_0 = \frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}[(\partial_t + \Omega\partial_\phi)\Phi]^2 - (1-x^2)(\sqrt{\epsilon}\,\partial_x\Phi)^2 - \frac{(r_+ + r_-x^2)^2(\sqrt{\epsilon}\,\partial_\phi\Phi)^2}{(r_+ + r_-)^2(1-x^2)}, \quad (3.7)$$

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[4]We will suggest later that the boundary actually carries all the energy of the black hole.

where $\mu = r_+/(8\pi)$ and we have thrown away all boundary terms. The minus sign in front of the $\mathcal{L}_0$ integral is consistent with that the boundary should be treated as a matter contribution, as was assumed in (2.8). The derivatives ∂_x and ∂_ϕ only appear in $\mathcal{L}_0$ through the combinations $\sqrt{\epsilon}\,\partial_x\Phi$ and $\sqrt{\epsilon}\,\partial_\phi\Phi$.[5] Given the smallness of ϵ, this means that only the large wave number modes can have significant contributions.

The $(\partial_t + \Omega\partial_\phi)$-terms in S_B consist a total derivative and drop out of the integral, which means that the action S_B does not predict any interesting dynamics of the boundary. Normally this would be the end of the story. But then we notice an interesting feature. That is, instead of $r = r_+\{1 + \epsilon[1 + \Phi(x,\phi,t)]\}$, if we look at the average location of the brane, $r = r_0 = r_+(1 + \epsilon)$, we indeed find well defined dynamics,

$$S_H = -\frac{r_+ - r_-}{4} + \frac{\mu}{2}\int_H dx d\phi dt\Big[\mathcal{L}_0 + \mathcal{O}(\Phi^3)\Big] + \mathcal{O}(\sqrt{\epsilon})\,, \qquad (3.8)$$

where the subscript H means the integral is taken over the surface $r_0 = r_+(1 + \epsilon)$. We have included an extra minus sign in the boundary action so that S_H is in the proper form of a matter contribution.

One may ask if there is any physical reason to believe in (3.8). A possible explanation comes from the approximation that we have made when deriving (3.7) and (3.8). When the boundary fluctuates, the metric near the boundary will fluctuate accordingly. But for technical reasons we have to rely on the average field approximation and use the average metric (3.1), instead of the real time metric, in the calculations. Although a concrete proof is hard to get at the moment, this approximation suggests that it could make more sense that we also derive the effective action at the average location of the boundary, just as in (3.8). In any case, this is still a weak link in the whole construction that we wish to improve in future works.

For the moment, we make the extra assumption that (3.8) is the correct effective action for the boundary. (Hopefully we can eliminate the need for this extra assumption in the near future!) And we will show in the following that this action does lead to physically reasonable results.

For the charges (2.10) the situation is similar. There is no dynamics at $r = r_+[1 + \epsilon(1 + \Phi)]$, while at $r = r_0$ we find

$$H = -\Omega J_\phi + \frac{\mu}{2}\int_H dx d\phi dt\bigg\{\frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}[(\partial_t + \Omega\,\partial_\phi)\Phi]^2 - \mathcal{L}_0\bigg\}\,, \qquad (3.9)$$

$$J_\phi = \frac{\mu}{2}\int_H dx d\phi dt\bigg\{\frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}(\partial_t + \Omega\,\partial_\phi)\Phi\,\partial_\phi\Phi$$
$$+ \Omega\frac{(r_+ + r_-)^2(1 - x^2)}{(r_+ + r_- x^2)^2}\Big[3r_+^2 - r_-^2 x^2 + r_+ r_-(1 + x^2)\Big]\bigg\}$$
$$= J + \frac{\mu}{2}\int_H dx d\phi dt\bigg[\frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}(\partial_t + \Omega\,\partial_\phi)\Phi\,\partial_\phi\Phi\bigg]\,, \qquad (3.10)$$

where $J = \frac{1}{2}(r_+ + r_-)\sqrt{r_+ r_-}$ is the angular momentum of the black hole.

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[5] Note one can always let $\partial_t + \Omega\partial_\phi \to \partial_t$ by a coordinate redefinition $\phi \to \phi + \Omega t$.

Not considering the constant terms, one can infer from (3.10) that the canonical momentum conjugating to Φ is

$$\Pi_\Phi = \mu \frac{r_+(r_+ + r_-)^2}{r_+ - r_-}(\partial_t + \Omega\,\partial_\phi)\Phi\,. \qquad (3.11)$$

From (3.9) one can also read off the Hamiltonian density

$$\begin{aligned}
\mathcal{H} &= -\Omega\Pi_\Phi\partial_\phi\Phi + \frac{\mu}{2}\left\{\frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}[(\partial_t + \Omega\,\partial_\phi)\Phi]^2 - \mathcal{L}_0\right\} \\
&= \frac{\mu}{2}\left\{\frac{2r_+(r_+ + r_-)^2}{r_+ - r_-}(\partial_t + \Omega\,\partial_\phi)\Phi\partial_t\Phi - \mathcal{L}_0\right\}, \qquad (3.12)
\end{aligned}$$

which is in the familiar form $\mathcal{H} = \Pi_\Phi\partial_t\Phi - \frac{1}{2}\mu\mathcal{L}_0$.

For an isolated system, we expect $\Pi_\Phi = \frac{\delta S_H}{\delta(\partial_t\Phi)}$. For a boundary, however, the contribution from the bulk action modifies the relation. In the present case, we actually find

$$\Pi_\Phi = \frac{1}{2}\frac{\delta S_H}{\delta(\partial_t\Phi)}\,. \qquad (3.13)$$

This unusual relation will not cause any problem for our calculations in the following.

4 Classical solutions and quantization

Let's now look at the solutions of (3.8). Instead of considering the general case, lets focus on the small rotation limit ($\Omega \to 0 \Rightarrow \rho \equiv r_-/r_+ \to 0$). In this case,

$$\begin{aligned}
\mathcal{L}_0 = \ &2r_+^2(1 + 3\rho)[(\partial_t + \Omega\,\partial_\phi)\Phi]^2 - (1 - x^2)(\sqrt{\epsilon}\,\partial_x\Phi)^2 \\
&-\left(\frac{1}{1 - x^2} - 2\rho\right)(\sqrt{\epsilon}\,\partial_\phi\Phi)^2\,, \qquad (4.1)
\end{aligned}$$

where we have preserved terms up to the subleading order in $\rho \to 0$. The Hamiltonian and the angular momentum are

$$H = -\Omega J_\phi + \frac{\mu}{2}\int_H dxd\phi\left\{(1 - x^2)(\sqrt{\epsilon}\,\partial_x\Phi)^2 + \left(\frac{1}{1 - x^2} - 2\rho\right)(\sqrt{\epsilon}\,\partial_\phi\Phi)^2\right\}, \quad (4.2)$$

$$J_\phi = J + \int_H dxd\phi\,\mu\,r_+^2(1 + 3\rho)(\partial_t + \Omega\,\partial_\phi)\Phi\,\partial_\phi\Phi\,. \qquad (4.3)$$

With $\Phi = f_\ell^m(x)\exp\{i[m(\phi - \Omega t) - \mathcal{E}_{\ell,m}t]\}$, the equation of motion from (3.8) is

$$2r_+^2(1 + 3\rho)\mathcal{E}_{\ell,m}^2 f_\ell^m + \epsilon\,\partial_x\Big[(1 - x^2)\partial_x f_\ell^m\Big] - \left(\frac{1}{1 - x^2} - 2\rho\right)\epsilon\,m^2 f_\ell^m = 0\,. \qquad (4.4)$$

This equation can be solved by the associated Legendre polynomials $f_\ell^m = P_\ell^m(x)$, with

$$\begin{aligned}
\mathcal{E}_{\ell,m} &= \sqrt{\epsilon\frac{\ell(\ell+1) - 2m^2\rho}{2r_+^2(1 + 3\rho)}} \approx \frac{\ell}{r_+}\sqrt{\frac{\epsilon}{2}}\left[1 - \rho\left(\frac{3}{2} + \frac{m^2}{\ell^2}\right)\right], \\
&\ell = 0, 1, \cdots, \infty\,, \quad m = -\ell, \cdots, \ell\,. \qquad (4.5)
\end{aligned}$$

(Here and in the following, when making approximations, we always preserve terms up to the subleading order in $\rho \to 0$ and up to the leading order in $|m| \sim \ell \to \infty$.) The full solution can be expanded as $\Phi = \sum_{\ell,m} \Phi_\ell^m$, where

$$\Phi_\ell^m = N_\ell^m P_\ell^m(x) \left\{ a_{\ell,m} e^{i[m(\phi-\Omega t) - \mathcal{E}_{\ell,m} t]} + a_{\ell,m}^\dagger e^{-i[m(\phi-\Omega t) - \mathcal{E}_{\ell,m} t]} \right\}, \qquad (4.6)$$

$$N_\ell^m = \sqrt{\frac{1}{2\mu\, r_+^2\,(1+3\rho)\mathcal{E}_{\ell,m}} \frac{2\ell+1}{4\pi} \frac{(\ell-m)!}{(\ell+m)!}}\,. \qquad (4.7)$$

In quantization, one promotes $a_{\ell,m}$ and $a_{\ell,m}^\dagger$ to operators $\hat{a}_{\ell,m}$ and $\hat{a}_{\ell,m}^\dagger$, yielding $\hat{\Phi}$ and $\hat{\Pi}_\Phi$. Then one imposes the following equal time commutation relations,

$$[\hat{\Phi}(x,\phi,t), \hat{\Pi}_\Phi(x',\phi',t)] = i\delta(x-x')\delta(\phi-\phi')\,,$$
$$[\hat{\Pi}_\Phi(x,\phi,t), \hat{\Pi}_\Phi(x',\phi',t)] = [\hat{\Phi}(x,\phi,t), \hat{\Phi}(x',\phi',t)] = 0\,, \qquad (4.8)$$

which, with (4.6) and (4.7), lead to

$$[\hat{a}_{\ell,m}\,, \hat{a}_{p,q}^\dagger] = \delta_{\ell,p}\,\delta_{m,q}\,, \quad [\hat{a}_{\ell,m}\,, \hat{a}_{p,q}] = [\hat{a}_{\ell,m}^\dagger\,, \hat{a}_{p,q}^\dagger] = 0\,. \qquad (4.9)$$

As usual, $\hat{a}_{\ell,m}$ annihilates the vacuum state in the Fock space, $\hat{a}_{\ell,m}|0\rangle = 0$, and $\hat{a}_{\ell,m}^\dagger$ generates multiparticle states, $|N_{\ell,m}\rangle \sim (\hat{a}_{\ell,m}^\dagger)^{N_{\ell,m}}|0\rangle$, which have the following properties

$$\hat{N}_{\ell,m}|N_{\ell',m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}N_{\ell,m}|N_{\ell,m}\rangle\,, \quad \langle N_{\ell,m}|N_{\ell',m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}\,. \qquad (4.10)$$

Here $\hat{N}_{\ell,m} = \hat{a}_{\ell,m}^\dagger \hat{a}_{\ell,m}$, and $N_{\ell,m}$ is a non-negative integer standing for the number of corresponding particles. The Hamiltonian (4.2) and the angular momentum (4.3) become

$$\hat{H} = -\Omega \hat{J}_\phi + \sum_{\ell,m} \mathcal{E}_{\ell,m} \left(\hat{N}_{\ell,m} + \frac{1}{2}\hat{N}_0 \right)$$
$$= \left(-\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} \right) \hat{N}_0 + \sum_{\ell,m} \mathcal{E}'_{\ell,m} \hat{N}_{\ell,m}\,, \qquad (4.11)$$

$$\hat{J}_\phi = J\hat{N}_0 - \sum_{\ell,m} m\hat{N}_{\ell,m}\,, \qquad (4.12)$$

where $\mathcal{E}'_{\ell,m} = \mathcal{E}_{\ell,m} + m\Omega$, and $\hat{N}_0$ is the number operator for the vacuum state,

$$\hat{N}_0|0\rangle = |0\rangle\,, \quad \hat{N}_0|N_{\ell,m}\rangle = 0\,. \qquad (4.13)$$

We have made the presence of $\hat{N}_0$ explicit so that both (4.12) and (4.11) are well defined operator equations. The fact that $\hat{H}$ and $\hat{J}_\phi$ have the expected structures supports the result for the stress energy tensor (2.8) and the result for the canonical momentum (3.11).

When there is gravity, we expect every piece of the Hamiltonian to contribute to the total energy. That is why we keep both the constant term and the terms of the zero point energy in (4.11). In order for the contribution from the zero point energy to be finite, there must be a cutoff on the physically available modes. As mentioned before, the existence

of two dimensional conformal symmetries on the horizon hints at the possibility that the horizon is a boundary separating two different phases of the spacetime. This can be taken as indicating the existence of substructures of the spacetime. In this case, the presence of a cutoff (say N_c) is very natural, which is directly related to the minimal lattice spacing (say a) of the substructures along the direction of the boundary,

$$N_c = \ell_{\max} = m_{\max} \approx \frac{2\pi r_+}{a} \,. \tag{4.14}$$

With the cutoff, the contribution from the zero point energy is

$$M_0 = \sum_{\ell=0}^{N_c} \sum_{m=-\ell}^{\ell} \frac{\mathcal{E}_{\ell,m}}{2} \approx \frac{N_c^3}{3r_+} \sqrt{\frac{\epsilon}{2}} \left(1 - \frac{11\rho}{6}\right). \tag{4.15}$$

5 Statistics

The thermal state of our scalar system with temperature T and angular velocity Ω is

$$
\begin{aligned}
|\Psi\rangle &= |0\rangle + \sum_{\ell,m} \sum_{N_{\ell,m}} \left(\frac{e^{-N_{\ell,m}(\beta\mathcal{E}'_{\ell,m}+\alpha m)}}{\Xi_{\ell,m}}\right)^{1/2} |N_{\ell,m}\rangle \\
&= |0\rangle + \sum_{\ell,m} \sum_{N_{\ell,m}} \left(\frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}}\right)^{1/2} |N_{\ell,m}\rangle \,,
\end{aligned}
\tag{5.1}
$$

where $\Xi_{\ell,m} = \sum_{N_{\ell,m}} e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}} = 1/(1 - e^{-\beta\mathcal{E}_{\ell,m}})$, $\beta = 1/T$ and $\alpha = -\beta\Omega$. (The minus sign in α is due to the fact that $\beta\Omega$ is a chemical potential.) By definition, the thermal sate (5.1) is space and time independent, and it assigns the expected occupancy number to each particle (including the vacuum),

$$\langle\Psi|\hat{N}_0|\Psi\rangle = 1 \,, \quad \langle\Psi|\hat{N}_{\ell,m}|\Psi\rangle = \sum_{N_{\ell,m}} \frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}} N_{\ell,m} \,. \tag{5.2}$$

As shown in (4.5), $\mathcal{E}_{\ell,m}$ is an even function in m. So "particles" with $m > 0$ and $m < 0$ are evenly excited. One can then derive from (4.11), (4.12) and (5.2) that

$$
\begin{aligned}
E = \langle\Psi|\hat{H}|\Psi\rangle &= -\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} + \sum_{\ell,m} \sum_{N_{\ell,m}} \mathcal{E}'_{\ell,m} N_{\ell,m} \frac{e^{-N_{\ell,m}\beta\mathcal{E}_{\ell,m}}}{\Xi_{\ell,m}} \\
&= -\Omega J + \sum_{\ell,m} \frac{\mathcal{E}_{\ell,m}}{2} - \partial_\beta \sum_{\ell,m} \ln \Xi_{\ell,m} \,, \\
J_\phi = \langle\Psi|\hat{J}_\phi|\Psi\rangle &= J \,,
\end{aligned}
\tag{5.3}
$$

where the second line for E is possible because the terms linear in m cancel among themselves in $\mathcal{E}'_{\ell,m}$. The result for J_ϕ suggests that all the angular momentum of the black hole is carried by the boundary, i.e., the horizon.

Now the key quantity to calculate is the the partition function,

$$\ln \Xi = \sum_{\ell,m} \ln \Xi_{\ell,m} = -\sum_{\ell=0}^{N_c} \sum_{m=-\ell}^{\ell} \ln \left(1 - e^{-\beta \mathcal{E}_{\ell,m}} \right). \tag{5.4}$$

An explicit result is possible in the small rotation limit. In this limit the black hole thermodynamical quantities can be expanded as

$$M \approx \frac{r_+}{2}(1+\rho), \quad J \approx \frac{r_+^2}{2}\sqrt{\rho}\,(1+\rho), \quad \Omega \approx \frac{\sqrt{\rho}}{r_+}(1-\rho), \quad T \approx \frac{1-2\rho}{4\pi r_+}, \quad S \approx \pi r_+^2(1+\rho). \tag{5.5}$$

We firstly expand (5.4) around $\rho \to 0$, assuming that β is an unknown constant. Then we sum over m and replace $\sum_{\ell=0}^{N_c}$ by an integral. The result is (note $y = kn$ and $k = \frac{\beta}{r_+}\sqrt{\frac{\epsilon}{2}}$)

$$\ln \Xi \approx -\frac{2}{k^2} \int_0^{kN_c} dy f_1(y) = -\frac{2}{k^2}\left[f_2(kN_c) - f_2(0) \right],$$

$$f_1(y) = f_2'(y) = y\ln(1-e^{-y}) - \frac{11\rho y^2}{6(e^y-1)}, \quad f_1(0) = 0,$$

$$f_2(y) = y Li_2(e^{-y}) + Li_3(e^{-y}) + \frac{11\rho}{3}f_3(y), \quad f_2(0) = \zeta(3)\left(1 + \frac{11\rho}{3}\right),$$

$$f_3(y) = y Li_2(e^{-y}) + Li_3(e^{-y}) - \frac{y^2}{2}\ln(1-e^{-y}), \quad f_3(0) = \zeta(3), \tag{5.6}$$

where $Li_s(z)$ is the polylogarithm. The total energy and the entropy are

$$E = -\Omega J + M_0 - \partial_\beta \ln \Xi$$

$$\approx -\frac{r_-}{2} + \frac{N_c^3}{3r_+}\sqrt{\frac{\epsilon}{2}}\left(1 - \frac{11\rho}{6}\right) + \frac{k}{\beta}\left\{ \frac{2}{k^2}f_1(kN_c)N_c - \frac{4}{b^3}\left[f_2(kN_c) - f_2(0)\right] \right\}$$

$$= \frac{\ell_p^2}{4\pi^3 r_+^2 \epsilon_0}\left[\zeta(3) - f_3(c_0) + \frac{c_0^3}{12} + \mathcal{O}(\rho) \right] M, \tag{5.7}$$

$$S' = (1 - \beta\partial_\beta)\ln \Xi$$

$$\approx -\frac{2}{k^2}\left[f_2(kN_c) - f_2(0)\right] + k\left\{ \frac{2}{k^2}f_1(kN_c)N_c - \frac{4}{k^3}\left[f_2(kN_c) - f_2(0)\right] \right\}$$

$$= \frac{3\ell_p^2}{4\pi^3 r_+^2 \epsilon_0}\left[\zeta(3) - f_3(c_0) - \frac{c_0^2}{6}\ln(1-e^{-c_0}) + \mathcal{O}(\rho) \right] S, \tag{5.8}$$

where we have let $\epsilon = \epsilon_0 + \epsilon_1\rho + \mathcal{O}(\rho^2)$, $N_c = N_0 + N_1\rho + \mathcal{O}(\rho^2)$, and $c_0 = 2\pi\sqrt{2\epsilon_0}\,N_0$. We have also written explicitly the Planck length ℓ_p so that the ratios $\frac{E}{M}$ and $\frac{S'}{S}$ are obviously dimensionless.

In our setup, the parameter ϵ and the cutoff N_c are a priori not known. From (5.3) we see that the boundary carries all the angular momentum of the black hole. This hints at an interesting possibility that all other charges of the black hole are also carried by the horizon. We use this to get an idea on what the values of N_c and ϵ might be.

From the mass and the entropy there are two equations, $E = M$ and $S' = S$. With two free parameters a solution is then possible.[6] This can be done order by order in the

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[6]This point is not so trivial as it looks. For example, if we do not include the contribution from the zero point energy, it is then NOT possible to satisfy both $E = M$ and $S' = S$ simultaneously.

expansion of the small rotation limit. Up to the subleading order, we find

$$\sqrt{\epsilon_0} \approx \frac{\ell_p}{7.27 r_+}\,, \quad N_0 \approx \frac{2\pi r_+}{2.84 \ell_p}\,, \quad \frac{\epsilon_1}{\epsilon_0} \approx 0.22\,, \quad \frac{N_1}{N_0} \approx 1.61\,. \tag{5.9}$$

A few comments are in order:

- Firstly, the putative lattice spacing introduced in (4.14) is

$$a \approx 2.84 \ell_p\,, \tag{5.10}$$

  which is independent of the properties of the black hole.

- Secondly, r_0 differs from the background horizon r_+ by

$$\delta \approx r_+ \epsilon_0 \sim \frac{\ell_p^2}{r_+}\,, \tag{5.11}$$

  which is qualitative the same as (1.1). From (3.1), the physical distance between r_0 and r_+ is

$$\delta' \approx r_+ \sqrt{\epsilon_0} \approx \frac{\ell_p}{7.27}\,, \tag{5.12}$$

  which is also independent of the properties of the black hole.

- Thirdly, the energy at the cutoff is approximately

$$\mathcal{E}_{N_0,m} \approx \frac{N_0}{r_+} \sqrt{\frac{\epsilon_0}{2}} \approx \frac{0.22}{r_+}\,. \tag{5.13}$$

Since the cutoff energy (5.13) is calculated by using the canonical time t, it is natural to interpret it as the energy measured by an observer at the spatial infinity. Translating back to the boundary, this means that the cutoff energy is

$$\mathcal{E}'_{N_0,m} \approx \frac{\mathcal{E}_{N_0,m}}{\sqrt{\epsilon_0}} \sim \frac{1}{\ell_p}\,, \tag{5.14}$$

which is at the Planck scale. This result fits well with our naive expectations and it also, interestingly, suggests the presence of a true firewall.

6 Summary

Apart from some very special cases (see, e.g. [38]), most of our understanding of black hole entropy comes without an explicit knowledge of the physical entities that actually carry the thermodynamical properties of the black hole. In view of the fact that the horizon acts as a (at least, causal) boundary to the spacetime, we ask what happens if it is in fact a physical and dynamical boundary to the spacetime, and then we study its contribution to the black hole thermodynamics.

The dynamics of a physical boundary is naturally governed by the boundary action, which can be identified by requiring that the variational principle is well defined. If the bulk

theory is Einstein-Hilbert, then the boundary action is given by the Gibbons-Hawking-York term. Using the Kerr black hole as an example, we find that the boundary action on the average location of the physical horizon effectively describes a massless Klein-Gordon field. Quantum mechanically, this inevitably leads to fluctuations of the boundary. We note that only large wave number modes have significant contributions to the effective action. The full spectrum of the system can be found in the small rotation limit of the black hole.

We then look at the contribution of the boundary to the black hole thermodynamics. It turns out that the angular momentum of the boundary matches exactly with that of the black hole. It is then tempting to suggest that all charges of the black hole are carried by the horizon. We use this as an assumption to fix the two unknown parameters in our model. As a result, the lattice spacing of the putative substructures of spacetime along the boundary is found to be (5.10), the physical distance from the real boundary to the background horizon r_+ is found to be (5.12). We also determined the cutoff energy of the excitations in (5.13).

Phenomenologically, our simple model is already capable of a detailed explanation of the statistical origin of the black hole entropy. At the deeper level, however, the model relies on two unusual assumptions which require further investigation. Let's finish by listing them:

- Firstly, our model assumes that the black hole horizon is a *physical* boundary to the spacetime. If there is indeed substructures to the spacetime, then the existence of a boundary means that the substructures are in two different phases across the horizon.

  Although it is widely accepted that no one can report to us any information from behind the horizon of a black hole, people also often assume that one can pass through the horizon unimpeded (see, however, [32, 33]). Our assumption implies that the spacetime behind the horizon is significantly different from what is predicted by the usual black hole metric.

- Secondly, the zero point energy of the quantum modes (4.11) plays an indispensable role in the calculation. The problem of zero point energy is not so crucial for a theory in flat spacetime. And its role in the case of gravity is not clear. Our model suggests that black holes may provide the first evidence that zero point energy is physically relevant in curved spacetime.

  It is possible that the horizon is only one of the many physical entities that contribute to the black hole thermodynamics. If we do not induce the contribution from the zero point energy, other sources will have to be included to make up the total black hole mass. In that case, the simplicity pertained to the present model will be lost. In particular, it is possible that one may need even more unusual assumptions in order to explain the unusual relation between the black hole mass and entropy.

Acknowledgments

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