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Comments on "A Local Interpretation of Heisenberg's Transfer Theory" by A. Muschinski and R. Roth, in Beitr. Phys. Atmosph. **66,** 335–346

and "On Relations between Constants in Homogeneous Turbulence Models and Heisenberg's Spectral Model" by U. Schumann, Beitr. Phys. Atmosph. **67,** this issue

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1 Introduction

Recently, Muschinski and Roth (1993) derived relationships between various constants in homogeneous turbulence closure models based on Heisenberg's theory of spectral eddy viscosity. Moreover, Muschinski and Roth claim that the logarithmic law of the wall emerges as corollary of Heisenberg's theory and that they have deduced an algebraic relation between Kolmogoroff's constant of turbulence spectra and the von-Kármán constant.

Schumann (1994) commented on Muschinski and Roth's paper and showed that Muschinski and Roth's basic results can be obtained by using only the interial subrange theory of Kolmogoroff for local equilibrium without referring to the viscous subrange. Furthermore, Schumann pointed out that there is no satisfactory theory of dissipation spectra.

Here, I would like to comment on two problems. Firstly, I would like to question that the logarithmic law of the wall emerges as corollary of Heisenberg's spectrum. Instead, I will argue that this relation, as well as the relation between Kolmogoroff's constant and von-Kármán's constant, is a consequence of scaling arguments concerning the energy-containing range of the spectrum. Secondly, I would like to ask whether we can expect any relation between the von-Kármán constant and the viscous subrange. Generally, I agree with Schumann that we do not anticipate such relation in Newtonian flow at infinitly large Reynolds number. But I would like to stimulate the discussion by citing an earlier paper of Malkus (1975).

2 Critique of Muschinski and Roth's Corollary

Following an idea proposed earlier by Roth (1972), Muschinski and Roth discuss a local interpretation of Heisenberg's eddy-viscosity model. They approximate the spectrum of turbulent kinetic energy (TKE), including the energy-containing range, by a $k'^{-5/3}$-law. Since Muschinski and Roth do not properly take into account the energy feeding of turbulence on the mean flow beyond the inertial subrange, their spectrum increases unrealistically without bound as the wavenumber k' becomes small. To circumvent this problem, Muschinski and Roth propose to cut off the spectrum at $k' = k = q/z$. By relating the shear stress to the dissipation of TKE and by computing the dissipation from Heisenberg's spectrum, Muschinski and Roth arrive at the logarithmic law of the wall.

It do not want to critizise Muschinski and Roth's quite reasonable assumption of a cut-off wavenumber k. The cut-off wavenumber was – to my knowledge – first introduced by Businger (1961). It is a valuable and, of course, "legal" trick for constructing conceptual spectral models. In particular, it allows for exploring the idea of scaling within the energy-containing range. The problem, I would like to point at, is that assuming $k = q/z$ is just a scaling argument – which is, as stated by Schumann, a standard assumption near walls in large-eddy simulations. And it is because of this special assumption on k which – together with Heisenberg's or Kolmogoroff's spectrum – leads to the logarithmic law of the wall and, ultimately, to a relation between the Kolmogoroff constant and the von-Kármán constant. The latter relation depends on the unknown constant q in $k = q/z$.

Muschinski and Roth argue that $q = \pi/2$ which, again, seems quite plausible when following their line of argument. On the other hand, there is no reason why q could not be a little bit larger or smaller. In fact, data of Kaimal et al. (1972) which are also cited in Muschinski and Roth and shown in their Figure 2 suggest that q should be larger. This would, in turn, imply a von-Kármán constant even smaller than Muschinski and Roth's value of 0.34 which is already at the lower end of empirical estimates.

In an earlier paper (Claussen, 1985 – for details see Claussen, 1984), I formulated a closure hypothesis of the energy-containing range to study the relations between turbulence spectra and mean flow. Since this closure hypothesis was based on similarity arguments I also could not deduce the von-Kárman constant from turbulence spectra (which was, in fact, not the scope of my study). But I could demonstrate that any turbulence theory which yields a $k'^{-5/3}$-law in the inertial subrange together with any plausible scaling argument for the energy-containing range eventually leads to a logarithmic law of the wall. Hence I doubt that the logarithmic law of the wall is a corollary of Heisenberg's theory, but it is a consequence of proper scaling arguments. Likewise, by using scaling arguments one cannot deduce a quantitative relation between constants of the turbulence spectrum and the mean flow, but just qualitative relations. There always remains at least one unknown factor which has to be determined by experiments or by add-hoc arguments.

3 The von-Kármán Constant and the Viscous Subrange

As shown by Schumann, it is not necessary to refer to the full turbulence spectrum to arrive at relations between constants in homogeneous turbulence models. This is also confirmed by my earlier study (Claussen, 1985). Obviously, the viscous subrange and the von-Kármán constant have nothing in common which can be anticipated for Newtonian flow at infinitly large Reynolds number. The question remains whether this is true in general. To stimulate the discussion I would like to cite an earlier paper by Malkus (1975).

Malkus presumes that the observed positive curvature of the mean velocity profile in turbulent channel flow has its mechanistic origin in the brief and violent instabilities which are the principal agents of the momentum transfer process and which act to remove transient inflexion point instabilities. Malkus shows that a velocity-defect law results from the single requirement of Reynolds-stress spectral smoothness. Malkus defines a spectrum I(f) by $(z_h^2/u_* U'') = I^* I$ where $U'' = d^2U/dz^2$, z_h is the half-width of the channel, u_* is the friction velocity, and $I(f) = \Sigma I_k e^{ikf}$, $k = 1, \infty$, with $f = \pi(z + z_h)/z_h$. If one assumes the spectrum I to be smooth in the sense that the difference between successive Fourier components I_k is small, then a velocity-defect law emerges which is parabolic in the mid-region and logarithmic near the boundary. Data suggest that $|I_0|$ is approximately constant, and is inversely related to the square of the von-Kármán constant. In his 1975 paper, Malkus just isolates certain a mechanistic consequences of the finite amplitude stabilzating process and does not deduce a value of I_0. This was done earlier (Malkus, 1956).

Malkus assumes that spectral smoothness may be less plausible at those wavenumbers where viscous effects first become as important as nonlinear effects because, at this scale of motion, the smallest finite amplitude instabilities are observed to occur. Moreover, Malkus argues that at these wavenumbers, drag-reducing additives are presumably to be most effective in producing non-Newtonian effects. In order to construct a spectral tail Malkus requires that the spectral tail shop drop off faster than any finite negative power of k′ – which is in keeping with the discussion of the viscous subrange by Schumann. Secondly, Malkus requires that any presumed spectral tail should be continuous with and matches the smoothness condition at the wavenumber where the tail joints the inertially controlled lower part of the spectrum. As a result (for details, the reader is referred to the original paper of Malkus, 1975) a spectrum emerges from which a mean velocity profiles can be deduced which exhibits a double logarithmic structure. The inner logarithmic law appears to have a larger slope than the outer one. The precise form of the change from one logarithmic law to the other depends on the constants implicit in the spectral tail. '

Obviously, in non-Newtonian flow, the von-Kármán constant has to change with height, if it is accepted to describe the logarithmic profile of mean velocity in the same manner as in Newtonian flow. Of course, it is rather tempting to speculate whether the atmosphere does not behave quite Newtonian to explain differences in the data of the von-Kármán constant. However, I assume that problems in experimental set up and deviation from ideal conditions of homogeneity, stationarity, and neutral stratification are better candidates which these differences can be blamed on (e.g. Wieringa, 1980). Here, I just would like to point out that there is some relation between mean flow and the tail of turbulence spectra quite generally.

4 Concluding Remarks

Muschinski and Roth present a paper which is quite valuable concerning practical applications and theoretical considerations of turbulence modelling. It is also valuable because Muschinski and Roth carefully discuss the limitations and assumptions implicit in their model. Therefore, it seems justified to add to their discussion. Here, I have critically reassessed the consequences of introducing a cut-off wavenumber as $k = q/z$. This assumption seems quite plausible. Nevertheless it is this particular assumption which eventually leads to a logarithmic law of the wall; any other would not. Likewise, also the specific value of the von-Kármán constant depends on the specific value of q which Muschinski and Roth presribe as $\pi/2$, but there is no reason that q could not be a little bit larger. Secondly, Muschinski and Roth use the full turbulence spectrum to deduce their results. Schumann argues that one can arrive at the same relations with referring the inertial subrange of the spectrum only – which is in line with earlier arguments of mine (Claussen, 1985).

Although it seems plausible that the von-Kármán constant has nothing to do with the spectral subrange of turbulence it is rather tempting for me to speculate that it does – quite generally. I have cited a paper of Malkus (1975) who discusses the peculiar double logarithmic structure of mean velocity profiles in non-Newtonian flow and I have interpreted Malkus' result as indication that there is a relation between mean flow and the spectral tail of turbulence. At the moment, I cannot find any better argument to justify my speculation except that it should add to Schumann's call upon a discussion of spectral models for the turbulence at dissipating scales.

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