$$\frac{1}{2}\|\mathbb{A}\mathbb{x} - \mathbb{B}\|^2$$

Nonnegative least squares

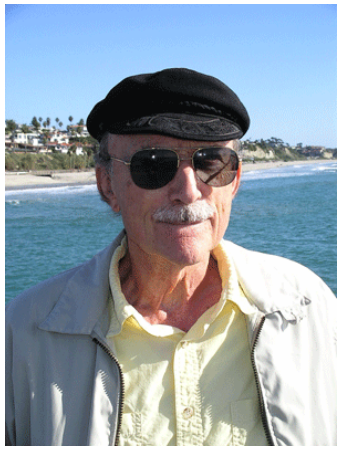
Suvrit Sra
ILAS (Pisa)
June 21–25, 2010
(based on joint work with D. Kim, I. Dhillon)

(MPI für biologische Kybernetik, Tübingen)

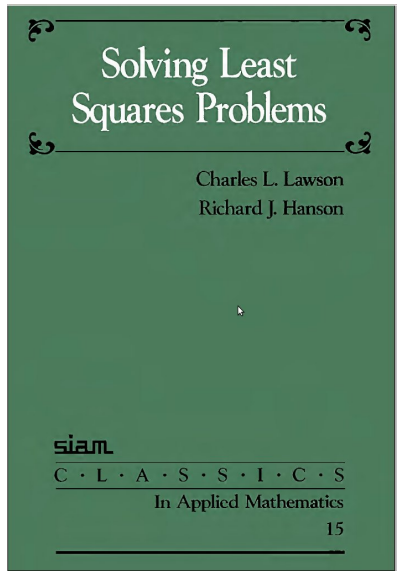
Introduction

1. Venerable history of least squares
2. Estimates optimal, but thanks to “cancellations” — so solution usually has positive and negative entries
3. Physical, modeling, or numerical reasons request nonnegative estimates

History



C. Lawson (2004)



The book

Applications

1. Astronomy [WP99, **?**, **?**]
2. Medicine (radio-therapy [PMB09, Gun99, MLV+04])
3. Drug delivery [Bur05]
4. Medical imaging (MRI [RMRG09, JVO09], PET [**?**], etc.)
5. Inverse problems, e.g., [ATB05, Par94, **?**]
6. Image Processing, computer graphics
7. Face recognition [VMC09]
8. Neural networks [Tim08]
9. Subproblem of NMA/NMF [**?**, CZPiA09]
10. Anomaly detection, e.g. in remote sensing [GB08]
11. Chemometrics [Gem08]
12. Bioinformatics [SGD05]
13. Machine learning (dual of SVM)
14. Fisheries acoustics [SM05]
15. Mechanical engineering [SI99]
16. Statistics (constrained least squares)

Algorithms

1. NNLS, by Lawson and Hanson (1976) – Widely used; matlab impl; slow
2. FNNLS Bro, Jong (19??) – faster
3. take from bbsg paper, and from pqnj paper

Our approach

1. large scale problems
2. low–medium accuracy solutions acceptable
3. generic algorithmic approaches, progress in optimization
4. gradient-projection

Numerical Results

Numerical Results

Summary

Questions?

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