Optical force acting on strongly driven atoms in free space or modified reservoirs

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We investigate the quantum dynamics of a moderately driven two-level particle in free space or modified electromagnetic field reservoir. Particularly, we calculate the optical force acting on the radiator in such an environment. We found that the modified environmental reservoir influences significantly the optical force. Very intense driving in free space also modifies the maximal force.

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I. INTRODUCTION

The force acting on a two-level atom in a resonance light field can be estimated as follows: in the field of a strong running wave, the atom absorbs a photon from the light beam and acquires the momentum $\hbar k$ of the photon. Respectively, a force $\hbar k\gamma$ acts on the atom, where 2γ is the spontaneous-decay rate of the upper level [1–3]. This force can be even stronger in a field of a standing wave. If the atom is accelerated in such a field only to a distance of half the wavelength, it acquires an energy greater than in the thermal case. The acceleration effect can be substantially enhanced if the frequency of one of the opposing waves varies with time. Acceleration of neutral particles was achieved in Ref. [4]. Furthermore, the scattering rate from a coherent stimulated process can be made arbitrarily large by detuning the optical field far from resonance and increasing the intensity. When detuned from resonance very large accelerations in the 10^{11}g range have been realized and this process has been termed optical Stark deceleration or acceleration [5]. Thus, the mechanical effect of a resonant or non-resonant field on atoms can be considerable. In an another context, laser acceleration of charged particles up to GeV energies was obtained as well [6, 7]. Previous force studies include dielectrics [8, 9] and plasmas [10, 11].

Another important related issue is the cooling and trapping in laser fields [12]. The experimental feasibility of Bose-Einstein condensation is already history [13]. Laser-cooled atoms are used, for example, as frequency standards [14], in quantum information processing [15] or in atomic clocks [16]. Therefore, it is not suprising that the optical force received a lot of attention. In particular, cooling of atoms with stimulated emission was observed in [17]. Laser cooling of atoms in squeezed vacumm was investigated in [18, 19], respectively. An overview on cold atoms and quantum control was given in Ref. [20] while laser cooling of atoms, ions or molecules by coherent scattering was studied in [21]. Adiabatic cooling of atoms by an intense standing wave was experimentally achieved in [22]. Further, in Ref. [23] trapping and cooling of atoms in a vacuum perturbed in a frequency-dependent manner was investigated. Light-pressure cooling of crystals was annalysed as well [24]. Stopping atoms with laser light was achieved in [25] while collective-emission-induced cooling of atoms in an optical cavity was observed in Ref. [26]. In the radiation field of an optical waveguide, the Rayleigh scattering of photons was shown to result in a strongly velocity-dependent force on atoms [27]. Finally, these techniques were exported to other systems such as mesoscopic systems. For instance, the resolved sideband laser cooling was used to cool a mesoscopic mechanical resonator to near its quantum ground state [28].

In this article we investigate the optical force acting on two-level atoms in moderately intense running laser fields in free space or modified electromagnetic field (EMF) reservoirs like low quality optical cavities. We show that modified reservoirs lead to a significant enhancement of optical forces. In contrast, very intense driving in free space contributes to a maximal force which is slightly smaller than for moderate pumping. This is somehow surprising as one may expect the force to be larger for bigger intensities. Note also that modified environmental reservoirs are responsible for recovering of the interference pattern [29], enhanced squeezing [30], population inversion [31] or thresholdless lasing [32].

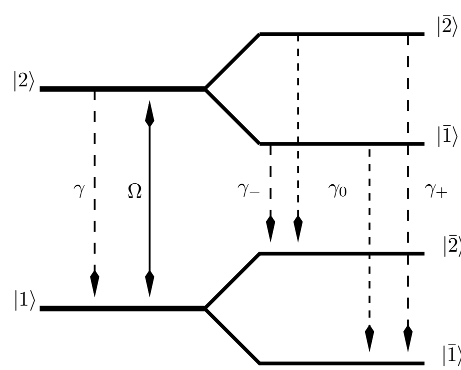

FIG. 1: The scematic picture shown the involved energy levels of a two-level atom. Here Ω is the Rabi frequency while γ is the bare state spontaneous decay rate. The dressed-state decay rates $\{\gamma_0, \gamma_\pm\}$ are different for a very intense laser field in free space or for a modified environmental electromagnetic field reservoir such as cavities.

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The article is organized as follows. In Section II we descibe the system of interest and obtain an expression for the optical force acting on strongly driven two-level atoms in free space or modified reservoirs. Section III deals with discussions of the obtained results. The Summary is given in Section IV.

II. QUANTUM DYNAMICS IN MODERATELY STRONG LASER FIELDS AND MODIFIED EMF RESERVOIR

We proceed by briefly introducing the main steps of the analytical formalism involved and then rigorously describing the obtained results. The Hamiltonian characterizing the interaction of a two-level particle possessing the frequency ω_0 with a coherent source of frequency ω_L, in a frame rotating at ω_L, is [33]: $H = H_0 + H_L + H_F$, where

$$H_0 = \sum_k \hbar(\omega_k - \omega_L)a^\dagger_k a_k + \hbar(\omega_0 - \omega_L)S_z, \tag{1a}$$

$$H_L = \hbar\Omega(S^+ e^{i\vec{k}_L\vec{r}} + S^- e^{-i\vec{k}_L\vec{r}}), \tag{1b}$$

$$H_F = i\sum_k(\vec{g}_k\cdot\vec{d})(a^\dagger_k S^- e^{-i\vec{k}\vec{r}} - a_k S^+ e^{i\vec{k}\vec{r}}). \tag{1c}$$

Here H_0 describes the free Hamiltonians of the electromagnetic field and the atomic subsystems, respectively. The interaction between the laser field with Rabi frequency Ω and wave vector $\vec{k}_L$, and the two-level radiator is given by H_L. H_F characterizes the interaction of the atom with the surrounded electromagnetic field reservoir and in free space $\vec{g}_k = \sqrt{2\pi\hbar\omega_k/V}\vec{e}_\lambda$ with $\vec{e}_\lambda$ being the photon polarization vector with $\lambda = 1, 2$ and V is the EMF quantization volume. Further, $S^+ = |2\rangle\langle 1|$ [$S^- = |1\rangle\langle 2|$] describes the excitation [deexcitation] of the two-level particle at position $\vec{r}$ and obeys the commutation relations for su(2) algebra: $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$. Here $S_z = (|2\rangle\langle 2| - |1\rangle\langle 1|)/2$ is the bare-state inversion operator. $a^\dagger_k$ and a_k are the creation and the annihilation operator of the EMF, respectively, and satisfy the standard bosonic commutation relations, i.e., $[a_k, a^\dagger_{k'}] = \delta_{kk'}$, and $[a_k, a_{k'}] = [a^\dagger_k, a^\dagger_{k'}] = 0$.

The optical force acting on two-level particles in a traveling laser wave is given by the following expression:

$$\vec{F} = -\nabla H_L(\vec{r}) = -i\vec{k}_L\hbar\Omega(S^+ - S^-), \tag{2}$$

where we considered that the particle is located at the origin, i.e. $e^{\pm i\vec{k}_L\vec{r}} \to 1$.

In what follows we shall describe our system using the dressed-states formalism (see Fig. 1):

$$|1\rangle = \cos\theta|\bar{1}\rangle + \sin\theta|\bar{2}\rangle, \tag{3a}$$

$$|2\rangle = \cos\theta|\bar{2}\rangle - \sin\theta|\bar{1}\rangle, \tag{3b}$$

with $\cot 2\theta = (\Delta/2)/\Omega$ and $\Delta = \omega_0 - \omega_L$. In this picture the force is given by the relation:

$$\vec{F} = -i\vec{k}_L\hbar\Omega(R^+ - R^-). \tag{4}$$

Here $R^+ = |\bar{2}\rangle\langle\bar{1}|$ and $R^- = |\bar{1}\rangle\langle\bar{2}|$ are new quasispin operators operating in the dressed state picture. They obey the same commutation relations as the old ones. To obtain the explicit expression for the force we need the equations of motion for the new dressed-state operators. Therefore we write the Hamiltonian in the dressed-state representation:

$$\begin{aligned} H &= \sum_k \hbar(\omega_k - \omega_L)a^\dagger_k a_k + \hbar\bar{\Omega}R_z \\ &+ i\sum_k\{(\sin 2\theta R_z/2 + \cos^2\theta R^- - \sin^2\theta R^+)a^\dagger_k \\ &- H.c.\}. \end{aligned} \tag{5}$$

Here $R_z = |\bar{2}\rangle\langle\bar{2}| - |\bar{1}\rangle\langle\bar{1}|$ is the dressed-state inversion operator while $\bar{\Omega} = \sqrt{\Omega^2 + (\Delta/2)^2}$ is the generalized Rabi frequency. The Heisenberg equation for an arbitrary dressed-state atomic operator Q is:

$$\frac{d}{dt}\langle Q(t)\rangle = \frac{i}{\hbar}\langle [H, Q(t)]\rangle. \tag{6}$$

Here the notation $\langle\cdots\rangle$ indicates averaging over the initial state of both the atoms and the EMF environmental reservoir.

Introducing the Hamiltonian (5) in Eq. (6) one arrives at:

$$\begin{aligned} &\frac{d}{dt}\langle Q\rangle - i\bar{\Omega}\langle [R_z, Q]\rangle = \\ &- \sum_k \frac{(\vec{g}_k\cdot\vec{d})}{\hbar}\langle a^\dagger_k[\sin 2\theta R_z/2 + \cos^2\theta R^- - \sin^2\theta R^+, Q]\rangle \\ &+ H.c., \end{aligned} \tag{7}$$

where for the in general non-Hermitian atomic operators Q, the H.c. terms should be evaluated without conjugating Q, i.e., by replacing Q^+ with Q in the Hermitian conjugate parts. Assuming that the atomic subsystem couples weakly to the surrounding EMF, i.e., in the bad-cavity limit, the EMF operators can be eliminated from the above equation of motion, Eq. (7). On solving formally the Heisenberg equations for the EMF field operators one can represent the solutions in the form:

$$\begin{aligned} a^\dagger_k(t) &= a^\dagger_k(0)e^{i\bar{\Delta}_k t} + \pi\frac{(\vec{g}_k\cdot\vec{d})}{\hbar}\{\sin 2\theta R_z(t)\delta(\omega_k - \omega_L)/2 \\ &+ \cos^2\theta R^+(t)\delta(\omega_k - \omega_L - 2\bar{\Omega}) \\ &- \sin^2\theta R^-(t)\delta(\omega_k - \omega_L + 2\bar{\Omega})\}, \end{aligned} \tag{8}$$

where $\bar{\Delta}_k = \omega_k - \omega_L$ and $a_k = [a^\dagger_k]^\dagger$. Here the contributions leading to a small Lamb shift were ignored. Substituting Eq. (8) in Eq. (7) and summing over k one arrives at the following master equation:

$$\frac{d}{dt}\langle Q\rangle - i\bar{\Omega}\langle[R_z, Q]\rangle = \\ -\langle(\gamma_0 \sin 2\theta R_z/2 + \gamma_+ \cos^2\theta R^+ - \gamma_- \sin^2\theta R^-) \\ \times[\sin 2\theta R_z/2 + \cos^2\theta R^- - \sin^2\theta R^+, Q]\rangle + H.c. \quad (9)$$

Here, in free space, $\gamma(\omega) = 2d^2\omega^3/(3\hbar c^3)$ and for γ_0 we have $\omega \equiv \omega_L$ while for $\gamma_\pm$ we have $\omega = \omega_L \pm 2\bar{\Omega}$. Note that for usual vacuum modes and moderate driving, i.e. $\bar{\Omega}/\omega_L \to 0$, we have $\gamma_0 = \gamma_+ = \gamma_-$. We anticipate that, for modified environmental reservoirs such as low quality optical cavities, or for very intense driving in free space with $\bar{\Omega}/\omega_L \ll 1$ but not zero, this is not the case, namely, $\gamma_0 \neq \gamma_+ \neq \gamma_-$ [34].

We emphasize here that the master equation (9) describes also, under the Born-Markov conditions, the case of a driven two-level particle that is damped by a modified reservoir, i.e., when the density of electromagnetic field modes is different at various dressed-states transitions. In this case $\gamma_\pm \propto g(\omega_L \pm 2\bar{\Omega})$ while $\gamma_0 \propto g(\omega_L)$, where $g(\omega)$ characterizes the atom-environment coupling strength [29–34]. In the next subsections, we shall obtain the equations of motion as well as the expression for the optical force acting on a strongly driven two-level particle in various environments.

A. Equations of motion

Using Eq. (9) one can obtain the equations of motion for the operators of interest:

$$\frac{d}{dt}\langle R_z\rangle = -2\gamma_p\langle R_z\rangle + \gamma_0 \sin 4\theta\langle R^+ + R^-\rangle/2 - 2\gamma_m, \quad (10a)$$

$$\frac{d}{dt}\langle R^+\rangle = \langle R^+\rangle[2i\bar{\Omega} - (\gamma_0 \sin^2 2\theta + \gamma_p)] \\ - (\gamma_+ + \gamma_-)\sin^2 2\theta\langle R^-\rangle/4 + \gamma_f \\ + \sin 2\theta\langle R_z\rangle(\gamma_+ \cos^2\theta - \gamma_- \sin^2\theta)/2, \quad (10b)$$

$$\frac{d}{dt}\langle R^-\rangle = -\langle R^-\rangle[2i\bar{\Omega} + (\gamma_0 \sin^2 2\theta + \gamma_p)] \\ - (\gamma_+ + \gamma_-)\sin^2 2\theta\langle R^+\rangle/4 + \gamma_f \\ + \sin 2\theta\langle R_z\rangle(\gamma_+ \cos^2\theta - \gamma_- \sin^2\theta)/2. \quad (10c)$$

Here $\gamma_p = \gamma_+ \cos^4\theta + \gamma_- \sin^4\theta$, $\gamma_m = \gamma_+ \cos^4\theta - \gamma_- \sin^4\theta$ while $\gamma_f = \sin 2\theta(\gamma_0 + \gamma_+ \cos^2\theta + \gamma_- \sin^2\theta)/2$. Further, $\cos^2\theta = (1 + (\Delta/2)/\bar{\Omega})/2$, $\sin^2\theta = (1 - (\Delta/2)/\bar{\Omega})/2$ and $\sin 2\theta = \Omega/\bar{\Omega}$.

B. Optical force

Substituting the steady-state solution of Eq. (10) in Eq. (4) we obtain the following mean expression for the optical force:

$$\langle F\rangle = k_L\hbar \frac{2\bar{\gamma}\Omega\bar{\Omega}\sin 2\theta}{4\gamma_p\bar{\Omega}^2 + \gamma_1(\gamma_2\gamma_p - \gamma_0\tilde{\gamma}\sin 2\theta \sin 4\theta/4)}. \quad (11)$$

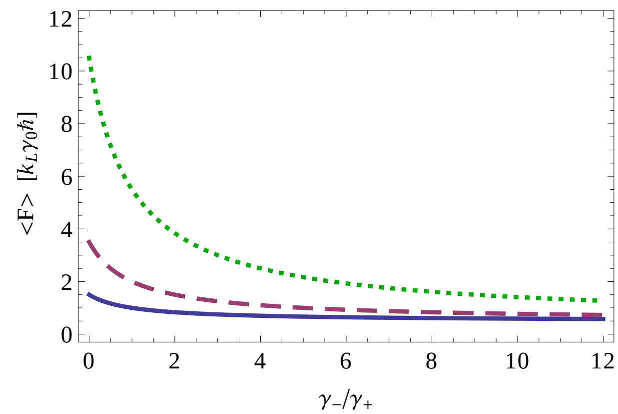

FIG. 2: (color online) The maximal optical force acting on a two-level particle in a modified environment. The solid line is for $\gamma_-/\gamma_0 = 1$, the dashed line corresponds to $\gamma_-/\gamma_0 = 3$ while the dotted ones to $\gamma_-/\gamma_0 = 10$.

where $\bar{\gamma} = \gamma_+ \cos^4\theta(\gamma_0 + 2\gamma_- \sin^2\theta) + \gamma_- \sin^4\theta(\gamma_0 + 2\gamma_+ \cos^2\theta)$, $\gamma_1 = \gamma_0 \sin^2 2\theta + \tilde{\gamma}\cos 2\theta$, $\gamma_2 = \gamma_0 \sin^2 2\theta + \gamma_+ \cos^2\theta + \gamma_- \sin^2\theta$ and $\tilde{\gamma} = \gamma_+ \cos^2\theta - \gamma_- \sin^2\theta$.

In the following section we shall analyze the optical force acting on a two-level particle in a running wave laser field in more details.

III. RESULTS AND DISCUSSIONS

In the case when the driven atom is surrounded by the usual vacuum modes and $2\bar{\Omega}/\omega_L \to 0$, the spontaneous decay rates corresponding to different dressed-state transitions are equal, i.e. $\gamma_0 = \gamma_+ = \gamma_- \equiv \gamma$. This will lead to the well-known expression for the optical force, that is:

$$\langle F\rangle = 2k_L\hbar\gamma \frac{\Omega^2}{\Delta^2 + \gamma^2 + 2\Omega^2}. \quad (12)$$

For strong fields, i.e. $\Omega^2 \gg \Delta^2 + \gamma^2$, one arrives at the maximal expression of the force:

$$\langle F\rangle = k_L\hbar\gamma. \quad (13)$$

Notice, that the particle velocity v can be included in Eq. (12) via the modified detuning, i.e., $\Delta \to \Delta - k_L v$.

In the following we shall obtain the corresponding expression for the maximal force in the strong-field limit and in the modified reservoirs. In the intense-field limit, that is $\Omega \gg \{\Delta, \gamma_\pm, \gamma_0\}$, we have $\cos^2\theta \approx \sin^2\theta \to 1/2$ while $\sin 2\theta \to 1$. Therefore, the optical force acting on a strongly driven two-level atom in a modified reservoir is:

$$\langle F\rangle = k_L\hbar\gamma_0\left(1/2 + \frac{\gamma_+\gamma_-}{\gamma_0(\gamma_+ + \gamma_-)}\right). \quad (14)$$

One can observe that the optical force in the strong-field limit depends on the spontaneous decay rates at particular dressed-state transitions. By modifying of these dressed-decay rates via a suitable environment reservoir one can influence the magnitude of the optical force. In particular, when $\gamma_+ \gg \gamma_-$ we have for the optical force the following expression

$$\langle F\rangle = k_L\hbar\gamma_0\biggl(1/2 + \frac{\gamma_-}{\gamma_0}\biggr). \tag{15}$$

Now if $\gamma_- \gg \gamma_0$ we obtain that the optical force is greater than $k_L\hbar\gamma_0$. Otherwise if $\gamma_- \ll \gamma_0$, we have $\langle F\rangle = k_L\hbar\gamma_0/2$.

Conversely, if $\gamma_- \gg \gamma_+$ we get for the force in Eq. (14):

$$\langle F\rangle = k_L\hbar\gamma_0\biggl(1/2 + \frac{\gamma_+}{\gamma_0}\biggr), \tag{16}$$

which is again much larger than $k_L\hbar\gamma_0$ when $\gamma_+ \gg \gamma_0$ or $\langle F\rangle = k_L\hbar\gamma_0/2$ if $\gamma_+ \ll \gamma_0$. Finally, if $\gamma_+ = \gamma_- \gg \gamma_0$ we again have an increase in the optical force. These features are shown in Figure (2) where Eq. (14) is plotted.

Further, we shall evaluate the maximal optical force when the two-level emitter is pumped with a very intense laser field in free space such that $2\bar{\Omega}/\omega_L \ll 1$ but not zero. In this case the dressed decay rates can be determined as follows:

$$\gamma_\pm = \gamma_0(1 \pm 2\bar{\Omega}/\omega_L)^3. \tag{17}$$

Substituting these expressions in Eq. (14) and to the second order in the small parameter $\bar{\Omega}/\omega_L$ one arrives at:

$$\langle F\rangle = k_L\hbar\gamma_0\biggl(1 - 3(2\bar{\Omega}/\omega_L)^2\biggr). \tag{18}$$

One can observe here that the maximal force in free space is smaller for very intense driving than for moderate pumping (see Eq. 13). In particular, if $2\bar{\Omega}/\omega_L = 0.1$ then the force in Eq. (18) is three procents smaller than the force (13). This result is somehow counterintuitive as one may expect the force to be larger for higher field intensities.

IV. SUMMARY

In summary, we have investigated the optical force acting on a two-level atom in a running wave laser and in a modified surrounding electromagnetic field reservoir. For this, we obtained the master equation describing this process and, correspondingly, we obtained the equations of motion for the dressed-state operators of interest which helped to get the optical force. The obtained optical force shows a strong dependence on the modifed electromagnetic reservoir via the dressed decay rates at particular frequencies. In particular, it can be much larger than the corresponding force in the free space. Finally, we evaluated the maximal optical force acting on a two-level emitter in very intense laser fields and in the free space.

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