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# First principles approach to leptogenesis

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# Von fundamentalen Prinzipien zu Leptogenese

Baryogenese durch Leptogenese liefert eine elegante Erklärung für den Ursprung der Baryonenasymmetrie des Universums durch den CP-verletzenden Zerfall von schweren rechtshändigen Neutrinos (im frühen Universum). Die nahe Verbindung zu Neutrinophysik hat in den letzten Jahrzehnten sehr zur Popularität dieses Szenarios beigetragen. Eine präzise Berechnung der Baryonenasymmetrie ist schwierig, da sie es erfordert, der Entwicklung des warmen frühen Universums im thermischen Ungleichgewicht zu folgen. In dieser Arbeit diskutieren wir Leptogenese mit Methoden der Nichtgleichgewichts-Quantenfeldtheorie. Diese Methode ist frei von zahlreichen Problemen der konventionellen Herangehensweise, welche auf der klassischen Boltzmann-Gleichung basiert. Wir leiten eine quanten-korrigierte Boltzmann-Gleichung für die Asymmetrie direkt aus den Grundprinzipien her. Die erhaltende Gleichung ist frei von dem Doppelzählungsproblem und beinhaltet konsistent die thermischen Korrekturen zu den Eigenschaften der Quasiteilchen, insbesondere deren thermische Masse und Breite. Effekte durch die begrenzte Breite werden durch eine modifizierte Quasiteilchennäherung berücksichtigt. Wir vergleichen numerisch die Ergebnisse dieses Nichtgleichgewichts-Quantenfeldtheorie-Ansatzes mit den konventionellen Methoden, und finden, dass die thermischen Effekte teilweise durch die thermischen Massen kompensiert werden.

## First principles approach to leptogenesis

Baryogenesis via leptogenesis offers an elegant explanation of the origin of the baryon asymmetry of the universe by means of the CP-violating decay of heavy right-handed neutrinos in the early universe. This scenario has become very popular over the past decades due to its connection with neutrino physics. A precise computation of the baryon asymmetry produced in the leptogenesis scenario is difficult since it requires to follow the out-of-equilibrium evolution of the hot early universe. We present here a nonequilibrium quantum field theory approach to leptogenesis. This method is free of many problems inherent to the conventional approach based on the classical Boltzmann equation. Starting from first principles we derive a quantum-corrected Boltzmann equation for the asymmetry. The obtained equation is free of the double counting problem and incorporates consistently thermal corrections to the quasiparticles properties, in particular thermal masses and thermal widths. Finite width effects are taken into account through the extended quasiparticle approximation. We compare numerically the reaction densities obtained from the conventional and nonequilibrium quantum field theory approaches. We find that the enhancement due to thermal effects is partially compensated by the suppression due to thermal masses.



# Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
1.1	Overview	7
1.2	Baryon asymmetry of the universe	9
1.2.1	Baryon asymmetry of the universe	9
1.2.2	Type-I seesaw Lagrangian	10
1.2.3	Production of baryon asymmetry in the leptogenesis scenario	12
<b>2</b>	<b>Boltzmann approach to leptogenesis</b>	<b>15</b>
2.1	Boltzmann equation	15
2.2	Boltzmann equation at $\mathcal{O}(h^4)$	17
2.3	Higgs mediated processes	23
<b>3</b>	<b>Nonequilibrium quantum field theory</b>	<b>27</b>
3.1	Kadanoff-Baym equations	27
3.2	Quantum kinetic equation	33
3.3	Boltzmann limit	41
3.4	Extended quasiparticle ansatz	44
<b>4</b>	<b>Nonequilibrium approach to leptogenesis</b>	<b>49</b>
4.1	Lepton asymmetry	49
4.2	Heavy neutrino decay	53
4.2.1	Tree-level contribution	53
4.2.2	Equilibrium solution for the heavy neutrino propagator	55
4.2.3	Self-energy contribution	57
4.2.4	Vertex contribution	59
4.3	$ \Delta L  = 2$ scattering processes	62
4.4	Higgs decay	67
4.4.1	Tree-level amplitude	68
4.4.2	Self-energy contribution	69
4.4.3	Vertex contribution	70
4.5	Higgs mediated scattering processes	71
4.5.1	Tree-level amplitude	73
4.5.2	Self-energy contribution	75
4.5.3	Vertex contribution	76
4.6	Heavy neutrino number density	78
<b>5</b>	<b>Comparison of the Boltzmann and nonequilibrium approaches</b>	<b>81</b>
5.1	Rate equations	81
5.2	Numerical comparison	88
5.2.1	Heavy neutrino decay	90

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5.2.2	$ \Delta L  = 2$ scattering processes . . . . .	91
5.2.3	Higgs decay . . . . .	95
5.2.4	Higgs mediated processes . . . . .	98
5.2.5	Heavy neutrino decay versus Higgs mediated processes . . . . .	104
<b>6</b>	<b>Conclusion</b>	<b>109</b>
	<b>Acknowledgements</b>	<b>113</b>
<b>A</b>	<b>Notation</b>	<b>117</b>
<b>B</b>	<b>Kinematics</b>	<b>119</b>
B.1	Two-body decay . . . . .	119
B.2	Two-body Scattering . . . . .	120
B.3	Three-body decay . . . . .	121
<b>C</b>	<b>Properties of the propagators</b>	<b>123</b>
C.1	Symmetry properties in coordinate representation . . . . .	123
C.2	Symmetry properties in Wigner representation . . . . .	124
C.3	CP-conjugated propagators in coordinate representations . . . . .	124
C.4	CP-conjugated propagators in Wigner representations . . . . .	125
<b>D</b>	<b>Wigner transform of a convolution product</b>	<b>127</b>
D.1	Properties of the diamond operator . . . . .	128
<b>E</b>	<b>2PI effective action and self-energies</b>	<b>129</b>
E.1	2PI effective action . . . . .	129
E.2	Lepton self-energy . . . . .	129
E.3	Right-handed neutrino self-energy . . . . .	132
E.4	Higgs self-energy . . . . .	136



# Chapter 1

## Introduction

### 1.1 Overview

At a fundamental level the vast majority of physical phenomena can be explained by the two basic theories of modern physics, the Standard Model (SM) of particle physics [1–3] and general relativity (GR). These theories have successfully passed numerous experimental tests over the past decades, the most recent one being the discovery at the Large Hadron Collider (LHC) of a Higgs boson [4–6] consistent with the SM predictions. Complementary to man-made experiments, the primordial universe constitutes a unique laboratory to test our current understanding of the laws of nature at energies that cannot be reached by existing or forthcoming facilities. When extrapolating the SM with the concordance model of cosmology ( $\Lambda$ CDM) back in the hot early universe one arrives at the conclusion that matter and antimatter would completely annihilate into radiation if they were present in exactly equal amount. An asymmetry between matter and antimatter in the primordial universe is therefore needed to explain the baryon asymmetry in the present universe. The generation of this asymmetry, *baryogenesis*, is one of the greatest challenges of modern physics. It is now widely accepted that the SM alone cannot be responsible for the matter-antimatter imbalance of the universe. The asymmetry as initial condition for the universe is very unsatisfactory and disfavoured when one includes a period of inflation in the very early universe to explain the flatness and horizon problems observed in the cosmic microwave background (CMB). Any scenario of a dynamical asymmetry generation must satisfy the three Sakharov conditions [7]:

- (i) Baryon number nonconservation,
- (ii) C- and CP-violation,
- (iii) Departure from thermal equilibrium.

Interestingly enough, these conditions are fulfilled within the SM. Baryon number is violated by the triangle anomaly. Non-perturbative baryon number violating processes, which are called *sphalerons*, involve nine left-handed quarks and three left-handed leptons. They conserve  $B-L$  ( $B$  is the baryon number and  $L$  the lepton number) but violate any other linear combinations. At zero temperature their amplitudes are highly suppressed [8] but become large at high temperature [9]. The chiral nature of the SM violates C (C denotes charge conjugation) and the phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix violate CP (P is the parity). Finally, the electroweak phase transition provides the out-of-equilibrium dynamics required by the condition (iii). However, in the SM, the CP-violation is too small and the departure from equilibrium not strong enough to generate the observed baryon asymmetry.

Over the past decades many theories beyond the SM have been proposed to generate the baryon asymmetry dynamically. Amongst the most studied ones are GUT baryogenesis [10–19], electroweak baryogenesis [20–22], Affleck-Dine mechanism [23, 24] and leptogenesis [25]. Because of its connection with neutrino physics, the mechanism of baryogenesis via leptogenesis has become very popular over the past few years. In this scenario an asymmetry is first produced in the lepton sector by the decay of heavy right-handed neutrinos, and then transferred to the baryon sector through the sphaleron processes.

A precise computation of the baryon asymmetry in any theory of baryogenesis is very challenging because of the third Sakharov condition (iii): one needs to track down the out-of-equilibrium evolution of the baryon number in the early universe. In leptogenesis the conventional way to tackle this problem is a semiclassical approach: *classical* Boltzmann equations with *vacuum* transition amplitudes computed in the S-matrix formalism are used to follow the number densities of the different particle species. It is however clear that this approach is questionable: baryon number generation is a pure *quantum* phenomenon which takes place in the *hot* early universe. Moreover, the Boltzmann equations suffer from the so-called *double counting problem*, which can be solved only in some limiting cases.

Even though a Boltzmann treatment of leptogenesis is not well justified, it has been used extensively to analyse many different aspects of leptogenesis, such as flavour effects [26–30], resonant effects [31, 32], the role of spectator processes [33, 34], supersymmetric extension of leptogenesis [35–37], soft leptogenesis [38–47] and  $N_2$ -dominated leptogenesis [48, 49]. The conventional method based on the Boltzmann equation with vacuum transition amplitudes has been improved [50, 51] by using transition amplitudes from thermal field theory. This approach was able to include thermal masses of the quasiparticles and finite density corrections to the amplitudes. However, it has been shown that these results are in contradiction with a first principles approach [52, 53].

Recently, many efforts towards a nonequilibrium quantum field theory (NEQFT) treatment of leptogenesis have been made [54–71]. NEQFT puts the thermal and quantum fluctuations on an equal footing and is therefore the appropriate formalism to describe the baryon number generation in the early universe. However, the resulting equations, the so-called *Kadanoff-Baym* (KB) equations, cannot be solved analytically except in some particular cases and are extremely difficult to handle numerically, even for simplistic models. For practical use the KB equations must be considerably simplified by applying a series of approximations.

In this work, *quantum-corrected* Boltzmann equations are derived from the KB equations for the case of leptogenesis. We focus here on the simplest scenario, i.e. unflavoured leptogenesis with hierarchical heavy neutrino mass spectrum. However, the method presented here can be easily generalised to more elaborate leptogenesis scenarios. Since the obtained equations describe the time evolution of the distribution functions they are very similar to the classical Boltzmann equations. However, unlike the classical ones, the quantum-corrected Boltzmann equations incorporate consistently the thermal corrections to the masses and to the amplitudes and are free of the double counting problem. They include decay and inverse decay of the right-handed neutrinos, lepton number violating scattering processes mediated by the heavy neutrinos, and Higgs mediated processes. They can also be easily extended to take into account other lepton number violating processes or lepton flavour effects.

We do not attempt to solve numerically the quantum-corrected Boltzmann equations obtained from first principles. In the conventional approach the asymmetry is usually computed by solving a system of *rate equations*. These are equations for the number density and can be

derived from the corresponding classical Boltzmann equations. They are governed by a set of *reaction densities*, which roughly correspond to thermally averaged transition amplitudes. We follow here a similar approach and derive rate equations from the quantum-corrected Boltzmann equations. In addition to the thermal corrections to the amplitudes, the reaction densities derived from the quantum-corrected Boltzmann equations contain quantum statistical factors, which are usually neglected in the conventional approach. The size of the thermal and medium corrections can be estimated by comparing the reaction densities obtained in the conventional approach with the ones obtained in the NEQFT formalism.

The aim of this work is to present a consistent way of studying the nonequilibrium production of baryon asymmetry in the early universe. We do not perform a phenomenological study of the parameter space of the leptogenesis Lagrangian. A parameter scan is only useful if one considers the *full* leptogenesis scenario, including e.g. lepton flavour effects and gauge boson interactions. This is beyond the scope of this work. However, we would like to stress that the formalism presented here can be used to incorporate these effects.

In the rest of this chapter we discuss the basic features of leptogenesis and its connection with neutrino physics. We then review the Boltzmann equations and the conventional approach to leptogenesis in chapter 2. A comprehensive overview of nonequilibrium quantum field theory and the derivation of quantum-corrected Boltzmann equations are presented in chapter 3. This formalism is applied to leptogenesis in chapter 4. In chapter 5 we derive from the quantum-corrected Boltzmann equations the set of rate equations and numerically compare the reaction densities obtained in this approach with the conventional ones. In chapter 6 we summarise our results and conclude.

Parts of this thesis are in print [72] or in preparation [73]. Major part of this work has been done in collaboration with Mathias Garny, Andreas Hohenegger, Alexander Kartavtsev and David Mitrouskas.

## 1.2 Baryon asymmetry of the universe

In this section we briefly review the experimental measurements of the baryon asymmetry of the universe, see e.g. [74] and references therein for a short review on the subject. We present then the main features of the leptogenesis scenario. Many reviews on leptogenesis can be found in the literature, see e.g. [75–78].

### 1.2.1 Baryon asymmetry of the universe

The baryon asymmetry of the universe is usually characterised by the baryon-to-photon ratio  $\eta$ ,

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma}, \quad (1.1)$$

where  $n_B$  ( $n_{\bar{B}}$ ) is the baryon (antibaryon) number density and  $n_\gamma$  the photon number density. This number can be measured in two independent ways. The baryon-to-photon ratio can be extracted from the cosmic microwave background (CMB). CMB anisotropies are sensitive to  $\Omega_B h^2$ , the baryon energy density normalised by the critical energy density of the universe multiplied by the square of the reduced Hubble constant,  $h \equiv H_0/(100 \text{ km sec}^{-1} \text{ Mpc}^{-1})$ . The effect of baryon is to enhance the odd peaks in the CMB power spectrum. The latest mea-

surement of the CMB anisotropies from the Planck satellite [79] gives a baryon energy density (at 68% c.l.),

$$\Omega_B h^2 = 0.02207 \pm 0.00033. \quad (1.2)$$

Combining the Planck power spectrum measurement with the Wilkinson Microwave Anisotropy Probe 9 years (WMAP9) data [80] one finds at 68% c.l. [79],

$$\Omega_B h^2 = 0.02205 \pm 0.00028. \quad (1.3)$$

The baryon energy density  $\Omega_B h^2$  is related to baryon-to-photon ratio through  $\eta = 2.74 \cdot 10^{-8} \Omega_B h^2$ . In terms of  $\eta$  the combined result (1.3) reads,

$$\eta = (6.0417 \pm 0.0077) \cdot 10^{-10}. \quad (1.4)$$

The second way of determining the baryon asymmetry of the universe is through the measurement of the abundance of light elements, D,  $^3\text{He}$ ,  $^4\text{He}$  and  $^7\text{Li}$ . These abundances are predicted by the theory of Big Bang nucleosynthesis (BBN) and depend only on the parameter  $\eta$ <sup>1</sup>. The synthesis of D and  $^3\text{He}$  are particularly sensitive to the baryon-to-photon ratio. The duration of nucleosynthesis depends strongly on the expansion rate of the universe, which is determined amongst others by the parameter  $\eta$ . The baryon-to-photon ratio predicted by BBN as given by [81] is (at 68% c.l.),

$$\eta = (5.80 \pm 0.27) \cdot 10^{-10}. \quad (1.5)$$

It is important to notice that the physics involved in the BBN computation and the physics affecting the CMB power spectrum are completely different since BBN takes place at  $T \sim 2$  MeV and the photon decoupling at  $T \sim 0.3$  eV. Therefore the measurements of  $\eta$  from the CMB power spectrum and from BBN, which are in perfect agreement, are independent of each other and are therefore a reliable confirmation of the value of the baryon asymmetry of the universe. Moreover they also constitute a solid evidence for the  $\Lambda\text{CDM}$  model below  $T \lesssim 1$  MeV.

## 1.2.2 Type-I seesaw Lagrangian

The simplest and probably most popular realisation of leptogenesis takes place in the type-I seesaw extension of the SM. In addition to the SM particles it contains three<sup>2</sup> right-handed Majorana neutrinos  $N_i$  which are singlet under the SM gauge group,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2} \bar{N}_i (i\not{\partial} - M_i) N_i - h_{\alpha i} \bar{\ell}_\alpha \tilde{\phi} P_R N_i - h_{\alpha i}^* \bar{N}_i \tilde{\phi}^\dagger P_L \ell_\alpha, \quad (1.6)$$

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<sup>1</sup>This is true only for standard BBN. The abundances of light elements also depend on other parameters, such as the number of relativistic degrees of freedom at the time of BBN (dark radiation) and the lepton asymmetry. The standard assumptions are that only photons and the three SM neutrino species contribute to the radiation energy density and that no sizeable lepton asymmetry is present.

<sup>2</sup>For leptogenesis one needs at least two right-handed neutrinos. Neutrino oscillation experiments also require the presence of at least two right-handed neutrinos. However, the case of three generations is interesting from an aesthetic point of view since it restores the symmetry between leptons and quarks.

where  $\ell_\alpha$  are the lepton doublets and  $\tilde{\phi} \equiv i\sigma_2\phi^*$  the conjugate of the Higgs doublets. The heavy neutrinos (in this work we use interchangeably the terminology *right-handed* or *heavy neutrino* to designate  $N_i$ ) are Majorana particles, i.e. they satisfy  $N_i = N_i^c \equiv i\gamma^2\gamma^0\bar{N}_i^T$ . Without loss of generality we choose the right-handed neutrino mass matrix  $M_i$  to be diagonal with positive entries.

The Lagrangian (1.6) is well known in neutrino physics since it provides a naturally small mass term to the SM neutrinos through the seesaw formula. In the SM the (left-handed) neutrinos are strictly massless. The observations of neutrino oscillations, i.e. flavour conversion during the neutrino propagation, can be explained by introducing a neutrino mass matrix which is not diagonal in flavour basis, see e.g. [82] for a review on the subject. Neutrino oscillation experiments are sensitive in particular to the neutrino mass square differences. Up to now, only two mass square differences have been measured. In the case of normal hierarchy, their best-fit value as given by [83] reads (at 68% c.l.),<sup>1</sup>

$$\Delta m_{\text{sol}}^2 = 7.54_{-0.22}^{+0.26} \cdot 10^{-5} \text{ eV}^2, \quad (1.7a)$$

$$\Delta m_{\text{atm}}^2 = 2.43_{-0.10}^{+0.06} \cdot 10^{-3} \text{ eV}^2. \quad (1.7b)$$

Oscillation experiments cannot measure the absolute neutrino mass scale. The best constraint on the mass scale comes from cosmology. Combining the Planck and WMAP9 data one finds for the upper bound on the sum of neutrino masses [79] (at 95% c.l.),

$$\sum_i m_{\nu_i} < 0.933 \text{ eV}. \quad (1.8)$$

The tiny neutrino masses can be elegantly explained by the model (1.6). Much below the electroweak scale one can approximate the Higgs field by its vacuum expectation value (vev),  $v = 174 \text{ GeV}$ . Then (1.6) takes the form,

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2}\bar{N}_i(i\not{\partial} - M_i)N_i - (m_D)_{\alpha i}\bar{\nu}_\alpha P_R N_i - (m_D^*)_{\alpha i}\bar{N}_i P_L \nu_\alpha, \quad (1.9)$$

where  $\nu_\alpha$  are the active (SM) neutrinos and we have defined the Dirac mass term,  $(m_D)_{\alpha i} \equiv v h_{\alpha i}$ . The neutrino mass matrix (the hat denotes matrices),

$$\begin{pmatrix} 0 & \hat{m}_D \\ \hat{m}_D^T & \hat{M} \end{pmatrix}, \quad (1.10)$$

can be block diagonalized in the limit  $\hat{m}_D \ll \hat{M}$ . One obtains two distinct sets of eigenvalues. The first three eigenvalues are suppressed by a factor  $\hat{m}_D\hat{M}^{-1}$  and give naturally small masses to active neutrinos. They are found by diagonalizing the mass matrix obtained from the seesaw formula [84–87],

$$\hat{m}_\nu = -\hat{m}_D\hat{M}^{-1}\hat{m}_D^T. \quad (1.11)$$

The second set of eigenvalues is approximatively given by the masses  $M_i$  and corresponds to the physical mass of the right-handed neutrinos. Assuming the Yukawa couplings to be of

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<sup>1</sup>In this work we are using natural units where  $c = \hbar = k_B = 1$ . This implies that  $[\text{mass}] = [\text{energy}] = [\text{temperature}] = [\text{time}]^{-1} = [\text{length}]^{-1}$ .

the order  $\mathcal{O}(1)$ , we find from the active neutrino mass matrix (1.11) that the right-handed neutrino must be very heavy,  $M_i \sim 10^{13}$  GeV, if we want the active neutrinos mass to be of the right order of magnitude,  $m_\nu \sim 0.1$  eV, as suggested by neutrino oscillation experiments. However, the masses of right-handed neutrino can be much smaller if the Yukawa couplings are suppressed, see e.g. [88, 89]. In thermal leptogenesis, for a hierarchical right-handed neutrino mass spectrum ( $M_1 \ll M_{i \neq 1}$ ), the mass of the lightest right-handed neutrino ( $M_1$ ) is bounded from below if one wants to explain the baryon asymmetry of the universe [90]. This is the Davidson-Ibarra bound,

$$M_i \gtrsim 10^9 \text{ GeV}. \quad (1.12)$$

Such large masses make the production of right-handed neutrinos impossible in present or forthcoming accelerator facilities. This is the main drawback of the leptogenesis scenario. It can only be tested indirectly. For example the observation of neutrinoless double  $\beta$ -decay (see e.g. [91] for a review), which is consequence of the Majorana nature of the neutrinos, would give us an indirect hint of the existence of the right-handed neutrinos.

There are many ways of lowering the right-handed neutrino masses to testable scales. For example, in the case of quasidegeneracy between the right-handed neutrino masses (*resonant* leptogenesis [31]), the CP-violating parameter (see (1.13)) is resonantly enhanced. In this way the Davidson-Ibarra bound can be relaxed and the right-handed neutrinos can have masses in the TeV range [31]. In this work we only consider leptogenesis with a hierarchical mass spectrum, with  $M_1 \sim 10^9$  GeV.

### 1.2.3 Production of baryon asymmetry in the leptogenesis scenario

In the leptogenesis scenario an asymmetry is first produced in the lepton sector and then transferred to the baryons, hence the name *leptogenesis*. In the  $SU(2)_L$ -symmetric phase the heavy neutrinos can decay into a lepton-Higgs pair,  $N_i \rightarrow \ell\phi$ , or an antilepton-antihiggs<sup>1</sup> pair,  $N_i \rightarrow \bar{\ell}\bar{\phi}$ . In the presence of CP-violating phases in the Yukawa couplings  $h_{\alpha i}$  these decay rates can be unequal. In that case a lepton asymmetry is produced by the decay of the right-handed neutrinos. The CP-asymmetry is usually characterised by the so-called *CP-violating parameter*,

$$\epsilon_i \equiv \frac{\Gamma_{N_i \rightarrow \ell\phi} - \Gamma_{N_i \rightarrow \bar{\ell}\bar{\phi}}}{\Gamma_{N_i \rightarrow \ell\phi} + \Gamma_{N_i \rightarrow \bar{\ell}\bar{\phi}}}, \quad (1.13)$$

where  $\Gamma_{N_i \rightarrow \ell\phi}$  ( $\Gamma_{N_i \rightarrow \bar{\ell}\bar{\phi}}$ ) is the decay rate into a lepton-Higgs (antilepton-antihiggs) pair. The CP-violating parameter  $\epsilon_i$  corresponds to the average lepton asymmetry produced by each decay of  $N_i$  in vacuum. As mentioned in section 1.1 the baryon and lepton numbers are not conserved separately at high temperature due to the sphaleron processes [9]. The latter are non-perturbative electroweak processes which connect different vacua. They violate  $B + L$  but conserve  $B - L$ . Sphaleron processes are in equilibrium (i.e. their rates are larger than the expansion rate of the universe) in the temperature range  $140 \text{ GeV} \lesssim T \lesssim 10^{12} \text{ GeV}$  [92] for a Higgs mass  $m_\phi \approx 125 \text{ GeV}$ . The baryon ( $B$ ) and lepton ( $L$ ) numbers are related through

<sup>1</sup>Since we are working in the  $SU(2)_L$ -symmetric phase the Higgs field  $\phi$  is complex. In this work the terminology *Higgs* refers to the doublet  $\phi$  and *antihiggs* to its hermitian conjugate.

[93, 94],

$$B = -\frac{28}{51}L. \quad (1.14)$$

In this work we focus exclusively on the scenario of *thermal leptogenesis* in which the heavy neutrinos are *thermally* produced from the plasma. This scenario requires the reheating temperature  $T_{\text{re}}$  to be larger than the mass of the lightest right-handed neutrino  $N_1$ ,  $T_{\text{re}} \gtrsim M_1$ . If  $T_{\text{re}} \lesssim M_1$  the production rate of the right-handed neutrinos is Boltzmann suppressed,  $\propto e^{-M_i/T}$ . In this case the heavy neutrinos cannot be produced in sufficient amount and another production mechanism is needed, e.g. through inflaton decays [95]. Thermal production, together with the Davidson-Ibarra bound (1.12), implies  $T_{\text{re}} \gtrsim 10^9$  GeV<sup>1</sup>. Once the temperature drops below  $M_1$ , the equilibrium number density of the lightest right-handed neutrinos is exponentially suppressed. If the interaction rate of  $N_1$  with the plasma is not strong enough the number density of the right-handed neutrino cannot follow the equilibrium one. The third Sakharov condition, departure from thermal equilibrium, is therefore satisfied. If the interactions of  $N_1$  are CP-violating, the three Sakharov conditions are fulfilled and an asymmetry can be generated. The asymmetry produced in the lepton sector by the heavy neutrinos is then partially transferred (see (1.14)) to the baryon sector through the sphaleron processes.

At very low temperature, when  $T \ll M_1$ , the number densities of the heavy neutrinos are Boltzmann suppressed,  $\sim e^{-M_i/T}$ . The CP- and lepton number violating interactions become negligible since they require the presence of the right-handed neutrinos, and the  $B-L$  asymmetry freezes out. At even lower temperature,  $T \lesssim 140$  GeV, the sphaleron processes become ineffective. Then baryon and lepton numbers are separately conserved and the baryon asymmetry of the universe survives until now.

Computing of the baryon asymmetry produced by the leptogenesis mechanism requires to track down the number density of the different particles involved in the generation of the asymmetry. In the canonical approach this is done with the help of the Boltzmann equation, see chapter 2.

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<sup>1</sup>In the supersymmetric extension of leptogenesis, such a high reheating temperature leads to an overproduction of gravitinos which spoils BBN predictions.





# Chapter 2

## Boltzmann approach to leptogenesis

The Boltzmann equation has been used extensively in the literature to study leptogenesis (for a review, see e.g. [75, 77]). Different aspects of leptogenesis have been treated in the Boltzmann framework within the past few years, in particular flavour effects [26–30], resonant effects [31, 32], the role of spectator processes [33, 34] and  $N_2$  dominated leptogenesis [48, 49]. We first briefly review the Boltzmann equation in an expanding universe, and then apply it to the most simple regime, unflavoured leptogenesis with a hierarchical right-handed neutrino mass spectrum. This computation will reveal the problems inherent to the Boltzmann treatment of leptogenesis and will be used as a comparison with the more sophisticated approach based on nonequilibrium quantum field theory.

### 2.1 Boltzmann equation

Equilibrium thermodynamic ensembles can be well described by macroscopic quantities such as temperature, pressure or chemical potential. However, as required by the third Sakharov condition, the production of an asymmetry requires a departure from equilibrium. In order to capture the physics of a nonequilibrium system one needs to follow the time evolution of the one-particle distribution function  $f_\psi(x^\mu, p^\mu)$  for each particle species  $\psi$ . The distribution function satisfies the Boltzmann equation, which can be schematically written as [96],

$$\hat{\mathbf{L}}[f_\psi] = \mathbf{C}[f_\psi], \quad (2.1)$$

where  $\hat{\mathbf{L}}$  is the Liouville operator and  $\mathbf{C}$  is the collision operator. The Liouville operator can be written in terms of the covariant derivative,  $\mathcal{D}_\mu$ ,

$$\hat{\mathbf{L}}[f_\psi] = p^\mu \mathcal{D}_\mu f_\psi. \quad (2.2)$$

In a Friedmann-Robertson-Walker (FRW) universe the distribution function depends only on the time and energy variable,  $f_\psi = f_\psi(E, t)$ , and the Liouville operator simplifies to

$$\hat{\mathbf{L}}[f_\psi] = \left( E \frac{\partial}{\partial t} - H |\vec{p}|^2 \frac{\partial}{\partial E} \right) f_\psi(E, t), \quad (2.3)$$

where  $H$  is the Hubble rate. In the radiation dominated epoch, where leptogenesis takes place,  $H = 1/(2t) = \sqrt{4\pi^3 g_*/45} T^2 / M_{pl}$ . Here  $g_* = 106.75$  (in the SM without the heavy neutrinos) is the number of relativistic degrees of freedom and  $M_{pl} = 1.22 \cdot 10^{19}$  GeV is the Planck mass. For notational convenience we will often suppress the time argument and write the energy (or, equivalently, the momentum) as a subscript,  $f_\psi^p \equiv f_\psi^E \equiv f_\psi(E, t)$ .

The collision operator can be written as a sum of collision terms for each process  $\psi + a + \dots \leftrightarrow i + \dots$ ,

$$\mathbf{C}[f_\psi] = \sum_{\{a\}, \{i\}} \mathbf{C}_{\psi+a+\dots \leftrightarrow i+\dots}[f_\psi], \quad (2.4)$$

with

$$\begin{aligned} \mathbf{C}_{\psi+a+\dots \leftrightarrow i+\dots}[f_\psi] = & \frac{1}{2} \int d\Pi_{a\dots i\dots}^{p_a\dots p_i\dots} (2\pi)^4 \delta(p_\psi + p_a + \dots - p_i - \dots) \\ & \times \left[ |\mathcal{M}_{i+\dots \rightarrow \psi+a+\dots}^2 \left( \prod_i f_i^{p_i} \right) (1 \pm f_\psi^{p_\psi}) \left( \prod_a (1 \pm f_a^{p_a}) \right) \right. \\ & \left. - |\mathcal{M}_{\psi+a+\dots \rightarrow i+\dots}^2 f_\psi^{p_\psi} \left( \prod_a f_a^{p_a} \right) \left( \prod_i (1 \pm f_i^{p_i}) \right) \right], \quad (2.5) \end{aligned}$$

where  $|\mathcal{M}_{i+\dots \rightarrow \psi+a+\dots}^2$  is the transition amplitude summed over the internal degrees of freedom of the particle species  $\{a\}$  and  $\{i\}$ , and  $d\Pi_{ab\dots}^{p_a p_b \dots}$  is a product of Lorentz invariant integral measure,

$$d\Pi_{ab\dots}^{p_a p_b \dots} = \frac{d^3 \vec{p}_a}{2E_a^{p_a} (2\pi)^3} \frac{d^3 \vec{p}_b}{2E_b^{p_b} (2\pi)^3} \dots \quad (2.6)$$

The factors  $(1 \pm f)$ , where the  $+$  sign applied to bosons, and  $-$  sign to fermions, are called the *quantum statistical factors*, and are manifestation of the Bose enhancement or Pauli blocking. The first term in the square bracket of (2.5) is the so-called *gain term* as it increases the number of  $\psi$ -particle, and the second one is called the *loss term* since it decreases the number of  $\psi$ -particle. Since the transition amplitudes in (2.5) are computed in the S-matrix formalism we will often refer to this approach as the *S-matrix approach*.

The particle number density  $n_\psi$  is given by the distribution function integrated over the momentum,

$$n_\psi(t) = \sum_{s_\psi} \int \frac{d^3 \vec{p}}{(2\pi)^3} f_\psi(E, t), \quad (2.7)$$

where the sum is over the internal degrees of freedom of  $\psi$ . Integrating the RHS of the Liouville operator (2.3) with the measure  $d^3 \vec{p} / [E_\psi^p (2\pi)^3]$  and summing over the internal degrees of freedom, we find  $dn_\psi/dt + 3Hn_\psi$  and write the Boltzmann equation as,

$$\begin{aligned} \frac{dn_\psi}{dt} + 3Hn_\psi = & \sum_{\{a\}, \{i\}} \int d\Pi_{\psi a \dots i \dots}^{p_\psi p_a \dots p_i \dots} (2\pi)^4 \delta(p_\psi + p_a + \dots - p_i - \dots) \\ & \times \left[ \Xi_{i+\dots \rightarrow \psi+a+\dots} \prod_i f_i^{p_i} (1 \pm f_\psi^{p_\psi}) \prod_a (1 \pm f_a^{p_a}) \right. \\ & \left. - \Xi_{\psi+a+\dots \rightarrow i+\dots} f_\psi^{p_\psi} \prod_a f_a^{p_a} \prod_i (1 \pm f_i^{p_i}) \right], \quad (2.8) \end{aligned}$$

where we have defined the transition amplitude summed over *all* internal degrees of freedom

as,

$$\Xi_{\psi+a+\dots\rightarrow i+\dots} \equiv \sum_{s_\psi} |\mathcal{M}|_{\psi+a+\dots\rightarrow i+\dots}^2. \quad (2.9)$$

The LHS of (2.8) is often written in terms of the abundance  $Y_\psi \equiv n_\psi/s$ , where  $s = \frac{2\pi^2}{45} g_* T^3$  is the comoving entropy density,

$$\frac{dn_\psi}{dt} + 3Hn_\psi = s \frac{dY_\psi}{dt}. \quad (2.10)$$

## 2.2 Boltzmann equation at $\mathcal{O}(h^4)$

We now apply the Boltzmann equation (2.8) to leptogenesis. We consider here only the terms up to  $\mathcal{O}(h^4)$ . The inclusion of the top quark, which contributes to the production of the asymmetry at  $\mathcal{O}(h^4 \lambda_t^2)$ , is performed in the next section.

At this order the processes contributing to the Boltzmann equation for the lepton asymmetry are the (inverse) decay of the heavy Majorana neutrinos and the  $|\Delta L| = 2$  scattering processes mediated by the right-handed neutrinos.

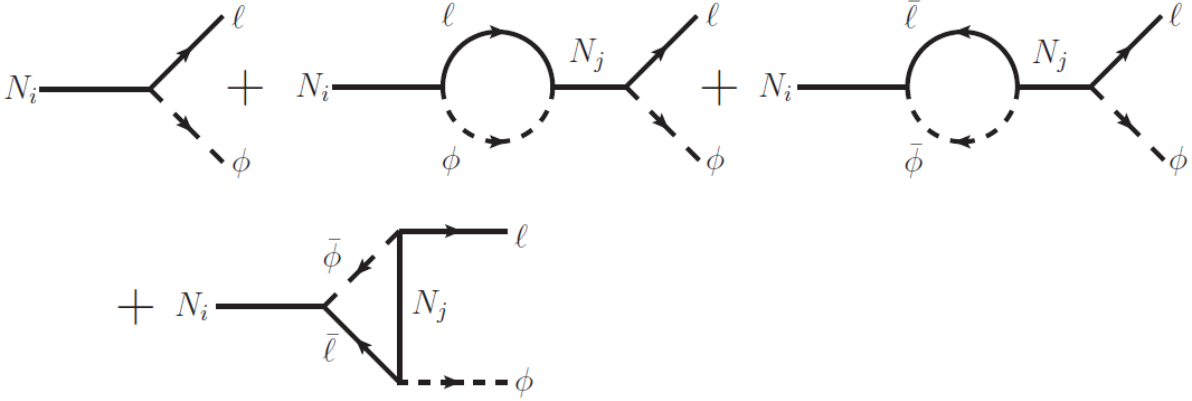


Figure 2.1: Decay amplitude  $\Xi_{N_i \rightarrow \ell \phi}$  at one-loop order. The second and third diagrams are the self-energy or wave-function diagrams, and the last one is the vertex diagram. Note that the second diagram is CP-conserving and does not contribute to the production of the total lepton asymmetry, but is relevant for the study of flavour leptogenesis [26–30].

Let us first consider the (inverse) decay. At  $\mathcal{O}(h^4)$  we need to consider the one-loop corrections to the decay amplitudes, see Fig. 2.1. We follow here the approach of [31], where effective Yukawa couplings are used to take into account the loop corrections,

$$\lambda_{+,\alpha i} \equiv h_{\alpha i} - i h_{\alpha j} (h^\dagger h)_{ji}^* f_{ij}, \quad (2.11a)$$

$$\lambda_{-,\alpha i} \equiv h_{\alpha i}^* - i h_{\alpha j}^* (h^\dagger h)_{ji} f_{ij}, \quad (2.11b)$$

where  $f_{ij}$  is the loop function,

$$f_{ij} \equiv \frac{1}{16\pi} \frac{M_i M_j}{M_i^2 - M_j^2} + \frac{1}{16\pi} \frac{M_j}{M_i} \left[ 1 - \left( 1 + \frac{M_j^2}{M_i^2} \right) \ln \left( 1 + \frac{M_i^2}{M_j^2} \right) \right]. \quad (2.12)$$

The first term in (2.12) describes the self-energy (or wave-function diagram) [97–99], and the second one the vertex diagram [25]. In the hierarchical limit,  $M_i \ll M_j$ , the loop function reads,

$$f_{ij} \approx -\frac{3}{32\pi} \frac{M_i}{M_j}. \quad (2.13)$$

Then the decay amplitudes at one-loop take the same form as the tree-level ones, but with the effective Yukawa couplings,

$$\Xi_{N_i \rightarrow \ell \phi} = 2g_w (\lambda_+^\dagger \lambda_+)_{ii}(pq), \quad (2.14a)$$

$$\Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}} = 2g_w (\lambda_-^\dagger \lambda_-)_{ii}(pq), \quad (2.14b)$$

where the factor  $g_w = 2$  comes from the summation over the  $SU(2)_L$  indices and  $(pq) = M_i^2/2$  under the constraints of the energy conserving delta-function. We are using here the notation  $\Xi_{X \rightarrow Y}$  for amplitude summed over all internal degrees of freedom. The decay rates in the rest frame of the parent particle are then given by,

$$\Gamma_{N_i \rightarrow \ell \phi} = \frac{\Xi_{N_i \rightarrow \ell \phi}}{32\pi M_i} = \frac{g_w}{32\pi} (\lambda_+^\dagger \lambda_+)_{ii} M_i, \quad (2.15a)$$

$$\Gamma_{N_i \rightarrow \bar{\ell} \bar{\phi}} = \frac{\Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}}}{32\pi M_i} = \frac{g_w}{32\pi} (\lambda_-^\dagger \lambda_-)_{ii} M_i, \quad (2.15b)$$

where we have neglected the mass of the final particle. Note that the effective Yukawa couplings are not complex conjugate of each other,  $\lambda_{+,\alpha i} \neq \lambda_{-,\alpha i}^*$ , which reflects the fact that the decay of the right-handed neutrino is CP-violating at one-loop level. The CP-asymmetry is usually parametrised by the CP-violating parameters,

$$\epsilon_i \equiv \frac{\Gamma_{N_i \rightarrow \ell \phi} - \Gamma_{N_i \rightarrow \bar{\ell} \bar{\phi}}}{\Gamma_{N_i}} = \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}} 2f_{ij}, \quad (2.16)$$

where we have defined the total decay rate,  $\Gamma_{N_i} \equiv \Gamma_{N_i \rightarrow \ell \phi} + \Gamma_{N_i \rightarrow \bar{\ell} \bar{\phi}}$ .  $\epsilon_i$  corresponds to the average fraction of lepton number produced by each decay of  $N_i$  and plays a central role in the study of leptogenesis.

Using (2.14) we can write the Boltzmann equation for the lepton as,

$$s \frac{dY_\ell}{dt} = \sum_i \int d\Pi_{\ell N_i \phi}^{pqk} (2\pi)^4 \delta(q - p - k) \times \left[ \Xi_{N_i \rightarrow \ell \phi} f_{N_i}^q (1 - f_\ell^p) (1 + f_\phi^k) - \Xi_{\ell \phi \rightarrow N_i} f_\ell^p f_\phi^k (1 - f_{N_i}^q) \right], \quad (2.17)$$

and a similar equation for the antilepton,

$$s \frac{dY_{\bar{\ell}}}{dt} = \sum_i \int d\Pi_{\bar{\ell} N_i \bar{\phi}}^{pqk} (2\pi)^4 \delta(q - p - k) \times \left[ \Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}} f_{N_i}^q (1 - f_{\bar{\ell}}^p) (1 + f_{\bar{\phi}}^k) - \Xi_{\bar{\ell} \bar{\phi} \rightarrow N_i} f_{\bar{\ell}}^p f_{\bar{\phi}}^k (1 - f_{N_i}^q) \right]. \quad (2.18)$$

The inverse decay amplitudes are related the decay amplitude by the CPT symmetry,

$$\Xi_{\ell \phi \rightarrow N_i} = \Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}}, \quad \text{and} \quad \Xi_{\bar{\ell} \bar{\phi} \rightarrow N_i} = \Xi_{N_i \rightarrow \ell \phi}. \quad (2.19a)$$

Equations (2.17) and (2.18) describe the time evolution of the lepton and antilepton number density in the FRW universe. However they are incomplete as we will see by the following computation. In thermal equilibrium, the distribution functions of particle and antiparticle are equal and satisfy the detailed balance condition,

$$f_{N_i}^{\text{eq},q} (1 - f_{\ell}^{\text{eq},p}) (1 + f_{\phi}^{\text{eq},k}) = f_{\ell}^{\text{eq},p} f_{\phi}^{\text{eq},k} (1 - f_{N_i}^{\text{eq},q}), \quad (2.20)$$

where we have used the energy conservation. The equilibrium distribution functions in (2.20) are the Bose-Einstein or Fermi-Dirac distribution functions,

$$f_a^{\text{eq},p} = \frac{1}{e^{E_a^p/T} - 1}, \quad \text{and} \quad f_a^{\text{eq},p} = \frac{1}{e^{E_a^p/T} + 1}, \quad (2.21)$$

for bosons or fermions, respectively. The time evolution of the lepton asymmetry abundance,  $Y_L = Y_{\ell} - Y_{\bar{\ell}}$ , is obtained by subtracting (2.17) and (2.18),

$$s \frac{dY_L}{dt} = 2 \sum_i \int d\Pi_{\ell N_i \phi}^{pqk} (2\pi)^4 \delta(q - p - k) 2\epsilon_i \Gamma_{N_i} f_{N_i}^{\text{eq},q} (1 - f_{\ell}^{\text{eq},p}) (1 + f_{\phi}^{\text{eq},k}), \quad (2.22)$$

where we have used (2.16) and assumed that the antiparticle dispersion relation is the same as its corresponding particle. Neglecting the quantum statistical factor,  $(1 - f_{\ell}^{\text{eq},p}) (1 + f_{\phi}^{\text{eq},k}) \approx 1$ , and using the Maxwell-Boltzmann distribution function for the right-handed neutrinos,  $f_{N_i}^{\text{eq},q} \approx \exp(-E_{N_i}^q/T)$ , we can show by integrating over the phase space that the RHS of the above equation is non-zero (see appendix B for details about the phase space integration),

$$s \frac{dY_L}{dt} = \sum_i \frac{2\epsilon_i \Gamma_{N_i} T M_i^2}{\pi} K_1\left(\frac{M_i}{T}\right) \neq 0, \quad (2.23)$$

where  $K_1$  is the modified Bessel function of the second kind. This implies that non-zero asymmetry can be produced even in thermal equilibrium, which is in contradiction with the third Sakharov condition. One could think that this problem arises because we did not consider the  $|\Delta L| = 2$  scattering processes, which are of the same order in the Yukawa coupling. However, including the tree-level scattering does not solve this problem because the tree-level scattering processes, which are at CP-conserving, do not contribute to the RHS of (2.22) and cannot cancel the contribution of the RHS of (2.23).

This problem is a consequence of the fact that the process consisting of an inverse decay followed by the decay is counted twice in the Boltzmann equation, once in the decay process, and once in the s-channel scattering amplitude where the intermediate state is on-shell (RIS,

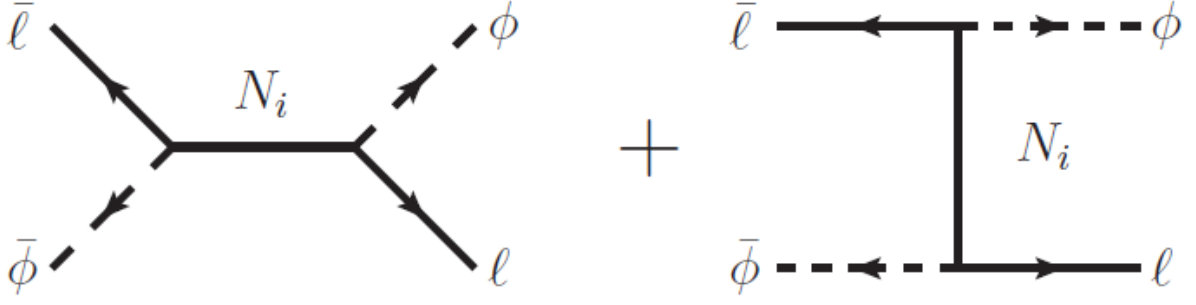


Figure 2.2:  $s$ - and  $t$ -channel contribution to the tree-level scattering amplitude  $\Xi_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi}$ . This process is lepton-violating but CP-conserving at tree-level.

or real intermediate state). This is the so-called *double counting* problem.

Let us check that these two contributions are indeed equal. The only processes which can exhibit a RIS are the scattering processes  $\bar{\ell}\bar{\phi} \leftrightarrow \ell\phi$ , see Fig. 2.2. Their amplitudes squared are given by [31]:

$$\begin{aligned} \Xi_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi} = 2g_w(p_1p_2) \sum_{i,j} M_i M_j & \left[ g_w(\lambda_+^\dagger \lambda_+)^2_{ij} P_i^*(s) P_j(s) + g_w(h^\dagger h)_{ij} P_i^*(t) P_j(t) \right. \\ & \left. + (\lambda_+^\dagger h)^2_{ij} P_i^*(s) P_j(t) + (h^\dagger \lambda_+)^2_{ij} P_i^*(t) P_j(s) \right], \end{aligned} \quad (2.24a)$$

$$\begin{aligned} \Xi_{\ell\phi\rightarrow\bar{\ell}\bar{\phi}} = 2g_w(p_1p_2) \sum_{i,j} M_i M_j & \left[ g_w(\lambda_-^\dagger \lambda_-)^2_{ij} P_i^*(s) P_j(s) + g_w(h^\dagger h)_{ij} P_i^*(t) P_j(t) \right. \\ & \left. + (\lambda_-^\dagger h)^2_{ij} P_i^*(s) P_j(t) + (h^\dagger \lambda_-)^2_{ij} P_i^*(t) P_j(s) \right], \end{aligned} \quad (2.24b)$$

where  $p_{1,2}$  are the momenta of the initial and final (anti)leptons,  $s$  and  $t$  are the usual Mandelstam variables, and we have defined the “scalar” heavy Majorana propagators,

$$P_i(q^2) = \frac{1}{q^2 - M_i^2 + i\theta(q^2)M_i\Gamma_{N_i}}, \quad (2.25)$$

where  $\theta(\cdot)$  is the Heaviside step function. In (2.25) we have resummed an infinite series of self-energy diagrams to take into account the finite width of the right-handed neutrino. Note that we include the next-to-leading order in the  $s$ -channel amplitude in (2.24) by using the effective Yukawa couplings. This is crucial in order to obtain amplitude at  $\mathcal{O}(h^4)$  when the intermediate neutrino goes on-shell. In (2.24a) the only terms where both propagators  $P_i^*$  and  $P_j$  can be simultaneously on-shell are the  $s \times s$  interference channel with  $i = j$ . We can therefore write the RIS part of the scattering amplitude (2.24a) as,

$$\begin{aligned} \Xi_{\bar{\ell}\bar{\phi}\rightarrow N_i \rightarrow \ell\phi} & = 2(p_1p_2)g_w^2(\lambda_+^\dagger \lambda_+)^2_{ii} |P_i(s)|^2 \\ & = \frac{2(p_1p_2)}{M_i^2} \Xi_{\bar{\ell}\bar{\phi}\rightarrow N_i} \frac{1}{(s - M_i)^2 + (M_i\Gamma_{N_i})^2} \Xi_{N_i \rightarrow \ell\phi}. \end{aligned} \quad (2.26)$$

In the narrow width approximation we can approximate the Breit-Wigner function by a Dirac

delta using the representation,

$$\lim_{\epsilon \rightarrow 0^+} \frac{\epsilon}{\omega^2 + \epsilon^2} = \pi \delta(\omega), \quad (2.27)$$

and write,

$$\Xi_{\bar{\ell}\bar{\phi} \rightarrow N_i \rightarrow \ell\phi} \approx \frac{2(p_1 p_2)}{M_i^2} \Xi_{\bar{\ell}\bar{\phi} \rightarrow N_i} \frac{\pi \delta(s - M_i)}{M_i \Gamma_{N_i}} \Xi_{N_i \rightarrow \ell\phi}. \quad (2.28)$$

We see from the appearance of  $\Gamma_{N_i}$  in the denominator of the above expression the importance of computing the  $s$ -channel amplitude at next-to-leading order. Similarly the RIS part of the scattering amplitude (2.24b) is given by,

$$\Xi_{\ell\phi \rightarrow N_i \rightarrow \bar{\ell}\bar{\phi}} \approx \frac{2(p_1 p_2)}{M_i^2} \Xi_{\ell\phi \rightarrow N_i} \frac{\pi \delta(s - M_i)}{M_i \Gamma_{N_i}} \Xi_{N_i \rightarrow \bar{\ell}\bar{\phi}}. \quad (2.29)$$

The RIS parts (2.28) and (2.29) give the following contribution to the Boltzmann equation for the lepton number,

$$\begin{aligned} s \frac{dY_L}{dt} = & 2 \sum_i \int d\Pi_{\ell\phi\ell\phi}^{p_1 k_1 p_2 k_2} (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \\ & \times \left[ \Xi_{\bar{\ell}\bar{\phi} \rightarrow N_i \rightarrow \ell\phi} f_{\bar{\ell}}^{p_1} f_{\bar{\phi}}^{k_1} (1 - f_{\ell}^{p_2}) (1 + f_{\phi}^{k_2}) - \Xi_{\ell\phi \rightarrow N_i \rightarrow \bar{\ell}\bar{\phi}} f_{\ell}^{p_1} f_{\phi}^{k_1} (1 - f_{\bar{\ell}}^{p_2}) (1 + f_{\bar{\phi}}^{k_2}) \right]. \end{aligned} \quad (2.30)$$

The overall factor of 2 comes from the fact that the processes (2.24) violate lepton number by two units. Using equilibrium distribution function and neglecting the quantum statistical factors we can rewrite (2.30) as,

$$\begin{aligned} s \frac{dY_L}{dt} = & 2^{12} \pi^3 \sum_i \frac{\Gamma_{N_i} \epsilon_i}{M_i} \int d\Pi_{\ell\phi\ell\phi}^{p_1 k_1 p_2 k_2} (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \\ & \times f_{\ell}^{\text{eq}, p_1} f_{\phi}^{\text{eq}, k_1} \delta((p_1 + k_1)^2 - M_i^2) (p_1 p_2). \end{aligned} \quad (2.31)$$

The integration over the phase-space can be performed analytically (see appendix B) and we recover exactly the extra production term from the decay, see the RHS of (2.23). This computation shows that the decay term producing an asymmetry in equilibrium can be exactly cancelled by subtracting the RIS part of the  $s$ -channel scattering amplitude. We therefore define the RIS-subtracted amplitude (which is denoted by a prime) as the difference between the tree-level scattering amplitude,  $\Xi_{\bar{\ell}\bar{\phi} \rightarrow \ell\phi}^{(T)}$ , and the RIS part,

$$\begin{aligned} \Xi'_{\bar{\ell}\bar{\phi} \rightarrow \ell\phi} & \equiv \Xi_{\bar{\ell}\bar{\phi} \rightarrow \ell\phi}^{(T)} - \sum_i \Xi_{\bar{\ell}\bar{\phi} \rightarrow N_i \rightarrow \ell\phi} \\ & = 2g_w (p_1 p_2) \sum_{i,j} M_i M_j (h^\dagger h)_{ij}^2 \left[ g_w P_i^*(s) P_j(s) + g_w P_i^*(t) P_j(t) + P_i^*(s) P_j(t) + P_i^*(t) P_j(s) \right] \\ & \quad - \sum_i 2g_w^2 (p_1 p_2) (\lambda_+^\dagger \lambda_+)_{ii}^2 \frac{\pi \delta(s - M_i)}{M_i \Gamma_{N_i}}, \end{aligned} \quad (2.32)$$

and a similar definition for  $\Xi'_{\ell\phi \rightarrow \bar{\ell}\bar{\phi}}$  with  $\lambda_+$  replaced by  $\lambda_-$ . It is then easy to see that the

RHS of the Boltzmann equation for the lepton asymmetry will vanish in thermal equilibrium if we use the RIS-subtracted amplitude (2.32) instead of the naive scattering amplitude. Since  $\Xi_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi}^{(T)}$  and  $\Xi_{\ell\phi\rightarrow\bar{\ell}\bar{\phi}}^{(T)}$  are CP-conserving they are equal and will cancel each other in thermal equilibrium. Therefore we will recover the RHS of (2.31), which cancel the RHS of (2.23). The sum of the RIS-amplitudes  $\Xi'_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi}$  and  $\Xi'_{\ell\phi\rightarrow\bar{\ell}\bar{\phi}}$  contributes to the washout of the lepton asymmetry and is given by,

$$\begin{aligned}\Xi'_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi} &\equiv \frac{1}{2} \left[ \Xi'_{\bar{\ell}\bar{\phi}\rightarrow\ell\phi} + \Xi'_{\ell\phi\rightarrow\bar{\ell}\bar{\phi}} \right] \\ &= 2g_w(p_1 p_2) \sum_{i,j} M_i M_j (h^\dagger h)_{ij} \left[ g_w \mathcal{P}_{ij}^2(s) + g_w P_i^*(t) P_j(t) + P_i^*(s) P_j(t) + P_i^*(t) P_j(s) \right],\end{aligned}\quad (2.33)$$

where we have defined the RIS-subtracted propagator squared,

$$\mathcal{P}_{ij}^2(q^2) = \begin{cases} P_i^*(q^2) P_j(q^2) & \text{for } i \neq j, \\ |P_i(q)|^2 - \frac{\pi}{M_i \Gamma_{N_i}} \delta(q^2 - M_i^2) & \text{for } i = j. \end{cases}\quad (2.34)$$

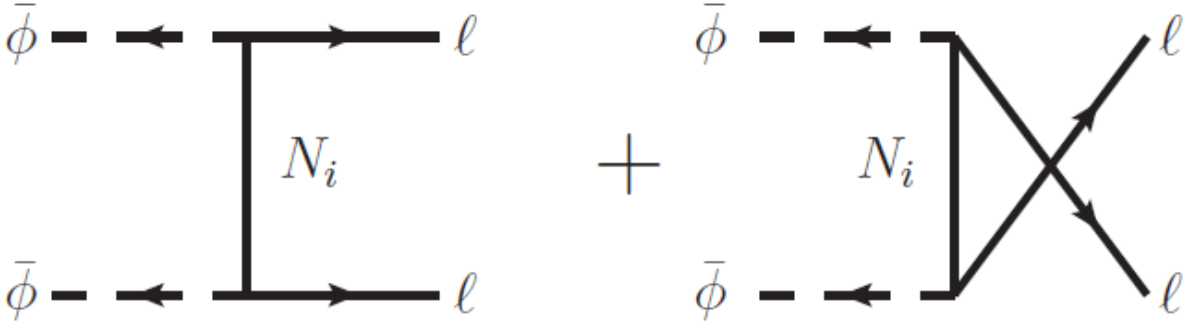


Figure 2.3:  $t$ - and  $u$ -channel contributions to the tree-level scattering amplitude  $\Xi_{\bar{\phi}\bar{\phi}\rightarrow\ell\ell}$ .

The other  $|\Delta L| = 2$  scattering amplitudes do not need to be RIS-subtracted since they are product of  $t$ - and  $u$ -channels only, see Fig. 2.3. At tree-level they read,

$$\begin{aligned}\Xi_{\bar{\phi}\bar{\phi}\rightarrow\ell\ell}^{(T)} = \Xi_{\ell\ell\rightarrow\bar{\phi}\bar{\phi}}^{(T)} &= \frac{2g_w(p_1 p_2)}{2!2!} \sum_{ij} M_i M_j (h^\dagger h)_{ij} \left[ g_w P_i(t) P_j^*(t) + g_w P_i(u) P_j^*(u) \right. \\ &\quad \left. + P_i(t) P_j^*(u) + P_i(u) P_j^*(t) \right],\end{aligned}\quad (2.35)$$

where  $p_{1,2}$  are the momenta of the (anti)leptons and the factors  $1/(2!)$  are the symmetry factors due to identical particles in the initial and final states. Finally we can write the Boltzmann



equation for the lepton abundance at order  $\mathcal{O}(h^4)$ ,

$$\begin{aligned}
 s \frac{dY_\ell}{dt} = & \sum_i \int d\Pi_{\ell N_i \phi}^{pqk} (2\pi)^4 \delta(q - p - k) \left[ \Xi_{N_i \rightarrow \ell \phi}^q f_{N_i}^q - \Xi_{\ell \phi \rightarrow N_i}^p f_\ell^p f_\phi^k \right] \\
 & + \int d\Pi_{\phi \ell \phi}^{p_1 k_1 p_2 k_2} (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \left[ \Xi'_{\bar{\ell} \phi \rightarrow \ell \phi} f_\ell^{p_1} f_\phi^{k_1} - \Xi'_{\ell \phi \rightarrow \bar{\ell} \phi} f_\ell^{p_1} f_\phi^{k_1} \right] \\
 & + 2 \int d\Pi_{\phi \phi \ell \ell}^{k_1 k_2 p_1 p_2} (2\pi)^4 \delta(k_1 + k_2 - p_1 - p_2) \Xi_{\phi \phi \rightarrow \ell \ell}^{(T)} \left[ f_\phi^{k_1} f_\phi^{k_2} - f_\ell^{p_1} f_\ell^{p_2} \right],
 \end{aligned} \tag{2.36}$$

where the factor of 2 in the last line comes from the fact that the scattering  $\Xi_{\phi \phi \rightleftharpoons \ell \ell}$  produce (destroy) two leptons. This equation must be supplemented by the Boltzmann equations for the heavy Majorana neutrinos,

$$s \frac{dY_{N_i}}{dt} = \int d\Pi_{N_i \ell \phi}^{qpk} (2\pi)^4 \delta(q - p - k) \left[ \Xi_{\ell \phi \rightarrow N_i} \left( f_\ell^q f_\phi^k - f_{N_i}^q \right) + \Xi_{\bar{\ell} \phi \rightarrow N_i} \left( f_\ell^p f_\phi^k - f_{N_i}^q \right) \right]. \tag{2.37}$$

In (2.36) and (2.37) we have neglected the quantum statistical factors to ensure consistency with the RIS-subtraction procedure.

Let us now summarise the procedure that we used above to obtain a consistent set of Boltzmann equations. We first saw that the Boltzmann equations for the lepton and antilepton, used with amplitudes computed in the S-matrix formalism, are inconsistent in the sense that a lepton asymmetry is produced even in thermal equilibrium. We identified the problem as a double counting of some processes, which need therefore to be subtracted from the amplitudes. This double counting comes from the fact that, in the S-matrix formalism, the initial and final states are assumed to be stable, or long-lived enough, to be well described by in- or out-states. However, in leptogenesis, the most relevant processes involve the heavy Majorana neutrinos, which are unstable. One need therefore to correct the S-matrix amplitudes by subtracting the real intermediate state.

Even if the obtained Boltzmann equation are consistent with the third Sakharov condition they are not completely satisfying. The RIS-subtraction corrects the Boltzmann equation in *equilibrium* but we are interested in the out-of-equilibrium evolution of the lepton asymmetry. The procedure performed above does not ensure that the Boltzmann equation (2.36) is correct when non-equilibrium distribution functions are used on the RHS. Moreover, our derivation of the RIS-subtracted amplitudes neglected the quantum statistical factors and used the equality,  $f_\ell^{\text{eq},p} f_\phi^{\text{eq},k} = f_{N_i}^{\text{eq},q}$ , which is only true when Maxwell-Boltzmann statistics are used. The RIS-subtraction with quantum statistics is more involved and can be performed using the optical theorem [72] but will not be discussed here.

We will see in chapter 4 that nonequilibrium quantum field theory can solved the above problem in a consistent way. The equation for the lepton asymmetry within this formalism automatically satisfies the third Sakharov condition. We therefore do not need to performed any RIS-subtraction procedures and the quantum statistical factors can be kept.

## 2.3 Higgs mediated processes

Other important processes for leptogenesis are the  $|\Delta L| = 1$  scatterings with the top quark or the gauge bosons mediated by the Higgs doublet. We perform here only the computation with the top quarks. This computation will be used for comparison with the NEQFT treatment.

In the S-matrix formalism the scatterings with the gauge bosons are treated in a similar way. However, when thermal effects are taking into account, the contribution of the gauge bosons must be treated in a different way, as multiple interactions with the gauge boson must be resummed and contribute at leading order [71, 100, 101]. A systematic treatment of the gauge boson interactions in leptogenesis is still missing in the literature. For simplicity, in this work we take into account the effects of the gauge interactions only through thermal masses of the SM particles, and neglect all other effects.

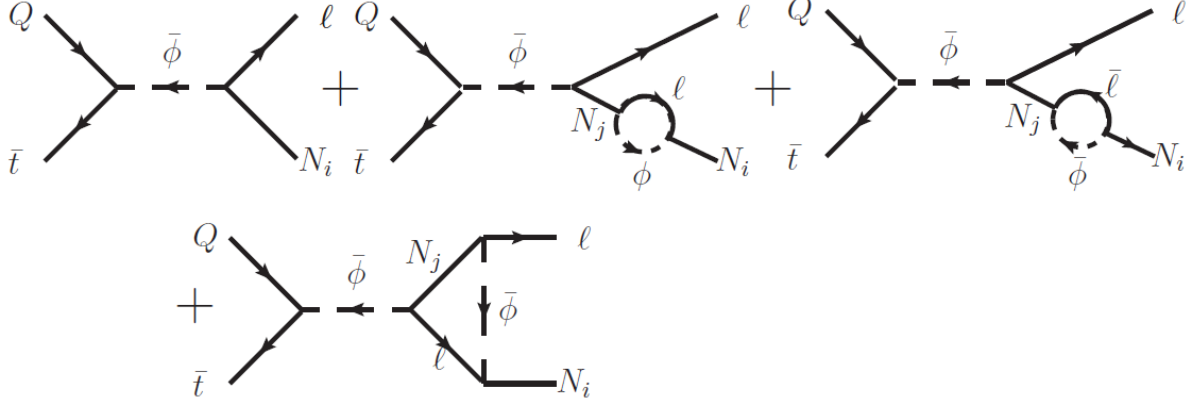


Figure 2.4: One-loop scattering amplitude  $\Xi_{\bar{t}Q \rightarrow N_i \ell}$ . Similarly to the one-loop heavy neutrino decay the second and third diagrams are the self-energy or wave-function corrections, and the last one is the vertex correction. The second diagram is CP-conserving and does not contribute to the generation of the lepton asymmetry.

The only relevant SM Yukawa coupling is the one of the top quark,

$$\mathcal{L}_{SM} \supset -\lambda_t \bar{Q} \tilde{\phi} P_R t - \lambda_t^* \bar{t} P_L \tilde{\phi}^\dagger Q, \quad (2.38)$$

where  $Q$  is the quark doublet of the third generation and  $t$  is the top quark singlet, and  $\lambda_t \approx 1$  is the top Yukawa coupling. The lepton violating scattering processes involving the top are the  $s$ -channel scattering processes  $\bar{t}Q \rightleftharpoons N_i \ell$  (see Fig. 2.4), and the  $t$ -channel processes  $N_i Q \rightleftharpoons \ell t$  (see Fig. 2.5) and  $N_i \bar{t} \rightleftharpoons \ell \bar{Q}$  (see Fig. 2.6). For a hierarchical mass spectrum the one-loop  $s$ -channel amplitudes read,

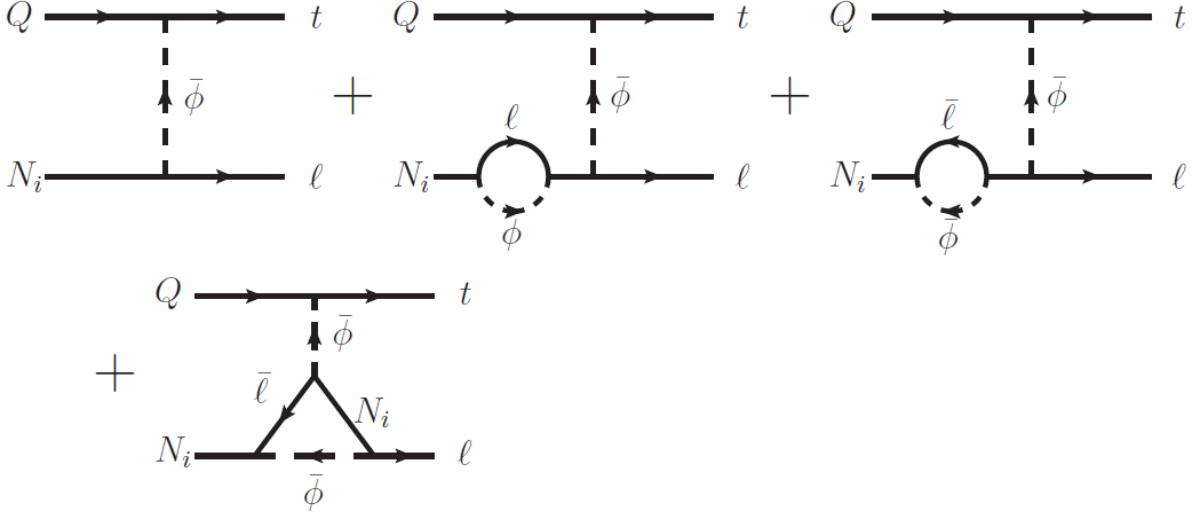
$$\Xi_{\bar{t}Q \rightarrow N_i \ell} = \Xi_{\bar{t}Q \rightarrow \bar{\phi}} \times \Delta_{\phi, F}^2(p+q) \times \Xi_{\bar{\phi} \ell \rightarrow N_i}, \quad (2.39a)$$

$$\Xi_{N_i \ell \rightarrow \bar{t}Q} = \Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}} \times \Delta_{\phi, F}^2(p+q) \times \Xi_{\bar{\phi} \rightarrow \bar{t}Q}, \quad (2.39b)$$

where  $\Delta_{\phi, F}^2(k) \approx 1/(k^2 - m_\phi^2)$  is the Feynman (or time-ordered) Higgs propagator,  $\Xi_{\bar{\phi} \ell \rightarrow N_i}$  and  $\Xi_{N_i \rightarrow \bar{\ell} \bar{\phi}}$  are given by (2.14a) and (2.14b), respectively, and

$$\Xi_{\bar{t}Q \rightarrow \bar{\phi}} = \Xi_{\bar{\phi} \rightarrow \bar{t}Q} = 2g_q |\lambda_t|^2 (p_Q p_t). \quad (2.40)$$

Here  $g_q = 3$  is the  $SU(3)_C$  factor, and  $q$ ,  $p$ ,  $p_t$  and  $p_Q$  are the momenta of the heavy neutrino, lepton, top quark and quark doublet, respectively. Note that (2.40) neglects the CP-violation in the top Yukawa coupling. Similarly, the  $t$ -channel scattering amplitudes  $\Xi_{N_i Q \rightleftharpoons \ell t}$  are given


 Figure 2.5: One-loop level scattering amplitude  $\Xi_{N_i Q \rightarrow t \ell}$ . See comments under Fig. 2.4.

by,

$$\Xi_{N_i Q \rightarrow t \ell} = \Xi_{N_i \bar{t} \rightarrow \ell \bar{Q}} = \Xi_{N_i \rightarrow \ell \phi} \times \Delta_{\phi, F}^2(q-p) \times \Xi_{Q \phi \rightarrow t}, \quad (2.41a)$$

$$\Xi_{\ell t \rightarrow N_i Q} = \Xi_{\ell \bar{Q} \rightarrow N_i \bar{t}} = \Xi_{\ell \phi \rightarrow N_i} \times \Delta_{\phi, F}^2(q-p) \times \Xi_{t \rightarrow Q \phi}, \quad (2.41b)$$

where  $\Xi_{Q \phi \rightarrow t}$  and  $\Xi_{t \rightarrow Q \phi}$  are both given by (2.40). The other lepton violating processes involving the top quark are the 3-body (inverse) decays,

$$\Xi_{N_i \rightarrow t \ell \bar{Q}} = \Xi_{N_i \rightarrow \ell \phi} \times \Delta_{\phi, F}^2(q-p) \times \Xi_{\phi \rightarrow t \bar{Q}}, \quad (2.42a)$$

$$\Xi_{\ell t \bar{Q} \rightarrow N_i} = \Xi_{\ell \phi \rightarrow N_i} \times \Delta_{\phi, F}^2(q-p) \times \Xi_{t \bar{Q} \rightarrow \phi}. \quad (2.42b)$$

Note that the factorisation of the amplitudes as in (2.39), (2.41) and (2.42) only works for a hierarchical mass spectrum  $M_i \ll M_j$ .

The Higgs-mediated processes contribute to the RHS of the Boltzmann equation for the lepton abundance,

$$s \frac{dY_\ell}{dt} = \sum_i \int d\Pi_{N_i t \ell Q}^{q p p t p Q} (2\pi)^4 \left( \begin{aligned} & \delta(q + p_t - p - p_Q) \left[ \Xi_{N_i \bar{t} \rightarrow \ell \bar{Q}} f_{N_i}^q f_{\bar{t}}^{p_t} (1 - f_\ell^p) (1 - f_Q^{p_Q}) - \Xi_{\ell \bar{Q} \rightarrow N_i \bar{t}} f_\ell^p f_{\bar{Q}}^{p_Q} (1 - f_{N_i}^q) (1 - f_{\bar{t}}^{p_t}) \right] \\ & + \delta(q + p - p_t - p_Q) \left[ \Xi_{\bar{t} Q \rightarrow N_i \ell} f_{\bar{t}}^{p_t} f_Q^{p_Q} (1 - f_{N_i}^q) (1 - f_\ell^p) - \Xi_{\ell N_i \rightarrow \bar{t} Q} f_\ell^p f_{N_i}^q (1 - f_{\bar{t}}^{p_t}) (1 - f_Q^{p_Q}) \right] \\ & + \delta(q + p_Q - p - p_t) \left[ \Xi_{N_i Q \rightarrow t \ell} f_{N_i}^q f_Q^{p_Q} (1 - f_\ell^p) (1 - f_t^{p_t}) - \Xi_{\ell t \rightarrow N_i Q} f_\ell^p f_t^{p_t} (1 - f_{N_i}^q) (1 - f_Q^{p_Q}) \right] \\ & + \delta(q - p - p_t - p_Q) \left[ \Xi_{N_i \rightarrow t \ell \bar{Q}} f_{N_i}^q (1 - f_\ell^p) (1 - f_t^{p_t}) (1 - f_Q^{p_Q}) - \Xi_{\ell t \bar{Q} \rightarrow N_i} f_\ell^p f_t^{p_t} f_Q^{p_Q} (1 - f_{N_i}^q) \right] \end{aligned} \right). \quad (2.43)$$

As for the case of (inverse) decay of heavy neutrinos the difference of the above Boltzmann

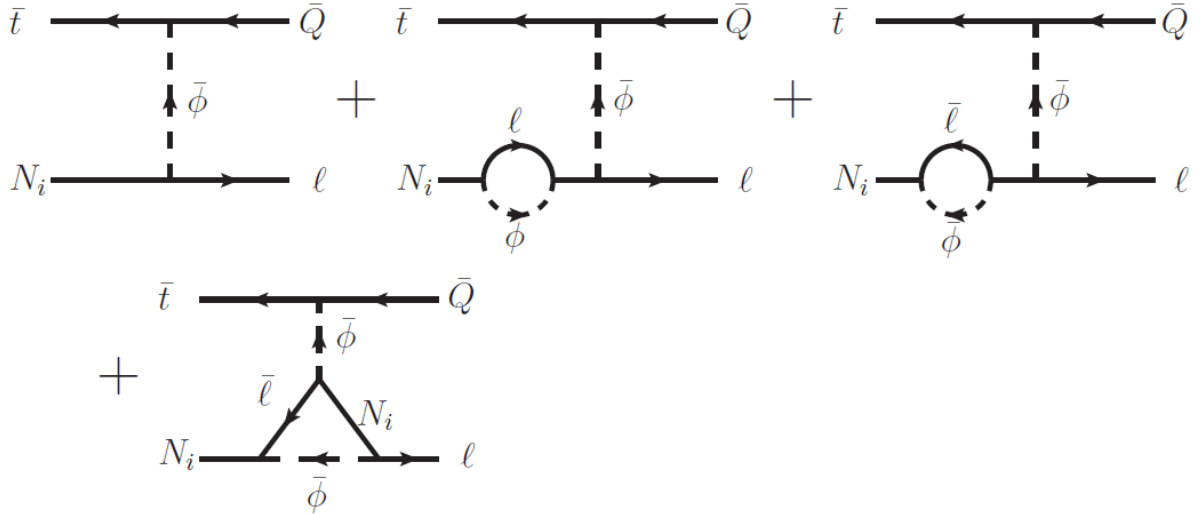


Figure 2.6: One-loop level scattering amplitude  $\Xi_{N_i \bar{t} \rightarrow l \bar{Q}}$ . See comments under Fig. 2.4.

equation and the one for the antilepton abundance is nonzero in thermal equilibrium, which is in contradiction with the third Sakharov condition. One would therefore need to perform a RIS-subtraction in a similar way as in the previous section. This can be done by including the  $2 \rightleftharpoons 3$  scattering processes, and subtracting their RIS part. We will however not perform here the RIS-subtraction in the S-matrix formalism which can be found in [102].

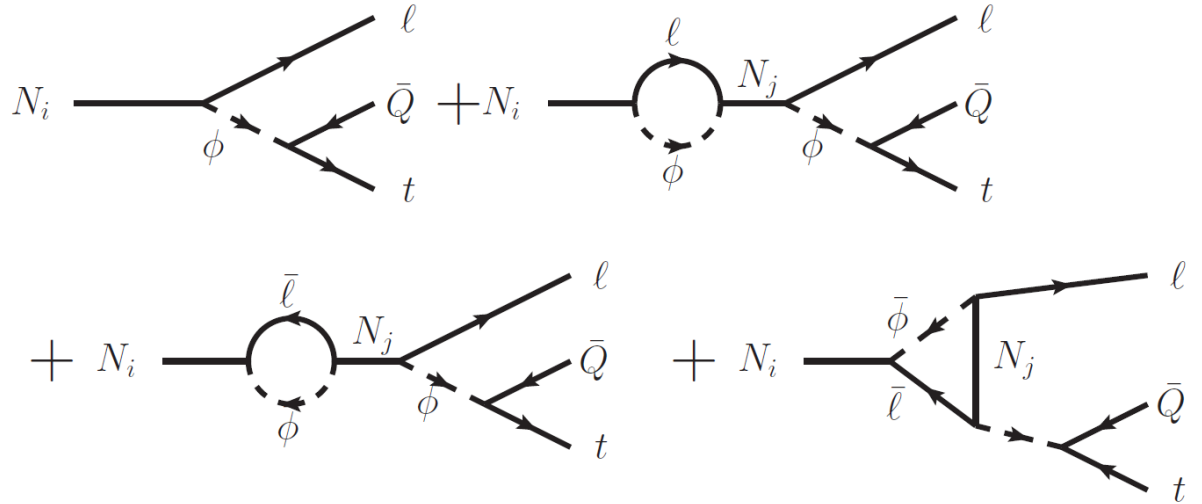


Figure 2.7: One-loop level decay amplitude  $\Xi_{N_i \rightarrow l \bar{t} \bar{Q}}$ . See comments under Fig. 2.4.

# Chapter 3

## Nonequilibrium quantum field theory

We sketch here the basic formalism of nonequilibrium quantum field theory (NEQFT) [103–107]. After defining the different quantities of interest we derive the Kadanoff-Baym equations for a complex scalar, a chiral and a Majorana field. We show then how we can recover the Boltzmann equations from the Kadanoff-Baym equations [108–112]. We are paying a special attention to the approximations made during the computation, and to the range of validity of the obtained equations. The complex scalar, chiral and Majorana fields correspond of course to the Higgs doublet, the lepton and the heavy neutrino fields in the leptogenesis scenario. We will however perform the computation in a general way as much as possible, without referring to the special case of leptogenesis. The application of the formalism derived below will be used for leptogenesis in the chapter 4. The entire computation will be performed in a flat space-time. The effect of an expanding universe will be implemented by the replacement of the usual derivative by the covariant one [113, 114].

### 3.1 Kadanoff-Baym equations

In NEQFT the basic objects under study are the expectation value of the two-point functions of the fields with the time argument belonging to the closed time path (CTP) [103], see Fig. 3.1. For a massless complex scalar (the Higgs), a massless chiral field (the leptons) and a Majorana field (the heavy neutrinos) the propagators are defined as,

$$\Delta_{ab}(x, y) = \langle T_C \phi^a(x) \phi^{b\dagger}(y) \rangle, \quad (3.1a)$$

$$S_{ab}^{\alpha\beta}(x, y) = \langle T_C \ell_\alpha^a(x) \bar{\ell}_\beta^b(y) \rangle, \quad (3.1b)$$

$$\mathcal{S}^{ij}(x, y) = \langle T_C N_i(x) \bar{N}_j(y) \rangle, \quad (3.1c)$$

where  $T_C$  denotes path-ordering along the CTP, the Latin subscripts are  $SU(2)_L$  indices, the

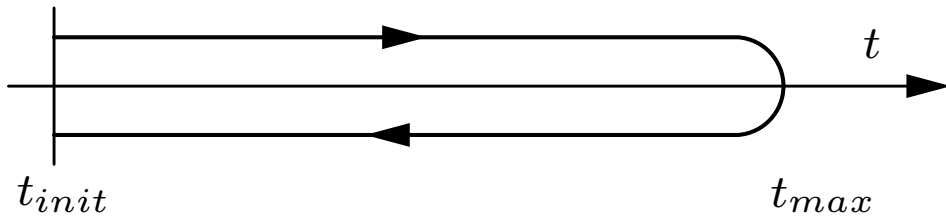


Figure 3.1: Closed time path.

Greek superscripts are lepton flavour indices, the latin superscripts are heavy neutrino flavour

indices, and we omitted the spinor indices for the lepton and heavy neutrino field. Here  $\langle O \rangle$  denotes the expectation value of an Heisenberg operator  $O$  and is given by,

$$\langle O \rangle = \text{tr}(O\rho(t_0)), \quad (3.2)$$

where  $\rho(t_0)$  is the density matrix at the initial time. In order to conveniently express the path ordering  $T_C$  we generalise the Heaviside step function to time argument belonging to the CTP [115],

$$\theta_C(x^0, y^0) \equiv \begin{cases} 1 & \text{if } x^0 \text{ occurs later than } y^0 \text{ on the CTP,} \\ 0 & \text{if } y^0 \text{ occurs later than } x^0 \text{ on the CTP,} \end{cases} \quad (3.3)$$

and rewrite (3.1) as,

$$\Delta_{ab}(x, y) = \theta_C(x^0, y^0) \langle \phi^a(x) \phi^{b\dagger}(y) \rangle + \theta_C(y^0, x^0) \langle \phi^{b\dagger}(y) \phi^a(x) \rangle, \quad (3.4a)$$

$$S_{ab}^{\alpha\beta}(x, y) = \theta_C(x^0, y^0) \langle \ell_\alpha^a(x) \bar{\ell}_\beta^b(y) \rangle - \theta_C(y^0, x^0) \langle \bar{\ell}_\beta^b(y) \ell_\alpha^a(x) \rangle, \quad (3.4b)$$

$$\mathcal{S}^{ij}(x, y) = \theta_C(x^0, y^0) \langle N_i(x) \bar{N}_j(y) \rangle - \theta_C(y^0, x^0) \langle \bar{N}_j(y) N_i(x) \rangle. \quad (3.4c)$$

Note that the path ordering features the usual minus sign for fermions. We define similarly the sign function and the Dirac delta on the CTP,

$$\text{sign}_C(x^0, y^0) \equiv 2\theta_C(x^0, y^0) - 1, \quad (3.5a)$$

$$\delta_C(x^0, y^0) \equiv \frac{d}{dx^0} \theta_C(x^0, y^0), \quad (3.5b)$$

$$\delta_C(x, y) \equiv \delta_C(x^0, y^0) \delta(\vec{x} - \vec{y}). \quad (3.5c)$$

For notational convenience we will often omit the numerous indices and use a matrix notation for the propagators,

$$\hat{\Delta}(x, y) = \langle T_C \phi(x) \phi^\dagger(y) \rangle, \quad (3.6a)$$

$$\hat{S}(x, y) = \langle T_C \ell(x) \bar{\ell}(y) \rangle, \quad (3.6b)$$

$$\hat{\mathcal{S}}(x, y) = \langle T_C N(x) \bar{N}(y) \rangle, \quad (3.6c)$$

where the hat over the two-point functions denotes a matrix.

The propagators (3.6) satisfy the Schwinger-Dyson equation,

$$\hat{\Delta}^{-1}(x, y) = \hat{\Delta}_0^{-1}(x, y) - \hat{\Omega}(x, y), \quad (3.7a)$$

$$\hat{S}^{-1}(x, y) = \hat{S}_0^{-1}(x, y) - \hat{\Sigma}(x, y), \quad (3.7b)$$

$$\hat{\mathcal{S}}^{-1}(x, y) = \hat{\mathcal{S}}_0^{-1}(x, y) - \hat{\Pi}(x, y), \quad (3.7c)$$

where  $\hat{\Omega}(x, y)$ ,  $\hat{\Sigma}(x, y)$  and  $\hat{\Pi}(x, y)$  are the self-energies, and  $\hat{\Delta}_0$ ,  $\hat{S}_0$  and  $\hat{\mathcal{S}}_0$  are the free

propagators of the corresponding fields,<sup>1</sup>

$$\hat{\Delta}_0^{-1}(x, y) = i\Box_x \delta_{\mathcal{C}}(x, y), \quad (3.8a)$$

$$\hat{S}_0^{-1}(x, y) = \not{\partial}_x P_L \delta_{\mathcal{C}}(x, y), \quad (3.8b)$$

$$\hat{\mathcal{S}}_0^{-1}(x, y) = -i \left( i\not{\partial}_x - \hat{M} \right) \delta_{\mathcal{C}}(x, y), \quad (3.8c)$$

where  $\Box_x$  is the d'Alembertian. The self-energies are obtained by functional differentiation of the 2PI effective action [116] and are functionals of the *full* propagators of each species. It is useful to trade the propagator with time arguments on the CTP for two propagators with time arguments living on the positive branch of the CTP. To this end we decompose the two-point function  $\hat{G}(x, y)$  (where  $\hat{G}(x, y)$  stands for any propagators in (3.6)) into a *spectral* function  $\hat{G}_\rho(x, y)$  and a *statistical* function  $\hat{G}_F(x, y)$ ,

$$\hat{G}(x, y) = \hat{G}_F(x, y) - \frac{i}{2} \text{sign}_{\mathcal{C}}(x^0, y^0) \hat{G}_\rho(x, y). \quad (3.9)$$

Using (3.4) and (3.5) we find the statistical and spectral functions for the Higgs,

$$\Delta_{abF}(x, y) = \frac{1}{2} \langle [\phi^a(x), \phi^{b\dagger}(y)]_+ \rangle, \quad (3.10a)$$

$$\Delta_{ab\rho}(x, y) = i \langle [\phi^a(x), \phi^{b\dagger}(y)]_- \rangle, \quad (3.10b)$$

where  $[\cdot, \cdot]_{\pm}$  are the commutator and anticommutator, respectively. For the fermions we find equations similar to (3.10), with commutator replaced by anticommutator, and vice versa. It will be proven useful to introduce the Wightman propagators, which are linear combination of the statistical and spectral functions,

$$\hat{G}_{\gtrless}(x, y) \equiv \hat{G}_F(x, y) \mp \frac{i}{2} \hat{G}_\rho(x, y). \quad (3.11)$$

We define next the retarded and advanced propagators,

$$\hat{G}_R(x, y) \equiv \theta(x^0 - y^0) \hat{G}_\rho(x, y), \quad (3.12a)$$

$$\hat{G}_A(x, y) \equiv -\theta(y^0 - x^0) \hat{G}_\rho(x, y), \quad (3.12b)$$

and finally the hermitian propagator,

$$\hat{G}_h(x, y) \equiv \frac{1}{2} (\hat{G}_R(x, y) + \hat{G}_A(x, y)). \quad (3.13)$$

Note that only two propagators for each field are needed to fully describe the system. However, using the above definitions simplifies significantly the notation. The different propagators satisfy many symmetry properties which are summarised in appendix C. In particular the

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<sup>1</sup>We are using a superfluous four-component notation for the lepton field. Here the inverse of the lepton propagator is defined by  $\int d^4z S(x, z) S^{-1}(z, y) = P_L$  and  $\int d^4z S^{-1}(x, z) S(z, y) = P_R$ .

Wightmann propagators satisfy,

$$\hat{\Delta}_{\geq}(x, y) = \hat{\Delta}_{\geq}^{\dagger}(y, x), \quad (3.14a)$$

$$\hat{S}_{\geq}(x, y) = \gamma^0 \hat{S}_{\geq}^{\dagger}(y, x) \gamma^0 = P_L \hat{S}_{\geq}(x, y) P_R, \quad (3.14b)$$

$$\hat{\mathcal{J}}_{\geq}(x, y) = \gamma^0 \hat{\mathcal{J}}_{\geq}^{\dagger}(y, x) \gamma^0 = C \hat{\mathcal{J}}_{\geq}^T(y, x) C^{-1}. \quad (3.14c)$$

The non-local part of the self-energy can be decomposed in a similar way,

$$\hat{\Pi}(x, y) \equiv -i\delta_C(x, y) \hat{\Pi}_{loc}(x) + \hat{\Pi}_F(x, y) - \frac{i}{2} \text{sign}_C(x^0, y^0) \hat{\Pi}_{\rho}(x, y), \quad (3.15a)$$

$$\hat{\Pi}_{\geq}(x, y) \equiv \hat{\Pi}_F(x, y) \mp \frac{i}{2} \hat{\Pi}_{\rho}(x, y), \quad (3.15b)$$

$$\hat{\Pi}_R(x, y) \equiv \theta(x^0 - y^0) \hat{\Pi}_{\rho}(x, y), \quad (3.15c)$$

$$\hat{\Pi}_A(x, y) \equiv -\theta(y^0 - x^0) \hat{\Pi}_{\rho}(x, y), \quad (3.15d)$$

$$\hat{\Pi}_h(x, y) \equiv \frac{1}{2} \left( \hat{\Pi}_R(x, y) + \hat{\Pi}_A(x, y) \right), \quad (3.15e)$$

where  $\hat{\Pi}(x, y)$  stands for any self-energies in (3.7) and  $\hat{\Pi}_{loc}(x)$  is the local self-energy. The self-energies satisfy symmetry properties similar to their corresponding propagators, see appendix C.

Finally, we will also need the CP-conjugated propagators. Under a CP-transformation the Higgs, lepton and heavy neutrino fields transforms as,

$$\phi^a(x) \xrightarrow{CP} \phi^{a\dagger}(\bar{x}), \quad (3.16a)$$

$$\ell_{\alpha}^a(x) \xrightarrow{CP} CP \bar{\ell}_{\alpha}^{aT}(\bar{x}), \quad (3.16b)$$

$$N_i(x) \xrightarrow{CP} CP \bar{N}_i^T(\bar{x}), \quad (3.16c)$$

where the transposition acts on the spinor indices. Here  $\bar{x} \equiv (x^0, -\vec{x})$ , and  $C \equiv i\gamma^2\gamma^0$  and  $P \equiv \gamma^0$  are the charge conjugation and parity matrices, respectively. The CP-conjugated propagators on the CTP are given by,

$$\bar{\Delta}_{ab}(x, y) = \Delta_{ba}(\bar{y}, \bar{x}), \quad (3.17a)$$

$$\bar{S}_{ab}^{\alpha\beta}(x, y) = (CP) S_{ba}^{\beta\alpha}(\bar{y}, \bar{x}) (CP)^{-1}, \quad (3.17b)$$

$$\bar{\mathcal{J}}_{ij}(x, y) = (CP) \mathcal{J}_{ba}(\bar{y}, \bar{x}) (CP)^{-1}, \quad (3.17c)$$

and the CP-conjugated self-energies are defined in a similar way. The CP-conjugate of the statistical, spectral and other two-point functions immediately follow from their definition and the CP-conjugated propagators (3.17) on the CTP and are listed in appendix C.

It is interesting to investigate the properties of the propagators in exact thermal equilibrium. A system in thermal equilibrium is characterised by the density matrix,

$$\rho^{\text{eq}} = \frac{1}{Z} e^{-\beta H}, \quad (3.18)$$

where  $Z$  is a normalisation factor,  $\beta = 1/T$  the inverse temperature and  $H$  the Hamiltonian



of the system. In equilibrium the Wightman Higgs propagator reads,

$$\begin{aligned}\Delta_{ab,>}^{\text{eq}}(x,y) &= \langle \phi^a(x) \phi^{b\dagger}(y) \rangle = \frac{1}{Z} \text{tr} [e^{-\beta H} \phi^a(x) \phi^{b\dagger}(y)] \\ &= \frac{1}{Z} \text{tr} [e^{i(i\beta)H} \phi^a(x) e^{-i(i\beta)H} e^{i(i\beta)H} \phi^{b\dagger}(y)].\end{aligned}\quad (3.19)$$

In the Heisenberg picture, when the Hamiltonian is time independent (which is obviously the case in thermal equilibrium), the time evolution of the field takes a simple form,

$$\phi^a(x^0 + t, \vec{x}) = e^{iHt} \phi^a(x^0, \vec{x}) e^{-iHt}.\quad (3.20)$$

We can therefore interpret the inverse temperature as an imaginary time [117], and write,

$$\begin{aligned}\Delta_{ab,>}^{\text{eq}}(x,y) &= \frac{1}{Z} \text{tr} [\phi^a(x^0 + i\beta, \vec{x}) e^{i(i\beta)H} \phi^{b\dagger}(y)] \\ &= \frac{1}{Z} \text{tr} [e^{i(i\beta)H} \phi^{b\dagger}(y) \phi^a(x^0 + i\beta, \vec{x})] = \Delta_{ab,<}^{\text{eq}}(x + i\beta, y),\end{aligned}\quad (3.21)$$

where we have used the cyclicity property of the trace, and the simplified notation  $(x + i\beta)$  for  $(x^0 + i\beta, \vec{x})$ . The relations of the type of (3.21) among the propagators are known as the Kubo-Martin-Schwinger (KMS) relations, and play an important role in the study of equilibrium field theory (see e.g. [118, 119]). We will use the KMS relations in the next section to motivate a particular ansatz. The KMS relation for fermions features an additional minus sign coming from the definition of the Wightman propagators,

$$\hat{S}_{>}^{\text{eq}}(x,y) = -\hat{S}_{<}^{\text{eq}}(x + i\beta, y),\quad (3.22a)$$

$$\hat{\mathcal{S}}_{>}^{\text{eq}}(x,y) = -\hat{\mathcal{S}}_{<}^{\text{eq}}(x + i\beta, y).\quad (3.22b)$$

Note that the self-energies also satisfy the KMS relations in thermal equilibrium.

We derive then the Kadanoff-Baym equations from the Schwinger-Dyson equation (3.7). Let us first focus on the Higgs field and take the convolution of (3.7a) with the full propagator  $\hat{\Delta}$  from the right,

$$i\Box_x \hat{\Delta}(x,y) = \delta_{\mathcal{C}}(x,y) + \int_{\mathcal{C}} d^4z \hat{\Omega}(x,z) \hat{\Delta}(z,y),\quad (3.23)$$

where we used the notation  $\int_{\mathcal{C}} d^4z \equiv \int_{\mathcal{C}} dz^0 \int d^3\vec{z}$ . We insert then the statistical and spectral decomposition of the propagator and self-energy into the above equation. On the LHS we obtain,

$$\begin{aligned}i\Box_x \left( \hat{\Delta}_F(x,y) - \frac{i}{2} \text{sign}_{\mathcal{C}}(x^0, y^0) \hat{\Delta}_{\rho}(x,y) \right) &= i\Box_x \hat{\Delta}_F(x,y) + \frac{1}{2} \text{sign}_{\mathcal{C}}(x^0, y^0) \Box_x \hat{\Delta}_{\rho}(x,y) \\ &\quad + \delta_{\mathcal{C}}(x^0, y^0) \partial_{x^0} \hat{\Delta}_{\rho}(x,y),\end{aligned}\quad (3.24)$$

where we have used the relations (3.5). Using the canonical (equal-time) commutation relation we write the last term in (3.24),

$$\lim_{x^0 \rightarrow y^0} \partial_{x^0} \Delta_{ab\rho}(x,y) = i \lim_{x^0 \rightarrow y^0} \langle [\partial_{x^0} \phi^a(x), \phi^{b\dagger}(y)]_- \rangle = \delta_{ab} \delta(\vec{x} - \vec{y}),\quad (3.25)$$

which exactly cancels the delta function on the RHS of (3.23). In terms of the statistical and spectral functions the integral over the CTP on the RHS of (3.23) reads,

$$\begin{aligned} \int_{\mathcal{C}} d^4 z \hat{\Omega}(x, z) \hat{\Delta}(z, y) &= \int_{\mathcal{C}} d^4 z \left[ \hat{\Omega}_F(x, z) \hat{\Delta}_F(z, y) - \frac{i}{2} \text{sign}_{\mathcal{C}}(x^0, z^0) \hat{\Omega}_{\rho}(x, z) \hat{\Delta}_F(z, y) \right. \\ &\quad \left. - \frac{i}{2} \text{sign}_{\mathcal{C}}(z^0, y^0) \hat{\Omega}_F(x, z) \hat{\Delta}_{\rho}(z, y) - \frac{1}{4} \text{sign}_{\mathcal{C}}(x^0, z^0) \text{sign}_{\mathcal{C}}(z^0, y^0) \hat{\Omega}_{\rho}(x, z) \hat{\Delta}_{\rho}(z, y) \right]. \end{aligned} \quad (3.26)$$

The integrals over the CTP on the RHS of (3.26) together with the sign functions can be expressed as integrals over the positive branch only,

$$\begin{aligned} \int_{\mathcal{C}} d^4 z \dots &= 0, \\ \int_{\mathcal{C}} d^4 z \text{sign}_{\mathcal{C}}(x^0, z^0) \dots &= 2 \int_0^{x^0} d^4 z \dots, \\ \int_{\mathcal{C}} d^4 z \text{sign}_{\mathcal{C}}(z^0, y^0) \dots &= -2 \int_0^{y^0} d^4 z \dots, \\ \int_{\mathcal{C}} d^4 z \text{sign}_{\mathcal{C}}(x^0, z^0) \text{sign}_{\mathcal{C}}(z^0, y^0) \dots &= 2 \text{sign}_{\mathcal{C}}(x^0, y^0) \int_{y^0}^{x^0} d^4 z \dots, \end{aligned} \quad (3.27)$$

where we have set the initial time on the CTP at  $t = 0$ . In (3.27) the ellipsis represents any functions with time arguments living on the positive branch. Using (3.24) and (3.27) into (3.23) we can split the equation into a “statistical” part (independent of the CTP branch) and a “spectral” part (proportional to  $\text{sign}_{\mathcal{C}}(x^0, y^0)$ ),

$$- \left( \square_x + \hat{\Omega}_{loc}(x) \right) \hat{\Delta}_F(x, y) = \int_0^{x^0} d^4 z \hat{\Omega}_{\rho}(x, z) \hat{\Delta}_F(z, y) - \int_0^{y^0} d^4 z \hat{\Omega}_F(x, z) \hat{\Delta}_{\rho}(z, y), \quad (3.28a)$$

$$- \left( \square_x + \hat{\Omega}_{loc}(x) \right) \hat{\Delta}_{\rho}(x, y) = \int_{y^0}^{x^0} d^4 z \hat{\Omega}_{\rho}(x, z) \hat{\Delta}_{\rho}(z, y). \quad (3.28b)$$

The above equations are known as the *Kadanoff-Baym* (KB) equations. We can see here that the local self-energy acts as an effective,  $x$ -dependent mass term. The KB equations for the statistical and spectral functions (3.28) can be expressed more conveniently using the retarded and advanced two-point functions (3.12),

$$\begin{aligned} - \left( \square_x + \hat{\Omega}_{loc}(x) \right) \hat{\Delta}_{F(\rho)}(x, y) &= \int d^4 z \theta(z^0) \left[ \hat{\Omega}_R(x, z) \hat{\Delta}_{F(\rho)}(z, y) \right. \\ &\quad \left. + \hat{\Omega}_{F(\rho)}(x, z) \hat{\Delta}_A(z, y) \right], \end{aligned} \quad (3.29)$$

where we extend the integration domain to the whole  $z$  plane with the help of the step function. Similarly we find for the lepton and heavy neutrino,

$$i \not{\partial}_x \hat{S}_{F(\rho)}(x, y) = \int d^4 z \theta(z^0) \left[ \hat{\Sigma}_R(x, z) \hat{S}_{F(\rho)}(z, y) + \hat{\Sigma}_{F(\rho)}(x, z) \hat{S}_A(z, y) \right], \quad (3.30)$$

$$\left( i \not{\partial}_x - \hat{M} \right) \hat{\mathcal{S}}_{F(\rho)}(x, y) = \int d^4 z \theta(z^0) \left[ \hat{\Pi}_R(x, z) \hat{\mathcal{S}}_{F(\rho)}(z, y) + \hat{\Pi}_{F(\rho)}(x, z) \hat{\mathcal{S}}_A(z, y) \right]. \quad (3.31)$$

Instead of the spectral function it is often convenient to use as dynamical quantities the retarded and advanced propagators which satisfy,

$$-\left(\square_x + \hat{\Omega}_{loc}(x)\right) \hat{\Delta}_{R(A)}(x, y) = -\delta(x - y)\mathbb{1} + \int d^4 z \hat{\Omega}_{R(A)}(x, z) \hat{\Delta}_{R(A)}(z, y), \quad (3.32a)$$

$$i\hat{\not{\partial}}_x \hat{S}_{R(A)}(x, y) = -\delta(x - y)\mathbb{1} + \int d^4 z \hat{\Sigma}_{R(A)}(x, z) \hat{S}_{R(A)}(z, y), \quad (3.32b)$$

$$\left(i\hat{\not{\partial}}_x - \hat{M}\right) \hat{\mathcal{S}}_{R(A)}(x, y) = -\delta(x - y)\mathbb{1} + \int d^4 z \hat{\Pi}_{R(A)}(x, z) \hat{\mathcal{S}}_{R(A)}(z, y), \quad (3.32c)$$

where the unit matrix is defined by,

$$\mathbb{1} = \begin{cases} \delta_{ab} & \text{for the Higgs field,} \\ \delta_{ab} \delta^{\alpha\beta} P_R & \text{for the lepton field,} \\ \delta_{ij} & \text{for the heavy neutrino field.} \end{cases} \quad (3.33)$$

Together with the functional form of the self-energies the coupled system of Kadanoff-Baym equations (3.29), (3.30) and (3.31) represent the starting point for the study of a nonequilibrium system. These equations are *exact* coupled integro-differential equations for the statistical and spectral functions. They can be seen as the quantum field theory generalisation of the Boltzmann equation. However, unlike the Boltzmann equation, they are non-Markovian, i.e. the evolution of the propagators depends on the whole history of the system. This property makes the KB equations extremely difficult to solve numerically [55].

Even though the KB equations are exact, the self-energies can only be computed perturbatively. This will give us a perturbative expansion of the RHS of the KB equations. However, as the self-energies are functional of the full propagators, they take into account higher order terms which have been resummed into the full propagators. This resummation is a key feature to avoid the secular terms [107]. Note that a recent development in nonequilibrium quantum field theory [120, 121] based on perturbation theory avoids the pinch singularity by considering finite time effects.

## 3.2 Quantum kinetic equation

In this section we derive from the Kadanoff-Baym equations the quantum kinetic equations [108–112]. They are the quantum analogue of the Boltzmann equations. The latter are equations for the one-particle distribution functions which depend on  $(x, \vec{p})$ . We need therefore to change the representation of the two-point functions if we want to recover the Boltzmann limit. To this end we perform a change of the variables from  $(x, y)$  to the central coordinate  $X \equiv \frac{1}{2}(x + y)$  and the relative coordinate  $s \equiv x - y$ ,

$$\hat{G}_{F(\rho)}(x, y) \rightarrow \hat{G}_{F(\rho)}(X, s), \quad (3.34)$$

and define the Wigner transform as a Fourier transform with respect to the relative coordinate,

$$\hat{G}_{F(\gtrless,R,A,h)}(X,p) \equiv \int d^4s e^{ips} \hat{G}_{F(\gtrless,R,A,h)}(X,s), \quad (3.35a)$$

$$\hat{G}_\rho(X,p) \equiv -i \int d^4s e^{ips} \hat{G}_\rho(X,s), \quad (3.35b)$$

where the factor  $-i$  for the spectral function is a convention and ensures that the Wigner transforms of the spectral and statistical functions have the same hermicity properties (see appendix C). We use here the same notation for the propagators in the  $(x,y)$  representation and its Wigner transform. The arguments of the two-point functions will distinguish in which representation we are working. In Wigner space the different propagators introduced in the previous section are related through,

$$\hat{G}_{\gtrless}(X,p) = \hat{G}_F(X,p) \pm \frac{1}{2} \hat{G}_\rho(X,p), \quad (3.36a)$$

$$i\hat{G}_\rho(X,p) = \hat{G}_R(X,p) - \hat{G}_A(X,p), \quad (3.36b)$$

$$\hat{G}_h(X,p) = \frac{1}{2}(\hat{G}_R(X,p) + \hat{G}_A(X,p)), \quad (3.36c)$$

and satisfy symmetry properties different from the one in coordinate space, see appendix C. The Wigner transforms of the self-energies are defined similarly,

$$\hat{\Pi}_{F(\gtrless,R,A,h)}(X,p) \equiv \int d^4s e^{ips} \hat{\Pi}_{F(\gtrless,R,A,h)}(X,s), \quad (3.37a)$$

$$\hat{\Pi}_\rho(X,p) \equiv -i \int d^4s e^{ips} \hat{\Pi}_\rho(X,s). \quad (3.37b)$$

We want now to write the KB equations in Wigner space. For the sake of clarity we concentrate on the Higgs field and neglect from now on the local self-energy. As we will see, the local self-energy of the Higgs plays no role for the generation of the lepton asymmetry. Then the Wigner transform of the LHS of (3.29) can be easily computed,

$$\begin{aligned} \int d^4(x-y) e^{iks} (-\square_x) \hat{\Delta}_F(x,y) &= \int d^4s e^{iks} \left( -\frac{1}{2} \square_X - \square_s - \partial_s \partial_X \right) \hat{\Delta}_F(X,s) \\ &= \int d^4s e^{iks} \left( k^2 + ik\partial_X - \frac{1}{4} \square_X \right) \hat{\Delta}_F(X,s) \\ &= \left( k^2 + ik\partial_X - \frac{1}{4} \square_X \right) \hat{\Delta}_F(X,k), \end{aligned} \quad (3.38)$$

where we integrated by part to obtain the second line. It is useful to compute the Wigner transform of a convolution product,

$$\hat{A}(x,y) = \int d^4z \hat{B}(x,z) \hat{C}(z,y), \quad (3.39)$$

where  $\hat{A}$ ,  $\hat{B}$  and  $\hat{C}$  are arbitrary matrices. The Wigner transform of (3.39) can be formally written as an infinite sum of derivatives (see appendix D),

$$\hat{A}(X, k) = e^{-\frac{i}{2}\diamond} \{\hat{B}(X, k)\} \{\hat{C}(X, k)\}, \quad (3.40)$$

where the diamond operator is defined as the generalised Poisson bracket,

$$\diamond\{\hat{B}(X, k)\} \{\hat{C}(X, k)\} \equiv \partial_X \hat{B}(X, k) \partial_k \hat{C}(X, k) - \partial_k \hat{B}(X, k) \partial_X \hat{C}(X, k). \quad (3.41)$$

We assumed in (3.40) that the two-point functions transform like the statistical propagators, i.e. without the  $-i$  factor. If any of the functions  $\hat{A}(X, s)$ ,  $\hat{B}(X, s)$  or  $\hat{C}(X, s)$  transform like the spectral function, additional  $-i$  factors should be added. The diamond operator satisfies many useful properties which can be derived from its definition (3.41). They are summarised in appendix D.1.

We can now write the KB equations (3.29), (3.30) and (3.31) in Wigner space. We first drop the Heaviside step function  $\theta(z^0)$  and extend the  $z^0$  integration over the complete real axis. This means that we send the initial time to  $-\infty$  but we still specify the initial conditions at some finite time, see [122] for more details about this approximation. Performing the Wigner transform of the KB equations for the statistical and spectral Higgs functions we find,

$$\begin{aligned} \left(k^2 + ik\partial_X - \frac{1}{4}\square_X\right) \hat{\Delta}_{F(\rho)}(X, k) &= e^{-\frac{i}{2}\diamond} \{\hat{\Omega}_{F(\rho)}(X, k)\} \{\hat{\Delta}_A(X, k)\} \\ &+ e^{-\frac{i}{2}\diamond} \{\hat{\Omega}_R(X, k)\} \{\hat{\Delta}_{F(\rho)}(X, k)\}. \end{aligned} \quad (3.42)$$

Similarly the advanced and causal propagators satisfy,

$$\left(k^2 + ik\partial_X - \frac{1}{4}\square_X\right) \hat{\Delta}_{R(A)}(X, k) = -\mathbf{1} + e^{-\frac{i}{2}\diamond} \{\hat{\Omega}_{R(A)}(X, k)\} \{\hat{\Delta}_{R(A)}(X, k)\}. \quad (3.43)$$

The Wigner transform of the KB equations of the lepton and heavy neutrino fields are computed in the same manner,

$$\begin{aligned} \left(\not{p} + \frac{i}{2}\not{\partial}_X\right) \hat{S}_{F(\rho)}(X, p) &= e^{-\frac{i}{2}\diamond} \{\hat{\Sigma}_{F(\rho)}(X, p)\} \{\hat{S}_A(X, p)\} \\ &+ e^{-\frac{i}{2}\diamond} \{\hat{\Sigma}_R(X, p)\} \{\hat{S}_{F(\rho)}(X, p)\}, \end{aligned} \quad (3.44a)$$

$$\left(\not{p} + \frac{i}{2}\not{\partial}_X\right) \hat{S}_{R(A)}(X, p) = -\mathbf{1} + e^{-\frac{i}{2}\diamond} \{\hat{\Sigma}_{R(A)}(X, p)\} \{\hat{S}_{R(A)}(X, p)\}, \quad (3.44b)$$

and

$$\begin{aligned} \left(\not{q} + \frac{i}{2}\not{\partial}_X - \hat{M}\right) \hat{\mathcal{S}}_{F(\rho)}(X, q) &= e^{-\frac{i}{2}\diamond} \{\hat{\Pi}_{F(\rho)}(X, q)\} \{\hat{\mathcal{S}}_A(X, q)\} \\ &+ e^{-\frac{i}{2}\diamond} \{\hat{\Pi}_R(X, q)\} \{\hat{\mathcal{S}}_{F(\rho)}(X, q)\}, \end{aligned} \quad (3.45a)$$

$$\left(\not{q} + \frac{i}{2}\not{\partial}_X - \hat{M}\right) \hat{\mathcal{S}}_{R(A)}(X, q) = -\mathbf{1} + e^{-\frac{i}{2}\diamond} \{\hat{\Pi}_{R(A)}(X, q)\} \{\hat{\mathcal{S}}_{R(A)}(X, q)\}. \quad (3.45b)$$

For notational clarity we use in the above equations different momenta for the heavy neutrino, lepton and Higgs fields. Here and in the rest of this work  $q$ ,  $p$  and  $k$  denote the right-handed neutrino, lepton and Higgs momenta, respectively.

Note that the above equations are local, i.e. the two-point functions are all evaluated at the same point in Wigner space. However the product of an infinite number of derivatives from the exponential of the diamond operator contains informations on the whole history of the system, similarly to the memory integrals. The KB equations in Wigner space are still exact (up to the fact that we send the initial time to  $-\infty$ ). However, due to the presence of the exponential of the diamond operator, they are of no practical utility. We need therefore to cut at a certain order the series of diamond operators. This is known as a *gradient expansion*. Mathematically, it corresponds to an expansion in  $\mathcal{O}(\partial_X \partial_p)$ . Physically, it corresponds to an expansion in slow relative to fast time-scales [72]. The slow time-scale is characterised by the macroscopic time evolution of the system, i.e. its departure from thermal equilibrium. In the leptogenesis scenario the macroscopic time is typically governed by the Hubble rate  $H$  or the decay rate of the Majorana neutrino  $\Gamma_{N_i}$ . These rates affect mainly the central coordinate  $X$ . We can write, schematically,

$$\partial_X \sim H, \Gamma_{N_i}. \quad (3.46)$$

On the other hand, the relative coordinate  $s$  is mainly associated with the hard scales, such as the energy or momentum of the particles in the thermal bath. In leptogenesis the hard scales correspond to the mass of the heavy neutrino  $M_i$  or the temperature  $T$  of the thermal bath. Schematically,

$$\partial_s \sim M_i, T. \quad (3.47)$$

Therefore in leptogenesis the gradient expansion corresponds to an expansion in  $H/T$  or  $\Gamma_{N_i}/M_i$ ,

$$\partial_X \partial_p \sim \partial_X / \partial s \sim H/T \text{ or } \Gamma_{N_i}/M_i. \quad (3.48)$$

In the radiation dominated epoch of the universe,  $H/T \sim T/M_{pl}$ . We can therefore safely perform a gradient expansion and only keep the first order terms in the diamond operator as long as  $T \ll M_{pl}$ . The effects of higher order terms have been investigated in [64]. At first order in the gradient expansion the Kadanoff-Baym equations for the Higgs, lepton and heavy neutrino read

$$\begin{aligned} \hat{\mathcal{D}}_X(p) \hat{G}_{F(\rho)}(X, p) &= \hat{\mathbf{\Pi}}_{F(\rho)}(X, p) \hat{G}_A(X, p) + \hat{\mathbf{\Pi}}_R(X, p) \hat{G}_{F(\rho)}(X, p) \\ &\quad - \frac{i}{2} \diamond \left\{ \hat{\mathbf{\Pi}}_{F(\rho)}(X, p) \right\} \left\{ \hat{G}_A(X, p) \right\} - \frac{i}{2} \diamond \left\{ \hat{\mathbf{\Pi}}_R(X, p) \right\} \left\{ \hat{G}_{F(\rho)}(X, p) \right\}, \end{aligned} \quad (3.49)$$

$$\begin{aligned} \hat{\mathcal{D}}_X(p) \hat{G}_{R(A)}(X, p) &= -\mathbf{1} + \hat{\mathbf{\Pi}}_{R(A)}(X, p) \hat{G}_{R(A)}(X, p) \\ &\quad - \frac{i}{2} \diamond \left\{ \hat{\mathbf{\Pi}}_{R(A)}(X, p) \right\} \left\{ \hat{G}_{R(A)}(X, p) \right\}, \end{aligned} \quad (3.50)$$

where we have defined the differential operator,

$$\hat{\mathcal{D}}_X(p) = \begin{cases} p^2 + ip\partial_X - \frac{1}{4}\square_X & \text{for the Higgs field,} \\ \not{p} + \frac{i}{2}\not{\partial}_X & \text{for the lepton field,} \\ \not{p} - \hat{M} + \frac{i}{2}\not{\partial}_X & \text{for the heavy neutrino field.} \end{cases} \quad (3.51)$$

Equations (3.49) and (3.50) can be written more compactly. We first note that the d'Alembertian  $\square_X$  in the differential operator (3.51) for the Higgs field is of second order in the diamond operator,

$$\square_X \hat{\Delta}_{F(\rho,R,A)}(X, k) = \frac{1}{2} \diamond^2 \{k^2\} \left\{ \hat{\Delta}_{F(\rho,R,A)}(X, k) \right\}, \quad (3.52)$$

and should therefore be neglected in a consistent first order gradient approximation. Defining the functions

$$\hat{\omega}_{R(A)}(X, p) = \begin{cases} p^2 - \hat{\Omega}_{R(A)}(X, p) & \text{for the Higgs field,} \\ \not{p} - \hat{\Sigma}_{R(A)}(X, p) & \text{for the lepton field,} \\ \not{p} - \hat{M} - \hat{\Pi}_{R(A)}(X, p) & \text{for the heavy neutrino field,} \end{cases} \quad (3.53)$$

we rewrite equations (3.49) and (3.50),

$$-\frac{i}{2} \diamond \left\{ \hat{\omega}_R \right\} \left\{ \hat{G}_{F(\rho)} \right\} + \hat{\omega}_R \hat{G}_{F(\rho)} = \hat{\Pi}_{F(\rho)} \hat{G}_A - \frac{i}{2} \diamond \left\{ \hat{\Pi}_{F(\rho)} \right\} \left\{ \hat{G}_A \right\}, \quad (3.54)$$

$$-\frac{i}{2} \diamond \left\{ \hat{\omega}_{R(A)} \right\} \left\{ \hat{G}_{R(A)} \right\} + \hat{\omega}_{R(A)} \hat{G}_{R(A)} = -\mathbb{1}, \quad (3.55)$$

where we have suppressed the arguments  $(X, p)$  of the two-point functions for notational convenience.

The hermitian and antihermitian parts of (3.54) or (3.55) are often referred to as *constraint* and *kinetic* equations, respectively. Note that they do not feature the memory integral as the KB equations, and are therefore local in space and time. The propagator at a space-time coordinate  $X$  only depends on the configuration of the system infinitesimally close to  $X$ . Any non-local behaviours have been neglected by the gradient approximation.

The equation (3.55) for the causal propagators can be solved algebraically. Up to first order in the gradient expansion the solutions of (3.55) are given by [123],

$$\begin{aligned} \hat{G}_{R(A)} &= -\hat{\omega}_{R(A)}^{-1} \mathbb{1} - \frac{i}{2} \hat{\omega}_{R(A)}^{-1} \diamond \left\{ \hat{\omega}_{R(A)} \right\} \left\{ \hat{\omega}_{R(A)}^{-1} \right\} \mathbb{1} \\ &= -\hat{\omega}_{R(A)}^{-1} \mathbb{1} + \frac{i}{2} \diamond \left\{ \hat{\omega}_{R(A)}^{-1} \right\} \left\{ \hat{\omega}_{R(A)} \right\} \hat{\omega}_{R(A)}^{-1} \mathbb{1}. \end{aligned} \quad (3.56)$$

It can be easily checked that (3.56) is a solution of the equations for the causal propagators (3.55) by inserting it into (3.55) and neglecting the terms of second order in the diamond operator. To derive the second equality in (3.56) we used the property of the diamond operator,

$$\hat{A}(X, p) \diamond \left\{ \hat{A}^{-1}(X, p) \right\} \left\{ \hat{A}(X, p) \right\} = - \diamond \left\{ \hat{A}(X, p) \right\} \left\{ \hat{A}^{-1}(X, p) \right\} \hat{A}(X, p), \quad (3.57)$$

which holds for any invertible matrix  $\hat{A}(X, p)$ . The property (3.57) can be derived by using the matrix identity

$$\partial_X \hat{A}^{-1} = -\hat{A}^{-1} (\partial_X \hat{A}) \hat{A}^{-1}. \quad (3.58)$$

Let us now investigate further the solutions (3.56) for the Higgs field. Since the early universe was in a  $SU(2)_L$ -symmetric state the matrix structure of the Higgs propagators and self-energies is trivial,

$$\begin{aligned} \Delta_{ab}(X, k) &= \delta_{ab} \Delta(X, k), \\ \Omega_{ab}(X, k) &= \delta_{ab} \Omega(X, k), \end{aligned} \quad (3.59)$$

where  $\Delta_{ab}(X, p)$  ( $\Omega_{ab}(X, k)$ ) stands for any of the Higgs propagators (self-energies). The Higgs two-point functions  $\Delta(X, k)$  and  $\Omega(X, k)$  without a hat or indices denote scalar functions without matrix structure. Since they commute the diamond operator in (3.56) vanishes,

$$\begin{aligned} \diamond \left\{ \omega_{R(A)} \right\} \left\{ \omega_{R(A)}^{-1} \right\} &= \left( \partial_X \omega_{R(A)} \right) \left( \partial_k \omega_{R(A)}^{-1} \right) - \left( \partial_k \omega_{R(A)} \right) \left( \partial_X \omega_{R(A)}^{-1} \right) \\ &= \left( \partial_X \omega_{R(A)} \right) \left( -\omega_{R(A)}^{-2} \partial_k \omega_{R(A)} \right) - \left( \partial_k \omega_{R(A)} \right) \left( -\omega_{R(A)}^{-2} \partial_X \omega_{R(A)} \right) = 0. \end{aligned} \quad (3.60)$$

The retarded and advanced Higgs propagators are then simply given by,

$$\Delta_{R(A)}(X, k) = -\omega_{R(A)}^{-1}(X, k) = \frac{-1}{k^2 - \Omega_{R(A)}(X, k)}. \quad (3.61)$$

The spectral function can be computed from the causal propagators,

$$\Delta_\rho = -i \left( \Delta_R - \Delta_A \right) = \frac{-\Omega_\rho}{(k^2 - \Omega_h)^2 + \left( \frac{1}{2} \Omega_\rho \right)^2}, \quad (3.62)$$

where we have suppressed the argument  $(X, k)$  for notational convenience. Note that the solution for the spectral function (3.62) is only formal since the self-energies  $\Omega_{\rho(h)}$  can be functionals of the spectral propagator. The solution (3.62) is useful if the back reactions are negligible. From (3.62) we see that the hermitian self-energy  $\Omega_h$  can be interpreted as an effective mass induced by the medium, and the spectral self-energy  $\Omega_\rho$  interpreted as an effective thermal width. The terminology ‘‘spectral’’ function becomes now clear: it underlines the fact that  $\Delta_\rho$  determines the spectral properties of the system at leading order in the gradient approximation. Similarly, the hermitian propagator reads,

$$\Delta_h = \frac{1}{2} \left( \Delta_R + \Delta_A \right) = \frac{-(k^2 - \Omega_h)}{(k^2 - \Omega_h)^2 + \left( \frac{1}{2} \Omega_\rho \right)^2}. \quad (3.63)$$

We now turn to the lepton field. Similarly to the Higgs field the  $SU(2)_L$  matrix structure of the lepton propagators and self-energies is trivial in the early universe. However the lepton two-point functions contain also lepton flavour indices. For simplicity we neglect here any correlation between the different flavours, and assume that the lepton flavours are well-described



by only one two-point function,

$$S_{ab}^{\alpha\beta}(X, p) = \delta_{ab}\delta^{\alpha\beta}S(X, p), \quad (3.64a)$$

$$\Sigma_{ab}^{\alpha\beta}(X, p) = \delta_{ab}\delta^{\alpha\beta}\Sigma(X, p). \quad (3.64b)$$

where  $S(X, p)$  and  $\Sigma(X, p)$  (without a hat and  $SU(2)_L$  and flavour indices) are now matrices in spinor space only. In leptogenesis the approximation (3.64) is valid if the flavour coherence of the lepton state produced by the decay of heavy neutrino is maintained [77]. This is the case only at high temperature,  $T \gtrsim 10^{12}$  GeV. In general the flavour effects should not be neglected in leptogenesis and could play an important role [26–30, 56, 124–134]. The remaining spinor structure of the lepton two-point functions implies that they do not commute in general. The matrix  $\omega_{R(A)}(X, p)$  can be easily inverted,

$$\omega_{R(A)}^{-1} = P_L \frac{\not{p} - \not{\Sigma}_{R(A)}}{(p - \Sigma_{R(A)})^2} + P_R \frac{\not{p}}{p^2}, \quad (3.65)$$

where we have used that the only spinor component of the lepton self-energy consistent with its chiral nature is given by  $\Sigma_{R(A)} = P_R \not{\Sigma}_{R(A)} P_L$ . Note that due to the spinor nature of the lepton propagators the second term in (3.56) does not vanish. However, this term is subleading and can be neglected. In this approximation the causal lepton propagators read,

$$S_{R(A)} \approx -\omega_{R(A)}^{-1} P_R = -\frac{\not{p} - \not{\Sigma}_{R(A)}}{(p - \Sigma_{R(A)})^2} P_R. \quad (3.66)$$

At leading order in the coupling the spectral function is then given by,

$$\begin{aligned} S_\rho &= -i(S_R - S_A) \\ &\approx -(\not{p} - \not{\Sigma}_h) P_R \frac{2p\Sigma_\rho}{(p^2 - 2p\Sigma_h)^2 + (p\Sigma_\rho)^2} + \not{\Sigma}_\rho P_R \frac{p^2 - 2p\Sigma_h}{(p^2 - 2p\Sigma_h)^2 + (p\Sigma_\rho)^2}. \end{aligned} \quad (3.67)$$

In the vicinity of the mass-shell, we have  $p^2 - 2p\Sigma_h \approx 0$  and the second term in (3.67) can be neglected,

$$S_\rho \approx -(\not{p} - \not{\Sigma}_h) P_R \frac{2p\Sigma_\rho}{(p^2 - 2p\Sigma_h)^2 + (p\Sigma_\rho)^2}. \quad (3.68)$$

In the same approximation the hermitian propagator is given by,

$$S_h \approx -(\not{p} - \not{\Sigma}_h) P_R \frac{p^2 - 2p\Sigma_h}{(p^2 - 2p\Sigma_h)^2 + (p\Sigma_\rho)^2}. \quad (3.69)$$

We derive next the spectral functions of the heavy neutrino field. We assume here that the heavy neutrino two-point functions are diagonal in flavour space, i.e.

$$\mathcal{S}^{ij}(X, q) = \mathcal{S}^{ii}(X, q)\delta^{ij}, \quad \Pi^{ij}(X, q) = \Pi^{ii}(X, q)\delta^{ij}. \quad (3.70)$$

As we will see later this approximation is not justified in leptogenesis and completely neglect the self-energy correction to the heavy neutrino decay amplitude. However this approximation will be useful for the “diagonal” part of the full heavy neutrino propagator, see chapter 4. In general the heavy neutrino propagator has a complicated spinor structure with scalar, pseudoscalar, vector, pseudovector and tensor components. Let us assume here that the retarded and advanced diagonal self-energies can be decomposed into scalar and vector components only,

$$\Pi_{R(A)}^{ii}(X, q) = \Pi_h^{s,ii}(x, q) + \mathbb{M}_h^{v,ii}(X, q) \pm \frac{i}{2} \mathbb{M}_\rho^{v,ii}(X, q). \quad (3.71)$$

This approximation holds at one-loop level in a CP-symmetric medium. Since the scalar part is induced by the on-shell renormalisation procedure the spectral part, which is finite at one-loop level, contains only a vector component. As in the lepton case we neglect the diamond operator in (3.56) and write the causal heavy neutrino propagators as,

$$\begin{aligned} \mathcal{S}_{R(A)}^{ii} &= -[\not{q} - M_i - \Pi_{R(A)}^{ii}]^{-1} \\ &\approx -(\not{q} - \mathbb{M}_h^{v,ii} + M_i + \Pi_h^{s,ii} \mp \frac{i}{2} \mathbb{M}_\rho^{v,ii}) \frac{q^2 - \mathbf{M}_i^2 \pm iq\Pi_\rho^{v,ii}}{(q^2 - \mathbf{M}_i^2)^2 + (q\Pi_\rho^{v,ii})^2}, \end{aligned} \quad (3.72)$$

where we have neglected higher order term in the Yukawa coupling and defined an *effective*  $q$ -dependent mass,

$$\mathbf{M}_i(X, q) \approx M_i + \frac{q\Pi_h^{v,ii}(X, q)}{M_i} + \Pi_h^{s,ii}(X, q). \quad (3.73)$$

Similarly to the lepton case the spectral function reads,

$$\begin{aligned} \mathcal{S}_\rho^{ii} &= (\not{q} - \mathbb{M}_h^{v,ii} + M_i + \Pi_h^{s,ii}) \frac{-2q\Pi_\rho^{v,ii}}{(q^2 - \mathbf{M}_i^2)^2 + (q\Pi_\rho^{v,ii})^2} + \frac{(q^2 - \mathbf{M}_i^2)\mathbb{M}_\rho^{v,ii}}{(q^2 - \mathbf{M}_i^2)^2 + (q\Pi_\rho^{v,ii})^2} \\ &\approx (\not{q} - \mathbb{M}_h^{v,ii} + M_i + \Pi_h^{s,ii}) \frac{-2q\Pi_\rho^{v,ii}}{(q^2 - \mathbf{M}_i^2)^2 + (q\Pi_\rho^{v,ii})^2}, \end{aligned} \quad (3.74)$$

where the second equality holds near the mass-shell. For a hierarchical mass spectrum we can safely neglect the thermal correction to the heavy neutrino mass, and write,

$$\mathcal{S}_\rho^{ii} \approx (\not{q} + M_i) \frac{-2q\Pi_\rho^{v,ii}}{(q^2 - M_i^2)^2 + (q\Pi_\rho^{v,ii})^2}. \quad (3.75)$$

This approximation is valid if  $|M_i - M_j| \ll |\Pi_{h(\rho)}|$ . For a quasidegenerate mass spectrum one needs to take into account the thermal mass of the heavy neutrino. In the same approximation the heavy neutrino hermitian propagator is given by,

$$\mathcal{S}_h^{ii} = (\not{q} + M_i) \frac{-(q^2 - M_i^2)}{(q^2 - M_i^2)^2 + (q\Pi_\rho^{v,ii})^2}. \quad (3.76)$$

The above solutions for the spectral and hermitian functions of the Higgs, lepton and Ma-

jorana fields are valid up to first order in the gradient expansion. The spectral functions determine the spectrum of the theory. They incorporate thermal mass through the hermitian self-energy and thermal width through the spectral self-energy. In general the constraint and kinetic equations for the statistical two-point functions cannot be solved algebraically. We will show in the next section how the Boltzmann equation can be recovered from the kinetic equation for the statistical function.

### 3.3 Boltzmann limit

We derive here the Boltzmann equation from the kinetic equation for the statistical functions. The first step is to introduce a pole approximation for the spectral functions. Expanding (3.62), (3.68) and (3.75) around the poles and taking the limit of vanishing width (see [69] for more details), we find,

$$\Delta_\rho(X, k) = \text{sign}(k^0)(2\pi)\delta(k^2 - m_\phi^2), \quad (3.77a)$$

$$S_\rho(X, p) = P_L \not{p} \text{sign}(p^0)(2\pi)\delta(p^2 - m_\ell^2), \quad (3.77b)$$

$$\mathcal{S}_\rho^{ii}(X, q) = (\not{q} + M_i) \text{sign}(q^0)(2\pi)\delta(q^2 - M_i^2), \quad (3.77c)$$

where we have used (2.27) and the fact that the effective decay width and the zero component of the momentum have opposite sign (hence the sign function). Here  $m_\phi$  and  $m_\ell$  are the effective thermal masses of the Higgs and lepton respectively, which are the real part of the poles of the spectral functions. We assume here that the thermal masses are independent of the spatial momentum of the particle. In (3.77b) we assumed that the lepton field obeys a conventional dispersion relation, which may not be satisfied in the early universe [135–137]. The approximations (3.77) are known as the *quasiparticle* approximation.

Since the Boltzmann equation describes the time evolution of the distribution function we need to express the statistical propagator in terms of the distribution function. The equilibrium case will help us to find such an ansatz. The KMS relations (3.21) and (3.22) in Wigner space read,

$$\Delta_{>}^{\text{eq}}(k) = e^{\beta k^0} \Delta_{<}^{\text{eq}}(k), \quad (3.78a)$$

$$S_{>}^{\text{eq}}(p) = -e^{\beta p^0} S_{<}^{\text{eq}}(p), \quad (3.78b)$$

$$\mathcal{S}_{>}^{\text{eq}, ii}(q) = -e^{\beta q^0} \mathcal{S}_{<}^{\text{eq}, ii}(q), \quad (3.78c)$$

where we used the fact that the equilibrium two-point functions are independent of the central

coordinate  $X$ . In terms of the statistical and spectral functions the above relations read,

$$\begin{aligned}\Delta_F^{\text{eq}}(k) &= \left(\frac{1}{2} + \frac{1}{e^{\beta k^0} - 1}\right) \Delta_\rho^{\text{eq}}(k) \\ &= \left(\theta(k^0) \left[\frac{1}{2} + \frac{1}{e^{\beta|k^0|} - 1}\right] - \theta(-k^0) \left[\frac{1}{2} + \frac{1}{e^{\beta|k^0|} - 1}\right]\right) \Delta_\rho^{\text{eq}}(k),\end{aligned}\quad (3.79a)$$

$$\begin{aligned}S_F^{\text{eq}}(p) &= \left(\frac{1}{2} - \frac{1}{e^{\beta p^0} + 1}\right) S_\rho^{\text{eq}}(p) \\ &= \left(\theta(p^0) \left[\frac{1}{2} - \frac{1}{e^{\beta|p^0|} + 1}\right] - \theta(-p^0) \left[\frac{1}{2} - \frac{1}{e^{\beta|p^0|} + 1}\right]\right) S_\rho^{\text{eq}}(p),\end{aligned}\quad (3.79b)$$

$$\begin{aligned}\mathcal{S}_F^{\text{eq},ii}(q) &= \left(\frac{1}{2} - \frac{1}{e^{\beta q^0} + 1}\right) \mathcal{S}_\rho^{\text{eq},ii}(q) \\ &= \left(\theta(q^0) \left[\frac{1}{2} - \frac{1}{e^{\beta|q^0|} + 1}\right] - \theta(-q^0) \left[\frac{1}{2} - \frac{1}{e^{\beta|q^0|} + 1}\right]\right) \mathcal{S}_\rho^{\text{eq},ii}(q),\end{aligned}\quad (3.79c)$$

where we can recognise the Bose-Einstein or Fermi-Dirac distribution functions in the square bracket. Since the positive and negative frequencies describe respectively particle and antiparticle (this can be inferred from the CP-conjugated propagators), we may rewrite the above equations as,

$$\Delta_F^{\text{eq}}(k) = \left(\theta(k^0) \left[\frac{1}{2} + f_\phi^{\text{eq},k}\right] - \theta(-k^0) \left[\frac{1}{2} + f_{\bar{\phi}}^{\text{eq},k}\right]\right) \Delta_\rho^{\text{eq}}(k), \quad (3.80a)$$

$$S_F^{\text{eq}}(p) = \left(\theta(p^0) \left[\frac{1}{2} - f_\ell^{\text{eq},p}\right] - \theta(-p^0) \left[\frac{1}{2} - f_{\bar{\ell}}^{\text{eq},p}\right]\right) S_\rho^{\text{eq}}(p), \quad (3.80b)$$

$$\mathcal{S}_F^{\text{eq},ii}(q) = \left(\theta(q^0) \left[\frac{1}{2} - f_{N_i}^{\text{eq},q}\right] - \theta(-q^0) \left[\frac{1}{2} + f_{N_i}^{\text{eq},q}\right]\right) \mathcal{S}_\rho^{\text{eq},ii}(q), \quad (3.80c)$$

with the equilibrium distribution functions,

$$f_\phi^{\text{eq},k} \equiv f_{\bar{\phi}}^{\text{eq},k} \equiv \frac{1}{e^{\beta E_\phi^k} - 1}, \quad (3.81a)$$

$$f_\ell^{\text{eq},p} \equiv f_{\bar{\ell}}^{\text{eq},p} \equiv \frac{1}{e^{\beta E_\ell^p} + 1}, \quad (3.81b)$$

$$f_{N_i}^{\text{eq},q} \equiv \frac{1}{e^{\beta E_{N_i}^q} + 1}. \quad (3.81c)$$

The equations (3.80) motivate the *Kadanoff-Baym ansatz*,

$$\Delta_F(X, k) = \left(\theta(k^0) \left[\frac{1}{2} + f_\phi(X, \vec{k})\right] - \theta(-k^0) \left[\frac{1}{2} + f_{\bar{\phi}}(X, \vec{k})\right]\right) \Delta_\rho(X, k), \quad (3.82a)$$

$$S_F(X, p) = \left(\theta(p^0) \left[\frac{1}{2} - f_\ell(X, \vec{p})\right] - \theta(-p^0) \left[\frac{1}{2} - f_{\bar{\ell}}(X, \vec{p})\right]\right) S_\rho(X, p), \quad (3.82b)$$

$$\mathcal{S}_F^{\text{eq},ii}(X, q) = \left(\theta(q^0) \left[\frac{1}{2} - f_{N_i}(X, \vec{q})\right] - \theta(-q^0) \left[\frac{1}{2} - f_{N_i}(X, \vec{q})\right]\right) \mathcal{S}_\rho^{\text{eq},ii}(X, q), \quad (3.82c)$$

where  $f_\phi$  ( $f_{\bar{\phi}}$ ),  $f_\ell$  ( $f_{\bar{\ell}}$ ) and  $f_{N_i}$  are the *out-of-equilibrium* distribution functions of the (anti)-Higgs, (anti)lepton and heavy neutrino respectively. Note that the distribution functions do not depend on  $p^0$ , i.e. they are on-shell functions. We do not introduce a distribution function for the right-handed antineutrino due to its Majorana nature. This is consistent with its

symmetry property under transposition, see appendix C. The ansatz (3.82) assumes that the different spin states can be well described by a single distribution function. In order to simplify the notation we introduce the generalised distribution function for any species  $a$ ,

$$F_a(X, p) = \theta(p^0) f_a(X, \vec{p}) \mp \theta(-p^0) [1 \pm f_a(X, \vec{p})], \quad (3.83)$$

where the upper (lower) sign applies to boson (fermion). The KB ansatz takes the form:

$$\Delta_F(X, k) = \left( \frac{1}{2} + F_\phi(X, k) \right) \Delta_\rho(X, k), \quad (3.84a)$$

$$S_F(X, p) = \left( \frac{1}{2} - F_\ell(X, p) \right) S_\rho(X, p), \quad (3.84b)$$

$$\mathcal{S}_F^{ii}(X, q) = \left( \frac{1}{2} - F_{N_i}(X, q) \right) \mathcal{S}_\rho^{ii}(X, q). \quad (3.84c)$$

It is often more convenient to work with the Wightmann two-point functions. In terms of these propagators the KB ansatz reads,

$$\Delta_{>} = (1 + F_\phi) \Delta_\rho, \quad \Delta_{<} = F_\phi \Delta_\rho, \quad (3.85a)$$

$$S_{>} = (1 - F_\ell) S_\rho, \quad S_{<} = -F_\ell S_\rho, \quad (3.85b)$$

$$\mathcal{S}_{>}^{ii} = (1 - F_{N_i}) \mathcal{S}_\rho^{ii}, \quad \mathcal{S}_{<}^{ii} = -F_{N_i} \mathcal{S}_\rho^{ii}. \quad (3.85c)$$

We can now derive the kinetic equation for the distribution functions. We focus first on the Higgs field. In a  $SU(2)_L$  symmetric state equation (3.59) applies and the constraint and kinetic equations read,

$$\left( k^2 - \Omega_h \right) \Delta_{\geq} = \Omega_{\geq} \Delta_h + \frac{1}{4} \diamond \left\{ \Omega_\rho \right\} \left\{ \Delta_{\geq} \right\} + \frac{1}{4} \diamond \left\{ \Omega_{\geq} \right\} \left\{ \Delta_\rho \right\}, \quad (3.86a)$$

$$- \diamond \left\{ k^2 - \Omega_h \right\} \left\{ \Delta_{\geq} \right\} = \Omega_{>} \Delta_{<} - \Omega_{<} \Delta_{>} - \diamond \left\{ \Omega_{\geq} \right\} \left\{ \Delta_h \right\}. \quad (3.86b)$$

The ansatz (3.85a) is a solution of the constraint equation when the on-shell condition is satisfied and when the diamond operator on the RHS is neglected. In the same approximation we also neglect the diamond operator on the RHS of the kinetic equation (3.86b). We will see in the next section that this term contributes to the drift term on the LHS, and should not be neglected. This contribution will be taken into account by a new quasiparticle ansatz. Let us first neglect this term. Inserting the KB ansatz (3.85) and the quasiparticle approximation (3.77) into (3.86b) we find,

$$- \diamond \left\{ k^2 - \Omega_h \right\} \left\{ (1 + F_\phi) \Delta_\rho \right\} = \Omega_{>} F_\phi \Delta_\rho - \Omega_{<} (1 + F_\phi) \Delta_\rho. \quad (3.87)$$

We neglect then for simplicity the derivative of the hermitian self-energy on the LHS, and integrate over the positive frequency,

$$2k \partial_X f_\phi(X, k) = \Omega_{>}(X, k) f_\phi(X, k) - \Omega_{<}(X, k) (1 + f_\phi(X, k)) \quad \text{with } k^0 = \sqrt{\vec{k}^2 + m_\phi^2}, \quad (3.88)$$

where we have used the definition (3.83). We have worked until now in a flat space time. The effect of a curved space time can be implemented by replacing the derivative on the LHS by

the covariant derivative [138],

$$k\partial_X \rightarrow k\mathcal{D}_X. \quad (3.89)$$

We recover then the Boltzmann equation (2.2) if we identify the RHS of (3.88) with the collision terms. From the factor  $f_\phi$  and  $(1 + f_\phi)$  we can infer that  $-\Omega_>f_\phi$  corresponds to the loss term and  $-\Omega_<(1 + f_\phi)$  to the gain term.

The derivation of the Boltzmann equation for the lepton and heavy Majorana neutrino can be performed in a similar way. We will not present this computation here, and but will derive the Boltzmann equation for the fermions with the new quasiparticle ansatz in section 3.4.

The above derivation of the Boltzmann equation is not very satisfying [139, 140]. Neglecting the diamond operator on the RHS of (3.86b) but keeping it on the LHS is not well justified. Moreover the KB ansatz (3.85) together with the quasiparticle approximation (3.77) completely neglect the off-shell part of the Wightmann propagators. In the next section we will see that these problems are related and can be solved by introducing a new quasiparticle ansatz.

### 3.4 Extended quasiparticle ansatz

We have seen in the last section that the quasiparticle and the KB ansatzes are very crude approximations even though they lead to the correct Boltzmann equation at leading order. We will see here how this ansatz can be improved. This new ansatz is crucial in leptogenesis in order to obtain the correct scattering amplitudes.

The basic idea is to represent the Wightman two-point functions as a sum of two terms [123, 141–144],

$$\Delta_{\geq}(X, k) \equiv \tilde{\Delta}_{\geq}(X, k) + \Delta_{\geq}^{\text{off-shell}}(X, k), \quad (3.90)$$

where  $\tilde{\Delta}_{\geq}(X, k)$  describes the particle behaviour of the propagator, and  $\Delta_{\geq}^{\text{off-shell}}(X, k)$  its off-shell part. We can derive explicitly the two terms of the propagator (3.90) by demanding that the particle part of the propagator satisfies a transport equation. The first step is to notice that we can rewrite the diamond operator term on the LHS of (3.86b) as,

$$\begin{aligned} \diamond\{\Omega_{\geq}\}\{\Delta_h\} &= \diamond\{\Omega_{\geq}\}\left\{\frac{1}{2}(\Delta_R + \Delta_A)\right\} \\ &= -\frac{1}{2}\diamond\{\Delta_R\Omega_{\geq}\Delta_R\}\{\Delta_R^{-1}\} - \frac{1}{2}\diamond\{\Delta_A\Omega_{\geq}\Delta_A\}\{\Delta_A^{-1}\} \\ &= \diamond\left\{\frac{1}{2}\Delta_R\Omega_{\geq}\Delta_R + \frac{1}{2}\Delta_A\Omega_{\geq}\Delta_A\right\}\{k^2 - \Omega_h\} \\ &\quad - \frac{i}{4}\diamond\{\Delta_R\Omega_{\geq}\Delta_R - \Delta_A\Omega_{\geq}\Delta_A\}\{\Omega_\rho\}, \end{aligned} \quad (3.91)$$

where we have used the solution (3.61) and the identity (3.57). We can then rewrite (3.86b) as,

$$\begin{aligned} -\diamond\{k^2 - \Omega_h\}\left\{\Delta_{\geq} + \frac{1}{2}(\Delta_R\Omega_{\geq}\Delta_R + \Delta_A\Omega_{\geq}\Delta_A)\right\} \\ - \frac{i}{4}\diamond\{\Delta_R\Omega_{\geq}\Delta_R - \Delta_A\Omega_{\geq}\Delta_A\}\{\Omega_\rho\} = \Omega_>\Delta_< - \Omega_<\Delta_>. \end{aligned} \quad (3.92)$$

Noting that the operator  $\diamond\{k^2 - \Omega_h\}\{\cdot\}$  gives us the LHS of the Boltzmann equation in the on-shell limit we define the off-shell part of the Wightmann propagators as,

$$\Delta_{\gtrless}^{\text{off-shell}} = -\frac{1}{2}(\Delta_R\Omega_{\gtrless}\Delta_R + \Delta_A\Omega_{\gtrless}\Delta_A). \quad (3.93)$$

Inserting the decomposition (3.90) with (3.93) into the kinetic equation (3.92) we find that only the ‘‘particle’’ part of the propagator enters the equation,

$$-\diamond\{k^2 - \Omega_h\}\{\tilde{\Delta}_{\gtrless}\} - \frac{i}{4}\diamond\{\Delta_R\Omega_{\gtrless}\Delta_R - \Delta_A\Omega_{\gtrless}\Delta_A\}\{\Omega_\rho\} = \Omega_{>}\tilde{\Delta}_{<} - \Omega_{<}\tilde{\Delta}_{>}. \quad (3.94)$$

We interpret  $\tilde{\Delta}_{\gtrless}$  as the quasiparticle part of the Wightman functions. The extra term,  $\Delta_{\gtrless}^{\text{off-shell}}$ , describes the off-shell part, and is completely neglected in the quasiparticle approximation studied in the previous section. We make now an ansatz similar to KB ansatz, but only for the quasiparticle part of the Wightman functions,

$$\begin{aligned} \tilde{\Delta}_{>}(X, k) &= \left(1 + F_\phi(X, k)\right)\tilde{\Delta}_\rho(X, k), \\ \tilde{\Delta}_{<}(X, k) &= F_\phi(X, k)\tilde{\Delta}_\rho(X, k). \end{aligned} \quad (3.95)$$

$$(3.96)$$

This ansatz is referred to as *extended quasiparticle* (eQP) approximation. The generalised distribution function  $F_\phi$  is defined similarly to (3.83). The eQP spectral function in (3.95) is given by,

$$\tilde{\Delta}_\rho \equiv \tilde{\Delta}_{>} - \tilde{\Delta}_{<} = \Delta_\rho + \frac{1}{2}\Delta_R\Omega_\rho\Delta_R + \frac{1}{2}\Delta_A\Omega_\rho\Delta_A. \quad (3.97)$$

After some algebra we find,

$$\tilde{\Delta}_\rho = -\frac{1}{2}\Delta_R\Omega_\rho\Delta_R\Omega_\rho\Delta_A\Omega_\rho\Delta_A = -\frac{1}{2}\frac{\Omega_\rho^3}{\left[(k^2 - \Omega_h)^2 + \left(\frac{1}{2}\Omega_\rho\right)^2\right]^2}. \quad (3.98)$$

Finally the Wightman functions are given by,

$$\Delta_{>} = \left(1 + F_\phi\right)\tilde{\Delta}_\rho - \frac{1}{2}(\Delta_R\Omega_{>}\Delta_R + \Delta_A\Omega_{>}\Delta_A), \quad (3.99a)$$

$$\Delta_{<} = F_\phi\tilde{\Delta}_\rho - \frac{1}{2}(\Delta_R\Omega_{<}\Delta_R + \Delta_A\Omega_{<}\Delta_A). \quad (3.99b)$$

As in (3.77a) the extended spectral function can be approximated by a delta function in the small width limit ,

$$\tilde{\Delta}_\rho \approx \lim_{\Omega_\rho \rightarrow 0} \left(-\frac{1}{2}\right) \frac{\Omega_\rho^3}{\left[(k^2 - \Omega_h)^2 + \left(\frac{1}{2}\Omega_\rho\right)^2\right]^2} = \text{sign}(k^0)(2\pi)\delta(k^2 - m_\phi), \quad (3.100)$$

where we have used the representation of the delta function,

$$\lim_{\epsilon \rightarrow 0^+} \frac{2\epsilon^3}{[\omega^2 + \epsilon^2]^2} = \pi\delta(\omega). \quad (3.101)$$

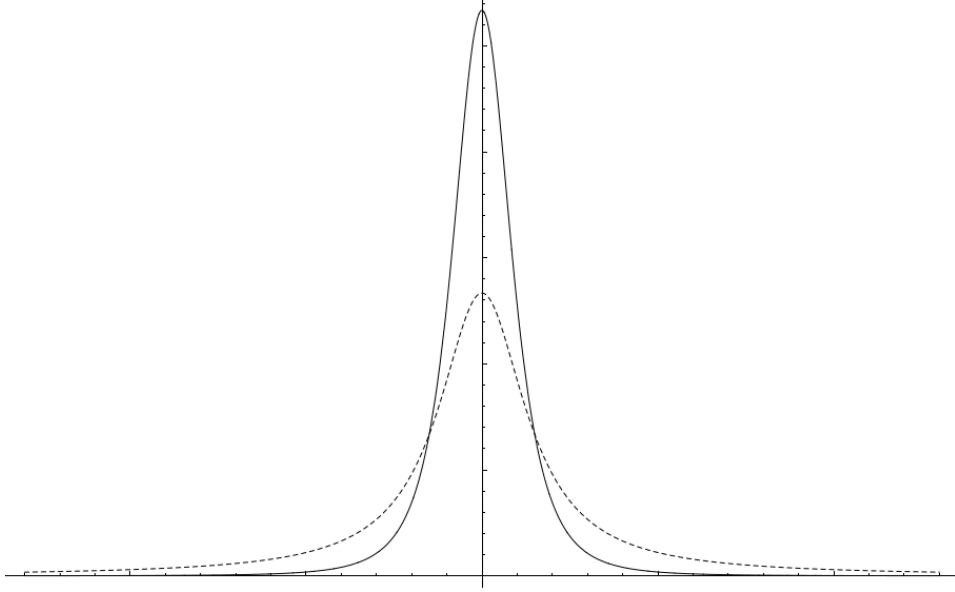


Figure 3.2: Schematic representation of the eQP spectral function (solid line) and QP spectral function (dashed line). The eQP spectral function peaks much sharper around the pole than the QP spectral function. Note that both spectral functions peak at the same pole.

Note that the QP and the eQP approximations pick up the same pole. However the eQP spectral function is much sharper around the pole than the QP spectral function, see Fig. 3.2. The difference between them, the off-pole contribution, is identified with off-shell effects and is neglected in the QP approximation. We will see that these terms describe the Higgs-mediated processes and  $|\Delta L| = 2$  scattering processes in leptogenesis.

Inserting the ansatz (3.99) into (3.92), integrating over the positive frequency and neglecting the momentum and time dependence of the hermitian self-energy, we find that the distribution function satisfies exactly the same equation as in the QP approximation [145, 146],

$$2k\partial_X f_\phi(X, \vec{k}) = \Omega_>(X, k)f_\phi(X, \vec{k}) - \Omega_<(X, k)(1 + f_\phi(X, \vec{k})) \text{ with } k^0 = \sqrt{\vec{k}^2 + m_\phi^2}. \quad (3.102)$$

The difference between the QP and eQP approximations comes from the fact that the self-energies on the RHS of (3.102) and (3.88) are functional of the propagators which are given by different expressions in the QP or eQP approximations. The difference between the two ansatzes only arises at next-to-leading order in the coupling.

In [146] it has been shown that the constraint equation (3.86a) is automatically satisfied if the kinetic equation holds. Therefore the eQP ansatz is consistent with the first order gradient expansion.

We can similarly derive the eQP ansatz for the fermion field. We focus here only on the heavy neutrino field since the lepton eQP can be obtained in the same manner. The kinetic



equation for the heavy neutrino reads,

$$\begin{aligned}
 -\frac{i}{2} \diamond \left\{ \hat{\omega}_R \right\} \left\{ \hat{\Pi}_{\geq} \right\} + \frac{i}{2} \diamond \left\{ \hat{\mathcal{S}}_{\geq} \right\} \left\{ \hat{\omega}_A \right\} + \hat{\omega}_R \hat{\mathcal{S}}_{\geq} - \hat{\mathcal{S}}_{\geq} \hat{\omega}_A = \hat{\Pi}_{\geq} \hat{\mathcal{S}}_A - \hat{\mathcal{S}}_R \hat{\Pi}_{\geq} \\
 - \frac{i}{2} \diamond \left\{ \hat{\Pi}_{\geq} \right\} \left\{ \hat{\mathcal{S}}_A \right\} + \frac{i}{2} \diamond \left\{ \hat{\mathcal{S}}_R \right\} \left\{ \hat{\Pi}_{\geq} \right\}. \quad (3.103)
 \end{aligned}$$

Due to the matrix structure of the right-handed neutrino two-point functions many terms in (3.103) do not cancel with each other. We approximate here the two-point functions by their diagonal components,

$$\mathcal{S}^{ij} \approx \mathcal{S}^{ii} \delta^{ij}, \quad \Pi^{ij} \approx \Pi^{ii} \delta^{ij}. \quad (3.104)$$

We stress here again that this approximation is not suitable for leptogenesis. This computation will however be useful later. Since we are only interested in the total particle number, and not in the spin distribution, we take the trace over the spinor indices in the kinetic equation. Using the identities of the diamond operator given in appendix D.1 we rearrange the terms in the kinetic equations similarly to the scalar case and obtain,

$$\begin{aligned}
 \text{tr} \left[ -\diamond \left\{ \not{q} - \Pi_h^{ii} \right\} \left\{ \mathcal{S}_{\geq}^{ii} + \frac{1}{2} (\mathcal{S}_R^{ii} \Pi_{\geq}^{ii} \mathcal{S}_R^{ii} + \mathcal{S}_A^{ii} \Pi_{\geq}^{ii} \mathcal{S}_A^{ii}) \right\} \right] \\
 - \text{tr} \left[ \frac{i}{4} \diamond \left\{ \mathcal{S}_R^{ii} \Pi_{\geq}^{ii} \mathcal{S}_R^{ii} - \mathcal{S}_A^{ii} \Pi_{\geq}^{ii} \mathcal{S}_A^{ii} \right\} \left\{ \Pi_{\rho}^{ii} \right\} \right] = \text{tr} \left[ \Pi_{>}^{ii} \mathcal{S}_{<}^{ii} - \Pi_{<}^{ii} \mathcal{S}_{>}^{ii} \right]. \quad (3.105)
 \end{aligned}$$

We define then the eQP ansatz for the heavy neutrino,

$$\mathcal{S}_{>}^{ii} = (1 - F_{N_i}) \tilde{\mathcal{S}}_{\rho}^{ii} - \frac{1}{2} (\mathcal{S}_R^{ii} \Pi_{>}^{ii} \mathcal{S}_R^{ii} + \mathcal{S}_A^{ii} \Pi_{>}^{ii} \mathcal{S}_A^{ii}), \quad (3.106a)$$

$$\mathcal{S}_{<}^{ii} = -F_{N_i} \tilde{\mathcal{S}}_{\rho}^{ii} - \frac{1}{2} (\mathcal{S}_R^{ii} \Pi_{<}^{ii} \mathcal{S}_R^{ii} + \mathcal{S}_A^{ii} \Pi_{<}^{ii} \mathcal{S}_A^{ii}), \quad (3.106b)$$

with the generalised distribution function  $F_{N_i}$  given by (3.83) and the eQP spectral function,

$$\tilde{\mathcal{S}}_{\rho}^{ii} = -\frac{1}{2} \Pi_R^{ii} \mathcal{S}_{\rho}^{ii} \Pi_R^{ii} \mathcal{S}_{\rho}^{ii} \Pi_A^{ii} \mathcal{S}_{\rho}^{ii} \Pi_A^{ii}. \quad (3.107)$$

Inserting the self-energy (3.71) and the causal propagators (3.72) into the eQP spectral function we find at leading order in the coupling,

$$\begin{aligned}
 \tilde{\mathcal{S}}_{\rho}^{ii} \approx (\not{q} - \mathbb{M}_h^{v,ii} + M_i + \Pi_h^{s,ii}) \frac{-4(q \Pi_{\rho}^{v,ii})^3}{\left[ (q^2 - \mathbf{M}_i^2)^2 + (q \Pi_{\rho}^{v,ii})^2 \right]^2} \\
 \approx (\not{q} + M_i) \text{sign}(q^0) (2\pi) \delta(q^2 - M_i^2), \quad (3.108)
 \end{aligned}$$

where we have neglected contributions which are tiny on the mass-shell. To obtain the second equality we have neglected the thermal correction to the mass and taken the limit of vanishing width. The transport equation for the right-handed neutrino distribution function is obtained by substituting the eQP ansatz (3.106) into the kinetic equation (3.105) and performing the same step as in the scalar case,

$$2q \partial_X f_{N_i} = \frac{1}{2} \text{tr} \left[ \Pi_{>}^{ii} (\not{q} + M_i) f_{N_i} + \Pi_{<}^{ii} (\not{q} + M_i) (1 - f_{N_i}) \right]. \quad (3.109)$$

When the spinor decomposition of the Wightman self-energy has only a vector component,  $\Pi_{\geq}^{ii} = \mathbb{1}_{\geq}^{v,ii}$ , which is the case at one-loop level in a CP-symmetric medium, the equation (3.109) simplifies to,

$$2q\partial_X f_{N_i} = 2q\Pi_{>}^{v,ii} f_{N_i} + 2q\Pi_{<}^{v,ii} (1 - f_{N_i}). \quad (3.110)$$

For completeness we write the eQP approximation for the lepton field in the unflavoured approximation which is obtained in complete analogy with the heavy neutrino case,

$$S_{>} = (1 - F_\ell)\tilde{S}_\rho - \frac{1}{2}(S_R\Sigma_{>}S_R + S_R\Sigma_{>}S_R), \quad (3.111a)$$

$$S_{<} = -F_\ell\tilde{S}_\rho - \frac{1}{2}(S_R\Sigma_{<}S_R + S_R\Sigma_{<}S_R), \quad (3.111b)$$

and the eQP spectral function,

$$\tilde{S}_\rho = -P_L(\not{p} - \not{Z}_h) \frac{4(p\Sigma_\rho)^3}{\left[(p^2 - 2p\Sigma_h)^2 + (p\Sigma_\rho)^2\right]^2} \approx P_L\not{p}\text{sign}(p^0)(2\pi)\delta(p^2 - m_\ell^2), \quad (3.112)$$

where the second equality is valid for conventional dispersion relation and for vanishing width.

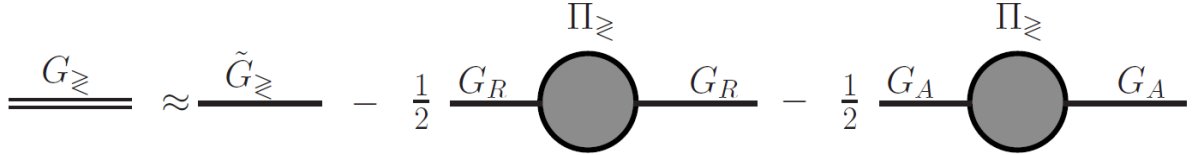


Figure 3.3: Diagrammatic representation of the eQP approximation. The full Wightmann propagators,  $G_{\geq}$ , are splitted into an on-shell part,  $\tilde{G}_{\geq}$ , which describes the particle behaviour of the propagator, and an off-shell part,  $-\frac{1}{2}(G_R\Pi_{\geq}G_R + G_A\Pi_{\geq}G_A)$ . The usual KB ansatz completely neglects the off-shell term.

The eQP approximation for bosons or fermions is diagrammatically represented in Fig. 3.3. The first term,  $\tilde{G}_{\geq}$ , describes on-shell particles. Due to the delta-function in  $\tilde{G}_\rho$  the on-shell term can be interpreted as “cut-propagator” which describes on-shell particles created from or absorbed by the medium [112]. The off-shell term,  $-\frac{1}{2}(G_R\Pi_{\geq}G_R + G_A\Pi_{\geq}G_A)$ , describes intermediate off-shell particles. In leptogenesis this term is needed to describe the  $|\Delta L| = 2$  scattering processes mediated by the heavy neutrino and the  $|\Delta L| = 1$  processes mediated by the Higgs. We will see that the intermediate propagator, which results from the eQP approximation, is automatically RIS-subtracted.

# Chapter 4

## Nonequilibrium approach to leptogenesis

In this section we apply to leptogenesis the formalism of NEQFT discussed in the previous chapter. This framework has been shown recently to be suitable for the derivation of quantum dynamic equations for the lepton asymmetry within a first-principle approach, and capable of incorporating medium, off-shell, coherence and possibly further quantum effects in a self-consistent way [54–71]. This chapter is based on work presented in [72] and on work in preparation [73].

### 4.1 Lepton asymmetry

The lepton asymmetry is given by the  $\mu = 0$ -component of the expectation value of the lepton-current operator:

$$j_L^\mu(x) = \sum_{\alpha,a} \langle \bar{\ell}_\alpha^a(x) \gamma^\mu \ell_\alpha^a(x) \rangle. \quad (4.1)$$

It can be expressed in terms of the lepton two-point function introduced in chapter 3,

$$j_L^\mu(x) = - \sum_{\alpha,a} \text{tr} [\gamma^\mu S_{<,aa}^{\alpha\alpha}(x,x)], \quad (4.2)$$

where the trace is over the spinor indices only. The minus sign in (4.2) comes from the definition of the lepton Wightman function,

$$S_{<,ab}^{\alpha\beta}(x,y) = - \langle \bar{\ell}_b^\beta(y) \ell_a^\alpha(x) \rangle. \quad (4.3)$$

An equation of motion for the lepton asymmetry can be derived by considering the divergence of the lepton current  $\mathcal{D}_\mu j_L^\mu(x)$ , where  $\mathcal{D}_\mu$  is the covariant derivative. Using the KB equations for the lepton field (3.30) one obtains:

$$\begin{aligned} \mathcal{D}_\mu j_L^\mu(x) &= - \sum_{\alpha,a} \text{tr} [(\mathcal{D}_\mu^x + \mathcal{D}_\mu^y) \gamma^\mu S_{<,aa}^{\alpha\alpha}(x,y)] \Big|_{x=y} \\ &= -i \sum_{\alpha,\beta,a,b} \int \mathcal{D}^4 z \theta(z^0) \text{tr} [\Sigma_{R,ab}^{\alpha\beta}(x,z) S_{<,ba}^{\beta\alpha}(z,x) + \Sigma_{<,ab}^{\alpha\beta}(x,z) S_{A,ba}^{\beta\alpha}(z,x) \\ &\quad - S_{<,ab}^{\alpha\beta}(x,z) \Sigma_{A,ba}^{\beta\alpha}(z,x) - S_{R,ab}^{\alpha\beta}(x,z) \Sigma_{<,ba}^{\beta\alpha}(z,x)]. \end{aligned} \quad (4.4)$$

We assume here that in a FRW space-time the effects of the universe expansion can be captured, to the required accuracy, by introducing the invariant integration measure  $\mathcal{D}^4 z \equiv \sqrt{-g} d^4 z$  and

using the covariant derivative  $\mathcal{D}_\mu$ . As has been demonstrated in [138], this is the case for scalar fields. A manifestly covariant generalisation of center and relative coordinates  $X$  and  $s$  to curved space-time can be found in [147].

The equation (4.4) represents the quantum field theory generalisation of the Boltzmann equation for the lepton asymmetry. Thus, it may be considered as the master equation for a quantum field theoretical treatment of leptogenesis [54, 56].

Following the discussion above (3.49) we perform a leading order gradient expansion of the time evolution equation for the lepton asymmetry,

$$\mathcal{D}_\mu j_L^\mu(X) = \sum_{\alpha,\beta,a,b} \int \frac{d^4 p}{(2\pi)^4} \text{tr} [\Sigma_{<,ab}^{\alpha\beta}(X,p) S_{>,ba}^{\beta\alpha}(X,p) - \Sigma_{>,ab}^{\alpha\beta}(X,p) S_{<,ba}^{\beta\alpha}(X,p)], \quad (4.5)$$

where  $p$  corresponds to the *physical* momentum [138]. In a homogeneous and isotropic medium, which the early universe was to a very good approximation, the two-point functions depend only on the time coordinate  $X^0 = t$  and on the momentum  $p = (p^0, |\vec{p}|)$ ,

$$\mathcal{D}_\mu j_L^\mu(t) = g_w \sum_{\alpha,\beta} \int \frac{d^4 p}{(2\pi)^4} \text{tr} [\Sigma_{<}^{\alpha\beta}(t,p) S_{>}^{\beta\alpha}(t,p) - \Sigma_{>}^{\alpha\beta}(t,p) S_{<}^{\beta\alpha}(t,p)], \quad (4.6)$$

where we have used the fact the early universe was in a  $SU(2)_L$  symmetric state and explicitly performed the summation over the  $SU(2)_L$  indices (hence the factor  $g_w = 2$ ). The physical meaning of the above equation becomes clear when we write it as an integral over positive frequency only,

$$\begin{aligned} \mathcal{D}_\mu j_L^\mu(t) = g_w \sum_{\alpha,\beta} \int_0^\infty \frac{dp^0}{(2\pi)} \int \frac{d^3 \vec{p}}{(2\pi)^3} \text{tr} [(\Sigma_{<}^{\alpha\beta}(t,p) S_{>}^{\beta\alpha}(t,p) - \Sigma_{>}^{\alpha\beta}(t,p) S_{<}^{\beta\alpha}(t,p)) \\ - (\bar{\Sigma}_{<}^{\alpha\beta}(t,p) \bar{S}_{>}^{\beta\alpha}(t,p) - \bar{\Sigma}_{>}^{\alpha\beta}(t,p) \bar{S}_{<}^{\beta\alpha}(t,p))], \end{aligned} \quad (4.7)$$

where the bar over the two-point functions denotes CP-conjugation, see appendix C. It is clear that (4.7) is the difference of the Boltzmann equation for leptons and the one for antileptons. Even if the physical meaning of (4.7) is clearer it is more convenient to work with (4.6).

It can be easily seen that the time evolution equation for the lepton asymmetry is automatically consistent with the third Sakharov condition by inserting the KMS relations for the lepton Wightman propagators and self-energies,

$$\begin{aligned} \hat{S}_{>}^{\text{eq}}(X,p) &= e^{\beta p^0} \hat{S}_{<}^{\text{eq}}(X,p), \\ \hat{\Sigma}_{>}^{\text{eq}}(X,p) &= e^{\beta p^0} \hat{\Sigma}_{<}^{\text{eq}}(X,p). \end{aligned} \quad (4.8)$$

We expect therefore that no need of RIS-subtraction will arise in the NEQFT approach. We will see in the following sections that this is indeed the case.

In the unflavoured regime considered here, the lepton propagators are proportional to the unit matrix in flavour space,  $S^{\alpha\beta} = \delta^{\alpha\beta} S$ . It is then convenient to perform the summation over the flavour indices of the self-energy,  $\delta^{\alpha\beta} \Sigma^{\alpha\beta} \equiv \Sigma$ . We use the eQP ansatz for the unflavoured

Wightman propagators (3.111),

$$S_{>} = (1 - F_\ell) \tilde{S}_\rho - \frac{1}{2} (S_R \Sigma_{>} S_R + S_R \Sigma_{>} S_R), \quad (4.9a)$$

$$S_{<} = -F_\ell \tilde{S}_\rho - \frac{1}{2} (S_R \Sigma_{<} S_R + S_R \Sigma_{<} S_R), \quad (4.9b)$$

$$(4.9c)$$

with the eQP spectral function,

$$\tilde{S}_\rho = P_L \not{p} P_R \text{sign}(p^0) (2\pi) \delta(p^2 - m_\ell^2) \equiv P_L \not{p} P_R \tilde{\mathbf{S}}_\rho, \quad (4.10)$$

where we have defined the “scalar” part of the spectral propagator  $\tilde{\mathbf{S}}_\rho$ . We refer the reader to appendix A, where the numerous propagators used in this chapter and their definition are summarised. Inserting the eQP ansatz (4.9) with (4.10) into the equation for the lepton asymmetry (4.6), we find,

$$\mathcal{D}_{\mu j_L}^\mu(t) = g_w \int \frac{d^4 p}{(2\pi)^4} \left( \text{tr}[\Sigma_{<} P_L \not{p} P_R] (1 - F_\ell) - \text{tr}[\Sigma_{>} P_L \not{p} P_R] (-F_\ell) \right) \tilde{\mathbf{S}}_\rho, \quad (4.11)$$

where we have suppressed the argument  $(t, p)$  of the two-point functions for notational convenience. As expected the off-shell part of the lepton propagator, which is lepton number conserving, cancels out on the RHS of the above equation.

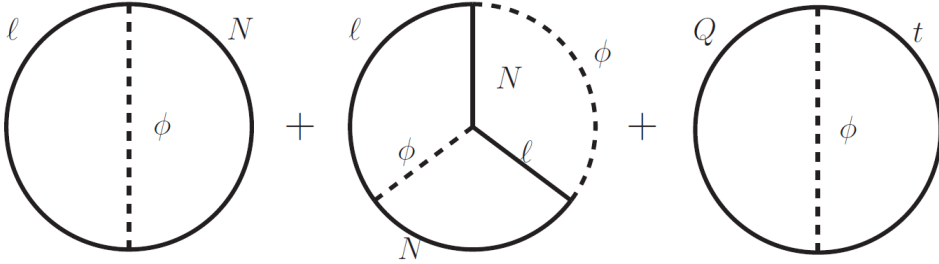


Figure 4.1: Two- and three-loop contributions to the 2PI effective action.

We need now to specify the Wightman component of the lepton self-energy. They are computed by functional differentiation of the 2PI effective action (see Fig. 4.1) with respect to the lepton propagator, see appendix (E). Loosely speaking, it corresponds to “cutting” a lepton line in the 2PI diagrams. At one loop-level, in Wigner space they read, (see Fig. 4.2),

$$\Sigma_{\geq}^{(1)}(t, p) = - \int d\Pi_{qk}^4 (2\pi)^4 \delta(q - p - k) (h^\dagger h)_{ji} P_R \mathcal{S}_{\geq}^{ij}(t, q) P_L \Delta_{\leq}(t, k), \quad (4.12)$$

where we have performed a summation over the lepton flavour indices, and defined the integral measure  $d\Pi_{p_a p_b \dots}^4 \equiv \frac{d^4 p_a}{(2\pi)^4} \frac{d^4 p_b}{(2\pi)^4} \dots$ . The explicit expression for the two-loop contribution, Fig. 4.3, is rather lengthy and it is convenient to split it into three distinct terms:

$$\Sigma_{\geq}^{(2)} = \Sigma_{\geq}^{(2.1)} + \Sigma_{\geq}^{(2.2)} + \Sigma_{\geq}^{(2.3)}. \quad (4.13)$$

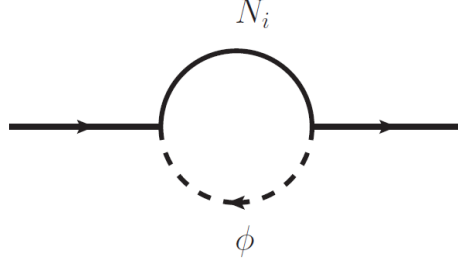


Figure 4.2: One-loop lepton self-energy.

The first term on the right-hand side reads

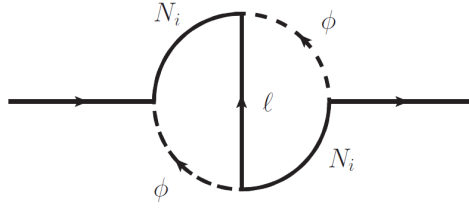


Figure 4.3: Two-loop lepton self-energy.

$$\begin{aligned} \Sigma_{\gtrless}^{(2,1)}(t, p) = & \int d\Pi_{qk}^4 (2\pi)^4 \delta(p + k - q) \left[ (h^\dagger h)_{in} (h^\dagger h)_{jm} \Lambda_{mn}(t, q, k) P_L C \mathcal{S}_{\gtrless}^{ij}(t, q) P_L \Delta_{\lesseqgtr}(t, k) \right. \\ & \left. + (h^\dagger h)_{ni} (h^\dagger h)_{mj} P_R \mathcal{S}_{\gtrless}^{ji}(t, q) C P_R V_{nm}(t, q, k) \Delta_{\lesseqgtr}(t, k) \right], \end{aligned} \quad (4.14)$$

where we have introduced two functions containing loop corrections:

$$\begin{aligned} \Lambda_{mn}(t, q, k) \equiv & \int d\Pi_{k_1 p_1 q_1}^4 (2\pi)^4 \delta(q + k_1 + p_1) (2\pi)^4 \delta(k + p_1 - q_1) \\ & \times \left[ P_R \mathcal{S}_R^{mn}(t, -q_1) C P_R S_F^T(t, p_1) \Delta_A(t, k_1) + P_R \mathcal{S}_F^{mn}(t, -q_1) C P_R S_R^T(t, p_1) \Delta_A(t, k_1) \right. \\ & \left. + P_R \mathcal{S}_R^{mn}(t, -q_1) C P_R S_A^T(t, p_1) \Delta_F(t, k_1) \right], \end{aligned} \quad (4.15)$$

and  $V_{nm}(t, q, k) \equiv \gamma^0 \Lambda_{nm}^\dagger(t, q, k) \gamma^0$  to shorten the notation. Comparing (4.12) and (4.14) we see that they have a very similar structure. First, the integration is over momenta of the Higgs and right-handed neutrino and the delta-function contains the same combination of the momenta. Second, both self-energies include one Wightman propagator of the Higgs field and one Wightman propagator of the Majorana field. Therefore the one-loop correction (4.14) describe the same processes as the tree-level self-energy (4.12).

The second term in (4.13) is given by,

$$\begin{aligned} \Sigma_{\gtrless}^{(2,2)}(t, p) = & \int d\Pi_{p_1 k_1 k_2}^4 (2\pi)^4 \delta(p + k_1 - p_1 - k_2) (h^\dagger h)_{ni} (h^\dagger h)_{mj} \\ & \times P_R \mathcal{S}_R^{ij}(t, p_1 + k_2) C P_R S_{\lesseqgtr}^T(t, -p_1) P_L C \mathcal{S}_A^{mn}(t, p_1 - k_1) P_L \Delta_{\lesseqgtr}(t, k_1) \Delta_{\lesseqgtr}(t, -k_2). \end{aligned} \quad (4.16)$$

Since it contains the causal propagators  $\mathcal{S}_{R(A)}^{ij}(t, q)$  it describes scattering processes mediated by the right-handed neutrino. Finally the last term in (4.13) reads,

$$\begin{aligned} \Sigma_{\gtrless}^{(2,3)}(t, p) &= \int d\Pi_{p_1 q_1 q_2}^4 (2\pi)^4 \delta(p + q_1 - p_1 - q_2) (h^\dagger h)_{ij} (h^\dagger h)_{lk} \\ &\times P_R \mathcal{S}_{\gtrless}^{jk}(t, -q_1) C P_R S_{\gtrless}^T(t, p_1) P_L C \mathcal{S}_{\gtrless}^{li}(t, q_2) P_L \Delta_A(t, -p_1 - q_2) \Delta_R(t, q_1 - p_1). \end{aligned} \quad (4.17)$$

This term describes lepton number conserving processes and does not contribute to the divergence of the lepton current.

## 4.2 Heavy neutrino decay

We investigate here the contribution to the lepton asymmetry coming from the heavy neutrino decay, i.e. we consider the case where  $M_i > m_\ell + m_\phi$ . As  $m_\ell \approx 0.2T$  and  $m_\phi \approx 0.4T$  the right-handed neutrino decay is kinematically allowed if  $M_i/T \gtrsim 0.6$ . At higher temperature the heavy neutrino decay is forbidden, and the Higgs decay becomes kinematically allowed when  $m_\phi > M_i + m_\ell$ . The Higgs decay is studied in the next section.

### 4.2.1 Tree-level contribution

We consider first the leading order contribution to the decay amplitude. As the CP-violation only arises at one-loop level we expect that no asymmetry is produced in that case. Let us check that this is indeed the case.

The leading order contribution to the RHS of (4.11) comes from the one-loop lepton self-energy (4.12). For the Higgs Wightman propagators we use the eQP approximation (3.99) and neglect its off-shell part, which is formally of higher order in the coupling,

$$\Delta_{\gtrless}(t, k) \approx F_\phi^{\gtrless}(t, k) \tilde{\Delta}_\rho(t, k) = F_\phi^{\gtrless}(t, k) \text{sign}(k^0) (2\pi) \delta(k^2 - m_\phi^2), \quad (4.18)$$

where we have introduced the generalised Wightman distribution function,

$$F_a^{\gtrless} \equiv 1 \pm F_a, \quad F_a^< \equiv \pm F_a, \quad (4.19)$$

where the upper (lower) sign refers to bosons (fermions). At leading order in the Yukawa coupling the heavy neutrino propagator is diagonal in flavour space. Assuming the eQP approximation (3.106) for the diagonal components of the right-handed neutrino propagator and keeping only its on-shell part,

$$\begin{aligned} \mathcal{S}_{\gtrless}^{ij}(t, q) &\approx \delta^{ij} F_{N_i}^{\gtrless} \tilde{\mathcal{S}}_\rho^{ii}(t, q) = \delta^{ij} F_{N_i}^{\gtrless} (q + M_i) \text{sign}(q^0) (2\pi) \delta(q^2 - M_i^2) \\ &\equiv \delta^{ij} F_{N_i}^{\gtrless} (q + M_i) \tilde{\mathcal{S}}_\rho^{ii}(t, p), \end{aligned} \quad (4.20)$$

we write the one-loop lepton self-energy as,

$$\Sigma_{\gtrless}^{(1)}(t, p) = -(h^\dagger h)_{ii} \int d\Pi_{qp}^4 (2\pi)^4 \delta(q - p - k) \tilde{\mathcal{S}}_\rho^{ii}(t, q) \tilde{\Delta}_\rho(t, k) F_{N_i}^{\gtrless}(t, q) F_\phi^{\gtrless}(t, k) P_R \not{q}, \quad (4.21)$$

where the ‘‘scalar’’ part of the heavy Majorana propagator  $\tilde{\mathcal{S}}_\rho^{ii}$  is defined by (4.20). Inserting

(4.21) into the time evolution equation for the lepton asymmetry (4.11) we find the tree-level contribution,

$$\begin{aligned} \mathcal{D}_\mu j_L^\mu(t)|_{(T)} &= \sum_i (h^\dagger h)_{ii} \int d\Pi_{pqk}^4 (2\pi)^4 \delta(q-p-k) \tilde{\mathcal{S}}_\rho^{ii}(t, q) \tilde{\mathbf{S}}_\rho(t, p) \tilde{\Delta}_\rho(t, k) 2g_w pq \\ &\quad \times (-1) \left[ F_{N_i}^<(t, q) F_\ell^>(t, p) F_\phi^>(t, k) - F_{N_i}^>(t, q) F_\ell^<(t, p) F_\phi^<(t, k) \right]. \end{aligned} \quad (4.22)$$

The eQP spectral functions are given by a sum of two delta functions, one with positive and one with negative frequency. Therefore the product of the heavy neutrino, lepton and Higgs eQP spectral functions in (4.22) gives raise to 8 terms, but only two of them satisfy the remaining delta function ensuring energy conservation. The generalised distribution functions  $F_a^<(p)$ , when evaluated on positive (negative) frequency, give the particle (antiparticle) distribution functions  $f_a(\vec{p})$  ( $f_{\bar{a}}(\vec{p})$ ), see (3.83),

$$F_a^<(p) \Big|_{p^0 > 0} = \pm f_a(\vec{p}) \equiv \pm f_a^p, \quad (4.23a)$$

$$F_a^<(p) \Big|_{p^0 < 0} = -(1 \pm f_{\bar{a}}(\vec{p})) \equiv -(1 \pm f_{\bar{a}}^p). \quad (4.23b)$$

Similarly,  $F_a^>(p)$  satisfies,

$$F_a^>(p) \Big|_{p^0 > 0} = 1 \pm f_a(\vec{p}) \equiv 1 \pm f_a^p, \quad (4.24a)$$

$$F_a^>(p) \Big|_{p^0 < 0} = \mp f_{\bar{a}}(\vec{p}) \equiv \mp f_{\bar{a}}^p. \quad (4.24b)$$

Note that the particle and antiparticle distribution functions  $f_a^p$  and  $f_{\bar{a}}^p$ , unlike the generalised distribution function  $F_a(p)$ , do not depend on  $p^0$ . Performing the frequency integrals we find,

$$\mathcal{D}_\mu j_L^\mu(t)|_{(T)} = \sum_i \int d\Pi_{N_i \ell \phi}^{qp k} \left[ \Xi_{N_i \leftrightarrow \ell \phi}^{(T)} \mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q;pk} - \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{q;pk} \right], \quad (4.25)$$

where we have defined the effective tree-level amplitudes (squared) summed over all internal degrees of freedom,

$$\Xi_{N_i \leftrightarrow \ell \phi}^{(T)} \equiv 2g_w (h^\dagger h)_{ii} pq, \quad (4.26a)$$

$$\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)} \equiv 2g_w (h^\dagger h)_{ii} pq, \quad (4.26b)$$

and a combination of distribution functions,

$$\mathcal{F}_{ab \dots \leftrightarrow ij \dots}^{p_a p_b \dots; p_i p_j \dots} \equiv (2\pi)^4 \delta\left(\sum_a p_a - \sum_i p_i\right) \left[ \prod_a f_a^{p_a} \prod_i (1 \pm f_i^{p_i}) - \prod_i f_i^{p_i} \prod_a (1 \pm f_a^{p_a}) \right]. \quad (4.27)$$

Note that in (4.25) and (4.26) the momenta are evaluated on the mass-shell of their corresponding particle. The first and second terms in square bracket of (4.25) describe decay and inverse decay of a right-handed neutrino into a lepton-Higgs pair and into a antilepton-antiHiggs pair, respectively.

At tree-level the amplitude (4.26a) and its CP-conjugate (4.26b) are equal. This implies that



the RHS of (4.25) is identically zero when the lepton and Higgs are in thermal equilibrium. Therefore no asymmetry is produced at tree-level, which is in agreement with the vacuum computation. In the presence of a nonzero lepton asymmetry  $f_\ell \neq f_{\bar{\ell}}$  and  $f_\phi \neq f_{\bar{\phi}}$ . In that case  $\mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q;pk} \neq \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{q;pk}$  and this leads to a washout of the asymmetry.

Note that the effective amplitudes (4.26) obtained within NEQFT and the corresponding amplitudes in the S-matrix formalism are equal at tree-level. The difference between the two approaches appears at NLO.

## 4.2.2 Equilibrium solution for the heavy neutrino propagator

As has been demonstrated in [60] for a toy-model, it is important to take the matrix structure in flavour space of the right-handed neutrino propagator into account. In the computation at tree-level we have completely neglected the off-diagonal components in flavour space of the heavy neutrino propagator, see (4.20). We want here to take into account the off-diagonal components by performing a resummation of the full propagator, and express the full propagator in flavour space as a function of the ‘‘diagonal’’ propagator and off-diagonal self-energy. Our starting point is the Schwinger-Dyson equation (3.7) for the heavy neutrino two-point function:

$$\hat{\mathcal{S}}^{-1}(x, y) = \hat{\mathcal{S}}_0^{-1}(x, y) - \hat{\Pi}(x, y). \quad (4.28)$$

Let us split the self-energy into its diagonal  $\hat{\Pi}^d(x, y)$  and off-diagonal  $\hat{\Pi}^{od}(x, y)$  components. We introduce then a new heavy neutrino propagator  $\hat{\mathcal{S}}^{-1}(x, y)$  defined by,

$$\hat{\mathcal{S}}^{-1}(x, y) = \hat{\mathcal{S}}_0^{-1}(x, y) - \hat{\Pi}^d(x, y). \quad (4.29)$$

Note that  $\hat{\mathcal{S}}^{-1}(x, y)$  is diagonal in flavour space since both  $\hat{\mathcal{S}}_0^{-1}(x, y)$  and  $\hat{\Pi}^d(x, y)$  are diagonal. For a hierarchical mass spectrum the poles of the full propagator  $\hat{\mathcal{S}}$  are well approximated by the poles of the ‘‘diagonal’’ propagator  $\hat{\mathcal{S}}$ . This means that in this regime the ‘‘diagonal’’ propagator describes the quasiparticle excitations. If the mass splitting is of the same order as the self-energy, i.e. in the case of a quasidegenerate mass spectrum, the ‘‘diagonal’’ propagator does not provide a good approximation for the poles of the full propagator.

In terms of the propagator (4.29) the Schwinger-Dyson equation reads,

$$\hat{\mathcal{S}}^{-1}(x, y) = \hat{\mathcal{S}}^{-1}(x, y) - \hat{\Pi}^{od}(x, y). \quad (4.30)$$

Multiplying the Schwinger-Dyson equation (4.30) by  $\hat{\mathcal{S}}^{-1}$  from the left and by  $\hat{\mathcal{S}}^{-1}(x, y)$  from the right and integrating over the CTP, we obtain a formal solution for the full nonequilibrium propagator,

$$\hat{\mathcal{S}}(x, y) = \hat{\mathcal{S}}(x, y) + \int_{\mathcal{C}} \mathcal{D}^4 u \mathcal{D}^4 v \hat{\mathcal{S}}(x, u) \hat{\Pi}^{od}(u, v) \hat{\mathcal{S}}(u, y). \quad (4.31)$$

After decomposing the propagators and self-energies into their Wightman components, we

rewrite (4.31) in the form,

$$\begin{aligned} \hat{\mathcal{S}}_{\gtrless}(x, y) = \hat{\mathcal{S}}_{\gtrless}(x, y) - \int \mathcal{D}^4 u \mathcal{D}^4 v \theta(u^0) \theta(v^0) & \left[ \hat{\mathcal{S}}_R(x, u) \hat{\Pi}_R^{od}(u, v) \hat{\mathcal{S}}_{\gtrless}(u, y) \right. \\ & \left. + \hat{\mathcal{S}}_R(x, u) \hat{\Pi}_{\gtrless}^{od}(u, v) \hat{\mathcal{S}}_A(u, y) + \hat{\mathcal{S}}_{\gtrless}(x, u) \hat{\Pi}_A^{od}(u, v) \hat{\mathcal{S}}_A(u, y) \right], \end{aligned} \quad (4.32)$$

where we have used the causal two-point functions to extend the integration over the whole  $uv$  plane. Similarly, the retarded and advanced propagators satisfy,

$$\hat{\mathcal{S}}_{R(A)}(x, y) = \hat{\mathcal{S}}_{R(A)}(x, y) - \int \mathcal{D}^4 u \mathcal{D}^4 v \theta(u^0) \theta(v^0) \hat{\mathcal{S}}_{R(A)}(x, u) \hat{\Pi}_{R(A)}^{od}(u, v) \hat{\mathcal{S}}_{(A)}(u, y). \quad (4.33)$$

We perform next a Wigner transform of (4.32) and (4.33). Analogously to the derivation of the KB equations in Wigner space we drop the Heaviside step function. At leading order in the gradient expansion the Wigner transforms of (4.32) and (4.33) read,

$$\begin{aligned} \hat{\mathcal{S}}_{\gtrless}(X, q) = \hat{\mathcal{S}}_{\gtrless}(X, q) - \hat{\mathcal{S}}_R(X, q) \hat{\Pi}_R^{od}(X, q) \hat{\mathcal{S}}_{\gtrless}(X, q) - \hat{\mathcal{S}}_R(X, q) \hat{\Pi}_{\gtrless}^{od}(X, q) \hat{\mathcal{S}}_A(X, q) \\ - \hat{\mathcal{S}}_{\gtrless}(X, q) \hat{\Pi}_A^{od}(X, q) \hat{\mathcal{S}}_A(X, q), \end{aligned} \quad (4.34a)$$

$$\hat{\mathcal{S}}_{R(A)}(X, q) = \hat{\mathcal{S}}_{R(A)}(X, q) - \hat{\mathcal{S}}_{R(A)}(X, q) \hat{\Pi}_{R(A)}^{od}(X, q) \hat{\mathcal{S}}_{(A)}(X, q). \quad (4.34b)$$

Strictly speaking the above equations are only valid in equilibrium since we neglect any  $X$ -dependence by cutting the gradient expansion at zeroth order. We assume however that the equations (4.34) are also valid close-to-equilibrium. The equation for the Wightman two-point functions (4.34a) can be solved algebraically for the *full* propagator in terms of the “diagonal” one, and off-diagonal self-energy,

$$\hat{\mathcal{S}}_{\gtrless}(X, q) = \hat{\Theta}_R(X, q) \left[ \hat{\mathcal{S}}_{\gtrless}(X, q) - \hat{\mathcal{S}}_R(X, q) \hat{\Pi}_{\gtrless}^{od}(X, q) \hat{\mathcal{S}}_A(X, q) \right] \hat{\Theta}_A(X, q), \quad (4.35)$$

where we have defined the functions  $\hat{\Theta}_{R(A)}(X, q)$ ,

$$\hat{\Theta}_R(X, q) \equiv \left( \mathbf{1} + \hat{\mathcal{S}}_R(X, q) \hat{\Pi}_R^{od}(X, q) \right)^{-1}, \quad (4.36a)$$

$$\hat{\Theta}_A(X, q) \equiv \left( \mathbf{1} + \hat{\Pi}_A^{od}(X, q) \hat{\mathcal{S}}_A(X, q) \right)^{-1}. \quad (4.36b)$$

Here  $\mathbf{1}$  corresponds to the unit matrix in flavour and spinor space. The two matrices  $\hat{\Theta}_R(X, q)$  and  $\hat{\Theta}_A(X, q)$  are related by,

$$\hat{\Theta}_A(X, q) = \gamma^0 \hat{\Theta}_R(X, q)^\dagger \gamma^0, \quad (4.37)$$

which is a consequence of the symmetry properties of the heavy neutrino two-point functions, see appendix C.

For the diagonal propagator  $\hat{\mathcal{S}}_{\gtrless}(X, q)$  we use the eQP approximation,

$$\begin{aligned} \mathcal{S}_{\gtrless}^{ii}(X, q) = F_{N_i}^{\gtrless}(X, q) \tilde{\mathcal{S}}_p^{ii}(X, q) \\ - \frac{1}{2} \left( \mathcal{S}_R^{ii}(X, q) \Pi_{\gtrless}^{ii}(X, q) \mathcal{S}_R^{ii}(X, q) + \mathcal{S}_A^{ii}(X, q) \Pi_{\gtrless}^{ii}(X, q) \mathcal{S}_A^{ii}(X, q) \right), \end{aligned} \quad (4.38)$$

with the eQP spectral function given by,

$$\tilde{\mathcal{S}}_\rho^{ii}(X, q) = (\not{q} + M_i) \text{sign}(q^0) (2\pi) \delta(q^2 - M_i^2) \equiv (\not{q} + M_i) \tilde{\mathcal{S}}_\rho^{ii}(X, q). \quad (4.39)$$

Similarly to the lepton case we have extracted the ‘‘scalar’’ part  $\tilde{\mathcal{S}}_\rho^{ii}(X, q)$  of the right-handed neutrino propagator. Putting (4.35) and (4.38) together we find,

$$\begin{aligned} \hat{\mathcal{S}}_\rho^\triangleright &= \hat{\Theta}_R \left[ \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho - \frac{1}{2} \left( \hat{\mathcal{S}}_R \hat{\Pi}_R^d \hat{\mathcal{S}}_R + \hat{\mathcal{S}}_A \hat{\Pi}_A^d \hat{\mathcal{S}}_A \right) - \hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{\mathcal{S}}_A \right] \hat{\Theta}_A \\ &\approx \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho - \hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho - \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho \hat{\Pi}_A^{od} \hat{\mathcal{S}}_A - \frac{1}{2} \left( \hat{\mathcal{S}}_R \hat{\Pi}_R^d \hat{\mathcal{S}}_R + \hat{\mathcal{S}}_A \hat{\Pi}_A^d \hat{\mathcal{S}}_A \right) - \hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{\mathcal{S}}_A, \end{aligned} \quad (4.40)$$

where we have suppressed the argument  $(X, q)$  of the two-point functions and used a matrix notation for the distribution function  $\hat{F}_N^{\triangleright} \equiv \text{diag}(F_{N_i}^{\triangleright})$ . In the second line of (4.40) we have approximated the matrices  $\hat{\Theta}_{R(A)}$  by

$$\hat{\Theta}_R \approx \mathbf{1} - \hat{\mathcal{S}}_R \hat{\Pi}_R^{od}, \quad \hat{\Theta}_A \approx \mathbf{1} - \hat{\Pi}_A^{od} \hat{\mathcal{S}}_A, \quad (4.41)$$

and only kept terms of leading order in the Yukawa coupling.

The solution (4.35) represents the starting point for the study of the self-energy correction to the decay amplitude. As mentioned below (4.34) the solution is valid only in thermal equilibrium. We assume however that it is also valid close to thermal equilibrium.

We have already seen that the first term in the second line of (4.40),  $\hat{\mathcal{S}}_\rho^{\triangleright(T)} \equiv \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho$  gives the tree-level decay amplitude when inserted into the one-loop lepton self-energy. The second and third terms,

$$\hat{\mathcal{S}}_\rho^{\triangleright(SE)} \equiv -\hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho - \hat{F}_N^{\triangleright} \hat{\mathcal{S}}_\rho \hat{\Pi}_A^{od} \hat{\mathcal{S}}_A, \quad (4.42)$$

give the self-energy contribution to the decay amplitude, see subsection 4.2.3. The remaining terms,

$$\hat{\mathcal{S}}_\rho^{\triangleright(Sc)} \equiv -\frac{1}{2} \left( \hat{\mathcal{S}}_R \hat{\Pi}_R^d \hat{\mathcal{S}}_R + \hat{\mathcal{S}}_A \hat{\Pi}_A^d \hat{\mathcal{S}}_A \right) - \hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{\mathcal{S}}_A, \quad (4.43)$$

describe the  $|\Delta L| = 2$  scattering processes mediated by the Higgs and are analysed in section 4.5.

### 4.2.3 Self-energy contribution

We investigate here the self-energy correction to the decay amplitude. As mentioned above this contribution comes from the term  $\hat{\mathcal{S}}_\rho^{\triangleright(SE)}$  of the heavy neutrino propagator. Inserting the one-loop lepton self-energy (4.12) with (4.42) into the lepton current (4.11), we find after some algebra:

$$\begin{aligned} \mathcal{D}_{\mu j_L}^\mu(t)|_{(S)} &= \sum_{i,j} \int d\Pi_{qpk}^4 (2\pi)^4 \delta(q - p - k) 2g_w \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ P_R (\not{q} + M_i) \Pi_A^{od,ij}(q) \mathcal{S}_A^{jj}(q) P_L \not{p} \right] \right] \\ &\quad \times \hat{\mathcal{S}}_\rho^{\triangleright(ii)}(q) \tilde{\mathcal{S}}_\rho(p) \tilde{\Delta}_\rho(k) \left[ F_{N_i}^<(q) F_\ell^>(p) F_\phi^>(k) - F_{N_i}^>(q) F_\ell^<(p) F_\phi^<(k) \right], \end{aligned} \quad (4.44)$$

where we have suppressed the time argument on the RHS. We integrate then over the frequency using the delta-functions from the spectral functions as explained above (4.25) and find an equation similar to the tree-level case,

$$\mathcal{D}_\mu j_L^\mu(t)|_{(S)} = \sum_i \int d\Pi_{N_i \ell \phi}^{qpk} \left[ \Xi_{N_i \leftrightarrow \ell \phi}^{(S)} \mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q;pk} - \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(S)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{q;pk} \right] \quad (4.45)$$

but with the tree-level amplitudes replaced by the self-energy correction,

$$\Xi_{N_i \leftrightarrow \ell \phi}^{(S)} \equiv -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ P_R (\not{q} + M_i) \Pi_A^{od,ij}(q) \mathcal{S}_A^{jj}(q) \not{p} \right] \right], \quad (4.46a)$$

$$\begin{aligned} \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(S)} &\equiv -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ P_R (-\not{q} + M_i) \Pi_A^{od,ij}(-q) \mathcal{S}_A^{jj}(-q) (-\not{p}) \right] \right], \quad (4.46b) \\ &= -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ \not{p} \mathcal{S}_R^{jj}(q) \Pi_R^{od,ji}(q) (\not{q} + M_i) P_R \right] \right]. \end{aligned}$$

In the second line of (4.46b) we have used the symmetry properties of the Majorana two-point functions (see appendix C) to rewrite the amplitude with positive momenta. Note that in the self-energy corrections to the effective amplitudes (4.46) the momenta of the lepton and heavy neutrino are evaluated on-shell. In particular the momentum of the right-handed neutrino is evaluated on the mass-shell of the  $i$ -particle,  $q^2 = M_i^2$ . For a hierarchical mass spectrum this implies that we can approximate the causal propagators of the heavy neutrino by its hermitian part,

$$\mathcal{S}_{R(A)}^{jj}(q) \approx \mathcal{S}_h^{jj}(q) \quad \text{at } q^2 = M_i^2, \quad i \neq j. \quad (4.47)$$

We need then to specify the heavy neutrino self-energy. At one-loop level in a CP-symmetric medium the spectral part is given by (see appendix E.3),

$$\Pi_\rho^{ij}(t, q) = -\frac{g_w}{16\pi} [(h^\dagger h)_{ij} P_L + (h^\dagger h)_{ij}^* P_R] L_\rho(t, q), \quad (4.48)$$

with,

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) [\mathbf{S}_>(t, p) \Delta_>(t, k) - \mathbf{S}_<(t, p) \Delta_<(t, k)] \not{p}. \quad (4.49)$$

Inserting the leading order term of the eQP approximation for the lepton and Higgs propagators into the loop integral (4.49) and integrating over the frequencies we find (assuming  $q^2 > 0$  and  $q^0 > 0$ ),

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^{\ell\phi} (2\pi)^4 \delta(q - p - k) [1 - f_\ell^{\text{eq},p} + f_\phi^{\text{eq},k}] \not{p}, \quad (4.50)$$

where we have used the equilibrium distribution functions for the lepton and Higgs. The function  $L_\rho$  encodes the medium corrections to the decay width. We do not need to specify the hermitian self-energy as it does not enter the CP-violating parameter. Similarly to the

vacuum case the self-energy CP-violating parameter is defined as,

$$\epsilon_i^{(S)} \equiv \frac{\Xi_{N_i \leftrightarrow \ell \phi}^{(S)} - \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(S)}}{\Xi_{N_i \leftrightarrow \ell \phi}^{(T)} + \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)}}. \quad (4.51)$$

Using (4.46a) with (4.47) and (4.48) we find for the CP-violating parameter,

$$\epsilon_i^{(S)} \equiv \sum_{j \neq i} \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i \Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + (\Gamma_{N_j}/M_j q L_\rho)^2} \frac{p L_\rho}{pq}, \quad (4.52)$$

where we have introduced the vacuum total decay width in the rest frame of the decaying particle,  $\Gamma_{N_i} \equiv g_w (h^\dagger h)_{ii} M_i / (16\pi)$ . In vacuum,  $L_\rho^\mu = \theta(q^2) \text{sign}(q^0) q^\mu$  and the CP-violating parameter takes the form,

$$\epsilon_i^{(S),vac} \equiv \sum_{j \neq i} \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i \Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + (\Gamma_{N_j} M_i^2 / M_j)^2}. \quad (4.53)$$

The ‘regulator’ in the denominator of (4.53) differs from the result  $M_i \Gamma_j$  found in [31, 36] by the ratio of the masses. For a hierarchical neutrino mass spectrum the ‘regulator’ term is subdominant and this difference is numerically small. Note also that although (4.52) does not diverge in the limit of vanishing mass difference the approximations made in the course of its derivation are not applicable for a quasidegenerate mass spectrum [60]. For a consistent treatment of resonant enhancement within NEQFT we refer to [148].

#### 4.2.4 Vertex contribution

We have so far only used the one-loop lepton self-energy. We investigate here contribution coming from the two-loop correction. We consider in particular the first term in (4.13),

$$\begin{aligned} \Sigma_{\gtrless}^{(2.1)}(p) = & \int d\Pi_{qk}^4 (2\pi)^4 \delta(p+k-q) \left[ (h^\dagger h)_{in} (h^\dagger h)_{jm} A_{mn}(q, k) P_L C \mathcal{S}_{\gtrless}^{ij}(q) P_L \Delta_{\lesseqgtr}(k) \right. \\ & \left. + (h^\dagger h)_{ni} (h^\dagger h)_{mj} P_R \mathcal{S}_{\gtrless}^{ji}(q) C P_R V_{nm}(q, k) \Delta_{\lesseqgtr}(k) \right], \end{aligned} \quad (4.54)$$

where we have suppressed the time argument. Since the above self-energy contribution is already of the fourth order in the Yukawa we can approximate the heavy neutrino propagator by its leading order term,  $\hat{\mathcal{S}}_{\gtrless}^{(T)} \equiv \hat{F}_N^{\gtrless} \hat{\mathcal{S}}_\rho$ . Inserting it in the lepton two-loop self-energy (4.54) with the leading order eQP approximation for the Higgs field we find,

$$\begin{aligned} \Sigma_{\gtrless}^{(2.1)}(p) = & \int d\Pi_{qk}^4 (2\pi)^4 \delta(p+k-q) \tilde{\mathcal{S}}_\rho^{ii}(q) \tilde{\Delta}_\rho(k) F_{N_i}^{\gtrless}(q) F_\phi^{\lesseqgtr}(k) \\ & \times M_i \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(q, k) P_L C + (h^\dagger h)_{ji}^2 C P_R \mathbf{V}_{jj}(q, k) \right], \end{aligned} \quad (4.55)$$

with the “diagonal” loop function given by,

$$\begin{aligned} \mathbf{A}_{jj}(q, k) \equiv & \int d\Pi_{k_1 p_1 q_1}^4 (2\pi)^4 \delta(q + k_1 + p_1) (2\pi)^4 \delta(k + p_1 - q_1) \\ & \times \left[ P_R \mathcal{S}_R^{jj}(-q_1) C P_R \mathcal{S}_F^T(p_1) \Delta_A(k_1) + P_R \mathcal{S}_F^{jj}(-q_1) C P_R \mathcal{S}_R^T(p_1) \Delta_A(k_1) \right. \\ & \left. + P_R \mathcal{S}_R^{jj}(-q_1) C P_R \mathcal{S}_A^T(p_1) \Delta_F(k_1) \right], \end{aligned} \quad (4.56)$$

and  $\mathbf{V}_{jj}(t, q, k) \equiv \gamma^0 \mathbf{A}_{jj}^\dagger(t, q, k) \gamma^0$  to shorten the notation. The vertex correction to the decay amplitudes are found by inserting the two-loop self-energy (4.54) into the lepton current (4.11) and integrating over the frequencies,

$$\Xi_{N_i \leftrightarrow \ell \phi}^{(V)} = -2g_w M_i \sum_j \text{Re} \left[ \text{tr} \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(q, k) C P_L \not{p} \right] \right], \quad (4.57a)$$

$$\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(V)} = -2g_w M_i \sum_j \text{Re} \left[ \text{tr} \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(-q, -k) C P_L (-\not{\phi}) \right] \right], \quad (4.57b)$$

where the momenta  $q$ ,  $p$  and  $k$  are evaluated on the mass-shell of the  $i$ th heavy neutrino, lepton and Higgs, respectively. Since we assume the medium to be almost CP-symmetric we can use, at leading order, CP-symmetric two-point functions in the loop integral  $\mathbf{A}_{jj}$ . Then, at leading order in the Yukawa couplings, we find for the vertex contribution to the CP-violating parameter:

$$\begin{aligned} \epsilon_i^{(V)} = & - \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}} \frac{M_i M_j}{pq} \int d\Pi_{q_1 p_1 k_1}^4 (2\pi)^4 \delta(q + k_1 + p_1) (2\pi)^4 \delta(k + p_1 - q_1) p p_1 \\ & \left[ \mathcal{S}_h^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_\rho(k_1) + \mathcal{S}_h^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_F(k_1) \right. \\ & - \mathcal{S}_F^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_h(k_1) + \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_h(k_1) \\ & \left. + \mathcal{S}_F^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_\rho(k_1) + \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_F(k_1) \right]. \end{aligned} \quad (4.58)$$

At leading order in the coupling we can replace the statistical and spectral two-point functions by their on-shell term of the eQP approximation. Therefore two of the three propagators in each term in square brackets in (4.58) correspond to on-shell particles. Each line in (4.58) can therefore be interpreted as cut through lines of the vertex diagram.

The first line corresponds to a cut through the lepton and Higgs propagators and reads,

$$\epsilon_i^{(V,1)} = -\frac{1}{g_w} \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \frac{M_i \Gamma_{N_j}}{M_j^2} \frac{p K_j^{(1)}(q, k)}{pq}, \quad (4.59)$$

where we introduced the loop function,

$$K_j^{(1)\mu}(q, k) \equiv 16\pi \int d\Pi_{p_1 k_1}^{\ell\phi} (2\pi)^4 \delta(q - k_1 - p_1) p_1^\mu \frac{M_j^2}{M_j^2 - (k - p_2)^2} [1 - f_\ell^{\text{eq}, p_1} + f_\phi^{\text{eq}, k_1}]. \quad (4.60)$$

In vacuum  $K_j$  can be computed explicitly (see appendix B) and we recover the well-known

result [25]:

$$\epsilon_i^{(V),vac} = \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{\Gamma_{N_j}}{M_i} \left[ 1 - \left( 1 + \frac{M_j^2}{M_i^2} \right) \ln \left( 1 + \frac{M_i^2}{M_j^2} \right) \right]. \quad (4.61)$$

Adding up (4.61) and (4.53) we obtain the canonical expression for the vacuum CP-violating parameter, (2.16). The two other cuts in (4.58) take the same form as (4.59) but with a different loop function,

$$K_j^{(2)\mu}(q, k) \equiv 16\pi \int d\Pi_{q_1 p_1}^{N_j \ell} (2\pi)^4 \delta(q_1 - k - p_1) p_1^\mu \frac{M_j^2}{m_\phi^2 - (q + p_1)^2} [f_{N_j}^{q_1} - f_\ell^{\text{eq}, p_1}], \quad (4.62a)$$

$$K_j^{(3)\mu}(q, k) \equiv 16\pi \int d\Pi_{q_1 k_1}^{N_j \phi} (2\pi)^4 \delta(q + k_1 - q_1 - k) (q + k_1)^\mu \times \frac{M_j^2}{m_\ell^2 - (q + k_1)^2} [f_{N_j}^{q_1} + f_\phi^{\text{eq}, k_1}]. \quad (4.62b)$$

Since they are proportional to  $(f_{N_j} - f_\ell^{\text{eq}})$  and  $(f_{N_j} + f_\phi^{\text{eq}})$ , respectively, they vanish in the zero temperature limit. At finite temperature they are usually Boltzmann-suppressed, but can be relevant in specific cases [66].

We finally obtain the heavy neutrino decay amplitude at one-loop by adding up the tree-level (4.26a), the self-energy (4.46) and vertex correction (4.57),

$$\Xi_{N_i \leftrightarrow \ell \phi}^{(T+S+V)} = \Xi_{N_i \leftrightarrow \ell \phi}^{(T)} + \Xi_{N_i \leftrightarrow \ell \phi}^{(S)} + \Xi_{N_i \leftrightarrow \ell \phi}^{(V)}, \quad (4.63a)$$

$$\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T+S+V)} = \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)} + \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(S)} + \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(V)}, \quad (4.63b)$$

which give the decay contribution to the lepton asymmetry,

$$\mathcal{D}_\mu j_L^\mu(t)|_{(D)} = \sum_i \int d\Pi_{N_i \ell \phi}^{qpk} \left[ \Xi_{N_i \leftrightarrow \ell \phi}^{(T+S+V)} \mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q:pk} - \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T+S+V)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{q:pk} \right]. \quad (4.64)$$

Equations of the type of the above equation, which are derived from first principles, will be referred to as *quantum-corrected Boltzmann equations*. Although the equation (4.64) looks very similar to the Boltzmann equations (2.17) and (2.18), there is a fundamental difference between them. Due to the fact that the decay ( $N_i \rightarrow \ell \phi$ ) and inverse decay ( $\ell \phi \rightarrow N_i$ ) amplitudes in (4.64) are equal (hence the double arrow in the definition of the effective amplitudes (4.63)), they both multiply the same combination of distribution functions, namely,

$$\mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q:pk} = (2\pi)^4 \delta(q - p - k) [f_{N_i}^q (1 - f_\ell^p) (1 + f_\phi^k) - f_\ell^p f_\phi^k (1 - f_{N_i}^q)]. \quad (4.65)$$

In equilibrium the Bose-Einstein and Fermi-Dirac distribution functions satisfy the important identity,

$$1 \pm f_a^{\text{eq}, p_a} = 1 \pm \frac{1}{e^{\beta E_a^{p_a}} \mp 1} = \frac{e^{\beta E_a^{p_a}}}{e^{\beta E_a^{p_a}} \mp 1} = e^{\beta E_a^{p_a}} f_a^{\text{eq}, p_a}, \quad (4.66)$$

which implies that the product of distribution function (4.65) exactly vanishes in thermal

equilibrium,

$$\begin{aligned} & (2\pi)^4 \delta(q-p-k) [f_{N_i}^{\text{eq},q} (1 - f_\ell^{\text{eq},p}) (1 + f_\phi^{\text{eq},k}) - f_\ell^{\text{eq},p} f_\phi^{\text{eq},k} (1 - f_{N_i}^{\text{eq},q})] \\ & = (2\pi)^4 \delta(q-p-k) f_{N_i}^{\text{eq},q} (1 - f_\ell^{\text{eq},p}) (1 + f_\phi^{\text{eq},k}) [1 - e^{\beta(E_{N_i}^q - E_\ell^p - E_\phi^k)}] = 0, \end{aligned} \quad (4.67)$$

where we have used the energy conserving delta-function in the second line. Therefore the Boltzmann equation (4.64) obtained within NEQFT does not suffer from the double-counting problem, and is automatically consistent with the third Sakharov condition.

### 4.3 $|\Delta L| = 2$ scattering processes

In the canonical approach the inclusion of modified  $|\Delta L| = 2$  scattering amplitudes is required in order to obtain a Boltzmann equation which is consistent with the third Sakharov condition. The modifications of the vacuum amplitudes can be recast into a RIS-subtracted propagator (2.34),

$$\mathcal{P}_{ij}^2(q^2) = \begin{cases} P_i^*(q^2) P_j(q^2) & \text{for } i \neq j, \\ |P_i(q)|^2 - \frac{\pi}{M_i \Gamma_{N_i}} \delta(q^2 - M_i^2) & \text{for } i = j, \end{cases} \quad (4.68)$$

where  $P_i(q^2) = [q^2 - M_i^2 + i\theta(q^2)M_i\Gamma_{N_i}]^{-1}$  is the heavy neutrino Feynman propagator. The Boltzmann equation (4.64) obtained within NEQFT is consistent with equilibrium consideration and there is no need to include the  $|\Delta L| = 2$  scattering processes. It is however interesting to compute the scattering amplitudes using NEQFT technique to check if the vacuum RIS procedure is valid, and to determine to what extent thermal and nonequilibrium effects affect the vacuum RIS procedure.

In the previous section we have disregarded the off-shell contributions to the resummed heavy neutrino propagator, (4.43),

$$\hat{\mathcal{S}}_{\gtrless}^{(Sc)} \equiv -\frac{1}{2} \left( \hat{\mathcal{S}}_R \hat{\Pi}_{\gtrless}^d \hat{\mathcal{S}}_R + \hat{\mathcal{S}}_A \hat{\Pi}_{\gtrless}^d \hat{\mathcal{S}}_A \right) - \hat{\mathcal{S}}_R \hat{\Pi}_{\gtrless}^{od} \hat{\mathcal{S}}_A, \quad (4.69)$$

and the second term in the lepton two-loop self-energy (4.13),

$$\begin{aligned} \Sigma_{\gtrless}^{(2,2)}(t, p) &= \int d\Pi_{p_1 k_1 k_2}^4 (2\pi)^4 \delta(p + k_1 - p_1 - k_2) (h^\dagger h)_{ni} (h^\dagger h)_{mj} \\ &\times P_R \mathcal{S}_R^{ij}(t, p_1 + k_2) C P_R S_{\gtrless}^T(t, -p_1) P_L C \mathcal{S}_A^{mn}(t, p_1 - k_1) P_L \Delta_{\gtrless}(t, k_1) \Delta_{\gtrless}(t, -k_2). \end{aligned} \quad (4.70)$$

These two terms involve the retarded and advanced heavy neutrino propagators, which represent off-shell state. Therefore they describe the  $|\Delta L| = 2$  scattering processes mediated by the heavy neutrino, see Figs. 2.2 and 2.3.

We first investigate the consequence of the off-shell heavy neutrino propagator (4.69). It contains the Wightman self-energies, which read, at one-loop level (see appendix E.3),

$$\begin{aligned} \Pi_{\gtrless}^{ij}(q) &= -g_w \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) \\ &\times [(h^\dagger h)_{ij} P_L S_{\gtrless}(p) P_R \Delta_{\gtrless}(k) + (h^\dagger h)_{ij}^* P_R P \bar{S}_{\gtrless}(\bar{p}) P P_L \bar{\Delta}_{\gtrless}(\bar{k})], \end{aligned} \quad (4.71)$$



where we have suppressed the time argument for notational convenience, and  $\bar{S}_{\geq}$  and  $\bar{\Delta}_{\geq}$  are the CP-conjugated propagators of the lepton and Higgs fields, respectively. Inserting into (4.71) the on-shell term of the eQP approximation for the lepton and Higgs Wightman propagators, we find,

$$\begin{aligned} \Pi_{\geq}^{ij}(q) = & -g_w \int d\Pi_{pk}^4 (2\pi)^4 \delta(q-p-k) \tilde{\mathbf{S}}_{\rho}(p) \tilde{\Delta}_{\rho}(k) \\ & \times [(h^\dagger h)_{ij} P_L \not{p} P_R F_{\ell}^{\geq}(p) F_{\phi}^{\geq}(k) + (h^\dagger h)_{ij}^* P_R \not{p} P_L F_{\ell}^{\geq}(p) F_{\phi}^{\geq}(k)], \end{aligned} \quad (4.72)$$

where we have used the fact that the distribution functions and the eQP spectral functions do not depend on the spatial direction of the momentum due to isotropy and rewritten  $P\not{p}P = \not{p}$  in the second term in square bracket. We also assume that particle and its corresponding antiparticle share the same eQP spectral function. Substituting (4.72) into the heavy neutrino off-shell propagator (4.69) we can split it into lepton number conserving (LC) and lepton number violating (LV) terms,

$$\begin{aligned} \mathcal{S}_{\geq}^{(Sc),ij} = & \int d\Pi_{pk}^4 (2\pi)^4 \delta(q-p-k) \tilde{\mathbf{S}}_{\rho}(p) \tilde{\Delta}_{\rho}(k) \\ & \times [F_{\ell}^{\geq}(p) F_{\phi}^{\geq}(k) \mathcal{S}_{LC}^{ij}(p, q) + F_{\ell}^{\geq}(p) F_{\phi}^{\geq}(k) \mathcal{S}_{LV}^{ij}(p, q)], \end{aligned} \quad (4.73)$$

where we have defined,

$$\begin{aligned} \mathcal{S}_{LC}^{ij}(p, q) \equiv & (h^\dagger h)_{ij} \left[ (1 - \delta^{ij}) \mathcal{S}_R^{ii}(q) P_L \not{p} P_R \mathcal{S}_A^{jj}(q) \right. \\ & \left. + \frac{1}{2} \delta^{ij} (\mathcal{S}_R^{ii}(q) P_L \not{p} P_R \mathcal{S}_R^{jj}(q) + \mathcal{S}_A^{ii}(q) P_L \not{p} P_R \mathcal{S}_A^{jj}(q)) \right], \end{aligned} \quad (4.74a)$$

$$\begin{aligned} \mathcal{S}_{LV}^{ij}(p, q) \equiv & (h^\dagger h)_{ji} \left[ (1 - \delta^{ij}) \mathcal{S}_R^{ii}(q) P_R \not{p} P_L \mathcal{S}_A^{jj}(q) \right. \\ & \left. + \frac{1}{2} \delta^{ij} (\mathcal{S}_R^{ii}(q) P_R \not{p} P_L \mathcal{S}_R^{jj}(q) + \mathcal{S}_A^{ii}(q) P_R \not{p} P_L \mathcal{S}_A^{jj}(q)) \right]. \end{aligned} \quad (4.74b)$$

Substituting the heavy neutrino propagator (4.73) with the lepton one-loop self-energy into the divergence of the lepton current (4.11), we find,

$$\begin{aligned} \mathcal{D}_{\mu} j_L^{\mu}(t)|_{(Sc1)} = & \sum_i \int d\Pi_{pkqp'k'}^4 (2\pi)^4 \delta(q-k-p) (2\pi)^4 \delta(q-k'-p') \tilde{\mathbf{S}}_{\rho}(p) \tilde{\mathbf{S}}_{\rho}(p') \tilde{\Delta}_{\rho}(k) \tilde{\Delta}_{\rho}(k') \\ & \times g_w^2 (h^\dagger h)_{ji} \left( \left[ F_{\ell}^{<}(p) F_{\phi}^{<}(k) F_{\ell}^{>}(p') F_{\phi}^{>}(k') - F_{\ell}^{>}(p) F_{\phi}^{>}(k) F_{\ell}^{<}(p') F_{\phi}^{<}(k') \right] \text{tr} [\mathcal{S}_{LC}^{ij}(p, q') P_L \not{p}] \right. \\ & \left. + \left[ F_{\ell}^{<}(p) F_{\phi}^{<}(k) F_{\ell}^{>}(p') F_{\phi}^{>}(k') - F_{\ell}^{>}(p) F_{\phi}^{>}(k) F_{\ell}^{<}(p') F_{\phi}^{<}(k') \right] \text{tr} [\mathcal{S}_{LV}^{ij}(p, q') P_L \not{p}] \right). \end{aligned} \quad (4.75)$$

It is convenient to factor out the ‘‘spinor’’ structure of the right-handed neutrino causal propagator, see (3.72),

$$\mathcal{S}_{R(A)}^{ii}(q) = (\not{q} + M_i) \mathcal{S}_{R(A)}^{ii}(q), \quad (4.76)$$

with

$$\mathcal{S}_{R(A)}^{ii}(q) \equiv \frac{-1}{q^2 - M_i^2 - 2q\Pi_{R(A)}^{v,ii}(q)}. \quad (4.77)$$

Here  $\Pi_{R(A)}^{v,ii}(q)$  corresponds to the vector part of the self-energy. At one-loop level the scalar part of the heavy neutrino self-energy is induced by on-shell renormalisation scheme and is negligible. The traces in (4.75) are easily evaluated by using the decomposition (4.76),

$$\text{tr}[\mathcal{S}_{LC}^{ij}(p, q')P_L\not{p}] = 2(h^\dagger h)_{ij}\mathcal{P}_{ji}^2(q)(2(qp')(qp) - q^2(pp')), \quad (4.78a)$$

$$\text{tr}[\mathcal{S}_{LV}^{ij}(p, q')P_L\not{p}] = 2(h^\dagger h)_{ji}\mathcal{P}_{ji}^2(q)M_iM_j(pp'), \quad (4.78b)$$

where we defined a ‘‘RIS-subtracted’’ propagator analogously to the vacuum case,

$$\mathcal{P}_{ij}^2(q) \equiv \mathcal{S}_A^{ii}(q)\mathcal{S}_R^{jj}(q) - \frac{1}{2}\delta^{ij}\mathcal{S}_\rho^{ii}(q)\mathcal{S}_\rho^{ii}(q), \quad (4.79)$$

with  $\mathcal{S}_\rho^{ii}(q) = -i(\mathcal{S}_R^{ii}(q) - \mathcal{S}_A^{ii}(q))$ . Note that the above ‘‘RIS-subtracted’’ propagator is not obtained by any ad-hoc RIS-subtraction procedure, but automatically results from the heavy neutrino propagator (4.69).

The symmetry of the traces (4.78) under the exchange of the lepton momenta  $p \leftrightarrow p'$  implies that the lepton conserving term in (4.75) vanishes since the combination of distribution functions,

$$F_\ell^<(p)F_\phi^<(k)F_\ell^>(p')F_\phi^>(k') - F_\ell^>(p)F_\phi^>(k)F_\ell^<(p')F_\phi^<(k'), \quad (4.80)$$

is antisymmetric under the exchange  $p \leftrightarrow p'$ . These terms correspond to scattering processes such as  $\ell\bar{\ell} \leftrightarrow \phi\bar{\phi}$  which do not contribute to the Boltzmann equation for the lepton asymmetry since they conserve lepton number.

Many integrals in (4.75) can be performed using the momentum-energy conserving delta-functions and the one in the spectral functions. We first integrate over the heavy neutrino momentum using the first delta-function in (4.75). The integrals over the frequencies of the lepton and Higgs fields are performed similarly to the decay amplitude. We have here four eQP spectral functions, each of them consisting of a sum of two delta-functions, which give raise to  $2^4$  terms with different sign assignments for the frequencies. Only six terms satisfy the remaining delta-function, they read,

$$\begin{aligned} \mathcal{D}_{\mu}j_L^\mu(t)|_{(Sc1)} = 2 \int d\Pi_{\ell\ell\phi\phi}^{pp'kk'} & \left[ (\Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(s\times s)} + \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(t\times t)})\mathcal{F}_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{pk;p'k'} + (\Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(t\times t)} + \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(u\times u)})\mathcal{F}_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{pp';kk'} \right. \\ & \left. + (\Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(t\times t)} + \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(u\times u)})\mathcal{F}_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{kk';pp'} \right], \end{aligned} \quad (4.81)$$

where we have defined the effective scattering amplitudes (squared) depending on the momen-

tum transfer,

$$\Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(s\times s)} + \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(t\times t)} = 2g_w^2(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{P}_{ij}^2(q_s) + \mathcal{P}_{ij}^2(q_t) \right), \quad (4.82a)$$

$$\Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(t\times t)} + \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(u\times u)} = \frac{g_w^2}{2}(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{P}_{ij}^2(q_t) + \mathcal{P}_{ij}^2(q_u) \right), \quad (4.82b)$$

$$\Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(t\times t)} + \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(u\times u)} = \frac{g_w^2}{2}(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{P}_{ij}^2(q_t) + \mathcal{P}_{ij}^2(q_u) \right). \quad (4.82c)$$

Here the intermediate heavy neutrino momenta are given by  $q_s \equiv p + k$ ,  $q_t \equiv p - k'$  and  $q_u \equiv p - k$ . In (4.81) the overall factor 2 comes from the fact that the processes (4.82) violate lepton number by two units. From (4.82) we see that the obtained amplitudes contain only  $s \times s$  and  $t \times t$  interference terms. Indeed, in the products of the Majorana propagators in (4.79) both of them depend on the same momentum.

The missing terms,  $s \times t$  and  $u \times t$  interference channels, come from the two-loop lepton self-energy, (4.70). Since it is already of the fourth order in the Yukawa we use the tree-level approximation for the heavy neutrino propagator. Substituting it into the lepton current (4.11) we find,

$$\begin{aligned} \mathcal{D}_{\mu} j_L^\mu(t)|_{(Sc2)} &= 2 \sum_i \int d\Pi_{pkp'k'}^4 (2\pi)^4 \delta(p + k - p' - k') \tilde{\mathbf{S}}_\rho(p) \tilde{\mathbf{S}}_\rho(p') \tilde{\Delta}_\rho(k) \tilde{\Delta}_\rho(k') \\ &\quad \times g_w (h^\dagger h)_{ji}^2 M_i M_j (pp') \mathcal{S}_R^{ii}(p' + k') \mathcal{S}_A^{jj}(p' - k) \\ &\quad \times (-1) \left[ F_\ell^<(p') F_\phi^<(k') F_\ell^>(p) F_\phi^>(k) - F_\ell^<(p) F_\phi^<(k) F_\ell^>(p') F_\phi^>(k') \right]. \end{aligned} \quad (4.83)$$

We integrate then over the frequencies as explained above (4.81) and obtain,

$$\begin{aligned} \mathcal{D}_{\mu} j_L^\mu(t)|_{(Sc2)} &= 2 \int d\Pi_{\ell\ell\phi\phi}^{pp'kk'} \left[ \left( \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(s\times t)} + \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(t\times s)} \right) \mathcal{F}_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{pk;p'k'} + \left( \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(t\times u)} + \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(u\times t)} \right) \mathcal{F}_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{pp';kk'} \right. \\ &\quad \left. + \left( \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(t\times u)} + \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(u\times t)} \right) \mathcal{F}_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{kk';pp'} \right], \end{aligned} \quad (4.84)$$

where we defined the  $s \times t$  and  $t \times u$  interference amplitudes,

$$\Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(s\times t)} + \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi}^{(t\times s)} = 2g_w(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{S}_A^{ii}(q_s) \mathcal{S}_R^{jj}(q_t) + \mathcal{S}_A^{ii}(q_t) \mathcal{S}_R^{jj}(q_s) \right), \quad (4.85a)$$

$$\Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(t\times u)} + \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi}^{(u\times t)} = \frac{g_w}{2}(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{S}_A^{ii}(q_t) \mathcal{S}_R^{jj}(q_u) + \mathcal{S}_A^{ii}(q_u) \mathcal{S}_R^{jj}(q_t) \right), \quad (4.85b)$$

$$\Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(t\times u)} + \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell}^{(u\times t)} = \frac{g_w}{2}(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( \mathcal{S}_A^{ii}(q_t) \mathcal{S}_R^{jj}(q_u) + \mathcal{S}_A^{ii}(q_u) \mathcal{S}_R^{jj}(q_t) \right). \quad (4.85c)$$

Combining (4.82) and (4.85) we obtain for the effective scattering amplitudes:

$$\begin{aligned} \Xi_{\bar{\ell}\bar{\phi}\leftrightarrow\ell\phi} = 2g_w(pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( g_w \mathcal{P}_{ij}^2(q_s) + g_w \mathcal{P}_{ij}^2(q_t) \right. \\ \left. + \mathcal{S}_A^{ii}(q_s) \mathcal{S}_R^{jj}(q_t) + \mathcal{S}_A^{ii}(q_t) \mathcal{S}_R^{jj}(q_s) \right), \end{aligned} \quad (4.86a)$$

$$\begin{aligned} \Xi_{\bar{\ell}\bar{\ell}\leftrightarrow\phi\phi} = \Xi_{\bar{\phi}\bar{\phi}\leftrightarrow\ell\ell} = \frac{g_w}{2} (pp') \sum_{i,j} (h^\dagger h)_{ij}^2 M_i M_j \left( g_w \mathcal{P}_{ij}^2(q_t) + g_w \mathcal{P}_{ij}^2(q_u) \right. \\ \left. + \mathcal{S}_A^{ii}(q_t) \mathcal{S}_R^{jj}(q_u) + \mathcal{S}_A^{ii}(q_u) \mathcal{S}_R^{jj}(q_t) \right). \end{aligned} \quad (4.86b)$$

Comparing (4.86) with the vacuum amplitudes (2.33) and (2.35) we see that they have a very similar structure. The vacuum amplitudes can be recovered from the one obtained within NEQFT in the zero temperature limit. In vacuum  $L_\rho(q) = \theta(q^2) \text{sign}(q^0) \not{q}$ , and the retarded and advanced heavy neutrino propagators reads (assuming  $q^0 > 0$ ),

$$\mathcal{S}_{R(A)}^{ii}(q) = \frac{-1}{q^2 - M_i^2 - 2q\Pi_{R(A)}^{ii}(q)} \approx \frac{-1}{q^2 - M_i^2 \pm i\theta(q^2)\Gamma_{N_i}/M_i q^2}, \quad (4.87)$$

where we have neglected the dispersive self-energy in the second equality. In the vicinity of the mass-shell of the respective heavy neutrino,  $q^2 \approx M_i^2$ , we find,

$$\mathcal{S}_R^{ii}(q) = -P_i(q), \quad \mathcal{S}_A^{ii}(q) = -P_i^*(q), \quad (4.88)$$

where  $P_i(q)$  is the vacuum Feynman propagator, see (2.25). Therefore the terms off-diagonal in flavour space ( $i \neq j$ ) in the scattering amplitudes (4.86) exactly coincide with the vacuum results, (2.33) and (2.35). For the diagonal terms we rewrite  $\mathcal{P}_{ii}^2(q)$  as,

$$\mathcal{P}_{ii}^2(q) = \mathcal{S}_A^{ii}(q) \mathcal{S}_R^{ii}(q) - \frac{1}{2} \mathcal{S}_\rho^{ii}(q) \mathcal{S}_\rho^{ii}(q) = \frac{1}{2} \left( \mathcal{S}_R^{ii}(q) \mathcal{S}_R^{ii}(q) + \mathcal{S}_A^{ii}(q) \mathcal{S}_A^{ii}(q) \right). \quad (4.89)$$

In vacuum we find,

$$\mathcal{P}_{ii}^2(q) \approx \frac{(q^2 - M_i^2)^2 - (\theta(q^2) \text{sign}(q^0) q^2 / M_i \Gamma_{N_i})^2}{\left[ (q^2 - M_i^2)^2 + (\theta(q^2) \text{sign}(q^0) q^2 / M_i \Gamma_{N_i})^2 \right]^2}. \quad (4.90)$$

In the  $t$ - and  $u$ -channel,  $q_t^2 < 0$  and  $q_u^2 < 0$  and the propagator (4.90) coincides with the vacuum one,

$$\mathcal{P}_{ii}^2(q_t) = \frac{1}{(q_t^2 - M_i^2)^2} = |P_i(q_t)|^2, \quad \mathcal{P}_{ii}^2(q_u) = \frac{1}{(q_u^2 - M_i^2)^2} = |P_i(q_u)|^2. \quad (4.91)$$

In the  $s$ -channel, in the vicinity of the mass-shell,  $q_s^2 \approx M_i^2$ , and the ‘‘RIS-propagator’’ obtained

within NEQFT is given by

$$\begin{aligned} \mathcal{P}_{ii}^2(q_s) &= \frac{(q_s^2 - M_i^2)^2 - (M_i \Gamma_{N_i})^2}{\left[ (q_s^2 - M_i^2)^2 + (M_i \Gamma_{N_i})^2 \right]^2} \\ &= \frac{1}{(q_s^2 - M_i^2)^2 + (M_i \Gamma_{N_i})^2} - \frac{1}{M_i \Gamma_{N_i}} \frac{2(M_i \Gamma_{N_i})^3}{\left[ (q_s^2 - M_i^2)^2 + (M_i \Gamma_{N_i})^2 \right]^2}. \end{aligned} \quad (4.92)$$

The first term in the second line correspond to the product of Feynman propagator  $|P_i(q_s)|^2$ . The second term can be identified with the eQP spectral function (3.108) that we approximated by a delta-function in the limit of vanishing width. In this approximation the diagonal ‘‘RIS-propagator’’ (4.92) reads,

$$\mathcal{P}_{ii}^2(q_s) \approx |P_i(q_s)|^2 - \frac{\pi}{M_i \Gamma_{N_i}} \delta(q_s^2 - M_i^2), \quad (4.93)$$

where we used the following representation of the delta-function,

$$\lim_{\epsilon \rightarrow 0^+} \frac{2\epsilon^3}{[\omega^2 + \epsilon^2]^2} = \pi \delta(\omega). \quad (4.94)$$

In this limit the effective amplitudes (4.86) exactly coincide with the vacuum result.

At finite temperatures  $L_\rho(t, q)$  is non zero even for  $t$ - and  $u$ -channels. In other words, the medium effects induce additional contributions to the effective decay amplitudes. However, these contributions are suppressed by the small Yukawa couplings since they are proportional to  $\Gamma_{N_i}/M_i = (h^\dagger h)_{ii}/(8\pi)$ . Numerical analysis shows that the additional correction typically do not have any sizable impact on the lepton production.

## 4.4 Higgs decay

At high temperature the phase-space of the decay of the heavy neutrino into a lepton-Higgs pair is suppressed due to thermal masses of the final states. At even higher temperature, when the thermal Higgs mass is large enough, the decay channel of the Higgs into a lepton and a right-handed neutrino becomes allowed. In the SM a rough estimate of the lepton and Higgs thermal masses are given by [136],

$$m_\ell(T) \approx 0.2T, \quad m_\phi(T) \approx 0.4T. \quad (4.95)$$

Therefore the decay of the heavy neutrino  $N_i$  is forbidden at  $T \gtrsim 1.3M_i$  and the Higgs decay is possible at temperature  $T \gtrsim 5M_i$ .

Similarly to the heavy neutrino decay the Higgs decay violates CP at one-loop level, see Fig. 4.4. Even if the decaying particle, the Higgs field, is very close to thermal equilibrium, a deviation from equilibrium is provided by the right-handed neutrino. The Sakharov conditions are therefore satisfied and a lepton asymmetry can be produced by the Higgs decay.

In this section we derive the tree-level Higgs decay amplitude and the corresponding self-energy and vertex corrections within NEQFT. Since this computation is quite similar to the one in the previous section we do not show the details of the derivation.

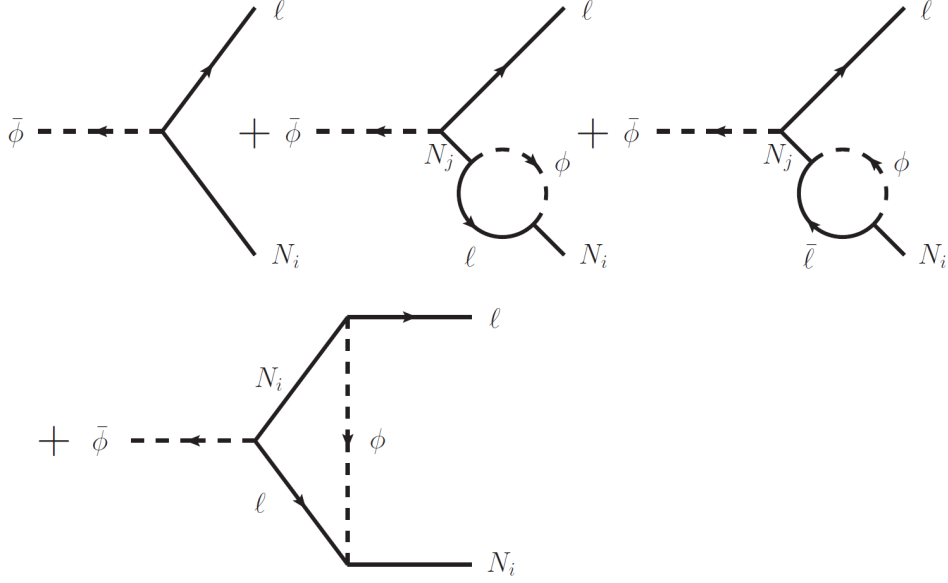


Figure 4.4: Tree-level and one-loop contributions to the decay amplitude  $\bar{\phi} \rightarrow N_i \ell$ . The second and third diagrams are the self-energy or wave-function diagrams, and the last one is the vertex diagram. Note that the second diagram is CP-conserving and does not contribute to the production of the total lepton asymmetry.

#### 4.4.1 Tree-level amplitude

The leading order contribution to the Higgs decay amplitude is obtained by inserting the one-loop lepton self-energy (4.12) into the divergence of the lepton current (4.11). It is convenient to rewrite the lepton self-energy as,

$$\Sigma_{\gtrless}^{(1)}(t, p) = - \int d\Pi_{qk}^4 (2\pi)^4 \delta(k - p - q) (h^\dagger h)_{ji} P_R C (\mathcal{S}_{\gtrless}^{ji})^T(t, q) C^{-1} P_L \bar{\Delta}_{\gtrless}(t, k), \quad (4.96)$$

where we have used a symmetry property of the Majorana propagator (see appendix C),  $\mathcal{S}_{\gtrless}^{ij}(t, -q) = C (\mathcal{S}_{\gtrless}^{ji})^T(t, q) C^{-1}$  and replaced the Higgs propagator by the CP-conjugated one,  $\Delta_{\gtrless}(t, -k) = \bar{\Delta}_{\gtrless}(t, k)$ , see appendix C. In (4.96) we have renamed the momenta of the Higgs and heavy neutrino so that the delta-function reflects the kinematics of a Higgs decay.

Substituting (4.96) into the divergence of the lepton current (4.11) with the leading order term of the eQP approximation for the Higgs and heavy neutrino propagators, we find,

$$\begin{aligned} \mathcal{D}_\mu j_L^\mu(t)|_{(\phi, T)} &= \sum_i \int d\Pi_{qpk}^4 (2\pi)^4 \delta(k - p - k) \tilde{\mathcal{S}}_\rho^{ii}(q) \tilde{\mathcal{S}}_\rho(p) \tilde{\Delta}_\rho(k) \\ &\quad \times 2g_w (h^\dagger h)_{ii} p q \left[ F_\phi^<(k) F_{N_i}^>(q) F_\ell^>(p) - F_{N_i}^<(q) F_\ell^<(p) F_\phi^>(k) \right]. \end{aligned} \quad (4.97)$$

Similarly to the heavy neutrino decay, we perform the integrals over the frequencies using the

delta-functions in the eQP spectral functions, and find,

$$\mathcal{D}_{\mu} j_L^{\mu}(t)|_{(\phi,T)} = \sum_i \int d\Pi_{qpk}^{N_i \ell \phi} \left[ \Xi_{\bar{\phi} \leftrightarrow N_i \ell}^{(T)} \mathcal{F}_{\bar{\phi} \leftrightarrow N_i \ell}^{k;qp} - \Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(T)} \mathcal{F}_{\phi \leftrightarrow N_i \bar{\ell}}^{k;qp} \right], \quad (4.98)$$

where we have defined the tree-level Higgs decay amplitudes,

$$\Xi_{\bar{\phi} \leftrightarrow N_i \ell}^{(T)} = 2g_w (h^\dagger h)_{ii} p q, \quad (4.99a)$$

$$\Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(T)} = 2g_w (h^\dagger h)_{ii} p q. \quad (4.99b)$$

As expected, the Higgs decay amplitudes are CP-even at tree-level. Therefore the amplitudes (4.99) do not contribute to the production of the lepton asymmetry but only to the washout of a preexisting asymmetry.

#### 4.4.2 Self-energy contribution

In complete analogy with the heavy neutrino decay computation, the self-energy correction to the decay amplitude is obtained by using the off-diagonal terms of the heavy neutrino propagator (4.42),

$$\hat{\mathcal{P}}_{\gtrless}^{(SE)} \equiv -\hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{F}_N^{\gtrless} \hat{\mathcal{S}}_\rho - \hat{F}_N^{\gtrless} \hat{\mathcal{S}}_\rho \hat{\Pi}_A^{od} \hat{\mathcal{S}}_A. \quad (4.100)$$

Substituting the lepton one-loop self-energy with the propagator (4.100) into the lepton current (4.11), we find,

$$\begin{aligned} \mathcal{D}_{\mu} j_L^{\mu}(t)|_{(\phi,S)} &= \sum_{i,j} \int d\Pi_{qpk}^A (2\pi)^4 \delta(k-p-q) \tilde{\mathcal{S}}_\rho^{ii}(q) \tilde{\mathcal{S}}_\rho(p) \tilde{\Delta}_\rho(k) \\ &\quad \times (-2g_w) \text{Re} \left[ (h^\dagger h)_{ij} \text{tr} \left[ (\not{q} + M_i) \Pi_A^{od,ij}(q) \mathcal{S}_A^{jj}(q) P_R \not{p} \right] \right] \\ &\quad \times \left[ F_\phi^<(k) F_{N_i}^>(q) F_\ell^>(p) - F_{N_i}^<(q) F_\ell^<(p) F_\phi^>(k) \right]. \end{aligned} \quad (4.101)$$

Similarly to the tree-level computation the frequency integrals are performed by using the eQP spectral functions, and we find an equation similar to (4.98) but with the tree-level amplitudes replaced by the one-loop corrected ones,

$$\Xi_{\bar{\phi} \leftrightarrow N_i \ell}^{(S)} = -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ij} \text{tr} \left[ (\not{q} + M_i) \Pi_A^{od,ij}(q) \mathcal{S}_A^{jj}(q) P_R \not{p} \right] \right], \quad (4.102a)$$

$$\Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(S)} = -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ij} \text{tr} \left[ (-\not{q} + M_i) \Pi_A^{od,ij}(-q) \mathcal{S}_A^{jj}(-q) P_R (-\not{p}) \right] \right]. \quad (4.102b)$$

The CP-violating parameter is defined similarly to the heavy neutrino decay (4.51),

$$\epsilon_{\phi,i}^{(S)} = \frac{\Xi_{\bar{\phi} \leftrightarrow N_i \ell}^{(S)} - \Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(S)}}{\Xi_{\bar{\phi} \leftrightarrow N_i \ell}^{(T)} + \Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(T)}}. \quad (4.103)$$

For a hierarchical mass spectrum we can approximate the causal heavy neutrino propagator in (4.102) by its hermitian part and neglect its spectral part. In this approximation the CP-violating parameter for the Higgs decay (4.103) takes the same form, up to the sign, as the one for the right-handed neutrino decay (4.52),

$$\epsilon_{\phi,i}^{(S)} \equiv - \sum_{j \neq i} \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + (\Gamma_{N_j}/M_j q L_\rho)^2} \frac{pL_\rho}{pq}. \quad (4.104)$$

However, they don't have the same magnitude since the function  $L_\rho$  depends on the kinematics and is therefore different for the Higgs and heavy neutrino decays. In the regime  $m_\phi > M_i + m_\ell$ , the loop function  $L_\rho$  reads (see appendix E),

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^{\ell\phi} (2\pi)^4 \delta(k - q - p) [f_\ell^{\text{eq},p}(t) + f_\phi^{\text{eq},k}(t)] \not{p}. \quad (4.105)$$

Note that the above result is different from the one presented in [75, 136, 149]. Instead of the  $f_\phi^{\text{eq}} - f_\ell^{\text{eq}} - 2f_\phi^{\text{eq}}f_\ell^{\text{eq}}$  dependence, it is proportional to a sum,  $f_\phi^{\text{eq}} + f_\ell^{\text{eq}}$ , of the two distribution functions. This dependence can also be obtained in the framework of real time thermal field theory using causal  $n$ -point functions, see [62].

### 4.4.3 Vertex contribution

In the preceding computation we only considered the one-loop lepton self-energy. We compute here the vertex correction to the Higgs decay amplitude, which results from the two-loop lepton self-energy (4.14),

$$\begin{aligned} \Sigma_{\geq}^{(2,1)}(p) = & \int d\Pi_{qk}^4 (2\pi)^4 \delta(k - q - p) \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(-q, -k) P_L \mathcal{S}_{\leq}^{ii}(t, q) C P_L \bar{\Delta}_{\geq}(k) \right. \\ & \left. + (h^\dagger h)_{ji}^2 P_R C \mathcal{S}_{\leq}^{ii}(t, q) P_R \mathbf{V}_{jj}(-q, -k) \bar{\Delta}_{\geq}(k) \right], \end{aligned} \quad (4.106)$$

where the ‘‘diagonal’’ loop functions  $\mathbf{A}_{jj}$  and  $\mathbf{V}_{jj}$  are given by (4.56). In (4.106) we neglected the off-diagonal components of the heavy neutrino propagator, which are of higher order in the Yukawa couplings. Inserting the two-loop lepton self-energy (4.106) into the divergence of the lepton current (4.11) and substituting the eQP approximation for the heavy neutrino and Higgs propagators, we find an equation similar to (4.98) but with the tree-level amplitudes replaced by the vertex corrections,

$$\Xi_{\phi \leftrightarrow N_i \ell}^{(V)} = -g_w M_i \sum_j \text{Re} \left[ (h^\dagger h)_{ij} \text{tr} \left[ \mathbf{A}_{jj}(-q, -k) C P_L \not{p} \right] \right], \quad (4.107a)$$

$$\Xi_{\phi \leftrightarrow N_i \bar{\ell}}^{(V)} = -g_w M_i \sum_j \text{Re} \left[ (h^\dagger h)_{ij} \text{tr} \left[ \mathbf{A}_{jj}(q, k) C P_L (-\not{p}) \right] \right]. \quad (4.107b)$$



The vertex CP-violating parameter is then given by,

$$\begin{aligned} \epsilon_{\phi,i}^{(V)} = \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}} \frac{M_i M_j}{pq} \int d\Pi_{q_1 p_1 k_1}^4 (2\pi)^4 \delta(q - k_1 - p_1) (2\pi)^4 \delta(k - p_1 - q_1) \\ \left[ \mathcal{S}_h^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_\rho(k_1) + \mathcal{S}_h^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_F(k_1) \right. \\ \left. - \mathcal{S}_F^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_h(k_1) - \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_h(k_1) \right. \\ \left. + \mathcal{S}_F^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_F(k_1) - \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_F(k_1) \right]. \end{aligned} \quad (4.108)$$

The three lines in square bracket of (4.108) correspond to the three possible cuts of the vertex graph. The relative magnitude of the different cuts depends on the kinematic regime. For definiteness, let us assume a strongly hierarchical mass spectrum,  $M_j \gg m_\phi > M_i$ . In that case the contribution of the two last lines are strongly Boltzmann suppressed. Integrating out the delta-functions we find for the contribution of the first line,

$$\epsilon_{\phi,i}^{(V,1)} = \frac{1}{g_w} \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \frac{M_i \Gamma_{N_j}}{M_j^2} \frac{p K_j^\phi(q, k)}{pq}, \quad (4.109)$$

where the loop function  $K_j^{\phi,\mu}(q, k)$  is defined as,

$$K_j^{\phi,\mu}(q, k) = 16\pi \int d\Pi_{p_1 k_1}^{\ell\phi} (2\pi)^4 \delta(k_1 - q - p_1) \frac{M_j^2}{M_j^2 - (k - p_1)^2} [f_\ell^{\text{eq},p_2} + f_\phi^{\text{eq},k_2}] p_1^\mu. \quad (4.110)$$

Note that the vertex CP-violating parameter (4.109) has an opposite sign relative to that for the right-handed neutrino decay. Similarly to the self-energy contribution we observe that the  $1 - f_\ell^{\text{eq}} + f_\phi^{\text{eq}}$  combination is replaced in (4.110) by  $f_\ell^{\text{eq}} + f_\phi^{\text{eq}}$ . For a milder mass hierarchy the two other cuts can become important. Their contributions are proportional to  $1 - f_\ell^{\text{eq}} - f_{N_j}$  and  $f_\phi^{\text{eq}} + f_{N_j}$  respectively.

## 4.5 Higgs mediated scattering processes

We have so far only used the on-shell part of the Higgs propagator and neglected the off-shell contribution. The off-shell part of the Wightman Higgs propagators is proportional to its self-energy,

$$\Delta_{\geq}^{\text{off-shell}}(t, k) = -\frac{1}{2} (\Delta_R^2(t, k) + \Delta_A^2(t, k)) \Omega_{\geq}(t, k). \quad (4.111)$$

Due to the gauge interactions and the large top Yukawa coupling the Higgs self-energy can potentially receive a large contribution. Therefore the off-shell part (4.111) is not suppressed by any small couplings compared to the on-shell part,  $\tilde{\Delta}_{\geq}(t, k)$ , and should not be disregarded for a consistent treatment of leptogenesis.

In this work we only consider the contribution to the Higgs Wightman self-energy coming from the top quark, and do not study the gauge interactions. The treatment of gauge interactions is more involved since multiple soft scatterings could give leading order contributions, as argued in [100, 101]. Here we take the gauge interactions into account only in the form of

the thermal masses of the SM particles. However, for a consistent treatment of leptogenesis, gauge interactions should also be included in the production and washout of the asymmetry, and in the production of the right-handed neutrino.

The causal propagators in the above expression indicate that this term describes processes with the Higgs field as an intermediate state. As we will see in this section the inclusion of the off-shell Higgs propagator (4.111) gives raise to the  $|\Delta L| = 1$  top scattering processes mediated by the Higgs, and to the  $|\Delta L| = 1$  decay of the heavy neutrino into a lepton, top quark and quark doublet.

### Quark propagators

We introduce here notation for the quark fields. The quark propagators are defined similarly to the lepton two-point functions,

$$S_t^{AB}(x, y) = \langle T_C t^A(x) \bar{t}^B(y) \rangle, \quad (4.112a)$$

$$S_{Qab}^{AB}(x, y) = \langle T_C Q_b^A(x) \bar{Q}_b^B(y) \rangle, \quad (4.112b)$$

where  $A, B$  are colour indices, and  $a, b$   $SU(2)_L$  indices. Since the top quark singlet is right-handed, its propagator satisfies,

$$\hat{S}_t(x, y) = P_R \hat{S}_t(x, y) P_L. \quad (4.113)$$

Similarly to the notation introduced in chapter 3 the hat over a quark two-point function denotes a matrix in colour, spinor and potentially  $SU(2)_L$  space. The quark doublet is left-handed and satisfies the same symmetry property as the lepton doublet,

$$\hat{S}_Q(x, y) = P_L \hat{S}_Q(x, y) P_R. \quad (4.114)$$

Since we are working in a  $SU(2)_L$  symmetric state the propagators (4.112) are proportional to the identity in  $SU(2)_L$  space. The matrix structure in colour space is also trivial, and therefore the propagators (4.112) read,

$$S_t^{AB}(x, y) \equiv \delta^{AB} S_t(x, y), \quad (4.115a)$$

$$S_{Qab}^{AB}(x, y) \equiv \delta_{ab} \delta^{AB} S_Q(x, y), \quad (4.115b)$$

where  $S_t(x, y)$  and  $S_Q(x, y)$  are now matrices in spinor space only. Up to the left- and right-handed projectors the above propagators satisfy the Schwinger-Dyson and Kadanoff-Baym equations similar to the lepton field. We can therefore use for the Wightman quark propagators an eQP approximation similar to the one for the leptons, see (3.111). We disregard its off-shell part since it is lepton number conserving at leading order, and only take into account the on-shell part of the quark two-point functions,

$$S_{t \gtrsim}(t, p_t) \approx \tilde{S}_{t \gtrsim}(t, p_t) = F_t^{\gtrsim}(t, p_t) \tilde{S}_{t\rho}(t, p_t), \quad (4.116a)$$

$$S_{Q \gtrsim}(t, p_Q) \approx \tilde{S}_{Q \gtrsim}(t, p_Q) = F_Q^{\gtrsim}(t, p_Q) \tilde{S}_{Q\rho}(t, p_Q), \quad (4.116b)$$

where the eQP spectral functions are given by a delta-function in the limit of a vanishing width,

$$\tilde{S}_{t\rho} = P_R \not{p}_t P_L \text{sign}(p_t^0) (2\pi) \delta(p_t^2 - m_t^2) \equiv P_R \not{p}_t P_L \tilde{\mathbf{S}}_{t\rho}, \quad (4.117a)$$

$$\tilde{S}_{Q\rho} = P_L \not{p}_Q P_R \text{sign}(p_Q^0) (2\pi) \delta(p_Q^2 - m_Q^2) \equiv P_L \not{p}_Q P_R \tilde{\mathbf{S}}_{Q\rho}. \quad (4.117b)$$

Here  $m_t$  and  $m_Q$  are the thermal masses of the quarks. They are approximatively given by  $m_t \approx m_Q \approx 0.4T$  [51]. Similarly to the lepton and heavy neutrino case we have factored out the ‘‘scalar’’ part of the quark propagators.

### 4.5.1 Tree-level amplitude

The leading contribution comes from the one-loop lepton self-energy,

$$\Sigma_{\gtrless}^{(1)}(t, p) = - \int d\Pi_{qk}^4 (2\pi)^4 \delta(q - p - k) (h^\dagger h)_{ji} P_R \mathcal{S}_{\gtrless}^{ij}(t, q) P_L \Delta_{\lesseqgtr}(t, k), \quad (4.118)$$

with the Higgs propagator replaced by its off-shell part (4.111). The latter is proportional to the Higgs Wightman self-energy. At one-loop level, in Wigner space, it reads (see appendix E),

$$\Omega_{\gtrless}(t, k) = g_q |\lambda_t|^2 \int d\Pi_{pQp_t}^4 (2\pi)^4 \delta(k + p_Q - p_t) \text{tr} [S_{Q\lesseqgtr}(t, p_Q) P_R S_{t\gtrless}(t, p_t) P_L], \quad (4.119)$$

where the factor  $g_q = 3$  comes from summation over the colour indices. Inserting the eQP ansatz (4.116) for the quarks we find,

$$\Omega_{\gtrless}(k) = g_q |\lambda_t|^2 \int d\Pi_{pQp_t}^4 (2\pi)^4 \delta(k + p_Q - p_t) \tilde{\mathbf{S}}_{t\rho}(p_t) \tilde{\mathbf{S}}_{Q\rho}(p_Q) 2p_t p_Q F_t^{\gtrless}(p_t) F_Q^{\lesseqgtr}(p_Q), \quad (4.120)$$

where we have suppressed the time argument for notational convenience. At leading order in the Yukawa couplings the right-handed neutrino propagator is diagonal,  $\mathcal{S}_{\gtrless}^{ij} \approx \tilde{S}_{\gtrless}^{ii} \delta^{ij} = F_{N_i}^{\gtrless} \tilde{S}_\rho^{ii} \delta^{ij}$ , and the one-loop lepton self-energy (4.118) reads,

$$\begin{aligned} \Sigma_{\gtrless}^{(1)}(p) &= g_q (h^\dagger h)_{ii} |\lambda_t|^2 \int d\Pi_{qpQp_t}^4 (2\pi)^4 \delta(q + p_Q - p - p_t) \tilde{\mathbf{S}}_\rho^i(q) \tilde{\mathbf{S}}_{t\rho}(p_t) \tilde{\mathbf{S}}_{Q\rho}(p_Q) \\ &\quad \times F_{N_i}^{\gtrless}(q) F_Q^{\gtrless}(p_Q) F_t^{\lesseqgtr}(p_t) \Delta_{R+A}^2(p - q) (2p_t p_Q) P_R \not{q}, \end{aligned} \quad (4.121)$$

where we have defined the Higgs propagators,

$$\Delta_{R+A}^2(k) \equiv \frac{1}{2} (\Delta_R^2(k) + \Delta_A^2(k)) = \Delta_R(k) \Delta_A(k) - \frac{1}{4} \Delta_\rho(k) \Delta_\rho(k). \quad (4.122)$$

Note the resemblance of the intermediate Higgs propagators (4.122) and the ‘‘RIS-subtracted’’ heavy neutrino propagator (4.89). They both describe an intermediate state and feature a RIS-subtraction. In a CP-symmetric, homogeneous and isotropic medium the propagator (4.122) is an even function of the momentum,

$$\Delta_{R+A}^2(-k) = \Delta_{R+A}^2(k). \quad (4.123)$$

The tree-level contribution is obtained by substituting the one-loop lepton self-energy (4.118) with the off-shell Higgs propagator into the divergence of the lepton current (4.11),

$$\begin{aligned} \mathcal{D}_{\mu} j_L^{\mu}(t)|_{(t,T)} &= \sum_i \int d\Pi_{qpp_t p_Q}^4 (2\pi)^4 \delta(q + p_Q - p - p_t) \tilde{\mathcal{S}}_{\rho}^i(q) \tilde{\mathcal{S}}_{t\rho}(p_t) \tilde{\mathcal{S}}_{Q\rho}(p_Q) \tilde{\mathcal{S}}_{\rho}(p) \\ &\quad \times g_q g_w (h^{\dagger} h)_{ii} |\lambda_t|^2 (2pq)(2p_t p_Q) \Delta_{R+A}^2(p_Q - p_t) \\ &\quad \times \left[ F_{N_i}^{<}(q) F_{\bar{Q}}^{<}(p_Q) F_{\ell}^{>}(p) F_t^{>}(p_t) - F_{\ell}^{<}(p) F_t^{<}(p_t) F_{N_i}^{>}(q) F_{\bar{Q}}^{>}(p_Q) \right]. \end{aligned} \quad (4.124)$$

Similarly to the heavy neutrino decay we perform then the trivial integration over the frequencies by using the delta-functions in the eQP spectral functions. The product of the four eQP spectral functions gives raise to  $2^4$  terms which correspond to  $2 \leftrightarrow 2$  scattering,  $1 \leftrightarrow 3$  (inverse)decay and unphysical  $0 \leftrightarrow 4$  processes. An additional constraint comes from the overall delta-function ensuring momentum-energy conservation. In the regime  $M_i > m_{\ell} + m_t + m_Q$  only 8 terms satisfy the energy conservation and read,

$$\begin{aligned} \mathcal{D}_{\mu} j_L^{\mu}(t)|_{(t,T)} &= \sum_i \int d\Pi_{N_i \ell t \bar{Q}}^{qpp_t p_Q} (2\pi)^4 \delta(q + p_Q - p - p_t) \\ &\quad \times \left[ \Xi_{N_i Q \leftrightarrow \ell t}^{(T)} \mathcal{F}_{N_i Q \leftrightarrow \ell t}^{qpQ:pp_t} - \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} t}^{(T)} \mathcal{F}_{N_i \bar{Q} \leftrightarrow \bar{\ell} t}^{qpQ:pp_t} \right. \\ &\quad + \Xi_{N_i \bar{t} \leftrightarrow \ell \bar{Q}}^{(T)} \mathcal{F}_{N_i \bar{t} \leftrightarrow \ell \bar{Q}}^{qp_t:pp_Q} - \Xi_{N_i t \leftrightarrow \bar{\ell} \bar{Q}}^{(T)} \mathcal{F}_{N_i t \leftrightarrow \bar{\ell} \bar{Q}}^{qp_t:pp_Q} \\ &\quad + \Xi_{\bar{t} Q \leftrightarrow \ell N_i}^{(T)} \mathcal{F}_{\bar{t} Q \leftrightarrow \ell N_i}^{p_t p_Q:qp} - \Xi_{t \bar{Q} \leftrightarrow \bar{\ell} N_i}^{(T)} \mathcal{F}_{t \bar{Q} \leftrightarrow \bar{\ell} N_i}^{p_t p_Q:qp} \\ &\quad \left. + \Xi_{N_i \leftrightarrow \ell \bar{t} \bar{Q}}^{(T)} \mathcal{F}_{N_i \leftrightarrow \ell \bar{t} \bar{Q}}^{q:pp_t p_Q} - \Xi_{N_i \leftrightarrow \bar{\ell} t Q}^{(T)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} t Q}^{q:pp_t p_Q} \right], \end{aligned} \quad (4.125)$$

where the tree-level amplitudes are given by,

$$\Xi_{N_i Q \leftrightarrow \ell t}^{(T)} = \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} t}^{(T)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(T)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.126a)$$

$$\Xi_{N_i \bar{t} \leftrightarrow \ell \bar{Q}}^{(T)} = \Xi_{N_i t \leftrightarrow \bar{\ell} \bar{Q}}^{(T)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(T)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.126b)$$

$$\Xi_{\bar{t} Q \leftrightarrow \ell N_i}^{(T)} = \Xi_{t \bar{Q} \leftrightarrow \bar{\ell} N_i}^{(T)} = \Xi_{\phi^* \leftrightarrow \ell N_i}^{(T)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.126c)$$

$$\Xi_{N_i \leftrightarrow \ell \bar{t} \bar{Q}}^{(T)} = \Xi_{N_i \leftrightarrow \bar{\ell} t Q}^{(T)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(T)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}. \quad (4.126d)$$

We have factored the amplitudes (4.126) into a top contribution,  $\Xi_{\phi^* Q \leftrightarrow t}^{(T)}$ , and the heavy neutrino contribution,  $\Xi_{N_i \leftrightarrow \ell \phi^*}^{(T)}$  or  $\Xi_{\phi^* \leftrightarrow \ell N_i}^{(T)}$ . They correspond to the tree-level (inverse) decay amplitudes with an off-shell Higgs, denoted by  $\phi^*$ , and read,

$$\Xi_{N_i \leftrightarrow \ell \phi^*}^{(T)} = \Xi_{\phi^* \leftrightarrow \ell N_i}^{(T)} = 2g_w (h^{\dagger} h)_{ii} pq, \quad (4.127a)$$

$$\Xi_{\phi^* Q \leftrightarrow t}^{(T)} = 2g_q |\lambda_t|^2 p_t p_Q. \quad (4.127b)$$

We have written (4.125) in such a way that each line in square brackets corresponds to the difference of a process and its CP-conjugate. They are equal at tree-level, see (4.126). Therefore the RHS of (4.125) vanishes when the SM particles are in thermal equilibrium and no asymmetry can be produced at tree-level.

The intermediate Higgs propagator reads,

$$\Delta_{R+A}^2(k) = \frac{1}{(k^2 - m_\phi^2)^2 + (\frac{1}{2}\Omega_\rho(k))^2} - \frac{1}{4} \frac{(\Omega_\rho(k))^2}{((k^2 - m_\phi^2)^2 + (\frac{1}{2}\Omega_\rho(k))^2)^2}. \quad (4.128)$$

In the regime  $m_\phi < m_t + m_Q$  the on-shell condition for the Higgs is never satisfied, and the second term in (4.128) is negligible. We can also neglect the ‘‘regulator’’ in the first term. Then the intermediate Higgs propagator reads,

$$\Delta_{R+A}^2(k) \approx \frac{1}{(k^2 - m_\phi^2)^2}. \quad (4.129)$$

In that limit the tree-level amplitudes (4.126) obtained within NEQFT and the ones computed in vacuum are equal.

### 4.5.2 Self-energy contribution

Similarly to the decay of heavy neutrino, the self-energy corrections to the amplitudes (4.126) are obtained by using the off-diagonal right-handed neutrino propagator (4.42),  $\hat{\mathcal{F}}_{\gtrless}^{(SE)} \equiv -\hat{\mathcal{S}}_R \hat{\Pi}_R^{od} \hat{F}_N^{\gtrless} \hat{\mathcal{S}}_\rho - \hat{F}_N^{\gtrless} \hat{\mathcal{S}}_\rho \hat{\Pi}_A^{od} \hat{\mathcal{S}}_A$ , instead of the leading order approximation. Performing the same steps as above we find the self-energy corrections to the scattering and three-body decay amplitudes,

$$\Xi_{N_i Q \leftrightarrow \ell t}^{(S)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.130a)$$

$$\Xi_{N_i \bar{t} \leftrightarrow \ell \bar{Q}}^{(S)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.130b)$$

$$\Xi_{\bar{t} Q \leftrightarrow \ell N_i}^{(S)} = \Xi_{\phi^* \leftrightarrow \ell N_i}^{(S)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.130c)$$

$$\Xi_{N_i \leftrightarrow \ell \bar{t} \bar{Q}}^{(S)} = \Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.130d)$$

where the corrections to the decay amplitudes  $\Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)}$  and  $\Xi_{\phi^* \leftrightarrow \ell N_i}^{(S)}$  are given by the same expression as in the heavy neutrino decay process since they do not depend on the momentum of the Higgs,

$$\Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)} = \Xi_{\phi^* \leftrightarrow \ell N_i}^{(S)} = -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ P_R (\not{q} + M_i) \Pi_A^{od,ij}(q) \mathcal{S}_A^{jj}(q) \not{p} \right] \right]. \quad (4.131)$$

The CP-conjugate of the amplitudes (4.130) are obtained from (4.130) by replacing the amplitude  $\Xi_{N_i \leftrightarrow \ell \phi^*}^{(S)}$  ( $\Xi_{\phi^* \leftrightarrow \ell N_i}^{(S)}$ ) by its CP-conjugate  $\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}^*}^{(S)}$  ( $\Xi_{\bar{\phi}^* \leftrightarrow \bar{\ell} N_i}^{(S)}$ ),

$$\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}^*}^{(S)} = \Xi_{\bar{\phi}^* \leftrightarrow \bar{\ell} N_i}^{(S)} = -2g_w \sum_{j \neq i} \text{Re} \left[ (h^\dagger h)_{ji} \text{tr} \left[ \not{p} \mathcal{S}_R^{jj}(q) \Pi_R^{od,ji}(q) (\not{q} + M_i) P_R \right] \right]. \quad (4.132)$$

Since the top-quark contributions to the amplitudes (4.130) factor out, it cancels out in the CP-parameters. Therefore, only one CP-parameter is needed to characterise the self-energy

correction to the Higgs-mediated processes. It is given by,

$$\begin{aligned} \epsilon_{t,i}^{(S)} &\equiv \frac{\Xi_{N_i Q \leftrightarrow \ell t}^{(S)} - \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} \bar{t}}^{(S)}}{\Xi_{N_i Q \leftrightarrow \ell t}^{(T)} - \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} \bar{t}}^{(T)}} \\ &= \sum_{j \neq i} \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2) M_i \Gamma_{N_j}}{(M_i^2 - M_j^2)^2 + (\Gamma_{N_j}/M_j q L_\rho)^2} \frac{p L_\rho}{pq}, \end{aligned} \quad (4.133)$$

and takes the same form as the corresponding CP-parameter of the heavy neutrino decay (4.52). However, since the kinematics of the two processes are different, they are not evaluated at the same momenta. In particular,  $pq \neq \frac{1}{2} M_i^2$  for the CP-violating parameter (4.133).

### 4.5.3 Vertex contribution

In complete analogy with the right-handed neutrino decay the two-loop lepton self-energy (4.14) together with the off-shell Higgs propagator (4.111) and on-shell heavy neutrino propagators gives the vertex correction to the amplitudes (4.126). After some algebra we find,

$$\Xi_{N_i Q \leftrightarrow \ell t}^{(V)} = \sum_{j \neq i} \Xi_{N_i \leftrightarrow \ell \phi^*}^{(V)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.134a)$$

$$\Xi_{N_i \bar{t} \leftrightarrow \bar{\ell} \bar{Q}}^{(V)} = \sum_{j \neq i} \Xi_{N_i \leftrightarrow \ell \phi^*}^{(V)} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.134b)$$

$$\Xi_{\bar{t} Q \leftrightarrow \ell N_i}^{(V)} = \sum_{j \neq i} \Xi_{\phi^* \leftrightarrow \ell N_i}^{(V)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.134c)$$

$$\Xi_{N_i \leftrightarrow \bar{\ell} \bar{t} \bar{Q}}^{(V)} = \sum_{j \neq i} \Xi_{N_i \leftrightarrow \ell \phi^*}^{(V)} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)}, \quad (4.134d)$$

with

$$\Xi_{N_i \leftrightarrow \ell \phi^*}^{(V)} \equiv -2g_w M_i \sum_j \text{Re} \left[ \text{tr} \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(q, q-p) C P_L \not{p} \right] \right], \quad (4.135a)$$

$$\Xi_{\phi^* \leftrightarrow \ell N_i}^{(V)} \equiv -2g_w M_i \sum_j \text{Re} \left[ \text{tr} \left[ (h^\dagger h)_{ij}^2 \mathbf{A}_{jj}(q, q+p) C P_L \not{p} \right] \right]. \quad (4.135b)$$

Since the two above amplitudes are not equal due to the sign difference of the lepton momentum, we can define two inequivalent CP-parameters [102],  $\epsilon_{t,i}^{(V^-)}$ , which corresponds to the amplitudes (4.134a), (4.134b) and (4.134d), and  $\epsilon_{t,i}^{(V^+)}$ , which corresponds to the amplitude

(4.134c). The two vertex CP-violating parameters read,

$$\begin{aligned}
 \epsilon_{t,i}^{(V^\mp)} &= - \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii}} \frac{M_i M_j}{pq} \int d\Pi_{q_1 p_1 k_1}^4 (2\pi)^4 \delta(q + k_1 + p_1) (2\pi)^4 \delta(q \mp p + p_1 - q_1) p p_1 \\
 &\quad \times \left[ \mathcal{S}_h^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_\rho(k_1) + \mathcal{S}_h^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_F(k_1) \right. \\
 &\quad \left. - \mathcal{S}_F^{jj}(q_1) \mathbf{S}_\rho(p_1) \Delta_h(k_1) + \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_F(p_1) \Delta_h(k_1) \right. \\
 &\quad \left. + \mathcal{S}_F^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_\rho(k_1) + \mathcal{S}_\rho^{jj}(q_1) \mathbf{S}_h(p_1) \Delta_F(k_1) \right] \\
 &\equiv - \frac{1}{g_w} \sum_j \frac{\text{Im}[(h^\dagger h)_{ij}^2]}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \frac{M_i \Gamma_{N_j}}{M_j^2} \frac{p K_{t,j}^{(\mp 1)}(q, p) + p K_{t,j}^{(\mp 2)}(q, p) + p K_{t,j}^{(\mp 3)}(q, p)}{pq}, \quad (4.136)
 \end{aligned}$$

where we have defined three loop functions for each CP-parameter which correspond to the three lines in square brackets. Inserting the on-shell term of the eQP approximation into (4.136) we find for the loop functions of  $\epsilon_{t,i}^{(V^-)}$ ,

$$K_{t,j}^{(-1),\mu}(q, p) \equiv 16\pi \int d\Pi_{\ell\phi}^{p_1 k_1} (2\pi)^4 \delta(q - p_1 - k_1) p_1^\mu \frac{M_j^2}{M_j^2 - (k_1 - p)^2} [1 - f_\ell^{p_1} + f_\phi^{k_1}], \quad (4.137a)$$

$$K_{t,j}^{(-2),\mu}(q, p) \equiv 16\pi \int d\Pi_{N_j \ell}^{q_1 p_1} (2\pi)^4 \delta(q + p_1 - q_1 - p) p_1^\mu \frac{M_j^2}{m_\phi^2 - (q + p_1)^2} [f_{N_j}^{q_1} - f_\ell^{p_1}] \quad (4.137b)$$

$$+ 16\pi \int d\Pi_{N_j \ell}^{q_1 p_1} (2\pi)^4 \delta(q + q_1 - p - p_1) p_1^\mu \frac{M_j^2}{m_\phi^2 - (q + p_1)^2} [f_{N_j}^{q_1} - f_\ell^{p_1}],$$

$$K_{t,j}^{(-3),\mu}(q, p) \equiv 16\pi \int d\Pi_{N_j \phi}^{q_1 k_1} (2\pi)^4 \delta(q_1 - p - k_1) (q + k_1)^\mu \frac{M_j^2}{m_\ell^2 - (q + k_1)^2} [f_{N_j}^{q_1} + f_\phi^{k_1}]. \quad (4.137c)$$

The loop functions for the second CP-parameter are given by,

$$K_{t,j}^{(+1),\mu}(q, p) \equiv 16\pi \int d\Pi_{\ell\phi}^{p_1 k_1} (2\pi)^4 \delta(q - p_1 - k_1) p_1^\mu \frac{M_j^2}{M_j^2 - (k_1 + p)^2} [1 - f_\ell^{p_1} + f_\phi^{k_1}], \quad (4.138a)$$

$$K_{t,j}^{(+2),\mu}(q, p) \equiv 16\pi \int d\Pi_{N_j \ell}^{q_1 p_1} (2\pi)^4 \delta(q + p + p_1 - q_1) p_1^\mu \frac{M_j^2}{m_\phi^2 - (q + p_1)^2} [f_{N_j}^{q_1} - f_\ell^{p_1}] \quad (4.138b)$$

$$- 16\pi \int d\Pi_{N_j \ell}^{q_1 p_1} (2\pi)^4 \delta(q + p - q_1 - p_1) p_1^\mu \frac{M_j^2}{m_\phi^2 - (q - p_1)^2} [1 - f_{N_j}^{q_1} - f_\ell^{p_1}],$$

$$K_{t,j}^{(+3),\mu}(q, p) \equiv 16\pi \int d\Pi_{N_j \phi}^{q_1 k_1} (2\pi)^4 \delta(q_1 - p - k_1) (k_1 - q)^\mu \frac{M_j^2}{m_\ell^2 - (q - k_1)^2} [f_{N_j}^{q_1} + f_\phi^{k_1}]. \quad (4.138c)$$

Similarly to the heavy neutrino decay case only the cut through  $\ell\phi$  gives a contribution to the CP-parameter  $\epsilon_{t,i}^{(V^-)}$  in the vacuum limit. The other CP-parameter,  $\epsilon_{t,i}^{(V^+)}$ , which corresponds to the  $\bar{t}Q \leftrightarrow \ell N_i$  scattering processes, receives two vacuum contributions [102]. One contribution comes from the cut through  $\ell\phi$ , (4.138a), and the other one corresponds to the cut through  $N_j \ell$ . The kinematics of the second contribution is the one of a  $N_i \ell \leftrightarrow N_j \ell$  scattering process. Therefore, in the divergence of the lepton current, the vacuum contribution coming from the

second cut is suppressed for a hierarchical mass spectrum since it requires the center-of-mass energy to be greater than  $M_j + m_\ell \gg M_i + m_\ell$ . For a mildly degenerate mass spectrum both contributions should be taken into account [102].

## 4.6 Heavy neutrino number density

We have for now only derived a quantum-corrected Boltzmann equation for the lepton asymmetry, see (4.64) (heavy neutrino decay contribution) and (4.125) (top quark contribution). These equations contain the heavy neutrino distribution function  $f_{N_i}$ . The latter satisfies the Boltzmann-like equation (see (3.109)),

$$\mathcal{D}_t f_{N_i}(t, q) = \frac{1}{4q^0} \text{tr} \left[ \Pi_{>}^{ii}(t, q) (\not{q} + M_i) f_{N_i}(t, q) + \Pi_{<}^{ii}(t, q) (\not{q} + M_i) (1 - f_{N_i}(t, q)) \right], \quad (4.139)$$

with the diagonal self-energy given by (see appendix E),

$$\begin{aligned} \Pi_{\gtrless}^{ii}(t, q) = & -g_w (h^\dagger h)_{ii} \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) [P_L S_{\gtrless}(t, p) P_R \Delta_{\gtrless}(t, k) \\ & + P_R P \bar{S}_{\gtrless}(t, \bar{p}) P P_L \bar{\Delta}_{\gtrless}(t, \bar{k})]. \end{aligned} \quad (4.140)$$

On the LHS of (4.139) we take into account the expansion of the universe by using the covariant derivative. The number density  $n_{N_i}$  is found by integrating the distribution function over the momentum and summing over the internal degrees of freedom,

$$n_{N_i}(t) \equiv 2 \int \frac{d^3 \vec{q}}{(2\pi)^3} f_{N_i}(t, q), \quad (4.141)$$

where the overall factor 2 comes from the summation over the helicity state. Similarly to the lepton asymmetry the time evolution of the heavy neutrino number density can be split between the contributions from the heavy neutrino decay and from the Higgs mediated processes,

$$\mathcal{D}_t n_{N_i}(t) = \mathcal{D}_t n_{N_i}(t)|_{(D)} + \mathcal{D}_t n_{N_i}(t)|_{(t)}. \quad (4.142)$$

We only need to consider the tree-level processes for the production or destruction of the heavy neutrino number.

### Heavy neutrino decay

We first consider the contribution from the heavy neutrino decay. Substituting the on-shell term of the eQP approximation for the lepton and Higgs in the heavy neutrino self-energy, we find for the heavy neutrino decay contribution,

$$\mathcal{D}_t n_{N_i}(t)|_{(D)} = - \int d\Pi_{N_i \ell \phi}^{qpk} \left[ \Xi_{N_i \leftrightarrow \ell \phi}^{(T)} \mathcal{F}_{N_i \leftrightarrow \ell \phi}^{q:pk} + \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{q:pk} \right], \quad (4.143)$$

where the tree-level amplitude  $\Xi_{N_i \leftrightarrow \ell \phi}^{(T)}$  and its CP-conjugate  $\Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)}$  are equal and given by,

$$\Xi_{N_i \leftrightarrow \ell \phi}^{(T)} = \Xi_{N_i \leftrightarrow \bar{\ell} \bar{\phi}}^{(T)} = 2g_w (h^\dagger h)_{ii} p q. \quad (4.144)$$



**Top quark contribution**

The top quark contributions are found by substituting the on-shell lepton and off-shell Higgs propagators into the one-loop heavy neutrino self-energy,

$$\begin{aligned}
 \mathcal{D}_t n_{N_i}(t) = & - \int d\Pi_{N_i \ell t Q}^{qpp_t p_Q} (2\pi)^4 \delta(q + p_Q - p - p_t) \\
 & \times \left[ \Xi_{N_i Q \leftrightarrow \ell t}^{(T)} \mathcal{F}_{N_i Q \leftrightarrow \ell t}^{q p_Q; p p_t} + \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} \bar{t}}^{(T)} \mathcal{F}_{N_i \bar{Q} \leftrightarrow \bar{\ell} \bar{t}}^{q p_Q; p p_t} \right. \\
 & + \Xi_{N_i \bar{t} \leftrightarrow \bar{\ell} \bar{Q}}^{(T)} \mathcal{F}_{N_i \bar{t} \leftrightarrow \bar{\ell} \bar{Q}}^{q p_t; p p_Q} + \Xi_{N_i t \leftrightarrow \bar{\ell} Q}^{(T)} \mathcal{F}_{N_i t \leftrightarrow \bar{\ell} Q}^{q p_t; p p_Q} \\
 & - \Xi_{\bar{t} Q \leftrightarrow \ell N_i}^{(T)} \mathcal{F}_{\bar{t} Q \leftrightarrow \ell N_i}^{p_t p_Q; q p} - \Xi_{t \bar{Q} \leftrightarrow \bar{\ell} N_i}^{(T)} \mathcal{F}_{t \bar{Q} \leftrightarrow \bar{\ell} N_i}^{p_t p_Q; q p} \\
 & \left. + \Xi_{N_i \leftrightarrow \bar{\ell} \bar{Q}}^{(T)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} \bar{Q}}^{q; p p_t p_Q} + \Xi_{N_i \leftrightarrow \bar{\ell} t Q}^{(T)} \mathcal{F}_{N_i \leftrightarrow \bar{\ell} t Q}^{q; p p_t p_Q} \right], \quad (4.145)
 \end{aligned}$$

where the tree-level amplitudes are given by (4.126).



# Chapter 5

## Comparison of the Boltzmann and nonequilibrium approaches

In this chapter we compare the amplitudes and CP-parameters obtained within NEQFT and the ones obtained in the conventional approach. However these quantities cannot be directly compared since the former depend on the momenta of the initial and final states in a non-trivial way. Therefore, we first obtain rate equations from the Boltzmann equations derived within NEQFT and from the conventional ones. These rate equations are governed by few averaged quantities, called *reaction densities*, which depend only on the temperature of the medium. The reaction densities can then be used to compare the NEQFT and conventional approaches. This chapter is based on the work presented in [72] and work in preparation [73].

### 5.1 Rate equations

Solving a system of Boltzmann equations in general requires the use of numerical codes capable of treating large systems of differential equations for the different momentum modes. This is a difficult task if one wants to study a wide range of model parameters. A commonly employed simplification is to approximate the Boltzmann equations by the corresponding system of rate equations for the abundances  $Y_a$ . In [150] it was shown that the two approaches, Boltzmann and the rate equations, give approximately equal result for the final asymmetry, up to 10% correction.

We first derive the rate equations from the Boltzmann equations obtained within NEQFT. They incorporate quantum statistical factors, which are usually neglected in the conventional approach for consistency with the RIS-subtraction procedure. The rate equations with Maxwell-Boltzmann statistics are derived for comparison with the improved result.

#### Rate equations from NEQFT

In the previous chapter we have derived from NEQFT an equation for the divergence of the lepton current. Its right-hand side can be represented as a combination of processes involving the right-handed neutrinos as initial or final states (e.g. heavy neutrino (inverse) decay  $N_i \leftrightarrow \ell\phi$ ) and processes with SM particles only (e.g. two-body scattering  $\bar{\ell}\bar{\phi} \leftrightarrow \ell\phi$ ),

$$\mathcal{D}_\mu j^\mu = \sum_{i,\{a\},\{j\}} \mathcal{D}_\mu j^\mu|_{N_i ab \dots \leftrightarrow jk \dots} + \sum_{\{a\},\{j\}} \mathcal{D}_\mu j^\mu|_{ab \dots \leftrightarrow jk \dots}, \quad (5.1)$$

where the sum runs over all lepton number violating processes  $N_i ab \dots \leftrightarrow jk \dots$  and  $ab \dots \leftrightarrow jk \dots$ . The contributions of the various processes can be generally represented in the form

(see (4.64) or (4.81)),

$$\mathcal{D}_\mu j^\mu|_{N_i ab \dots \leftrightarrow jk \dots} = \pm \int d\Pi_{N_i ab \dots jk \dots}^{qp_a p_b \dots p_j p_k \dots} \left[ \mathcal{F}_{N_i ab \dots \leftrightarrow jk \dots}^{qp_a p_b \dots p_j p_k \dots} \Xi_{N_i ab \dots \leftrightarrow jk \dots} - \mathcal{F}_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots}^{qp_a p_b \dots p_j p_k \dots} \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right], \quad (5.2a)$$

$$\mathcal{D}_\mu j^\mu|_{ab \dots \leftrightarrow jk \dots} = \pm \int d\Pi_{ab \dots jk \dots}^{p_a p_b \dots p_j p_k \dots} \left[ \mathcal{F}_{ab \dots \leftrightarrow jk \dots}^{p_a p_b \dots p_j p_k \dots} \Xi_{ab \dots \leftrightarrow jk \dots} - \mathcal{F}_{\bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots}^{p_a p_b \dots p_j p_k \dots} \Xi_{\bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right], \quad (5.2b)$$

where the plus sign applies if  $\ell \in \{jk \dots\}$  and the minus sign if  $\ell \in \{ab \dots\}$ . In a homogeneous and isotropic universe, the only non-trivial component of the lepton current is its zeroth entry, which is equal to the lepton number density:  $j^\mu = (n_L, \vec{0})$ . The LHS of (5.1) is conveniently written in terms of the lepton asymmetry abundance  $Y_L \equiv n_L/s$  (where  $s$  is the comoving entropy density, see above (2.10)),

$$\mathcal{D}_\mu j^\mu(t) = s \frac{dY_L(t)}{dt}. \quad (5.3)$$

The time argument is usually traded for the dimensionless inverse temperature  $z \equiv \frac{M_1}{T}$  ( $M_1$  is the *lightest* right-handed neutrino mass),

$$s \frac{dY_L}{dt} = \frac{s\mathcal{H}}{z} \frac{dY_L}{dz}, \quad (5.4)$$

where  $\mathcal{H} \equiv H(T = M_1)$  is the Hubble rate evaluated at  $T = M_1$ .

We also derived an equation for the heavy neutrino number density, see (4.143) and (4.145). Similarly to (5.1) it can be written as a sum over the different processes. In terms of the right-handed neutrino abundance  $Y_{N_i} \equiv n_{N_i}/s$ , it reads,

$$\frac{s\mathcal{H}}{z} \frac{dY_{N_i}}{dz} = - \sum_{\{a\}, \{j\}} \int d\Pi_{N_i ab \dots jk \dots}^{qp_a p_b \dots p_j p_k \dots} \left[ \mathcal{F}_{N_i ab \dots \leftrightarrow jk \dots}^{qp_a p_b \dots p_j p_k \dots} \Xi_{N_i ab \dots \leftrightarrow jk \dots} + \mathcal{F}_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots}^{qp_a p_b \dots p_j p_k \dots} \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right]. \quad (5.5)$$

The system of differential equations for the lepton asymmetry (5.1) and for the heavy neutrino abundance (5.5) does not form a closed set of differential equations since they are equations for the *number density* whereas their RHS depend on the *full* momentum dependence of the distribution functions  $f_a(t, p_a)$  of the different species. Therefore, the distribution functions  $f_a(t, p_a)$  must be expressed in terms of the lepton asymmetry and heavy neutrino abundance in order to close the system of differential equations (5.1) and (5.5). This can only be done by assuming a particular shape for the distribution functions. We assume that the SM particles are maintained in kinetic equilibrium by the fast gauge interactions. This means that their distribution function takes the simple form:

$$f_a(t, p_a) = \frac{1}{e^{\beta(E_a^{p_a} - \mu_a)} \mp 1}, \quad (5.6)$$

with a time- (or equivalently temperature-) dependent chemical potential  $\mu_a = \mu_a(t)$ . Here the upper (lower) sign corresponds to bosons (fermions). It is also useful to define the equilibrium

distribution function,

$$f_a^{\text{eq},p_a} = \frac{1}{e^{\beta E_a^{p_a}} \mp 1}. \quad (5.7)$$

The fast SM interactions and the sphalerons relate the chemical potentials of the leptons, quarks and Higgs fields, such that only one of them is independent. We can therefore express the chemical potential of the Higgs and quarks as a function of the lepton chemical potential [151–153],

$$\mu_\phi = \frac{4}{7}\mu_\ell \equiv c_{\phi\ell}\mu_\ell, \quad (5.8a)$$

$$\mu_t = \frac{5}{21}\mu_\ell \equiv c_{t\ell}\mu_\ell, \quad (5.8b)$$

$$\mu_Q = -\frac{1}{3}\mu_\ell \equiv c_{Q\ell}\mu_\ell. \quad (5.8c)$$

The chemical potential of an antiparticle is related to the one of the corresponding particle,  $\mu_{\bar{a}} = -\mu_a$ . Therefore, under the assumption of kinetic equilibrium, the distribution functions of the SM particles at a particular time are completely determined by a single parameter, which we choose to be the chemical potential of the lepton. The latter is related to the lepton asymmetry abundance by,

$$\frac{\mu_\ell}{T} \approx c_\ell \frac{Y_L}{2Y_\ell^{\text{eq}}}, \quad (5.9)$$

where  $Y_\ell^{\text{eq}}$  is the equilibrium lepton abundance and  $c_\ell$  is a numerical parameter which depends on the temperature and thermal mass of the lepton. For  $m_\ell/T \approx 0.2$  it can be very well approximated by the zero mass limit,  $c_\ell \approx 9\zeta(3)/\pi^2 \approx 1.1$  ( $\zeta$  is the Euler-Riemann zeta function).

Using the identity  $1 \pm f_a^{p_a} = e^{(E_a - \mu_a)/T} f_a^{p_a}$  and energy conservation we can rewrite the combinations of distribution functions appearing in (5.2a) and (5.5) as:

$$\begin{aligned} \mathcal{F}_{N_i ab \dots \leftrightarrow jk \dots}^{q p_a p_b \dots; p_j p_k \dots} &= (2\pi)^4 \delta \left( q + \sum_a p_a - \sum_j p_j \right) \frac{\prod_a f_a^{p_a} \prod_j (1 \pm f_j^{p_j})}{1 - f_{N_i}^{\text{eq},q}} \\ &\times [f_{N_i}^q - f_{N_i}^{\text{eq},q} - f_{N_i}^{\text{eq},q} (1 - f_{N_i}^q) \{ e^{\sum_j \mu_j/T - \sum_a \mu_a/T} - 1 \}]. \end{aligned} \quad (5.10)$$

Since the lepton chemical potential normalised to the temperature is of the order of the lepton asymmetry, the ratio  $\frac{\mu_\ell}{T} \sim Y_L \sim 10^{-10}$  is typically tiny during leptogenesis and we can safely expand (5.10) in the small chemical potentials. Assuming the Majorana neutrino to be close to equilibrium,  $f_{N_i}^q - f_{N_i}^{\text{eq},q} \sim \mathcal{O}(\mu_a)$ , we see that the term in square bracket in (5.10) is already of the first order in the chemical potential. We can therefore replace the distribution functions in the first line of (5.10) by the equilibrium ones:

$$\begin{aligned} \frac{\prod_a f_a^{p_a} \prod_j (1 \pm f_j^{p_j})}{1 - f_{N_i}^{\text{eq},q}} &\approx \frac{\prod_a f_a^{\text{eq},p_a} \prod_j (1 \pm f_j^{\text{eq},p_j})}{1 - f_{N_i}^{\text{eq},q}} = \prod_a f_a^{\text{eq},p_a} \prod_j (1 \pm f_j^{\text{eq},p_j}) \\ &+ \prod_j f_j^{\text{eq},p_j} \prod_a (1 \pm f_a^{\text{eq},p_a}), \end{aligned} \quad (5.11)$$

where we have used the energy conservation to obtain the second equality. Expanding in the small chemical potentials the square bracket in the second line of (5.10) and keeping only the leading order terms, we find,

$$\begin{aligned} & [f_{N_i}^q - f_{N_i}^{\text{eq},q} - f_{N_i}^{\text{eq},q}(1 - f_{N_i}^q)\{e^{\sum_j \mu_j/T - \sum_a \mu_a/T} - 1\}] \\ & \approx [f_{N_i}^q - f_{N_i}^{\text{eq},q} - f_{N_i}^{\text{eq},q}(1 - f_{N_i}^{\text{eq},q})\{\sum_j \mu_j/T - \sum_a \mu_a/T\}]. \end{aligned} \quad (5.12)$$

It is useful to introduce another combination of equilibrium distribution functions,

$$\begin{aligned} \tilde{\mathcal{F}}_{(X)ab\dots\leftrightarrow jk\dots}^{\text{eq}, p_X p_a p_b \dots; p_j p_k \dots} & \equiv (2\pi)\delta(p_X + \sum_a p_a - \sum_j p_j) \\ & \times \left[ \prod_a f_a^{\text{eq}, p_a} \prod_j (1 \pm f_j^{\text{eq}, p_j}) + \prod_j f_j^{\text{eq}, p_j} \prod_a (1 \pm f_a^{\text{eq}, p_a}) \right], \end{aligned} \quad (5.13)$$

and the numerical factor,

$$c_{ab\dots\leftrightarrow jk\dots} \equiv \frac{\sum_j \mu_j - \sum_a \mu_a}{\mu_\ell} = \sum_j c_{j\ell} - \sum_a c_{a\ell}, \quad (5.14)$$

where the quantities  $c_{j\ell}$  and  $c_{a\ell}$  are given in (5.8) for the Higgs and quark fields. Then, up to first order in the chemical potential, (5.10) takes the simple form,

$$\mathcal{F}_{N_i ab\dots\leftrightarrow jk\dots}^{q p_a p_b \dots; p_j p_k \dots} \approx \tilde{\mathcal{F}}_{(N_i)ab\dots\leftrightarrow jk\dots}^{\text{eq}, q p_a p_b \dots; p_j p_k \dots} [f_{N_i}^q - f_{N_i}^{\text{eq},q} - f_{N_i}^{\text{eq},q}(1 - f_{N_i}^{\text{eq},q})c_\ell \frac{Y_L}{2Y_\ell^{\text{eq}}} c_{ab\dots\leftrightarrow jk\dots}]. \quad (5.15)$$

The corresponding equation for the CP-conjugate process is obtained from the above one by replacing  $\mu_a \rightarrow -\mu_a$ ,

$$\mathcal{F}_{N_i ab\dots\leftrightarrow jk\dots}^{q p_a p_b \dots; p_j p_k \dots} \approx \tilde{\mathcal{F}}_{(N_i)ab\dots\leftrightarrow jk\dots}^{\text{eq}, q p_a p_b \dots; p_j p_k \dots} [f_{N_i}^q - f_{N_i}^{\text{eq},q} + f_{N_i}^{\text{eq},q}(1 - f_{N_i}^{\text{eq},q})c_\ell \frac{Y_L}{2Y_\ell^{\text{eq}}} c_{ab\dots\leftrightarrow jk\dots}]. \quad (5.16)$$

The last step is to assume that the distribution function of the right-handed neutrino is proportional to the equilibrium one,

$$f_{N_i}^q \approx \frac{Y_{N_i}(t)}{Y_{N_i}^{\text{eq}}(t)} f_{N_i}^{\text{eq},q}. \quad (5.17)$$

Putting everything together we get the conventional form of the rate equation (compare with [31, 75, 76, 136, 154]),

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab\dots\leftrightarrow jk\dots} & = \pm \langle \epsilon_{jk\dots}^{N_i ab\dots} \gamma_{jk\dots}^{N_i ab\dots} \rangle \left( \frac{Y_{N_i}}{Y_{N_i}^{\text{eq}}} - 1 \right) \\ & \quad - \langle \gamma_{jk\dots}^{N_i ab\dots} \rangle^W c_\ell (\pm c_{ab\dots\leftrightarrow jk\dots}) \frac{Y_L}{2Y_\ell^{\text{eq}}}, \end{aligned} \quad (5.18)$$

where we have defined the *CP-violating*  $\langle \epsilon_{jk\dots}^{N_i ab\dots} \gamma_{jk\dots}^{N_i ab\dots} \rangle$  and *washout*  $\langle \gamma_{jk\dots}^{N_i ab\dots} \rangle|_W$  reaction den-

sities,

$$\langle \epsilon_{jk\dots}^{N_i ab\dots} \gamma_{jk\dots}^{N_i ab\dots} \rangle \equiv \int d\Pi_{N_i ab\dots jk\dots}^{q p_a p_b \dots p_j p_k \dots} \tilde{\mathcal{F}}_{(N_i) ab\dots \leftrightarrow jk\dots}^{\text{eq}, q p_a p_b \dots; p_j p_k \dots} f_{N_i}^{\text{eq}, q} \times \epsilon_{N_i ab\dots \leftrightarrow jk\dots} \left( \Xi_{N_i ab\dots \leftrightarrow jk\dots} + \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right), \quad (5.19a)$$

$$\langle \gamma_{jk\dots}^{N_i ab\dots} \rangle|_W \equiv \int d\Pi_{N_i ab\dots jk\dots}^{q p_a p_b \dots p_j p_k \dots} \tilde{\mathcal{F}}_{(N_i) ab\dots \leftrightarrow jk\dots}^{\text{eq}, q p_a p_b \dots; p_j p_k \dots} f_{N_i}^{\text{eq}, q} (1 - f_{N_i}^{\text{eq}, q}) \times \left( \Xi_{N_i ab\dots \leftrightarrow jk\dots} + \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right). \quad (5.19b)$$

Note that the reaction densities defined in the above equations depend only on the temperature of the medium since they are thermally averaged quantities. The CP-violating parameter  $\epsilon_{N_i ab\dots \leftrightarrow jk\dots}$  in (5.19a) is defined in the usual way (see (4.52)),

$$\epsilon_{N_i ab\dots \leftrightarrow jk\dots} \equiv \frac{\Xi_{N_i ab\dots \leftrightarrow jk\dots} - \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots}}{\Xi_{N_i ab\dots \leftrightarrow jk\dots} + \Xi_{N_i \bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots}}. \quad (5.20)$$

The physical interpretation of the rate equation (5.18) is straightforward. The term proportional to  $\left(\frac{Y_{N_i}}{Y_{N_i}^{\text{eq}}} - 1\right)$  is the production term. It describe the out-of-equilibrium production of lepton or antilepton, and vanishes when the right-handed neutrinos are in thermal equilibrium. The second term in (5.18) does not depend on the heavy neutrino abundance. It is proportional to  $\frac{Y_L}{2Y_\ell^{\text{eq}}}$  and therefore vanishes in absence of any preexisting asymmetry. Since it is negative it leads to a washout of the lepton asymmetry.

The terms without the right-handed neutrino in the initial or final states are treated similarly. Up to first order in the chemical potential, the combination of the distribution functions appearing in (5.2b) reads,

$$\mathcal{F}_{ab\dots \leftrightarrow jk\dots}^{p_a p_b \dots; p_j p_k \dots} \approx -(2\pi)^4 \delta\left(\sum_a p_a - \sum_j p_j\right) \prod_a f_a^{\text{eq}, p_a} \prod_j (1 \pm f_j^{\text{eq}, p_j}) c_\ell \frac{Y_L}{2Y_\ell^{\text{eq}}} c_{ab\dots \leftrightarrow jk\dots}, \quad (5.21)$$

and a similar expression but with an opposite sign for the CP-conjugated process. Then equation (5.2b) takes the simple form,

$$\mathcal{D}_\mu j^\mu|_{ab\dots \leftrightarrow jk\dots} = -\langle \gamma_{jk\dots}^{ab\dots} \rangle c_\ell \frac{Y_L}{2Y_\ell^{\text{eq}}} (\pm c_{ab\dots \leftrightarrow jk\dots}), \quad (5.22)$$

where we have defined the reaction density,

$$\langle \gamma_{jk\dots}^{ab\dots} \rangle \equiv \int d\Pi_{ab\dots jk\dots}^{p_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta\left(\sum_a p_a - \sum_j p_j\right) \prod_a f_a^{\text{eq}, p_a} \prod_j (1 \pm f_j^{\text{eq}, p_j}) \times \left( \Xi_{ab\dots \leftrightarrow jk\dots} + \Xi_{\bar{a} \bar{b} \dots \leftrightarrow \bar{j} \bar{k} \dots} \right). \quad (5.23)$$

Therefore the RHS of (5.22) only contributes to the washout of the asymmetry.

Similarly the right-handed neutrino abundance satisfies,

$$\frac{s\mathcal{H}}{z} \frac{dY_{N_i}}{dz} = - \sum_{\{a\}, \{j\}} \langle \gamma_{jk\dots}^{N_i ab\dots} \rangle|_P \left( \frac{Y_{N_i}}{Y_{N_i}^{\text{eq}}} - 1 \right), \quad (5.24)$$

where the production reaction density is defined by,

$$\langle \gamma_{jk\dots}^{N_i ab\dots} \rangle|_P \equiv \int d\Pi_{N_i ab\dots jk\dots}^{qp_a p_b \dots p_j p_k \dots} \tilde{\mathcal{F}}_{N_i ab\dots jk\dots}^{\text{eq}, qp_a p_b \dots p_j p_k \dots} f_{N_i}^{\text{eq}, q} \left( \Xi_{N_i ab\dots jk\dots} + \Xi_{N_i \bar{a} \bar{b} \dots \bar{j} \bar{k} \dots} \right). \quad (5.25)$$

### Rate equations from classical Boltzmann equations

The Boltzmann equation for the lepton asymmetry obtained in the conventional approach can be splitted in a similar way as the one obtained within NEQFT (5.1). However the expressions for the various contributions take a different form,

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab\dots jk\dots} = & \pm \int d\Pi_{N_i ab\dots jk\dots}^{qp_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(q + \sum_a p_a - \sum_j p_j) \\ & \times \left[ f_{N_i}^q \prod_a f_a^{p_a} \Xi_{N_i ab\dots jk\dots} - \prod_j f_j^{p_j} \Xi_{jk\dots \rightarrow N_i ab\dots} \right. \\ & \left. - f_{N_i}^q \prod_a f_{\bar{a}}^{p_a} \Xi_{N_i \bar{a} \bar{b} \dots \bar{j} \bar{k} \dots} + \prod_j f_{\bar{j}}^{p_j} \Xi_{\bar{j} \bar{k} \dots \rightarrow N_i \bar{a} \bar{b} \dots} \right], \quad (5.26a) \end{aligned}$$

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{ab\dots \leftrightarrow jk\dots} = & \pm \int d\Pi_{N_i ab\dots jk\dots}^{p_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(\sum_a p_a - \sum_j p_j) \\ & \times \left[ \prod_a f_a^{p_a} \Xi_{ab\dots \rightarrow jk\dots} - \prod_j f_j^{p_j} \Xi_{jk\dots \rightarrow ab\dots} - \prod_a f_{\bar{a}}^{p_a} \Xi_{\bar{a} \bar{b} \dots \bar{j} \bar{k} \dots} + \prod_j f_{\bar{j}}^{p_j} \Xi_{\bar{j} \bar{k} \dots \rightarrow \bar{a} \bar{b} \dots} \right], \quad (5.26b) \end{aligned}$$

where the plus sign applies if  $\ell \in \{jk\dots\}$  and the minus sign if  $\ell \in \{ab\dots\}$ . Note that the amplitudes do not feature the double arrow as in the NEQFT approach. We first focus on the processes involving the right-handed neutrino as initial and final state, (5.26a). We can write it as a term similar to the one obtained within NEQFT and an extra term,

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab\dots \leftrightarrow jk\dots} = & \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab\dots \leftrightarrow jk\dots}^{\text{extra}} \pm \int d\Pi_{N_i ab\dots jk\dots}^{qp_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(q + \sum_a p_a - \sum_j p_j) \\ & \times \left[ \left( f_{N_i}^q \prod_a f_a^{p_a} - \prod_j f_j^{p_j} \right) \Xi_{N_i ab\dots \rightarrow jk\dots} - \left( f_{N_i}^q \prod_a f_{\bar{a}}^{p_a} - \prod_j f_{\bar{j}}^{p_j} \right) \Xi_{N_i \bar{a} \bar{b} \dots \rightarrow \bar{j} \bar{k} \dots} \right]. \quad (5.27) \end{aligned}$$

The extra term does not appear in the NEQFT and reads,

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab\dots \leftrightarrow jk\dots}^{\text{extra}} = & \pm \int d\Pi_{N_i ab\dots jk\dots}^{qp_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(q + \sum_a p_a - \sum_j p_j) \\ & \times \left( \Xi_{N_i ab\dots \rightarrow jk\dots} - \Xi_{jk\dots \rightarrow N_i ab\dots} \right) \left( \prod_j f_j^{p_j} + \prod_j f_{\bar{j}}^{p_j} \right), \quad (5.28) \end{aligned}$$

where we use the CPT symmetry to rewrite the amplitudes. In the canonical approach the assumption of kinetic equilibrium means that the distribution functions take the shape of a



Maxwell-Boltzmann distribution functions,

$$f_a^{p_a} = e^{-\beta(E_a^{p_a} - \mu_a)}, \quad (5.29)$$

with a time-dependent chemical potential  $\mu_a = \mu_a(t)$ . Similarly to (5.7) it is convenient to define the equilibrium distribution,  $f_a^{\text{eq}, p_a} = e^{-\beta E_a^{p_a}}$ . Note that we are using the same notation for the Maxwell-Boltzmann distribution function and for the Bose-Einstein or Fermi-Dirac ones. Inserting the ansatz (5.29) into (5.27) we write the distribution functions as,

$$\begin{aligned} (f_{N_i}^q \prod_a f_a^{p_a} - \prod_j f_j^{p_j}) &= \prod_a f_a^{p_a} [f_{N_i}^q - f_{N_i}^{\text{eq}, q} - f_{N_i}^{\text{eq}, q} (e^{\beta(\sum_j \mu_j - \sum_a \mu_a)} - 1)] \\ &\approx \prod_a f_a^{\text{eq}, p_a} [f_{N_i}^q - f_{N_i}^{\text{eq}, q} - f_{N_i}^{\text{eq}, q} \beta \mu_\ell c_{ab \dots \leftrightarrow jk \dots}]. \end{aligned} \quad (5.30)$$

In the second line we have assumed the heavy neutrino to be close to equilibrium and expanded to first order in the small chemical potential. Similarly to (5.17) we make the assumption that the right-handed neutrino distribution function is proportional to the equilibrium one. Then, (5.27) takes the same form as (5.18),

$$\begin{aligned} \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab \dots \leftrightarrow jk \dots} &= \pm \langle \epsilon_{jk \dots}^{N_i ab \dots} \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}} \left( \frac{Y_{N_i}}{Y_{N_i}^{\text{eq}}} - 1 \right) - \langle \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}} (\pm c_{\ell ab \dots \leftrightarrow jk \dots}) \frac{Y_L}{2Y_\ell^{\text{eq}}} \\ &+ \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{N_i ab \dots \leftrightarrow jk \dots}^{\text{extra}}, \end{aligned} \quad (5.31)$$

but with different reactions densities (in particular, the equilibrium distribution functions are the Maxwell-Boltzmann ones),

$$\begin{aligned} \langle \epsilon_{jk \dots}^{N_i ab \dots} \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}} &\equiv \int d\Pi_{N_i ab \dots jk \dots}^{q p_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(q + \sum_a p_a - \sum_j p_j) \prod_a f_a^{\text{eq}, p_a} f_{N_i}^{\text{eq}, q} \\ &\times \epsilon_{N_i ab \dots \rightarrow jk \dots} (\Xi_{N_i ab \dots \rightarrow jk \dots} + \Xi_{N_i \bar{a} \bar{b} \dots \rightarrow \bar{j} \bar{k} \dots}), \end{aligned} \quad (5.32a)$$

$$\begin{aligned} \langle \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}} &\equiv \int d\Pi_{N_i ab \dots jk \dots}^{q p_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(q + \sum_a p_a - \sum_j p_j) \prod_a f_a^{\text{eq}, p_a} f_{N_i}^{\text{eq}, q} \\ &\times (\Xi_{N_i ab \dots \rightarrow jk \dots} + \Xi_{N_i \bar{a} \bar{b} \dots \rightarrow \bar{j} \bar{k} \dots}). \end{aligned} \quad (5.32b)$$

The CP-violating parameter in (5.32a) is defined similarly to (5.20). The extra term (5.28) gives a contribution similar to the washout reaction density (5.32b) but is suppressed by an additional factor  $\epsilon_{N_i ab \dots \rightarrow jk \dots}$  and can therefore be neglected.

The processes which do not involve the right-handed neutrino in the initial or final states are treated in a similar way,

$$\frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{ab \dots \leftrightarrow jk \dots} = - \langle \gamma_{jk \dots}^{ab \dots} \rangle_{\text{MB}} c_\ell (\pm c_{ab \dots \leftrightarrow jk \dots}) \frac{Y_L}{2Y_\ell^{\text{eq}}} + \frac{s\mathcal{H}}{z} \frac{dY_L}{dz} \Big|_{ab \dots \leftrightarrow jk \dots}^{\text{extra}} \quad (5.33)$$

where the washout reaction density is given by an expression similar to (5.32b),

$$\langle \gamma_{jk \dots}^{ab \dots} \rangle_{\text{MB}} \equiv \int d\Pi_{ab \dots jk \dots}^{p_a p_b \dots p_j p_k \dots} (2\pi)^4 \delta(\sum_a p_a - \sum_j p_j) \prod_a f_a^{\text{eq}, p_a} (\Xi_{ab \dots \rightarrow jk \dots} + \Xi_{\bar{a} \bar{b} \dots \rightarrow \bar{j} \bar{k} \dots}). \quad (5.34)$$

The extra term in (5.33) is suppressed by the CP-violating parameter and can be neglected.

In the canonical approach the heavy neutrino satisfies the Boltzmann equation (2.37),

$$s \frac{dY_{N_i}}{dt} = - \sum_{\{a\}, \{j\}} \int d\Pi_{N_i ab \dots jk \dots}^{qp_a p_b \dots p_j p_k \dots} \left[ f_{N_i}^q \prod_a f_a^{p_a} \Xi_{N_i ab \dots \rightarrow jk \dots} - \prod_j f_j^{p_j} \Xi_{jk \dots \rightarrow N_i ab \dots} + f_{N_i}^q \prod_a f_a^{p_a} \Xi_{N_i \bar{a} b \dots \rightarrow \bar{j} \bar{k} \dots} - \prod_j f_j^{p_j} \Xi_{\bar{j} \bar{k} \dots \rightarrow N_i \bar{a} \bar{b} \dots} \right]. \quad (5.35)$$

Proceeding as above we find,

$$\frac{s\mathcal{H}}{z} \frac{dY_{N_i}}{dz} = - \sum_{\{a\}, \{j\}} \langle \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}} \left( \frac{Y_{N_i}}{Y_{N_i}^{\text{eq}}} - 1 \right), \quad (5.36)$$

where the reaction density  $\langle \gamma_{jk \dots}^{N_i ab \dots} \rangle_{\text{MB}}$  is given by (5.32b).

## 5.2 Numerical comparison

To perform the quantitative analysis we need to specify the Yukawa couplings and masses of the right-handed neutrinos. For simplicity we only consider the case of a very heavy third Majorana neutrino  $M_3 \gg M_2 > M_1$ . In that case the contribution from the third right-handed neutrino to the lepton asymmetry generation is completely negligible. In this limit the Yukawa couplings can be expressed in terms of the observed active neutrino masses, the mixing angles and only one complex additional free parameter  $\omega$ . In the Casas-Ibarra parameterisation [155–158] the Yukawa couplings are given by,

$$(h^\dagger h)_{11} = \frac{M_1}{v^2} (m_2 |1 - \omega^2| + m_3 |\omega^2|), \quad (5.37a)$$

$$(h^\dagger h)_{22} = \frac{M_2}{v^2} (m_3 |1 - \omega^2| + m_2 |\omega^2|), \quad (5.37b)$$

$$(h^\dagger h)_{12} = \frac{\sqrt{M_1 M_2}}{v^2} (m_2 \omega \sqrt{1 - \omega^{2*}} - m_3 \omega^* \sqrt{1 - \omega^2}), \quad (5.37c)$$

where  $v \approx 174$  GeV is the Higgs vev and we have assumed normal hierarchy. In this case the physical neutrino masses are given by

$$m_2 = (\Delta m_{sol}^2)^{\frac{1}{2}} \approx 8.71 \cdot 10^{-12} \text{ GeV}, \quad (5.38)$$

$$m_3 = (\Delta m_{sol}^2 + \Delta m_{atm}^2)^{\frac{1}{2}} \approx 5.0 \cdot 10^{-11} \text{ GeV}. \quad (5.39)$$

For illustration we choose the benchmark points  $\omega = \exp(-0.01 \cdot i)$  and  $\omega = \exp(-0.5 \cdot i)$  denoted by BM1 and BM2 in the plots. As masses of the right-handed neutrinos we choose  $M_1 = 10^9$  GeV and  $M_2 = \sqrt{10} M_1$ , respectively. This choice of parameters is such that effects related to the resonant enhancement will be unimportant but contributions from both heavy neutrinos,  $N_1$  and  $N_2$ , can be relevant.

The so-called *washout parameters* play an important role for the generation of the asymme-

try. They are defined by,

$$K_i \equiv \frac{\Gamma_{N_i}}{\mathcal{H}}, \quad (5.40)$$

where  $\mathcal{H}$  is the Hubble rate evaluated at  $T = M_1$ , see (5.4). They are often written in terms of the *effective neutrino mass*  $\tilde{m}_i$  and *equilibrium neutrino mass*  $m^*$  [76],

$$K_i = \frac{\tilde{m}_i}{m^*}, \quad (5.41)$$

with

$$\tilde{m}_i \equiv (h^\dagger h)_{ii} \frac{v^2 M_i}{M_1^2}, \quad (5.42a)$$

$$m^* \equiv \frac{32}{g_w} \sqrt{\frac{\pi^5 g_*}{45}} \frac{M_1^2}{M_{pl}}. \quad (5.42b)$$

If  $K_1 \gg 1$  we are in the so-called *strong washout regime*. Since  $\Gamma_{N_1} \gg \mathcal{H}$  the lightest right-handed neutrino reaches thermal equilibrium at  $T > M_1$  and therefore washes out any pre-existing lepton asymmetry. An asymmetry is then produced by its subsequent decay. In the *weak washout regime*  $K_1 \ll 1$  and the heavy neutrinos never enter thermal equilibrium, meaning that the value of the final asymmetry depends on the initial condition. Note that this classification is too simplistic and not realistic since it does not incorporate flavour effects. A lepton asymmetry can be protected from  $N_1$ -washout when it is stored in the lepton flavour perpendicular to the state which interacts with  $N_1$ . Since we are only considering non-flavoured leptogenesis we are not concerned with the aforementioned subtlety. Our choice of the parameters corresponds to the strong washout regime with the washout parameter given by

$$K_1|_{\text{BM1}} \approx 47, \quad (5.43a)$$

$$K_1|_{\text{BM2}} \approx 55, \quad (5.43b)$$

for the two benchmark sets. In that case the lepton asymmetry is typically produced at  $z \sim \ln K_1 \approx 4$ .

The SM particles acquire thermal mass from the fast SM interactions. In this work we only consider conventional dispersion relation for the fermion fields, see [135, 136, 159–161] for more details about modified dispersion relations. The thermal masses of the SM particles depend on the SM couplings which are itself temperature dependent. The lepton and Higgs masses are given by [162],

$$m_\ell^2 = \left( \frac{3}{16} g_2^2 + \frac{1}{16} g'^2 + \frac{1}{4} \lambda_t^2 + \frac{1}{2} \lambda \right) T^2, \quad (5.44a)$$

$$m_\phi^2 = \left( \frac{3}{32} g_2^2 + \frac{1}{32} g'^2 \right) T^2, \quad (5.44b)$$

where  $g_2$  and  $g'$  are the  $SU(2)_L$  and  $U(1)$  gauge couplings,  $\lambda$  is the Higgs self-coupling and  $\lambda_t$  is the top Yukawa coupling. A very useful approximation for the SM thermal masses reads

[136],

$$m_\ell \approx 0.2T, \quad (5.45a)$$

$$m_\phi \approx m_t \approx m_Q \approx 0.4T. \quad (5.45b)$$

For simplicity we use the approximation (5.45) to estimate the contributions of the Higgs mediated processes. The more involved expressions (5.44) are used for the heavy neutrino decay.

### 5.2.1 Heavy neutrino decay

We focus here on the contribution of the heavy neutrino decay to the reaction densities. The CP-violating and production reaction densities are given by (see (5.19a) and (5.25)),

$$\langle \epsilon_{\ell\phi}^{N_i} \gamma_{\ell\phi}^{N_i} \rangle = \int d\Pi_{qpk}^{N_i\ell\phi} (2\pi)^4 \delta(q-p-k) f_{N_i}^{\text{eq},q} (1 - f_\ell^{\text{eq},p} + f_\phi^{\text{eq},k}) \epsilon_i \left( \Xi_{N_i \leftrightarrow \ell\phi} + \Xi_{N_i \leftrightarrow \bar{\ell}\bar{\phi}} \right), \quad (5.46a)$$

$$\langle \gamma_{\ell\phi}^{N_i} \rangle_P = \int d\Pi_{qpk}^{N_i\ell\phi} (2\pi)^4 \delta(q-p-k) f_{N_i}^{\text{eq},q} (1 - f_\ell^{\text{eq},p} + f_\phi^{\text{eq},k}) \left( \Xi_{N_i \leftrightarrow \ell\phi} + \Xi_{N_i \leftrightarrow \bar{\ell}\bar{\phi}} \right). \quad (5.46b)$$

Here the equilibrium distribution functions  $f_a^{\text{eq}}$  denote Bose-Einstein or Fermi-Dirac distribution functions. The phase-space integral can be further simplified, see appendix B for more details. In the conventional approach the decay amplitudes do not depend on the momenta and the phase-space integration can be performed analytically. The corresponding reaction densities with amplitudes computed in the S-matrix formalism are given by,

$$\langle \epsilon_{\ell\phi}^{N_i} \gamma_{\ell\phi}^{N_i} \rangle_{\text{MB}} = \frac{\epsilon_i}{\pi^2} M_i^2 \Gamma_{N_i} T K_1 \left( \frac{M_i}{T} \right), \quad (5.47a)$$

$$\langle \gamma_{\ell\phi}^{N_i} \rangle_{\text{MB}} = \frac{1}{\pi^2} M_i^2 \Gamma_{N_i} T K_1 \left( \frac{M_i}{T} \right), \quad (5.47b)$$

where  $\epsilon_i$  is the vacuum CP-violating parameter (2.16) and  $K_1$  is the modified Bessel function of the second kind, see appendix B.

In Fig. 5.1 we show the ratio of the reaction densities (5.46) with the one in the conventional approach (5.47) as a function of the inverse temperature  $z$ . In order to identify separately the effects of the thermal masses we plot the reaction densities (5.46) without thermal masses (denoted by  $\langle \cdot \rangle_{m=0}$  in the plot) but with quantum statistical effects taken into account and the full results ( $\langle \cdot \rangle$  in the plot), including the thermal masses and quantum statistics. We plot these reaction densities for the two right-handed neutrinos (the red lines correspond to the lightest right-handed neutrino  $N_1$  and the green ones to  $N_2$ ). We observe as expected that both ratios (with or without thermal masses) approach unity at low temperature (high  $z$ ). When the thermal masses are neglected the CP-violating and production reaction densities grow with the temperature. The inclusion of the thermal masses leads to a suppression of the available phase-space, which reduces the reaction densities. At higher temperature the increase due to the thermal corrections are overcompensated by the shrinking of the phase-space, and the ratios decrease and reach zero at a certain temperature. This temperature can be extracted from the equation  $M_i = m_\ell(T) + m_\phi(T)$ . The asymmetry is typically produced at  $T \lesssim M_1$ . In that range of temperature we observe an enhancement of the CP-violating and production reaction densities up to 20%.

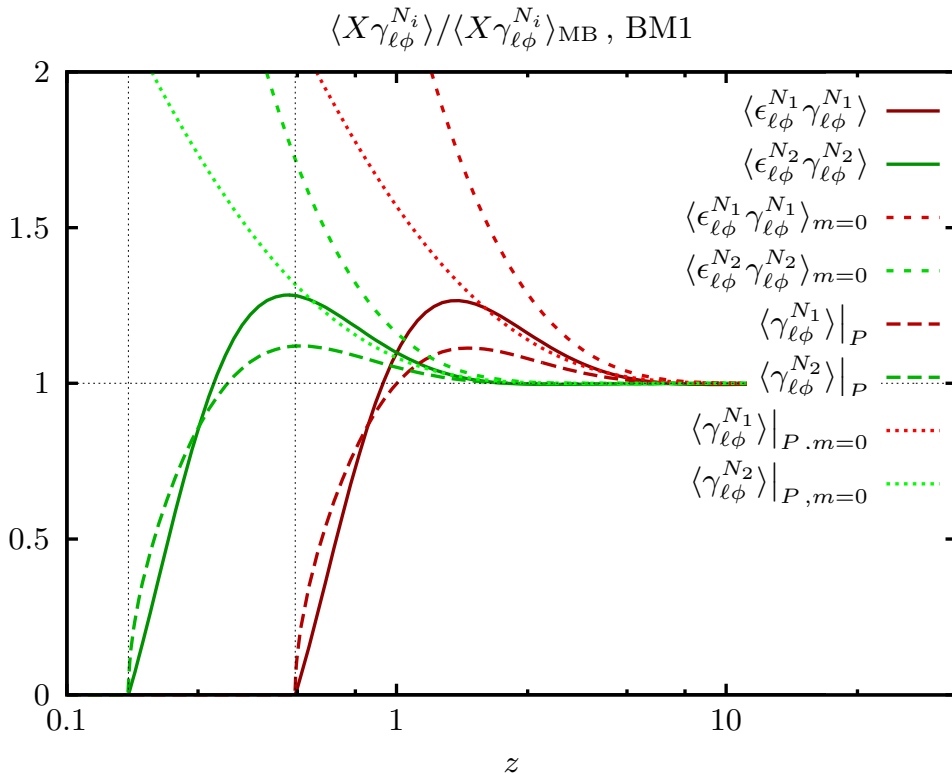


Figure 5.1: CP-violating and production reaction densities for the two heavy neutrinos normalised by the conventional ones. The red lines correspond to the lightest right-handed neutrino  $N_1$  and the green ones to  $N_2$ . Plotted are the CP-violating reaction densities with thermal masses (solid lines) and without thermal masses (short dashed lines), the production reaction densities with thermal mass (long dashed lines) and without thermal masses (dotted lines). We only consider the self-energy contribution to the CP-violating parameter. Benchmark set of parameters: BM1.

The (inverse) decay of the heavy neutrino also contributes to the washout of the asymmetry. The washout reaction density is given by (see (5.19b)),

$$\langle \gamma_{\ell\phi}^{N_i} \rangle_W = \int d\Pi_{qp k}^{N_i \ell\phi} (2\pi)^4 \delta(q - p - k) (1 - f_\ell^{\text{eq},p} + f_\phi^{\text{eq},k}) (1 - f_{N_i}^{\text{eq},q}) f_{N_i}^{\text{eq},q} \times \left( \Xi_{N_i \leftrightarrow \ell\phi} + \Xi_{N_i \leftrightarrow \bar{\ell}\bar{\phi}} \right). \quad (5.48)$$

This contribution is analysed in the next subsection together with the  $|\Delta L| = 2$  scattering processes since they both wash out the asymmetry.

### 5.2.2 $|\Delta L| = 2$ scattering processes

At  $\mathcal{O}(h^4)$  two different processes contribute to the washout of the asymmetry, namely the heavy neutrino (inverse) decay and the scattering processes mediated by the right-handed neutrinos. The latter violate lepton number by two units but is CP-conserving at  $\mathcal{O}(h^4)$ . Therefore they only contribute to the washout term. The reaction densities for the  $|\Delta L| = 2$  scattering

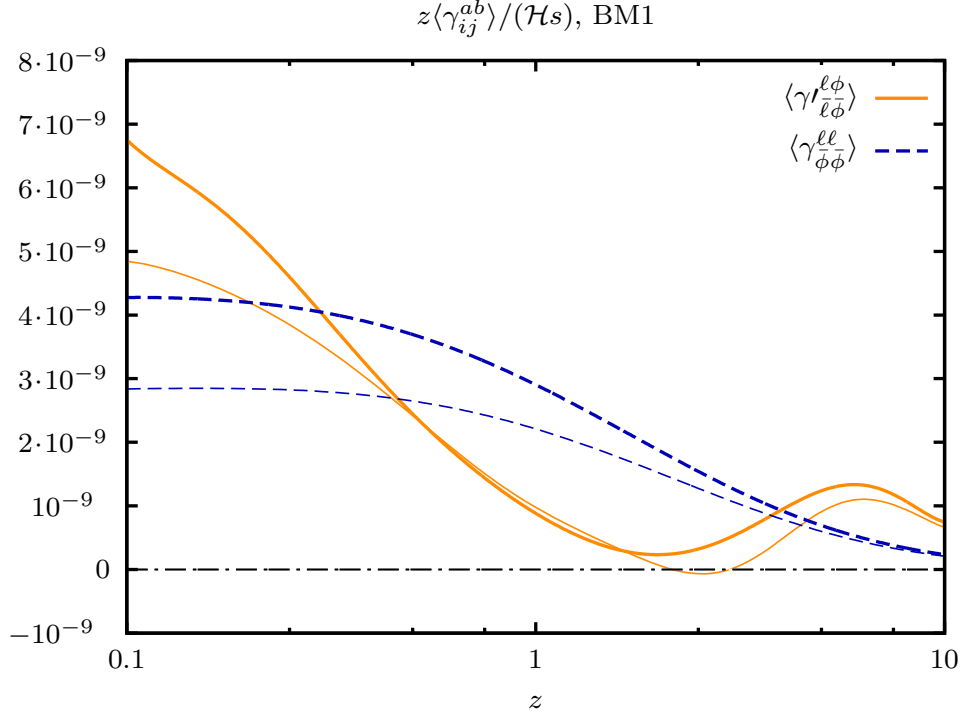


Figure 5.2: Washout reaction densities for the scattering processes  $\ell\phi \leftrightarrow \bar{l}\bar{\phi}$  (orange lines) and  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$  (blue lines). The thick lines correspond to the reaction densities with quantum statistical factors, and the thin line to the one with Maxwell-Boltzmann statistics. The thermal masses are neglected. Benchmark set of parameters: BM1.

processes read (see (5.23)),

$$\begin{aligned} \langle \gamma_{\bar{l}\bar{\phi}}^{\ell\phi} \rangle &= \int d\Pi_{p_1 k_1 p_2 k_2}^{\ell\phi\ell\phi} (2\pi)^4 \delta(p_1 + k_1 - p_2 - k_2) \\ &\quad \times f_\ell^{\text{eq}, p_1} f_\phi^{\text{eq}, k_1} (1 - f_\ell^{\text{eq}, p_2}) (1 + f_\phi^{\text{eq}, k_2}) \Xi_{\bar{l}\bar{\phi} \leftrightarrow \ell\phi}, \end{aligned} \quad (5.49a)$$

$$\begin{aligned} \langle \gamma_{\bar{\phi}\bar{\phi}}^{\ell\ell} \rangle &= \int d\Pi_{p_1 p_2 k_1 k_2}^{\ell\ell\phi\phi} (2\pi)^4 \delta(p_1 + p_2 - k_1 - k_2) \\ &\quad \times f_\ell^{\text{eq}, p_1} f_\ell^{\text{eq}, p_2} (1 + f_\phi^{\text{eq}, k_1}) (1 + f_\phi^{\text{eq}, k_2}) \left( \Xi_{\bar{\ell}\bar{\ell} \leftrightarrow \phi\phi} + \Xi_{\bar{\phi}\bar{\phi} \leftrightarrow \ell\ell} \right), \end{aligned} \quad (5.49b)$$

with the amplitudes given by (4.86). In (5.49a) we have written the reaction density with a prime to underline the fact that the amplitude is RIS-subtracted by construction. The corresponding reaction densities in the S-matrix formalism are given by,

$$\langle \gamma_{\bar{l}\bar{\phi}}^{\ell\phi} \rangle_{\text{MB}} = \frac{T}{64\pi^4} \int_{s_{\min}}^{\infty} ds \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right) \hat{\sigma}_{\bar{l}\bar{\phi}}^{\ell\phi}(s), \quad (5.50a)$$

$$\langle \gamma_{\bar{\phi}\bar{\phi}}^{\ell\ell} \rangle_{\text{MB}} = \frac{T}{64\pi^4} \int_{s_{\min}}^{\infty} ds \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right) \hat{\sigma}_{\bar{\phi}\bar{\phi}}^{\ell\ell}(s), \quad (5.50b)$$

where  $\hat{\sigma}_{\bar{\ell}\bar{\phi}}^{\ell\phi}(s)$  and  $\hat{\sigma}_{\bar{\phi}\bar{\phi}}^{\ell\ell}(s)$  are the so-called *reduced cross-section* (the integration limits can be found in appendix B),

$$\hat{\sigma}_{ij}^{ab}(s) \equiv \frac{1}{8\pi} \int_0^{2\pi} \frac{d\varphi_{ai}}{2\pi} \int_{t^-}^{t^+} \frac{dt}{s} (\Xi_{ab \rightarrow ij} + \Xi_{\bar{a}\bar{b} \rightarrow \bar{i}\bar{j}}). \quad (5.51)$$

Explicitly, they read,

$$\begin{aligned} \hat{\sigma}_{\bar{\ell}\bar{\phi}}^{\ell\phi}(s) = & \frac{1}{4\pi x} \sum_{i,j} \text{Re}[(h^\dagger h)_{ij}^2] \sqrt{a_i a_j} \left\{ x^2 \frac{(x-a_i)(x-a_j) + (1-2\delta_{ij})a_i a_j c_i c_j}{[(x-a_i)^2 + (a_i c_i)^2][(x-a_j)^2 + (a_j c_j)^2]} \right. \\ & + 2 \frac{x+a_i}{a_j - a_i} \ln\left(\frac{x+a_i}{a_i}\right) + \frac{x-a_i}{(x-a_i)^2 + (a_i c_i)^2} \left[ x - (x+a_j) \ln\left(\frac{x+a_j}{a_j}\right) \right] \\ & \left. + 2 \frac{x+a_j}{a_i - a_j} \ln\left(\frac{x+a_j}{a_j}\right) + \frac{x-a_j}{(x-a_j)^2 + (a_j c_j)^2} \left[ x - (x+a_i) \ln\left(\frac{x+a_i}{a_i}\right) \right] \right\}, \end{aligned} \quad (5.52a)$$

$$\begin{aligned} \hat{\sigma}_{\bar{\phi}\bar{\phi}}^{\ell\ell}(s) = & \frac{1}{2\pi} \sum_{i,j} \text{Re}[(h^\dagger h)_{ij}^2] \sqrt{a_i a_j} \left\{ \frac{1}{a_i - a_j} \ln\left(\frac{a_i(x+a_j)}{a_j(x+a_i)}\right) \right. \\ & \left. + \frac{1}{2} \frac{1}{x+a_i+a_j} \ln\left(\frac{(x+a_i)(x+a_j)}{a_i a_j}\right) \right\}. \end{aligned} \quad (5.52b)$$

In (5.52) we have introduced the dimensionless quantities  $x \equiv s/M_1^2$ ,  $a_i \equiv M_i^2/M_1^2$  and  $c_i \equiv \Gamma_{N_i}/M_i$ . The case  $i = j$  is included in the expressions (5.52) in the limiting sense  $a_j \rightarrow a_i$ .

In Figs. 5.2 and 5.3 we plot the quantities  $z \langle \gamma_{\bar{\ell}\bar{\phi}}^{\ell\phi} \rangle / (\mathcal{H}s)$  and  $z \langle \gamma_{\bar{\phi}\bar{\phi}}^{\ell\ell} \rangle / (\mathcal{H}s)$  as functions of the inverse temperature  $z$  for the benchmark points 1 and 2. The thick lines correspond to the reaction density computed with quantum statistical factors, and the thin lines with Maxwell-Boltzmann statistics. In both cases thermal masses are neglected. We observe that for the scattering process  $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$  the reaction density with quantum statistics (thick lines) is negative in the temperature range  $2 \lesssim z \lesssim 3$  for BM1 and  $0.5 \lesssim z \lesssim 1$  for BM2. This behaviour originates from the fact that the effective amplitude  $\Xi_{\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}}$  (4.92) is *negative* in the vicinity of the mass-shell of the intermediate neutrino. Since the amplitude is RIS-subtracted by construction it does not correspond to a transition *probability* and is therefore not constrained to be positive. For BM2 point the reaction density with Maxwell-Boltzmann statistics is also negative for some temperature range, whereas for BM1 it is positive at all temperatures. We would like to stress that this is merely a numerical coincidence. The vacuum effective amplitude  $\Xi'_{\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}}$  is also negative in the vicinity of the intermediate mass-shell since it is RIS-subtracted. Therefore the reaction density, which corresponds to the “integrated” amplitude weighted by the distribution functions, can be positive or negative depending on the precise numerical value of the amplitude. This is the reason why the reaction densities for the two choices of parameters show a different behaviour. Note that a similar behaviour has been observed in [31]. The effective amplitude for the other scattering process,  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$ , is always positive since it is not RIS-subtracted. At high temperature we observe a difference up to 50% between the reaction densities with quantum statistics and with Maxwell-Boltzmann statistics. As expected, the two results coincide at low temperature.

Thermal masses of the leptons and Higgs have been neglected in Figs. 5.2 and 5.3. The effects of thermal masses can be estimated from Fig. 5.4 where we plot the reaction density  $\langle \gamma_{\bar{\phi}\bar{\phi}}^{\ell\ell} \rangle$  with

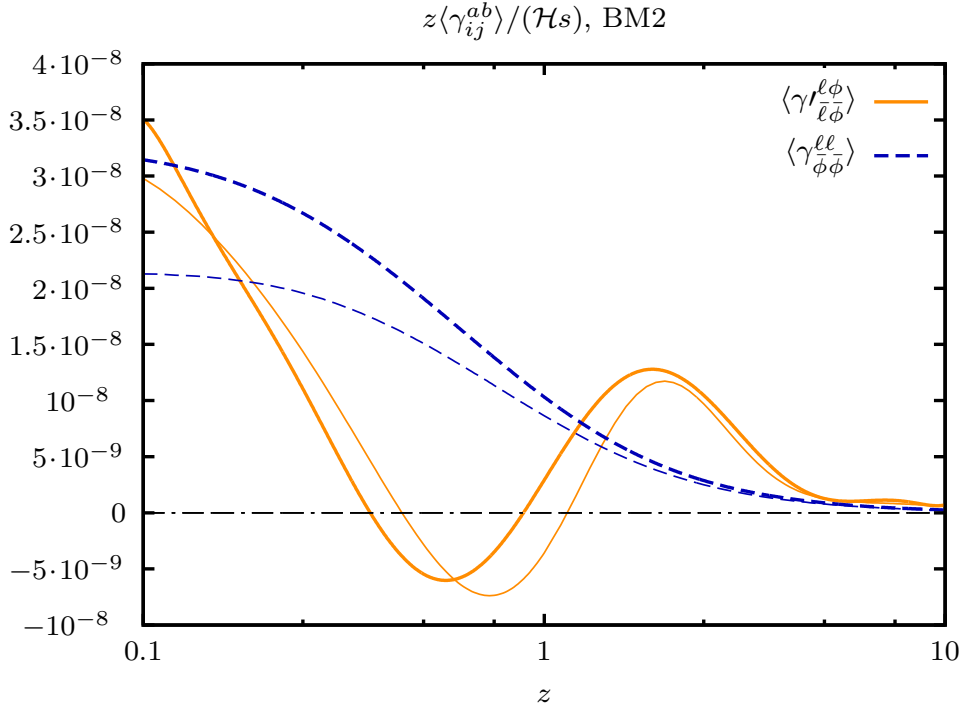


Figure 5.3: Washout reaction densities for the scattering processes  $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$  (orange lines) and  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$  (blue lines). The thick lines correspond the reaction densities with quantum statistical factors, and the thin lines to the ones with Maxwell-Boltzmann statistics. The thermal masses are neglected. Benchmark set of parameters: BM2.

thermal mass normalised by the conventional one (without thermal mass and with Maxwell-Boltzmann statistics) for the two benchmark points. We observe that the decrease of the reaction density due to the thermal masses is almost completely compensated by the increase due to quantum statistics. The full computation, with quantum statistics and thermal mass, differs from the conventional one, which uses Maxwell-Boltzmann statistics without thermal masses, by a factor of  $\sim 5\%$  at high temperature.

The  $|\Delta L| = 2$  scattering processes are not the only processes responsible for the washout of the asymmetry. Decay and inverse decay of heavy neutrinos also contribute to the washout with the reaction density given by (5.48). The total washout term reads,

$$\frac{s\mathcal{H}}{z} \frac{dY_L}{dz} = \dots - \frac{Y_L}{2Y_\ell^{eq}} c_\ell (1 + c_{\phi\ell}) \left( \sum_i \langle \gamma_{\ell\phi}^{N_i} \rangle|_W + 4\langle \gamma_{\ell\bar{\phi}}^{\ell\phi} \rangle + 4\langle \gamma_{\bar{\phi}\bar{\phi}}^{\ell\ell} \rangle \right). \quad (5.53)$$

The factors of 4 in front of the scattering reaction densities come from the different combinations of chemical potentials,

$$c_{\bar{\phi}\bar{\phi} \leftrightarrow \ell\ell} = c_{\bar{\ell}\bar{\phi} \leftrightarrow \ell\phi} = 2c_{\ell\phi} = 2(1 + c_{\phi\ell}), \quad (5.54)$$

and from the factor of 2 in front of the scattering amplitudes, see (4.81). To compare the relative importance of each terms in (5.53) we plot them separately in Figs. 5.5 and 5.6 as a function of the inverse temperature  $z$  for the benchmark points BM1 and BM2. We observe



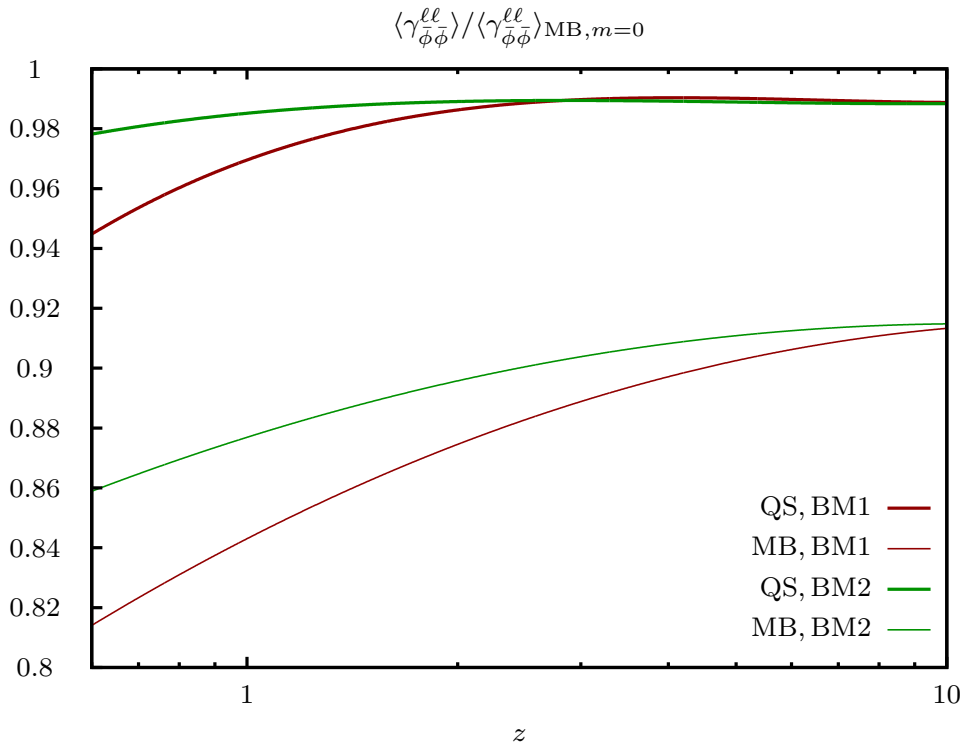


Figure 5.4: Washout reaction densities for the scattering processes  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$  with thermal masses normalized by the one without thermal masses and with Maxwell-Boltzmann statistics. The thick lines correspond to the reaction densities with quantum statistical (QS) factors, and the thin line to the one with Maxwell-Boltzmann (MB) statistics. Red lines: BM1. Green lines: BM2.

that the reaction densities for the decay of  $N_1$  and  $N_2$  (red and green lines) are larger than the one for the scattering processes (yellow and blue lines) by a few orders of magnitude in the whole range of temperature. As expected, the total washout term (black dashed-dotted line) for both benchmark points is always *positive*, i.e. it only depletes the asymmetry (note the overall minus sign in (5.53) in front of the washout term). Note that the normalisation of the reaction densities in Figs. 5.5 and 5.6, which is temperature independent, is different from the one in Figs. 5.2 and 5.3.

### 5.2.3 Higgs decay

At high temperature the process  $N_i \leftrightarrow \ell\phi$  becomes kinematically forbidden due to the large thermal masses of the lepton and Higgs. At even higher temperature, when  $m_\phi > M_i + m_\ell$ , the Higgs can decay into an antilepton and a heavy neutrino. The temperature at which the Higgs decay becomes kinematically allowed can be estimated from the relation (5.45) and is approximately given by  $z \approx 0.2$  for the lightest right-handed neutrino. The reaction densities

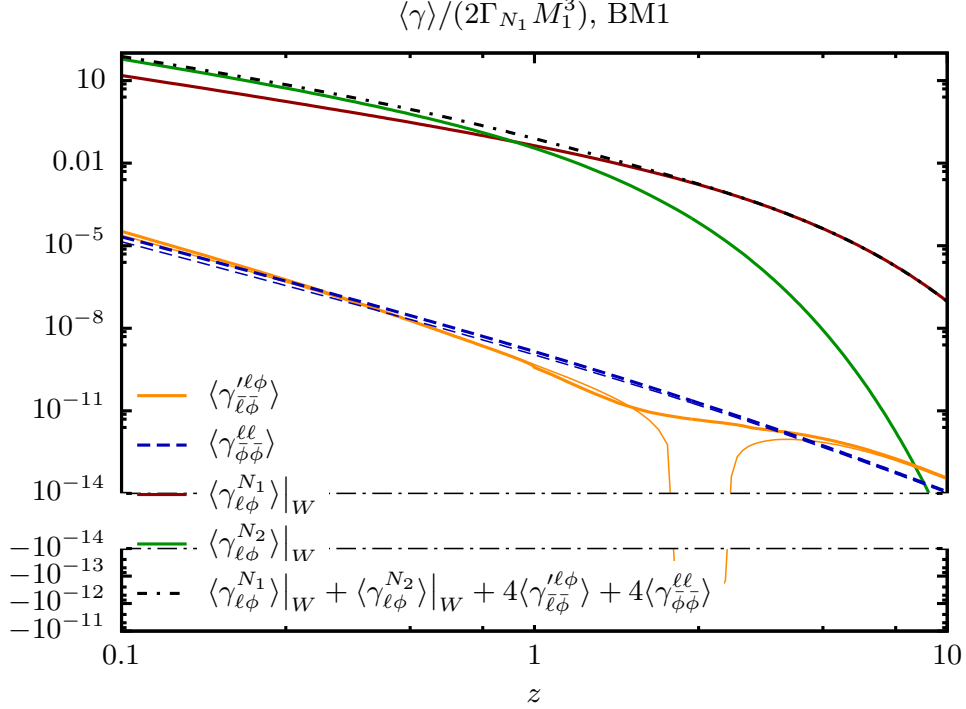


Figure 5.5: Comparison of the washout reaction densities for the scattering processes  $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$  (orange lines),  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$  (blue dash lines) and for the  $N_1$  (green line) and  $N_2$  (red line) decays. For the scatterings the thick (thin) lines correspond to the reaction densities computed with quantum (Maxwell-Boltzmann) statistics. Thermal masses are neglected. The total washout reaction density (black dash-dotted line) is well approximated by the decay reaction densities. Note that the normalisation is different from the one in Figs. 5.2 and 5.3. Benchmark set of parameters: BM1.

for the Higgs decay are obtained from (5.19a), (5.19b) and (5.25),

$$\langle \epsilon_{\bar{\phi}}^{N_i\ell} \gamma_{\bar{\phi}}^{N_i\ell} \rangle = \int d\Pi_{qpk}^{N_i\ell\phi} (2\pi)^4 \delta(k - q - p) f_{N_i}^{\text{eq},q} (f_{\ell}^{\text{eq},p} + f_{\phi}^{\text{eq},k}) \epsilon_{\phi,i} (\Xi_{N_i\ell\leftrightarrow\bar{\phi}} + \Xi_{N_i\bar{\ell}\leftrightarrow\phi}), \quad (5.55a)$$

$$\begin{aligned} \langle \gamma_{\bar{\phi}}^{N_i\ell} \rangle|_W &= \int d\Pi_{qpk}^{N_i\ell\phi} (2\pi)^4 \delta(k - q - p) f_{N_i}^{\text{eq},q} (1 - f_{N_i}^{\text{eq},q}) (f_{\ell}^{\text{eq},p} + f_{\phi}^{\text{eq},k}) \\ &\quad \times (\Xi_{N_i\ell\leftrightarrow\bar{\phi}} + \Xi_{N_i\bar{\ell}\leftrightarrow\phi}), \end{aligned} \quad (5.55b)$$

$$\langle \gamma_{\bar{\phi}}^{N_i\ell} \rangle|_P = \int d\Pi_{qpk}^{N_i\ell\phi} (2\pi)^4 \delta(k - q - p) f_{N_i}^{\text{eq},q} (f_{\ell}^{\text{eq},p} + f_{\phi}^{\text{eq},k}) (\Xi_{N_i\ell\leftrightarrow\bar{\phi}} + \Xi_{N_i\bar{\ell}\leftrightarrow\phi}), \quad (5.55c)$$

where  $\epsilon_{\phi,i}$  is the CP-violating parameter for the Higgs decay, see (4.104) and (4.108). As pointed out above (4.104), the CP-violating parameters for the Higgs decay and for the heavy neutrino decay have opposite sign. However this sign difference is compensated by the minus sign in the rate equation for the Higgs decay, see (5.18). Therefore, the asymmetries produced by both processes have the same sign. In order to compare the Higgs and heavy neutrino

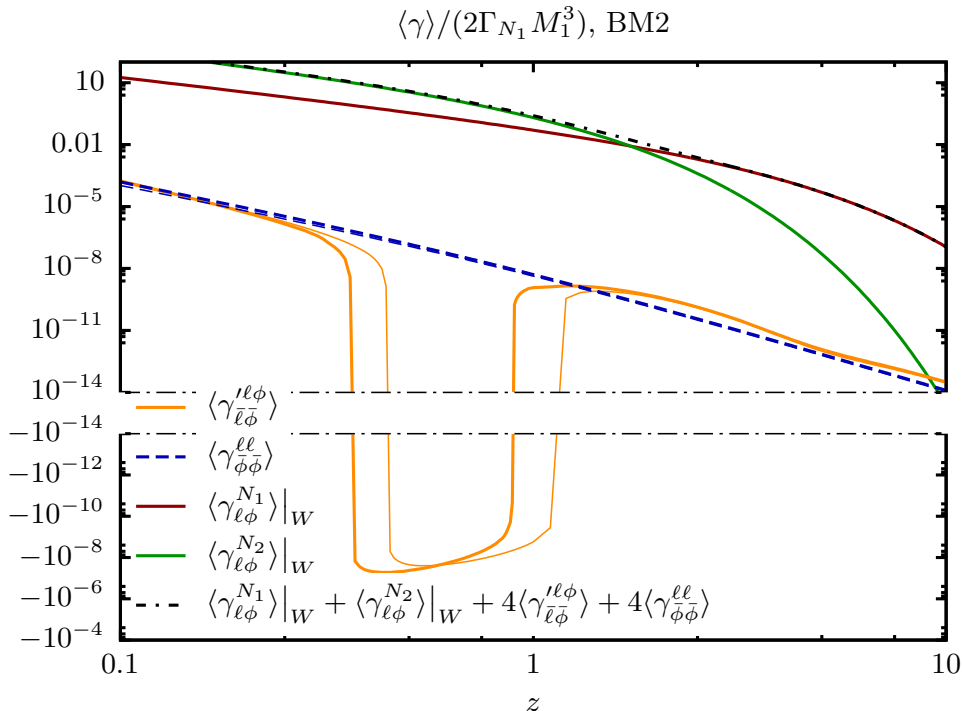


Figure 5.6: Comparison of the washout reaction densities for the scattering processes  $\ell\phi \leftrightarrow \bar{\ell}\bar{\phi}$  (orange lines),  $\ell\ell \leftrightarrow \bar{\phi}\bar{\phi}$  (blue dash lines) and for the  $N_1$  (green line) and  $N_2$  (red line) decays. For the scatterings the thick (thin) lines correspond to the reaction densities computed with quantum (Maxwell-Boltzmann) statistics. Thermal masses are neglected. The total washout reaction density (black dash-dotted line) is well approximated by the decay reaction densities. Note that the normalisation is different from the one in Figs. 5.2 and 5.3. Benchmark set of parameters: BM2.

decays we define the *averaged* CP-violating parameters by,

$$\langle \epsilon_i \rangle \equiv \begin{cases} \frac{\langle \epsilon_{\ell\phi}^{N_i} \gamma_{\ell\phi}^{N_i} \rangle}{\langle \gamma_{\ell\phi}^{N_i} \rangle_P}, & \text{when the heavy neutrino decay is possible,} \\ \frac{\langle \epsilon_{\bar{\phi}}^{N_i\ell} \gamma_{\bar{\phi}}^{N_i\ell} \rangle}{\langle \gamma_{\bar{\phi}}^{N_i\ell} \rangle_P}, & \text{when the Higgs decay is possible.} \end{cases} \quad (5.56)$$

The average CP-violating parameter (5.56) is equal to the CP-violating parameter if the latter is momentum independent. In particular, in the conventional approach this is the case for the heavy neutrino decay,

$$\langle \epsilon_i \rangle_{\text{MB}} \equiv \frac{\langle \epsilon_{\ell\phi}^{N_i} \gamma_{\ell\phi}^{N_i} \rangle_{\text{MB}}}{\langle \gamma_{\ell\phi}^{N_i} \rangle_{\text{MB}}} = \epsilon_{\ell\phi}^{N_i}. \quad (5.57)$$

We plot in Figs. 5.7 and 5.8 the average CP-violating parameters  $\langle \epsilon_1 \rangle$  (red dash line) and  $\langle \epsilon_2 \rangle$  (green solid line) as well as the vacuum ones (thin lines) as a function of the inverse temperature  $z$  for the two benchmark points. Note that we plot only the self-energy CP-

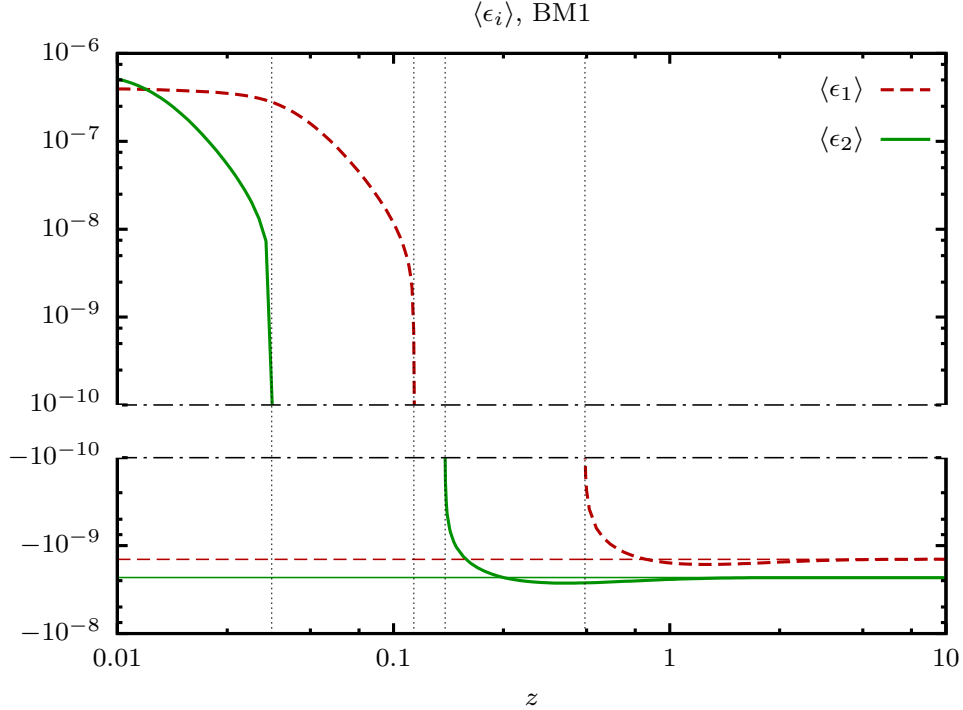


Figure 5.7: Averaged CP-violating parameter as given in (5.56) for  $N_1$  (red thick line) and  $N_2$  (green thick line). The corresponding thin lines are the vacuum CP-violating parameters. Only the self-energy contribution to the CP-violating parameters are plotted. Benchmark set of parameters: BM1.

violating parameters. At low temperature the average CP-violating parameter converges to the conventional one. At higher temperature the heavy neutrino decay is replaced the Higgs decay, and the averaged CP-violating parameter changes sign. We observe that averaged CP-violating parameter for the Higgs decay is a few orders of magnitude larger than the one for the heavy neutrino decay.

#### 5.2.4 Higgs mediated processes

We focus now on the Higgs mediated processes. In order to simplify the numerical analysis we consider here only the case of a hierarchical mass spectrum, i.e.  $M_1 \ll M_2$ . In that case the self-energy CP-violating parameter (4.133) is given by,

$$\epsilon_{t,i}^{(S)} \approx \epsilon_{t,i}^{vac,(S)} \frac{pL_\rho}{pq}, \quad (5.58)$$

where  $\epsilon_{t,i}^{vac,(S)}$  is the vacuum CP-violating parameter. In this subsection we only take into account the contribution to  $L_\rho$  from the right-handed neutrino decay, see subsection 5.2.5 for the inclusion of the Higgs mediated processes. The reaction densities for the Higgs mediated

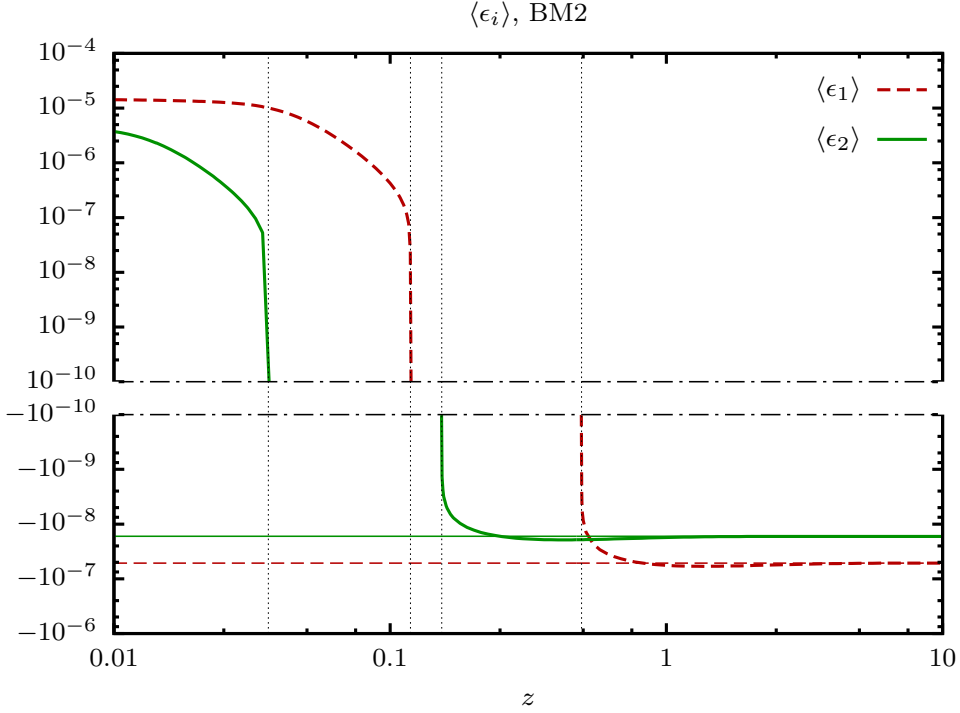


Figure 5.8: Averaged CP-violating parameter as given in (5.56) for  $N_1$  (red thick line) and  $N_2$  (green thick line). The corresponding thin lines are the vacuum CP-violating parameters. Only the self-energy contribution to the CP-violating parameters are plotted. Benchmark set of parameters: BM2.

scattering processes are given by (see (5.19a), (5.19b) and (5.25)),

$$\langle \epsilon_{\ell\bar{t}}^{N_i Q} \gamma_{\ell\bar{t}}^{N_i Q} \rangle = \int d\Pi_{N_i \ell\bar{t} Q}^{qpp\bar{t}pQ} (2\pi)^4 \delta(q + p - p_t - p_Q) \epsilon_{t,i} \left( \Xi_{N_i Q \leftrightarrow \ell\bar{t}} + \Xi_{N_i \bar{Q} \leftrightarrow \ell\bar{t}} \right) \quad (5.59a)$$

$$\times f_{N_i}^{\text{eq},q} [f_Q^{\text{eq},pQ} (1 - f_\ell^{\text{eq},p}) (1 - f_t^{\text{eq},p_t}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} (1 - f_Q^{\text{eq},pQ})],$$

$$\langle \gamma_{\ell\bar{t}}^{N_i Q} \rangle|_W = \int d\Pi_{N_i \ell\bar{t} Q}^{qpp\bar{t}pQ} (2\pi)^4 \delta(q + p - p_t - p_Q) \left( \Xi_{N_i Q \leftrightarrow \ell\bar{t}} + \Xi_{N_i \bar{Q} \leftrightarrow \ell\bar{t}} \right) \quad (5.59b)$$

$$\times f_{N_i}^{\text{eq},q} (1 - f_{N_i}^{\text{eq},q}) [f_Q^{\text{eq},pQ} (1 - f_\ell^{\text{eq},p}) (1 - f_t^{\text{eq},p_t}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} (1 - f_Q^{\text{eq},pQ})],$$

$$\langle \gamma_{\ell\bar{t}}^{N_i Q} \rangle|_P = \int d\Pi_{N_i \ell\bar{t} Q}^{qpp\bar{t}pQ} (2\pi)^4 \delta(q + p - p_t - p_Q) \left( \Xi_{N_i Q \leftrightarrow \ell\bar{t}} + \Xi_{N_i \bar{Q} \leftrightarrow \ell\bar{t}} \right) \quad (5.59c)$$

$$\times f_{N_i}^{\text{eq},q} [f_Q^{\text{eq},pQ} (1 - f_\ell^{\text{eq},p}) (1 - f_t^{\text{eq},p_t}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} (1 - f_Q^{\text{eq},pQ})],$$

and similar expressions for the scattering processes  $N_i \bar{t} \leftrightarrow \ell \bar{Q}$  and  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$ . Numerically the reaction densities for  $N_i Q \leftrightarrow \ell\bar{t}$  and  $N_i \bar{t} \leftrightarrow \ell \bar{Q}$  are equal since the thermal masses of the top quark and quark doublet are both given by  $m_t \approx m_Q \approx 0.4T$ . The three-body decay reaction

densities read,

$$\langle \epsilon_{\ell\bar{Q}}^{N_i} \gamma_{\ell\bar{Q}}^{N_i} \rangle = \int d\Pi_{N_i\ell\bar{Q}}^{pp_t p_Q} (2\pi)^4 \delta(q-p-p_t-p_Q) \epsilon_{t,i} \left( \Xi_{N_i \leftrightarrow \ell\bar{Q}} + \Xi_{N_i \leftrightarrow \bar{\ell}iQ} \right) \times f_{N_i}^{\text{eq},q} \left[ (1-f_\ell^{\text{eq},p})(1-f_t^{\text{eq},p_t})(1-f_Q^{\text{eq},p_Q}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} f_Q^{\text{eq},p_Q} \right], \quad (5.60a)$$

$$\langle \gamma_{\ell\bar{Q}}^{N_i} \rangle_W = \int d\Pi_{N_i\ell\bar{Q}}^{pp_t p_Q} (2\pi)^4 \delta(q-p-p_t-p_Q) \left( \Xi_{N_i \leftrightarrow \ell\bar{Q}} + \Xi_{N_i \leftrightarrow \bar{\ell}iQ} \right) \times f_{N_i}^{\text{eq},q} (1-f_{N_i}^{\text{eq},q}) \left[ (1-f_\ell^{\text{eq},p})(1-f_t^{\text{eq},p_t})(1-f_Q^{\text{eq},p_Q}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} f_Q^{\text{eq},p_Q} \right], \quad (5.60b)$$

$$\langle \gamma_{\ell\bar{Q}}^{N_i} \rangle_P = \int d\Pi_{N_i\ell\bar{Q}}^{pp_t p_Q} (2\pi)^4 \delta(q-p-p_t-p_Q) \left( \Xi_{N_i \leftrightarrow \ell\bar{Q}} + \Xi_{N_i \leftrightarrow \bar{\ell}iQ} \right) \times f_{N_i}^{\text{eq},q} \left[ (1-f_\ell^{\text{eq},p})(1-f_t^{\text{eq},p_t})(1-f_Q^{\text{eq},p_Q}) + f_\ell^{\text{eq},p} f_t^{\text{eq},p_t} f_Q^{\text{eq},p_Q} \right]. \quad (5.60c)$$

At leading order the thermal effects are negligible in the total effective amplitudes  $(\Xi_{N_i Q \leftrightarrow \ell t} + \Xi_{N_i \bar{Q} \leftrightarrow \bar{\ell} i})$  and  $(\Xi_{N_i \leftrightarrow \ell t \bar{Q}} + \Xi_{N_i \leftrightarrow \bar{\ell} i Q})$ . The thermal corrections only appear in the intermediate Higgs propagator,  $\Delta_{R+A}^2$ . In the kinematic region of interest the intermediate Higgs is never close to its mass-shell. Therefore the effective width of the Higgs is negligible and the thermal propagator can be safely replaced by the vacuum (time-ordered) one,  $\Delta_{R+A}^2 \rightarrow \Delta_{\phi,F}^2$ . Note that the above integrals can be further simplified, see appendix B for more details.

If one uses Maxwell-Boltzmann statistics then the reaction densities for the Higgs mediated scattering processes take the a form,

$$\langle \gamma_{jk}^{N_i a} \rangle_{\text{MB}} = \frac{T}{64\pi^4} \int_{s_{\min}}^{\infty} ds K_1\left(\frac{\sqrt{s}}{T}\right) \hat{\sigma}_{jk}^{N_i a}(s), \quad (5.61)$$

where  $s_{\min} = \max((M_i + m_a)^2, (m_j + m_k)^2)$  and  $\hat{\sigma}_{jk}^{N_i a}(s)$  is the reduced cross-section,

$$\hat{\sigma}_{jk}^{N_i a}(s) = \frac{1}{8\pi} \int_0^{2\pi} \frac{d\varphi_{ai}}{2\pi} \int_{t^-}^{t^+} \frac{dt}{s} (\Xi_{N_i a \leftrightarrow jk} + \Xi_{N_i \bar{a} \leftrightarrow \bar{j}\bar{k}}). \quad (5.62)$$

The integration limits can be found in appendix B. Neglecting the thermal masses of the initial and final SM particles but retaining the one of the intermediate Higgs (which must be kept to avoid infrared divergences) we obtain the standard expressions (see, e.g. [163, 164]) for the reduced cross sections,

$$\hat{\sigma}_{\ell\bar{Q}}^{N_i Q} = \sigma_{\ell\bar{Q}}^{N_i \bar{\ell}} = \frac{g_w g_q (h^\dagger h)_{ii} |\lambda_t|^2 \frac{x-a_i}{x}}{4\pi} \times \left[ \frac{x-2a_i+2a_\phi}{x-a_i+a_\phi} + \frac{a_i-2a_\phi}{x-a_i} \ln\left(\frac{x-a_i+a_\phi}{a_\phi}\right) \right], \quad (5.63a)$$

$$\hat{\sigma}_{\bar{Q}t}^{N_i \bar{\ell}} = \frac{g_w g_q (h^\dagger h)_{ii} |\lambda_t|^2 \frac{(x-a_i)^2}{(x-a_\phi)^2}}{4\pi}. \quad (5.63b)$$

We plot in Fig. 5.9 the washout (thick lines) and production (thin lines) reaction densities for the scattering processes normalised by the conventional one (without thermal masses and with Maxwell-Boltzmann statistics). The blues lines correspond to the scattering  $N_i Q \leftrightarrow \ell t$

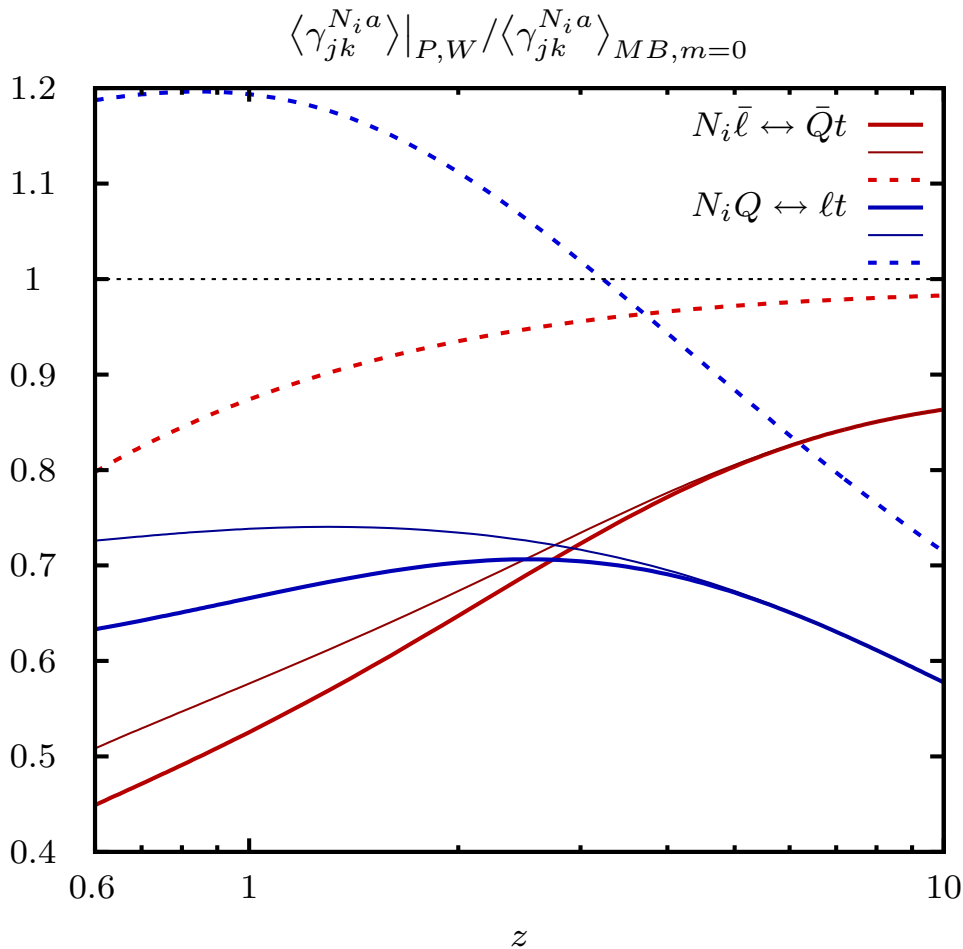


Figure 5.9: Production and washout reaction densities with thermal masses and Maxwell-Boltzmann statistics (dashed lines) and with thermal masses and quantum statistics (solid lines) normalised by the conventional ones (without thermal masses and with Maxwell-Boltzmann statistics). The blue lines correspond to the  $N_i Q \leftrightarrow \ell t$  scattering and the red ones to  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$ . The reaction densities for the process  $N_i \bar{\ell} \leftrightarrow \ell \bar{Q}$  are equal to the ones for the process  $N_i Q \leftrightarrow \ell t$ . Thick lines: washout reaction density (5.59b). Thin lines: production reaction density (5.59c).

and the red ones to  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$ . Interestingly enough, the washout and production reaction densities for the process  $N_i Q \leftrightarrow \ell t$  do not approach the conventional one at low temperature. This counterintuitive result can be understood as follows. The reaction density can be roughly approximated by evaluating them at the thermally averaged momenta. The ratio of reaction densities is then simply given by,

$$\frac{\langle \gamma_{\ell t}^{N_i Q} \rangle_{MB,m \neq 0}}{\langle \gamma_{\ell t}^{N_i Q} \rangle_{MB,m=0}} \sim \frac{e^{-\langle E_{N_i} \rangle/T} e^{-\langle E_Q \rangle/T}}{e^{-\langle E_{N_i} \rangle/T} e^{-\langle |\vec{p}_Q| \rangle/T}} \sim e^{-\sqrt{1 + \frac{m_Q^2}{\langle |\vec{p}_Q| \rangle^2}}}. \quad (5.64)$$

The thermally averaged momentum is proportional to the temperature  $\langle p_Q \rangle \sim T$ . Since

$m_Q \approx 0.4T$  the ratio (5.64) is independent of the  $T$  and does not approach unity at low temperature. For the process  $N_i\bar{\ell} \leftrightarrow t\bar{Q}$  we obtain a similar result, but with  $m_Q$  replaced by  $m_\ell$ . Since  $m_\ell \approx 0.2T < m_Q$  the Boltzmann suppression is less effective and the ratio  $\langle \gamma_{t\bar{Q}}^{N_i\bar{\ell}} \rangle_{\text{MB}, m \neq 0} / \langle \gamma_{t\bar{Q}}^{N_i\bar{\ell}} \rangle_{\text{MB}, m=0}$  gets closer to unity at low temperature. Note that for a (relatively) strong washout regime the asymmetry is typically produced at  $z \lesssim 10$ . Therefore the low temperature behaviour of the reaction densities does not affect significantly the production of the lepton asymmetry.

The dashed lines in Fig. 5.9 correspond to the reactions densities computed taking into account the thermal masses and Maxwell-Boltzmann statistics. The inclusion of thermal masses leads to an enhancement of the reaction density for the  $t$ -channel scattering ( $N_i Q \leftrightarrow \ell t$ ) at high temperature. For the  $s$ -channel scattering ( $N_i\bar{\ell} \leftrightarrow t\bar{Q}$ ) thermal masses decrease the reaction density in the whole range of temperature. Since the particles involved in the scattering processes are all fermions the reaction density with quantum statistics are smaller than with Maxwell-Boltzmann statistics due to Pauli blocking.

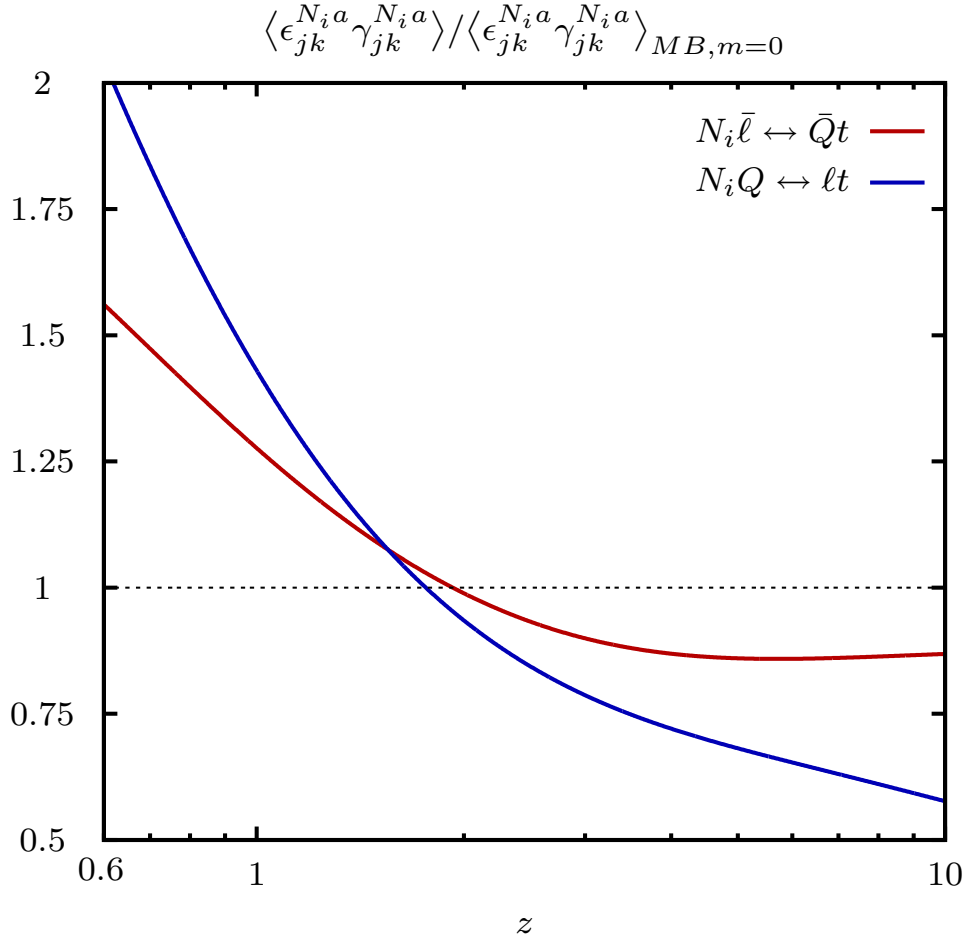


Figure 5.10: CP-violating reaction density (with thermal masses and quantum statistics) for the scattering processes  $N_i\bar{\ell} \leftrightarrow t\bar{Q}$  (red line) and  $N_i Q \leftrightarrow \ell t$  (blue line) normalised by the conventional one (without thermal masses and with Maxwell-Boltzmann statistics). Only the self-energy CP-violating parameters are shown.



In Fig. 5.10 we plot the CP-violating reaction density for the scattering processes  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$  (red line) and  $N_i Q \leftrightarrow lt$  (blue line) normalised by the conventional one as a function of the inverse temperature  $z$ . Only the self-energy CP-violating parameters are plotted. Similarly to the  $N_i$  decay the thermal corrections lead to an enhancement of the CP-violating reaction density at high temperature. However, in the case of the scattering processes, this enhancement is not overcompensated by the thermal masses. Therefore the resulting reaction densities (thermal masses plus quantum statistics) are increased by approximately 25% for the process  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$  (50% for  $N_i Q \leftrightarrow lt$ ) at  $z \sim 1$  in comparison with the conventional one (without thermal masses and with Maxwell-Boltzmann statistics). Note that similarly to the washout and production reaction densities, the ratio of the CP-violating reaction densities do not approach unity at low temperature.

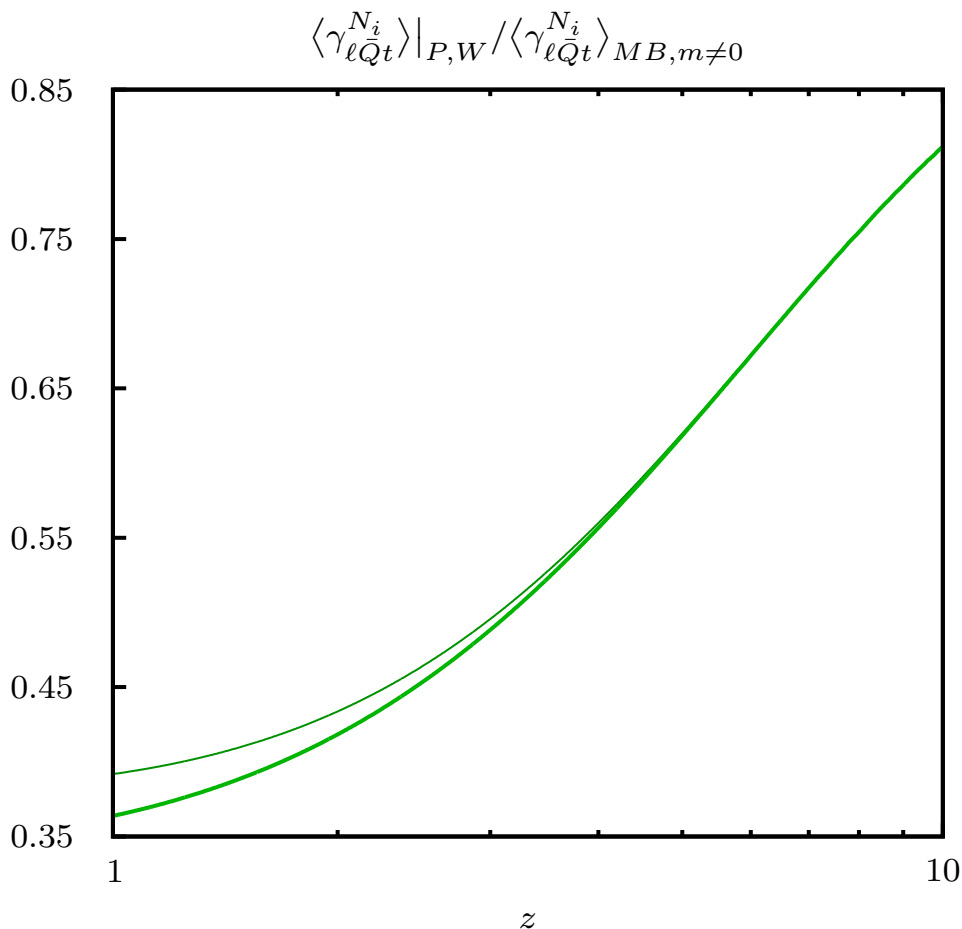


Figure 5.11: Production (thin line) and washout (thick line) reaction densities (with thermal masses and quantum statistics) for the three-body decay  $N_i \leftrightarrow lt\bar{Q}$  normalised by the conventional ones (without thermal masses and with Maxwell-Boltzmann statistics).

We plot next the reaction densities for the three-body decay process  $N_i \leftrightarrow lt\bar{Q}$ . This decay is kinematically allowed at  $z \lesssim 1$ . The production (thin line) and washout (thick line) reaction densities monotonically increase with the inverse temperature and approach the conventional one at low temperature, see Fig. 5.11. The CP-violating reaction density (Fig. 5.12) shows

a similar behaviour, except at high temperature where the thermal correction to the CP-parameter become important.

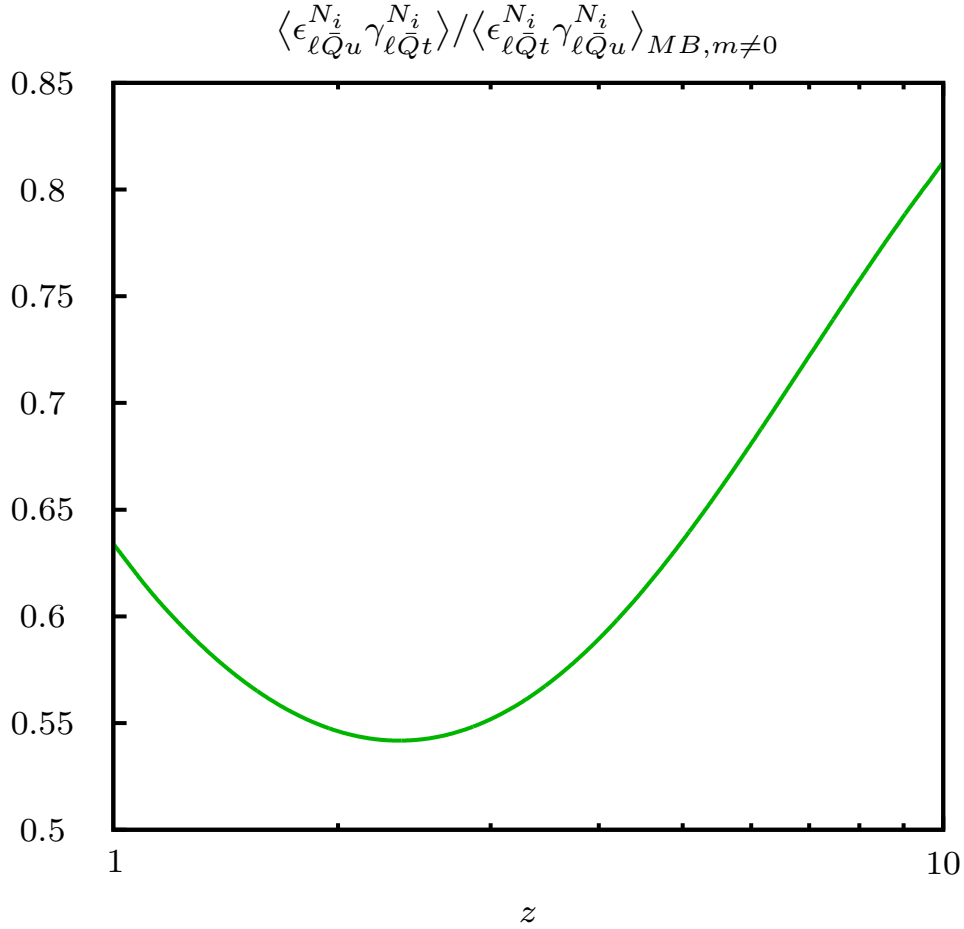


Figure 5.12: CP-violating reaction density (with thermal masses and with quantum statistics) for the three-body decay  $N_i \leftrightarrow lt\bar{Q}$  normalised by the conventional one (without thermal masses and with Maxwell-Boltzmann statistics). Only the self-energy CP-violating parameter is shown.

### 5.2.5 Heavy neutrino decay versus Higgs mediated processes

It is interesting to compare the relative importance of both processes. To this end we plot the reaction densities for the Higgs mediated processes divided by the ones for the heavy neutrino decay as a function of the inverse temperature  $z$ , see Fig. 5.13. The thick lines correspond to the reaction densities computed with thermal masses and quantum statistics, the thin lines assume thermal masses and Maxwell-Boltzmann statistics, and the dashed lines neglect thermal masses and take into account Maxwell-Boltzmann statistics. Plotted are the reaction densities for the scattering processes  $N_i\bar{l} \leftrightarrow t\bar{Q}$  (red lines) and  $N_iQ \leftrightarrow lt$  (blue line) and three-body decay  $N_i \leftrightarrow ltQ$  (green lines). We observe that the three-body decay is subdominant in the whole range of temperature. At  $z \sim 1$  the scattering reaction densities are about 10% of the one for the right-handed neutrino decay. For the processes with the heavy neutrino in the

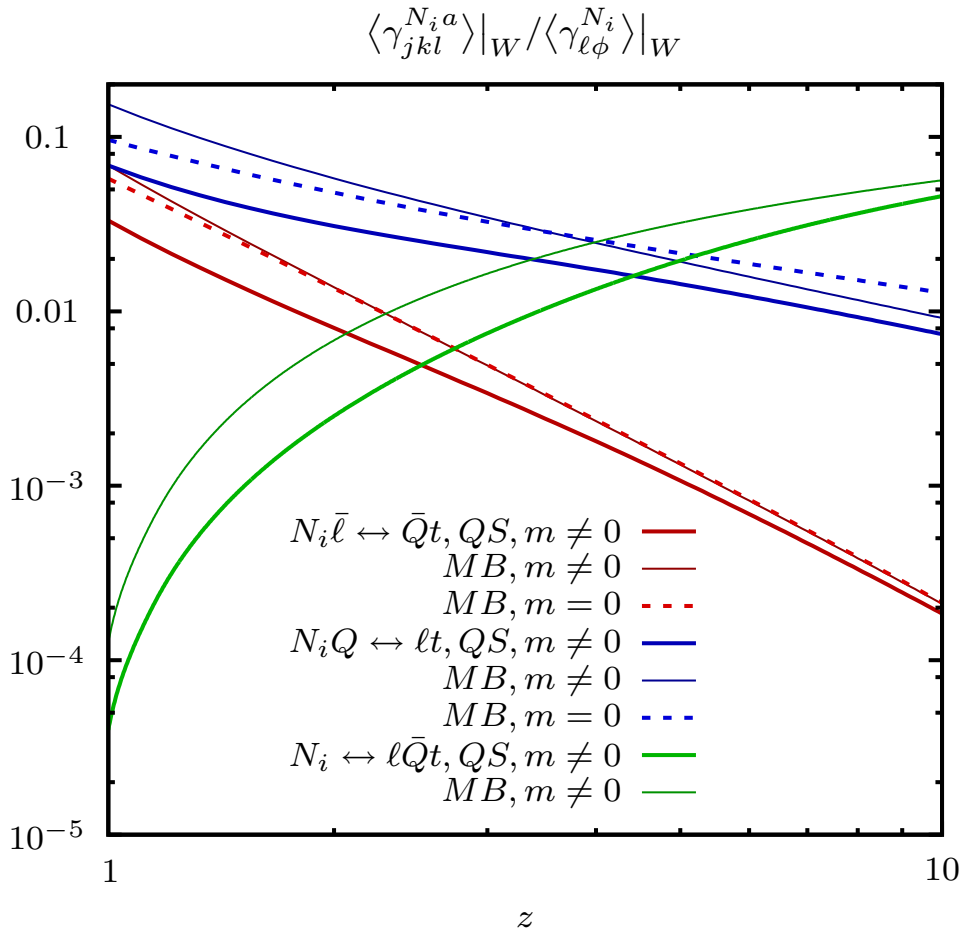


Figure 5.13: Higgs mediated reaction densities normalised by the ones for the heavy neutrino decay as a function of the inverse temperature  $z$ . Plotted are the reaction densities for the scattering processes  $N_i \bar{\ell} \leftrightarrow t \bar{Q}$  (red lines) and  $N_i Q \leftrightarrow \ell t$  (blue line) and three-body decay  $N_i \leftrightarrow \ell t \bar{Q}$  (green lines). The Higgs mediated processes are subdominant at low temperature. In the numerics we approximate the Yukawa coupling of the top quark by  $\lambda_t = 1$ .

initial or final states the inclusion of thermal masses has a similar effect on the scattering and two-body decay reaction densities in the temperature range considered here. Therefore the ratios plotted in Fig. 5.13 are not very sensitive to the thermal masses. The quantum statistics have much stronger effects on the scattering processes than on the two-body decay since the reaction densities for the latter involves boson and fermion distribution functions which partially cancel with each other. At higher temperature ( $T \gtrsim 1.6M_i$ , not shown in Fig. 5.13) the decay of the heavy neutrino into a lepton-Higgs pair becomes forbidden, and the Higgs mediated processes remain the only CP-violating source. At even higher temperature the Higgs decay becomes kinematically allowed and also contributes to the CP-violation.

So far we have only considered the heavy neutrino decay contribution to  $L_\rho$ ,

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^{\ell\phi} (2\pi)^4 \delta(q - p - k) [1 - f_\ell^{\text{eq},p} + f_\phi^{\text{eq},k}] \not{p}. \quad (5.65)$$

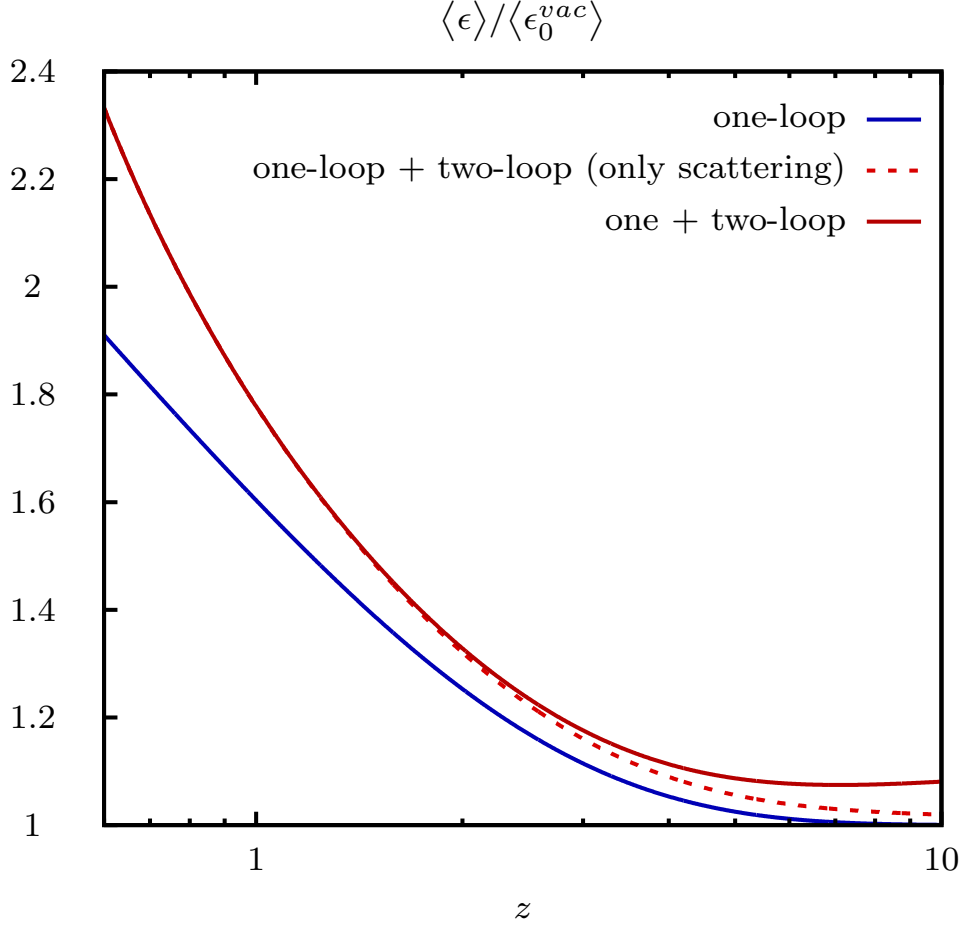


Figure 5.14: Thermally averaged CP-violating parameter for the heavy neutrino decay normalised by the vacuum one. The blue line includes only the heavy neutrino contributions to  $L_\rho$ , the red dashed line takes into account in addition the Higgs mediated scattering processes, and the red solid line corresponds to the full computation (heavy neutrino decay plus Higgs mediated scattering processes plus three-body decay). Thermal masses are neglected. In the numerics we approximate the Yukawa coupling of the top quark by  $\lambda_t = 1$ .

The contribution of the Higgs mediated processes to  $L_\rho$  can be found by substituting the off-shell Higgs propagator into (4.49). Integrating out the frequencies as usual we find (assuming  $q^0 > 0$ ),

$$\begin{aligned}
 L_\rho^t(t, q) = & 16\pi \int d\Pi_{\ell t Q}^{pp_t p_Q} \left[ \tilde{\mathcal{F}}_{(N_i)Q \leftrightarrow \ell t}^{q p_Q: p p_t} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} + \tilde{\mathcal{F}}_{(N_i)\bar{t} \leftrightarrow \ell \bar{Q}}^{q p_t: p p_Q} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} \right. \\
 & \left. + \tilde{\mathcal{F}}_{(N_i)\bar{\ell} \leftrightarrow t \bar{Q}}^{q p: p_t p_Q} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} + \tilde{\mathcal{F}}_{(N_i)\leftrightarrow \ell t \bar{Q}}^{q: p p_t p_Q} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} \right] \not{p}, \quad (5.66)
 \end{aligned}$$

see appendix E. The first three terms correspond to the scattering processes and the last one to the three-body decay. In the vacuum limit only the last term survives. In order to show the effects of the Higgs mediated processes on  $L_\rho$  we plot in Fig. 5.14 the thermally averaged CP-violating parameter for the heavy neutrino decay normalised by the vacuum one

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including in  $L_\rho$  the heavy neutrino decay only (blue line), the heavy neutrino decay plus the three scattering processes (red dashed line) and the heavy neutrino decay plus all the Higgs mediated contributions (red solid line). The ratios plotted in Fig. 5.14 correspond to  $\frac{\langle pL_\rho \rangle}{\langle pL_\rho + pL_\rho^t \rangle}$  as the vacuum CP-violating parameters cancel out. For simplicity we neglect thermal masses here. We observe that the full result (red solid line) does not approach unity at low temperature since the three-body decay contribution does not vanish in vacuum. The latter process gives sizable contribution only at low temperature. At  $z \sim 1$  the full and the one-loop results differ by approximately  $\sim 10\%$ .



# Chapter 6

## Conclusion

Explaining the matter-antimatter asymmetry of the universe is one of the biggest challenges of modern physics. The Standard Model of particle physics and the concordance model of cosmology, though both being very successful theories, cannot account for the baryon asymmetry of the universe. Over the past decades many theories of baryogenesis have been proposed. One of the most popular scenarios is leptogenesis. In this scenario the asymmetry is first produced in the lepton sector, and then transferred to the baryon sector through the sphaleron processes. This scenario postulates the existence of new heavy particles, the right-handed counterparts of the three Standard Model neutrinos. Leptogenesis is very attractive since, in addition to the baryon asymmetry of the universe, it naturally explains the small masses of the left-handed neutrinos.

Qualitatively, it is clear that the leptogenesis scenario can explain the observed baryon asymmetry of the universe. However, it is very difficult to compute the precise value of the produced baryon asymmetry. The main reason is that the generation of the asymmetry is a pure quantum phenomenon which takes place during the out-of-equilibrium evolution of the universe, as required by the third Sakharov condition. In the conventional approach that we have reviewed in chapter 2, the matter-antimatter asymmetry of the universe is computed by means of the classical Boltzmann equation with *vacuum* transition amplitudes in the collision term. Since leptogenesis takes place in the *hot* early universe, this approach is questionable. Over the past few years a lot of efforts have been made to improve the computation of the asymmetry generation in the leptogenesis scenario.

In this work we have studied leptogenesis using a first principles approach based on nonequilibrium quantum field theory. Starting from the Kadanoff-Baym equations we have derived in chapter 4 a quantum-corrected Boltzmann equation for the time evolution of the lepton asymmetry in the early universe. This equation is very similar to the classical Boltzmann equation. Its left-hand side corresponds to the Liouville operator, and its right-hand side to the collision term. Despite the similarities, the quantum-corrected Boltzmann equation and the classical one differ in a fundamental way. The latter, when used with transition amplitudes computed in the S-matrix formalism, generates an asymmetry even in thermal equilibrium. This problem can be solved by subtracting the contribution of the real intermediate states, which have been counted twice in the collision term. However, this procedure neglects the quantum statistical effects. The double counting problem does not arise in the quantum-corrected Boltzmann equation since its collision term vanishes in thermal equilibrium due to the KMS relations. In other words, the latter is automatically consistent with the third Sakharov condition and there is no need to perform an ad-hoc RIS subtraction as in the conventional approach. Another advantage of the equation obtained from first principles is that it incorporates *consistently* thermal corrections to the collision term and takes into account thermal corrections to the quasiparticle properties, in particular thermal masses.

In this work we have taken into account contributions to the collision term of the quantum-corrected Boltzmann equation for the lepton asymmetry up to order  $\mathcal{O}(h^4)$  and  $\mathcal{O}(h^4\lambda_t^2)$ . This implies that the equation derived from first principles includes decay and inverse decay of the right-handed neutrinos at tree- and one-loop level, lepton number violating scattering processes mediated by the heavy neutrinos at tree-level, and Higgs mediated processes at tree- and one-loop level. Technically, this is achieved by using the extended quasiparticle approximation. This ansatz represents the propagator as a sum of two terms, one of them describing the quasiparticle behaviour of the propagator, the other one corresponding to its off-shell part. The latter is of higher order in the coupling and is neglected in the quasiparticle approximation. However, this term is crucial for the consistent description of the heavy neutrino and Higgs mediated scattering processes. We have also taken into account the contribution from the Higgs decay, which is kinematically allowed at high temperature when one includes the thermal masses. The method presented here can be easily generalised to include other lepton number and CP-violating processes.

The derived quantum-corrected Boltzmann equation is valid for a hierarchical and mildly degenerate heavy neutrino mass spectrum, but not in the quasidegenerate case. For a quasidegenerate mass spectrum one cannot reduce the dynamics of two-point function of the right-handed neutrino to the dynamics of its diagonal components and we need to consider its full flavour structure. The thermal correction to the masses of the heavy neutrino must also be included in this case.

In this work we have not attempted to solve numerically the obtained quantum-corrected Boltzmann equations. In order to compare the first principles approach and the conventional one, we derived from the Boltzmann equations obtained in both approaches a system of rate equations. The latter are governed by a handful of reaction densities, which describe the production and washout of the asymmetry and the production of the right-handed neutrinos. In chapter 5 we have compared numerically the reaction densities obtained in the two approaches. When we neglect the thermal masses we observe that the improved reaction densities for the heavy neutrino decay are mildly enhanced compared to the conventional ones. This behaviour is due to the thermal corrections to the decay width of the right-handed neutrinos. The CP-violating parameter, being a loop effect, is stronger enhanced by the thermal corrections. The inclusion of thermal masses decreases the available phase-space for the right-handed neutrino decay. Therefore, when we consider the reaction densities with thermal masses, we observe that the enhancement of the effective amplitudes due to thermal corrections is overcompensated by the shrinking of the phase-space. At very high temperature the heavy neutrino decay becomes kinematically forbidden and is replaced by the Higgs decay. The CP-violating parameter for the Higgs decay is a few orders of magnitude larger than the one for the heavy neutrino decay.

The scattering amplitudes of the processes mediated by the heavy neutrinos are less affected by the thermal corrections since they are tree-level processes. The effects of thermal masses are also less important. The washout reaction densities for the scattering processes mediated by the heavy neutrinos are much smaller than the one for the heavy neutrino decay at  $z \lesssim 1$  and can be neglected at low temperature. Our computation confirmed the RIS-subtraction procedure performed in the S-matrix approach.

Finally we have studied the processes mediated by the Higgs. Due to the large top Yukawa coupling the contributions of these processes are not negligible. At low temperature they are suppressed in comparison with the heavy neutrino decay. At high temperature the reaction densities for the Higgs-mediated scattering processes become larger than the one for the heavy



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neutrino decay since the latter is suppressed due to the thermal masses. Moreover, the Higgs mediated processes affect indirectly the production of the asymmetry since they increase the relaxation rate of the heavy neutrino. This leads to an enhancement of the CP-violating parameter of the heavy neutrino decay by about 10% at  $z \sim 1$ . This is a genuine medium effect since in vacuum only the three-body decay, which is negligible, affects the relaxation rate of the heavy neutrino.

As a final word, we would like to stress that the method presented here can be used to compute consistently the asymmetry produced not only in the leptogenesis scenario, but also in other baryogenesis scenarios. It overcomes many problems inherent to the conventional approach based on the classical Boltzmann equation. Even though our method is computationally more demanding than the conventional approach, the formalism put forward by this work is needed for a precise computation of the baryon asymmetry produced in the leptogenesis scenario.



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I will not try to make a complete list of every people who contributed to this work. I thank globally all colleagues at the Max-Planck-Institut für Kernphysik who aroused my interest in physics.



# Appendix



# Appendix A

## Notation

<b>Heavy neutrino propagator</b>	
$\mathcal{S}^{ij}(x, y) = \langle T_C N_i(x) \bar{N}_j(y) \rangle$	propagator on the CTP
$\hat{\mathcal{S}}(x, y)$	propagator on the CTP (matrix notation)
$\mathcal{S}^{ii}(X, q)$	“diagonal” propagator
$\mathcal{S}^{ii}(X, q)$	scalar part of the “diagonal” propagator
$\tilde{\mathcal{S}}_\rho^{ii}(t, q) = (\not{q} + M_i) \text{sign}(q^0) (2\pi) \delta(q^2 - M_i^2)$	eQP spectral function
$\tilde{\mathcal{S}}_\rho^{ii}(t, q) = \text{sign}(q^0) (2\pi) \delta(q^2 - M_i^2)$	scalar part of the eQP spectral function
<b>Lepton propagator</b>	
$S_{ab}^{\alpha\beta}(x, y) = \langle T_C \ell_\alpha^a(x) \bar{\ell}_\beta^b(y) \rangle$	propagator on the CTP
$\hat{S}(x, y)$	propagator on the CTP (matrix notation)
$S(x, y)$	unflavoured, $SU(2)_L$ -symmetric propagator
$\tilde{S}_\rho(t, p) = P_L \not{p} \text{sign}(p^0) (2\pi) \delta(p^2 - m_\ell^2)$	eQP spectral function
$\tilde{\mathcal{S}}_\rho(t, p) = \text{sign}(p^0) (2\pi) \delta(p^2 - m_\ell^2)$	“scalar” part of eQP spectral function
<b>Higgs propagator</b>	
$\Delta_{ab}(x, y) = \langle T_C \phi^a(x) \phi^{b\dagger}(y) \rangle$	propagator on the CTP
$\hat{\Delta}(x, y)$	propagator on the CTP (matrix notation)
$\Delta(x, y)$	$SU(2)_L$ -symmetric propagator on the CTP
$\tilde{\Delta}_\rho(t, k) = \text{sign}(k^0) (2\pi) \delta(k^2 - m_\phi^2)$	eQP spectral function
<b>Top quark propagator</b>	
$S_t^{AB}(x, y) = \langle T_C t^A(x) \bar{t}^B(y) \rangle$	propagator on the CTP
$S_t(x, y)$	colourless propagator
$\tilde{S}_{t\rho}(t, p_t) = P_R \not{p}_t \text{sign}(p_t^0) (2\pi) \delta(p_t^2 - m_t^2)$	eQP spectral function
$\tilde{\mathcal{S}}_{t\rho}(t, p_t) = \text{sign}(p_t^0) (2\pi) \delta(p_t^2 - m_t^2)$	“scalar” part of eQP spectral function
<b>Quark doublet propagator</b>	
$S_{Qab}^{AB}(x, y) = \langle T_C Q_b^A(x) \bar{Q}_b^B(y) \rangle$	propagator on the CTP
$S_Q(x, y)$	colourless, $SU(2)_L$ -symmetric propagator
$\tilde{S}_{Q\rho}(t, p_Q) = P_L \not{p}_Q \text{sign}(p_Q^0) (2\pi) \delta(p_Q^2 - m_Q^2)$	eQP spectral function
$\tilde{\mathcal{S}}_{Q\rho}(t, p_Q) = \text{sign}(p_Q^0) (2\pi) \delta(p_Q^2 - m_Q^2)$	“scalar” part of eQP spectral function





# Appendix B

## Kinematics

We show here analytical simplification of the phase-space integration appearing in the reaction densities.

### B.1 Two-body decay

The reaction densities of the heavy neutrino decay can be generally written as,

$$\langle X \rangle^D \equiv \int d\Pi_{qp}^{N_i \ell \phi} (2\pi)^4 \delta(q - p - k) X, \quad (\text{B.1})$$

where  $X$  is any function of the momenta  $q$ ,  $p$  and  $k$ . Performing the integration over  $k$  we find,

$$\langle X \rangle^D = \int d\Pi_{qp}^{N_i \ell} (2\pi) \delta_+((p - k)^2 - m_\phi^2) X. \quad (\text{B.2})$$

Using spherical coordinate for  $\vec{p}$  and integrating over the azimuthal angle using the remaining delta function we find,

$$\langle X \rangle^D = \frac{1}{8\pi} \int d\Pi_q^{N_i} \frac{1}{|\vec{q}|} \int_{E_p^-}^{E_p^+} dE_p \int_0^{2\pi} \frac{d\varphi}{2\pi} X = \frac{1}{32\pi^3} \int_{M_i}^\infty dE_q \int \frac{d\Omega}{4\pi} \int_{E_p^-}^{E_p^+} dE_p \int_0^{2\pi} \frac{d\varphi}{2\pi} X, \quad (\text{B.3})$$

where  $\Omega$  is the solid angle of  $\vec{q}$ . The integral limits are given by,

$$E_p^\pm \equiv \frac{1}{2} [E_q(1 + x_\ell - x_\phi) \pm |\vec{q}| \lambda^{\frac{1}{2}}(1, x_\ell, x_\phi)], \quad (\text{B.4})$$

where  $x_\ell \equiv m_\ell^2/q^2$ ,  $x_\phi \equiv m_\phi^2/q^2$  and  $\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ . If the function  $X$  is independent of the angles we find,

$$\langle X \rangle^D = \frac{1}{32\pi^3} \int_{M_i}^\infty dE_q \int_{E_p^-}^{E_p^+} dE_p X. \quad (\text{B.5})$$

In vacuum the decay amplitude and CP-violating parameter do not depend on the momentum. In that case the above integral can be performed analytically. Neglecting the mass of the SM particles the CP-violating reaction density in vacuum is obtained from (B.5) by replacing

$$X \rightarrow 32\pi M_i \Gamma_{N_i} \epsilon_i e^{-E_q/T},$$

$$\begin{aligned} \langle \epsilon_i \gamma_{N_i}^D \rangle_{\text{MB}} &= \frac{\epsilon_i M_i \Gamma_{N_i}}{\pi^2} \int_{M_i}^{\infty} dE_q \int_{\frac{1}{2}(E_q - |\vec{q}|)}^{\frac{1}{2}(E_q + |\vec{q}|)} dE_p e^{-E_q/T} \\ &= \frac{\epsilon_i M_i \Gamma_{N_i}}{\pi^2} \int_{M_i}^{\infty} dE_q \sqrt{E_q^2 - M_i^2} e^{-E_q/T}. \end{aligned} \quad (\text{B.6})$$

The last integral corresponds to the modified Bessel function of the second kind,

$$K_1(z) \equiv \frac{1}{z} \int_z^{\infty} dx \sqrt{x^2 - z^2} e^{-x}. \quad (\text{B.7})$$

Then the reaction density (B.6) reads,

$$\langle \epsilon_i \gamma_{N_i}^D \rangle_{\text{MB}} = \frac{\epsilon_i M_i^2 \Gamma_{N_i} T}{\pi^2} K_1(M_i/T). \quad (\text{B.8})$$

Similarly the washout reaction density in vacuum reads,

$$\langle \gamma_{N_i}^D \rangle_{\text{MB}} = \frac{M_i^2 \Gamma_{N_i} T}{\pi^2} K_1(M_i/T). \quad (\text{B.9})$$

## B.2 Two-body Scattering

The reaction densities for scattering processes involve phase-space integrals of the type,

$$\langle X \rangle^S \equiv \int d\Pi_{p_a p_b p_j p_k}^{abjk} (2\pi)^4 \delta(p_a + p_b - p_j - p_k) X. \quad (\text{B.10})$$

Inserting the identity  $1 = \int ds d^4 p \delta(p - p_a - p_b) \delta(p^2 - s)$  into above equation, we rewrite it as,

$$\langle X \rangle^S \equiv \int ds \frac{d^4 p}{(2\pi)^4} \delta(p^2 - s) d\Pi_{p_a p_b}^{ab} (2\pi)^4 \delta(p_a + p_b - p) d\Pi_{p_j p_k}^{jk} (2\pi)^4 \delta(p - p_j - p_k) X. \quad (\text{B.11})$$

The integral over  $p_a$  and  $p_b$  are performed similarly to the two-body decay:

$$d\Pi_{p_a p_b}^{ab} (2\pi)^4 \delta(p_a + p_b - p) = \frac{1}{8\pi} \frac{1}{\sqrt{E_p^2 - s}} \int_{E_{p_a}^+}^{E_{p_a}^-} dE_{p_a} \frac{d\varphi_a}{2\pi}, \quad (\text{B.12})$$

with the integration limits,

$$E_{p_a}^{\pm} \equiv \frac{1}{2} [E_p (1 + x_a - x_b) \pm \sqrt{E_p^2 - s} \lambda^{\frac{1}{2}}(1, x_a, x_b)]. \quad (\text{B.13})$$

For the integration over  $p_j$  and  $p_k$  we use a different representation. Since it is a Lorentz-invariant quantity we can perform the integration in the center-of-mass frame, where  $\vec{p} = 0$ ,

$$d\Pi_{p_j p_k}^{jk} (2\pi)^4 \delta(p - p_j - p_k) = \frac{1}{8\pi} \frac{|\vec{p}_j|}{p^0} \int d\cos\theta \frac{d\varphi}{2\pi}, \quad (\text{B.14})$$

where  $\theta$  is the angle between  $\vec{p}_a$  and  $\vec{p}_j$ . The integration variable  $\theta$  is traded for the Mandelstam variable  $t = (p_a - p_j)^2$ ,

$$d \cos \theta = \frac{1}{2|\vec{p}_a||\vec{p}_j|} dt. \quad (\text{B.15})$$

Then, using the identity  $2p^0|\vec{p}_a| = s\lambda^{\frac{1}{2}}(1, m_a^2, m_b^2)$ , which is valid in the center-of-mass frame, we can write (B.14) with Lorentz invariant quantities only,

$$d\Pi_{p_j p_k}^{jk} (2\pi)^4 \delta(p - p_j - p_k) = \frac{1}{8\pi} \frac{1}{\lambda^{\frac{1}{2}}(1, m_a^2, m_b^2)} \int_{t^-}^{t^+} \frac{dt}{s} \frac{d\varphi_j}{2\pi}, \quad (\text{B.16})$$

with the integration limits given by,

$$t^\pm = m_a^2 + m_j^2 - \frac{s}{2} [(1 + x_a - x_b)/1 + x_j - x_k] \mp \lambda^{\frac{1}{2}}(1, m_a^2, m_b^2) \lambda^{\frac{1}{2}}(1, m_j^2, m_k^2). \quad (\text{B.17})$$

Putting everything together we find,

$$\langle X \rangle^S = \frac{1}{64\pi^4} \int_{s_{\min}}^{\infty} ds \int_{\sqrt{s}}^{\infty} dE_p \frac{d\Omega_p}{4\pi} \frac{1}{\lambda^{\frac{1}{2}}(1, m_a^2, m_b^2)} \int_{E_{p_a}^+}^{E_{p_a}^-} dE_{p_a} \frac{d\varphi_a}{2\pi} \times \frac{1}{8\pi} \int_{t^-}^{t^+} \frac{dt}{s} \frac{d\varphi_j}{2\pi} X, \quad (\text{B.18})$$

where  $s_{\min} = \max((m_a + m_b)^2, (m_j + m_k)^2)$ .

The above integral can be further simplified when one neglects the thermal masses and uses the Maxwell-Boltzmann distribution function. In that case the reaction density for the scattering process  $ab \leftrightarrow jk$  reads,

$$\langle \gamma_{jk}^{ab} \rangle_{\text{MB}} = \frac{T}{64\pi^4} \int_{s_{\min}}^{\infty} ds \sqrt{s} K_1\left(\frac{\sqrt{s}}{T}\right) \hat{\sigma}_{jk}^{ab}, \quad (\text{B.19})$$

where  $\hat{\sigma}_{jk}^{ab}$  is the so-called *reduced cross-section*,

$$\hat{\sigma}_{jk}^{ab} \equiv \frac{1}{8\pi} \int_0^{2\pi} \frac{d\varphi_a}{2\pi} \int_{t^-}^{t^+} \frac{dt}{s} \Xi_{ab \leftrightarrow jk}. \quad (\text{B.20})$$

### B.3 Three-body decay

The reaction density for a three-body decay is of the type,

$$\langle X \rangle^{3D} = \int d\Pi_{qpptpQ}^{N_i t t Q} (2\pi)^4 \delta(q - p - p_t - p_Q) X, \quad (\text{B.21})$$

where  $X$  is any function of the momenta. Similarly to the scattering we insert into (B.21) the identity in the form,

$$1 = \int ds d^4 k \delta(p_t + p_Q - k) \delta(k^2 - s). \quad (\text{B.22})$$

Then the integrals can be performed as in (B.3), and (B.21) reads,

$$\begin{aligned} \langle X \rangle^{3D} = & \frac{1}{64\pi^2} \int d\Pi_q^{N_i} \int_{(m_t+m_Q)^2}^{(M_i-m_\ell)^2} \frac{ds}{2\pi} \frac{1}{|\vec{q}|} \int_{E_p^-}^{E_p^+} dE_p \int_0^{2\pi} \frac{d\varphi_p}{2\pi} \\ & \times \frac{1}{\sqrt{(E_q - E_p)^2 - s}} \int_{E_{pQ}^-}^{E_{pQ}^+} dE_{pQ} \int_0^{2\pi} \frac{d\varphi_{pQ}}{2\pi} X, \end{aligned} \quad (\text{B.23})$$

where the integration limits are given by,

$$E_p^\pm = \frac{1}{2} [E_q(1 + x_\ell - x_s) \pm |\vec{q}|\lambda^{\frac{1}{2}}(1, x_\ell, x_s)], \quad (\text{B.24a})$$

$$E_{pQ}^\pm = \frac{1}{2} [E_q(1 + x_Q - x_t) \pm |\vec{q}|\lambda^{\frac{1}{2}}(1, x_Q, x_t)], \quad (\text{B.24b})$$

where  $x_s \equiv s/M_i^2$ .

# Appendix C

## Properties of the propagators

We list here the properties of the Higgs, lepton and heavy neutrino propagators. Here the hermitian conjugate and transpose act on the  $SU(2)_L$ , flavour and spinor indices.

### C.1 Symmetry properties in coordinate representation

#### Higgs field

$$\hat{\Delta}_F(x, y) = \hat{\Delta}_F^\dagger(y, x), \quad (\text{C.1a})$$

$$\hat{\Delta}_\rho(x, y) = -\hat{\Delta}_\rho^\dagger(y, x), \quad (\text{C.1b})$$

$$\hat{\Delta}_{\geq}(x, y) = \hat{\Delta}_{\geq}^\dagger(y, x), \quad (\text{C.1c})$$

$$\hat{\Delta}_R(x, y) = \hat{\Delta}_A^\dagger(y, x), \quad (\text{C.1d})$$

$$\hat{\Delta}_h(x, y) = \hat{\Delta}_h^\dagger(y, x). \quad (\text{C.1e})$$

#### Lepton field

$$\hat{S}_F(x, y) = \gamma^0 \hat{S}_F^\dagger(y, x) \gamma^0 = P_L \hat{S}_F(x, y) P_R, \quad (\text{C.2a})$$

$$\hat{S}_\rho(x, y) = -\gamma^0 \hat{S}_\rho^\dagger(y, x) \gamma^0 = P_L \hat{S}_\rho(x, y) P_R, \quad (\text{C.2b})$$

$$\hat{S}_{\geq}(x, y) = \gamma^0 \hat{S}_{\geq}^\dagger(y, x) \gamma^0 = P_L \hat{S}_{\geq}(x, y) P_R, \quad (\text{C.2c})$$

$$\hat{S}_R(x, y) = \gamma^0 \hat{S}_A^\dagger(y, x) \gamma^0 = P_L \hat{S}_R(x, y) P_R, \quad (\text{C.2d})$$

$$\hat{S}_h(x, y) = \gamma^0 \hat{S}_h^\dagger(y, x) \gamma^0 = P_L \hat{S}_h(x, y) P_R. \quad (\text{C.2e})$$

#### Heavy neutrino field

$$\hat{\mathcal{S}}_F(x, y) = \gamma^0 \hat{\mathcal{S}}_F^\dagger(y, x) \gamma^0 = C \hat{\mathcal{S}}_F^T(y, x) C^{-1}, \quad (\text{C.3a})$$

$$\hat{\mathcal{S}}_\rho(x, y) = -\gamma^0 \hat{\mathcal{S}}_\rho^\dagger(y, x) \gamma^0 = -C \hat{\mathcal{S}}_\rho^T(y, x) C^{-1}, \quad (\text{C.3b})$$

$$\hat{\mathcal{S}}_{\geq}(x, y) = \gamma^0 \hat{\mathcal{S}}_{\geq}^\dagger(y, x) \gamma^0 = C \hat{\mathcal{S}}_{\geq}^T(y, x) C^{-1}, \quad (\text{C.3c})$$

$$\hat{\mathcal{S}}_R(x, y) = \gamma^0 \hat{\mathcal{S}}_A^\dagger(y, x) \gamma^0 = C \hat{\mathcal{S}}_A^T(y, x) C^{-1}, \quad (\text{C.3d})$$

$$\hat{\mathcal{S}}_h(x, y) = \gamma^0 \hat{\mathcal{S}}_h^\dagger(y, x) \gamma^0 = C \hat{\mathcal{S}}_h^T(y, x) C^{-1}. \quad (\text{C.3e})$$

## C.2 Symmetry properties in Wigner representation

### Higgs field

$$\hat{\Delta}_F(X, k) = \hat{\Delta}_F^\dagger(X, k), \quad (\text{C.4a})$$

$$\hat{\Delta}_\rho(X, k) = \hat{\Delta}_\rho^\dagger(X, k), \quad (\text{C.4b})$$

$$\hat{\Delta}_{\geq}(X, k) = \hat{\Delta}_{\geq}^\dagger(X, k), \quad (\text{C.4c})$$

$$\hat{\Delta}_R(X, k) = \hat{\Delta}_A^\dagger(X, k), \quad (\text{C.4d})$$

$$\hat{\Delta}_h(X, k) = \hat{\Delta}_h^\dagger(X, k). \quad (\text{C.4e})$$

### Lepton field

$$\hat{S}_F(X, p) = \gamma^0 \hat{S}_F^\dagger(X, p) \gamma^0 = P_L \hat{S}_F(X, p) P_R, \quad (\text{C.5a})$$

$$\hat{S}_\rho(X, p) = \gamma^0 \hat{S}_\rho^\dagger(X, p) \gamma^0 = P_L \hat{S}_\rho(X, p) P_R, \quad (\text{C.5b})$$

$$\hat{S}_{\geq}(X, p) = \gamma^0 \hat{S}_{\geq}^\dagger(X, p) \gamma^0 = P_L \hat{S}_{\geq}(X, p) P_R \gamma^0, \quad (\text{C.5c})$$

$$\hat{S}_R(X, p) = \gamma^0 \hat{S}_A^\dagger(X, p) \gamma^0 = P_L \hat{S}_R(X, p) P_R, \quad (\text{C.5d})$$

$$\hat{S}_h(X, p) = \gamma^0 \hat{S}_h^\dagger(X, p) \gamma^0 = P_L \hat{S}_h(X, p) P_R. \quad (\text{C.5e})$$

### Heavy neutrino field

$$\hat{\mathcal{S}}_F(X, q) = \gamma^0 \hat{\mathcal{S}}_F^\dagger(X, q) \gamma^0 = C \hat{\mathcal{S}}_F^T(X, -q) C^{-1}, \quad (\text{C.6a})$$

$$\hat{\mathcal{S}}_\rho(X, q) = \gamma^0 \hat{\mathcal{S}}_\rho^\dagger(X, q) \gamma^0 = -C \hat{\mathcal{S}}_\rho^T(X, -q) C^{-1}, \quad (\text{C.6b})$$

$$\hat{\mathcal{S}}_{\geq}(X, q) = \gamma^0 \hat{\mathcal{S}}_{\geq}^\dagger(X, q) \gamma^0 = C \hat{\mathcal{S}}_{\geq}^T(X, -q) C^{-1}, \quad (\text{C.6c})$$

$$\hat{\mathcal{S}}_R(X, q) = \gamma^0 \hat{\mathcal{S}}_A^\dagger(X, q) \gamma^0 = C \hat{\mathcal{S}}_A^T(X, -q) C^{-1}, \quad (\text{C.6d})$$

$$\hat{\mathcal{S}}_h(X, q) = \gamma^0 \hat{\mathcal{S}}_h^\dagger(X, q) \gamma^0 = C \hat{\mathcal{S}}_h^T(X, -q) C^{-1}. \quad (\text{C.6e})$$

## C.3 CP-conjugated propagators in coordinate representations

### Higgs CP-conjugated propagators

$$\bar{\hat{\Delta}}_F(x, y) = \hat{\Delta}_F^T(\bar{y}, \bar{x}), \quad (\text{C.7a})$$

$$\bar{\hat{\Delta}}_\rho(x, y) = -\hat{\Delta}_\rho^T(\bar{y}, \bar{x}), \quad (\text{C.7b})$$

$$\bar{\hat{\Delta}}_{\geq}(x, y) = \hat{\Delta}_{\geq}^T(\bar{y}, \bar{x}), \quad (\text{C.7c})$$

$$\bar{\hat{\Delta}}_R(x, y) = \hat{\Delta}_A^T(\bar{y}, \bar{x}), \quad (\text{C.7d})$$

$$\bar{\hat{\Delta}}_h(x, y) = \hat{\Delta}_h^T(\bar{y}, \bar{x}). \quad (\text{C.7e})$$

**Lepton CP-conjugated propagators**

$$\bar{\hat{S}}_F(x, y) = (CP) \hat{S}_F^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.8a})$$

$$\bar{\hat{S}}_\rho(x, y) = -(CP) \hat{S}_\rho^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.8b})$$

$$\bar{\hat{S}}_{\geq}(x, y) = (CP) \hat{S}_{\leq}^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.8c})$$

$$\bar{\hat{S}}_R(x, y) = (CP) \hat{S}_A^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.8d})$$

$$\bar{\hat{S}}_h(x, y) = (CP) \hat{S}_h^T(\bar{y}, \bar{x}) (CP)^{-1}. \quad (\text{C.8e})$$

**Heavy neutrino CP-conjugated propagators**

$$\bar{\hat{\mathcal{S}}}_F(x, y) = (CP) \hat{\mathcal{S}}_F^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.9a})$$

$$\bar{\hat{\mathcal{S}}}_\rho(x, y) = -(CP) \hat{\mathcal{S}}_\rho^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.9b})$$

$$\bar{\hat{\mathcal{S}}}_{\geq}(x, y) = (CP) \hat{\mathcal{S}}_{\leq}^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.9c})$$

$$\bar{\hat{\mathcal{S}}}_R(x, y) = (CP) \hat{\mathcal{S}}_A^T(\bar{y}, \bar{x}) (CP)^{-1}, \quad (\text{C.9d})$$

$$\bar{\hat{\mathcal{S}}}_h(x, y) = (CP) \hat{\mathcal{S}}_h^T(\bar{y}, \bar{x}) (CP)^{-1}. \quad (\text{C.9e})$$

**C.4 CP-conjugated propagators in Wigner representations**
**Higgs CP-conjugated propagators**

$$\bar{\hat{\Delta}}_F(X, k) = \hat{\Delta}_F^T(\bar{X}, -\bar{k}), \quad (\text{C.10a})$$

$$\bar{\hat{\Delta}}_\rho(X, k) = -\hat{\Delta}_\rho^T(\bar{X}, -\bar{k}), \quad (\text{C.10b})$$

$$\bar{\hat{\Delta}}_{\geq}(X, k) = \hat{\Delta}_{\leq}^T(\bar{X}, -\bar{k}), \quad (\text{C.10c})$$

$$\bar{\hat{\Delta}}_R(X, k) = \hat{\Delta}_A^T(\bar{X}, -\bar{k}), \quad (\text{C.10d})$$

$$\bar{\hat{\Delta}}_h(X, k) = \hat{\Delta}_h^T(\bar{X}, -\bar{k}). \quad (\text{C.10e})$$

**Lepton CP-conjugated propagators**

$$\bar{\hat{S}}_F(X, p) = (CP) \hat{S}_F^T(\bar{X}, -\bar{p}) (CP)^{-1}, \quad (\text{C.11a})$$

$$\bar{\hat{S}}_\rho(X, p) = -(CP) \hat{S}_\rho^T(\bar{X}, -\bar{p}) (CP)^{-1}, \quad (\text{C.11b})$$

$$\bar{\hat{S}}_{\geq}(X, p) = (CP) \hat{S}_{\leq}^T(\bar{X}, -\bar{p}) (CP)^{-1}, \quad (\text{C.11c})$$

$$\bar{\hat{S}}_R(X, p) = (CP) \hat{S}_A^T(\bar{X}, -\bar{p}) (CP)^{-1}, \quad (\text{C.11d})$$

$$\bar{\hat{S}}_h(X, p) = (CP) \hat{S}_h^T(\bar{X}, -\bar{p}) (CP)^{-1}. \quad (\text{C.11e})$$

**Heavy neutrino CP-conjugated propagators**

$$\hat{\mathcal{P}}_{\bar{F}}(X, q) = (CP) \hat{\mathcal{P}}_F^T(\bar{X}, -\bar{q}) (CP)^{-1}, \quad (\text{C.12a})$$

$$\hat{\mathcal{P}}_{\bar{\rho}}(X, q) = -(CP) \hat{\mathcal{P}}_{\rho}^T(\bar{X}, -\bar{q}) (CP)^{-1}, \quad (\text{C.12b})$$

$$\hat{\mathcal{P}}_{\bar{\geq}}(X, q) = (CP) \hat{\mathcal{P}}_{\leq}^T(\bar{X}, -\bar{q}) (CP)^{-1}, \quad (\text{C.12c})$$

$$\hat{\mathcal{P}}_{\bar{R}}(X, q) = (CP) \hat{\mathcal{P}}_A^T(\bar{X}, -\bar{q}) (CP)^{-1}, \quad (\text{C.12d})$$

$$\hat{\mathcal{P}}_{\bar{h}}(X, q) = (CP) \hat{\mathcal{P}}_h^T(\bar{X}, -\bar{q}) (CP)^{-1}. \quad (\text{C.12e})$$



## Appendix D

### Wigner transform of a convolution product

We derive here the Wigner transform of a convolution product,

$$A(x, y) = \int d^4 z B(x, z) C(z, y). \quad (\text{D.1})$$

Using the central  $X = \frac{1}{2}(x + y)$  and relative  $s = x - y$  coordinates we can write the above equation as,

$$A(X, s) = \int d^4 z B\left(X + \frac{1}{2}s_{zy}, s_{xz}\right) C\left(X - \frac{1}{2}s_{xz}, s_{zy}\right), \quad (\text{D.2})$$

where we have defined  $s_{zy} = z - y$  and  $s_{xz} = x - z$ . Performing the Wigner transform separately on the two-point functions we find,

$$A(X, p) = \int d^4 s d^4 z \frac{d^4 p_B}{(2\pi)^4} \frac{d^4 p_C}{(2\pi)^4} e^{i(ps - p_B s_{xz} - p_C s_{zy})} B\left(X + \frac{1}{2}s_{zy}, p_B\right) C\left(X - \frac{1}{2}s_{xz}, p_C\right). \quad (\text{D.3})$$

We can now expand the functions  $B$  and  $C$  around the central coordinate  $X$ ,

$$B\left(X + \frac{1}{2}s_{zy}, p_B\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{1}{2}s_{zy} \partial_X\right)^n B(X, p_B), \quad (\text{D.4a})$$

$$C\left(X - \frac{1}{2}s_{xz}, p_C\right) = \sum_{m=0}^{\infty} \frac{1}{m!} \left(-\frac{1}{2}s_{xz} \partial_X\right)^m C(X, p_C). \quad (\text{D.4b})$$

Using the expansion (D.4) into (D.3) we find,

$$\begin{aligned}
 A(X, p) &= \int d^4 s d^4 z \frac{d^4 p_B}{(2\pi)^4} \frac{d^4 p_C}{(2\pi)^4} e^{i(ps - p_B s_{xz} - p_C s_{zy})} \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{1}{2} s_{zy} \partial_X \right)^n B(X, p_B) \right\} \\
 &\quad \times \left\{ \sum_{m=0}^{\infty} \frac{1}{m!} \left( -\frac{1}{2} s_{xz} \partial_X \right)^m C(X, p_C) \right\} \\
 &= \int d^4 s d^4 z \frac{d^4 p_B}{(2\pi)^4} \frac{d^4 p_C}{(2\pi)^4} e^{ips} \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{i}{2} \partial_{p_C} \partial_X \right)^n \left( e^{-ip_C s_{zy}} B(X, p_B) \right) \right\} \\
 &\quad \times \left\{ \sum_{m=0}^{\infty} \frac{1}{m!} \left( \frac{-i}{2} \partial_{p_B} \partial_X \right)^m \left( e^{-ip_B s_{xz}} C(X, p_C) \right) \right\} \\
 &= \int d^4 s d^4 z \frac{d^4 p_B}{(2\pi)^4} \frac{d^4 p_C}{(2\pi)^4} e^{i(ps - p_B s_{xz} - p_C s_{zy})} \\
 &\quad \times \left[ B(X, p_B) \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{i}{2} \overleftarrow{\partial}_X \overrightarrow{\partial}_{p_C} \right)^n \right\} \left\{ \sum_{m=0}^{\infty} \frac{1}{m!} \left( \frac{i}{2} \overleftarrow{\partial}_{p_B} \overrightarrow{\partial}_X \right)^m \right\} C(X, p_C) \right] \\
 &= B(X, p) \left\{ \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{i}{2} \overleftarrow{\partial}_X \overrightarrow{\partial}_p \right)^n \right\} \left\{ \sum_{m=0}^{\infty} \frac{1}{m!} \left( \frac{i}{2} \overleftarrow{\partial}_p \overrightarrow{\partial}_X \right)^m \right\} C(X, p) \\
 &= B(X, p) e^{-\frac{i}{2} \left( \overleftarrow{\partial}_X \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_X \right)} C(X, p) \equiv e^{-\frac{i}{2} \diamond} \{ B(X, p) \} \{ C(X, p) \}, \tag{D.5}
 \end{aligned}$$

where the arrows over the derivatives indicate on which direction they act.

## D.1 Properties of the diamond operator

We summarise here different properties of the diamond operator. One can easily verify these identities by using the definition of the diamond operator given in (D.5).

$$\left[ \diamond \left\{ \hat{A}(X, p) \right\} \left\{ \hat{B}(X, p) \right\} \right]^\dagger = -\diamond \left\{ \hat{B}^\dagger(X, p) \right\} \left\{ \hat{A}^\dagger(X, p) \right\}, \tag{D.6a}$$

$$\hat{A}(X, p) \diamond \left\{ \hat{A}^{-1}(X, p) \right\} \left\{ \hat{A}(X, p) \right\} = -\diamond \left\{ \hat{A}(X, p) \right\} \left\{ \hat{A}^{-1}(X, p) \right\} \hat{A}(X, p), \tag{D.6b}$$

$$\text{tr} \left[ \left\{ \hat{A}(X, p) \right\} \left\{ \hat{B}(X, p) \right\} \right] = -\text{tr} \left[ \left\{ \hat{B}(X, p) \right\} \left\{ \hat{A}(X, p) \right\} \right], \tag{D.6c}$$

$$\text{tr} \left[ \left\{ \hat{A}(X, p) \right\} \left\{ \hat{B}(X, p) \right\} \right] = -\text{tr} \left[ \left\{ \hat{A}^{-1}(X, p) \right\} \left\{ \hat{A}(X, p) \hat{B}(X, p) \hat{A}(X, p) \right\} \right]. \tag{D.6d}$$

# Appendix E

## 2PI effective action and self-energies

### E.1 2PI effective action

The 2PI effective action is defined as a functional of the one- and two-point functions consisting of an infinite sum of all connected 2PI vacuum diagrams [116]. In practice, its expansion can be characterised in terms of the number of loops appearing in each diagram:

$$i\Gamma_{2\text{PI}}[S, \mathcal{S}, \Delta, S_t, S_Q] = \sum_n i\Gamma_{2\text{PI}}^{(n)}[S, \mathcal{S}, \Delta, S_t, S_Q]. \quad (\text{E.1})$$

Two diagrams relevant for this work contribute to the two-loop 2PI effective action,

$$i\Gamma_{2\text{PI}}^{(2.1)} = - \int_{\mathcal{C}} d^4u d^4w \text{tr} [hP_R \hat{\mathcal{S}}(u, w) P_L h^\dagger \hat{S}(w, u) \epsilon \hat{\Delta}^*(u, w) \epsilon], \quad (\text{E.2a})$$

$$i\Gamma_{2\text{PI}}^{(2.2)} = -|\lambda_t|^2 \int_{\mathcal{C}} d^4u d^4v \text{tr} [\hat{S}_Q(v, u) P_R \hat{S}_t(u, v) \epsilon \hat{\Delta}^T(v, u) \epsilon], \quad (\text{E.2b})$$

At three-loop order only one diagram contributes to the 2PI effective action,

$$i\Gamma_{2\text{PI}}^{(3)} = \frac{1}{2} \int_{\mathcal{C}} d^4u d^4w d^4\eta d^4\xi \text{tr} [hP_R \hat{\mathcal{S}}(u, w) C P_R h^T \hat{S}^T(\eta, w) \epsilon \hat{\Delta}^*(w, \xi) \epsilon h^* P_L C \hat{\mathcal{S}}(\eta, \xi) P_L h^\dagger \hat{S}(\xi, u) \epsilon \hat{\Delta}^*(u, \eta) \epsilon]. \quad (\text{E.3})$$

In (E.2) and (E.3) the trace acts on flavour, colour,  $SU(2)_L$  and spinor space, and in (E.3) the trace acts only in spinor and flavour space. Note that the propagators in the 2PI effective action are the full propagators with the time arguments attached to the CTP.

### E.2 Lepton self-energy

#### Coordinate representation

By functional differentiation of the 2PI effective action with respect to the lepton propagator we obtain the corresponding self-energy which enters the Schwinger-Dyson equation:

$$\hat{\Sigma}^{(n-1)}(x, y) = -i \frac{\delta \Gamma_{2\text{PI}}^{(n)}}{\delta \hat{S}^T(y, x)}. \quad (\text{E.4})$$

Since we do not consider the flavour effects and the early universe was in an  $SU(2)_L$ -symmetric state, the  $SU(2)_L$  and flavour structure of the lepton and Higgs propagators is trivial,

$$S_{ab}^{\alpha\beta}(x, y) = \delta_{ab}\delta^{\alpha\beta}S(x, y), \quad \Delta_{ab}(x, y) = \delta_{ab}\Delta(x, y). \quad (\text{E.5})$$

Using the identity  $\epsilon_{ab}\epsilon_{bc} = -\delta_{ac}$  we find that the lepton self-energy is also proportional to the identity in  $SU(2)_L$  space,  $\Sigma_{ab}^{\alpha\beta}(x, y) = \delta_{ab}\Sigma_{\alpha\beta}(x, y)$ . Furthermore, in the unflavoured approximation it is convenient to take the trace over lepton flavours:  $\Sigma(x, y) \equiv \Sigma^{\alpha\alpha}(x, y)$ . Then the one- and two-loop lepton self-energy read,

$$\Sigma^{(1)}(x, y) = -(h^\dagger h)_{ji}P_R\mathcal{S}^{ij}(x, y)P_L\Delta(y, x), \quad (\text{E.6a})$$

$$\begin{aligned} \Sigma^{(2)}(x, y) = & -(h^\dagger h)_{ij}(h^\dagger h)_{lk} \int_C d^4w d^4\eta P_R\mathcal{S}^{jk}(x, w)CP_R S^T(\eta, w)P_L \\ & \times C\mathcal{S}^{li}(\eta, y)P_L\Delta(y, w)\Delta(\eta, x), \end{aligned} \quad (\text{E.6b})$$

Eventually, we want to find the Wightman components since they enter the gain and loss terms on the right-hand side of (4.11). To this end we insert into (E.6) the decomposition of the propagators into their Wightman components,

$$G(x, y) = \theta_C(x^0, y^0)G_>(x, y) \pm \theta_C(y^0, x^0)G_<(x, y), \quad (\text{E.7})$$

where the upper (lower) sign corresponds to boson (fermion). At one-loop level we find,

$$\Sigma_{\gtrless}^{(1)}(x, y) = -(h^\dagger h)_{ji}P_R\mathcal{S}_{\gtrless}^{ij}(x, y)P_L\Delta_{\lesseqgtr}(y, x), \quad (\text{E.8})$$

where the Wightman components of the self-energy are defined similarly to (E.7). In the case of the two-loop contribution (E.6b) the computation becomes slightly more elaborate. The complication is due to the appearance of 32 different terms after inserting the decomposition (E.7) for each of the five propagators into (E.6b) as well as due to the two remaining integrations over the internal space-time arguments  $w$  and  $\eta$ . The decomposition makes the path-ordering explicit and allows us to convert the integration along the CTP into an integration along the positive branch. The 32 terms contain different combinations of the  $\theta_C$ -functions. These can be rewritten by using relations similar to (3.27) which can be found in [59]. After some simple

but lengthy algebra we obtain for the two-loop Wightman components:

$$\begin{aligned}
 \Sigma_{\gtrless}^{(2)}(x, y) &= (h^\dagger h)_{ij}(h^\dagger h)_{lk} \int_{\mathcal{C}} d^4\omega d^4\eta \\
 &\times [P_R \mathcal{S}_R^{jk}(x, \omega) C P_R S_F^T(\eta, \omega) P_L C \mathcal{S}_{\gtrless}^{li}(\eta, y) P_L \Delta_{\lesseqgtr}(y, \omega) \Delta_A(\eta, x) \\
 &+ P_R \mathcal{S}_F^{jk}(x, \omega) C P_R S_R^T(\eta, \omega) P_L C \mathcal{S}_{\gtrless}^{li}(\eta, y) P_L \Delta_{\lesseqgtr}(y, \omega) \Delta_A(\eta, x) \\
 &+ P_R \mathcal{S}_R^{jk}(x, \omega) C P_R S_A^T(\eta, \omega) P_L C \mathcal{S}_{\gtrless}^{li}(\eta, y) P_L \Delta_{\lesseqgtr}(y, \omega) \Delta_F(\eta, x) \\
 &+ P_R \mathcal{S}_{\gtrless}^{jk}(x, \omega) C P_R S_R^T(\eta, \omega) P_L C \mathcal{S}_A^{li}(\eta, y) P_L \Delta_F(y, \omega) \Delta_{\lesseqgtr}(\eta, x) \\
 &+ P_R \mathcal{S}_{\gtrless}^{jk}(x, \omega) C P_R S_A^T(\eta, \omega) P_L C \mathcal{S}_F^{li}(\eta, y) P_L \Delta_R(y, \omega) \Delta_{\lesseqgtr}(\eta, x) \\
 &+ P_R \mathcal{S}_{\gtrless}^{jk}(x, \omega) C P_R S_F^T(\eta, \omega) P_L C \mathcal{S}_A^{li}(\eta, y) P_L \Delta_R(y, \omega) \Delta_{\lesseqgtr}(\eta, x) \\
 &+ P_R \mathcal{S}_R^{jk}(x, \omega) C P_R S_{\lesseqgtr}^T(\eta, \omega) P_L C \mathcal{S}_A^{li}(\eta, y) P_L \Delta_{\lesseqgtr}(y, \omega) \Delta_{\lesseqgtr}(\eta, x) \\
 &+ P_R \mathcal{S}_{\gtrless}^{jk}(x, \omega) C P_R S_{\lesseqgtr}^T(\eta, \omega) P_L C \mathcal{S}_{\gtrless}^{li}(\eta, y) P_L \Delta_R(y, \omega) \Delta_A(\eta, x)] . \quad (\text{E.9})
 \end{aligned}$$

### Wigner representation

We perform then a Wigner transformation defined in (3.35) on the lepton self-energy. The Wigner transform of the one-loop self-energy is obtained straightforwardly,

$$\Sigma_{\gtrless}(X, p) = -(h^\dagger h)_{ji} \int d\Pi_{qk}^4 (2\pi)^4 \delta(q - k - p) P_R \mathcal{S}_{\gtrless}^{ij}(X, q) P_L \Delta_{\lesseqgtr}(X, k) . \quad (\text{E.10})$$

The Wigner transform of the two-loop self-energy (E.9) is more involved. Due to the integration over  $\omega$  and  $\eta$  the Wigner transform can be expressed in term of an infinite sum of derivatives similarly to the convolution product, see appendix D. We neglect here the derivatives and only keep the leading order terms in the gradient expansion. It can be conveniently written as a sum of three terms,

$$\Sigma_{\gtrless}^{(2)}(X, p) = \Sigma_{\gtrless}^{(2.1)}(X, p) + \Sigma_{\gtrless}^{(2.2)}(X, p) + \Sigma_{\gtrless}^{(2.3)}(X, p) . \quad (\text{E.11})$$

The first term of the above equation, which corresponds to the first six terms in the square bracket of (E.9), reads,

$$\begin{aligned}
 \Sigma_{\gtrless}^{(2.1)}(X, p) &= \int d\Pi_{qk}^4 (2\pi)^4 \delta(p + k - q) \left[ (h^\dagger h)_{in} (h^\dagger h)_{jm} \Lambda_{mn}(X, q, k) P_L C \mathcal{S}_{\gtrless}^{ij}(t, q) P_L \Delta_{\lesseqgtr}(X, k) \right. \\
 &\quad \left. + (h^\dagger h)_{ni} (h^\dagger h)_{mj} P_R \mathcal{S}_{\gtrless}^{ji}(X, q) C P_R V_{nm}(X, q, p) \Delta_{\lesseqgtr}(X, k) \right] , \quad (\text{E.12})
 \end{aligned}$$

where we have introduced two functions containing loop corrections:

$$\begin{aligned}
 \Lambda_{mn}(X, q, k) &\equiv \int d\Pi_{k_1 p_1 q_1}^4 (2\pi)^4 \delta(q + k_1 + p_1) (2\pi)^4 \delta(k + p_1 - q_1) \\
 &\times \left[ P_R \mathcal{S}_R^{mn}(X, -q_1) C P_R S_F^T(t, p_1) \Delta_A(X, k_1) + P_R \mathcal{S}_F^{mn}(X, -q_1) C P_R S_R^T(X, p_1) \Delta_A(X, k_1) \right. \\
 &\quad \left. + P_R \mathcal{S}_R^{mn}(X, -q_1) C P_R S_A^T(X, p_1) \Delta_F(X, k_1) \right] , \quad (\text{E.13})
 \end{aligned}$$

and  $V_{nm}(X, q, k) \equiv P \Lambda_{nm}^\dagger(X, q, k) P$  to shorten the notation. The second term in (E.11) corresponds to the seventh term in (E.9) and is given by,

$$\begin{aligned} \Sigma_{\gtrless}^{(2.2)}(X, p) &= \int d\Pi_{p_1 k_1 k_2}^4 (2\pi)^4 \delta(p + k_1 - p_1 - k_2) (h^\dagger h)_{ni} (h^\dagger h)_{mj} \\ &\times P_R \mathcal{S}_R^{ij}(X, p_1 + k_2) C P_R S_{\gtrless}^T(X, -p_1) P_L C \mathcal{S}_A^{mn}(X, p_1 - k_1) P_L \Delta_{\gtrless}(X, k_1) \Delta_{\gtrless}(X, -k_2). \end{aligned} \quad (\text{E.14})$$

Finally the last term in (E.11) reads,

$$\begin{aligned} \Sigma_{\gtrless}^{(2.3)}(X, p) &= \int d\Pi_{p_1 q_1 q_2}^4 (2\pi)^4 \delta(p + q_1 - p_1 - q_2) (h^\dagger h)_{ij} (h^\dagger h)_{lk} \\ &\times P_R \mathcal{S}_{\gtrless}^{jk}(X, -q_1) C P_R S_{\gtrless}^T(X, p_1) P_L C \mathcal{S}_{\gtrless}^{li}(X, q_2) P_L \Delta_A(X, -p_1 - q_2) \Delta_R(X, q_1 - p_1). \end{aligned} \quad (\text{E.15})$$

### E.3 Right-handed neutrino self-energy

#### Coordinate representation

We consider here only the one-loop right-handed neutrino self-energy. Similarly to (E.4) the heavy neutrino self-energy is obtained by differentiating the 2PI effective action with respect to the corresponding propagator,

$$\hat{\Pi}(x, y) = -i \frac{\delta \Gamma_{2\text{PI}}^{(2)}}{\delta \hat{\mathcal{S}}^T(y, x)}. \quad (\text{E.16})$$

Due to its Majorana nature two terms contribute to the one-loop heavy neutrino self-energy,

$$\Pi^{ij}(x, y) = -g_w [(h^\dagger h)_{ij} P_L S(x, y) P_R \mathcal{S}(x, y) + (h^\dagger h)_{ij}^* P_R P \bar{S}(\bar{x}, \bar{y}) P P_L \bar{\Delta}(\bar{x}, \bar{y})], \quad (\text{E.17})$$

where we have assumed  $SU(2)_L$  symmetric medium and neglected the flavour effects. In (E.17) we have written the second term using the CP-conjugate lepton and Higgs propagators to emphasize the fact that the intermediate lines correspond to antiparticle. We decompose then the one-loop heavy neutrino self-energy into its Wightmann components,

$$\begin{aligned} \mathcal{S}_{\gtrless}^{ij}(x, y) &= -g_w [(h^\dagger h)_{ij} P_L S_{\gtrless}(x, y) P_R \mathcal{S}_{\gtrless}(x, y) \\ &+ (h^\dagger h)_{ij}^* P_R P \bar{S}_{\gtrless}(\bar{x}, \bar{y}) P P_L \bar{\Delta}_{\gtrless}(\bar{x}, \bar{y})]. \end{aligned} \quad (\text{E.18})$$

#### Wigner representation

Performing the Wigner transformation of (E.18) we find,

$$\begin{aligned} \Pi_{\gtrless}^{ij}(t, q) &= -g_w \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) [(h^\dagger h)_{ij} P_L S_{\gtrless}(t, p) P_R \Delta_{\gtrless}(t, k) \\ &+ (h^\dagger h)_{ij}^* P_R P \bar{S}_{\gtrless}(t, \bar{p}) P P_L \bar{\Delta}_{\gtrless}(t, \bar{k})], \end{aligned} \quad (\text{E.19})$$

where we used the fact the the propagators and self-energies are independent of  $\vec{X}$  in a homogeneous and isotropic medium. The quantity entering the CP-violating parameters is the

spectral self-energy. It can be easily read off from (E.19),

$$(E.20)$$

$$\Pi_\rho^{ij}(t, q) = -\frac{g_w}{16\pi} [(h^\dagger h)_{ij} P_L \Pi_\rho(t, q) P_R + (h^\dagger h)_{ij}^* P_L \bar{\Pi}_\rho(t, \bar{q}) P_R], \quad (E.21)$$

where we have defined the two functions

$$\Pi_\rho(t, q) \equiv 16\pi \int d\Pi_{pk}^4 (2\pi) \delta(q - p - k) [S_{>}(t, p) \Delta_{>}(t, k) - S_{<}(t, p) \Delta_{<}(t, k)], \quad (E.22a)$$

$$\bar{\Pi}_\rho(t, \bar{q}) \equiv 16\pi \int d\Pi_{pk}^4 (2\pi) \delta(q - p - k) [\bar{S}_{>}(t, \bar{p}) \bar{\Delta}_{>}(t, \bar{k}) - \bar{S}_{<}(t, \bar{p}) \bar{\Delta}_{<}(t, \bar{k})]. \quad (E.22b)$$

In a CP-symmetric medium, which the early universe was to a very good approximation, the lepton and Higgs propagators satisfy,

$$S_{\gtrless}(t, p) = \bar{S}_{\gtrless}(t, p), \quad \Delta_{\gtrless}(t, k) = \bar{\Delta}_{\gtrless}(t, k). \quad (E.23)$$

The homogeneity and isotropy of the early universe furthermore imply, that there is no dependence on the momentum direction and the spatial central coordinate so that  $\bar{\Delta}_{\gtrless}(t, \bar{k}) = \Delta_{\gtrless}(t, k)$ . In the lepton case we need to take into account the  $\not{p}$  term,

$$P_L \not{p} P_R = P_L P \not{p} P P_R = P P_R \not{p} P_L P. \quad (E.24)$$

Therefore, in a homogeneous, isotropic and CP-symmetric medium, we can rewrite (E.21) as a sum of left and right projections of the same ‘‘vector’’ integral  $L_\rho(t, q)$ ,

$$\Pi_\rho^{ij}(t, q) = -\frac{g_w}{16\pi} [(h^\dagger h)_{ij} P_L + (h^\dagger h)_{ij}^* P_R] L_\rho(t, q), \quad (E.25)$$

with,

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) [\mathbf{S}_{>}(t, p) \Delta_{>}(t, k) - \mathbf{S}_{<}(t, p) \Delta_{<}(t, k)] \not{p}, \quad (E.26)$$

where we have factored out the ‘‘scalar’’ part  $\mathbf{S}_{\gtrless}(t, p)$  of the lepton propagator. Inserting the leading order term of the eQP approximation for the lepton and Higgs propagators into the loop integral (E.26) we find,

$$L_\rho(t, q) = 16\pi \int d\Pi_{pk}^4 (2\pi)^4 \delta(q - p - k) \tilde{\mathbf{S}}_\rho(t, p) \tilde{\Delta}_\rho(t, k) \times [(1 - F_\ell(t, p))(1 + F_\phi(t, k)) + F_\ell(t, p)F_\phi(t, k)]. \quad (E.27)$$

In the quasiparticle approximation the eQP spectral functions of the lepton and Higgs fields are represented by a delta function,

$$\tilde{\mathbf{S}}_\rho(t, p) = \text{sign}(p^0) (2\pi) \delta(p^2 - m_\ell^2), \quad \tilde{\Delta}_\rho(t, k) = \text{sign}(k^0) (2\pi) \delta(k^2 - m_\phi^2). \quad (E.28)$$

The delta functions in (E.28) can be decomposed in a sum of two delta functions, one with positive frequency, and one with negative frequency. Therefore the product of the two spectral functions in (E.27) gives raise to four terms. We need then to specify the kinetic conditions

to determine which of the four terms satisfy the delta function ensuring energy conservation. We consider first the case where  $M_i > m_\ell + m_\phi$  and  $q^2$  positive. In that case the loop integral (E.27) takes the form,

$$L_\rho(t, q) = 16\pi \int d\Pi_{\ell\phi}^{pq} \tilde{\mathcal{F}}_{(N_i) \leftrightarrow \ell\phi}^{q;pk} \not{p}, \quad (\text{E.29})$$

where we assumed  $q^0 > 0$  and defined the combination of distribution function,

$$\begin{aligned} \tilde{\mathcal{F}}_{(X)ab\dots\leftrightarrow ij\dots}^{p_X p_a p_b \dots p_i p_j \dots} &\equiv (2\pi) \delta(p_X + \sum_a p_a - \sum_i p_i) \\ &\times \left[ \prod_a f_a^{p_a} \prod_i (1 \pm f_i^{p_i}) + \prod_i f_i^{p_i} \prod_a (1 \pm f_a^{p_a}) \right]. \end{aligned} \quad (\text{E.30})$$

Inserting the equilibrium distribution functions into (E.29) and integrating out the energy-momentum conserving delta-function we obtain for the Lorentz components of the loop integral [63],

$$\begin{aligned} L_\rho^0 &= \frac{2T}{y} I_1(y^0, y), \\ \vec{L}_\rho &= \frac{\vec{q}}{|\vec{q}|} \frac{2T}{y^2} \left[ y^0 I_1(y^0, y) - \frac{1}{2} (y^{02} - y^2) (1 + x_\ell - x_\phi) I_0(y^0, y) \right], \end{aligned} \quad (\text{E.31})$$

where  $y^0 \equiv q^0/T$ ,  $y \equiv |\vec{q}|/T$ ,  $x_\ell \equiv m_\ell^2/q^2$  and  $x_\phi \equiv m_\phi^2/q^2$ . The integral functions  $I_n(y^0, y)$  are defined by,

$$I_n(y^0, y) \equiv \int_{z^-}^{z^+} dz z^n \left( 1 + \frac{1}{e^{y^0-z} - 1} - \frac{1}{e^z + 1} \right), \quad (\text{E.32})$$

with the integration limits given by,

$$z^\pm \equiv \frac{1}{2} \left[ y^0 (1 + x_\ell - x_\phi) \pm y \lambda^{\frac{1}{2}} (1, x_\ell, y_\ell) \right]. \quad (\text{E.33})$$

For  $M_i > m_\ell + m_\phi$  and  $q^2 > 0$  but negative  $q^0$  the components of  $L_\rho$  are related to the one above by  $L_\rho^0(-q^0, \vec{q}) = L_\rho^0(q^0, \vec{q})$  and  $\vec{L}_\rho(-q^0, \vec{q}) = -\vec{L}_\rho(q^0, \vec{q})$ .

To evaluate the  $t$ - and  $u$ -channel processes we also need to calculate  $L_\rho$  for negative  $q^2$ . In that case the loop integral (E.27) reads,

$$L_\rho(t, q) = 16\pi \int d\Pi_{\ell\phi}^{pk} \left[ (2\pi)^4 \delta(q + k - p) + (2\pi)^4 \delta(q - k + p) \right] [f_\ell^p + f_\phi^k] \not{p}. \quad (\text{E.34})$$

Although it is in principle possible to retain the thermal masses of leptons and the Higgs in the calculation, the resulting expressions are quite lengthy in this case. Neglecting the thermal



masses we obtain for the Lorentz components of  $L_\rho$  in this regime:

$$\begin{aligned} L_\rho^0 &= \frac{2T}{y} \sum_{\pm} I_1^\pm(y_0, y), \\ \vec{L}_\rho &= \frac{\vec{q}}{|\vec{q}|} \frac{2T}{y^2} \sum_{\pm} \left[ y_0 I_1^\pm(y_0, y) - \frac{1}{2}(y_0^2 - y^2) I_0^\pm(y_0, y) \right], \end{aligned} \quad (\text{E.35})$$

where the integral functions are given by

$$I_n^\pm(y_0, y) \equiv \int_{\frac{1}{2}(y \pm y_0)}^{\infty} dz z^n \left( \frac{1}{e^z + 1} + \frac{1}{e^{z \mp y_0} - 1} \right).$$

Note that in this regime  $y > y_0$  and therefore the lower integration limit is positive.

To compute the scattering amplitude we need to calculate the product  $qL_\rho$ . For  $q^2 > 0$  we find:

$$qL_\rho = q^2(1 + x_\ell - x_\phi) y^{-1} I_0(y_0, y), \quad (\text{E.36})$$

whereas the corresponding expression for  $q^2 < 0$  reads

$$qL_\rho = q^2 y^{-1} \sum_{\pm} I_0^\pm(y_0, y). \quad (\text{E.37})$$

At low temperatures (E.37) is exponentially small and vanishes in the vacuum limit.

The top quark also contributes to the function  $L_\rho$  at NLO. Substituting the off-shell Higgs propagator into (E.26), we find,

$$\begin{aligned} L_\rho^t(t, q) &= 16\pi \int d\Pi_{pp_t p_Q}^4 (2\pi)^4 \delta(q + p_t - p - p_t) \tilde{\mathbf{S}}_\rho(p) \tilde{\mathbf{S}}_{t\rho}(p_t) \tilde{\mathbf{S}}_{Q\rho}(p_Q) g_q |\lambda_t|^2 (2p_t p_Q) \\ &\quad \times \Delta_{R+A}^2(p_t - p_Q) \left[ -F_Q^<(p_Q) F_\ell^>(p) F_t^>(p_t) + F_\ell^<(p) F_t^<(p_t) F_Q^>(p_Q) \right], \end{aligned} \quad (\text{E.38})$$

where we have used the eQP approximation for the lepton, top and quark fields. Integrating out the delta-functions in the eQP spectral functions, we can write (E.38) as a sum of four terms (assuming  $q^0 > 0$ ),

$$\begin{aligned} L_\rho^t(t, q) &= 16\pi \int d\Pi_{\ell t Q}^{pp_t p_Q} \left[ \tilde{\mathcal{F}}_{(N_i)_{Q \leftrightarrow \ell t}}^{qpQ:pp_t} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} + \tilde{\mathcal{F}}_{(N_i)_{\bar{t} \leftrightarrow \ell Q}}^{qp_t:pp_Q} \Delta_{R+A}^2(p_t - p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} \right. \\ &\quad \left. + \tilde{\mathcal{F}}_{(N_i)_{\bar{\ell} \leftrightarrow t Q}}^{qp:pt p_Q} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} + \tilde{\mathcal{F}}_{(N_i)_{\ell \leftrightarrow \bar{t} Q}}^{q:pp_t p_Q} \Delta_{R+A}^2(p_t + p_Q) \Xi_{\phi^* Q \leftrightarrow t}^{(T)} \right] \not{p}. \end{aligned} \quad (\text{E.39})$$

The first three terms correspond to the scattering processes and the last one to the three-body decay. In the vacuum limit only the last term survives.

We consider now the regime  $m_\phi > M_i + m_\ell$ , where the Higgs field can decay into a right-handed neutrino and a lepton. In that case the function  $L_\rho$  reads,

$$L_\rho(t, q) = 16\pi \int d\Pi_{\ell\phi}^{pk} \tilde{\mathcal{F}}_{(N_i)_{\ell \leftrightarrow \phi}}^{qp;k} \not{p}, \quad (\text{E.40})$$

where we assumed  $q^0 > 0$ . After integrating out the delta-function in  $\tilde{\mathcal{F}}_{(N_i)\ell\leftrightarrow\bar{\phi}}^{qp;k}$  we obtain,

$$\begin{aligned} L_\rho^0(t, q) &= \frac{2T}{y} J_1(y_0, y), \\ \vec{L}_\rho(t, q) &= \frac{\vec{q}}{|\vec{q}|} \frac{2T}{y^2} \left[ y_0 J_1(y_0, y) - \frac{1}{2} (x_\phi - x_\ell - 1) J_0(y_0, y) \right], \end{aligned} \quad (\text{E.41})$$

where the integral function is defined as,

$$J_n(y^0, y) \equiv \int_{z^-}^{z^+} dz z^n \left( \frac{1}{e^z + 1} + \frac{1}{e^{z+y^0} - 1} \right), \quad (\text{E.42})$$

with the integration limit,

$$z^\pm \equiv \frac{1}{2} \left[ y^0 (x_\phi - x_\ell - 1) \pm y \lambda^{\frac{1}{2}} (1, x_\ell, y_\ell) \right]. \quad (\text{E.43})$$

## E.4 Higgs self-energy

### Coordinate representation

At one-loop level the Higgs self-energy receives two contributions, one from the quark loop, and one from the lepton-heavy neutrino loop. The later is suppressed due to the smallness of the heavy neutrino Yukawa couplings  $h_{\alpha i}$  and is neglected here. In a  $SU(2)_L$  symmetric state, the Higgs, leptons and quark doublet propagators are proportional to the identity in the  $SU(2)_L$  space, which implies, in particular, that the  $SU(2)_L$  matrix structure of the Higgs self-energy is trivial. The top quark contribution is obtained by functional differentiation of the 2PI effective action (E.2b),

$$\Omega_{ab}(x, y) = \Omega(x, y) \delta_{ab} = \frac{\delta i \Gamma_{2PI}^{(2.2)}}{\Delta_{ba}(y, x)} = |\lambda_t|^2 \text{tr} [\hat{S}_Q(y, x) P_R \hat{S}_t(x, y) P_L] \delta_{ab}, \quad (\text{E.44})$$

where the trace acts on spinor and colour space. The propagators of the quarks are diagonal in colour space,

$$S_{QAB}(x, y) \equiv S_Q(x, y) \delta_{AB}, \quad S_{tAB}(x, y) \equiv S_t(x, y) \delta_{AB}. \quad (\text{E.45})$$

Performing explicitly the trace over the colour indices we obtain for the Higgs self-energy,

$$\Omega(x, y) = g_q |\lambda_t|^2 \text{tr} [S_Q(y, x) P_R S_t(x, y) P_L] \delta_{ab}, \quad (\text{E.46})$$

where  $g_q = \delta_{AB} \delta_{BA} = 3$ . The trace in (E.46) acts now only in spinor space. Its Wightman components of the Higgs self-energy (E.46) are given by,

$$\Omega_{\gtrless}(x, y) = g_q |\lambda_t|^2 \text{tr} [S_{Q\lesseqgtr}(y, x) P_R S_{t\gtrless}(x, y) P_L]. \quad (\text{E.47})$$

**Wigner representation**

Performing a Wigner transform of (E.47) we obtain,

$$\Omega_{\geq}(t, k) = g_q |\lambda_t|^2 \int d\Pi_{p_Q p_t}^4 (2\pi)^4 \delta(k + p_Q - p_t) \text{tr} [S_{Q\leq}(t, p_Q) P_R S_{t\geq}(t, p_t) P_L], \quad (\text{E.48})$$

where we used the fact that, in a homogeneous medium, the propagators and self-energies are independent of the spatial coordinate  $\vec{X}$ .



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