Mean Value Coordinates for Arbitrary Spherical Polygons and Polyhedra in $\mathbb{R}^3$

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Abstract

Since their introduction, mean value coordinates enjoy ever increasing popularity in computer graphics and computational mathematics because they exhibit a variety of good properties. Most importantly, they are defined in the whole plane which allows interpolation and extrapolation without restrictions. Recently, mean value coordinates were generalized to spheres and to $\mathbb{R}^3$. We show that these spherical and 3D mean value coordinates are well-defined on the whole sphere and the whole space $\mathbb{R}^3$, respectively.

Keywords

Barycentric coordinates, interpolation, extrapolation

Contents

1 Introduction

While certain types of generalized barycentric coordinates for planar polygons were already known for a long time (Wachspress coordinates [10]), a new interest in them was initiated by the invention of the mean value coordinates [1]. A general theory of barycentric coordinates was developed, and it could be shown that Wachspress and mean value coordinates are basically the only homogeneous three-point coordinates that are positive for all convex polygons [2]. Moreover, it turned out that mean value coordinates, unlike the Wachspress coordinates, can be defined in the whole plane, for convex and non-convex polygons [4]. All these properties made mean value coordinates an increasingly popular tool for mesh deformation and many other applications [5, 8, 9].

Naturally, several attempts were made to generalize mean value coordinates to higher dimensions. First, a definition for polyhedra with triangular faces was obtained [3, 6, 7]; later, constructions for spherical polygons and for arbitrary polyhedra were developed [8]. Nevertheless, a proof, that these spherical mean value coordinates and 3D mean value coordinates are defined for the whole sphere and the whole space $\mathbb{R}^3$, was not yet given. In the following chapters, we will derive such a proof.

2 Planar Mean Value Coordinates

In this chapter, we give a definition of the mean value coordinates and a proof that they are well-defined that is easy to generalize to higher dimensions.

2.1 Definition. A (planar) *polygon* P is given by a finite sequence of distinct vertices $\mathbf{v}_i \in \mathbb{R}^2$ such that its edges $(\mathbf{v}_i,\ \mathbf{v}_{i+1})$ do not intersect. For an edge $\mathbf{e}$, $V(\mathbf{e})$ denotes the set of indices i such that $\mathbf{v}_i$ is incident to $\mathbf{e}$, and $V(P)$ denotes the set of all vertex indices.

2.2 Algorithm (planar mean value coordinates). *Mean value coordinates for a point $\mathbf{x} \in \mathbb{R}^2$ with respect to a polygon P can be defined in the following way [1]:*

- *An edge vector $\mathbf{v_e}$ is assigned to each edge $\mathbf{e}$ of the polygon such that $\sum_{\mathbf{e}} \mathbf{v_e} = \mathbf{0}$. ($\mathbf{v_e}$ can be considered as some kind of edge normal.)*

  For an edge $\mathbf{e} = (\mathbf{v}_i,\ \mathbf{v}_{i+1})$, let $\mathbb{S}^1_{\mathbf{e}} \subset \mathbb{S}^1$ be the oriented circular arc with end points $\frac{\mathbf{v}_i - \mathbf{x}}{\|\mathbf{v}_i - \mathbf{x}\|}$ and $\frac{\mathbf{v}_{i+1} - \mathbf{x}}{\|\mathbf{v}_{i+1} - \mathbf{x}\|}$. Let $\mathbf{n} : \mathbb{S}^1 \to \mathbb{R}^2$ be the outward unit normal vector of the circle. Then $\mathbf{v_e}$ is defined as the integral of $\mathbf{n}$ over $\mathbb{S}^1_{\mathbf{e}}$, see Figure 2.1 (left):

$$\mathbf{v_e} := \int_{\mathbb{S}^1_{\mathbf{e}}} \mathbf{n} dS.$$

- *The edge vectors are distributed to their respective edge vertices by the unique weights $\mu_{\mathbf{e},j}$ such that $\mu_{\mathbf{e},i}(\mathbf{v}_i - \mathbf{x}) + \mu_{\mathbf{e},i+1}(\mathbf{v}_{i+1} - \mathbf{x}) = \mathbf{v_e}$ for an edge $\mathbf{e} = (\mathbf{v}_i,\ \mathbf{v}_{i+1})$.*

- *The weights at each vertex $\mathbf{v}_i$ are cumulated as $w_i := \mu_{\mathbf{e}_{i-1},i} + \mu_{\mathbf{e}_i,i}$ where $\mathbf{e}_i$ denotes the edge $(\mathbf{v}_i,\ \mathbf{v}_{i+1})$.*

- *The weights are normalized to form a partition of unity:*

$$\lambda_i := \frac{w_i}{\sum_j w_j}. \tag{2.1}$$

It is straightforward to show that the definition of $\mathbf{v_e}$ has the following closed form solution [1]

$$\mathbf{v_e} = \tan\frac{\alpha_i}{2}\Big(\frac{\mathbf{v}_i - \mathbf{x}}{\|\mathbf{v}_i - \mathbf{x}\|} + \frac{\mathbf{v}_{i+1} - \mathbf{x}}{\|\mathbf{v}_{i+1} - \mathbf{x}\|}\Big) \tag{2.2}$$

where α_i denotes the angle between $\mathbf{v}_i - \mathbf{x}$ and $\mathbf{v}_{i+1} - \mathbf{x}$. Distributing these edge vectors to the vertices yields the formula proposed earlier [1]:

$$w_i = \frac{\tan\frac{\alpha_{i-1}}{2} + \tan\frac{\alpha_i}{2}}{\|\mathbf{v}_i - \mathbf{x}\|}. \tag{2.3}$$

To show that this yields well-defined coordinates, it is necessary to show that the denominator in the normalization step (Equation (2.1)) cannot become zero. As shown in [4], this can be done in two steps:

- A refinement lemma is proven that states that the denominator does not change if we refine our polygon by including additional vertices.
- For a particular refinement of the polygon, it is shown that the denominator doesn't vanish.

We will give an alternative proof for the first step. It has the advantage of being more general than the original proof in [4], and it can easily be generalized to higher dimensions. The second step proceeds as in [4].

2.3 Lemma. *Let $\mathbf{u} \in \mathbb{R}^2$ be a point, let $\mathbf{v}_j \in \mathbb{R}^2$ be a set of points that lie on a common line $\mathbf{e}$, and let $\mathbf{u} = \sum_j \mu_j \mathbf{v}_j$ for some coefficients μ_j. Denote by $(\mathbf{n}_k)_{k=0,1}$ an orthonormal basis of $\mathbb{R}^2$ such that $\mathbf{n}_0$ is the normal vector of $\mathbf{e}$, and let ξ_k and ζ_{jk} be the coefficients of $\mathbf{u}$ and $\mathbf{v}_j$, respectively: $\mathbf{u} = \sum_k \xi_k \mathbf{n}_k$, $\mathbf{v}_j = \sum_k \zeta_{jk}\mathbf{n}_k$. We define $\zeta_0 := \zeta_{00} = \zeta_{10} > 0$. (The last inequality can be achieved by choosing the orientation of $\mathbf{n}_0$ appropriately.) This setting is sketched in Figure 2.1 (right).*

Then $\sum_j \mu_j = \frac{\xi_0}{\zeta_0}$.

Proof.

$$\mathbf{u} = \sum_j \mu_j \mathbf{v}_j = \sum_j \mu_j \zeta_0 \mathbf{n}_0 + \sum_j \mu_j \zeta_{j1}\mathbf{n}_1.$$

Since $\mathbf{u}$ has a unique representation in the basis $(\mathbf{n}_k)$, the claim follows. □

2.4 Lemma. *In the situation of Lemma 2.3, assume that $\mathbf{u}$ is given as the sum of points $\mathbf{u} = \sum_i \mathbf{u}_i$, $\mathbf{u}_i = \sum_j \mu_{ij}\mathbf{v}_j \in \mathbb{R}^2$.*

Then $\sum_j \mu_j = \sum_{ij} \mu_{ij}$.

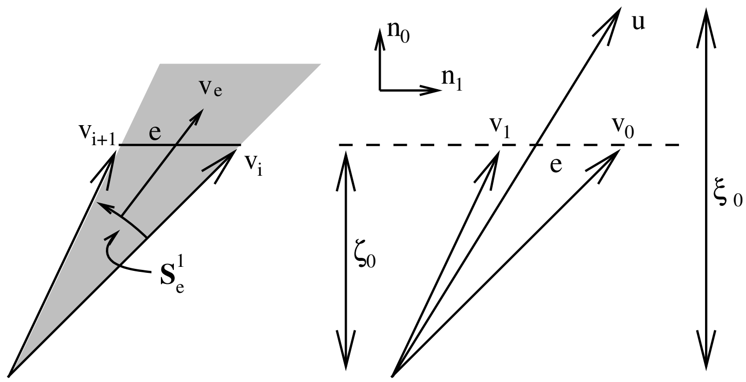

Figure 2.1: Left: Construction of the edge vector $\mathbf{v_e}$. Right: Some notation.

Proof. Let $\mathbf{u}_i = \sum_k \xi_{ik}\mathbf{n}_k$ be the representations of $\mathbf{u}_i$ in the basis $(\mathbf{n}_k)$. From the unique representation of $\mathbf{u} = \sum_k \xi_k \mathbf{n}_k$ in this basis and the fact that $\mathbf{u} = \sum_i \mathbf{u}_i$, we can conclude $\xi_0 = \sum_i \xi_{i0}$. Using Lemma 2.3 for $\mathbf{u}$ and the $\mathbf{u}_i$, we obtain

$$\sum_j \mu_j = \frac{\xi_0}{\zeta_0} = \sum_i \frac{\xi_{i0}}{\zeta_0} = \sum_{ij} \mu_{ij}.$$

□

2.5 Definition. A *refinement* $\widehat{P}$ of a polygon P is a polygon that contains all the vertices of P in the same order and additional vertices that lie on edges of P such that P and $\widehat{P}$ bound the same area in $\mathbb{R}^2$.

2.6 Lemma (refinement of planar polygons). *Let P be a polygon, and let $\widehat{P}$ be a refinement of P. Let w_i and $\widehat{w}_i$ be the weights in step 3 of Algorithm 2.2 for P and $\widehat{P}$.*

Then $\sum_{i\in V(P)} w_i = \sum_{i\in V(\widehat{P})} \widehat{w}_i$.

Proof. Let $\mathbf{e}$ be an edge of P, and let $\widehat{E}$ be the set of edges in $\widehat{P}$ that compose the refinement of $\mathbf{e}$. From the definition of $\mathbf{v_e}$, it is obvious that $\mathbf{v_e} = \sum_{\widehat{\mathbf{e}}\in\widehat{E}} \mathbf{v}_{\widehat{\mathbf{e}}}$. Therefore, Lemma 2.4 implies that $\sum_{i\in V(\mathbf{e})} \mu_{\mathbf{e},i} = \sum_{\widehat{\mathbf{e}}\in\widehat{E},\ i\in V(\widehat{\mathbf{e}})} \widehat{\mu}_{\widehat{\mathbf{e}},i}$. By taking the sum over all edges $\mathbf{e}$, we obtain the claim. □

Note, that this lemma is not restricted to mean value coordinates but applies to all kind of coordinates as defined in [7]. The reason is that the edge vector for an edge $\mathbf{e}$ can always be expressed as the sum of the edge vectors of its refinement. The following lemma, in contrast, is in general not true if applied to other coordinates than mean value coordinates.

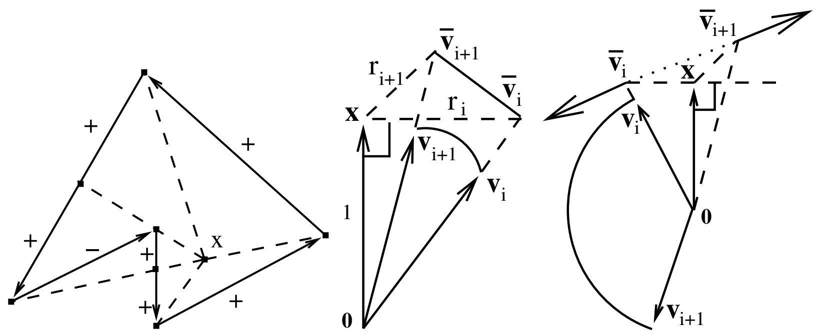

Figure 2.2: Left: Refinement of a polygon such that all edges with negative weight (-) are counterbalanced by an edge with positive weight (+). Middle and Right: Projection of an edge of a spherical polygon to the tangent plane at **x**.

2.7 Lemma. *Let* **e** *be an edge of a polygon P. Let $\mu_{\mathbf{e},i}$ be the coefficients of its edge vector* $\mathbf{v}_\mathbf{e}$.

Then $\sum_i \mu_{\mathbf{e},i}$ is greater than zero if and only if $\mathbb{S}^1_\mathbf{e}$ is positively oriented.

Proof. Without loss of generality let $\mathbb{S}^1_\mathbf{e}$ be positively oriented. Then $\mathbf{v}_\mathbf{e}$ is contained in the cone defined by the convex hull of the $\mathbf{v}_j - \mathbf{x}$. (The cone is indicated by the shaded area in the left part of Figure 2.1.) Since all the $\zeta_{j0} = \zeta_0$ are greater than zero by definition of $\mathbf{n}_0$, ξ_0 must be greater than zero as well. Therefore, the claim follows from Lemma 2.3. □

2.8 Theorem ([4]). *Let P be a polygon. Then the planar mean value coordinates with respect to P are well-defined in $\mathbb{R}^2$.*

Proof. Consider the denominator $W := \sum_i w_i$ in Equation (2.1). We have to show that W does not vanish if computed for an arbitrary $\mathbf{x} \in \mathbb{R}^2$. We refine P by adding all intersection points of rays from $\mathbf{x}$ through the vertices $\mathbf{v}_i$ with the edges of P, see the left hand side of Figure 2.2. According to Lemma 2.6, this does not change W. If we now split the denominator of the refined polygon into partial sums $\sum_{i \in V(\mathbf{e})} \mu_{\mathbf{e},i}$ associated to the edges **e**, we know from Lemma 2.7 that the sign of these partial sums depends only on the orientation of $\mathbb{S}^1_\mathbf{e}$. From Formula (2.2) we can see that the absolute value of the partial sums decreases with increasing distance to **x**. Therefore, W splits into sums of alternating sign and decreasing absolute value starting with a positive number if **x** is inside and a negative number if **x** is outside the polygon. Hence, W is positive inside and negative outside the polygon. In particular, W is not equal to zero, and the theorem is proven. (For points **x** on an edge exists a continuous extension of the definition in Algorithm 2.2.) For details see [4]. □

3 Spherical mean value coordinates

In this chapter, we show that the spherical mean value coordinates that we introduced in [8] are well-defined in the whole space $\mathbb{R}^3$.

3.1 Definition. A *spherical polygon* P consists of a finite sequence of distinct vertices $\mathbf{v}_i \in \mathbb{S}^2$ located on a sphere and a set of non-intersecting edges $(\mathbf{v}_i, \mathbf{v}_{i+1})$ that connect vertices $\mathbf{v}_i$ and $\mathbf{v}_{i+1}$ by geodesic lines (these are the arcs of great circles on the sphere).

It is *admissible* if it contains no antipodal points.

We consider a spherical polygon P on the unit sphere centered at the origin. Let $\mathbf{x}$ be a point on the sphere. To obtain its spherical coordinates, P is centrally projected to the tangent plane $T_{\mathbf{x}}\mathbb{S}^2$ at $\mathbf{x}$ to the sphere. Let $\overline{\mathbf{v}}_i$ be the intersection points of the line given by $\mathbf{v}_i$ and $T_{\mathbf{x}}\mathbb{S}^2$, see Figure 3.1 and Figure 2.2 (middle). The points $\overline{\mathbf{v}}_i$ determine a polygon $\overline{P}$ in the plane $T_{\mathbf{x}}\mathbb{S}^2$. Now, we can compute the planar mean value coordinates $\overline{\lambda}_i$ of $\mathbf{x}$ with respect to $\overline{P}$. The spherical mean value coordinates λ_i of $\mathbf{x}$ are defined by

$$\sum_i \lambda_i \mathbf{v}_i = \mathbf{x}, \qquad \lambda_i := \langle \mathbf{v}_i, \overline{\mathbf{v}}_i \rangle \overline{\lambda}_i$$

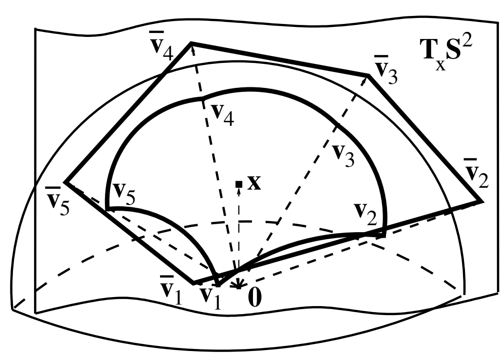

Figure 3.1: Construction of spherical barycentric coordinates.

where $\langle\cdot,\cdot\rangle$ denotes the usual scalar product in $\mathbb{R}^3$. Note that $\langle \mathbf{v}_i, \overline{\mathbf{v}}_i\rangle$ is just $\pm\|\overline{\mathbf{v}}_i\|$. Although this value becomes very large and finally undefined if the angle θ_i between $\mathbf{x}$ and $\mathbf{v}_i$ approaches $\frac{\pi}{2}$, this is compensated by a shrinkage of $\overline{\lambda}_i$ such that the definition of λ_i can be extended continuously to the case $\theta_i = \frac{\pi}{2}$. Note that these coordinates can be used for exact interpolation of spherical harmonics of degree one.

Finally, we remark that these coordinates can also be extended to vectors $\mathbf{v}_i$ and $\mathbf{x}$ of arbitrary length by defining

$$\lambda_i(\mathbf{x}) := \frac{\|\mathbf{x}\|}{\|\mathbf{v}_i\|}\langle \mathbf{v}_i, \overline{\mathbf{v}}_i\rangle \overline{\lambda}_i. \tag{3.1}$$

More details of this construction can be found in [8].

Since the planar mean value coordinates $\overline{\lambda}_i$ are well-defined and the spherical mean value coordinates are a scaled version of them, we can basically conclude that spherical mean value coordinates are well-defined, too. When doing so, one new difficulty occurs: when a spherical polygon is projected to the tangent plane, more general polygons may occur than considered so far, see Figure 2.2 (right). We need to show that the results from the previous chapter still hold in this case.

3.2 Definition. A *projective polygon* consists of a finite sequence of distinct vertices $\mathbf{v}_i \in \mathbb{R}^2$ and a sequence of non-intersecting edges, given either by $\mathbf{e}_i = \{\mathbf{x} = p\mathbf{v}_i + q\mathbf{v}_{i+1} \in \mathbb{R}^2 | p+q = 1,\ p \in [0,1]\}$ or by $\mathbf{e}_i = \{\mathbf{x} = p\mathbf{v}_i + q\mathbf{v}_{i+1} \in \mathbb{R}^2 | p+q = 1,\ p \in \mathbb{R}\setminus(0,1)\}$.

Such a polygon can be represented in polar coordinates as a sequence of distinct vertices $\mathbf{v}_j = r_j e^{i\phi_j}$ with $\phi_j \in [0, 2\pi)$ and $r_j \in \mathbb{R}$ (including the negative numbers). Using this notation, the complement of the line segment $(\mathbf{v}_j, \mathbf{v}_{j+1})$ is chosen as edge $\mathbf{e}_j$ if and only if the signs of r_j and r_{j+1} differ, see Figure 3.2. Without loss of generality let $\mathbf{x}$ be at the origin. Then we can define mean value coordinates for projective polygons just as in Algorithm 2.2, and Formula (2.3) becomes

$$w_i = \frac{\tan\frac{\alpha_{i-1}}{2} + \tan\frac{\alpha_i}{2}}{r_i}$$

with $\alpha_i = \phi_{i+1} - \phi_i$.

Since projective polygons should actually be defined for the real projective plane $\mathbb{RP}^2 \supset \mathbb{R}^2$, we can use the same proofs as in Chapter 2 if we take care of the infinite points. The idea is that infinite points have infinite norm and therefore weight zero. More accurately, we had to consider the limit of points that approach infinity on a given edge. Doing this, we find out that Lemmas 2.3–2.6 still hold. Lemma 2.7 has to be stated more precisely as

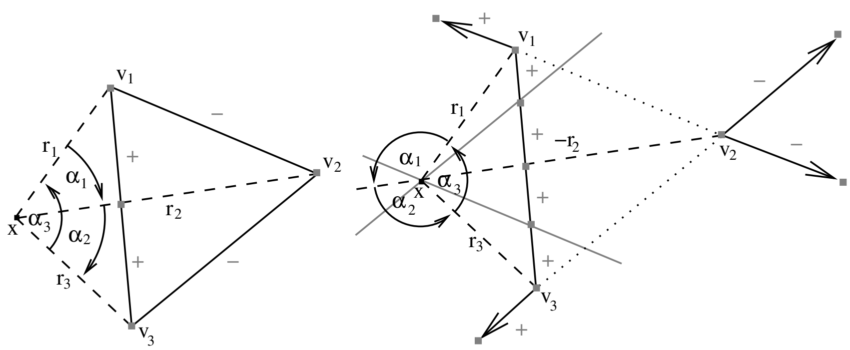

Figure 3.2: Two projective triangles with the same vertices but different edges. The refinement in Theorem 3.4 and the sign of the edge weights are indicated in grey.

3.3 Lemma. *Let* $\mathbf{e}$ *be an edge that does not contain an infinite point in its interior. Let* $\mu_{\mathbf{e},i}$ *be the coefficients of the respective edge vector* $\mathbf{v_e}$.

Then $\sum_i \mu_{\mathbf{e},i}$ *is greater than zero if and only if the respective angle* α_i *and the distances* r_i *and* r_{i+1} *have the same sign.*

We can now state

3.4 Theorem. *Planar mean value coordinates for projective polygons are well-defined.*

Proof. In a first step, the projective polygon is refined at all infinite points. Then, no edge contains an infinite point in its interior any more, and we can proceed as in the proof of Theorem 2.8 using Lemmas 2.6 and 3.3. □

3.5 Corollary. *Let* P *be an admissible spherical polygon. Then the spherical mean value coordinates with respect to* P *are well-defined on* $\mathbb{S}^2$.

Proof. Since P is admissible, its projection $\overline{P}$ is a projective polygon (in particular, the projection is still non-intersecting). Hence, P is well-defined by Theorem 3.4. □

4 3D mean value coordinates for arbitrary polyhedra

In this chapter, we show that 3D mean value coordinates are well-defined. It turns out that the proof can be given along the same lines as in Chapter 2 for planar mean value coordinates. Therefore, we indicate only the necessary changes.

4.1 Definition. A *polyhedron P* consists of a finite set of distinct vertices $\mathbf{v}_i \in \mathbb{R}^3$ and a set of non-intersecting faces. $F(\mathbf{v}_i)$ denotes the set of faces incident to $\mathbf{v}_i$. For a face $\mathbf{f}$, $V(\mathbf{f})$ denotes the set of indices i such that $\mathbf{v}_i$ is incident to $\mathbf{f}$, and $V(P)$ denotes the set of all vertex indices.

4.2 Algorithm (3D mean value coordinates). *Mean value coordinates for a point* $\mathbf{x} \in \mathbb{R}^3$ *with respect to a polyhedron P can be defined in the following way [8]:*

- *A face vector* $\mathbf{v}_\mathbf{f}$ *is assigned to each face* $\mathbf{f}$ *of the polyhedron such that* $\sum_\mathbf{f} \mathbf{v}_\mathbf{f} = \mathbf{0}$. *(*$\mathbf{v}_\mathbf{f}$ *can be considered as some kind of face normal.)*

  For a face $\mathbf{f}$*, let* $\mathbb{S}^2_\mathbf{f} \subset \mathbb{S}^2$ *be the spherical patch obtained by projecting* $\mathbf{f}$ *to the unit sphere with center* $\mathbf{x}$*. Choose the orientation of* $\mathbb{S}^2_\mathbf{f}$ *consistently with the orientation of the boundary of* $\mathbf{f}$*. Let* $\mathbf{n} : \mathbb{S}^2 \to \mathbb{R}^3$ *be the outward unit normal vector of the sphere. Then* $\mathbf{v}_\mathbf{f}$ *is defined as the integral of* $\mathbf{n}$ *over* $\mathbb{S}^2_\mathbf{f}$*:*

  $$\mathbf{v}_\mathbf{f} := \int_{\mathbb{S}^2_\mathbf{f}} \mathbf{n}\,dS.$$

  A closed form for this integral is given in [8].

- *For each face* $\mathbf{f}$*, its face vector* $\mathbf{v}_\mathbf{f}$ *is distributed to the respective face vertices using the spherical mean value coordinates* $\mu_{\mathbf{f},i}$ *with respect to the boundary polygon of* $\mathbf{f}$*:* $\sum_{i \in V(\mathbf{f})} \mu_{\mathbf{f},i}(\mathbf{v}_i - \mathbf{x}) = \mathbf{v}_\mathbf{f}$*. Since* $\mathbf{f}$ *is planar, its projection to the unit sphere centered at* $\mathbf{x}$ *(the boundary of* $\mathbb{S}^2_\mathbf{f}$*) is an admissible spherical polygon. Therefore, the* $\mu_{\mathbf{f},i}$ *are well-defined by Corollary 3.5.*

- *The weights at each vertex* $\mathbf{v}_i$ *are cumulated as* $w_i := \sum_{\mathbf{f}\in F(\mathbf{v}_i)} \mu_{\mathbf{f},i}$.
- *The weights are normalized to form a partition of unity (2.1).*

It is now easy to see that analogons of Lemmas 2.3–2.6 can be proven for polyhedra by replacing lines and edges by planes and faces and so on. We obtain

4.3 Definition. A *refinement* $\widehat{P}$ of a polyhedron P is a polyhedron that contains all the vertices of P and additional vertices and edges that lie on faces of P such that P and $\widehat{P}$ bound the same volume in $\mathbb{R}^3$.

4.4 Lemma (refinement of polyhedra). *Let* P *be a polyhedron, and let* $\widehat{P}$ *be a refinement of* P*. Let* w_i *and* $\widehat{w}_i$ *be the weights in step 3 of Algorithm 4.2 for* P *and* $\widehat{P}$.

Then $\sum_{i\in V(P)} w_i = \sum_{i\in V(\widehat{P})} \widehat{w}_i$.

The proofs for Lemma 2.7 and Theorem 2.8 carry over to the 3D case as well, and we arrive at

4.5 Theorem. *3D mean value coordinates are well-defined in* $\mathbb{R}^3$.

Note that this theorem holds also for non-convex polyhedra with multiple components (if these are alternatingly oriented, compare [4]). Also, we do not require the faces to be simply connected.

5 Conclusion

We showed that the spherical mean value coordinates and the 3D mean value coordinates that were very recently introduced in [8] are well-defined. As far as we know, these are the only known coordinates for polyhedra that are defined in the whole space $\mathbb{R}^3$. Extensions to polytopes in $\mathbb{S}^{n-1}$ and $\mathbb{R}^n$ are straightforward. We hope that this will foster the usage of these coordinates in the future.

Bibliography

[1] M. S. Floater. Mean value coordinates. *Computer Aided Geometric Design*, 20(1):19–27, 2003.

[2] M. S. Floater, K. Hormann, and G. Kós. A general construction of barycentric coordinates over convex polygons. *Advances in Computational Mathematics*, 24(1–4):311–331, Jan. 2006.

[3] M. S. Floater, G. Kós, and M. Reimers. Mean value coordinates in 3D. *Comp. Aided Geom. Design*, 22:623–631, 2005.

[4] K. Hormann and M. S. Floater. Mean value coordinates for arbitrary planar polygons. *ACM Transactions on Graphics*, Mar. 2006. Accepted.

[5] J. Huang, X. Shi, X. Liu, K. Zhou, L.-Y. Wei, S.-H. Teng, H. Bao, B. Guo, and H.-Y. Shum. Subspace gradient domain mesh deformation. *ACM Trans. Graph.*, 25(3):1126–1134, 2006.

[6] T. Ju, S. Schaefer, and J. Warren. Mean value coordinates for closed triangular meshes. *ACM Trans. Graph.*, 24(3):561–566, 2005.

[7] T. Ju and J. Warren. General constructions of barycentric coordinates in a convex triangular polyhedron. Technical report, Washington University in St. Louis, Nov. 2005.

[8] T. Langer, A. Belyaev, and H.-P. Seidel. Spherical barycentric coordinates. In A. Sheffer and K. Polthier, editors, *Fourth Eurographics Symposium on Geometry Processing*, pages 81–88. Eurographics Association, June 2006.

[9] O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rössl, and H.-P. Seidel. Laplacian surface editing. In R. Scopigno and D. Zorin, editors, *SGP 2004—Symposium on Geometry Processing*, pages 175–184. Eurographics Association, July 2004.

[10] E. L. Wachspress. *A Rational Finite Element Basis*, volume 114 of *Mathematics in Science and Engineering*. Academic Press, New York, 1975.

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