On Positive Influence and Negative Dependence

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MPI–I–98–1–018   August 1998

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Acknowledgements

This work was done partly when the authors were visiting the Max–Planck–Institut für Informatik, Saarbrücken, Germany, and Basic Research in Computer Science(BRICS), Århus, Denmark and was partially supported by the ESPRIT Basic Research Actions Program of the EC under contract No. 7141 (project ALCOM II).

Abstract

We study two notions of negative influence namely negative regression and negative association. We show that if a set of symmetric binary random variables are negatively regressed then they are necessarily negatively associated. The proof uses a lemma that is of independent interest and shows that every binary symmetric distribution has a variable of "positive influence". We also show that in general the notion of negative regression is different from that of negative association.

Keywords

Probabilistic Algorithms and Analyses; Negative Dependence; Negative Regression; Negative Dependence; Positive Influence; Symmetric Distributions.

1 Introduction

We discuss and compare two strong notions of negative dependence amongst random variables. Intuitively, a set of random variables is negatively dependent if they have the following property: if one subset of variables is "high" then other disjoint subsets are "low". The notion of negative dependence amongst random variables is important and useful in probabilistic analysis for a variety of reasons. For example, for purposes of stochastic bounds on the sum of variables, one can treat variables that are strongly negatively dependent as if they were independent; so, one can use tools like the Chernoff-Hoeffding bounds which are normally available only for independent variables. This allows simpler analysis for many probabilistic processes and algorithms. For some applications and examples, see [5].

Let $X_1, \ldots, X_n$ be a set of random variables and let I and J be disjoint subsets of $[n]$. One can formalize the intuitive notion of negative dependence,

$$X_i, i \in I \quad \text{are high} \quad \leftrightarrow \quad X_j, j \in J \quad \text{are low}, \tag{1.1}$$

in many ways. One of the simplest notions of negative dependence is negative correlation: $E[\prod_i X_i] \leq \prod_i E[X_i]$. Unfortunately, this notion is too weak for many applications. We next provide the definitions for two strong natural notions of negative dependence: *negative regression* and *negative association.* [1]

Definition 1 (Negative Regression and Association) *Random variables $X_1, \ldots, X_n$ satisfy:*

($-R$) *the* **negative regression** *condition if*

$$E[f(X_i, i \in I) \mid X_j = t_j, j \in J],$$

is non–increasing in each $t_j, j \in J$ for any disjoint $I, J \subseteq [n]$ and any non–decreasing function f.

($-A$) *the* **negative association** *condition if for every two disjoint index sets, $I, J \subseteq [n]$,*

$$E[f(X_i, i \in I)g(X_j, j \in J)] \leq E[f(X_i, i \in I)]E[g(X_j, j \in J)]$$

for all functions $f : \mathsf{R}^{|I|} \to \mathsf{R}$ and $g : \mathsf{R}^{|J|} \to \mathsf{R}$ that are both non–decreasing or both non–increasing [7].

Some other notions of negative dependence and some previous work on negative dependence can be found in [1, 8]. There are corresponding notions of *positive* regression and association. It is easy to show that positive regression implies positive association [6, Proposition 5.18]. However, the proof cannot be modified in a straight–forward way to show that negative regression implies negative association. In fact, the relationship between various notions of negative dependence is not entirely clear.

We present results pertaining to the relationship between negative regression and negative association. In § 2, we prove that for symmetric binary random

[1] We deal exclusively with finite-valued, discrete random variables. We shall assume that whenever we have conditional expectations or probabilities, the conditioning event is non–null.

variables, negative regression implies negative association thus establishing that the notion of negative regression is at least as strong as the notion of negative association for an important and frequently occuring class of distributions. In § 3, we give a counter–example to show that the two notions are not the same, in particular, that negative association does not imply negative regression in general. The same example shows also that negative regression, while seemingly a more attractive condition for negative dependence, does not share some nice robustness properties of negative association. In § 4, we state some open problems with some conjectures.

2 Binary Symmetric Variables

A special family of distributions are the *symmetric* or *exchangeable* distributions [9, Ch. 15].

Definition 2 (Symmetric Random Variables) *Random variables $X_1, \ldots, X_n$ are called* **symmetric** *or* **exchangeable** *if for every permutation $\sigma : [n] \to [n]$, and for all $a_1, \ldots, a_n \in \mathsf{R}$,*

$$\Pr[X_1 = a_1, \ldots, X_n = a_n] = \Pr[X_1 = a_{\sigma(1)}, \ldots, X_n = a_{\sigma(n)}].$$

Example 3 (Fermi–Dirac Statistics) Fermi–Dirac statistics are used in quantum statistical physics to describe the behaviour of an *ensemble* of particles distributed in a state space obeying the Pauli–exclusion principle. One can model this as an experiment where m balls are thrown into n $(m \leq n)$ bins with each bin containing at most one ball, and each distribution of balls among the bins is equally likely to occur. The *occupancy numbers*, $B_i, i \in [n]$ are 0/1 variables indicating if the corresponding bins are occupied. For $m_i \in \{0, 1\}$, $i \in [n]$, with $\sum_i m_i = m$,

$$\Pr(B_1 = m_1, \ldots, B_n = m_n) = \binom{n}{m}^{-1} .$$

Then $B_i's$ are symmetric random variables.

2.1 Positive Influence

We shall establish that for Binary Symmetric Random Variables, negative regression implies negative association. The proof is provided in the next subsection and, somewhat surprisingly, makes critical use of a lemma about "positive influence" that is of independent interest. The lemma shows that for every binary symmeric distribution and for every non-decreasing real-valued function there is a variable that has a positive influence on the function. Intuitively, a variable has a positive influence on a function if one expects the value of the function to increase if the value of the variable is increased, assuming that the other variables are chosen at random. To be more precise, we define positive influence as follows:

Definition 4 (Positive Influence) *Let $X_1 \ldots X_n$ be real-valued random variables and ψ be any probability distribution on $X_1 \ldots X_n$. Let f be any real-valued function of $X_1 \ldots X_n$. We say that X_j is a* **positive influence variable** *for $\langle f, \psi \rangle$ if $E_\psi[f(X_1, \ldots X_n) | X_j = t]$ is a non-decreasing function of t.*

Notions of influence of variables on functions have been defined and used before. In particular, it is instructive to compare our definition of positive influence with that of influence in Bourgain et al [2].

For simplification of notation, when it is clear what distribution we are talking about or we are talking about an arbitrary fixed distribution, we will often omit ψ from the subscript in the expectation. Also, note that for binary random variables, X_j is a variable of positive influence for $\langle f, \psi \rangle$ iff

$$E_\psi[f(X_1, \ldots X_n) \mid X_j = 0] \leq E_\psi[f(X_1, \ldots X_n) \mid X_j = 1].$$

A very natural question is: for which function-distribution pairs (f, ψ) do variables of positive influence exist? Below, we show that for every binary symmetric distribution ψ and for every non-decreasing real-valued function f there exists a variable with positive influence. This is sufficient for proving our main theorem. For further results on positive influence see [3] where amongst other results it is proved that for every symmetric non-decreasing function and every binary distribution there is a variable of positive influence.

Lemma 5 (Positive Influence Lemma: Symmetric Distributions) *Let $X_1, \ldots, X_n$ be symmetric binary random variables. Let f be any non–decreasing function of $X_1, \ldots, X_n$. Then there is a positive influence variable for f, that is there exists an $i \in [n]$ such that*

$$F_i(0) := E[f(X_1, \ldots, X_n) \mid X_i = 0] \leq E[f(X_1, \ldots, X_n) \mid X_i = 1] =: F_i(1).$$

Proof. Set $F(t) := \sum_i F_i(t)$; then it suffices to prove that $F(0) \leq F(1)$, since $F(1) - F(0) = \sum_i (F_i(1) - F_i(0))$.

We shall use some notation to make the proof more compact. For $\hat{a} = (a_1, \ldots a_n) \in \{0,1\}^n$, let $\mu(\hat{X} = \hat{a})$ denote $\Pr(X_1 = a_1, \ldots, X_n = a_n)$. Also, we shall identify a set with its characteristic vector *i.e.* we shall use subsets S of $[n]$ interchangably with the binary vector $(\chi_S(1), \ldots \chi_S(n))$. Hence, for example, for any subset S of $[n]$, $f(S)$ stands for $f(\chi_S(1), \ldots \chi_S(n))$. Moreover $\mu(S)$ will stand for $\mu(\hat{X} = S)$.

Let $p_0 := \Pr[X_i = 0]$. Note that $\Pr[X_i = 0]$ is same for each X_i as the variables are symmetric.

Now,

$$\begin{aligned}
F(0) &= \sum_i F_i(0) \\
&= \sum_i E[f(X_1, \ldots, X_n) \mid X_i = 0] \\
&= \sum_i \sum_{\hat{a} \in \{0,1\}^n} f(\hat{a}) \mu(\hat{X} = \hat{a} \mid X_i = 0) \\
&= \sum_i \sum_{\substack{\hat{a} \\ a_i = 0}} f(\hat{a}) \mu(\hat{X} = \hat{a}) / p_0 \\
&= \sum_i \sum_{i \notin S} f(S) \mu(S) / p_0
\end{aligned}$$

Noting that $p_0 = \sum_{i \notin S} \mu(S)$ we can continue,

$$
\begin{aligned}
F(0) &= n\frac{\sum_i \sum_{i \notin S} f(S)\mu(S)}{\sum_i \sum_{i \notin S} \mu(S)} \\
&= n\frac{\sum_S \sum_{i \notin S} f(S)\mu(S)}{\sum_S \sum_{i \notin S} \mu(S)} \\
&= n\frac{\sum_S f(S)\mu(S) \sum_{i \notin S} 1}{\sum_S \mu(S) \sum_{i \notin S} 1} \\
&= n\frac{\sum_S f(S)\mu(S)(n-|S|)}{\sum_S \mu(S)(n-|S|)}
\end{aligned}
$$

Similarly,

$$F(1) = n\frac{\sum_S f(S)\mu(S)|S|}{\sum_S \mu(S)|S|}.$$

Thus we need to show that

$$\frac{\sum_S f(S)\mu(S)(n-|S|)}{\sum_S \mu(S)(n-|S|)} \leq \frac{\sum_S f(S)\mu(S)|S|}{\sum_S \mu(S)|S|}. \tag{2.2}$$

We now use the fact that the variables are symmetric. So $\mu(S)$ dependes only on $|S|$ and we may denote it by $\mu_{|S|}$. Also, let $f_t := \sum_{|S|=t} f(S)$. Then,

$$
\begin{aligned}
\sum_S f(S)\mu(S)(n-|S|) &= \sum_{t=0}^{n} \sum_{|S|=t} f(S)\mu(S)(n-|S|) \\
&= \sum_t \sum_{|S|=t} f(S)(n-t)\mu_t \\
&= \sum_t (n-t)\mu_t \sum_{|S|=t} f(S) \\
&= \sum_t f_t(n-t)\mu_t \qquad (2.3)
\end{aligned}
$$

Similarly, we compute:

$$\sum_S \mu(S)(n-|S|) = \sum_t \binom{n}{t}(n-t)\mu_t. \tag{2.4}$$

$$\sum_S f(S)\mu(S)|S| = \sum_t f_t t\mu_t \tag{2.5}$$

$$\sum_S \mu(S)|S| = \sum_t \binom{n}{t} t\mu_t. \tag{2.6}$$

Plugging (2.3) through (2.6) into (2.2), we need to prove:

$$\frac{\sum_t f_t(n-t)\mu_t}{\sum_t \binom{n}{t}(n-t)\mu_t} \leq \frac{\sum_t f_t t\mu_t}{\sum_t \binom{n}{t} t\mu_t}.$$

Let $\overline{f}(t) := f_t/\binom{n}{t}$. $\overline{f}(t)$ is easily seen to be non–decreasing by an averaging argument. Furthermore, let $\mu_1(t) := \binom{n}{t}(n-t)\mu_t$ and $\mu_2(t) := \binom{n}{t}t\mu_t$.

It is easy to confirm that if $a \leq b$ then $\frac{\mu_1(b)}{\mu_1(a)} \leq \frac{\mu_2(b)}{\mu_2(a)}$. The desired result follows from an application of the one–dimensional version of Holley's generalisation of the FKG inequality (Theorem 6) to the function $\overline{f}$ and the measures μ_1, μ_2. □

Theorem 6 (One Dimensional Holley Inequality) *Let μ_1 and μ_2 be two measures on the real line such that $\frac{\mu_1(b)}{\mu_1(a)} \leq \frac{\mu_2(b)}{\mu_2(a)}$ if $a \leq b$. Then for any non–decreasing f,*

$$\frac{\sum_t f(t)\mu_1(t)}{\sum_t \mu_1(t)} \leq \frac{\sum_t f(t)\mu_2(t)}{\sum_t \mu_2(t)}.$$

Interestingly enough, it is not clear if the symmetry of the distribution is really necessary for a positive influence variable to exist.

2.2 The Main Theorem

Now, we are ready to prove our main theorem.

Theorem 7 *For symmetric binary random variables, the regression condition $(-R)$ implies the association condition $(-A)$.*

Proof. We shall prove the result by induction on the number of variables, n. The result is trivially true for $n = 1$. Now let $1 \leq k < n$ and let $f(X_1, \ldots X_k), g(X_{k+1} \ldots X_n)$ be any two non-decreasing functions. We shall show that, $E[fg] \leq E[f]E[g]$.

By the Positive Influence Lemma 5, it follows that there exists an $i \leq k$ such that $E[f(X_1, \ldots X_k)|X_i = t]$ is non-decreasing in t. Without loss of generality assume that $i = 1$. Then,

$$E[f(X_1 \ldots X_k)g(X_{k+1} \ldots X_n)] = \sum_{a=0,1} E[f(X_1 \ldots X_k)g(X_{k+1} \ldots X_n)|X_1 = a]\Pr[X_1 = a]. \tag{2.7}$$

Since $X_1 \ldots X_n$ are symmetric random variables, fixing $X_1 = 0$ or $X_1 = 1$ gives rise also to symmetric distributions (possibly different in the two cases) on $X_2, \ldots, X_n$. Moreover, by the definition of $(-R)$, each of these individually satisfies $(-R)$. Hence, by induction, we can continue from (2.7),

$$\begin{aligned} E[f(X_1 \ldots X_k)g(X_{k+1} \ldots X_n)] &\leq \\ &\sum_{a=0,1} E[f(X_1 \ldots X_k)|X_1 = a]E[g(X_{k+1} \ldots X_n)|X_1 = a]\Pr[X_1 = a] \\ &= \sum_{a=0,1} f_1(a)g_1(a)\mu(a), \end{aligned} \tag{2.8}$$

where we put:

$$f_1(a) = E[f(X_1 \ldots X_k)|X_1 = a], \quad g_1(a) = E[g(X_{k+1} \ldots X_n)|X_1 = a],$$

and

$$\mu(a) = \Pr[X_1 = a].$$

Then, f_1 is monotone non-decreasing by choice from the Positive Influence Lemma, g_1 is monotone non-increasing by the definition of $(-R)$. Applying the (one–dimensional) Chebyshev–FKG–Harris inequality for the second inequality below, we get, continuing from (2.8),

$$\begin{aligned} E[f(X_1 \dots X_k)g(X_{k+1} \dots X_n)] &\leq \sum_{a=0,1} f_1(a)g_1(a)\mu(a) \\ &\leq \sum_{a=0,1} f_1(a)\mu(a). \sum_{a=0,1} g_1(a)\mu(a) \\ &= E[f(X_1 \dots X_k)]E[g(X_{k+1} \dots X_n)]. \end{aligned}$$

□

It is worth noting that the inductive proof would essentially go through for any class of distributions for which one could establish a positive influence lemma for non-decreasing functions. One would like to conjecture that this is true for all distributions but it is shown to be false in [3].

Application 8 It is easy to show that the variables in Fermi-Dirac distribution are negatively regressed. From the theorem it follows that they are negatively associated. This is shown via another elegant method in [4].

3 $(-A)$ is different from $(-R)$

In this section, we construct a distribution on four random variables that will show that $(-A)$ is different from $(-R)$. Moreover, it shows that $(-R)$ does not possess many nice properties that $(-A)$ has.

Consider any distribution on four discrete finite-valued random variables X_1, X_2, Y_1, Y_2 that satisfies the following conditions:

- X_1, X_2 are independent of Y_1, Y_2; X_1 is independent of X_2.
- $X_1, X_2 \in \{0, 1, 2\}$.
- $Y_1, Y_2 \in \{0, 1\}$ and $Y_1 + Y_2 = 1$.
- All marginal probabilities are non–zero.
- The marginal distribution of X_2 satisfies the following strict *log–convexity* condition:
$$\Pr[X_2 = 1]^2 < \Pr[X_2 = 0]\Pr[X_2 = 2]. \tag{3.9}$$

Clearly, it is possible to construct such distributions on X_1, X_2, Y_1, Y_2. Moreover, it is easy to see that for any such distribution X_1, X_2, Y_1, Y_2 are negatively associated.

Now, let $Z_1 = X_1 + Y_1$ and $Z_2 = X_2 + Y_2$. Then Z_1, Z_2 are negatively associated. Below we show that Z_1, Z_2 are not negatively regressed.

Proposition 9 *Z_1, Z_2 do not satisfy $(-R)$.*

Proof. Consider the non–decreasing function $f(Z_1) = [Z_1 \geq 3]$. We compute,

$$\begin{aligned} E[f(Z_1)|Z_2 = 1] &= \Pr[Z_1 \geq 3|Z_2 = 1] \\ &= \Pr[Z_1 = 3|Z_2 = 1], \quad \text{since } Z_1 = X_1 + Y_1 \leq 3 \\ &= \Pr[Z_1 = 3, Z_2 = 1]/\Pr[Z_2 = 1] \\ &= \Pr[X_1 + Y_1 = 3, X_2 + Y_2 = 1]/\Pr[Z_2 = 1] \\ &= \Pr[X_1 = 2, Y_1 = 1, X_2 + Y_2 = 1]/\Pr[Z_2 = 1] \\ &\quad \text{since } X_1 + Y_1 = 3 \text{ iff } X_1 = 2, Y_1 = 1 \\ &= \Pr[X_1 = 2, Y_1 = 1, X_2 = 1]/\Pr[Z_2 = 1] \\ &\quad \text{since } Y_1 = 1 \leftrightarrow Y_2 = 0 \end{aligned}$$

Since X_1, X_2 and Y_1 are independent we get,

$$E[f(Z_1)|Z_2 = 1] = \Pr[X_1 = 2]\Pr[Y_1 = 1]\Pr[X_2 = 1]/\Pr[Z_2 = 1]. \tag{3.10}$$

Similarly, we have

$$E[f(Z_1)|Z_2 = 2] = \Pr[X_1 = 2]\Pr[Y_1 = 1]\Pr[X_2 = 2]/\Pr[Z_2 = 2]. \tag{3.11}$$

Thus, comparing (3.10) and (3.11), the negative regression condition fails if all marginal probabilities are non–zero and,

$$\frac{\Pr[Z_2 = 1]}{\Pr[X_2 = 1]} > \frac{\Pr[Z_2 = 2]}{\Pr[X_2 = 2]}. \tag{3.12}$$

Now, we have,

$$\begin{aligned} \frac{\Pr[Z_2 = 1]}{\Pr[X_2 = 1]} &= \Pr[Z_2 = 1 \mid X_2 = 1] + \Pr[Z_2 = 1 \mid X_2 = 0]\frac{\Pr[X_2 = 0]}{\Pr[X_2 = 1]} \\ &= \Pr[Y_2 = 0 \mid X_2 = 1] + \Pr[Y_2 = 1 \mid X_2 = 0]\frac{\Pr[X_2 = 0]}{\Pr[X_2 = 1]} \\ &= \Pr[Y_2 = 0] + \Pr[Y_2 = 1]\frac{\Pr[X_2 = 0]}{\Pr[X_2 = 1]}. \end{aligned} \tag{3.13}$$

Similarly,

$$\frac{\Pr[Z_2 = 2]}{\Pr[X_2 = 2]} = \Pr[Y_2 = 0] + \Pr[Y_2 = 1]\frac{\Pr[X_2 = 1]}{\Pr[X_2 = 2]} \tag{3.14}$$

From (3.12) through (3.14), the negative regression condition fails if (3.9) holds. □

A number of consequences follow:

Corollary 10 *The negative association condition* $(-A)$ *does not imply the negative regression condition* $(-R)$ *(even for two 4-valued variables).*

Corollary 11 *Sums of disjoint sets of variables that satisfy* $(-R)$ *do not necessarily satisfy* $(-R)$.

Remark 12 In contrast, the negative association property is preserved if we take *arbitrary* non–decreasing functions of disjoint sets of variables. This is the main reason that makes negative association much easier to work with in applications such as the one in the next remark.

Application 13 (Balls and Bins) Consider the classical balls and bins experimant: m balls are thrown independently (but not necessarily uniformly) into n bins, and we focus on the *occupancy numbers* $B_1, \ldots, B_n$ of the number of balls in each bin. It is intuitively obvious that these variables are negatively dependent. To make this precise, one can ask: which of the notions of negative dependence listed above obtain in this example?

The property of $(-A)$ mentioned above can be exploited to yield a short proof that the variables are negatively associated. For this, it is expedient to introduce the indicator variables $B_{i,k}$ for $i \in [n], k \in [m]$:

$$B_{i,k} := \begin{cases} 1, & \text{if ball } k \text{ goes into bin } i; \\ 0, & \text{otherwise.} \end{cases}$$

It is quite easy to show that the variables $(B_{i,k}, i \in [n], k \in [m]$ satisfy $(-A)$ as well as $(-R)$. Now, since $B_i = \sum_k B_{i,k}, i \in [n]$ are non–decreasing functions of disjoint variables, we conclude immediately that $B_1, \ldots, B_n$ satisfy $(-A)$.

Unfortunately, one cannot do the same for negative regression, since Proposition 9 shows that even sums do not preserve the $(-R)$ property.

Another possible approach that suggests itself is to proceed by induction on the set of balls. Let $B_i^K, i \in [n]$ denote the occupancy numbers corresponding to the restricted experiment with the balls labelled by the index set $K \subseteq [m]$. Note that for $k \notin K$, and $i \in [n]$,

$$B_i^{K \cup \{k\}} = B_i^K + B_{i,k}.$$

Suppose by induction that $B_i^K, i \in [n]$ satisfy $(-R)$. Then so would $B_i^{K \cup \{k\}}, i \in [n]$ if the following plausible–sounding assertion was true: Let $X_1, \ldots, X_n$ satisfy $(-R)$ and let $Y_1, \ldots, Y_n$ be 0/1 variables with $\sum_i Y_i = 1$. Then $Z_1 := X_1 + Y_1, \ldots, Z_n := X_n + Y_n$ also satisfy $(-R)$. Unfortunately, this assertion is also false in general as shown in Proposition 9.

Remark 14 It is shown in [5] that the B_i's are indeed negatively regressed but the proof is quite involved.

4 Conclusion, Open Problems and Conjectures

The goal of this research is to understand the relationship between $(-A)$ and $(-R)$, two strong notions of negative dependence. We have taken a step forward by showing that in the special, useful case of symmetric binary distributions, $(-R)$ implies $(-A)$ and that in general the two notions are different. Many open problems remain.

- The most interesting open problem is: does $(-R)$ imply $(-A)$ in general?
- An interesting sub-problem is: does $(-R)$ imply $(-A)$ for binary (not necessarily symmetric) random variables?

We conjecture that the answer to both these problems is yes. For the latter case, we even conjecture that the Positive Influence Lemma is true. Unfortunately however, one can construct an example to show that the averaging argument we gave above is not going to prove it.

- Another, interesting open problem is the reverse direction: does $(-A)$ imply $(-R)$ for binary random variables (with and/or without the assumption of symmetry)?

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