

Appendix

a. Solar radiation

The incoming solar radiation at the top of the atmosphere, F_{sun} , is partially reflected by clouds:

$$F_{rad}(W) = [1 - \alpha_W(W)] F_{sun} , \quad (1)$$

where $\alpha_W(W)$ depends on the amount of liquid water in the atmosphere,

$$\alpha_W(W) = \alpha_{cl} \frac{W}{W_0}$$

and α_{cl} is the cloud albedo coefficient.

b. Sensible heat flux

The sensible heat flux is given by the aerodynamic bulk formula,

$$Q_s = \rho_L c_p C_D |\bar{u}_s| (T_T - \theta_L) \quad (2)$$

where $|\bar{u}_s|$ is the mean wind velocity and C_D is the bulk aerodynamic drag coefficient. The flux is by convention positive when heating the atmosphere and cooling the soil.

c. Evaporation

Evaporation uplifts water from the soil surface to the PBL. The parameterization we choose is the following:

$$E(s_T, q_L) = E_0(s_T) \frac{q_{sat}(T_L) - q_L}{q_{sat}(T_L)} , \quad (3)$$

where

$$E_0(s_T) = \begin{cases} 0 & \text{if } s_T < s_w \\ E_{max} \frac{s_T - s_w}{1 - s_w} & \text{if } s_w \leq s_T \leq 1 \end{cases}$$

Here, E_{max} is the optimal evaporation rate, s_w is the wilting point below which no evaporation occurs (Baudena and Provenzale, 2008), and $q_{sat}(T_L)$ is the specific humidity of saturation, calculated at the surface pressure. Evaporation is maximum when the PBL is completely dry and zero when the PBL is saturated (D'Andrea et al. 2006).

d. Transpiration

Transpiration uplifts water from the deep soil reached by plant roots to the PBL. Analogously to evaporation, it is reduced by PBL humidity. We define transpiration as:

$$R(s_D, q_L) = R_0(s_D) \frac{q_{sat}(T_L) - q_L}{q_{sat}(T_L)} \quad (4)$$

where

$$R_0(s_D) = \begin{cases} 0 & \text{if } s_D < s_w \\ R_{max} \frac{s_D - s_w}{s^* - s_w} & \text{if } s_w \leq s_D < s^* \\ R_{max} & \text{if } s_D \geq s^* \end{cases}$$

Here s^* is the value of soil moisture below which plants close their stomata and start reducing transpiration, and R_{max} is the optimal transpiration for $s_D \geq s^*$.

e. Leakage and infiltration

Water can be lost from a soil layer because of two physical processes: leakage and infiltration. Leakage represents water that percolates at the bottom of each layer. Following Laio et al. (2001), we assume the leakage to be zero when soil moisture is below a certain threshold, i.e. soil field capacity s_{fc} , while when soil moisture is above s_{fc} we parameterize it as

$$L(s) = K_s \frac{e^{\beta_W(s-s_{fc})} - 1}{e^{\beta_W(1-s_{fc})} - 1}. \quad (5)$$

Instantaneous infiltration occurs when the amount of water available exceeds saturation, and it is immediately transferred to deeper soil layers. Following Baudena and Provenzale (2008), we define infiltration at time t as

$$\Delta I(s(t)) = \begin{cases} 0 & \text{if } s(t - \delta t) + \delta s < 1 \\ s(t - \delta t) + \delta s - 1 & \text{if } s(t - \delta t) + \delta s > 1 \end{cases} \quad (6)$$

where δt is the time step, and δs is the instantaneous increase/decrease of soil water due to all the other terms in the soil moisture equations besides the infiltration term itself.

f. Vegetation cover: colonization and mortality

Following Baudena and Provenzale (2008), we assume that the vegetation colonization ability is the product of two factors, namely the seed production ability of the existing vegetation and the seed germination probability. Thus, the vegetation colonization rate g depends on soil humidity both in the deep soil, as this determines plant conditions and thus seed production, g_p , and the humidity in the surface layer, as this determines the seed germination probability, g_g :

$$g(s_T, s_D) = g_0 g_p(s_D) g_g(s_T) \quad (7)$$

where g_0 is the maximum colonization rate of vegetation.

Seed germination is related to the plant establishment ability, and we assume it depends only on surface humidity, since seeds use only surface soil water. We model this term with a steep hyperbolic tangent centered around s^* ,

$$g_g(s_T) = \left(1 + \tanh \left(\frac{s_T - s^*}{d_s} \right) \right). \quad (8)$$

The production rate g_p is assumed to be close to zero when soil moisture is significantly below the fully-open-stomata threshold s^* ,

$$g_p(s_D) = \begin{cases} 0 & \text{if } s_D < s_w \\ \frac{s_D - s_w}{s^* - s_w} & \text{if } s_w \leq s_D < s^* \\ 1 & \text{if } s_D \geq s^* \end{cases}$$

When $s > s^*$, the colonization rate $g(s_T, s_D)$ approaches a constant value g_0 .

The extinction rate μ is assumed to depend on deep soil moisture, where the plant roots seek for water and nutrients, and it is maximal under wilting conditions. In this model, μ tends to the constant value μ_1 for $s < s_w$ and to the value μ_2 for $s > s_w$, where $\mu_2 < \mu_1$. Again, we assume a hyperbolic tangent shape, this time centered around s_w ,

$$\mu(s_D) = \frac{\mu_2 - \mu_1}{2} \left(1 - \tanh \left(\frac{s_D - s_w}{d_s} \right) \right) + \mu_1. \quad (9)$$

g. Precipitation: convective contribution

In Eq. (15) in the text, f_c represents the convective contribution to precipitation. As in D'Andrea et al. 2006, we represent it as:

$$f_c = f \rho h_L \frac{\widetilde{\Delta q}}{\delta t} \quad (10)$$

where f expresses the precipitation efficiency of convection. If $\rho h_L \widetilde{\Delta q} / \delta t$ is smaller than 1 mm/day, f assumes a minimum value $f = 0.2$. If, instead, the convective updraft is large, $f = 0.9$. Between the two values, f is joined smoothly. If convection is weak, we assume that only a small fraction of water precipitates locally, while if strong convective fluxes take place, much more water precipitates since the collision probability increases.

Table 1: Parameter values introduced in the Supporting information: symbols, names, values, and units

Symbol	Meaning	Value	Units
F_{sun}	Net radiation at the top of the atmosphere	340	W m^{-2}
α_{cl}	Albedo of clouds	0.6	
C_D	Bulk aerodynamic drag coefficient	0.008	
s_w	Soil wilting point	0.18	
s^*	Minimum soil moisture value with maximum plant fitness	0.46	
s_{fc}	Soil field capacity	0.56	
E_{max}	Maximum potential evaporation at s^*	$3.0 \cdot 10^{-4}$	$\text{kg m}^{-2} \text{s}^{-1}$
R_{max}	Maximum potential evaporation at s^*	$2.0 \cdot 10^{-4}$	$\text{kg m}^{-2} \text{s}^{-1}$
K_s	Saturated hydraulic conductivity	0.10	m d^{-1}
β_W	Water retention parameter	12.7	
τ_a	Relaxation time for PBL	3	d
	Season length	100	d
\bar{u}_s	Mean wind	6	m s^{-1}
f	Precipitation efficiency	0.2	
	weak water updraft ($< 1 \text{ mm d}^{-1}$)		
	strong water updraft ($> 3 \text{ mm d}^{-1}$)	0.9	
g_0	Colonization rate	0.8	y^{-1}
μ_2	Extinction rate for $s_D \ll s_w$	0.5	y^{-1}
μ_1	Extinction rate for $s_D \gg s_w$	0.1	y^{-1}
d_s	Hyperbolic tangent width	0.005	